Theoretical prediction for wavelet l_1-norm

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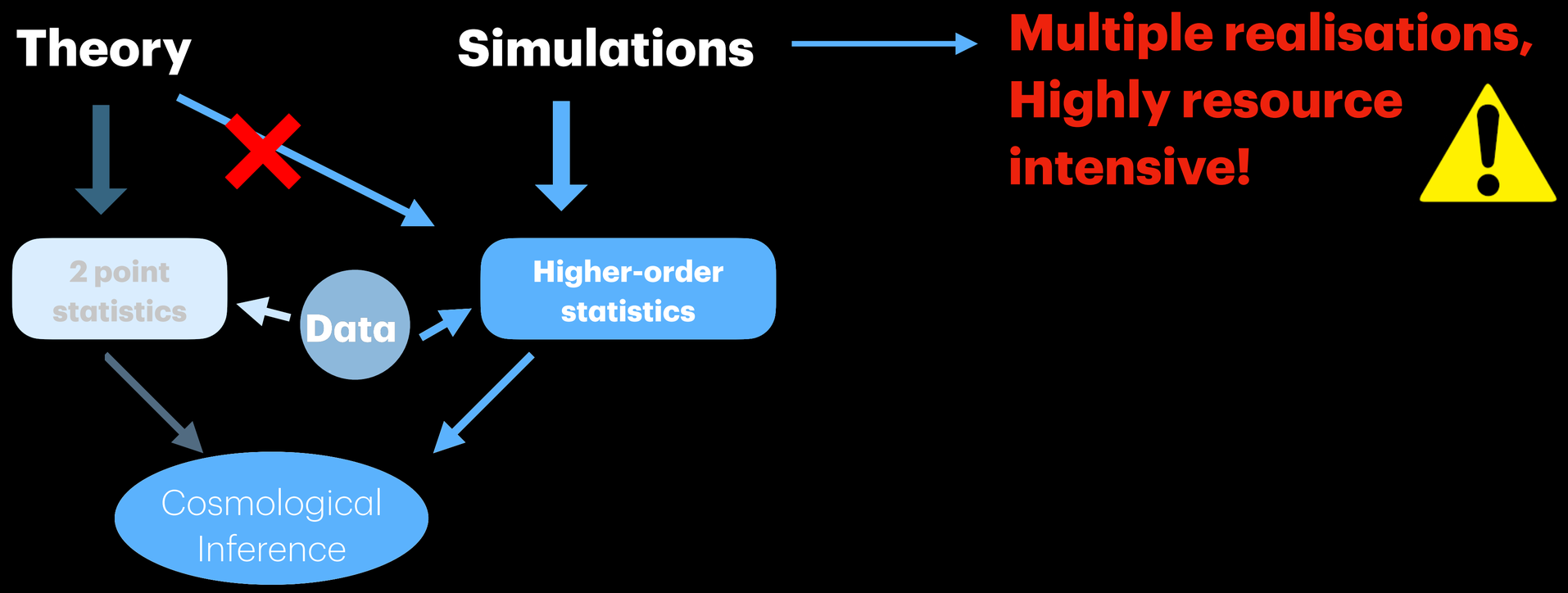
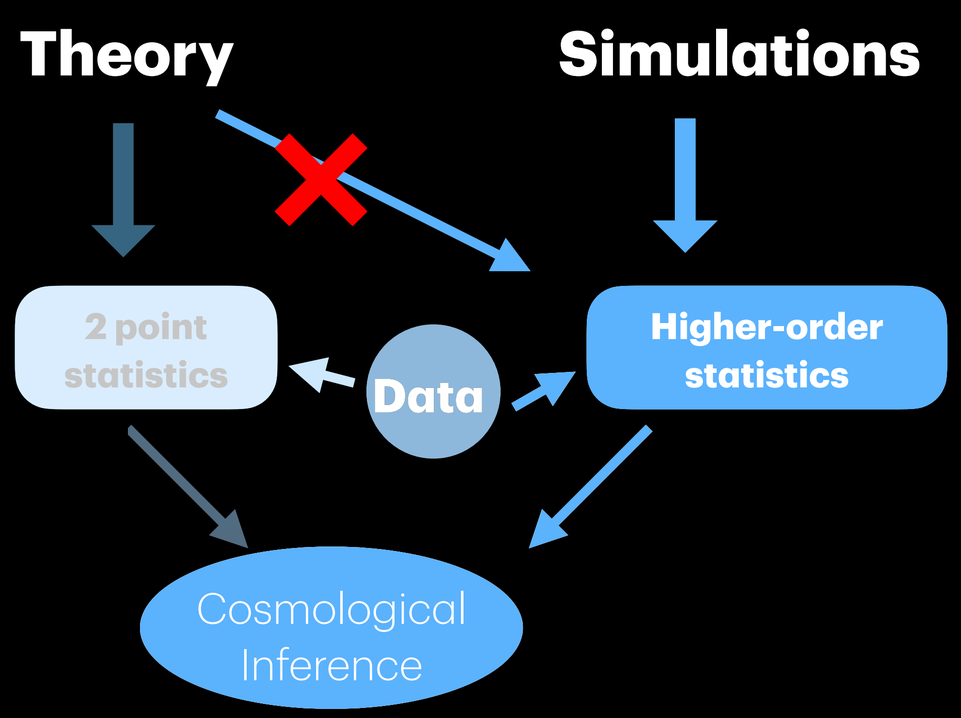
Beyond 2 point statistics:

Statistics	Tomo	Systematics	Params	Forecasts (with II order)	Real data	Survey	References
Summary statistics employed in the analysis	If a tomographic analysis was performed	m = multiplicative bias c = additive bias photo-z = photometric redshifts bar = baryonic effects IA = intrinsic alignment	The cosmological parameters that are constrained	Improvement w.r.t 2PCF %=single parameter Number = 2D FoM	Constraining power > = better ~ = similar < = worst	Survey specs, name or sky coverage + galaxy number density	First author + year.
PDF	no yes no	m, c no no	Ω_m, σ_8 M_ν, A_s M_ν, w_0	2 35%, 61% 27%,40%+Planck		DES-Y1 LSST Euclid	Patton + 2017 Liu, J.+ 2018 Boyle+ 2020
Bispectrum	yes yes yes	no no no	$\sigma_8, w_a, w_0, \Omega_\Lambda$ Ω_m, σ_8 M_ν, Ω_m, A_s	3 2 32%, 13%, 57%		4000 deg^2, 100 arcmin^{-2} Euclid LSST	Takada+ 2005 Bergé+ 2010 Coulton+ 2019
MF	yes no yes yes	no photo-z, m, c no IA, photo-z, m	Ω_m, σ_8, w_0 Ω_m, σ_8 M_ν, Ω_m, A_s Ω_m, σ_8	11%, 14%, 14% 4 4.2	biased (syst.)	LSST CFHTLenS LSST DES	Kratochvil+ 2012 Petri+2015 Marques+2018 Zürcher+ 2021
Moments	no yes yes	photo-z, m, c m, c bar, IA, photo-z, m	Ω_m, σ_8 Ω_m, σ_8 S_8	 2 20%	> 2PCF	CFHTLenS 3500 deg^2, 27 arcmin^{-2} DES-Y3	Petri+ 2015 Vicinanza+ 2018 Gatti+ 2019
Peaks	yes yes no yes yes yes	photo-z, m, c photo-z, m, c m,c, IA, boost, photo-z m,c, IA, photo-z, bar no no	Ω_m, σ_8 Ω_m, σ_8 Ω_m, σ_8 S_8 M_ν, Ω_m, A_s M_ν, Ω_m, A_s	 39%, 32%, 60% 63%, 40%, 72%	~ 2PCF > 2PCF (2) ~ 2PCF > 2PCF (20%)	CS82 CFHTLenS DES-Y1 KiDS-450 LSST Euclid	Liu X.+ 2015 Liu J.+ 2015 Kacprzak+ 2016 Martinet+ 2017 Li Z.+ 2018 Ajani+ 2020
Minima **Minima+Peaks** **Voids** **1D M_{ap}**	yes yes no yes	IA, photo-z, m bar no no	Ω_m, σ_8 M_ν, Ω_m, A_s Ω_m, S_8, h, w_0 Ω_m, S_8, w_0	2.8 44%, 11%, 63% $\gtrsim$ 2PCF 57%, 46%, 68%		DE LSST LSST Euclid	Zürcher+ 2021 Coulton+ 2020 Davies+ 2020 Martinet+2020
M. Learning	no no yes	no no photo-z, m, c, IA	Ω_m, σ_8 Ω_m, σ_8 S_8	5 ~45% (dep. noise)	> 2PCF (30%)	3500 deg^2, no noise KiDS-450 KiDS-450	Gupta+ 2018 Fluri 2018 Fluri 2019
Scattering T. **Starlet ℓ_1- norm**	yes yes	no no	M_ν, Ω_m, w_0 M_ν, Ω_m, A_s	40%, > 2PCF 72%, 60%, 75%		LSST Euclid	Cheng S.+ 2021 Ajani+ 2021

Source: Ajani 2021

Motivation:


Theory
Simulations
Multiple realisations, Highly resource intensive!
2 point statistics
Data
Higher-order statistics
Cosmological Inference



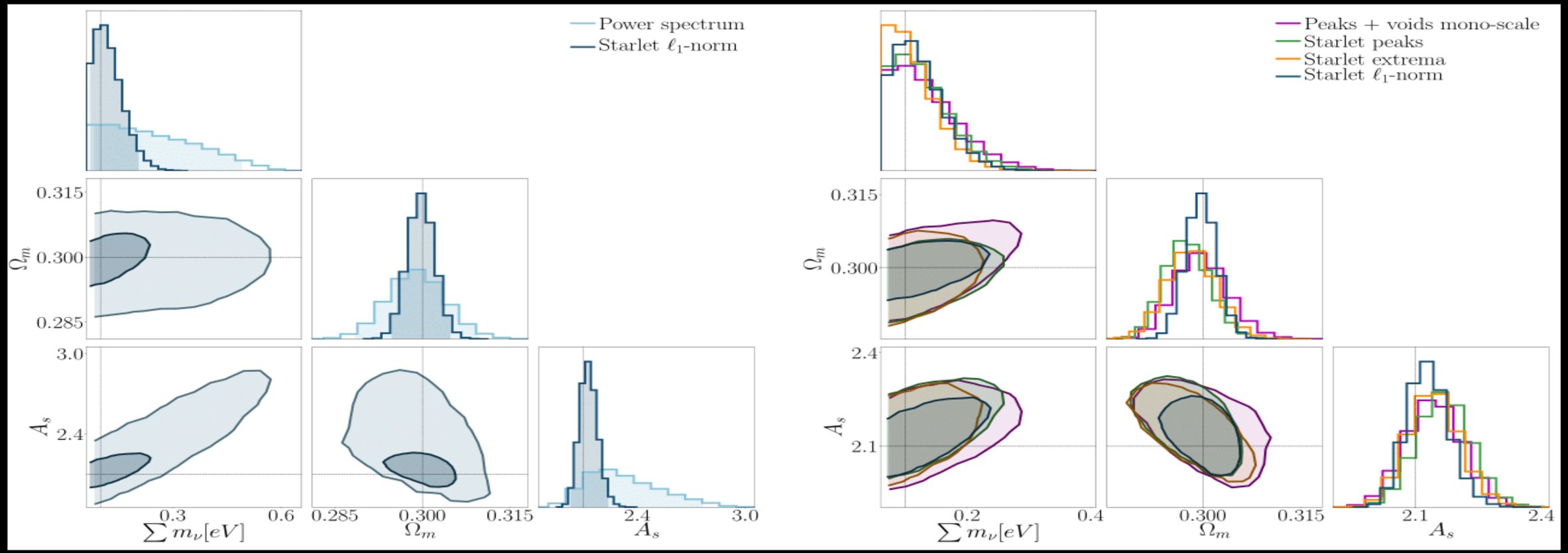
Multiple realisations, Highly resource intensive!

My work:

Have a theory for the wavelet l_1-norm

State of the art: Why Wavelet ℓ_1-norm?

- Shown in Ajani et al. (2021) that it remarkably outperforms commonly used summary statistics,



Wavelet ℓ_1-norm: mathematical formulation

$$w_j = < \kappa, \psi_j >$$
$$\psi_j = \varphi_j - \varphi_{j+1}$$
$$w_j = < \kappa, \varphi_j > - < \kappa, \varphi_{j+1} >$$

Scaling function

Where w_j is the wavelet coefficient for a scale j.

Wavelet ℓ_1-norm at scale j and bin B_i is

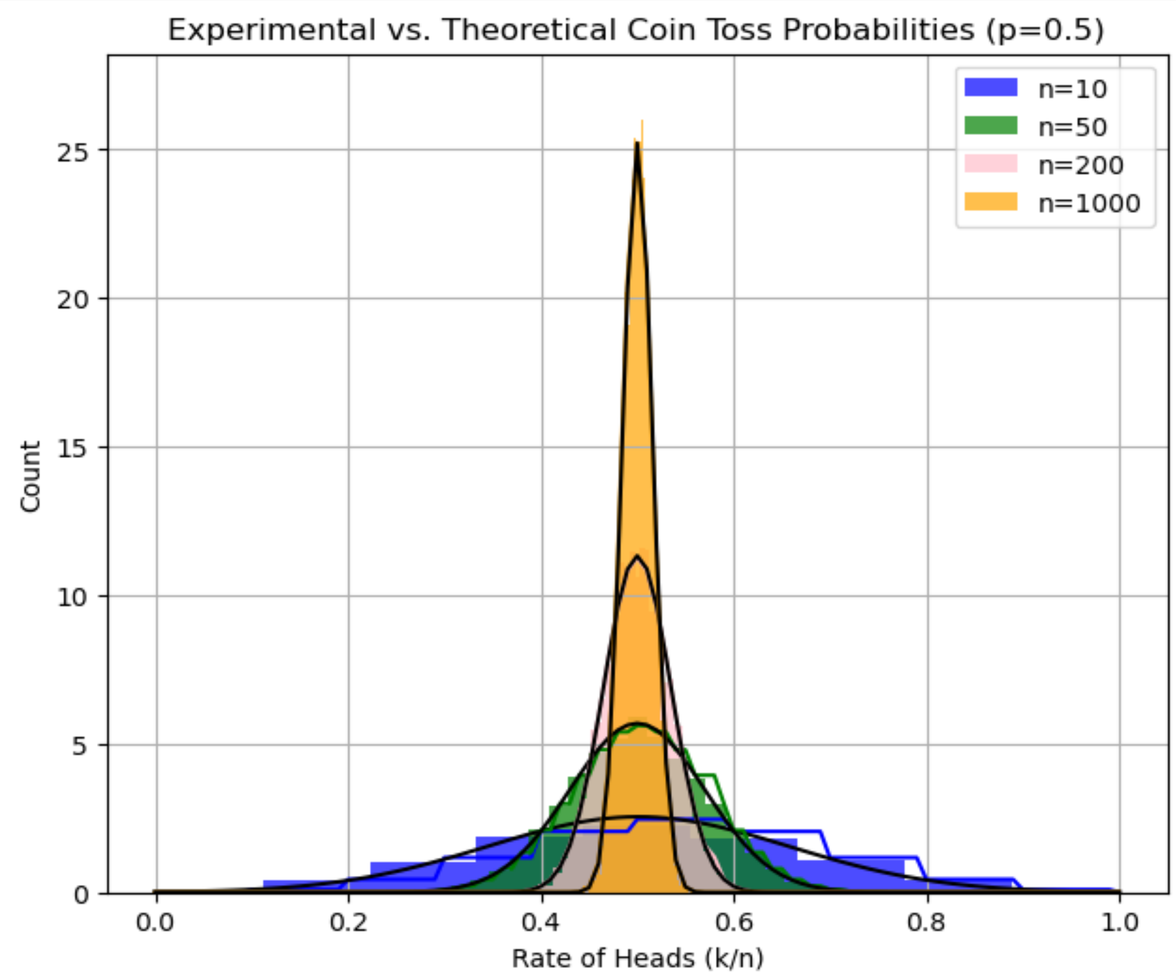
$$\ell_1^{j,i} = \sum_{u=1}^{\#coef(Sj,i)} |S_{j,i}[u]|$$

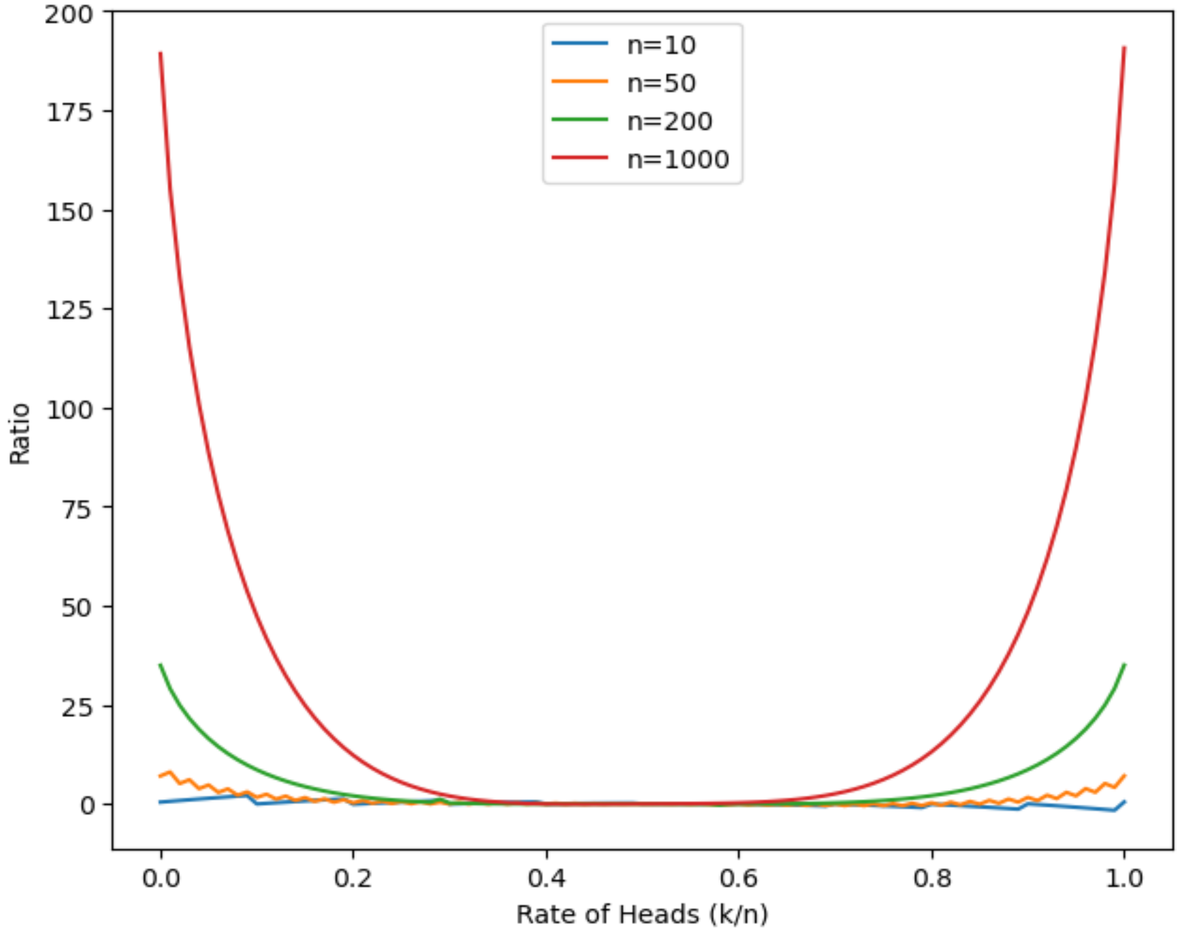
$$S_{j,i} = w_{j,k} / B_i < w_{j,k} < B_{i+1}$$

- information encoded in all pixels
- automatically includes peaks and voids
- multi-scale approach

Where the wavelet coefficient, with the pixel index (k) at scale j is: $w_{j,k}$

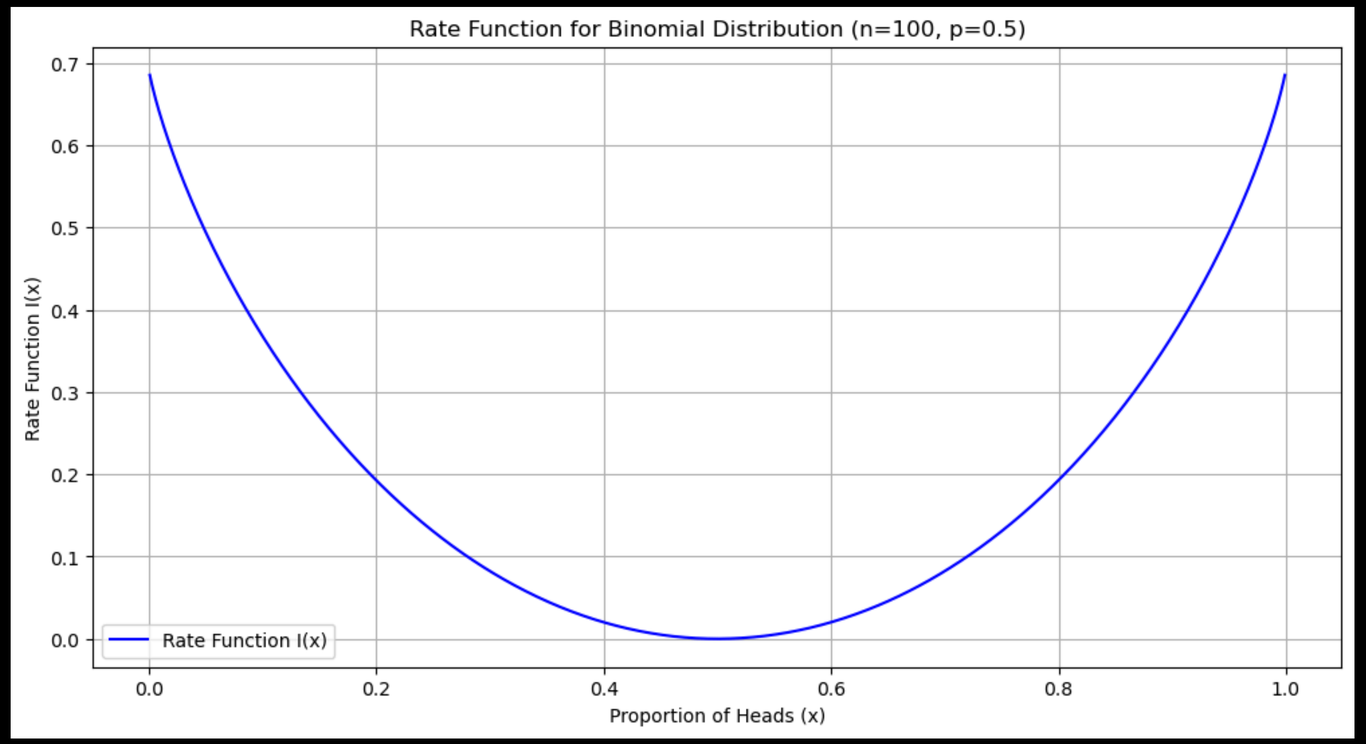
Large Deviation Theory: Intuition





LDT: The bits and pieces

- Rate function: Characterise the asymptotic behaviour of a variable



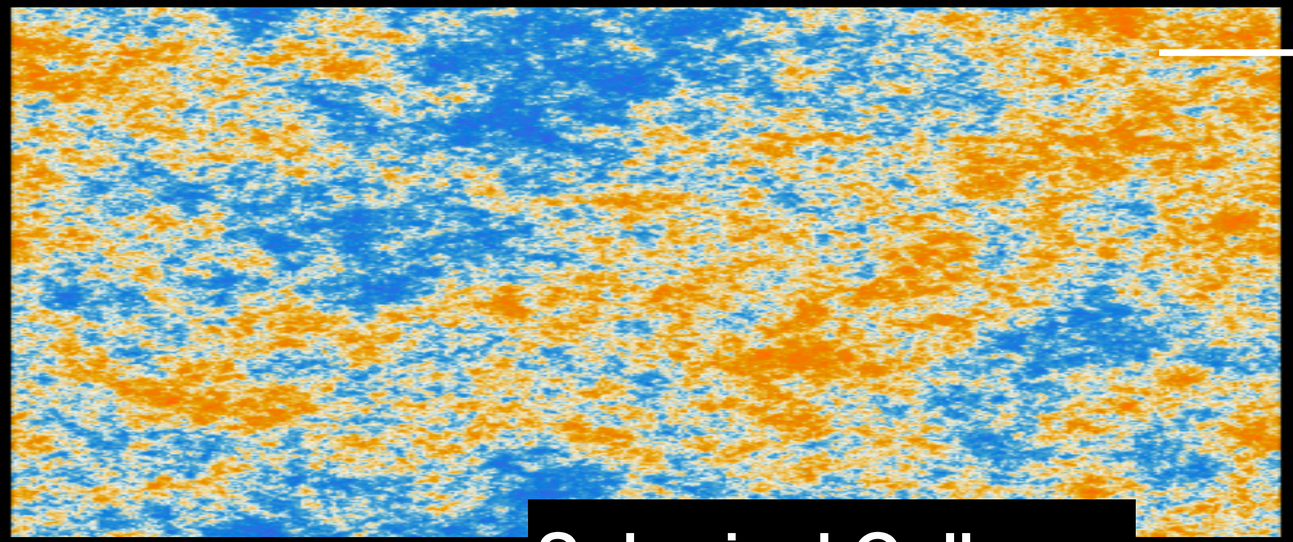
The approximation by the central limit theorem may not be accurate if x is far from E[X] and N is not sufficiently large.

LDP: The probability of rare events happening decreases exponentially with the size of the sample, and the rate function quantifies how unlikely these events are.

Rate function <——> Cumulant generating function <——>PDF

Cool! But…where is **Cosmology** used here?

Large Deviation Theory——>A framework to predict one-PDF in mildly non-linear regime from the 1st principles of Cosmology



We know the PDF

Spherical Collapse

LDT

Rate Function

Probability Density Function

Deriving wavelet ℓ_1-norm from PDF

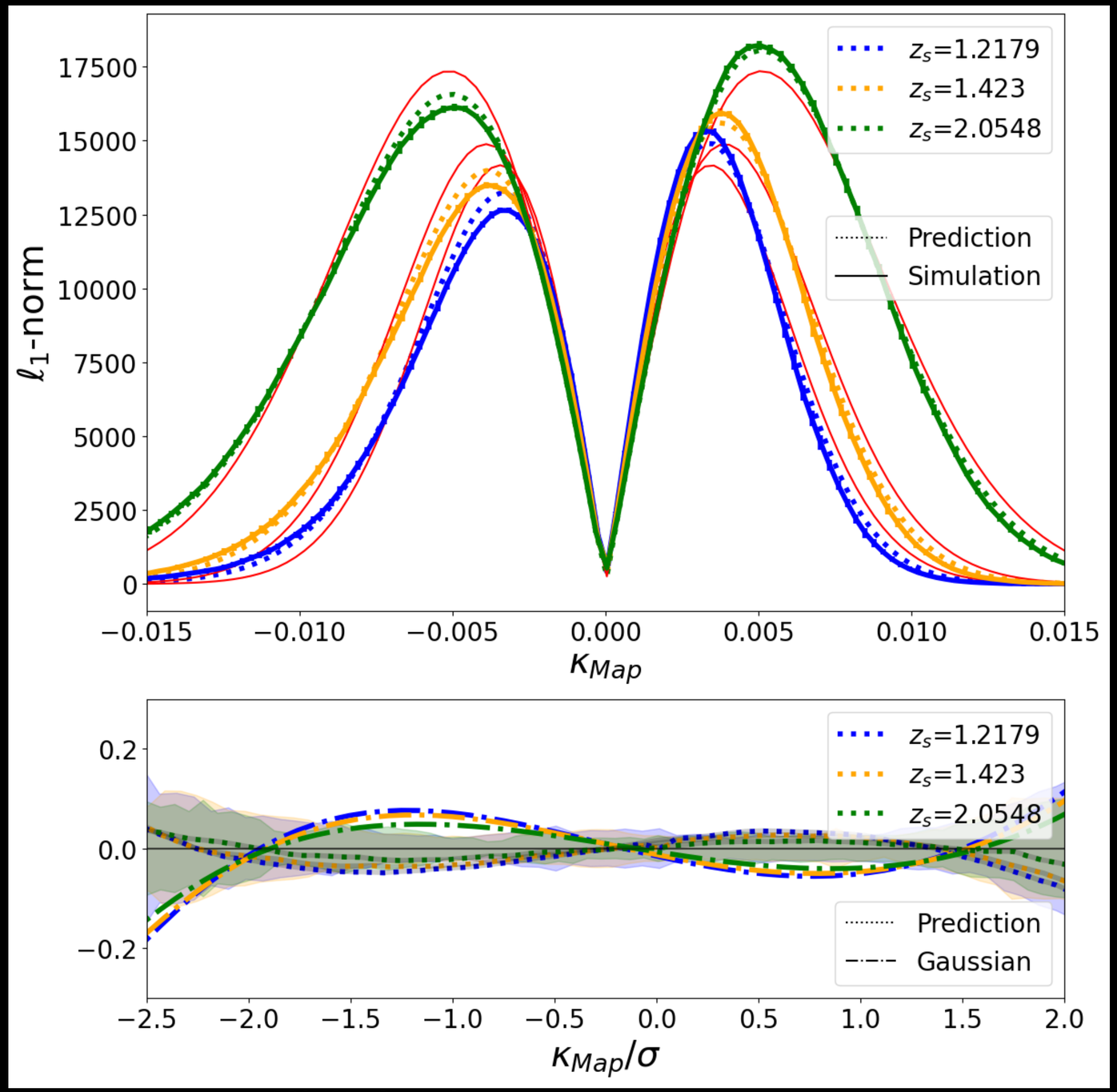
$$w_j = < \kappa, \varphi_{j+1} > - < \kappa, \varphi_j >$$

Apply this in the LDT framework to get the wavelet ℓ_1-norm of the wavelet coefficients w_j

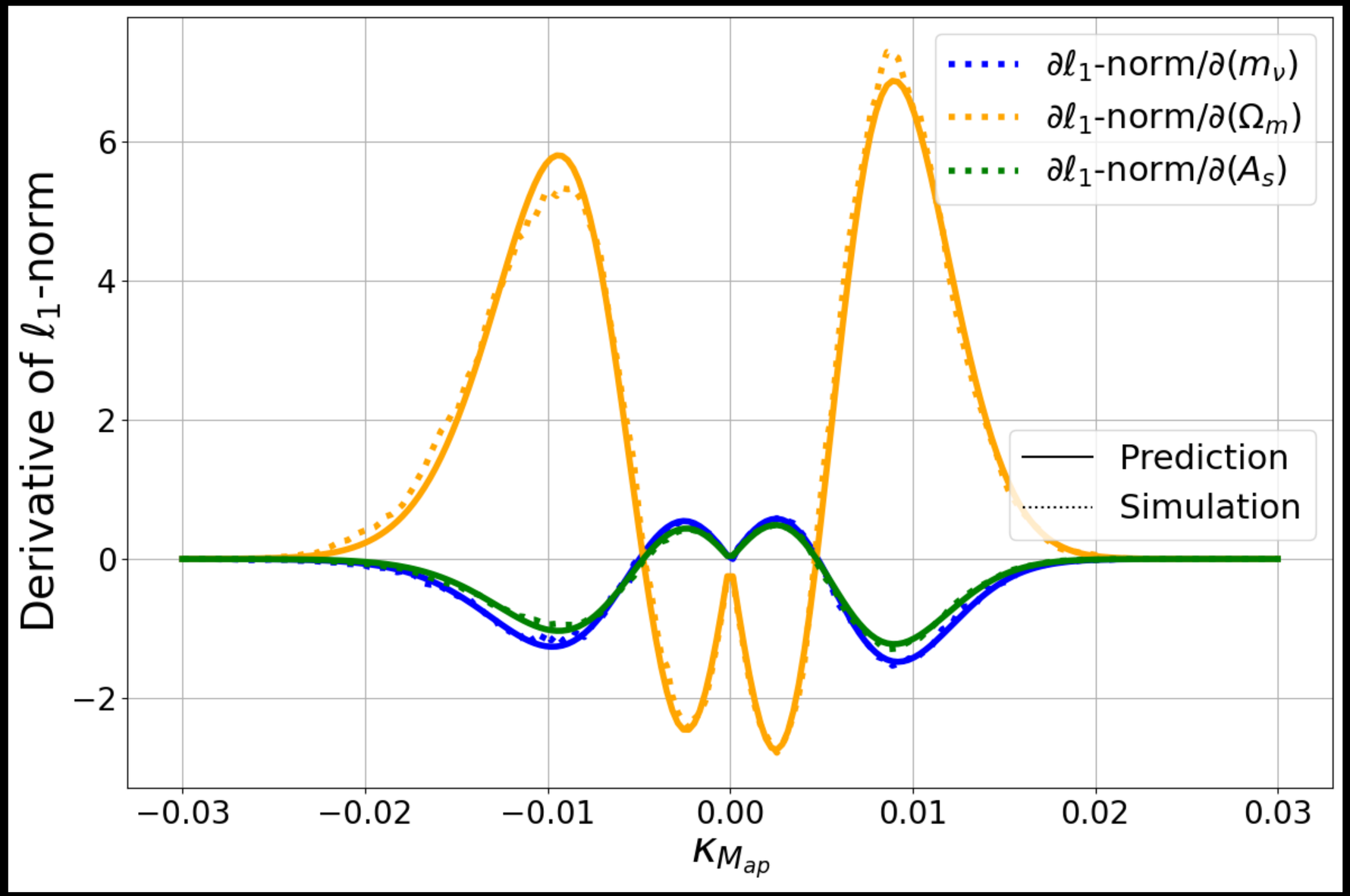
Using LDT first get the $P(w_j)$

$$\ell^{j,i}_{1,pred} = P_i(w_j) \times |B_i|$$ X scaling factor

$$\ell^{j,i}_{1,theory} = \sum_{u=1}^{\#coef(Sj,i)} |S_{j,i}[u]|$$



[Vilasini+, submitted]


$\partial\ell_1$-norm/$\partial(m_\nu)$
$\partial\ell_1$-norm/$\partial(\Omega_m)$
$\partial\ell_1$-norm/$\partial(A_s)$
Prediction
Simulation
Derivative of ℓ_1-norm
$\kappa_{M_{ap}}$
6
4
2
0
−2
−0.03
−0.02
−0.01
0.00
0.01
0.02
0.03

Conclusion

- We need different analytical methods to extract non-Gaussianities
  - Using Higher-Order statistics
  - Wavelet ℓ_1-norm is shown to be a better estimator in comparison to power spectrum, peaks and void statistics

Current methods use simulations based approach —> **Highly resource intensive**

- Need theoretical modelling

- Use LDT based approach to obtain the PDF for mass maps
  - Derived wavelet ℓ_1-norm from PDF

- **Currently working on: Extending to any wavelet filter and building an emulator with these constraints**

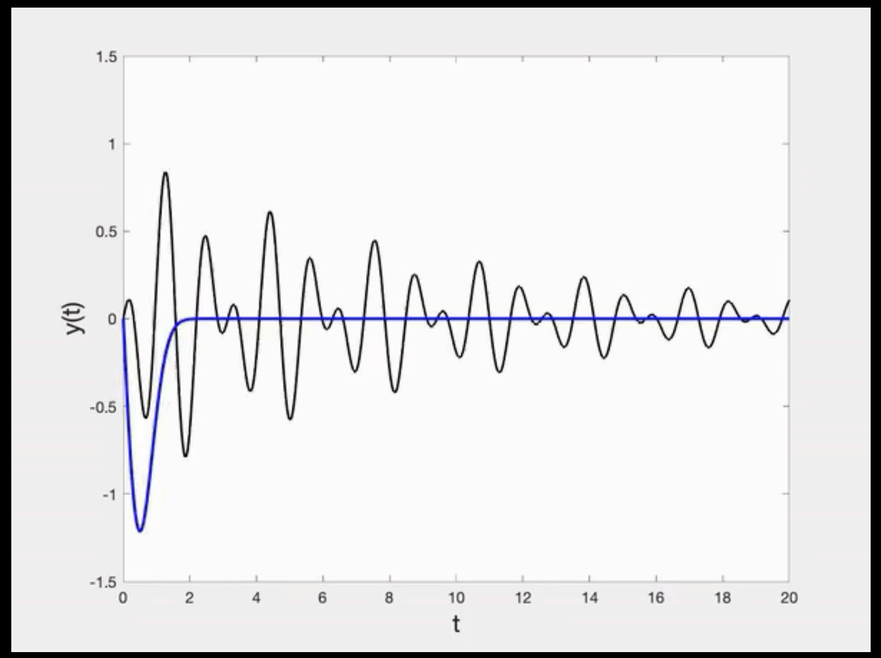
EXTRA SLIDES

Wavelets:

A set of mathematical function that is defined by the following properties:

- Highly localized in space/time
- Has a vanishing mean

  - A useful tool in analyzing signals where there are sharp spikes and discontinuities



The Continuous Wavelet Transform

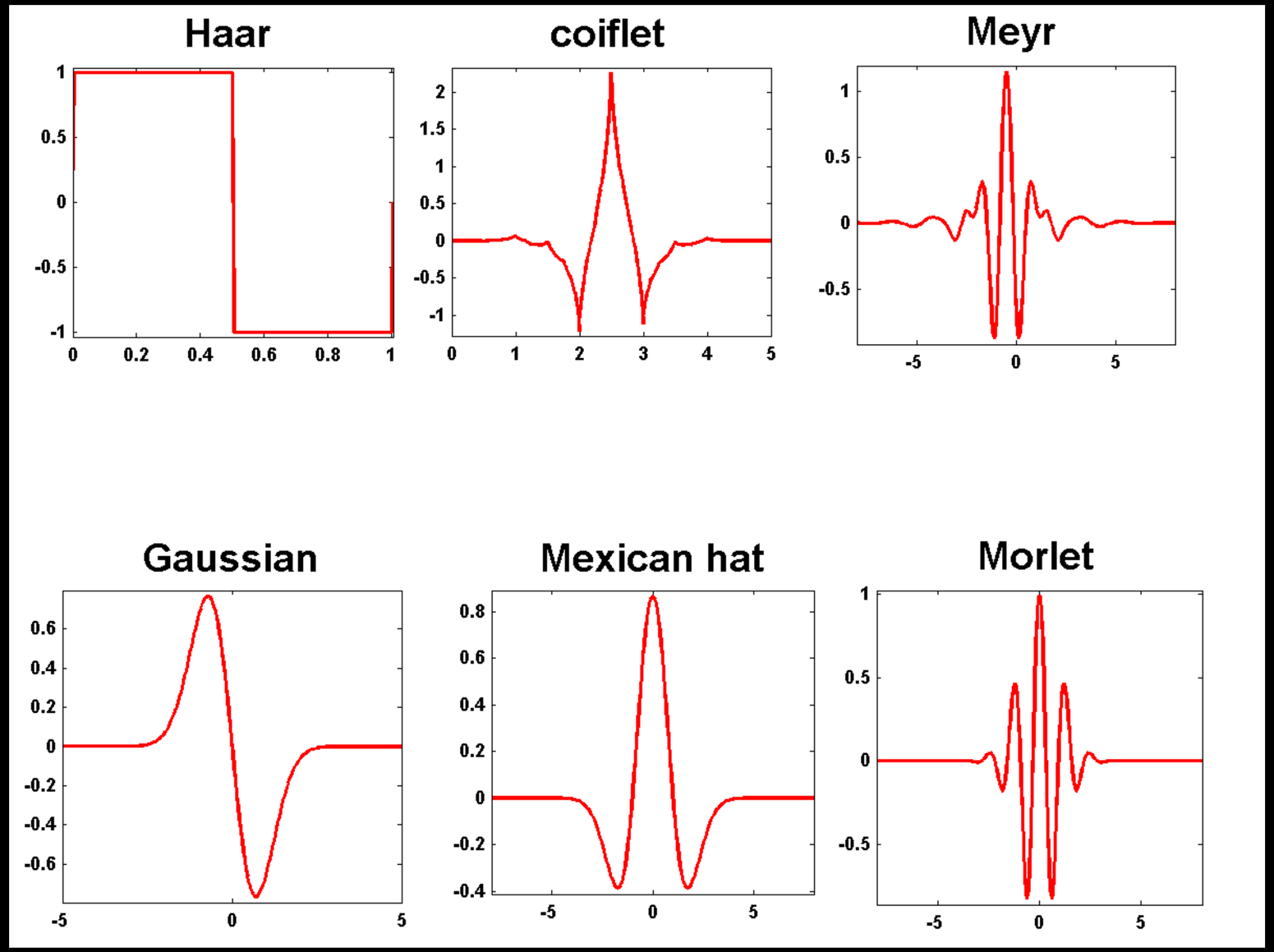
$$W(a,b) = K \int_{-\infty}^{+\infty} \psi^*(\frac{x-b}{a}) f(x) dx$$

where:

- $W(a,b)$ is the wavelet coefficient of the function $f(x)$
- $\psi(x)$ is the analyzing wavelet
- a (> 0) is the scale parameter
- b is the position parameter

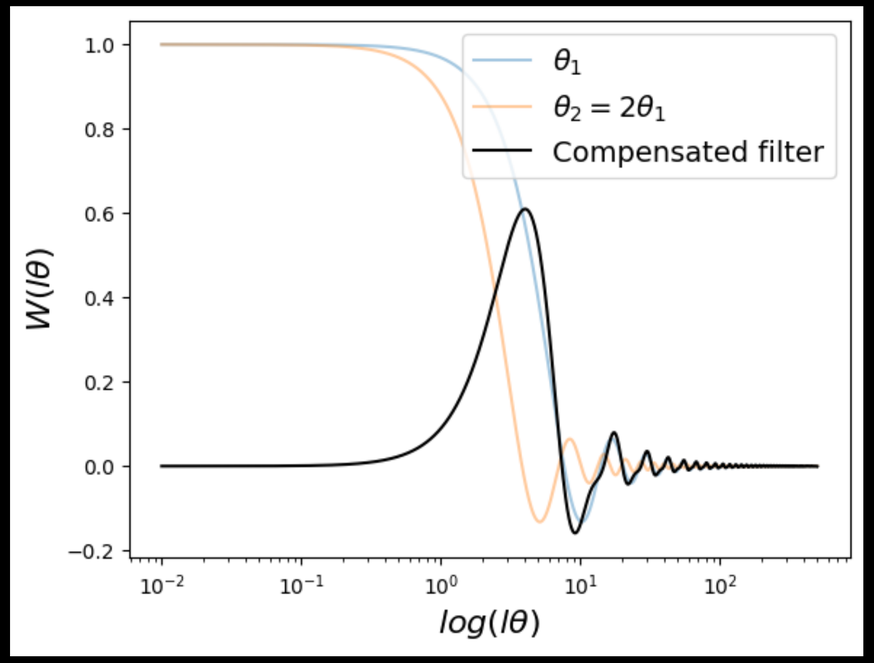
In Fourier space, we have: $\hat{W}(a,\nu) = \sqrt{a}\hat{f}(\nu)\hat{\psi}^*(a\nu)$
When the scale a varies, the filter $\hat{\psi}^*(a\nu)$ is only reduced or dilated while keeping the same pattern.

Some typical mother wavelets



Leonard et al. (2012)

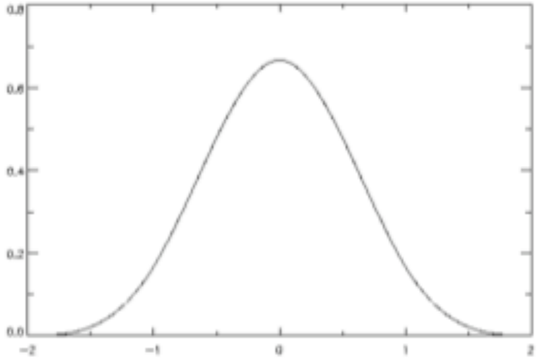
In this work:

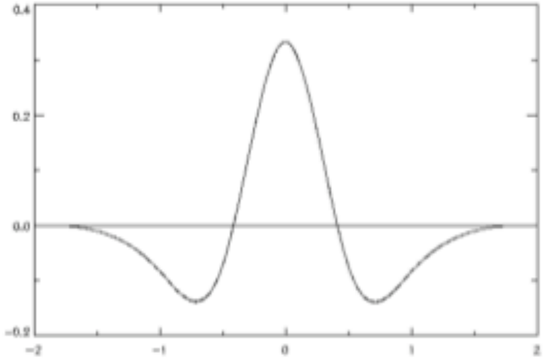


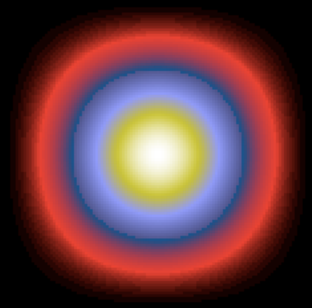
Wavelet Transform in Astronomy

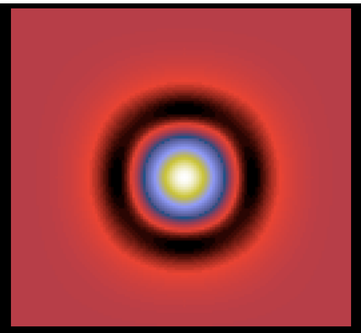
The Isotropic Wavelet and Scaling Functions

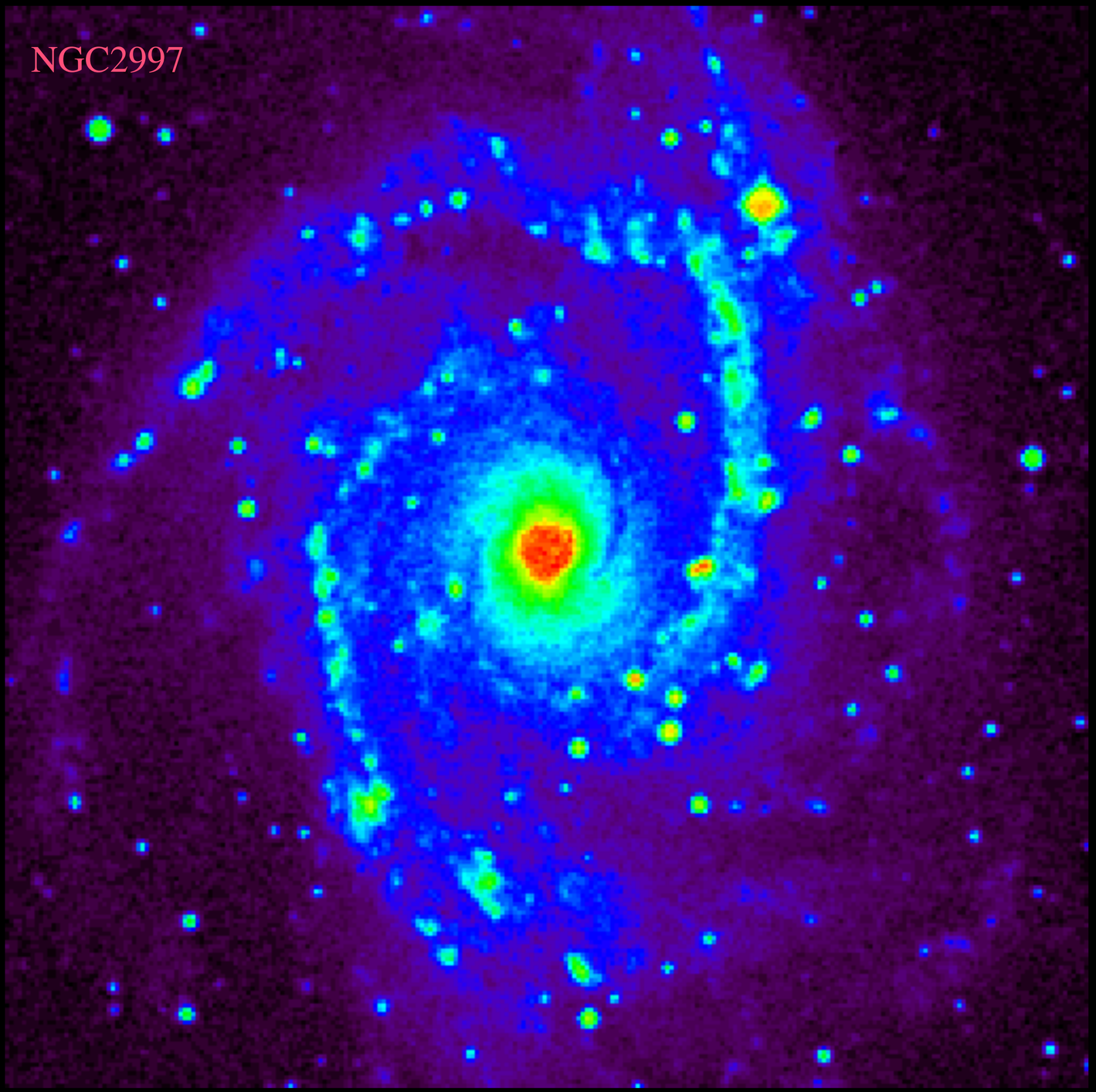
$$
\begin{aligned}
B_3(x) &= \frac{1}{12}(|x-2|^3 - 4|x-1|^3 + 6|x|^3 - 4|x+1|^3 + |x+2|^3) \\
\psi(x,y) &= B_3(x)B_3(y) \\
\frac{1}{4}\psi(\frac{x}{2},\frac{y}{2}) &= \phi(x,y) - \frac{1}{4}\phi(\frac{x}{2},\frac{y}{2})
\end{aligned}
$$

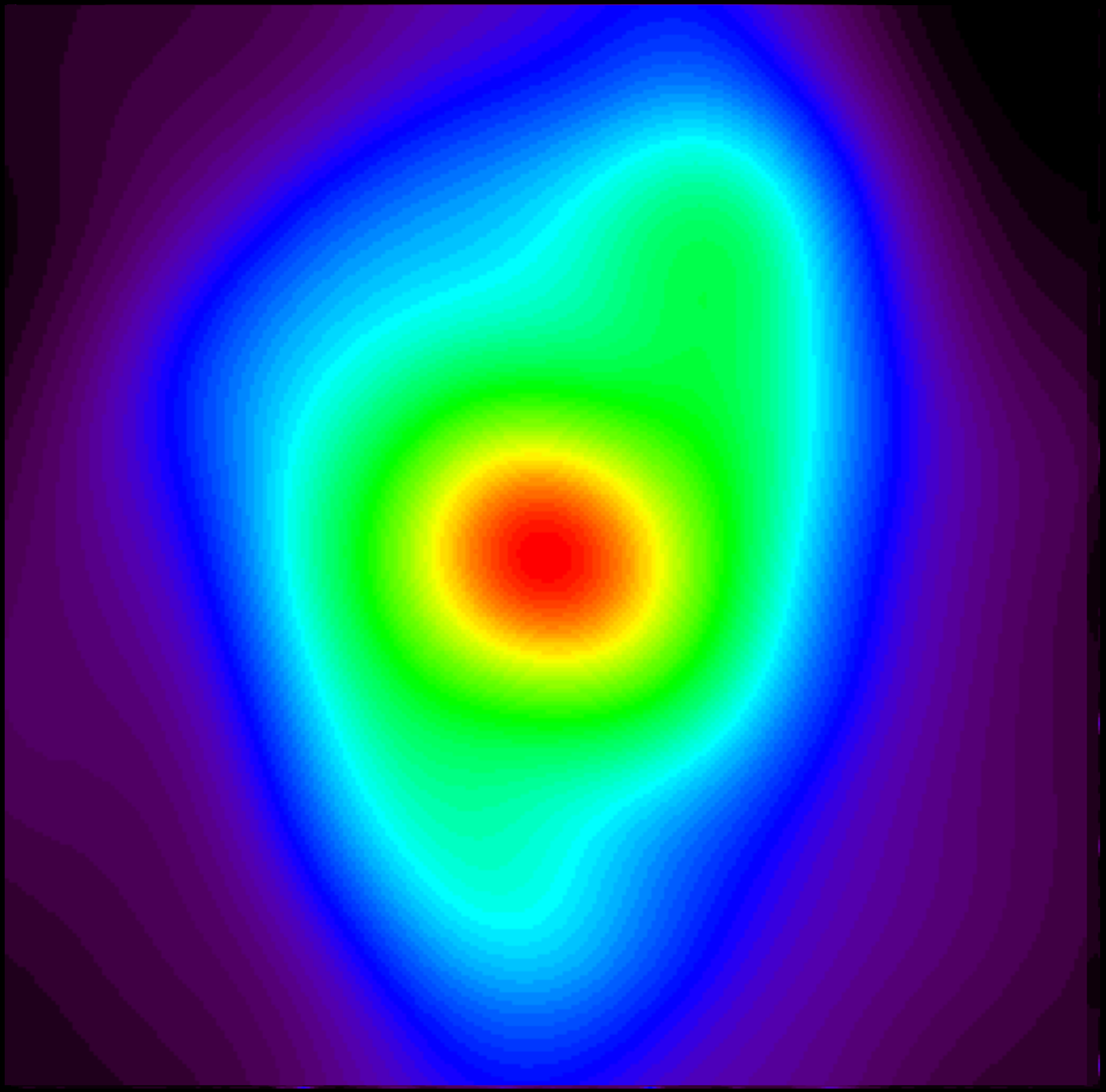






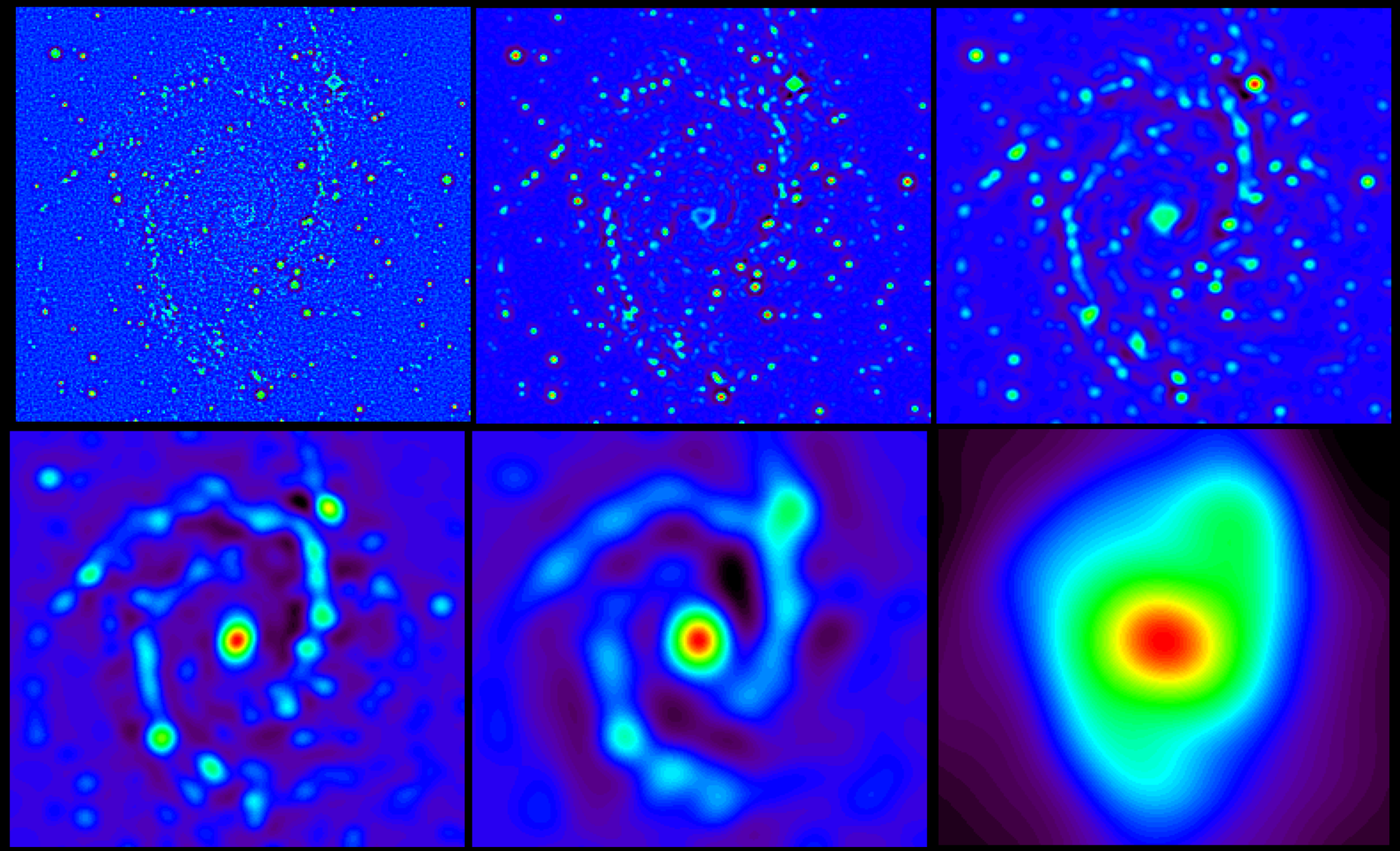



NGC2997



The STARLET Transform

Isotropic Undecimated Wavelet Transform (a trous algorithm)



Aperture Mass and Wavelets

⇒ Wavelets filters are formally **indentical** to Mass aperture

A. Leonard et al, "Fast Calculation of the Weak Lensing Aperture Mass Statistic", **MNRAS**, 423, pp 3405-3412, 2012.

but wavelets presents several advantages:
- compensated and **compact** support filters
- **all scales** processed in one step.
- **reconstruction** is possible
  ==> image restoration for peak counting

Fast calculation for both aperture and wavelet approaches if we grid the shear data and use the FFT.