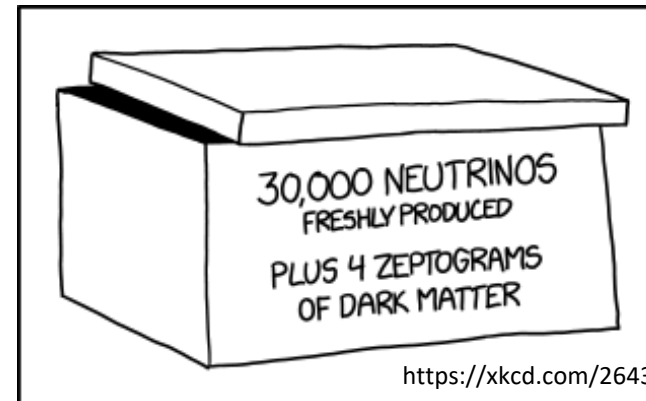


*Towards an Optimal Cosmological Detection  
of Neutrino Mass  
with Joint Analyses  
and Field-Level Inference*

Adrian Bayer

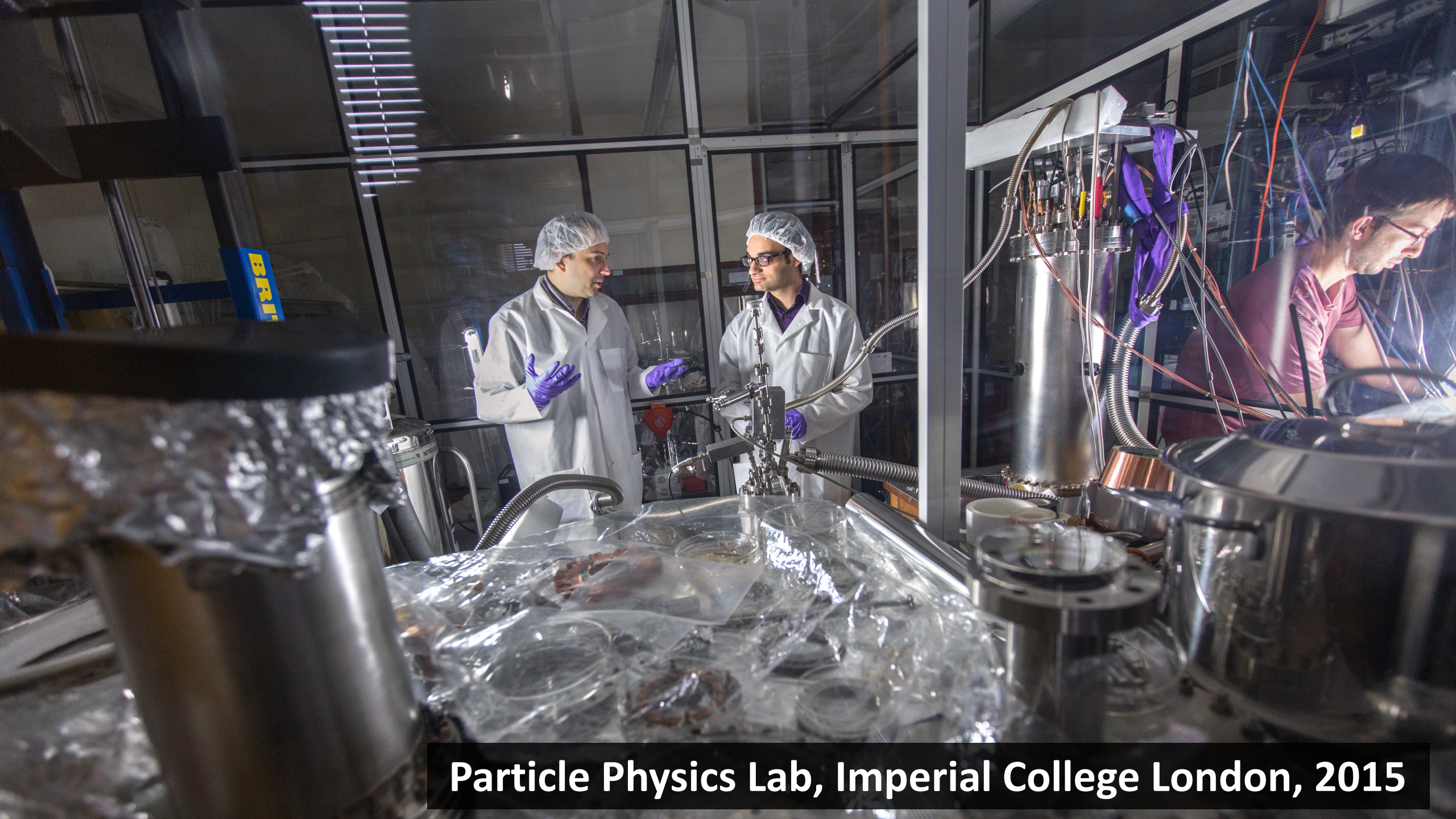
Princeton University / Simons Foundation

COSMO21  
Chania, Greece  
22 May 2024



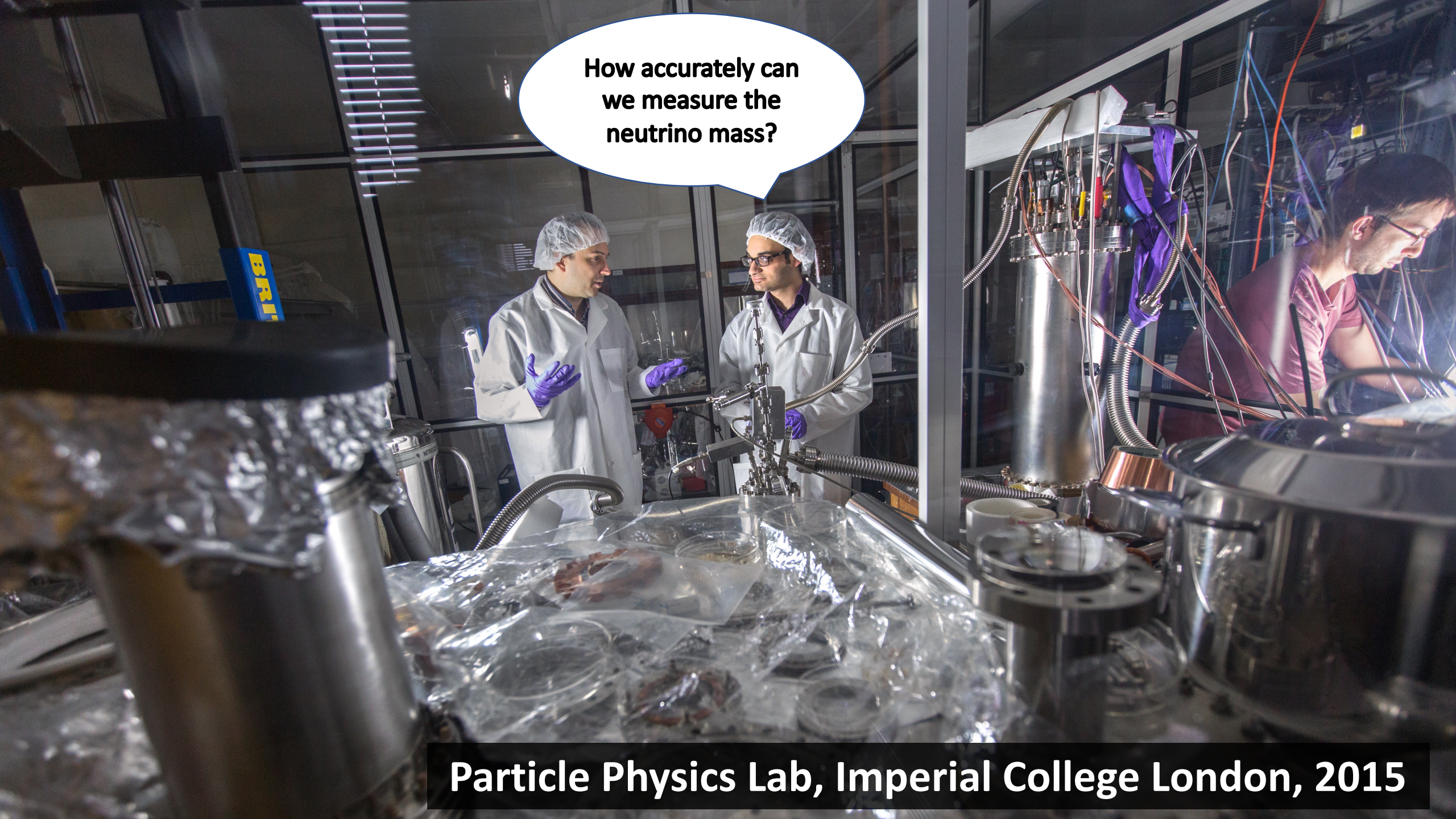
COSMOLOGISTS ARE EASY TO SHOP FOR  
BECAUSE YOU CAN JUST GET THEM A BOX.





**Particle Physics Lab, Imperial College London, 2015**





How accurately can  
we measure the  
neutrino mass?

**Particle Physics Lab, Imperial College London, 2015**



DISCLAIMER: This is not really what my advisor said

Adrian, perhaps  
you should switch  
to cosmology.

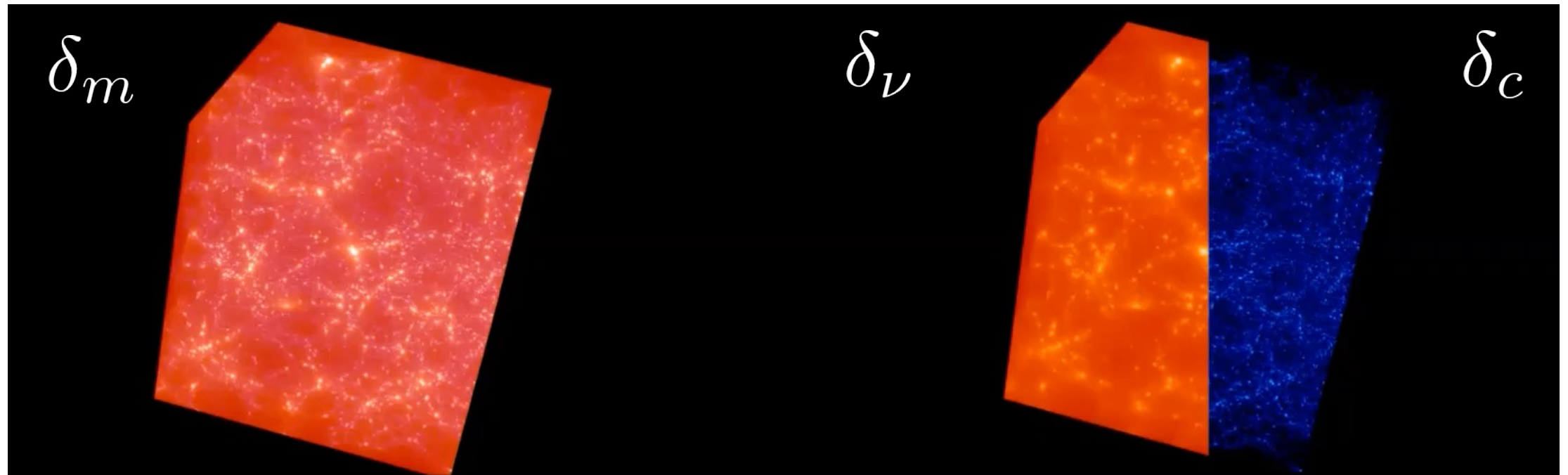
How accurately can  
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Particle Physics Lab, Imperial College London, 2015



# Cosmological simulations of massive neutrinos

$$\delta_m = (1 - f_\nu)\delta_c + f_\nu\delta_\nu$$



Animation Credit: Francisco Villaescusa-Navarro

Many methods to simulate massive neutrinos with different pros/cons:

Linear response approach: Ali-Haïmoud & Bird (2012), Hybrid approach: Bird, et al. (2019), FastPM: Bayer, Banerjee, Feng (2021), ...



$$\delta_m = (1 - f_\nu)\delta_c + f_\nu\delta_\nu$$



# Traditional cosmology uses 2-pt information

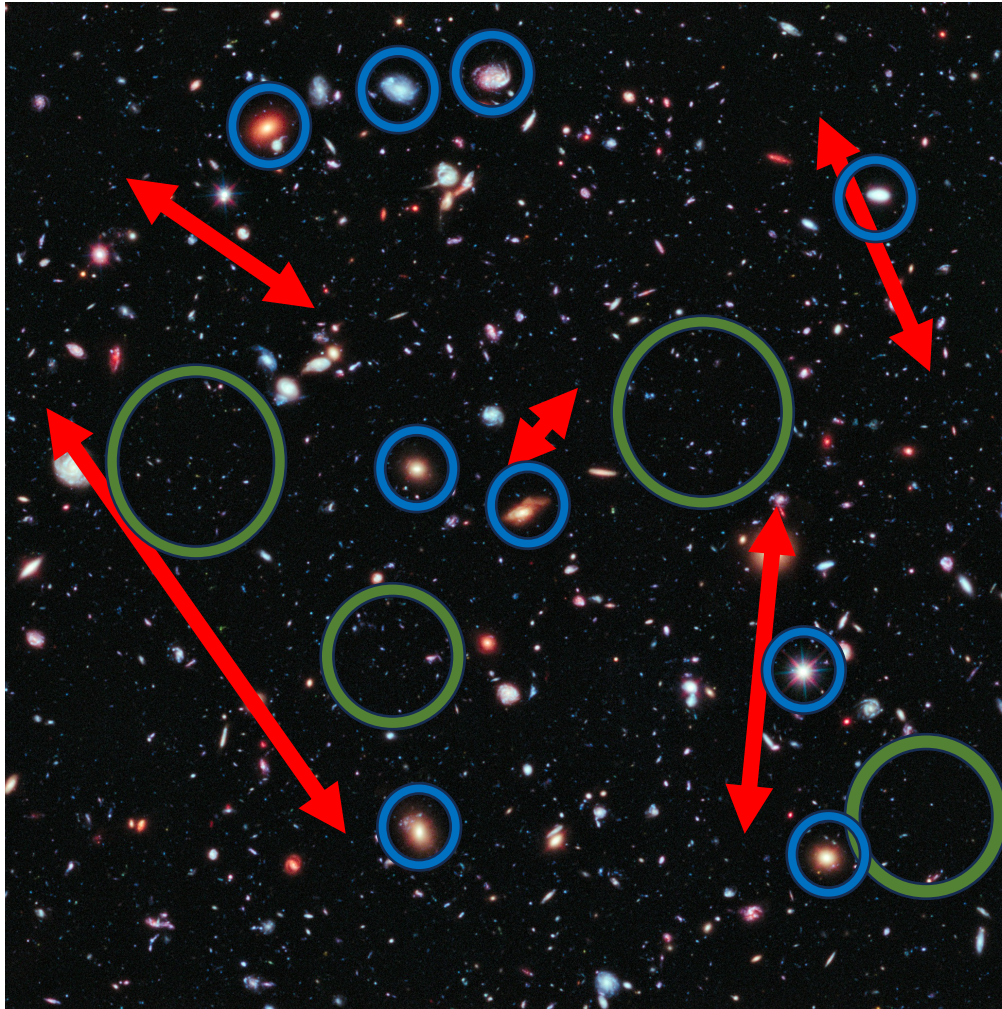
but this is no longer optimal as we probe smaller scales





# Higher-order statistics

can provide information beyond the 2-pt



Halo Mass Function

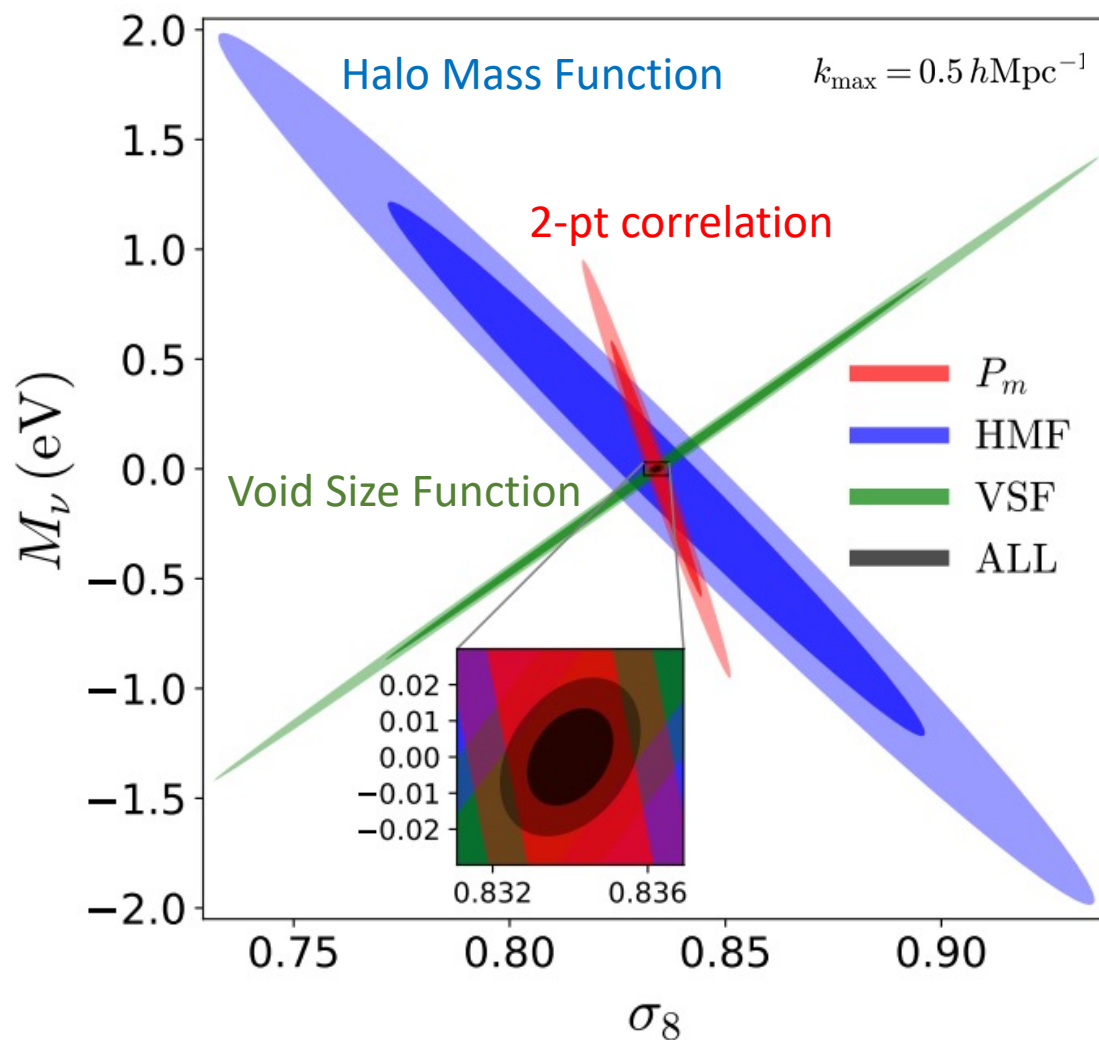
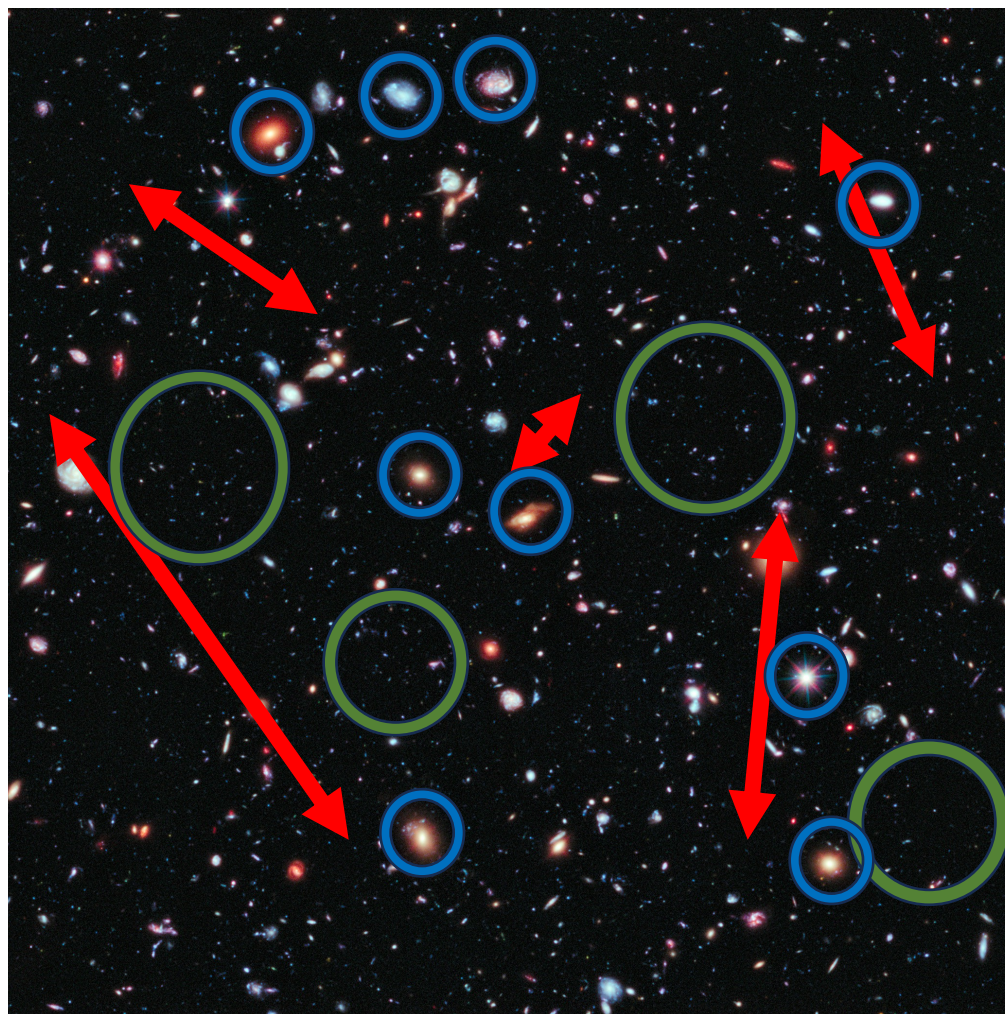
2-pt correlation

Void Size Function



# Higher-order statistics

can provide information beyond the 2-pt





# Many proposals for higher-order statistics

- Lensing bispectrum (Coulton+2018)
- Lensing Minkowski functionals (Marques+2018)
- Lensing peak counts (Li+2018, Ajani+2020)
- Lensing probability density function (Liu+2020)
- Matter probability density function (Uhlemann+2020)
- Redshift-space bispectrum (Hahn+2020)
- Marked power spectrum (Massara+2021)
- Wavelets (Cheng+2021, Valogiannis+2021)
- Voids (Bayer+2021, Kreisch+2021)
- ...

Constraining neutrino mass with tomographic weak lensing one-point probability distribution function and power spectrum  
Jia Liu\* and Mathew S. Madhavacheril

Constraining neutrino mass with weak lensing Minkowski Functionals  
Gabriela A. Marques,<sup>a,1</sup> Jia Liu,<sup>b</sup> José Manuel Zorrilla Matilla,<sup>c</sup> Zoltán Haiman,<sup>c</sup> Armando Bernui<sup>a</sup> and Camila P. Novaes<sup>a</sup>

**Fisher for complements: Extracting cosmology and neutrino mass from the counts-in-cells PDF**

Cora Uhlemann<sup>1,2</sup>, Oliver Friedrich<sup>3,4</sup>, Francisco Villaescusa-Navarro<sup>5,6</sup>, Arka Banerjee<sup>7,8,9</sup>, Sandrine Codis<sup>10</sup>

**Constraining  $M_\nu$  with the Bispectrum I: Breaking Parameter Degeneracies**

ChangHoon Hahn<sup>a,b</sup>, Francisco Villaescusa-Navarro<sup>c,d</sup>, Emanuele Castorina<sup>a,b</sup>, Roman Scoccimarro<sup>c</sup>

DETECTING NEUTRINO MASS BY COMBINING MATTER CLUSTERING, HALOS, AND VOIDS  
ADRIAN E. BAYER<sup>1,2,\*</sup>, FRANCISCO VILLAESCUSA-NAVARRO<sup>3,4,†</sup>, ELENA MASSARA<sup>5,4</sup>, JIA LIU<sup>1,2,6</sup>, DAVID N. SPERGEL<sup>3,4</sup>, LICIA VERDE<sup>7,8</sup>, BENJAMIN D. WANDELT<sup>9,10,4</sup>, MATTEO VIEL<sup>11,12,13,14</sup>, SHIRLEY HO<sup>4,3,15</sup>

**The GIGANTES dataset: precision cosmology from voids in the machine learning era**

CHRISTINA D. KREISCH<sup>1</sup>, ALICE PISANI<sup>1</sup>, FRANCISCO VILLAESCUSA-NAVARRO<sup>1</sup>, DAVID N. SPERGEL<sup>1,2</sup>, BENJAMIN D. WANDELT<sup>2,3,4</sup>, NICO HAMAUS<sup>5</sup> AND ADRIAN E. BAYER<sup>6,7</sup>

Constraining neutrino masses with weak-lensing multiscale peak counts

Virginia Ajani,<sup>1,\*</sup> Austin Peel,<sup>2</sup> Valeria Pettorino,<sup>1</sup> Jean-Luc Starck,<sup>1</sup> Zack Li,<sup>3</sup> and Jia Liu<sup>4,3</sup>

Constraining Neutrino Mass with the Tomographic Weak Lensing Bispectrum  
William R. Coulton

**Using the Marked Power Spectrum to Detect the Signature of Neutrinos in Large-Scale Structure**

Elena Massara,<sup>1,2,\*</sup> Francisco Villaescusa-Navarro,<sup>3,2</sup> Shirley Ho,<sup>2,3,4</sup> Neal Dalal,<sup>5</sup> and David N. Spergel<sup>2,3</sup>

Constraining neutrino mass with tomographic weak lensing peak counts

Zack Li\* and Jia Liu



# Many proposals for higher-order statistics

- Lensing bispectrum (Coulton+2018)
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- Lensing peak counts (Li+2018, Ajani+2021)
- Lensing probability density function
- Matter probability density function
- Redshift-space clustering
- Marked power spectrum (Li+2021)

**Where is the information coming from?  
Can we find it in real data?**

Constraining neutrino mass with tomographic weak lensing bispectrum  
Virginia Ajani,<sup>1,\*</sup> Austin Peel,<sup>2</sup> Valeria Pettorino,<sup>1</sup> Jean-Luc Starck,<sup>1</sup> Zack Li,<sup>3</sup> and Jia Liu<sup>4,3</sup>

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Neutrino mass and neutrino mass  
Francisco Villaescusa-Navarro<sup>5,6</sup>

NEUTRINO MASS BY COMBINING MATTER CLUSTERING, HALOS, AND VOIDS  
FRANCISCO VILLAESCUSA-NAVARRO<sup>3,4,†</sup>, ELENA MASSARA<sup>5,4</sup>, JIA LIU<sup>1,2,6</sup>, DAVID N. SPERGEL<sup>3,4</sup>, LUCIA VERDE<sup>7,8</sup>, BENJAMIN D. WANDELT<sup>9,10,4</sup>, MATTEO VIEL<sup>11,12,13,14</sup>, SHIRLEY HO<sup>4,3,15</sup>

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# What do surveys measure?

$$\delta_m = (1 - f_\nu)\delta_c + f_\nu\delta_\nu$$



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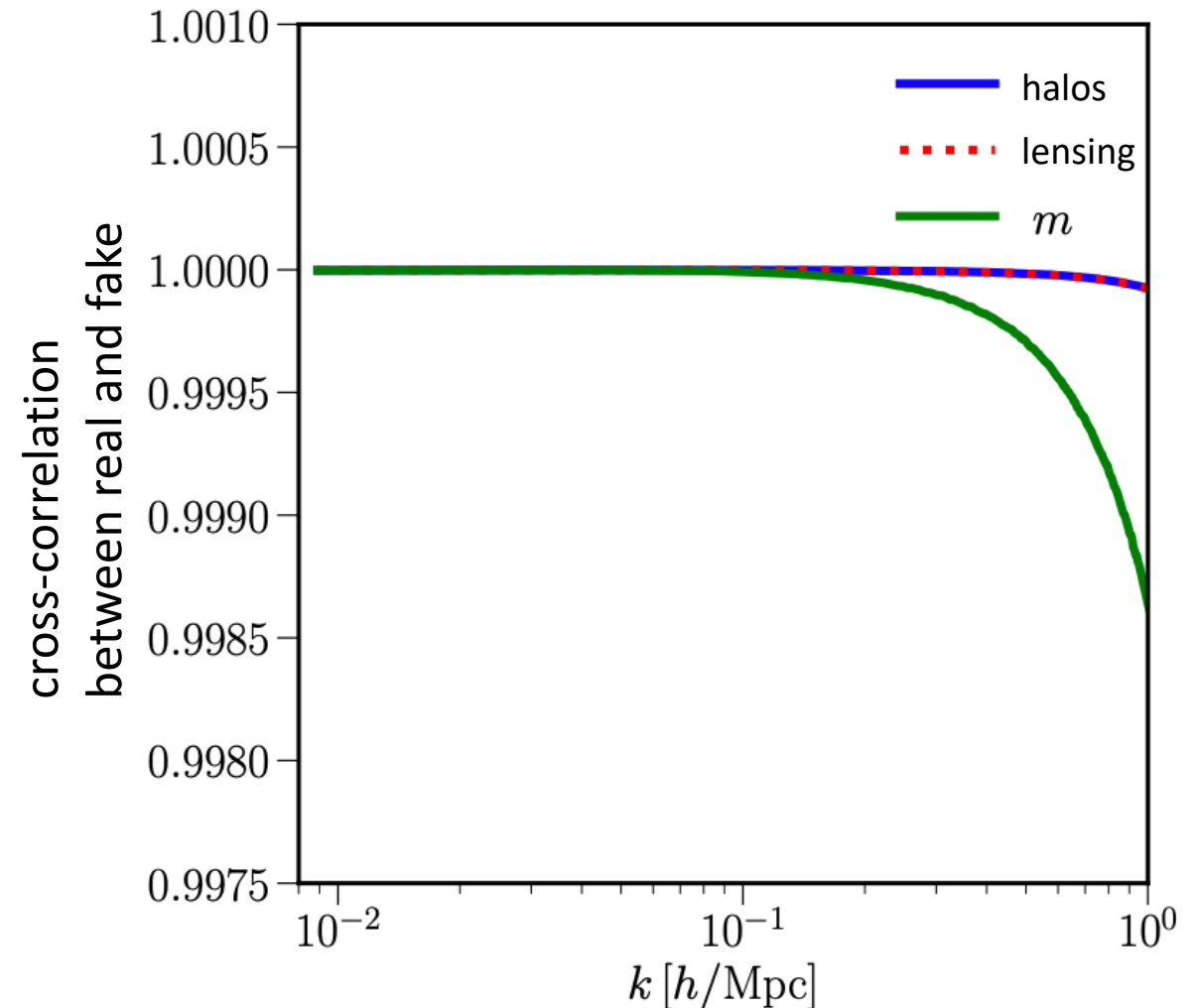
- Lensing:

$$\kappa(\chi_*, \hat{\mathbf{n}}) = \frac{3H_0^2\Omega_m}{2c^2} \int_0^{\chi_*} d\chi \frac{\chi}{a(\chi)} \left(1 - \frac{\chi}{\chi_*}\right) \delta_m(\chi\hat{\mathbf{n}})$$



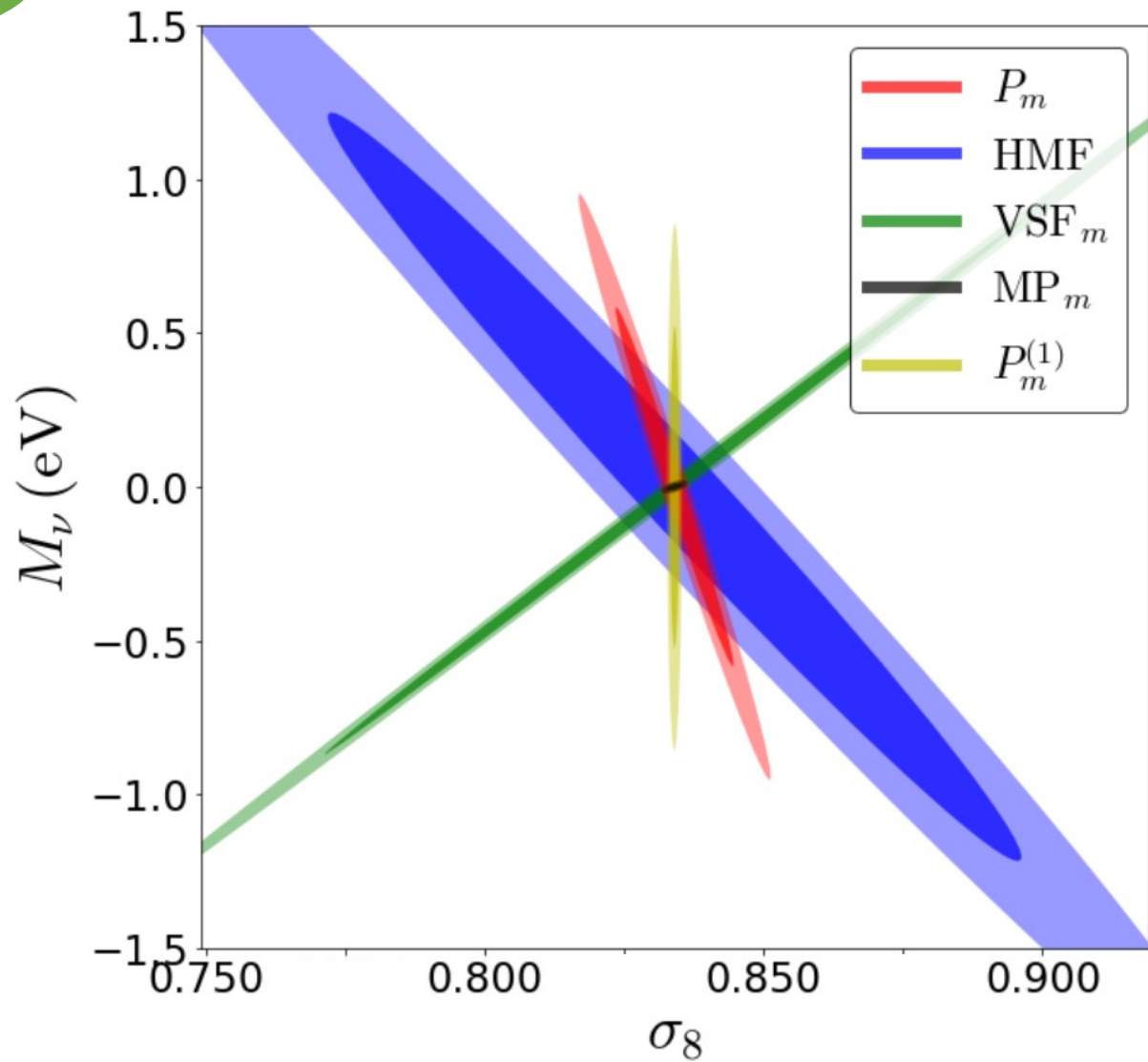
# Fake vs?

- Run two N-body simulations using **matched linear physics**:
  1. with massive neutrinos (**real**)
  2. without massive neutrinos (**fake**)
- Nonlinear effects of massive neutrinos at a **single redshift** can be **faked by CDM** for halos and lensing

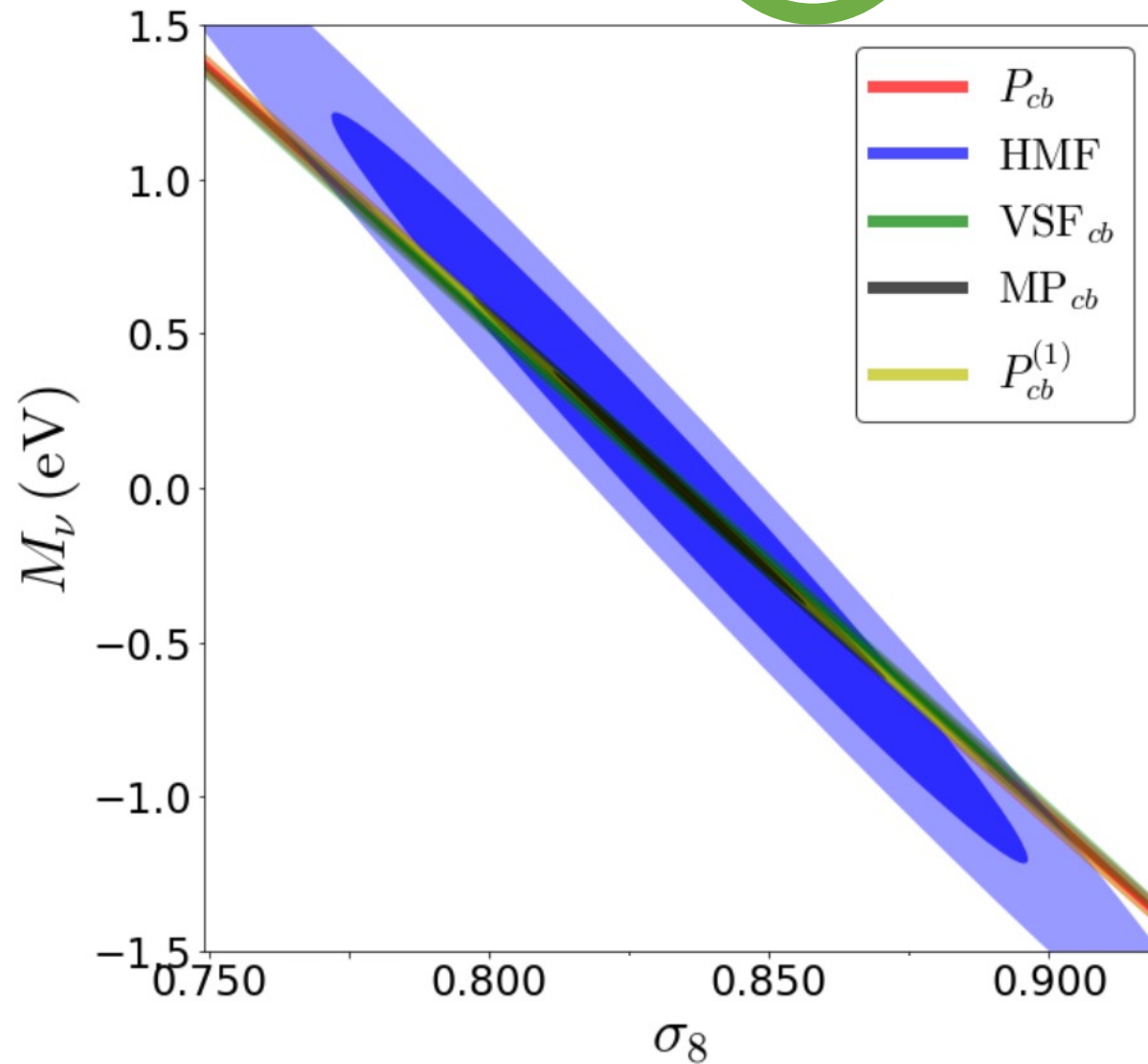




$$\delta_m = (1 - f_\nu)\delta_c + f_\nu\delta_\nu$$



$$\delta_m = (1 - f_\nu) \delta_c + f_\nu \delta_\nu$$

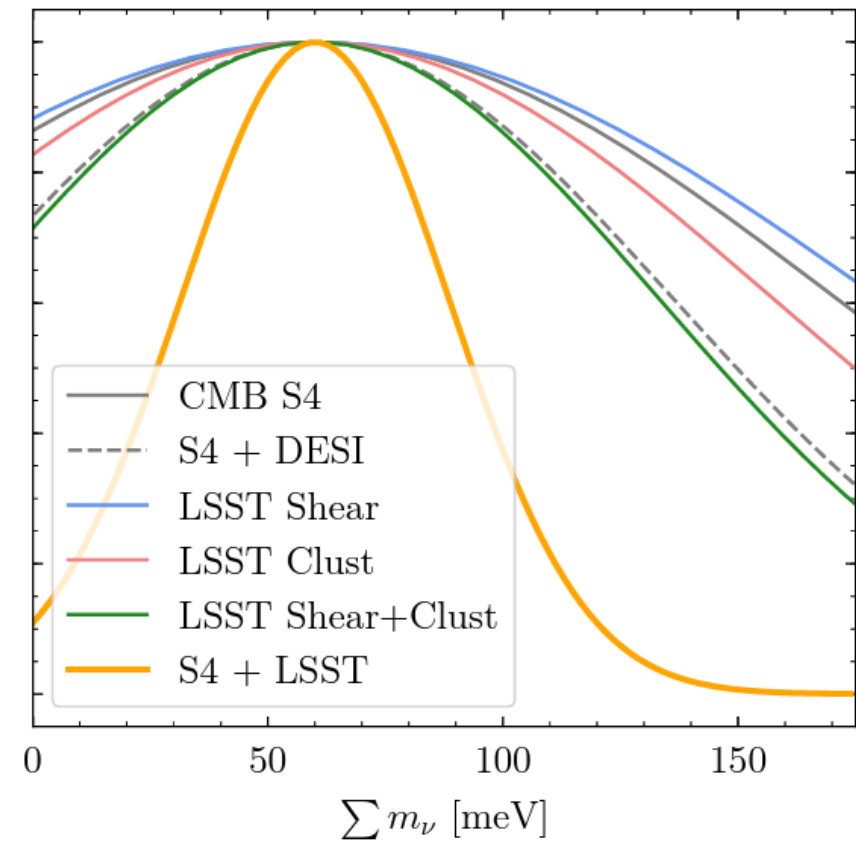




How to maximize the  $v$  information?

# How to maximize the $\nu$ information?

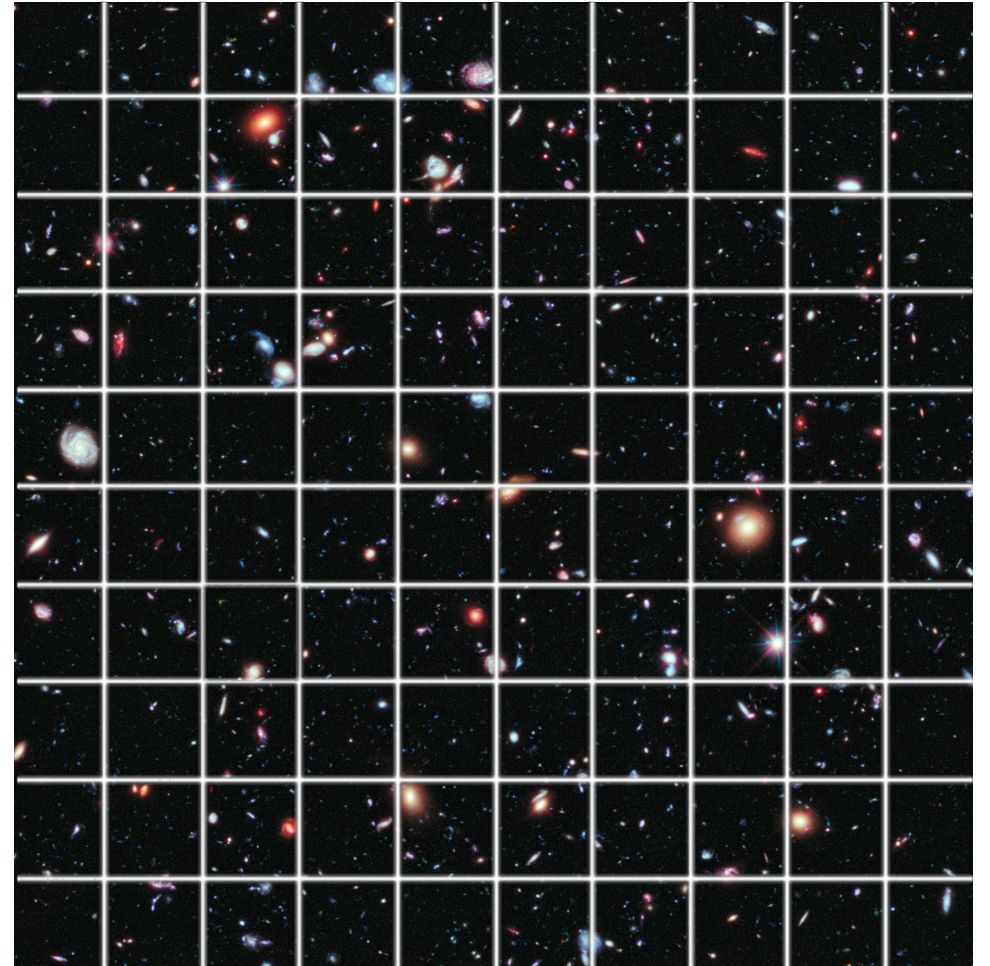
1. Jointly analyze **multiple redshifts** and **multiple tracers** to break degeneracies  
e.g. CMB x LSS





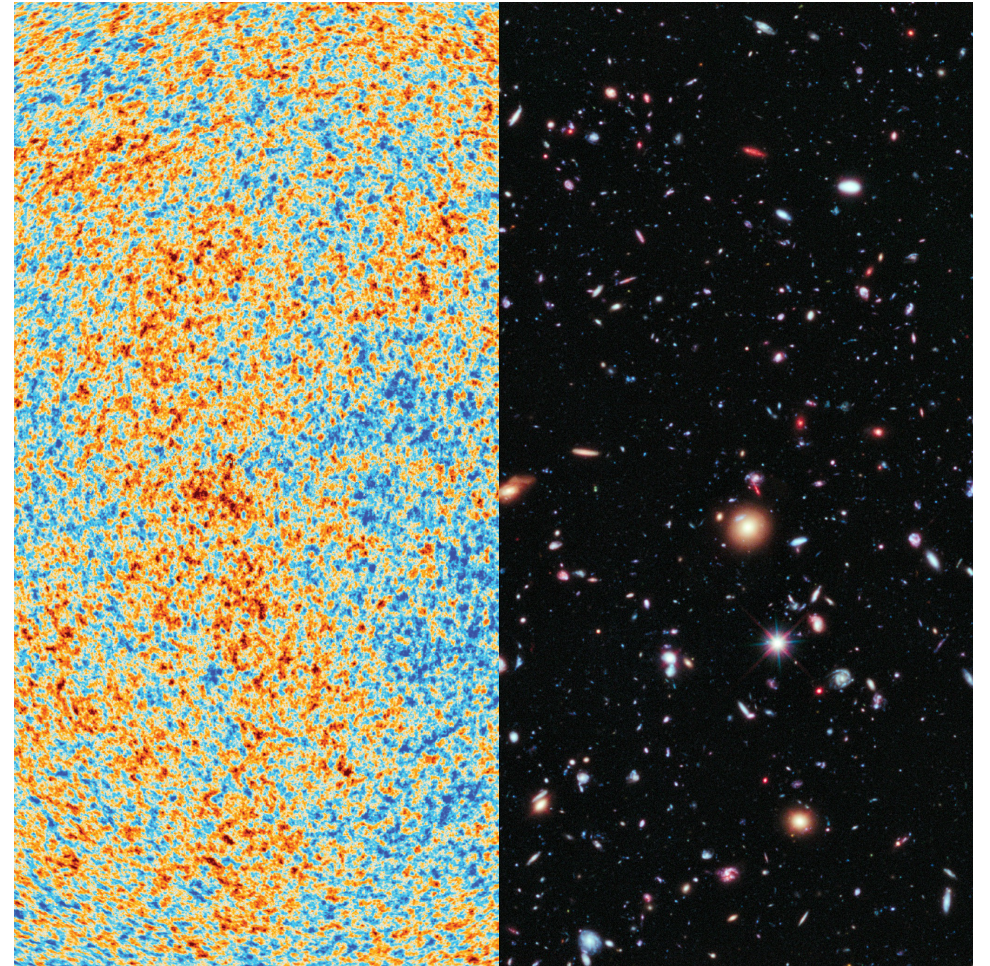
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e.g. field-level inference



# How to maximize the $\nu$ information?

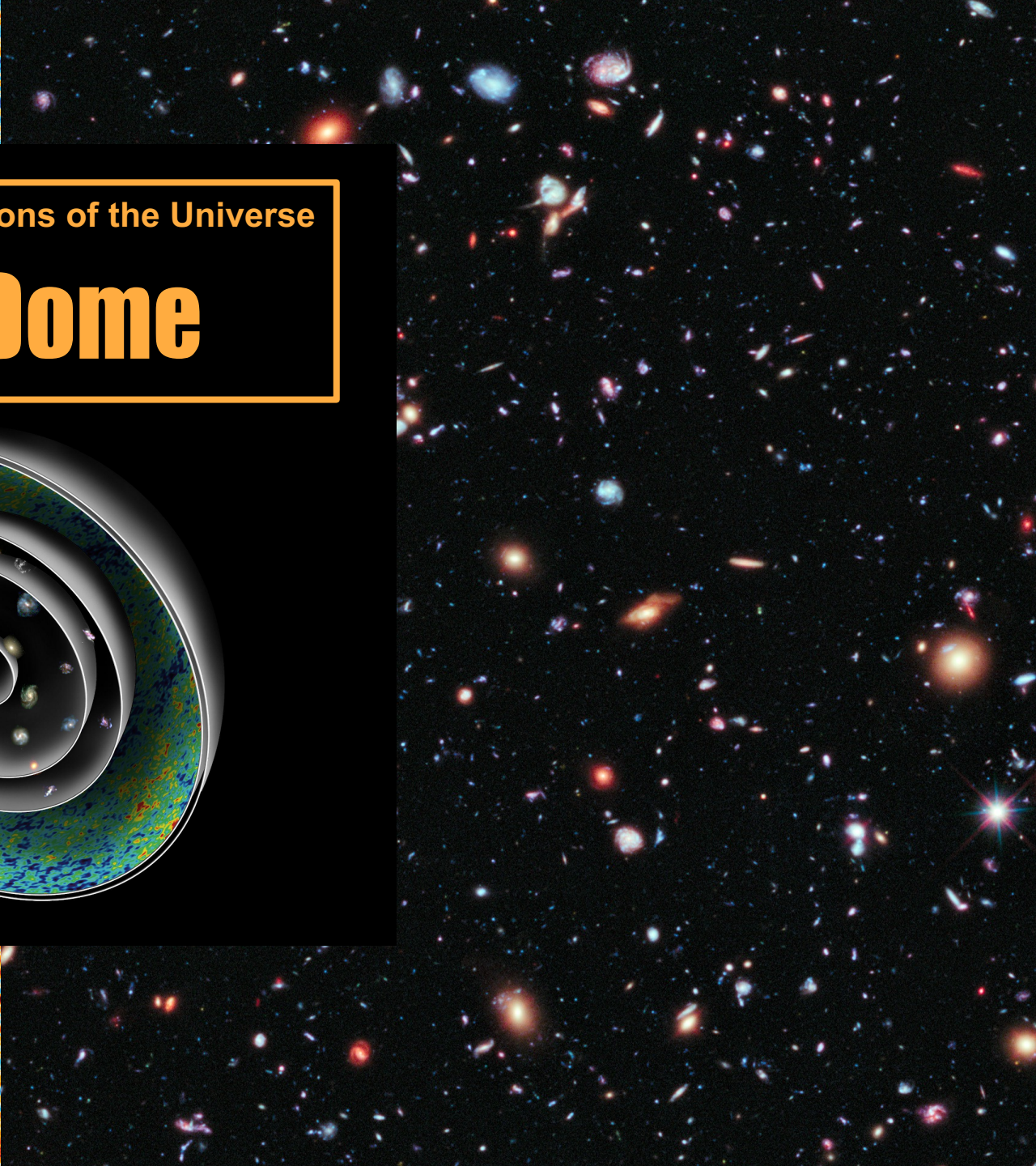
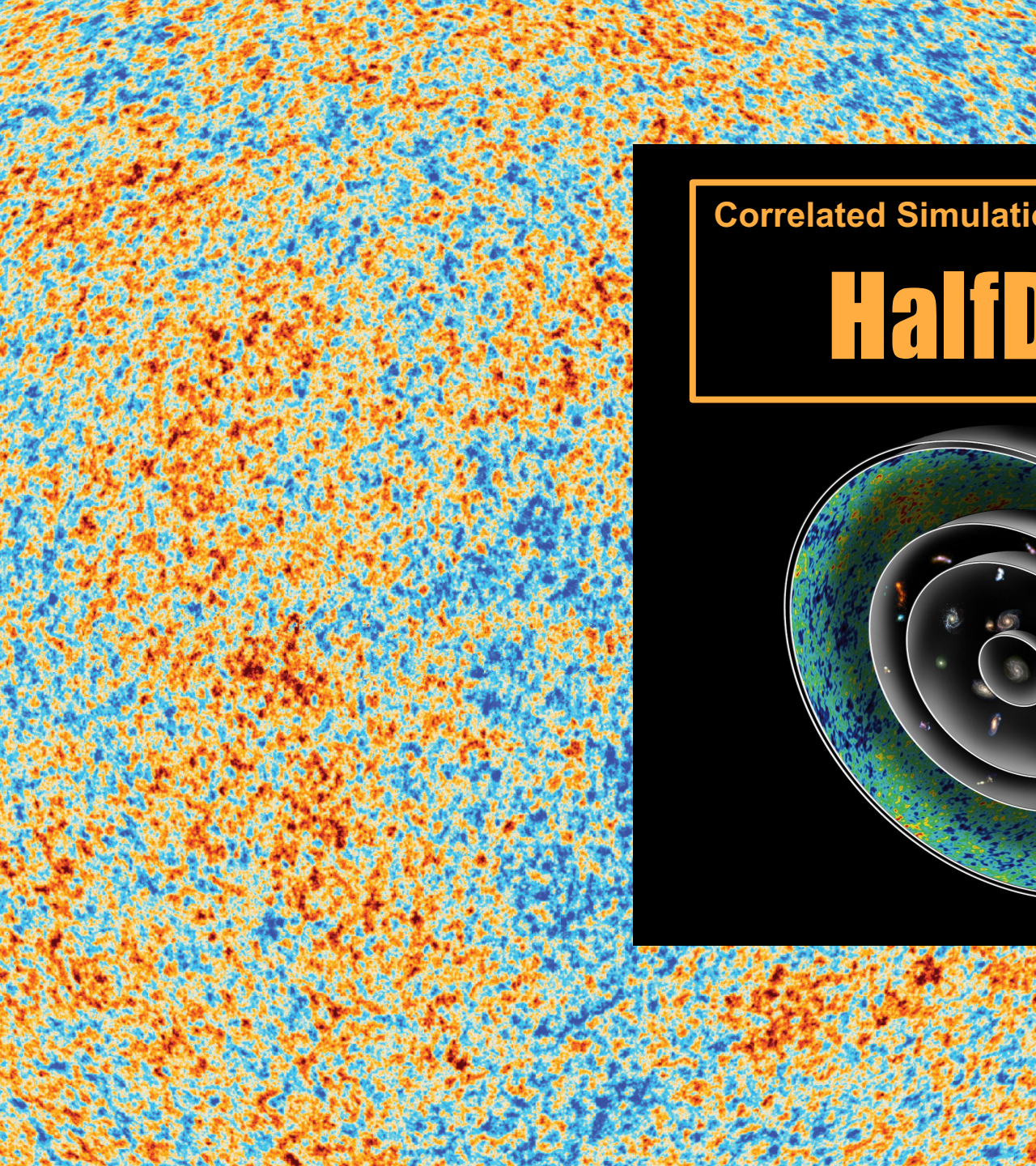
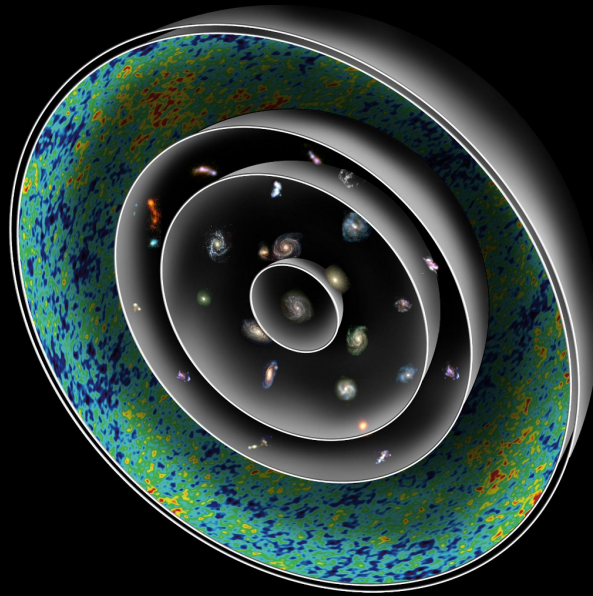
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Correlated Simulations of the Universe

# HalfDome





# Simulation Comparison

|                                    | Sehgal+2010         | <b>Websky</b><br>Stein+2020<br>Li+2022 | <b>Agora</b><br>Omori 2022                 | <b>Stage IV<br/>requirements*</b> |
|------------------------------------|---------------------|--|--|-----------------------------------|
| Box Size<br>$N_{\text{particles}}$ | 1 Gpc/h<br>$1024^3$ | 7.7 Gpc<br>$6144^3$                    | 1 Gpc/h<br>$3840^3$                        | a few Gpc/h                       |
| Min. $M_{\text{halo}}$             | $10^{13} M_{\odot}$ | $1.2 \times 10^{13} M_{\odot}$         | $1.5 \times 10^9 M_{\odot}/h$              | $10^{12} M_{\odot}/h$             |
| LSS<br>observables                 | None                | None                                   | $\kappa$ , clusters, LIM,<br>+more to come | $\kappa$ ,<br>galaxies, clusters  |
| Number of<br>realizations          | 1                   | 1                                      | 1  | 10–100                            |

\* Inputs from SO, CMB-S4, LSST, DESI, PFS, SPHEREx, Roman collaborators



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|------------------------------------|---------------------|--|--|----------------------------------|---|
| Box Size<br>$N_{\text{particles}}$ | 1 Gpc/h<br>$1024^3$ | 7.7 Gpc<br>$6144^3$                    | 1 Gpc/h<br>$3840^3$                        | a few Gpc/h                      | 3.5 Gpc/h,<br>$6144^3$                  |
| Min. $M_{\text{halo}}$             | $10^{13} M_{\odot}$ | $1.2 \times 10^{13} M_{\odot}$         | $1.5 \times 10^9 M_{\odot}/h$              | $10^{12} M_{\odot}/h$            | $10^{12} M_{\odot}/h$                   |
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| Number of realizations             | 1                   | 1                                      | 1  | 10–100                           | 11+ $1f_{\text{NL}}$<br>(more to come)  |

\* Inputs from SO, CMB-S4, LSST, DESI, PFS, SPHEREx, Roman collaborators



# The Team



[Adrian Bayer](#)

Princeton/CCA



[Jia Liu](#)

Kavli IPMU



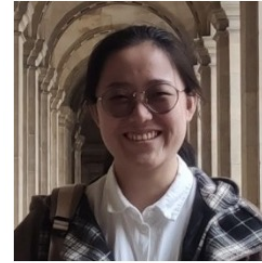
[Zack Li](#)

UC Berkeley/LBL



[Joe DeRose](#)

UC Berkeley/LBL



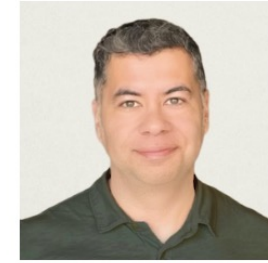
[Yici Zhong](#)

U Tokyo



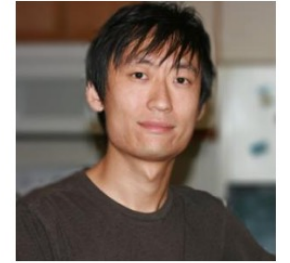
[Linda Blot](#)

Kavli IPMU



[Marcelo Alvarez](#)

LBL



[Yu Feng](#)

Google



[Junjie Xia](#)

Kavli IPMU



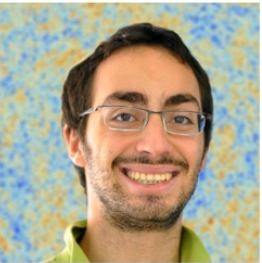
[Alex Laguë](#)

U Penn



[Will Coulton](#)

Cambridge



[Giuseppe Puglisi](#)

U Catania



[Hideki Tanimura](#)

Kavli IPMU



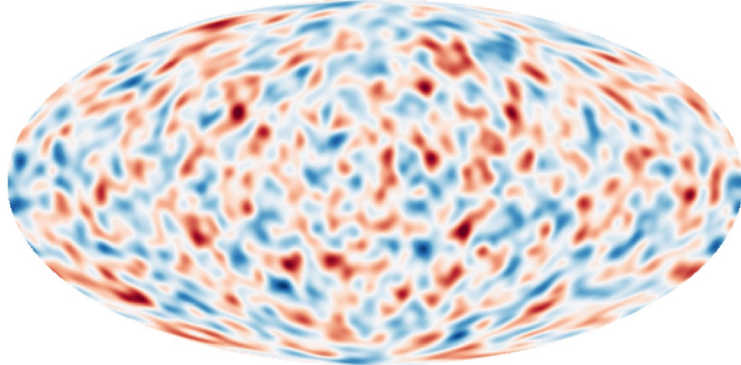
[Mathew Madhavacheril](#)

U Penn

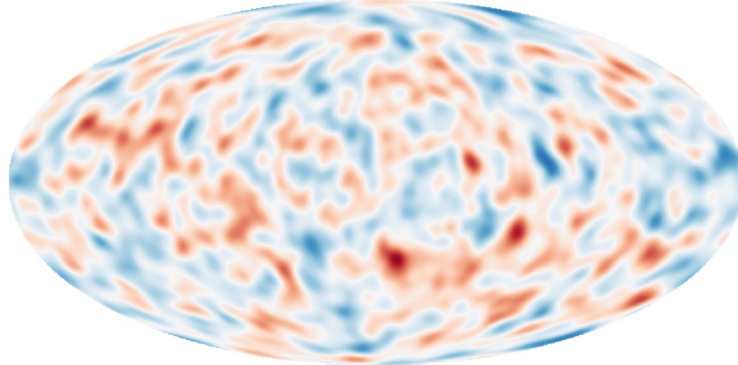
# Preliminary Maps

stay tuned for more!

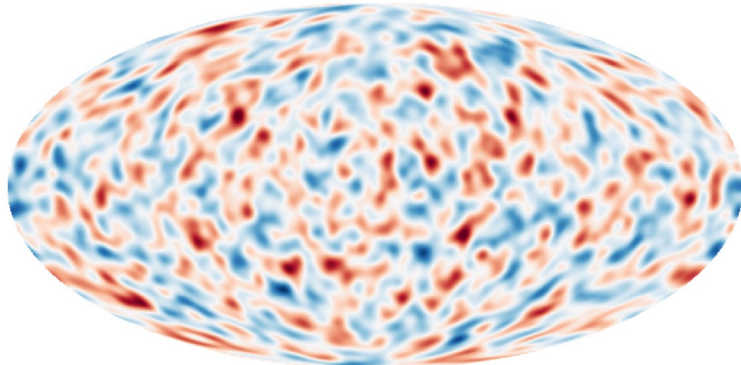
Matter Density



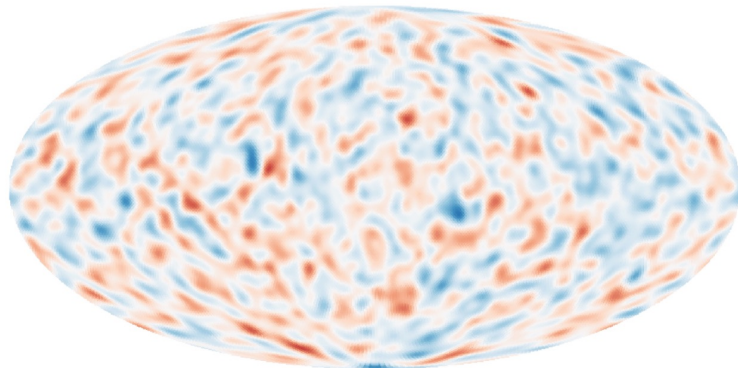
Velocity Sheet



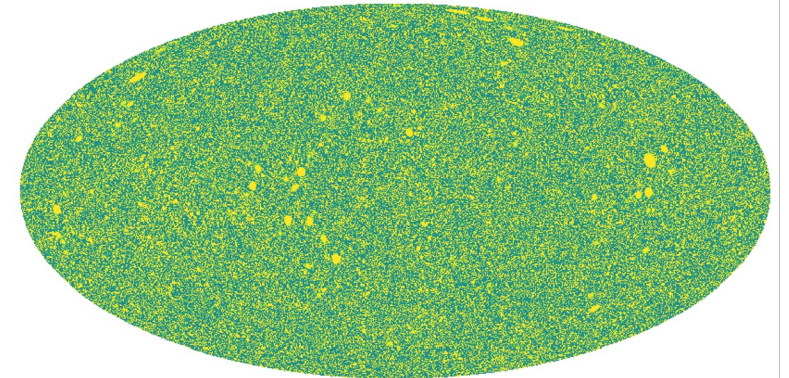
Mass Sheet



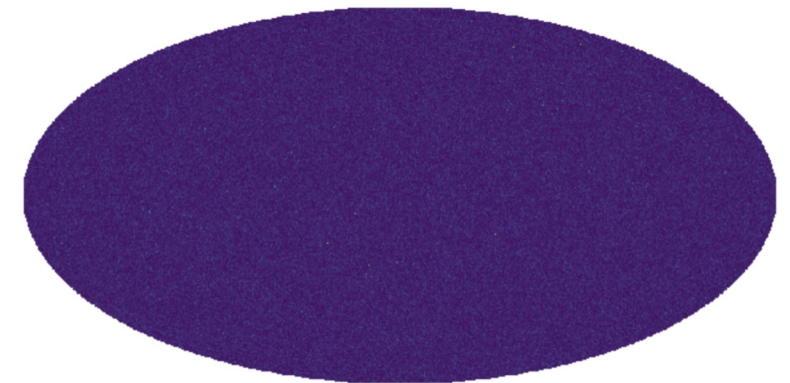
Halo Density



tSZ

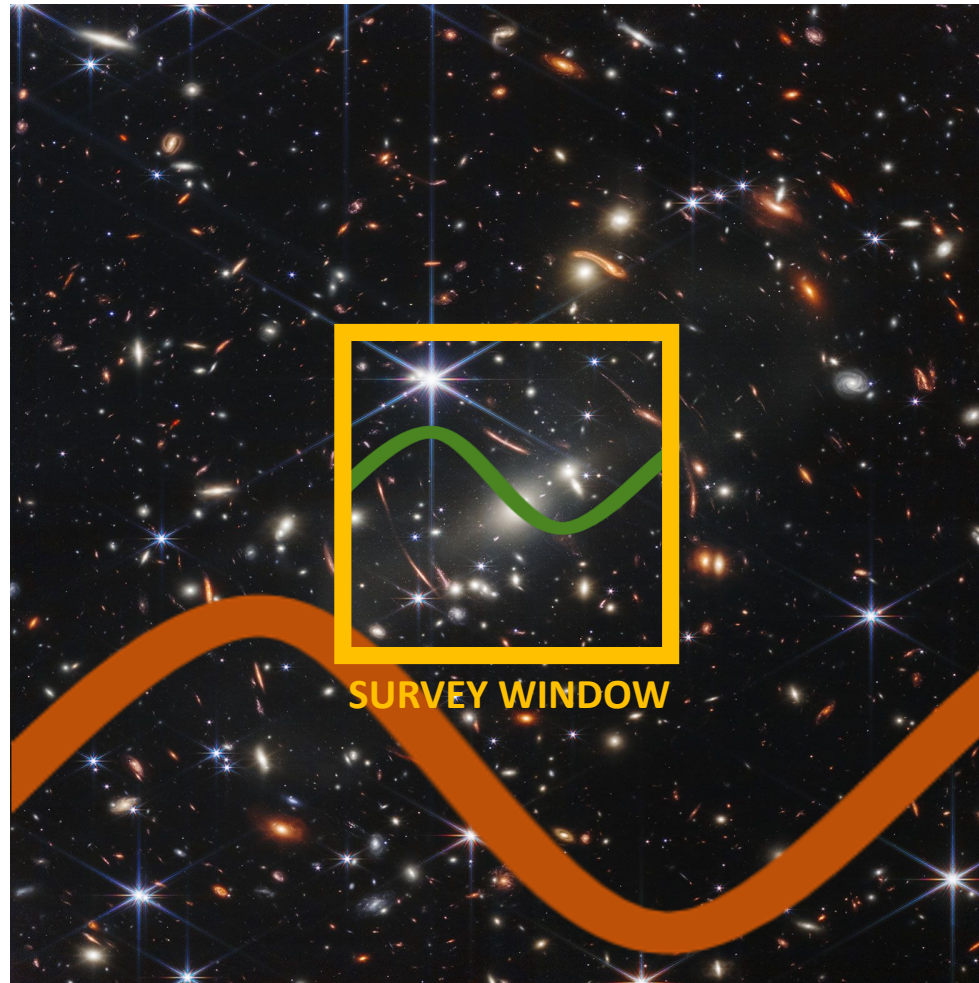


$z_s=3$  lensing map



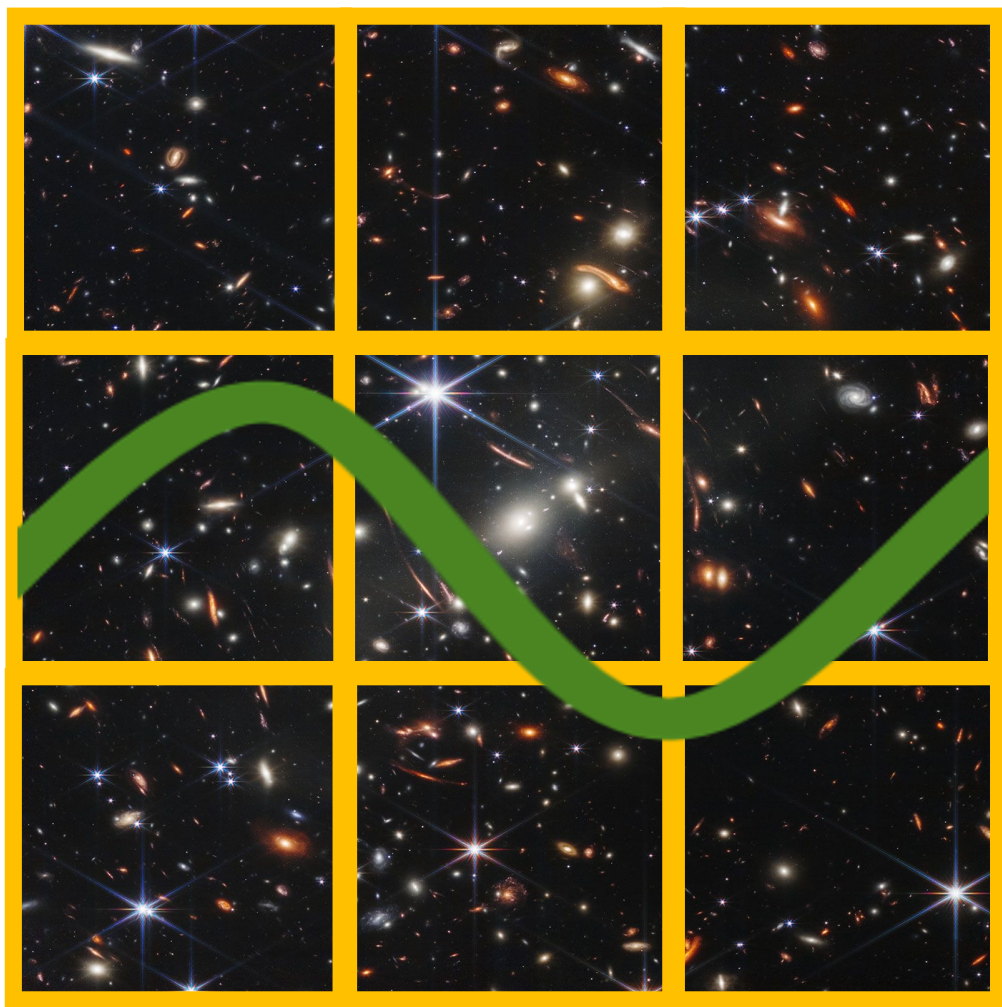


# Super-sample covariance

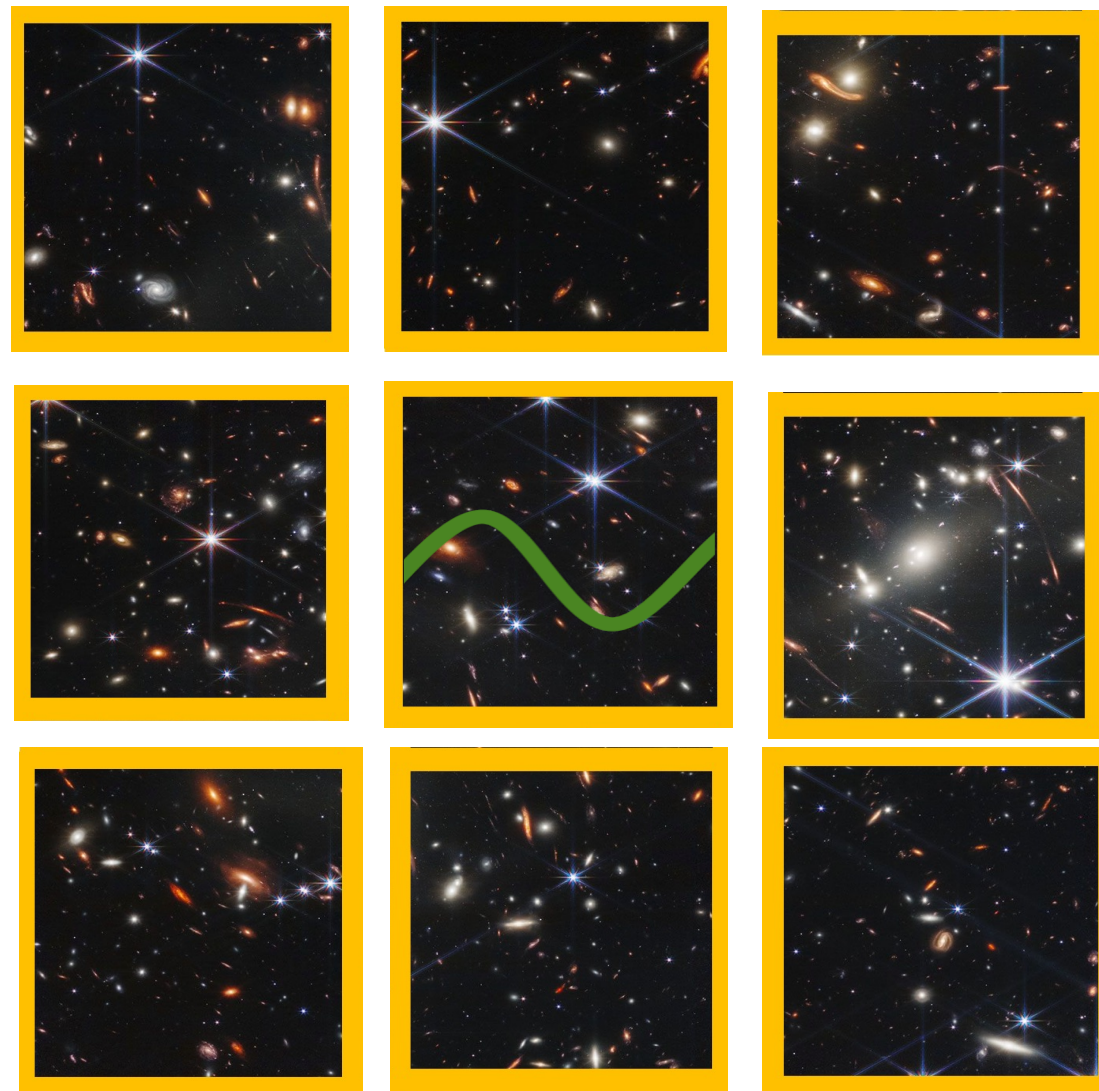


# Compare

sub-boxes



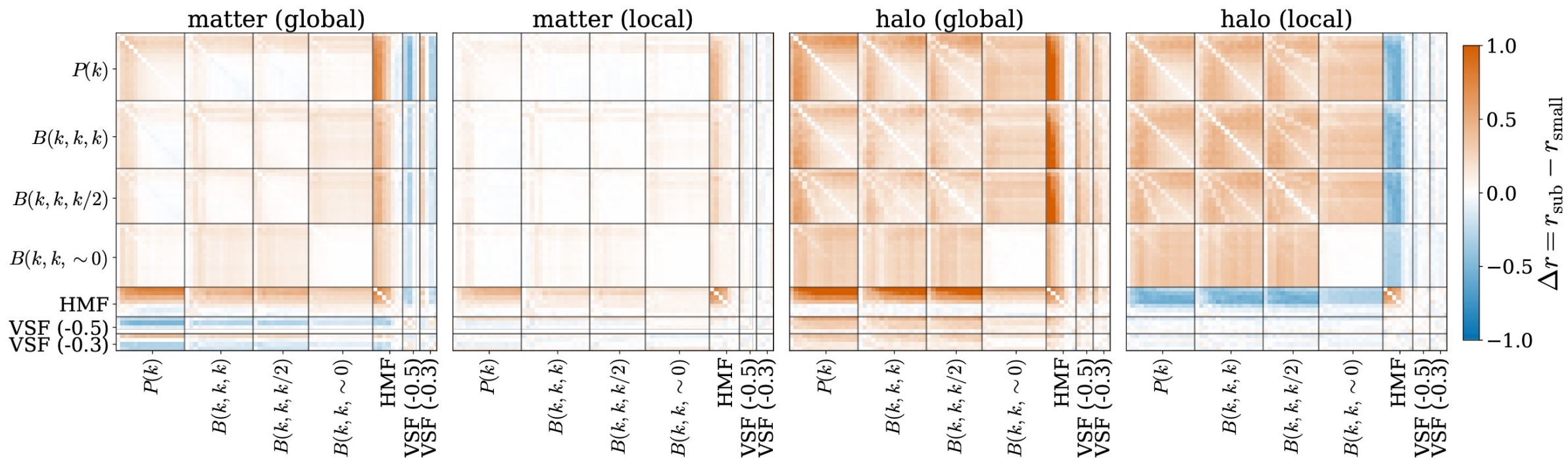
small boxes





# Cross-correlations

sub-box size 625 Mpc/h

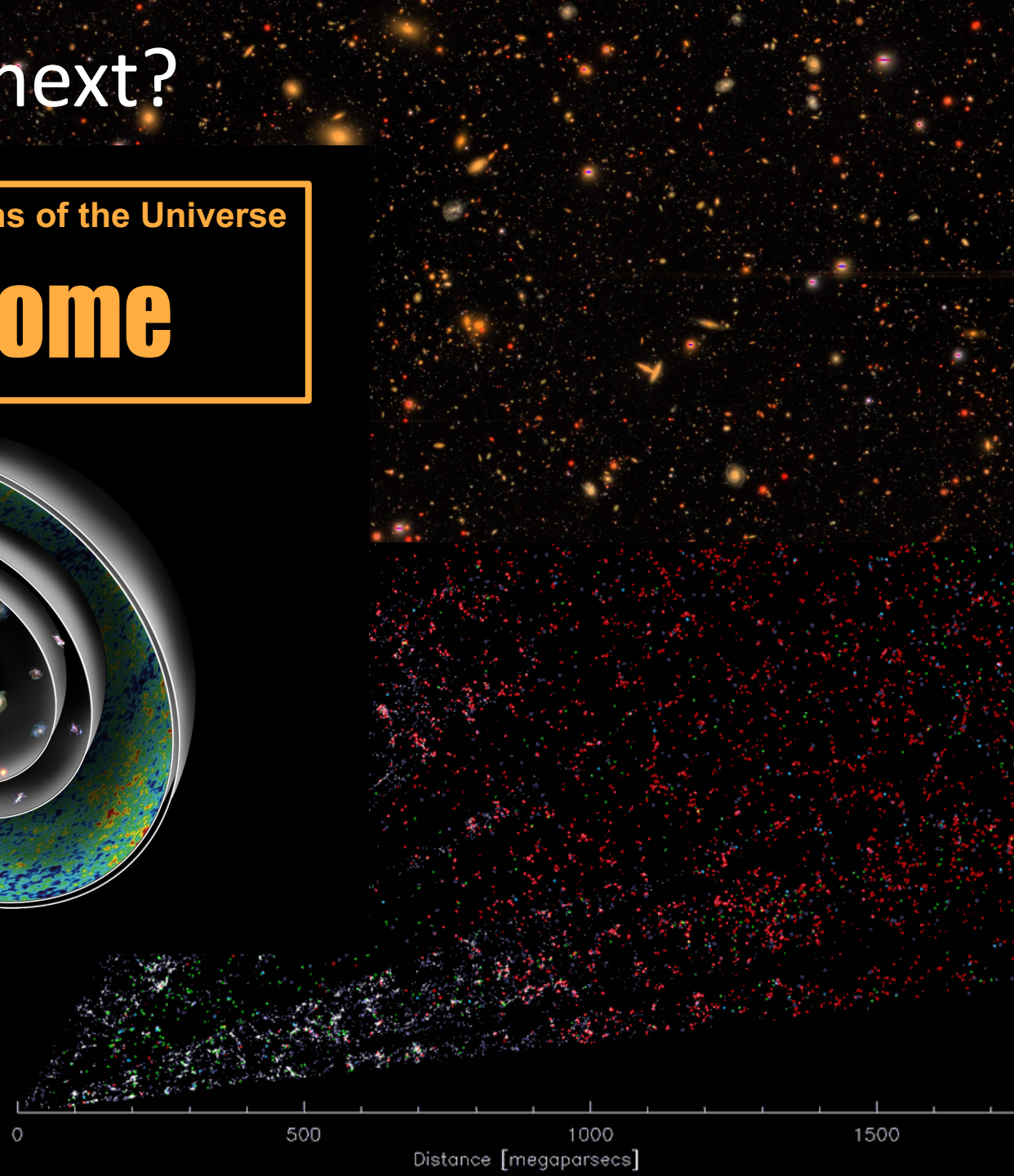
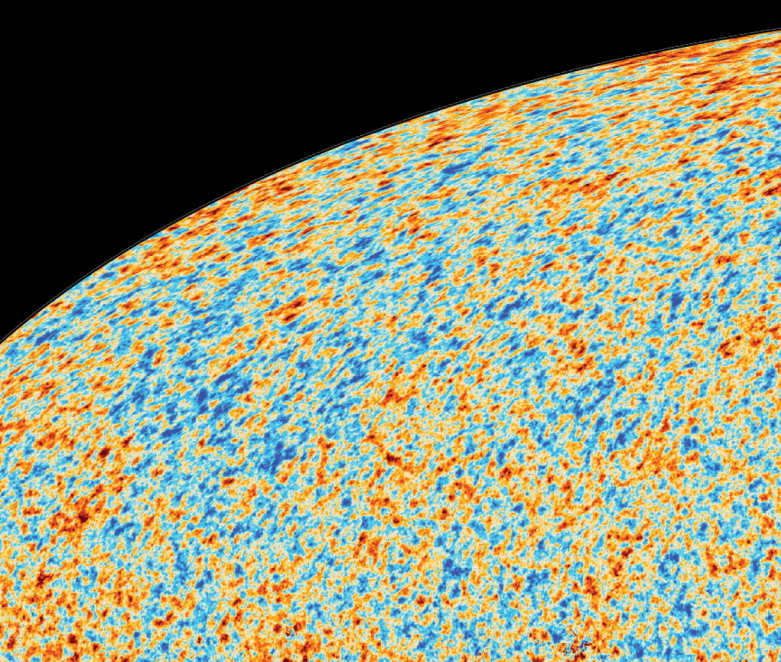
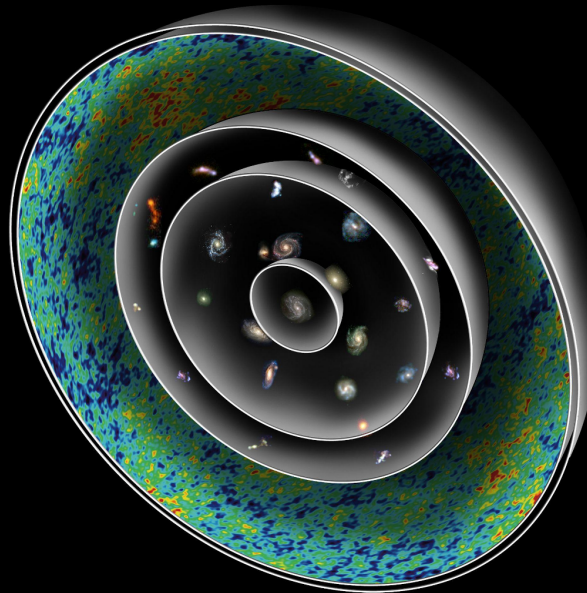




What's next?

Correlated Simulations of the Universe

# HalfDome



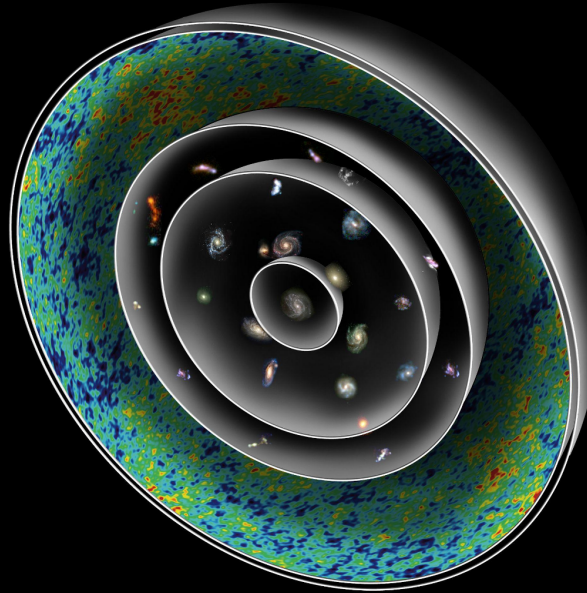


What's next?

Correlated Simulations of the Universe

# HalfDome

1. Complete production of tSZ, kSZ, CIB, radio, lensing, galaxy data by summer 2024



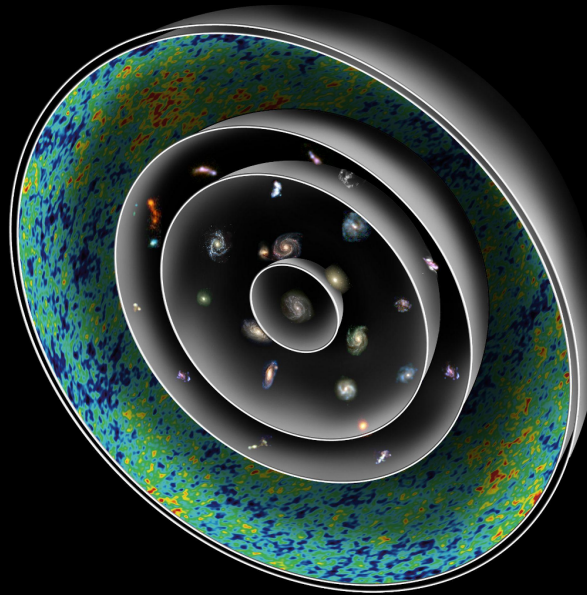
0 500 1000 1500  
Distance [megaparsecs]



# What's next?

Correlated Simulations of the Universe

## HalfDome



1. Complete production of tSZ, kSZ, CIB, radio, lensing, galaxy data by summer 2024

2. More simulations  
wCDM + neutrinos  
Full N-body



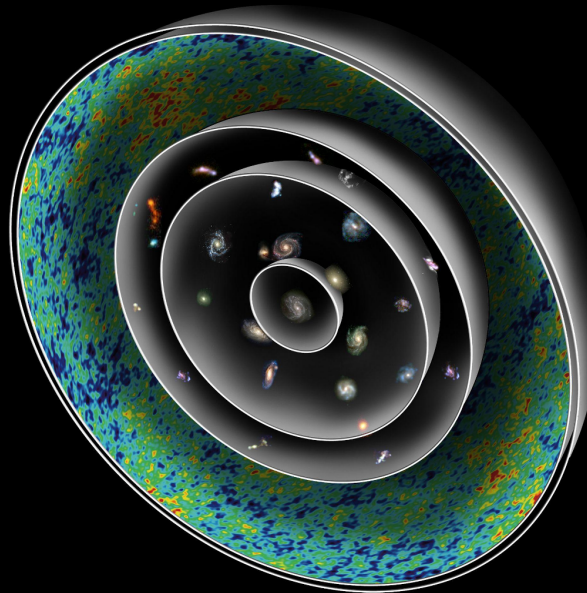
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Distance [megaparsecs]



# What's next?

Correlated Simulations of the Universe

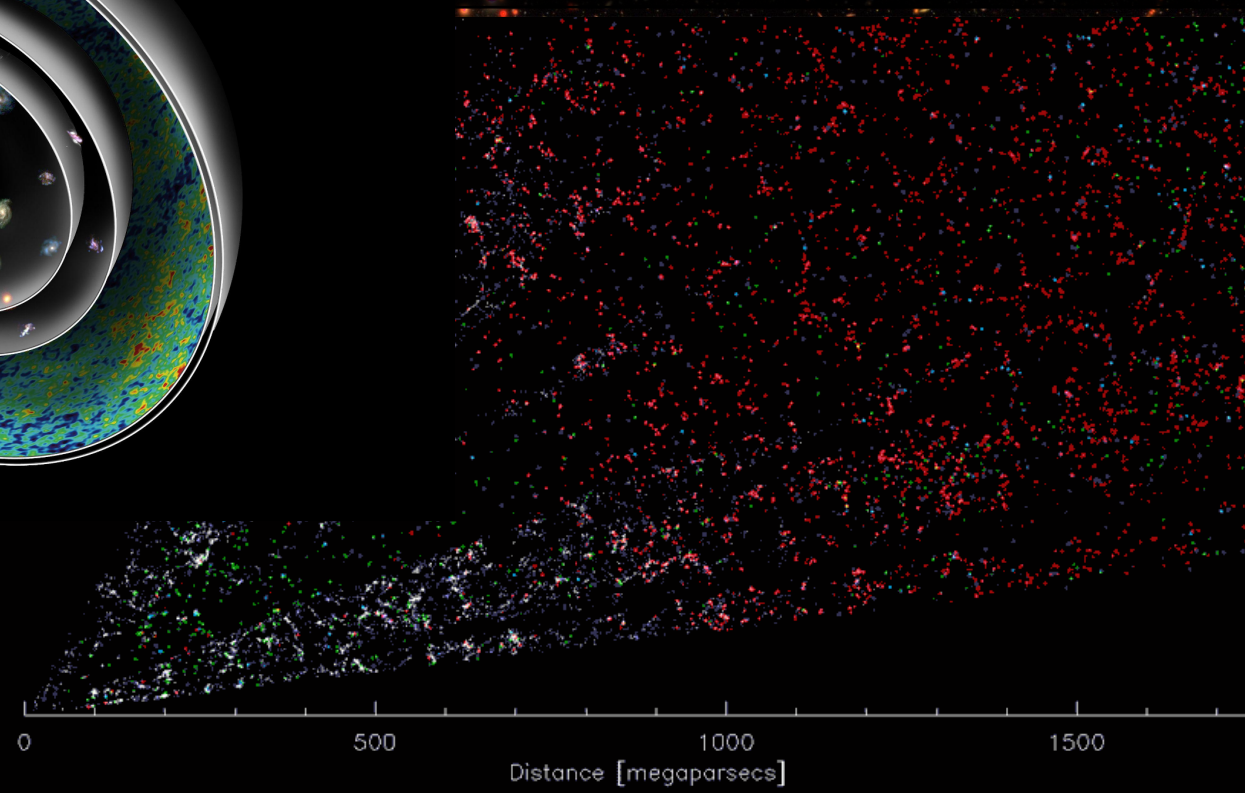
## HalfDome



1. Complete production of tSZ, kSZ, CIB, radio, lensing, galaxy data by summer 2024

2. More simulations  
wCDM + neutrinos  
Full N-body

3. Machine learning  
Super-resolution  
Field-level inference

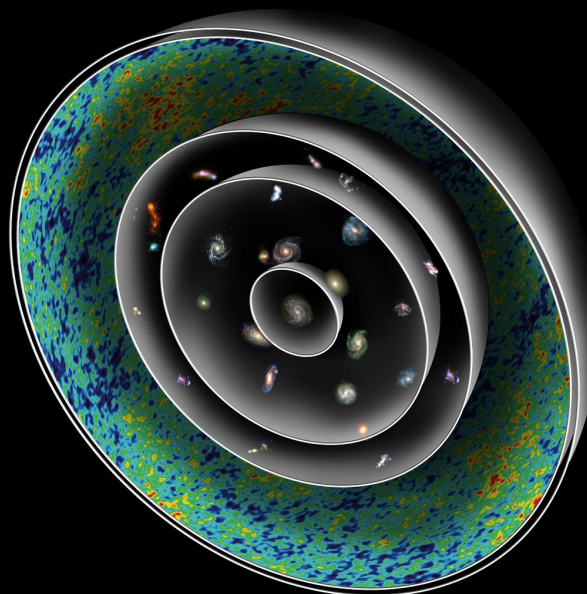




# What's next?

Correlated Simulations of the Universe

## HalfDome

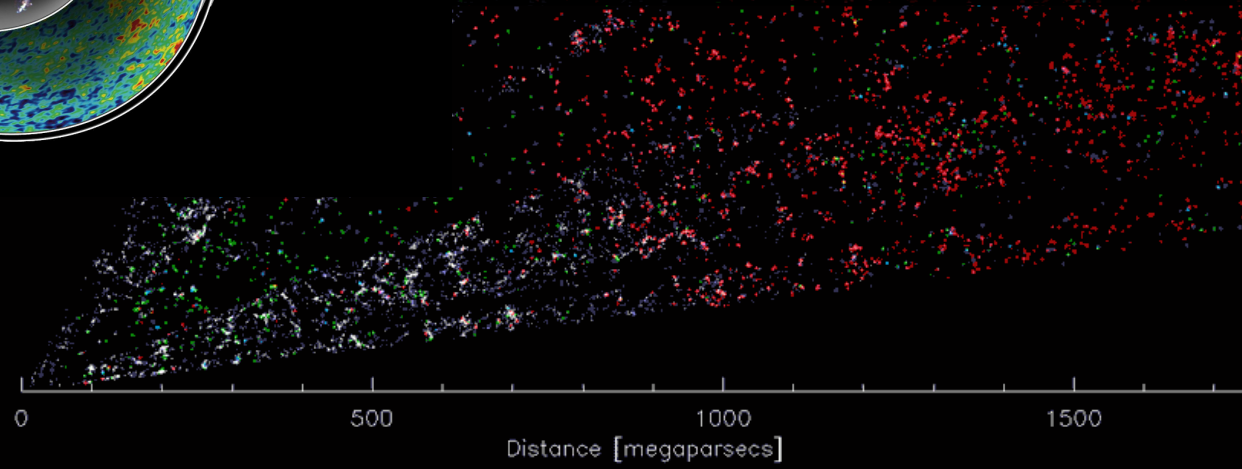


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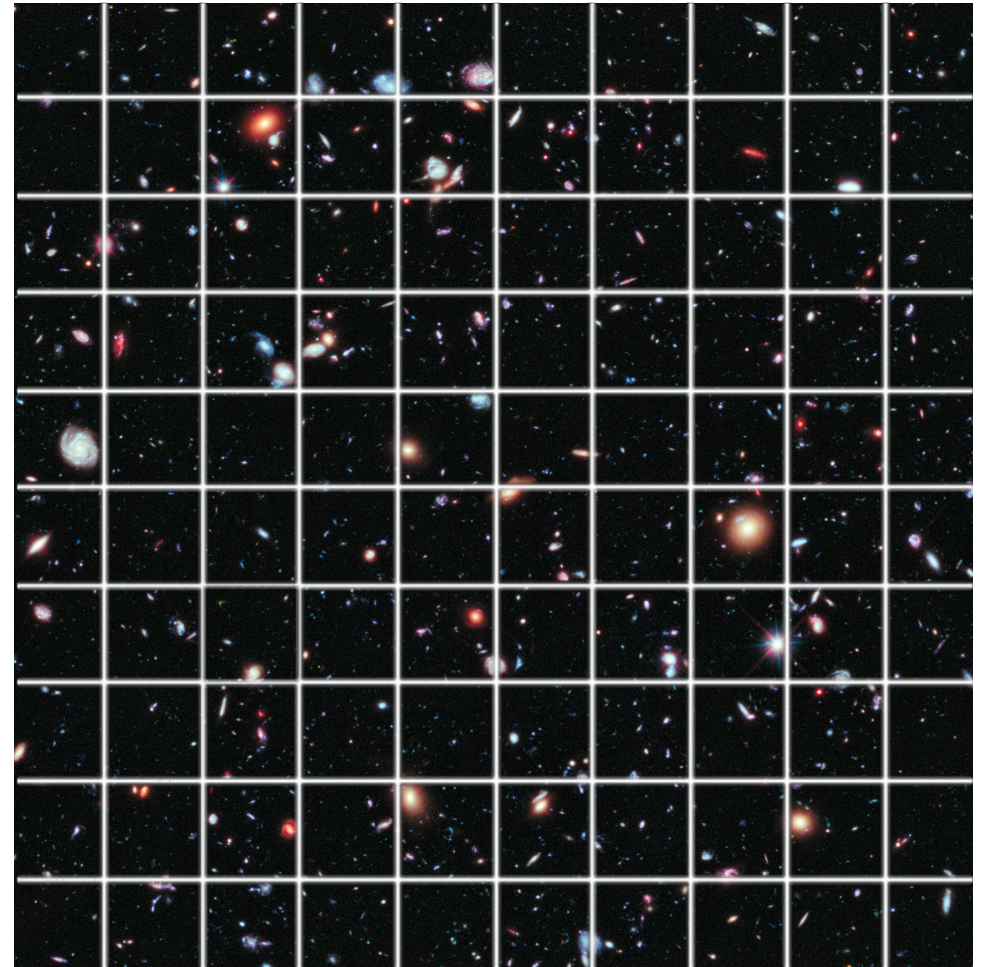
**Listen to you!**  
Interested in other science?  
Let us know!





# How to maximize the $\nu$ information?

1. Jointly analyze **multiple redshifts** and **multiple tracers** to break degeneracies  
e.g. CMB x LSS
2. Develop **statistical methods** to optimally extract information  
e.g. field-level inference







Field-Level Inference  
*with*  
Microcanonical  
Langevin Monte Carlo

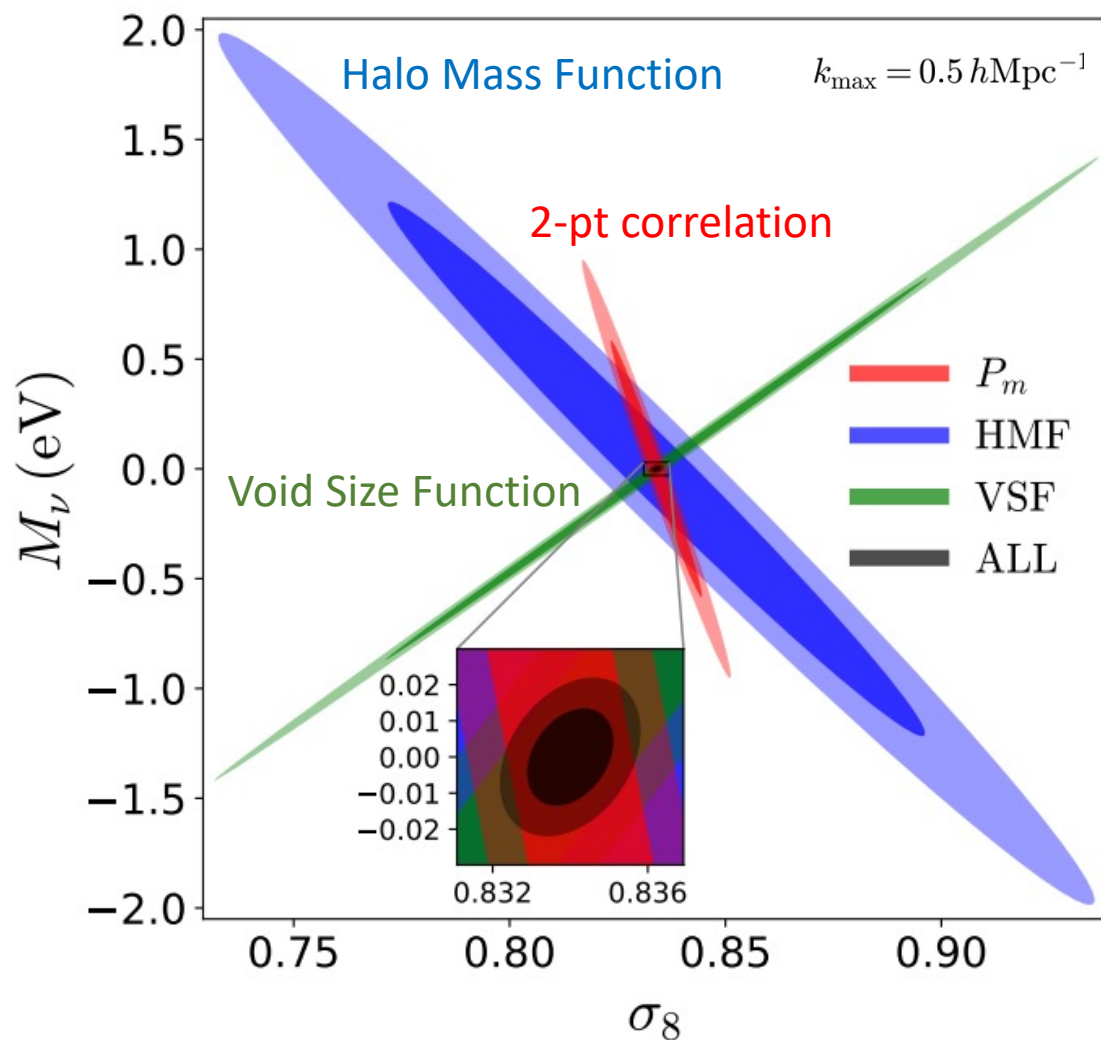
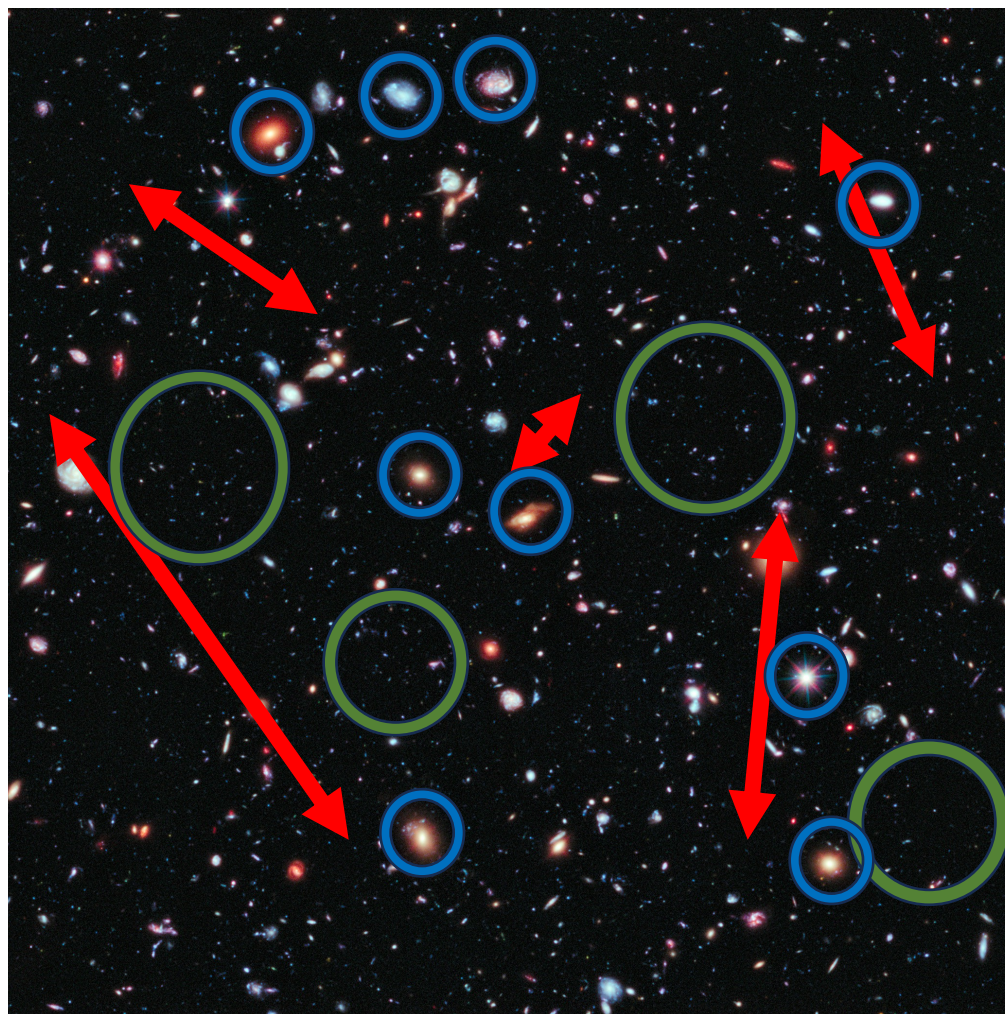


**Field-Level Inference**  
*with*  
Microcanonical  
Langevin Monte Carlo



# Higher-order statistics

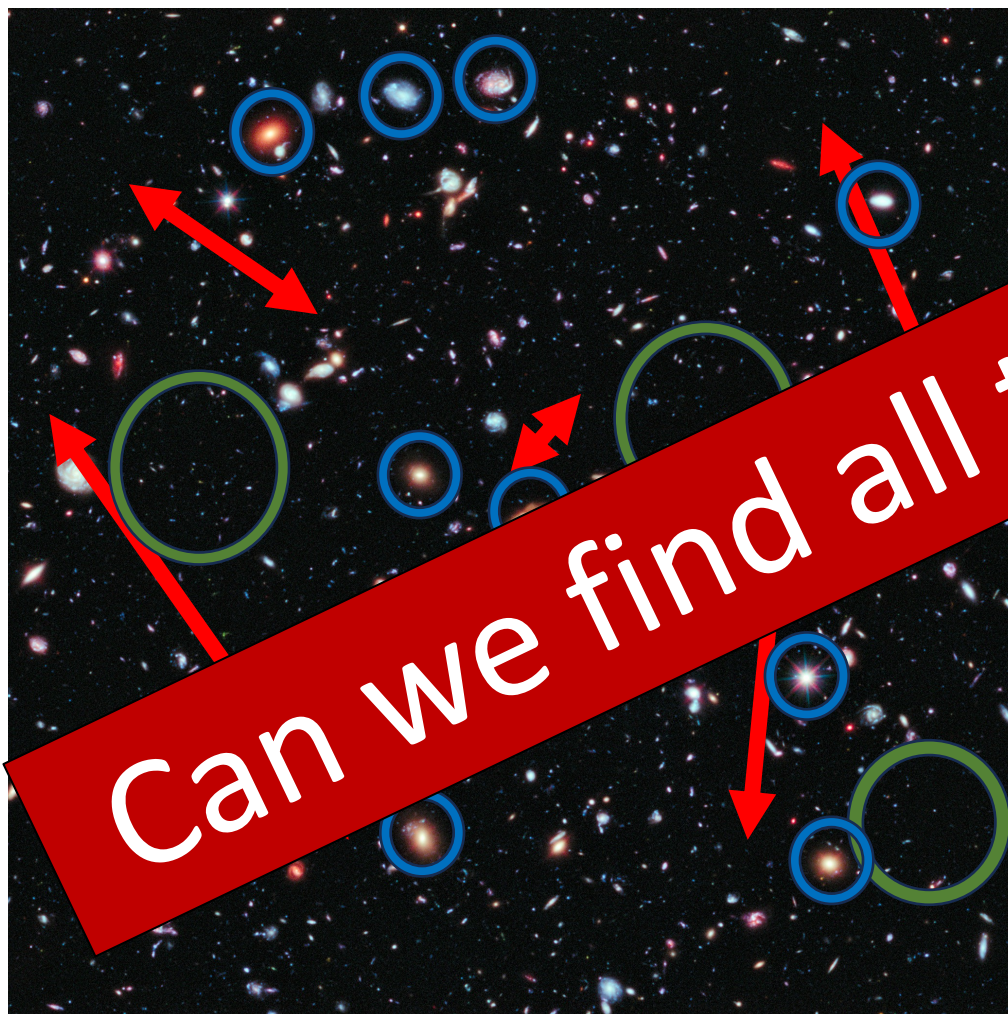
can provide information beyond the 2-pt



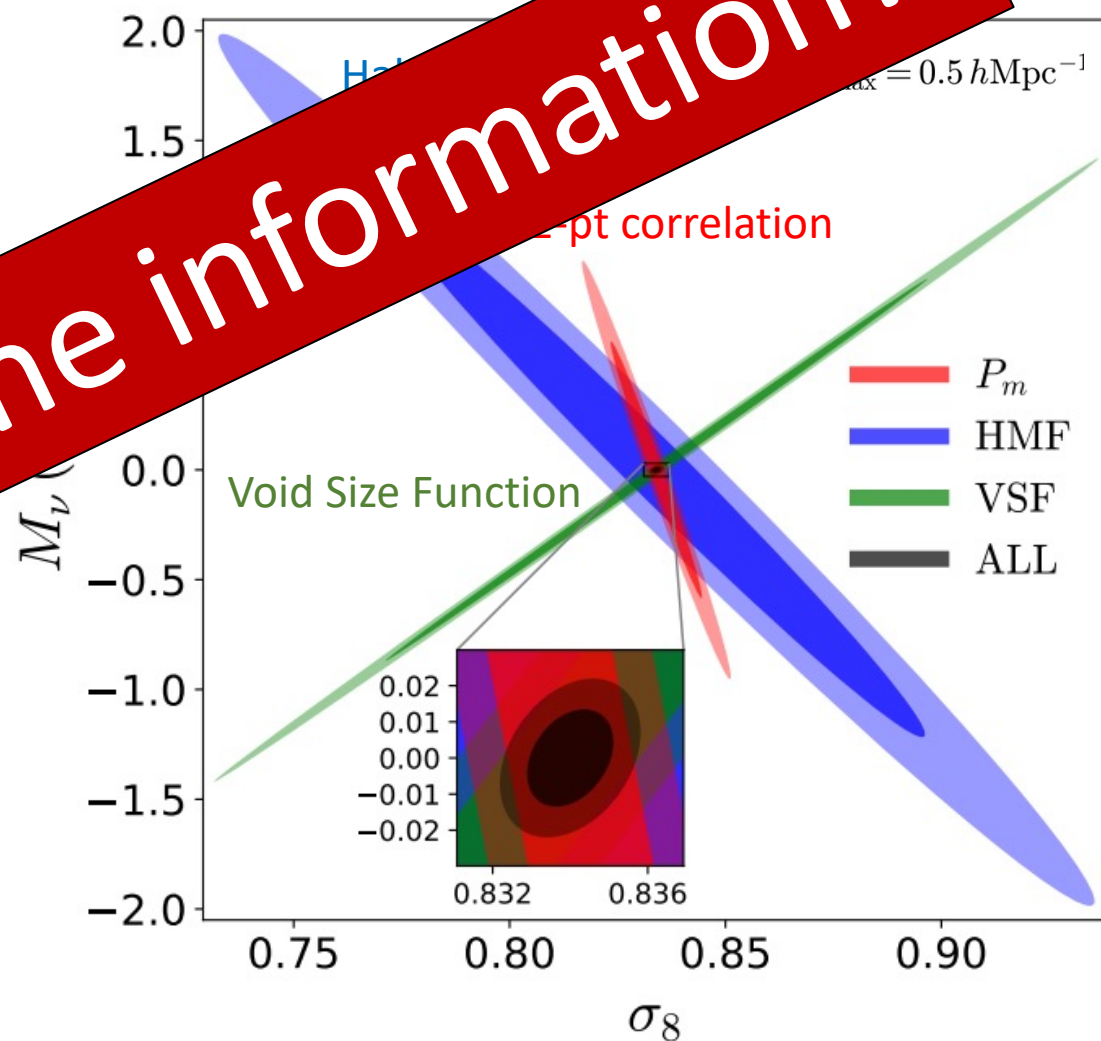


# Higher-order statistics

can provide information beyond the 2-pt

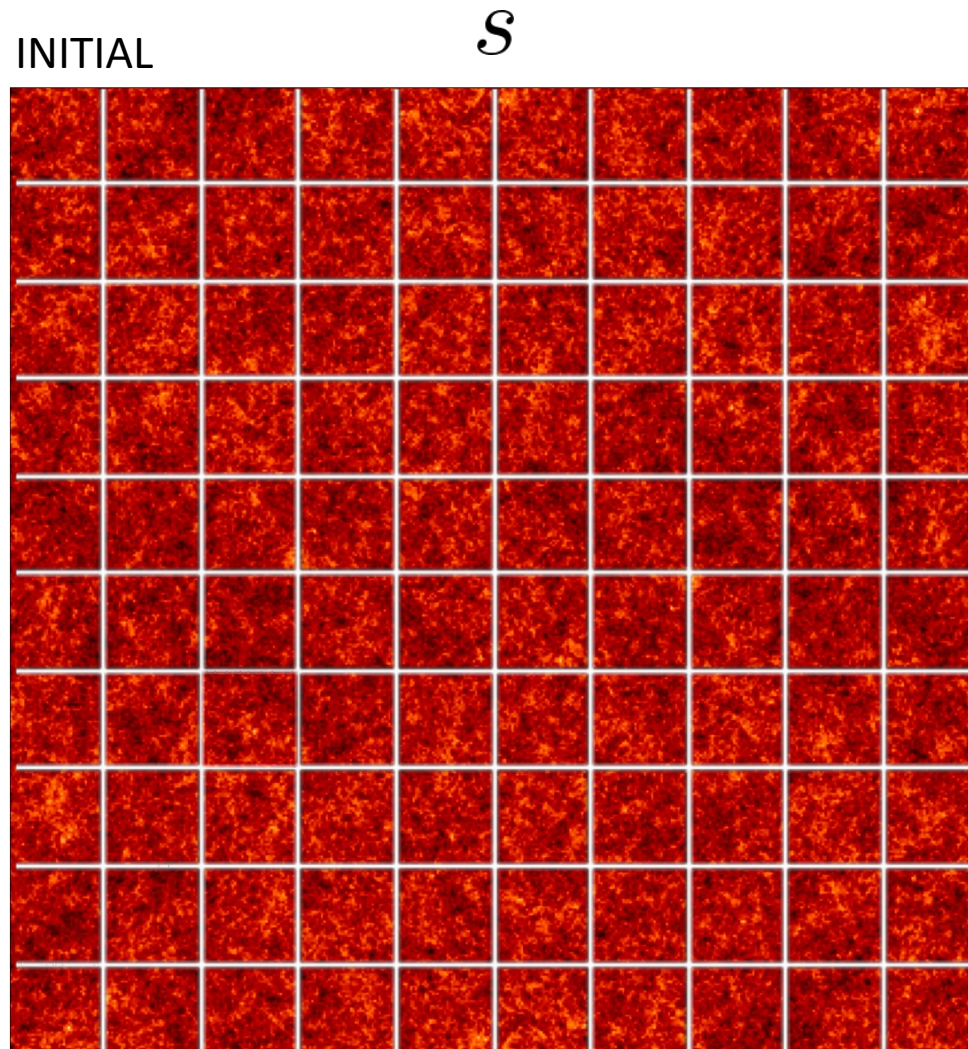


Can we find all the information?



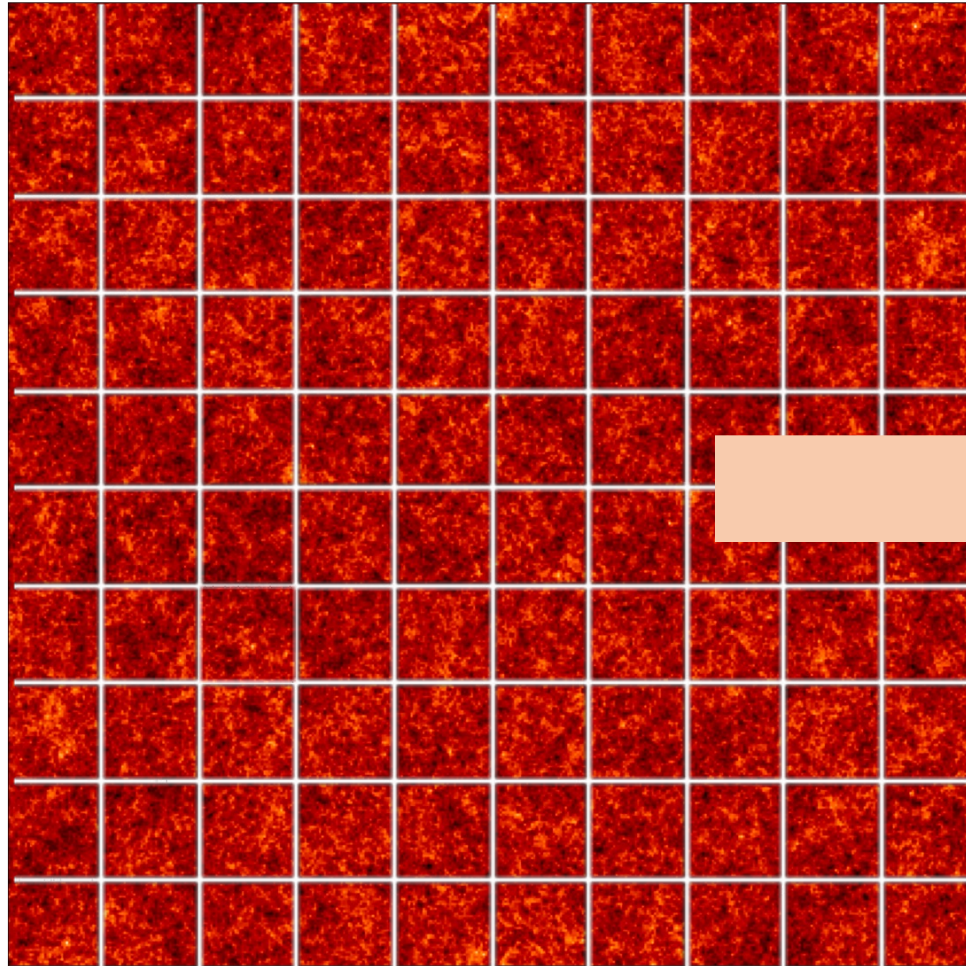


# Forward modeling



# Forward modeling

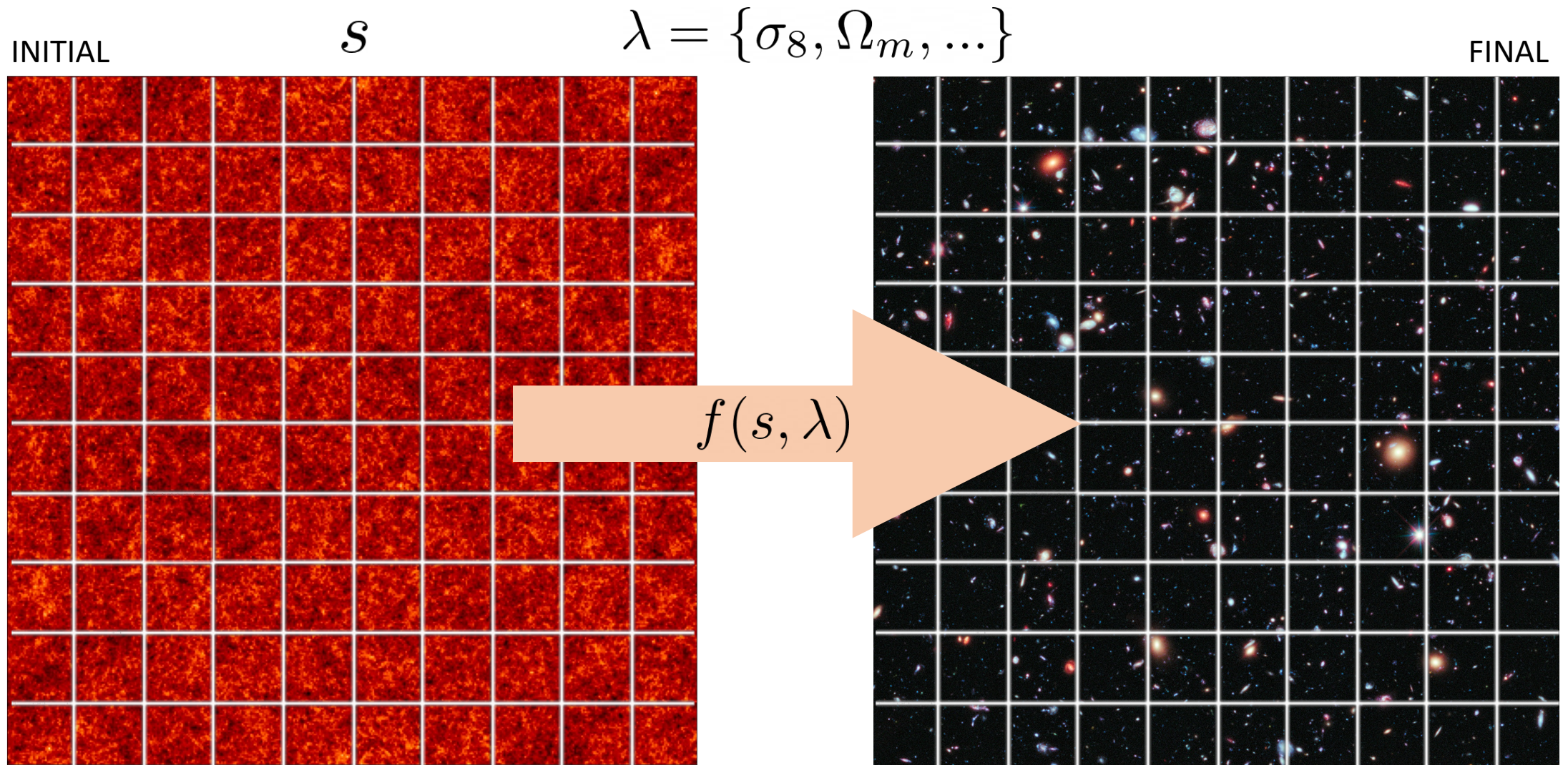
INITIAL  $s$   $\lambda = \{\sigma_8, \Omega_m, \dots\}$



$f(s, \lambda)$

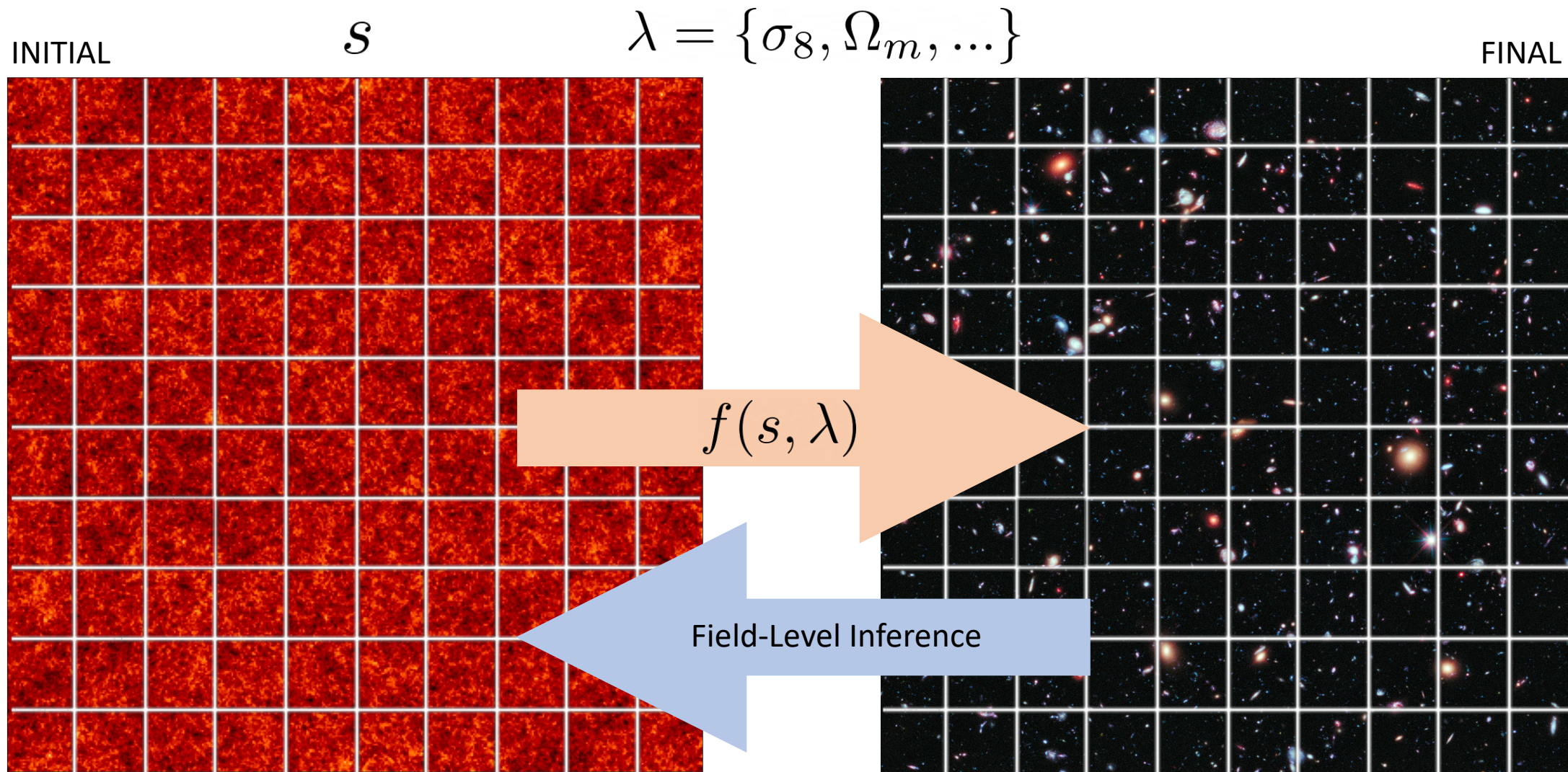


# Forward modeling





# Forward modeling







# Field-Level Inference

Given field data  $d$  and forward model  $f$  infer  
initial modes  $s$  and cosmological parameters  $\lambda$

$$-2 \log P(\mathbf{s}, \lambda | d) = \sum_{\vec{k}} \left[ \frac{|d - f(\mathbf{s}, \lambda)|^2}{N} + \frac{|\mathbf{s}|^2}{\mathcal{P}(\lambda)} \right]_{\vec{k}}$$

posterior                      likelihood                      prior

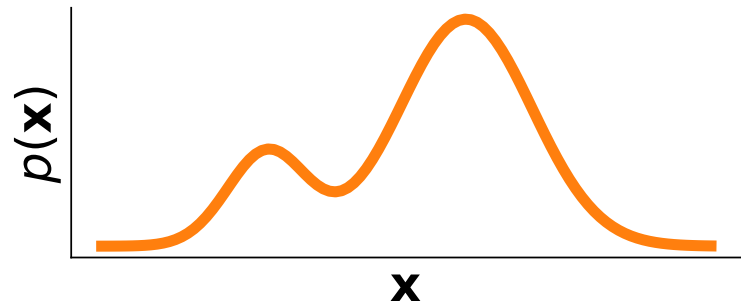
**CHALLENGE:** Multimillion dimensional parameter space!



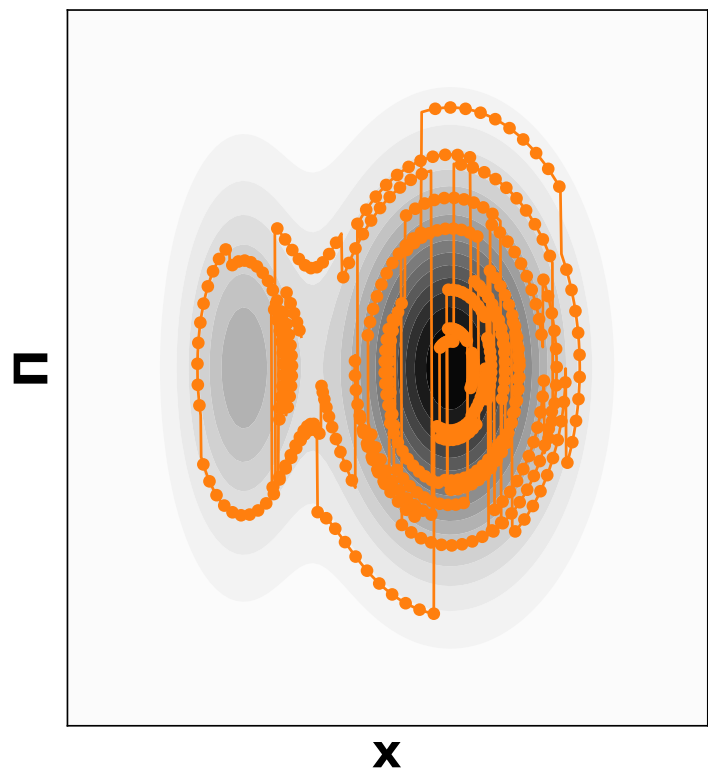


Field-Level Inference  
*with*  
**Microcanonical  
Langevin Monte Carlo**

# Canonical HMC

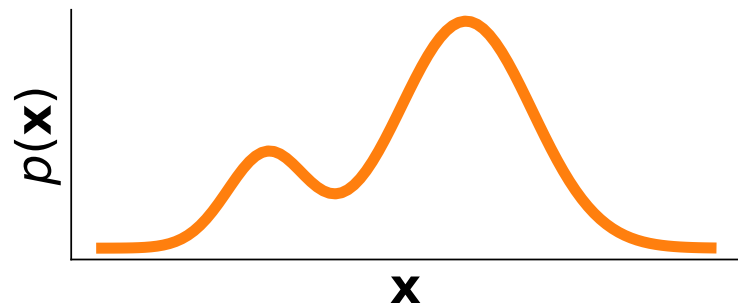


$$p(\mathbf{x}, \boldsymbol{\pi}) \propto e^{-H(\mathbf{x}, \boldsymbol{\pi})}$$

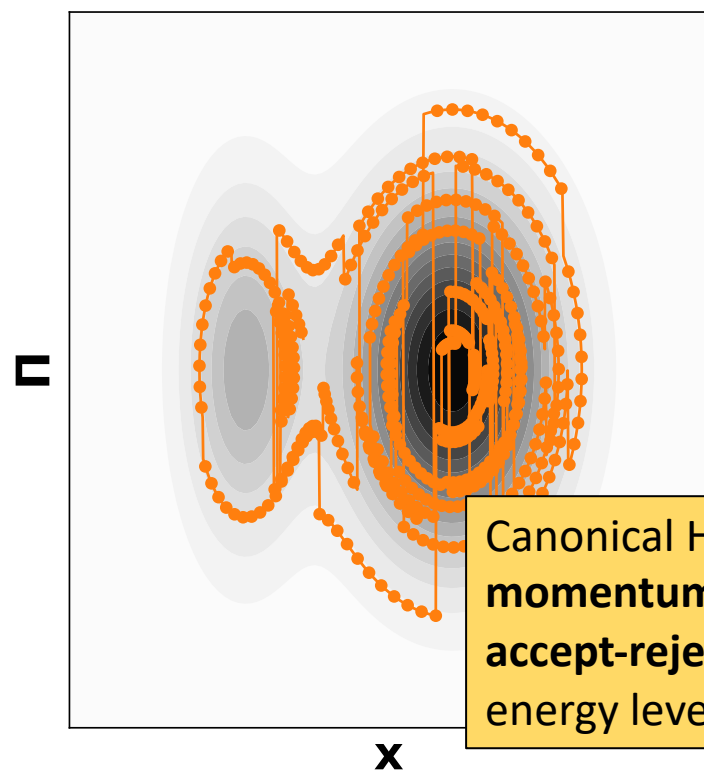




# Canonical HMC

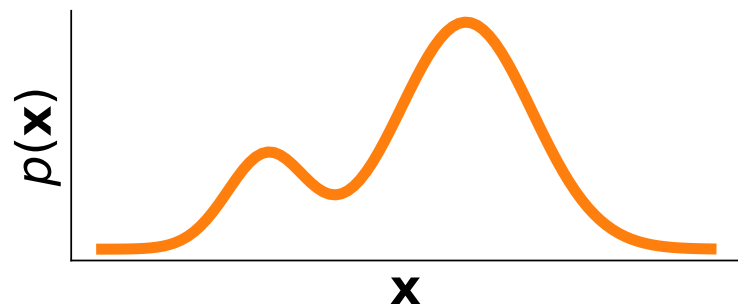


$$p(\mathbf{x}, \boldsymbol{\Pi}) \propto e^{-H(\mathbf{x}, \boldsymbol{\Pi})}$$

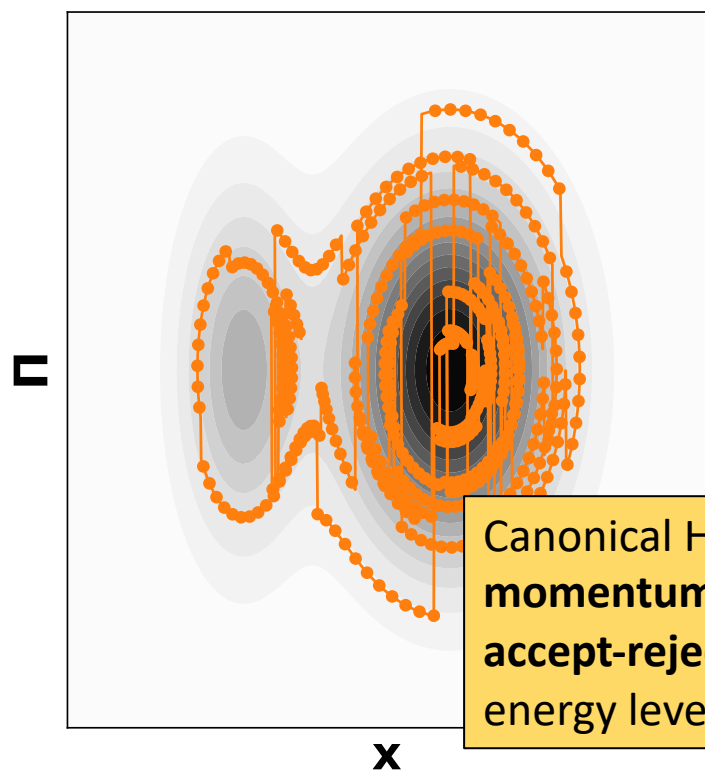


Canonical HMC requires **momentum resampling** and **accept-reject step** to change energy levels and converge

## Canonical HMC

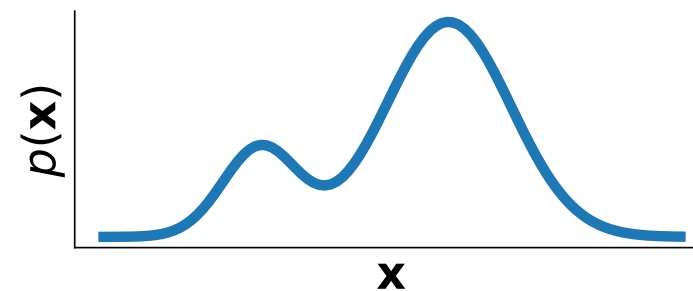


$$p(\mathbf{x}, \boldsymbol{\Pi}) \propto e^{-H(\mathbf{x}, \boldsymbol{\Pi})}$$

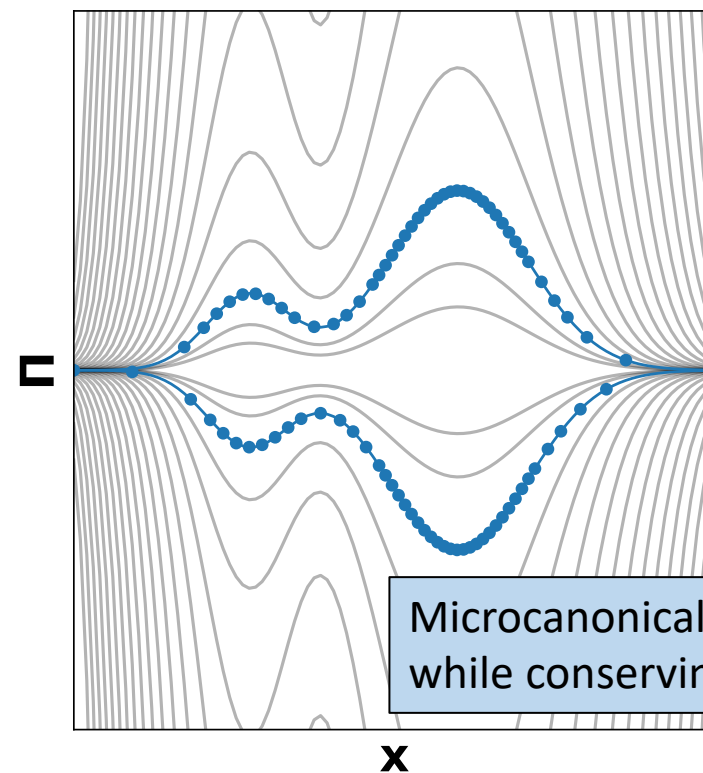


Canonical HMC requires **momentum resampling** and **accept-reject step** to change energy levels and converge

## Microcanonical HMC



$$p(\mathbf{x}, \boldsymbol{\Pi}) \propto \delta(H(\mathbf{x}, \boldsymbol{\Pi}) - E)$$



Microcanonical HMC converges while conserving energy



# Microcanonical Hamiltonian Monte Carlo

$$dz = u dt$$

$$du = - (d - 1)^{-1} (1 - uu^T) \nabla \mathcal{L}(z)$$

**MCHMC**

# Microcanonical Langevin Monte Carlo

$$dz = u dt$$

$$du = - (d-1)^{-1} (1 - uu^T) [\nabla \mathcal{L}(z) + \eta dW]$$

**MCHMC**

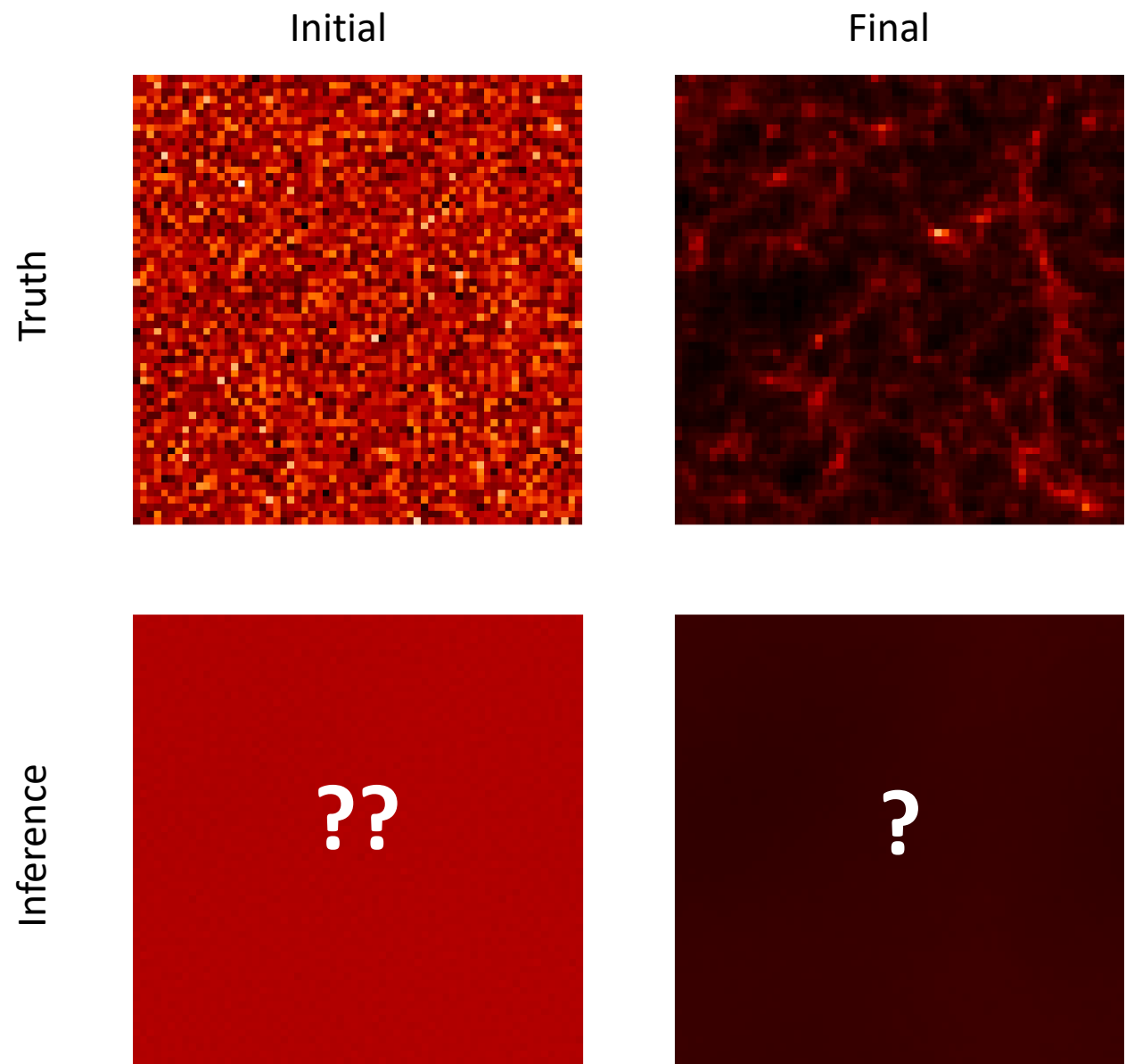
**MCLMC**

Improve ergodicity by including Langevin-like stochastic term

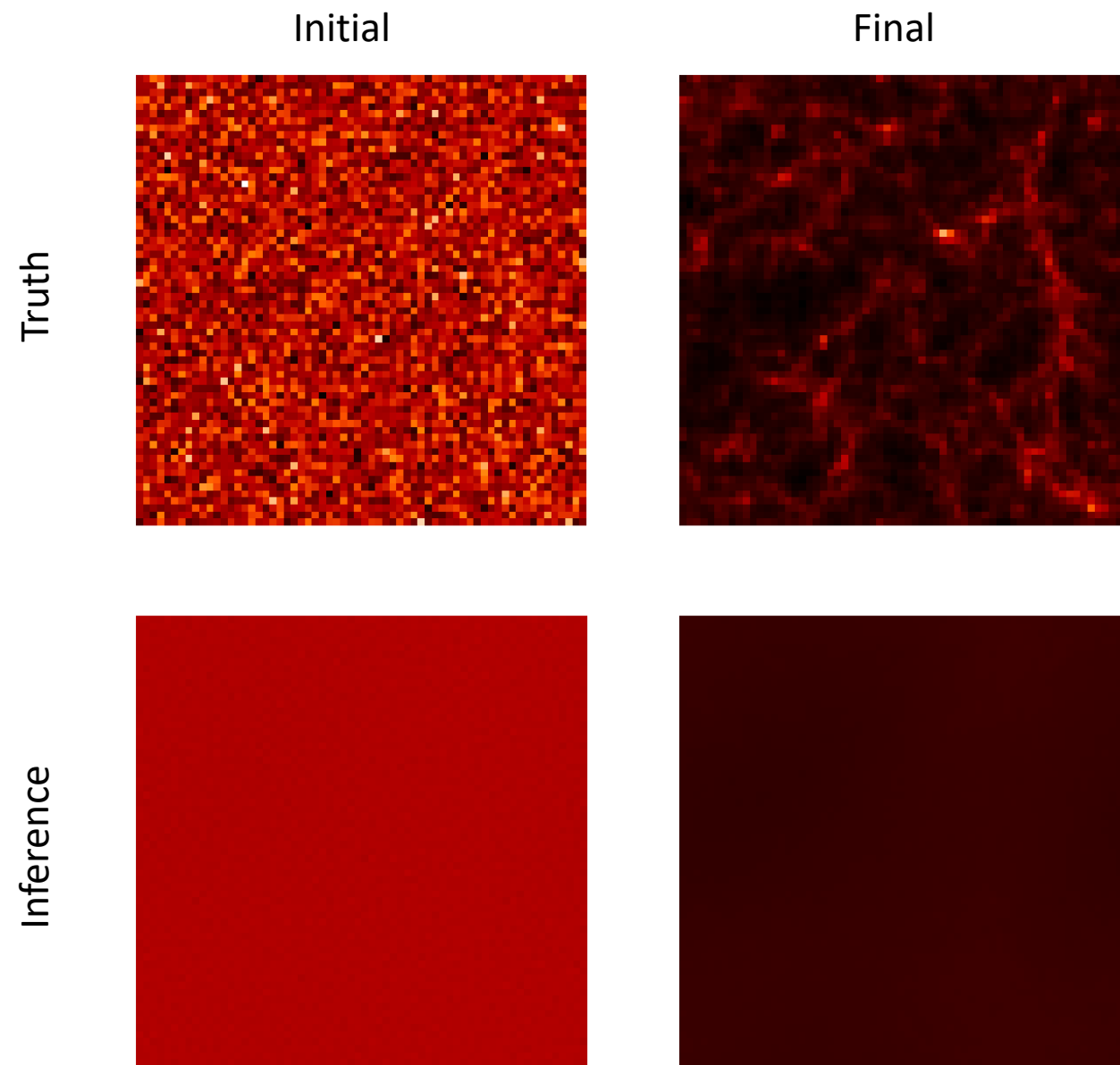




Field-Level Inference  
*with*  
Microcanonical  
Langevin Monte Carlo

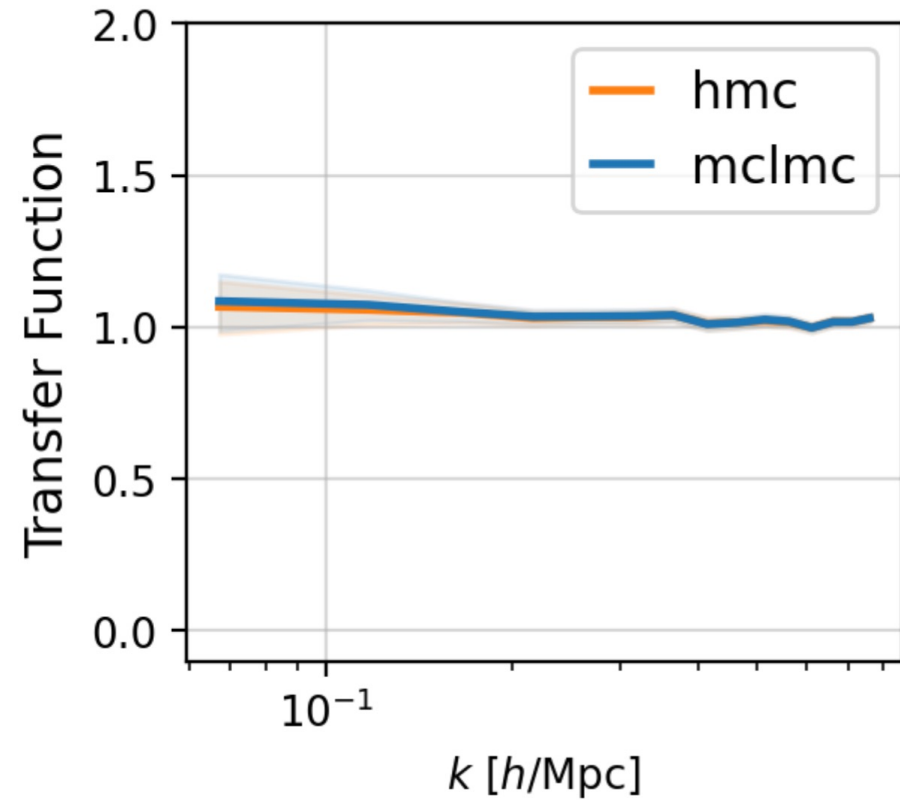
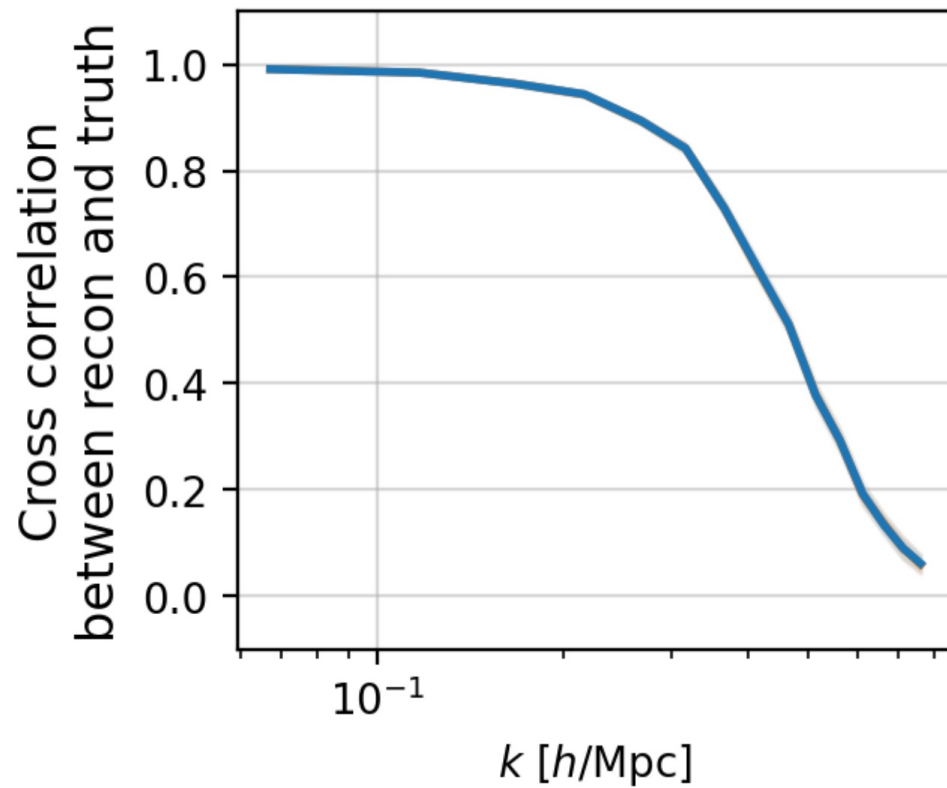






# Samples of Initial Modes

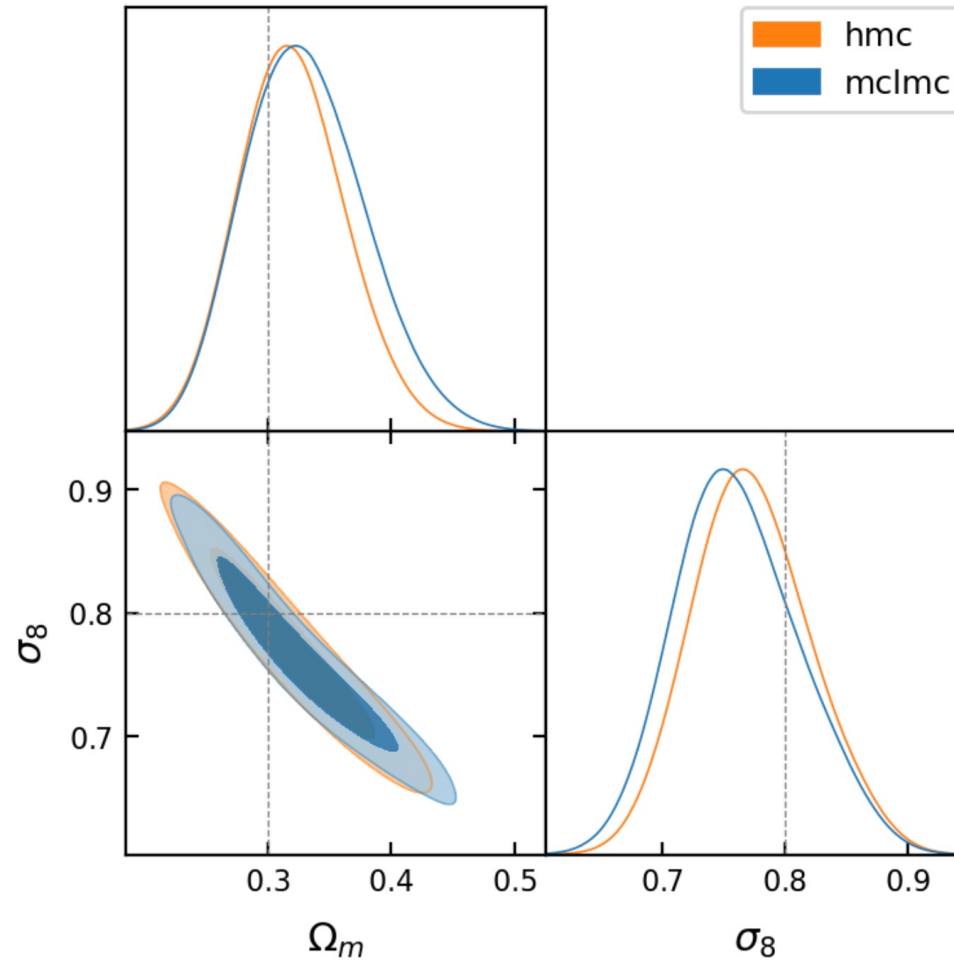
$$d = 32^3 + 2 = 32,770$$



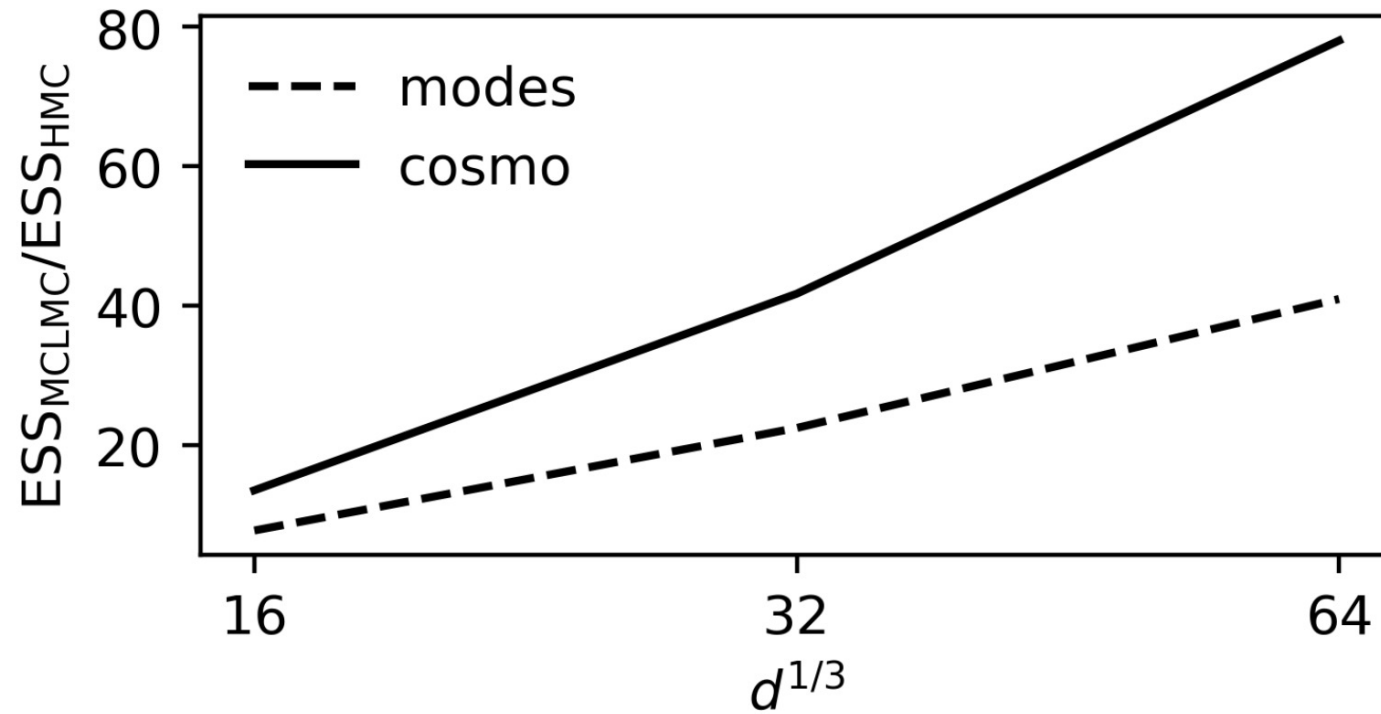


# Samples of Cosmological Parameters

$$d = 32^3 + 2 = 32,770$$



# Efficiency improves with dimensionality!





# Summary

## Neutrino mass from cosmology

**Simulation:** Bayer, Banerjee, Feng [[2007.13394](#)]

**Fisher Analysis:** Bayer, Villaescusa-Navarro, et al. [[2102.05049](#)]

**Fake vs:** Bayer, Banerjee, Seljak [[2108.04215](#)]

**Void Shape:** Bayer, Liu, et al. [[2405.12302](#) , today!]

## The HalfDome Simulations [in prep]

**Super-Sample Covariance:** Bayer, Liu, et al. [[2210.15647](#)]

## Field-level inference

**Microcanonical MC:** Bayer, Seljak, Modi [[2307.09504](#)]

**Joint Densities + Velocities:** Bayer, Modi, Ferraro [[2210.15649](#)]

**Thank you!** <http://adrianbayer.github.io>

No Time 😞

## Look-elsewhere effect

**Theory:** Bayer, Seljak [[2007.13821](#)]

**Theory:** Bayer, Seljak, Robnik [[2108.06333](#)]

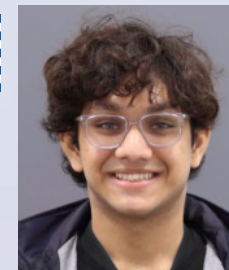
**BBH:** Robnik, Bayer, Charisi, et al. [in prep]



## What are CNNs learning?

Golshan, Bayer, Böhm [in prep]

Lahiry, Bayer, Villaescusa-Navarro [in prep]



Arnab Lahiry  
Graduate Student  
(FORTH)