The scattering transform: a CNN without training

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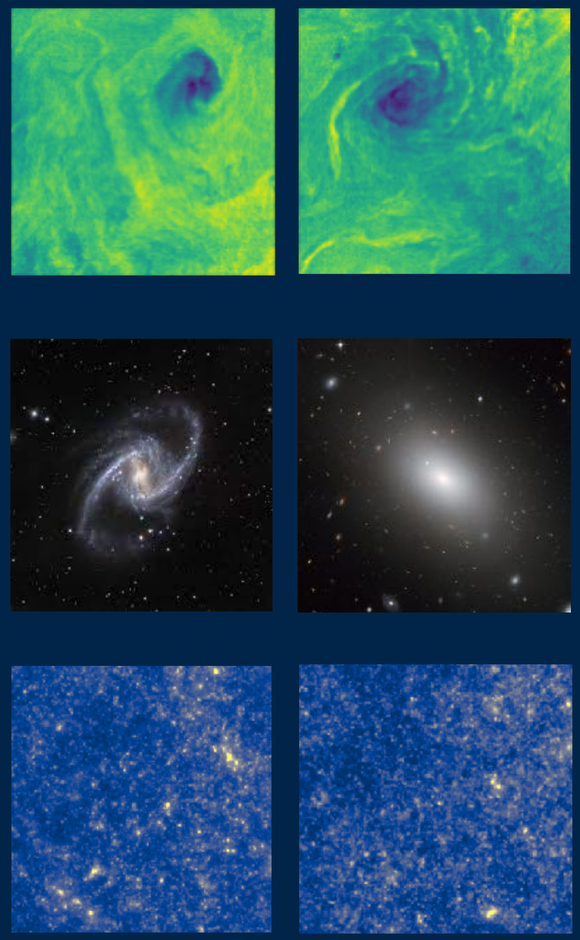
complex
data



a vocabulary

power spectrum P(k)?

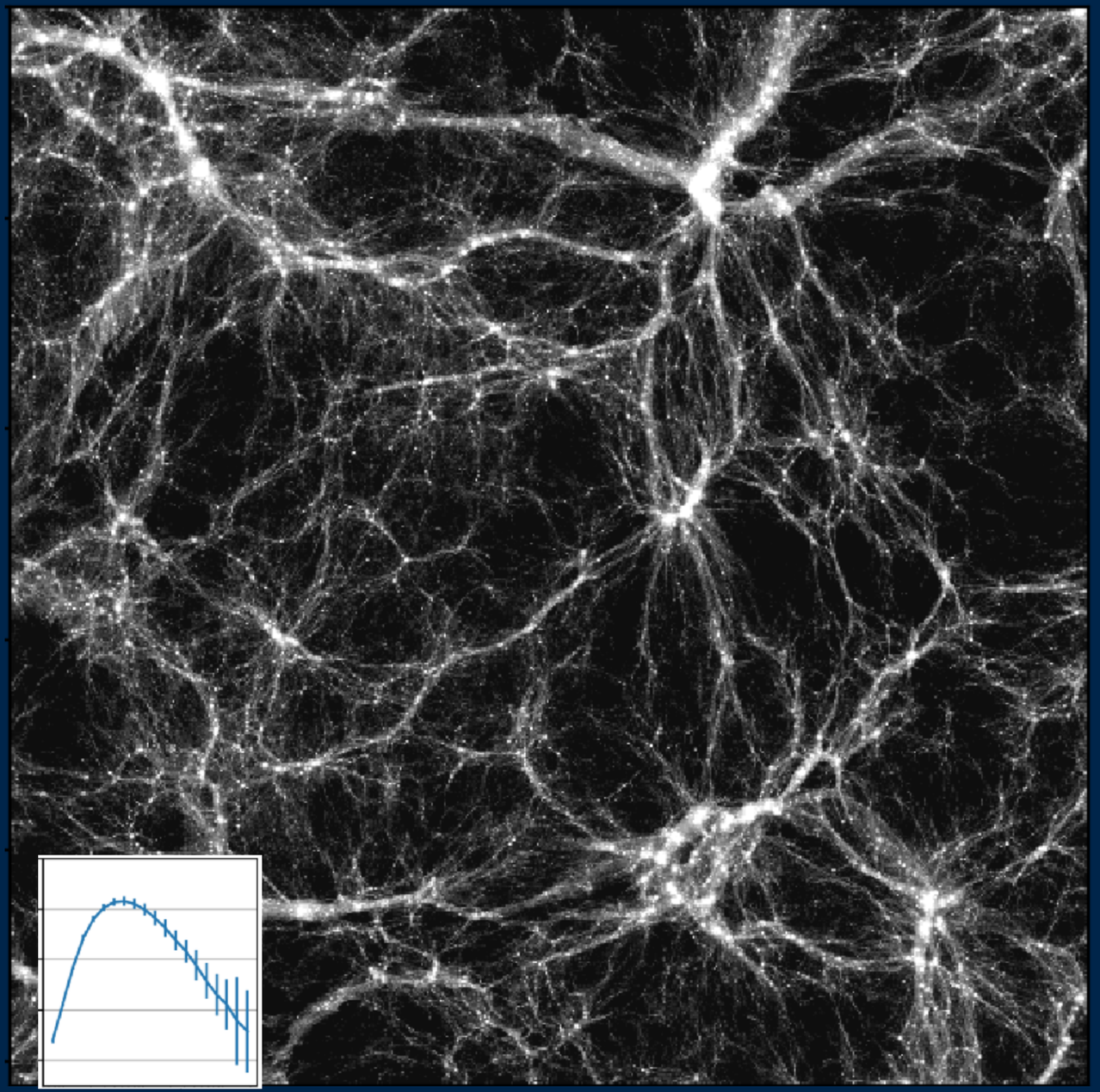
- informative
- robust
- compact

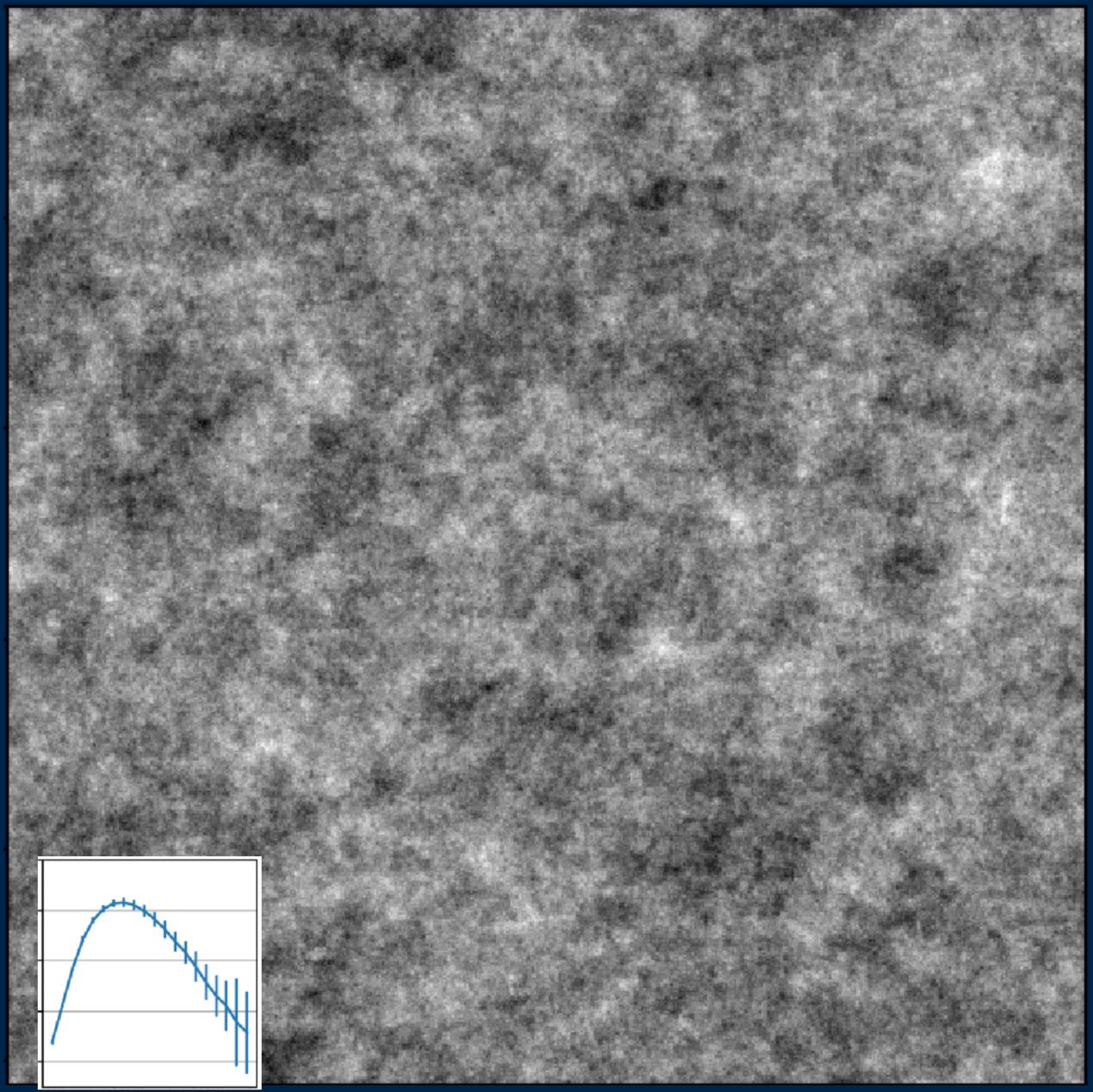
simple
information

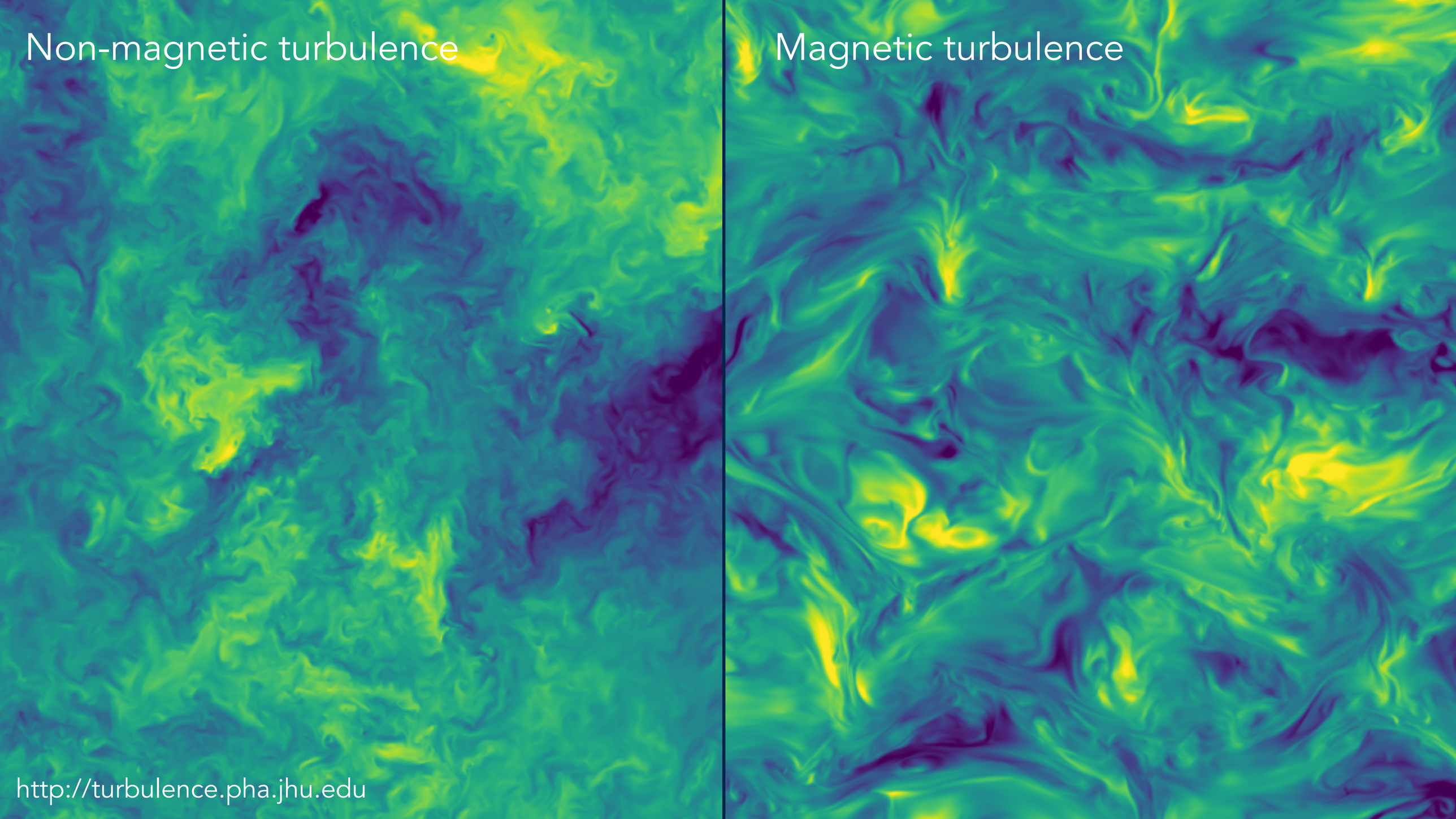
data exploration
no model

classification
discrete model

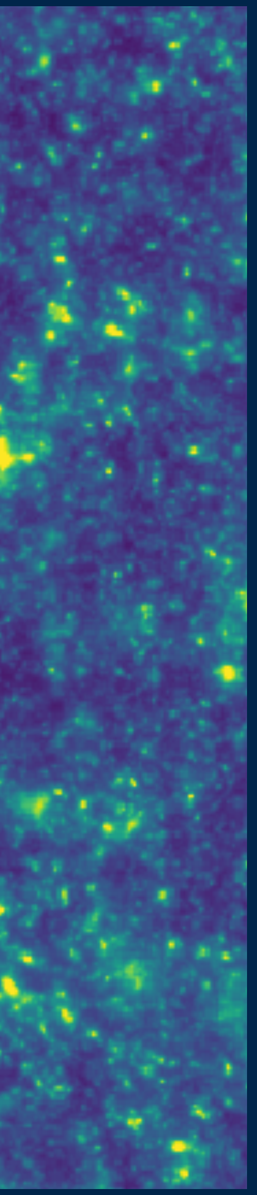
parameter inference
continuous model






Non-magnetic turbulence
Magnetic turbulence
http://turbulence.pha.jhu.edu

How do we characterize a field?



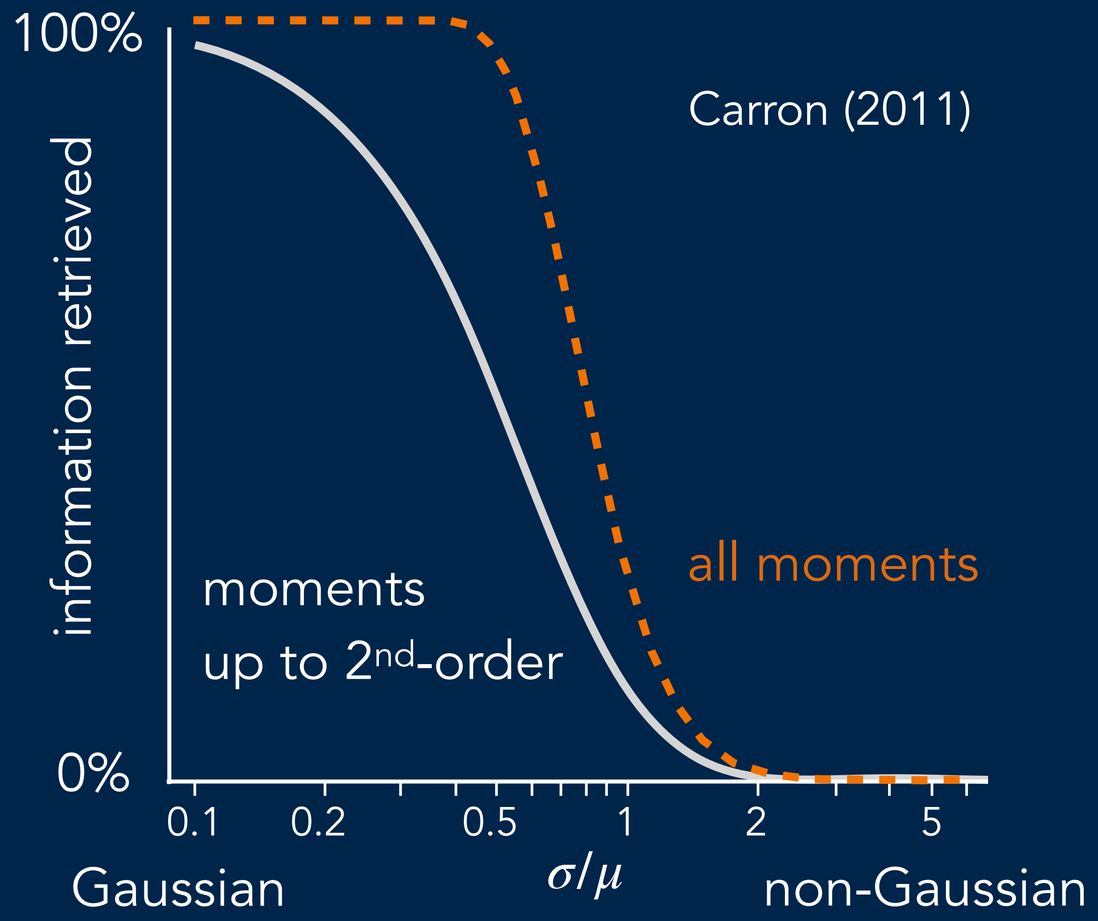
$\langle \delta_1 \delta_2 \ldots \delta_n \rangle$

power spectrum

plus high-order statistics

information

log-normal variable

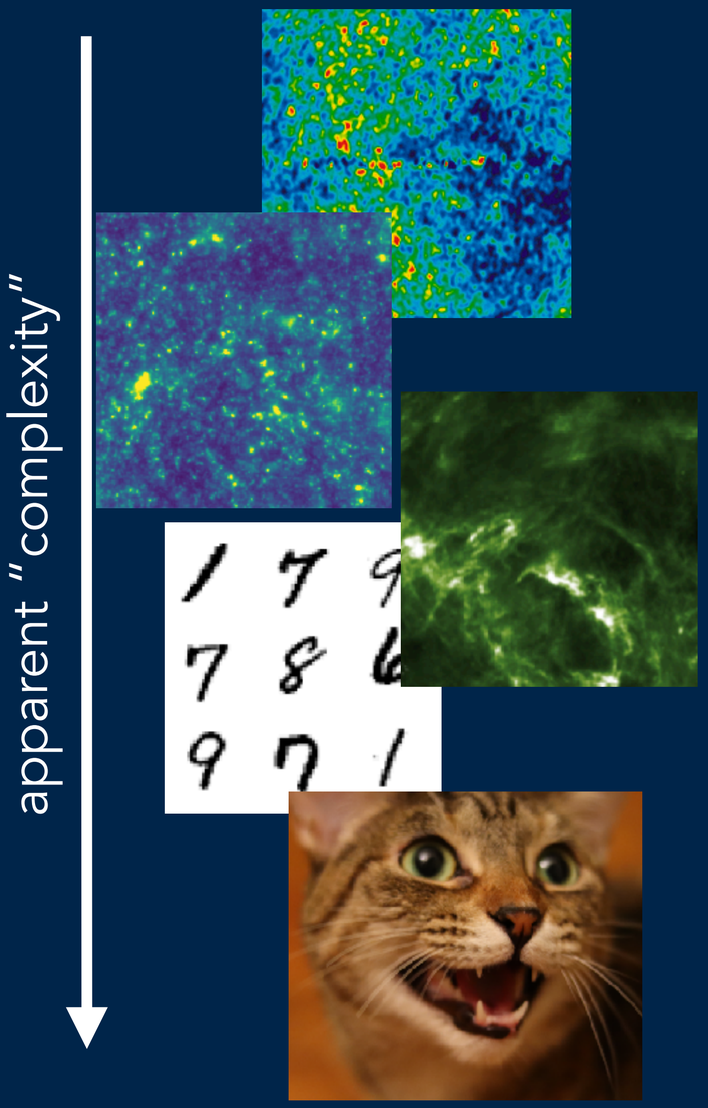


Carron (2011)

Limitations:

- lose information
- too many coefficients

How do we characterize a field?

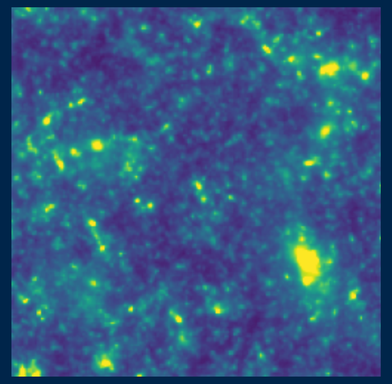


power spectrum

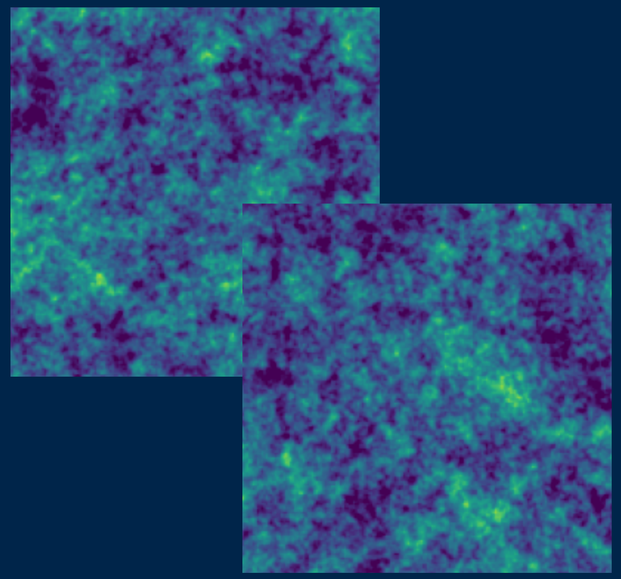
scattering transform

CNN

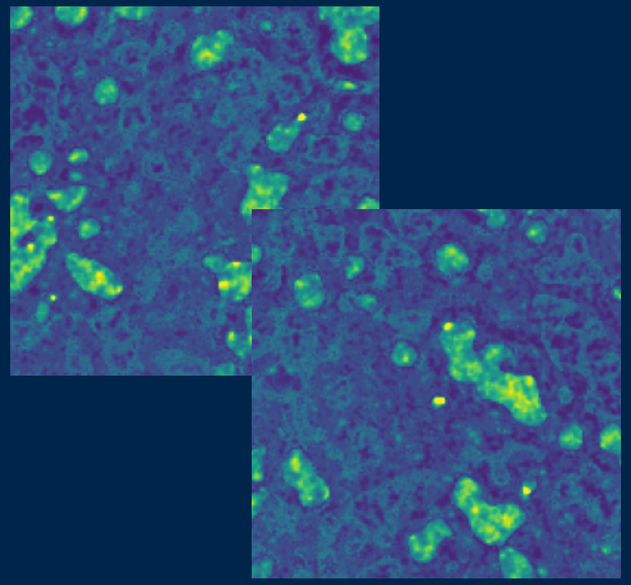
cosmic density map



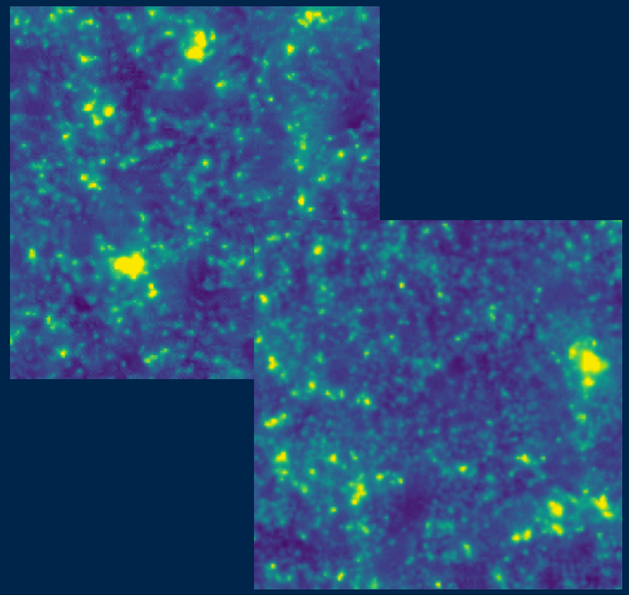
2-order statistics



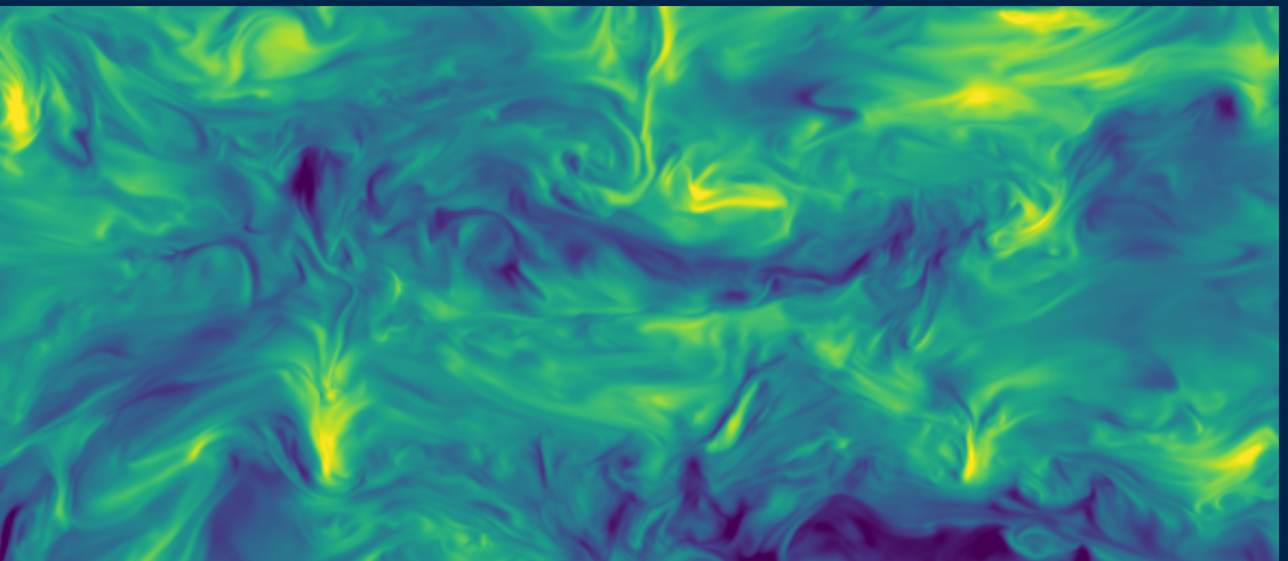
3-order statistics



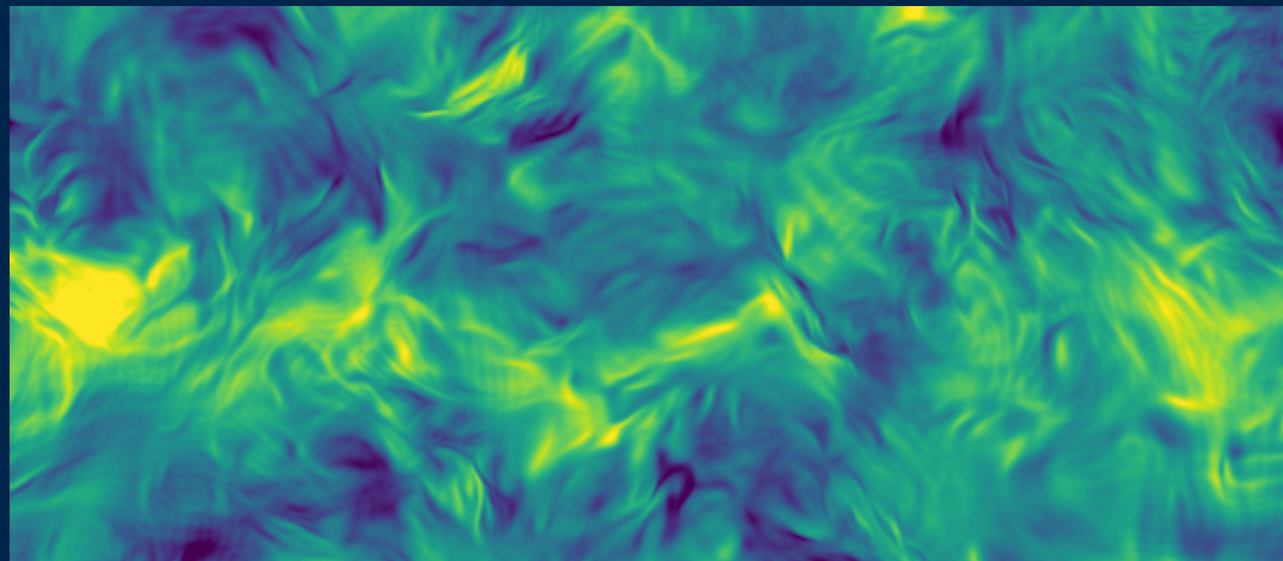
scattering statistics



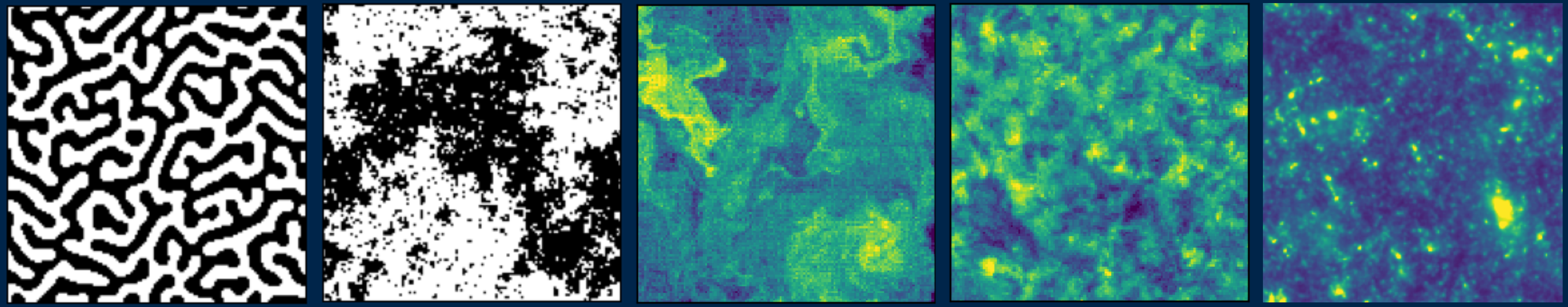
slices of 3D MHD simulation



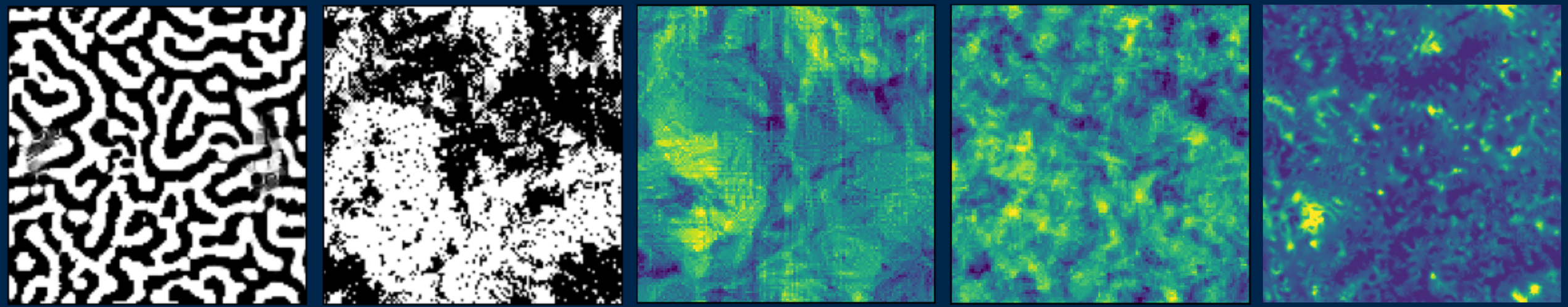
Synthesized with scattering statistics



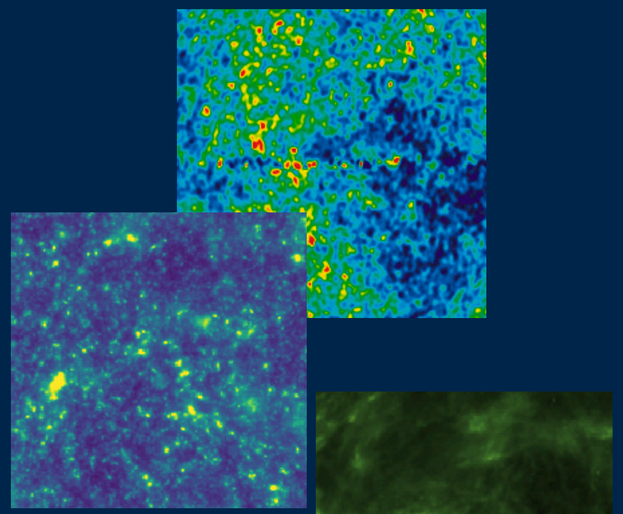
Turing pattern | Ising model | sea temperature | solar UV image | cosmic matter



generated with scattering statistics (translation invariant)



How do we characterize a field?



power spectrum

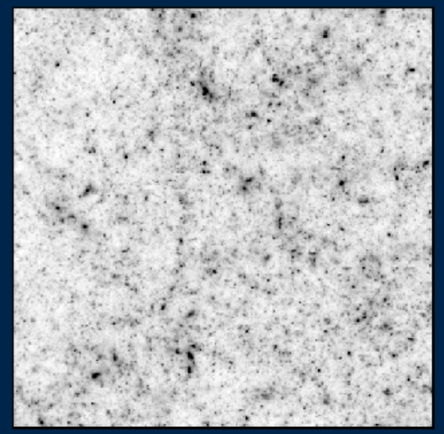
scattering transform

CNN

from power spectrum to scattering transform

$$P(k) \propto \left\langle \left| I \star e^{ikx} \right|^2 \right\rangle$$

$$\left\langle \cdot^2 \right\rangle = P(k)$$

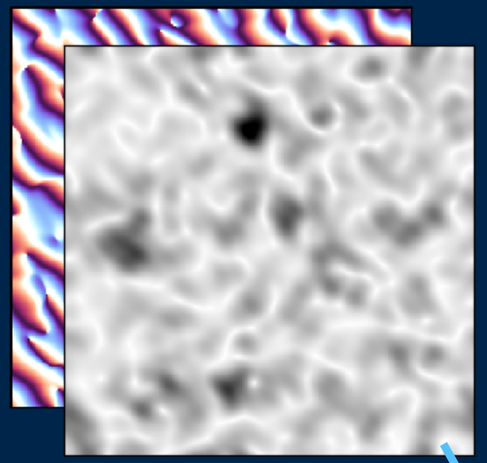


Fourier mode e^{ikx}

phase

modulus

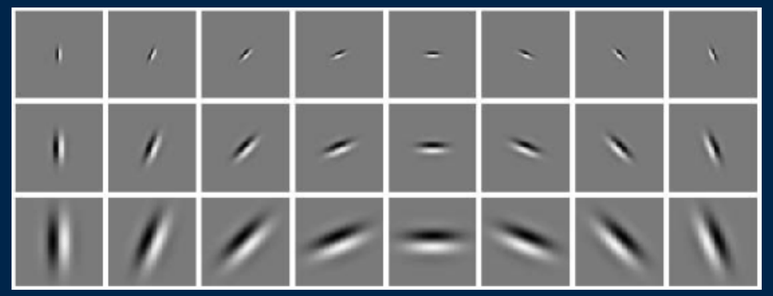
local kernel $\psi(x)$



$$S_1(k) = \left\langle \left| I \star \psi \right| \right\rangle$$

$$\left\langle \cdot \right\rangle = S_1(k)$$

convolutional network



scale $j \approx 1/k$

orientation $l \approx \theta$

I

scales 1

$I_1 = |I \star \psi|$

scales 2

$I_2 = ||I \star \psi_1| \star \psi_2|$

$\cdots$

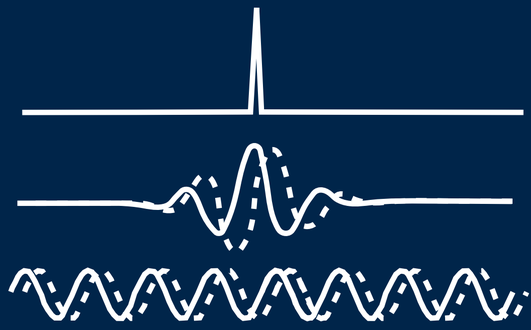
S_0

$S_1(j, l)$

$S_2(j_1, l_1, j_2, l_2)$

wavelets and sparsity

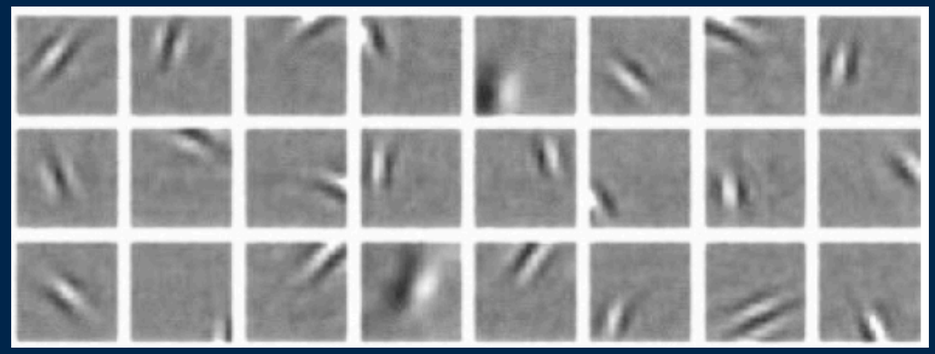
Dirac
wavelet
Fourier



vision cells in brain
(Hubel & Wiesel 1968)

close to Gabor wavelets

sparse representation of natural images
(Olshausen & Field 1996)

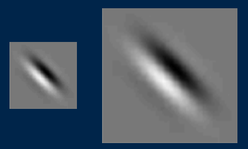


kernels learned in AlexNet
(Krizhevsky, Sutskever, & Hinton 2012)

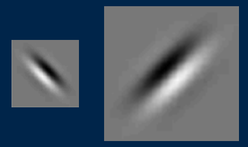
interpretation

modification of 2, 3, 4-point: log bin + stable non-linearity

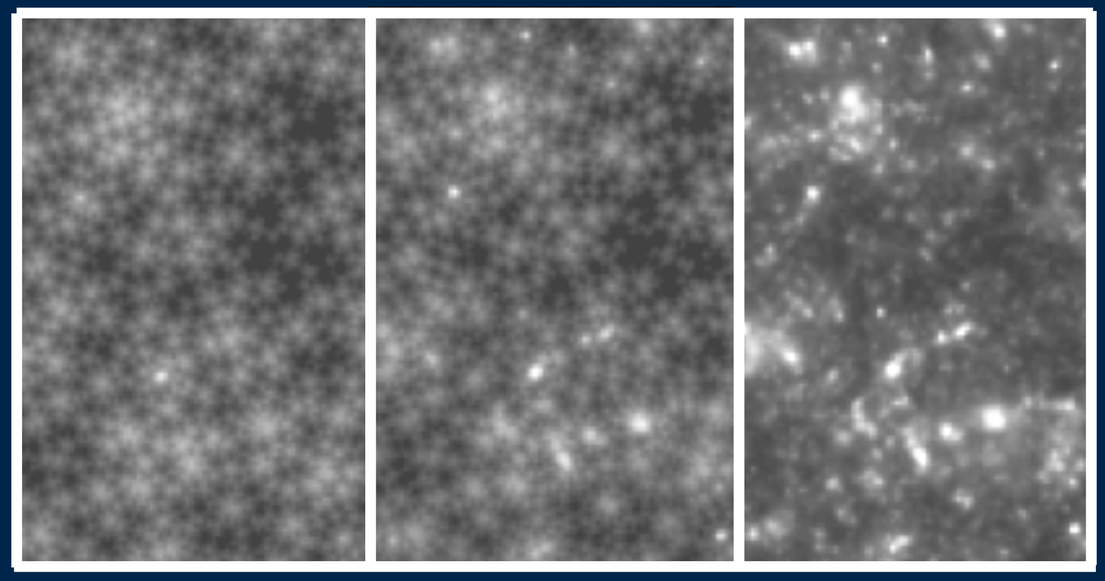
$l_1 \parallel l_2$:



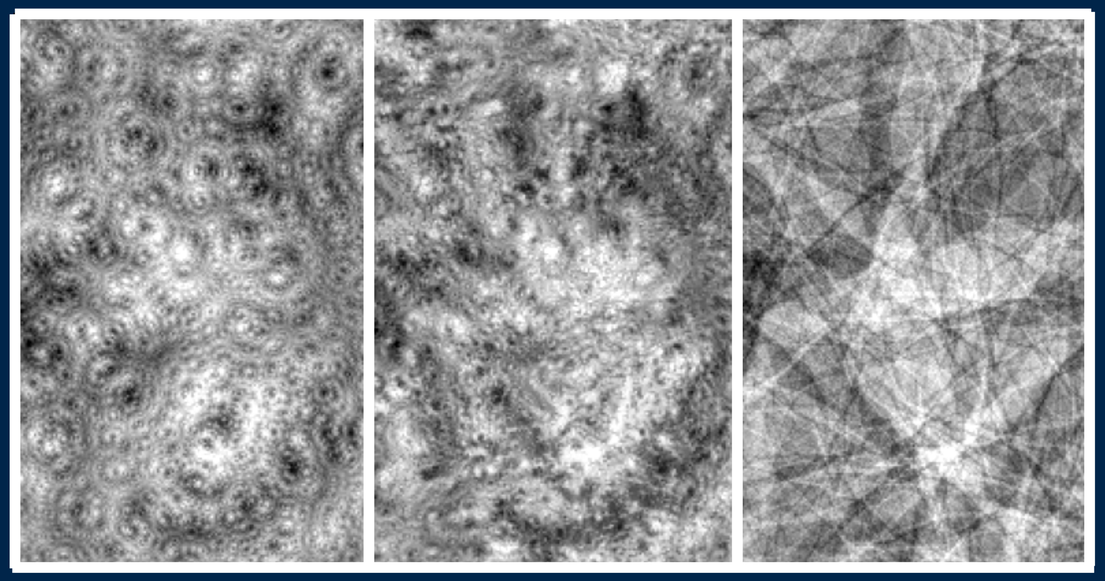
$l_1 \perp l_2$:



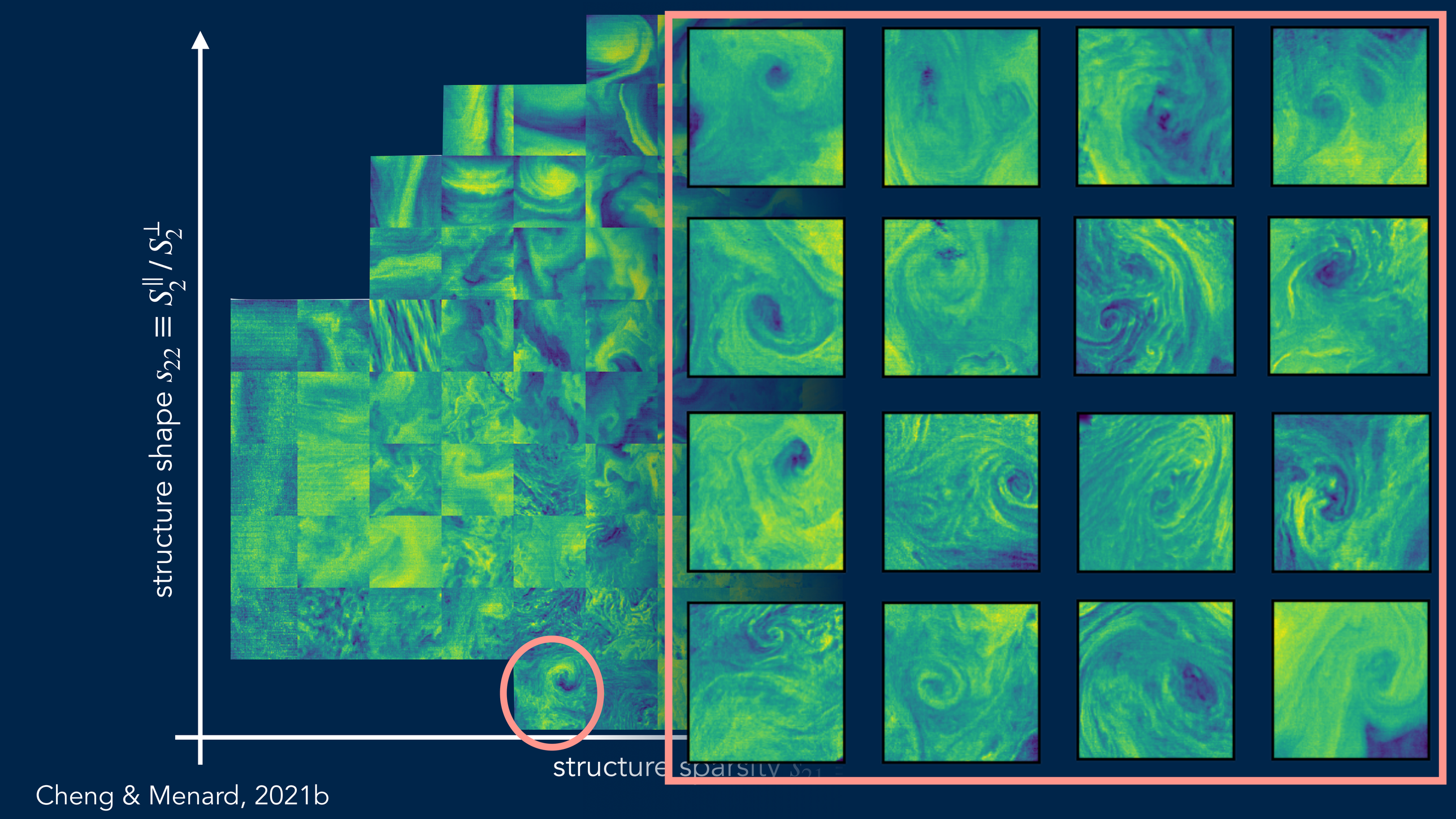
structure sparsity $s_{21} \equiv \overline{S_2} / S_1$



structure shape $s_{22} \equiv S_2^{\parallel} / S_2^{\perp}$

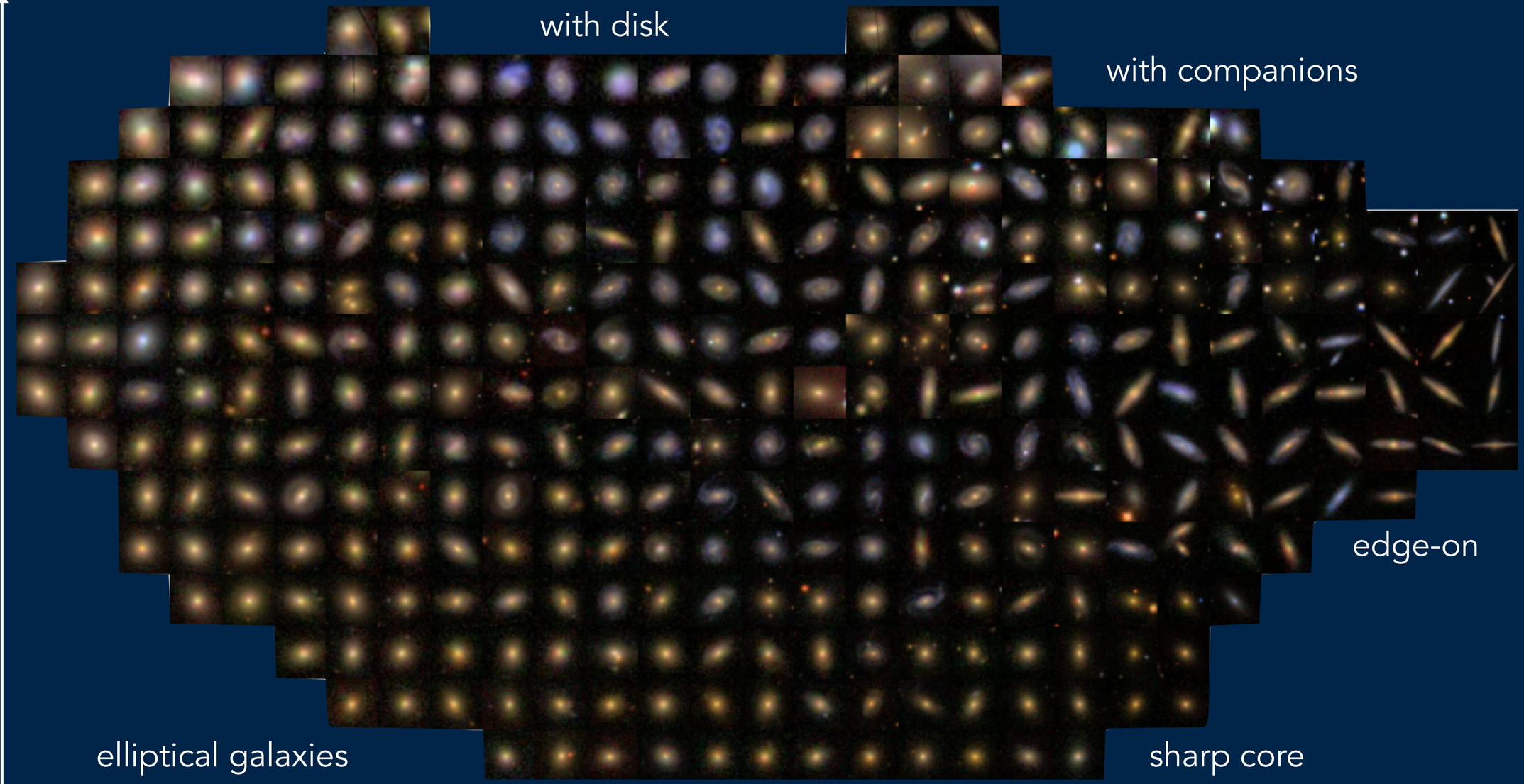


Cheng & Menard, 2021b



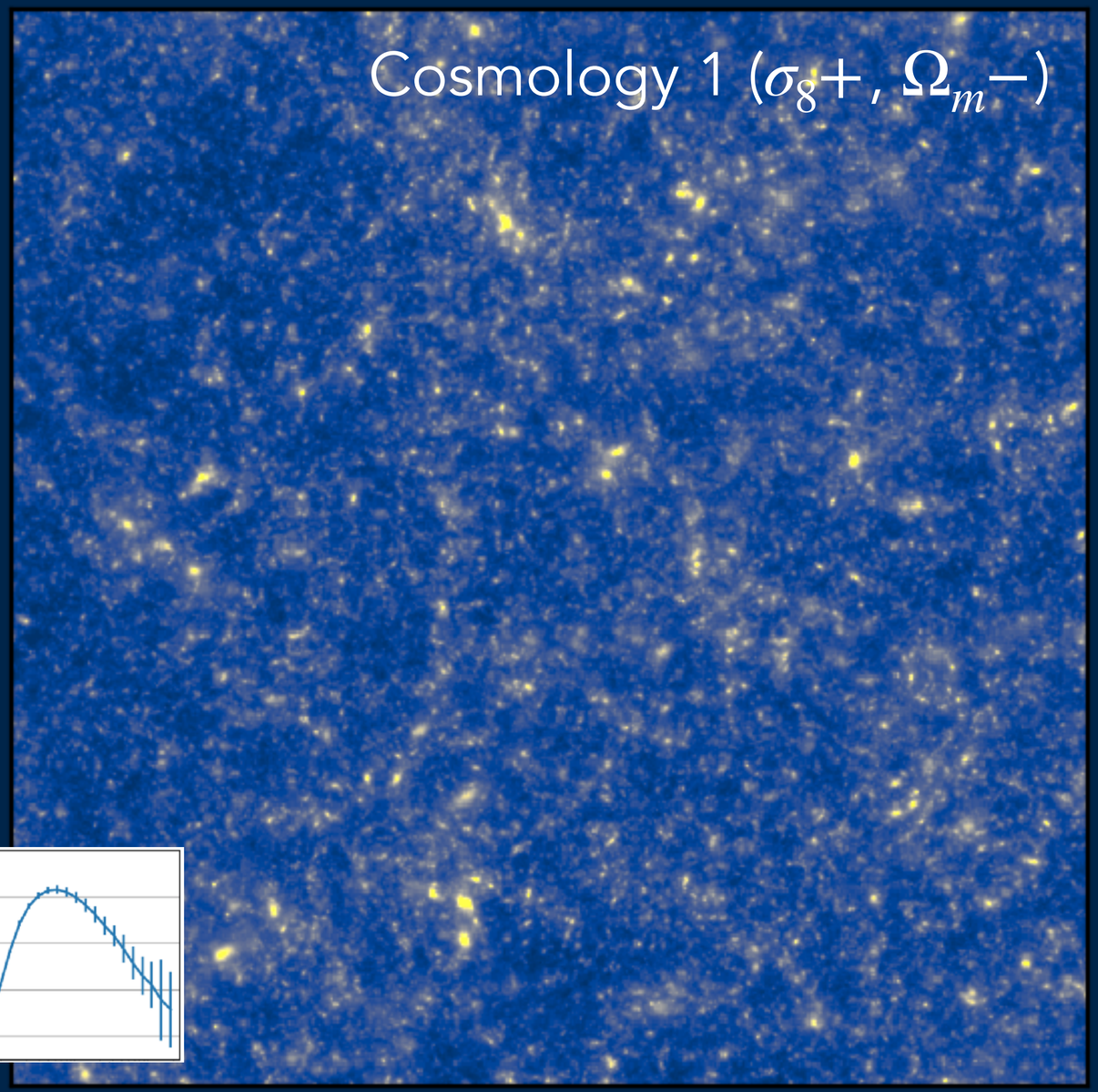
Cheng & Menard, 2021b

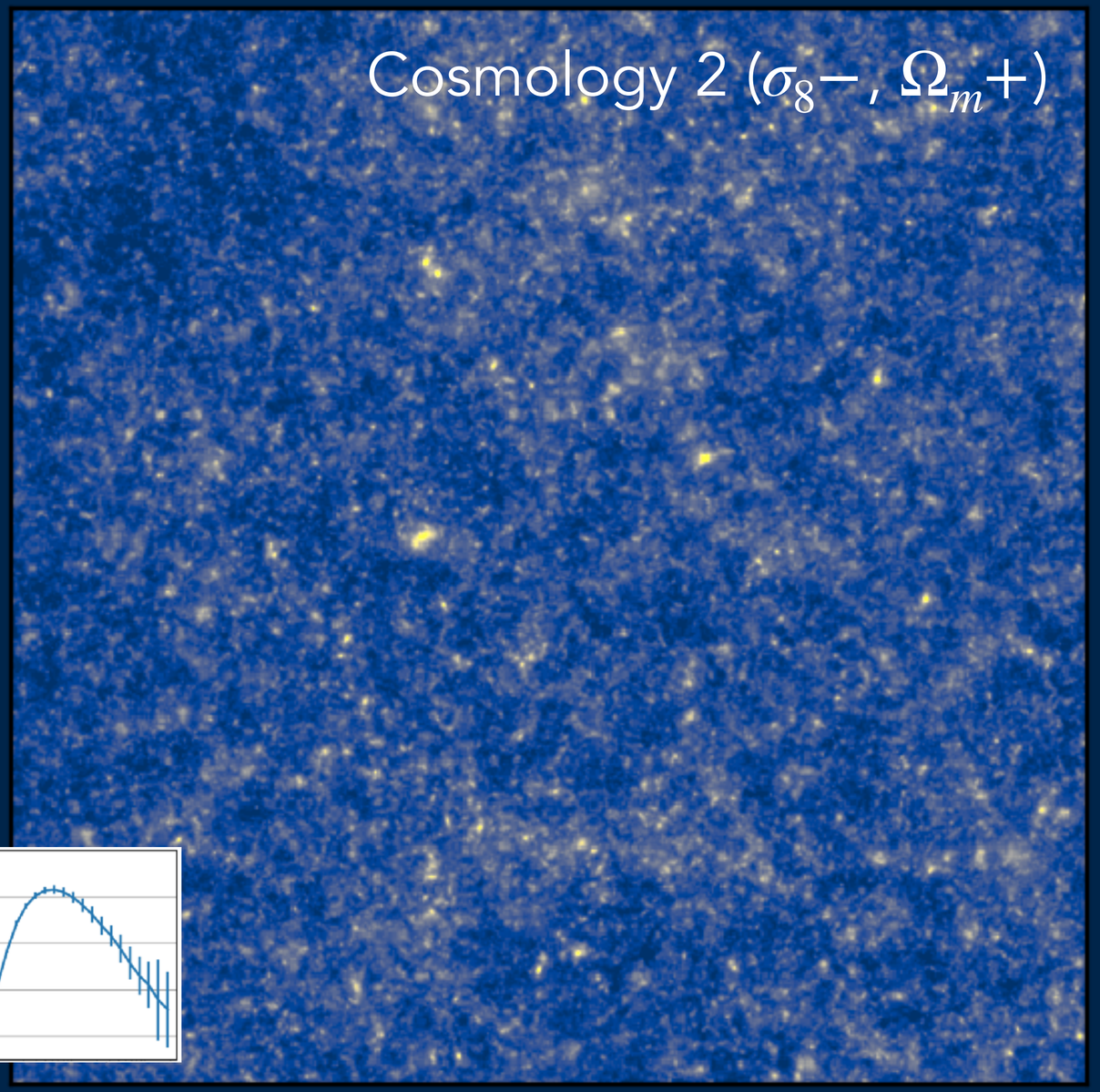
arranged by 2nd-order scattering coefficients



Cheng & Menard, 2021b

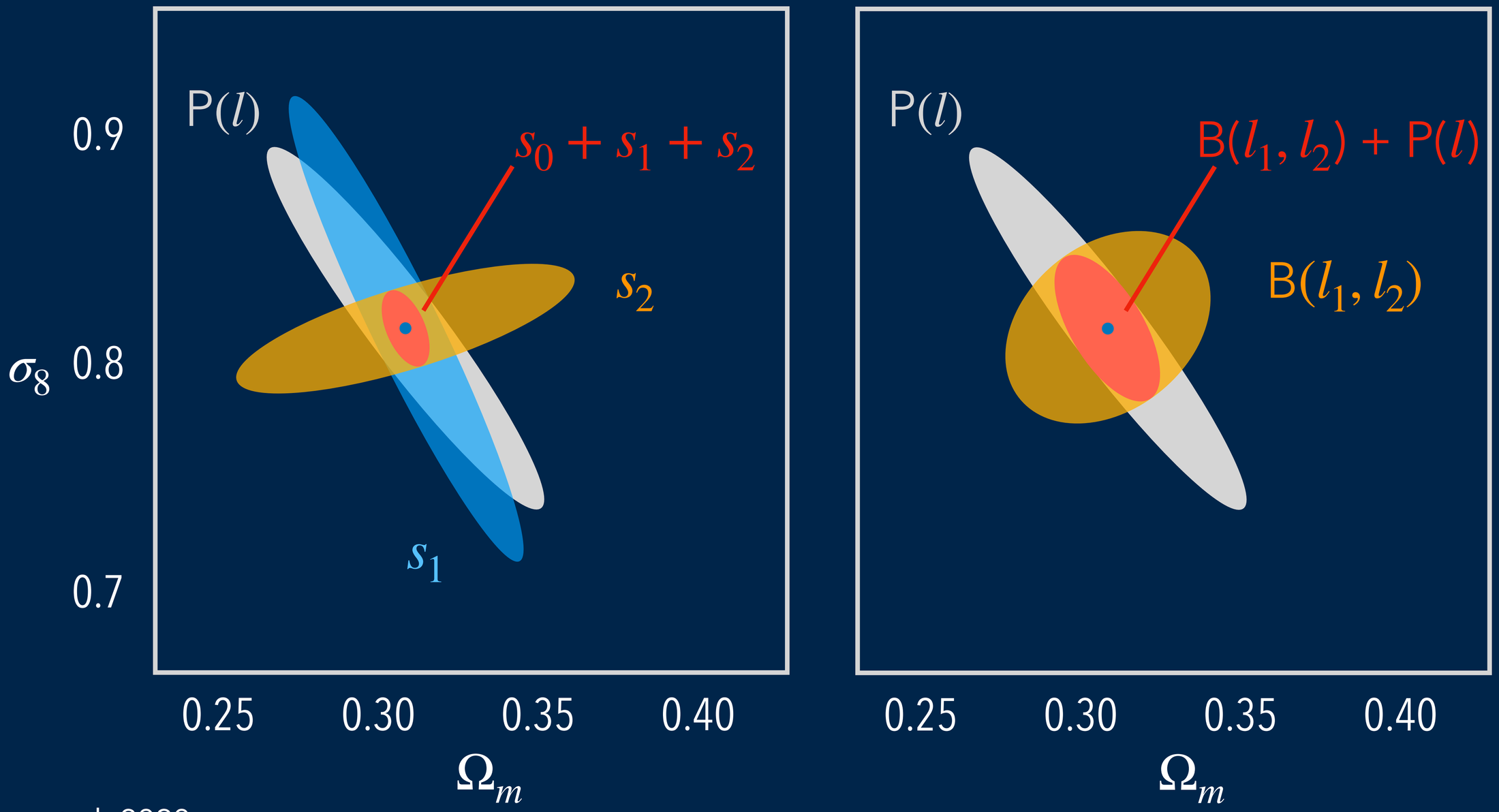
cosmological simulations





Matilla Zorrilla et al. 2016, Gupta et al. 2018 (Columbia lensing group)

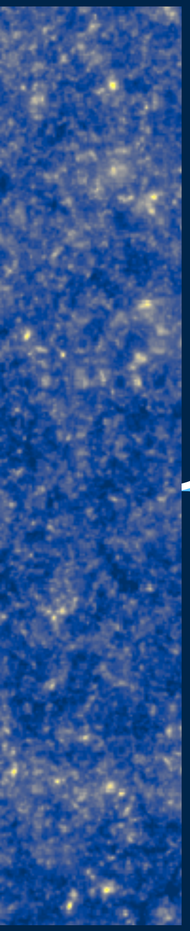
inferring cosmological parameters



Cheng et al. 2020

inferring cosmological parameters

- informative
- compact
- robust



$P(l)$

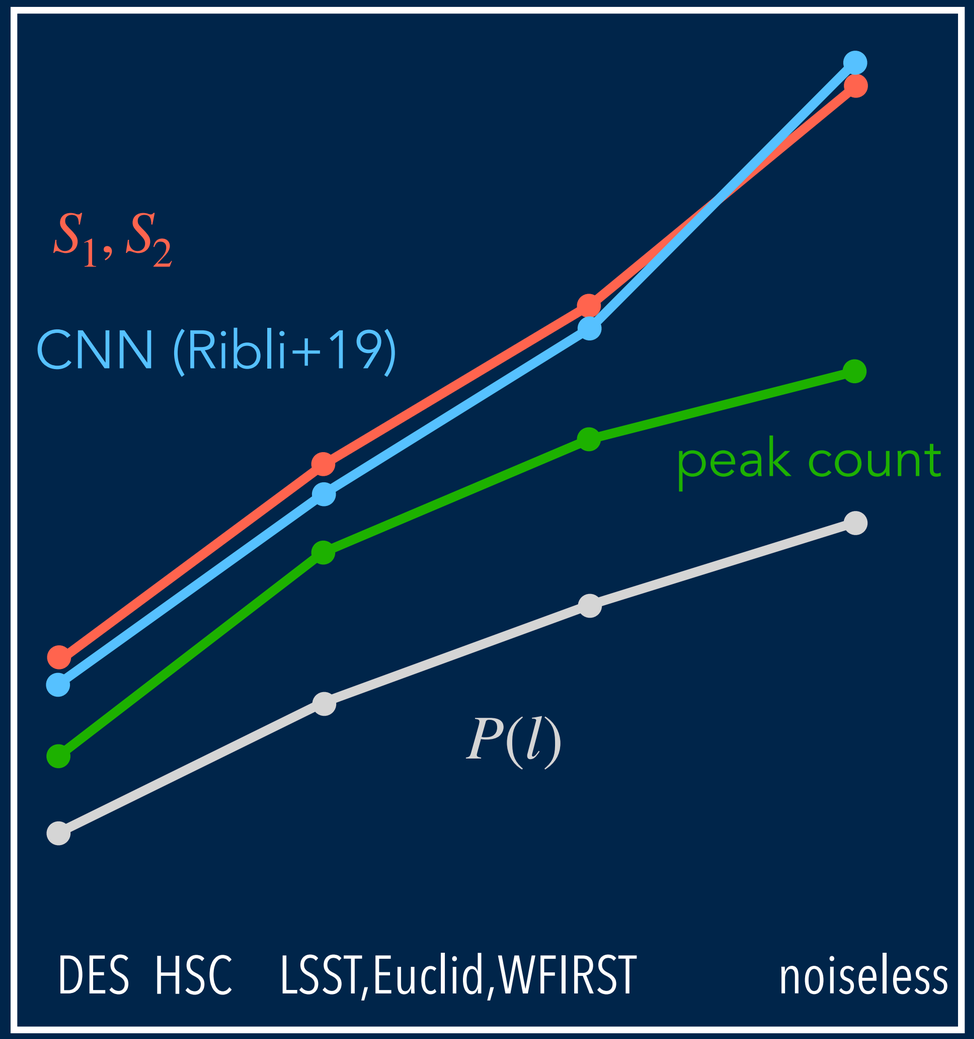
~20 coefficients

scattering coefficients

37 coefficients

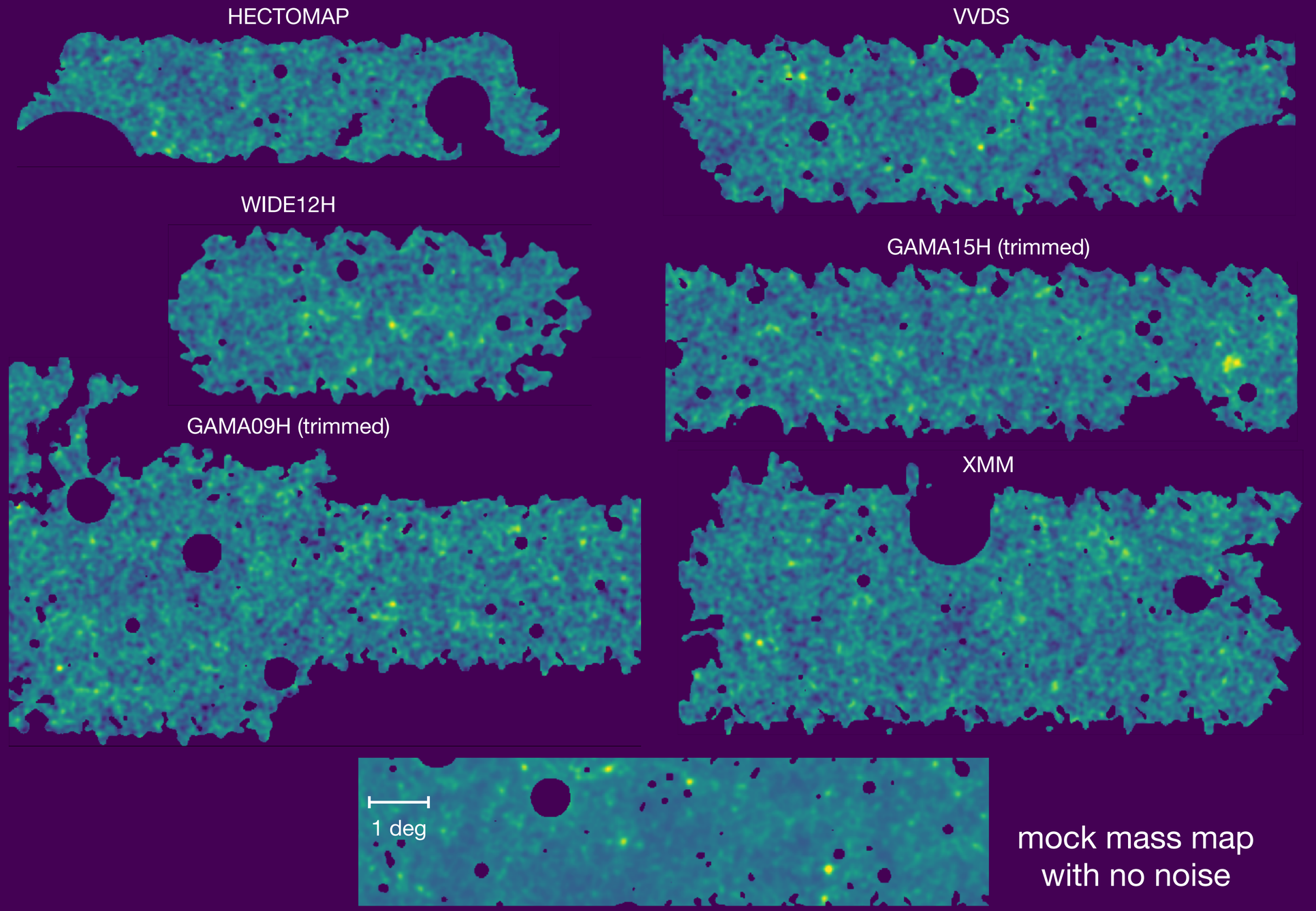
CNN

millions



Cheng et al. 2020

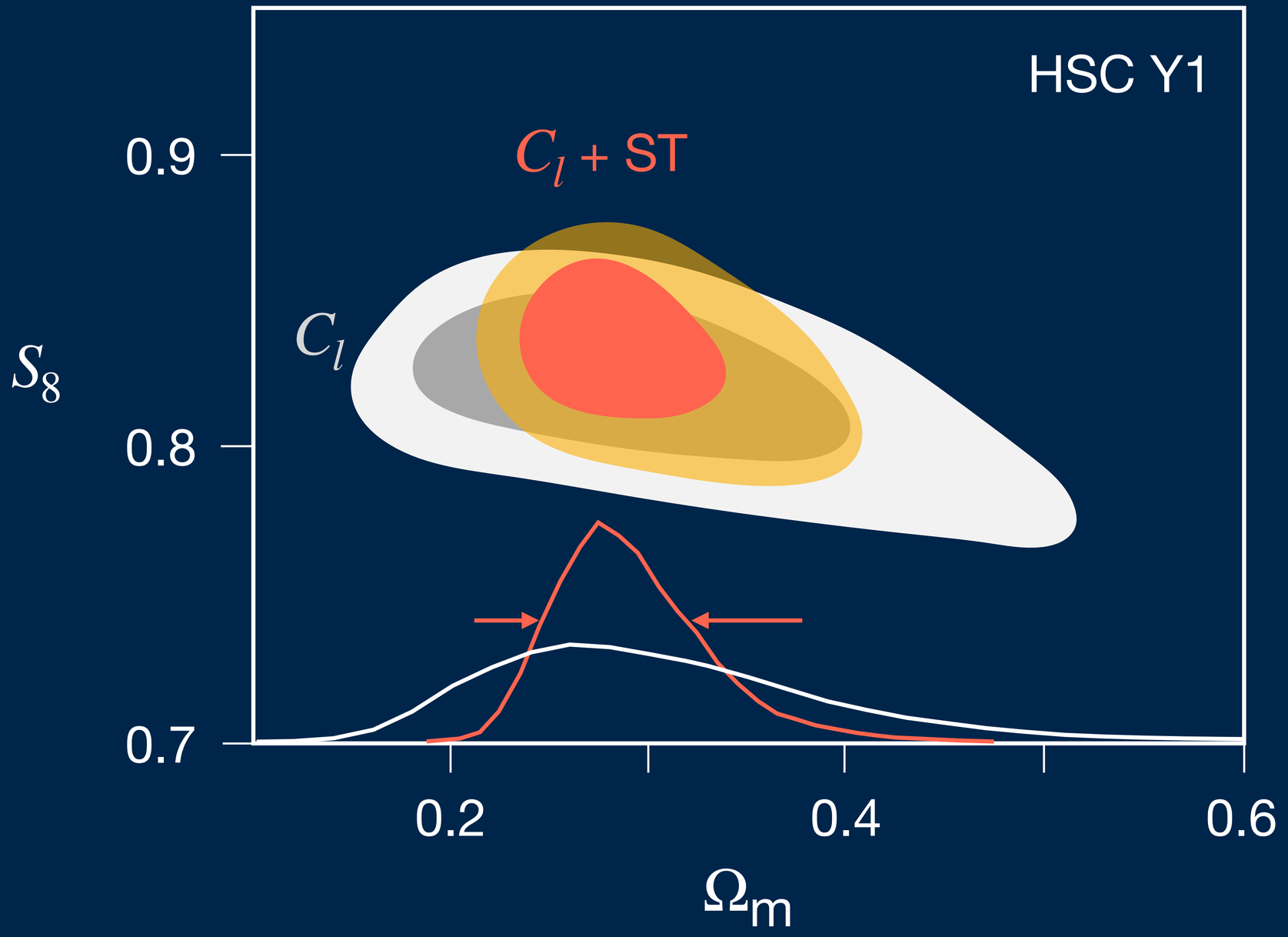
HSC year 1



mock mass map
with no noise

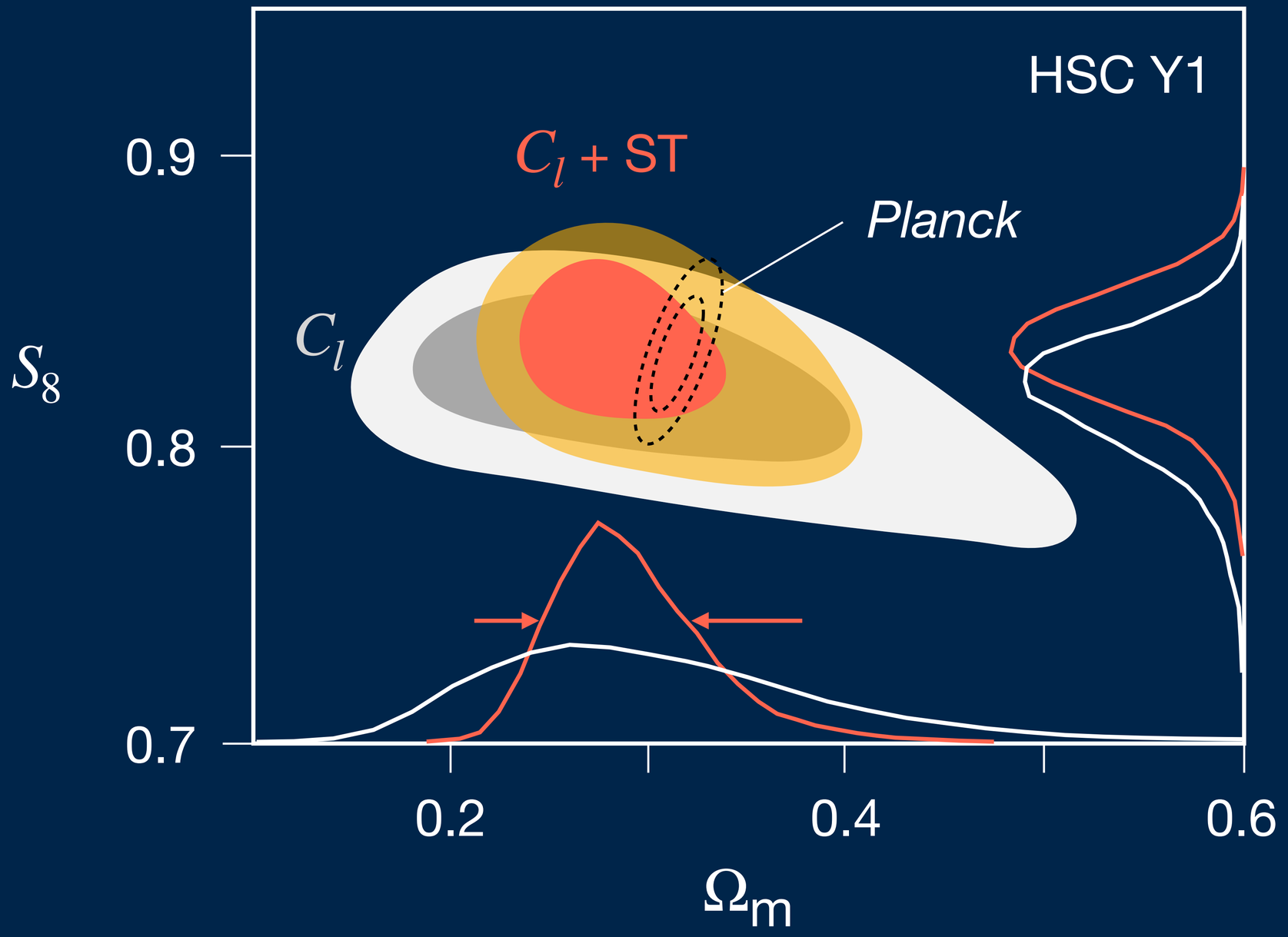
Cheng, Marques, Grandón, Thiele, Shirasaki, Ménard, Liu, 2024

improvement from ST



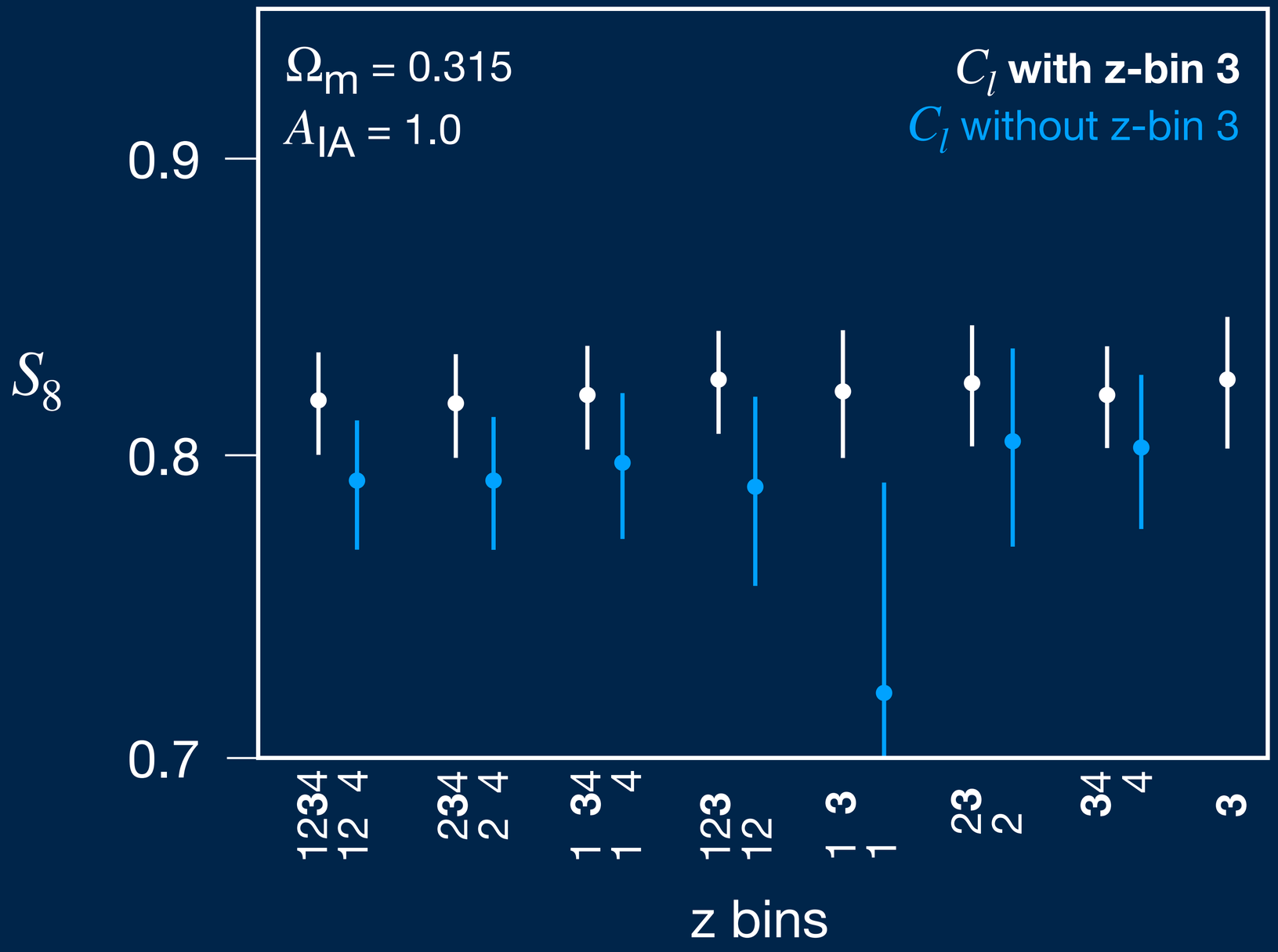
Cheng, Marques, Grandón, Thiele, Shirasaki, Ménard, Liu, 2024

high S8 value

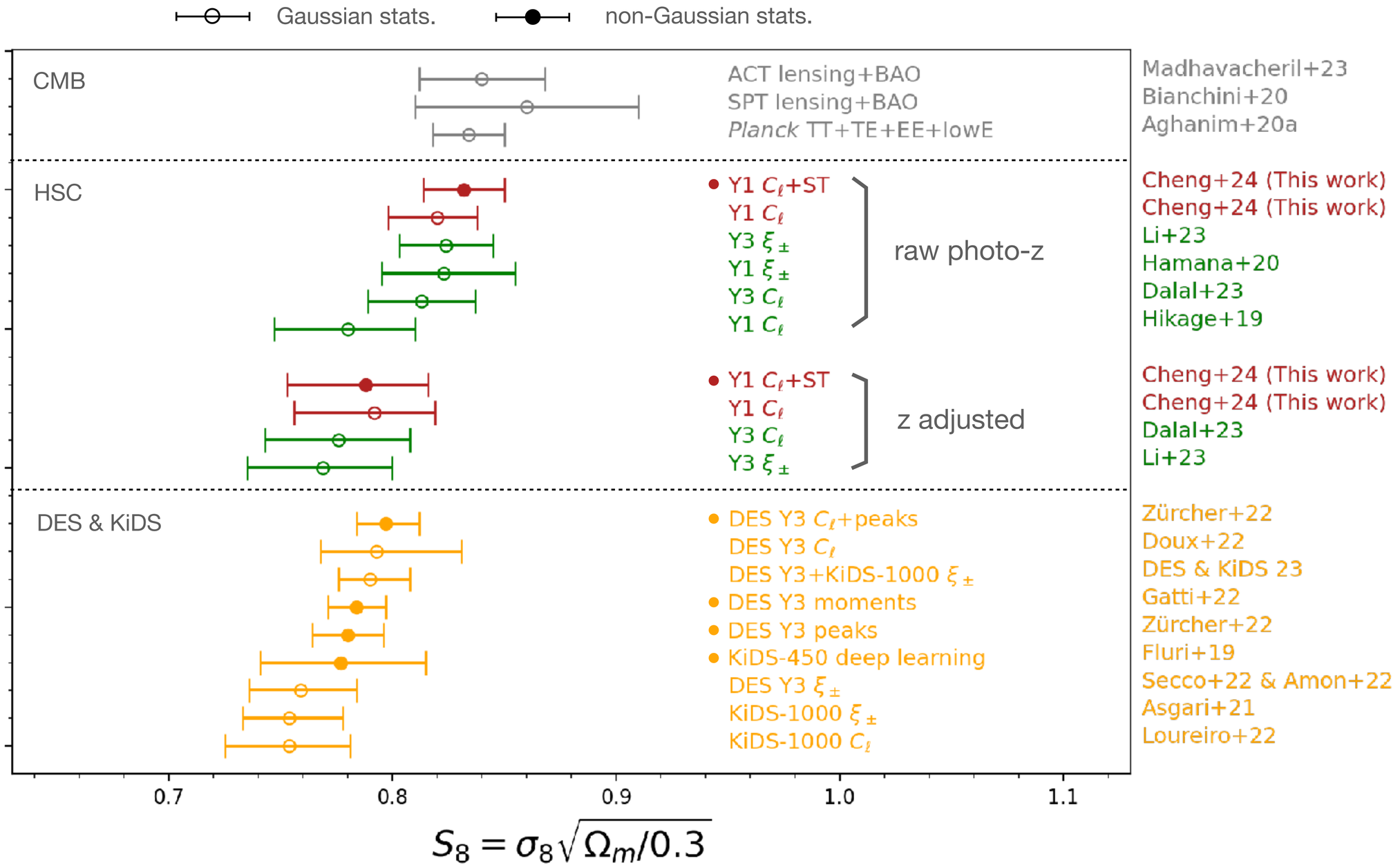


Cheng, Marques, Grandón, Thiele, Shirasaki, Ménard, Liu, 2024

photo-z issue

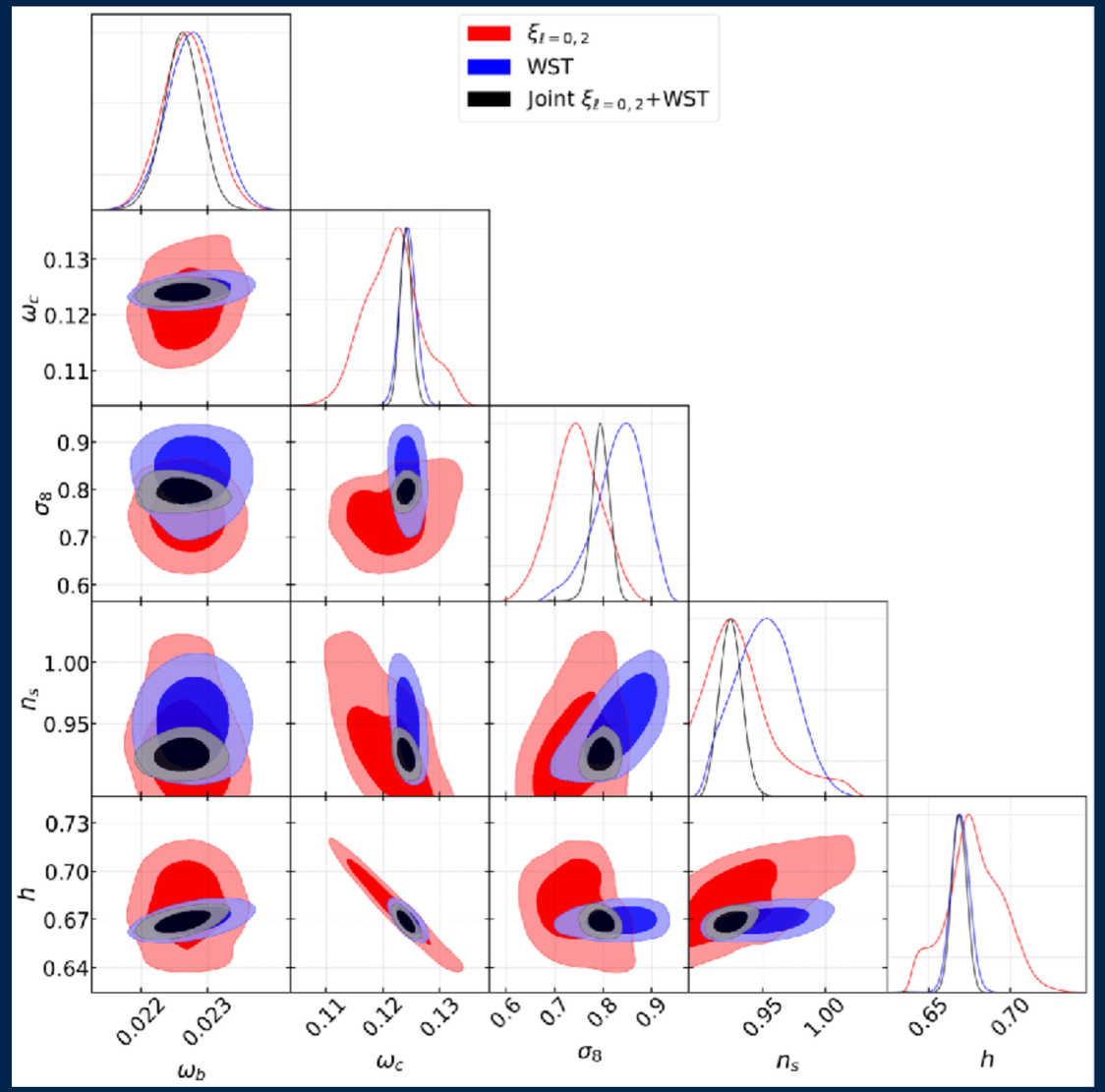


Cheng, Marques, Grandón, Thiele, Shirasaki, Ménard, Liu, 2024

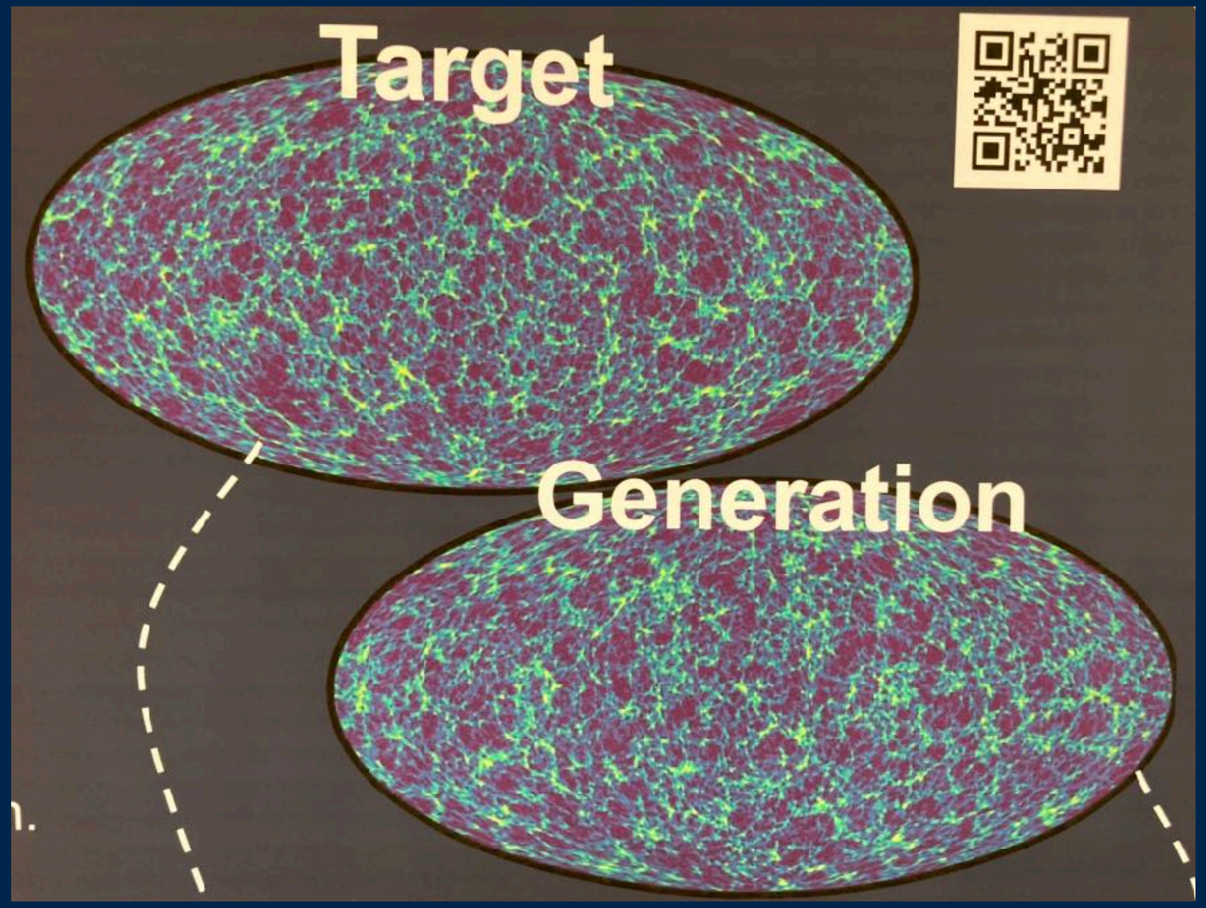


Cheng, Marques, Grandón, Thiele, Shirasaki, Ménard, Liu, 2024

BOSS galaxies (Georgias Valogiannis)



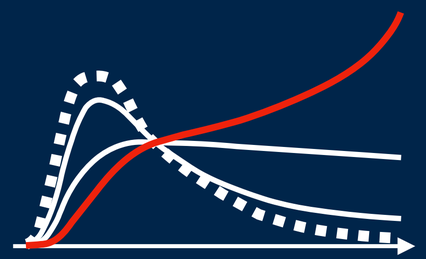
spherical generative model (Matt Price)



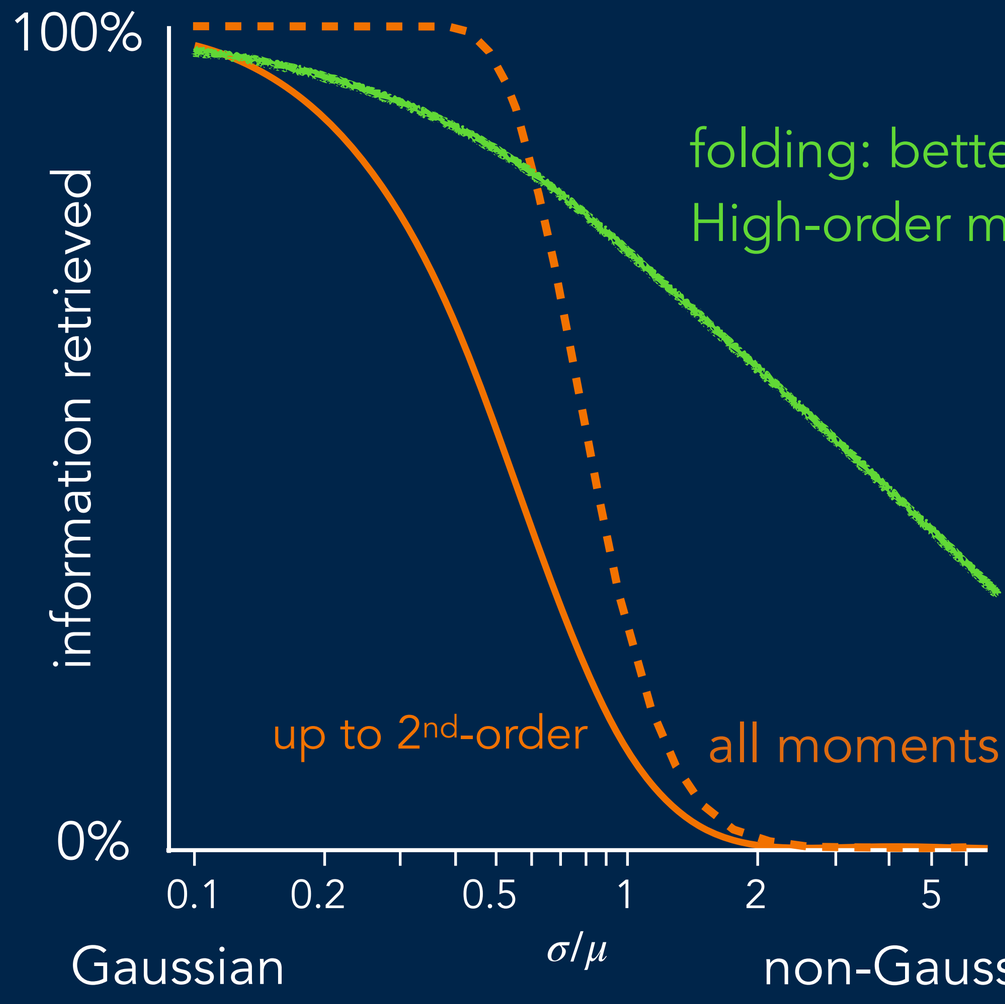
one-variable illustration: moments vs scattering

$\langle \delta_1 \delta_2 \,..\, \delta_n \rangle$

amplifying the tail

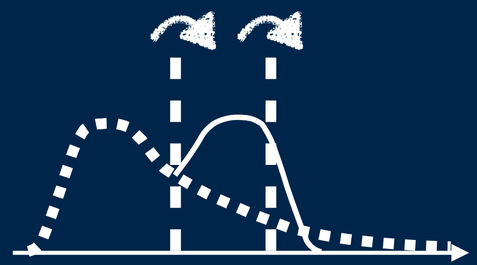


log-normal variable



$\langle \quad I \star \psi \quad \star \psi \quad \rangle$

folding the core



Cheng & Menard, 2021b

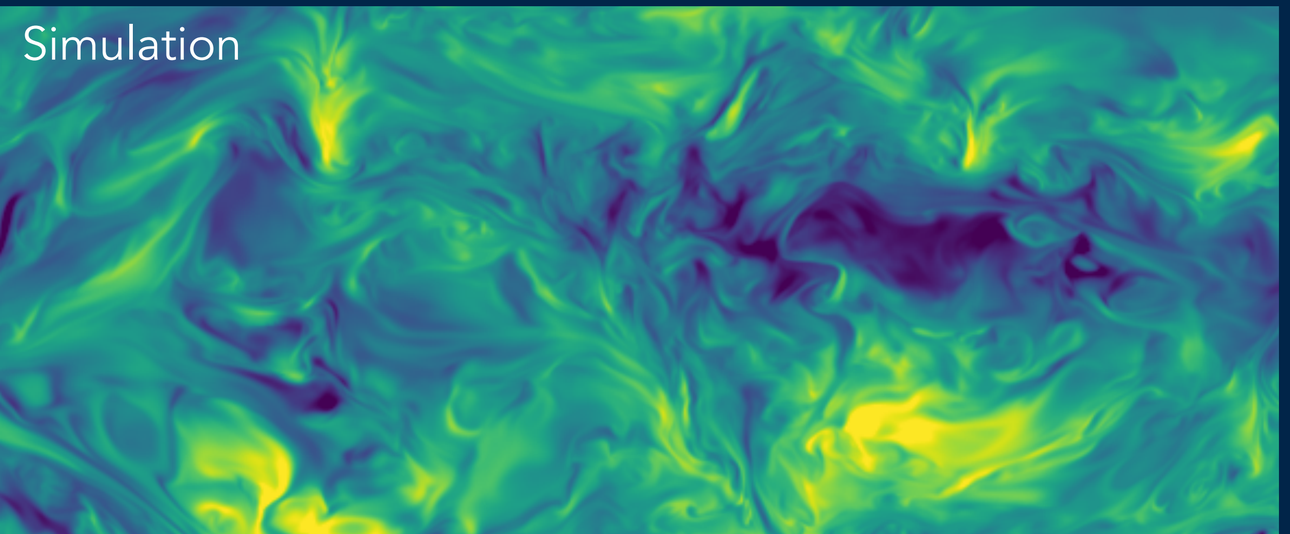
extension to cross-correlations

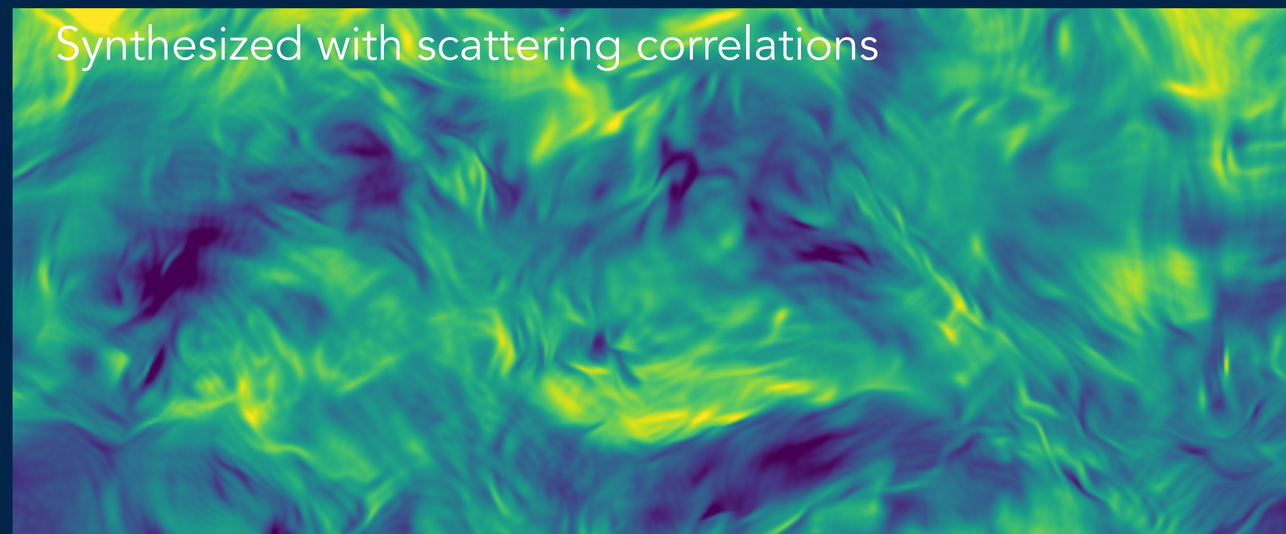
mean: scattering coefficients

I $\quad |I \star \psi_1|$ $\quad ||I \star \psi_1| \star \psi_2|$ $\quad \ldots$

$|I \star \psi_2|$

cross correlations



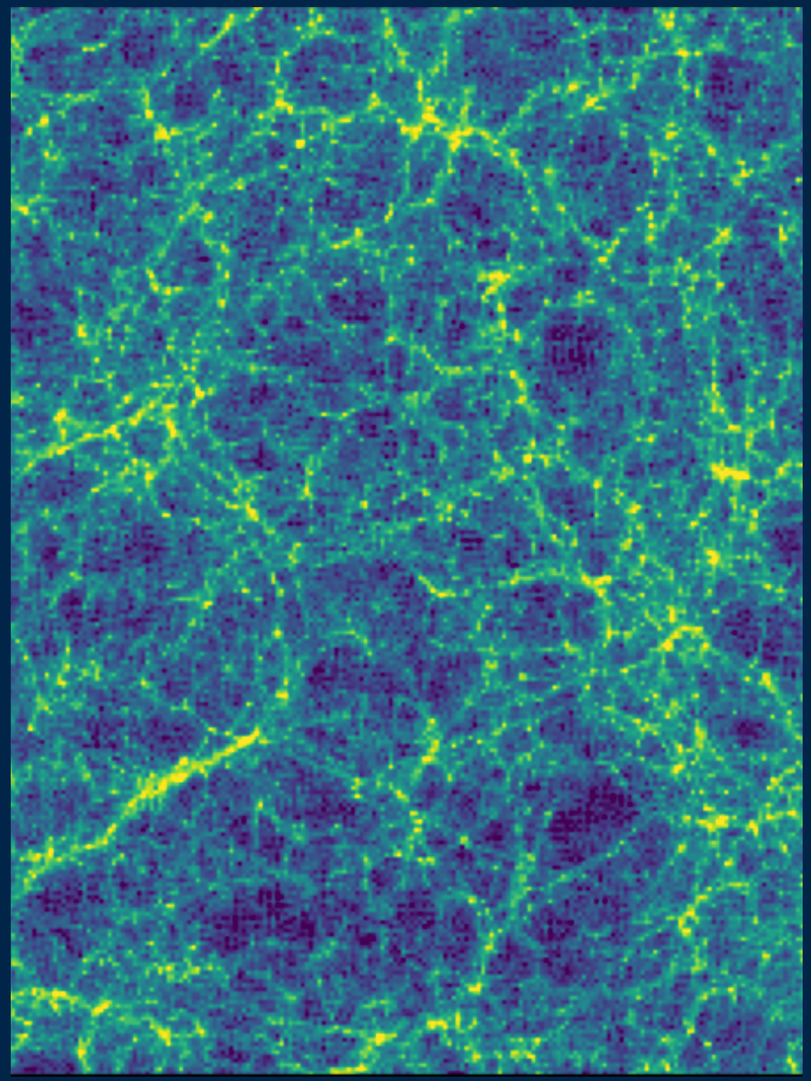


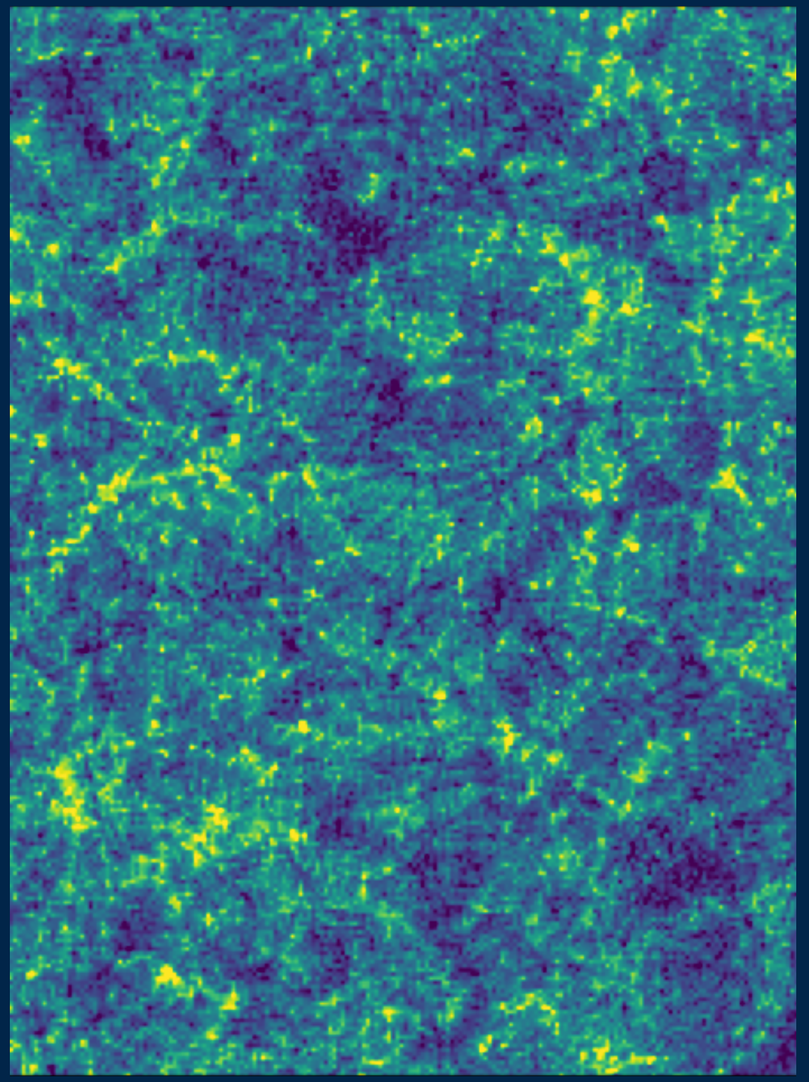
extension to cross-correlations

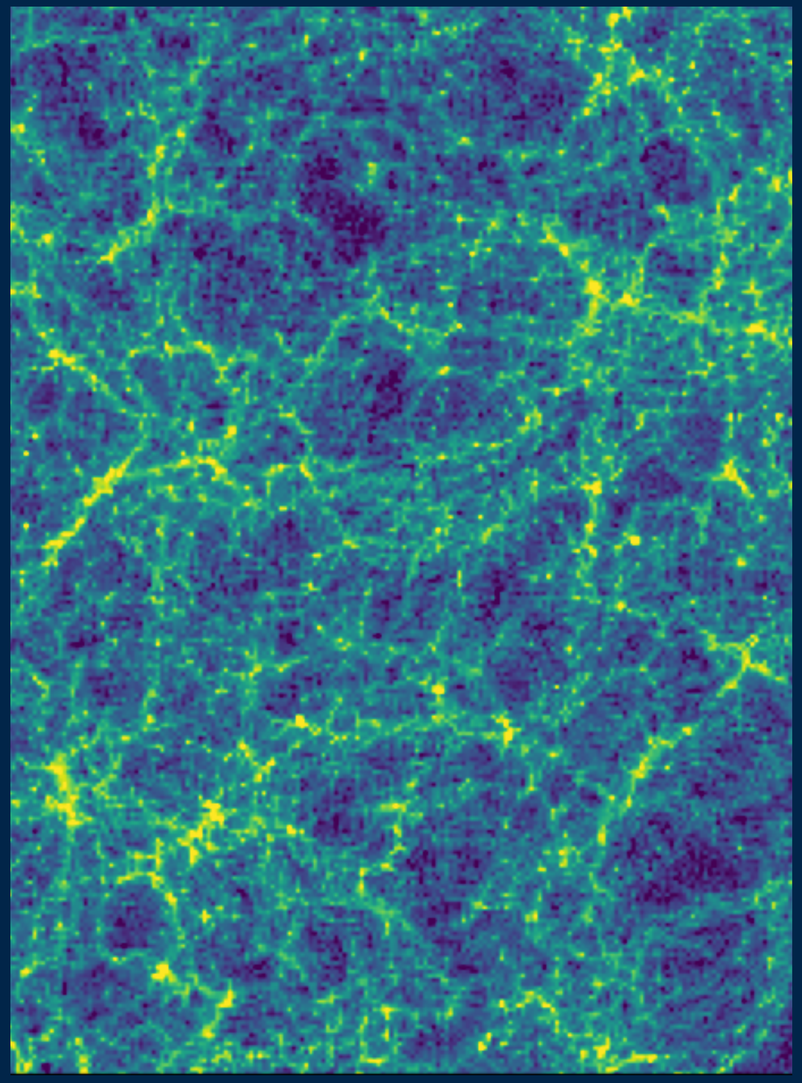
input image

synthesis from scattering transform alone

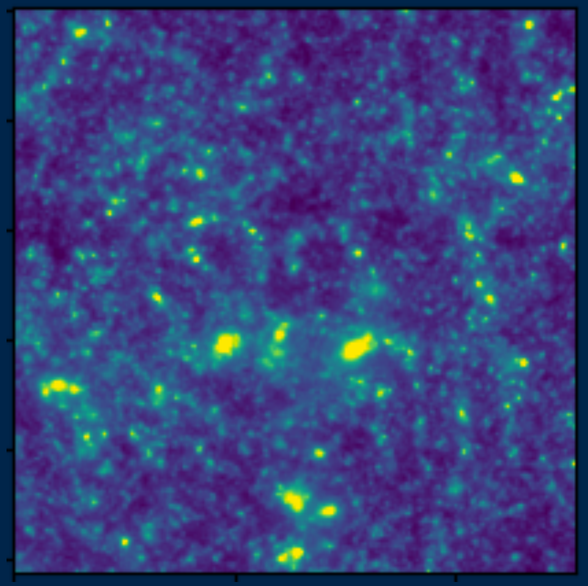
add cross-correlations $Corr(I,\ I \star \psi\ )$



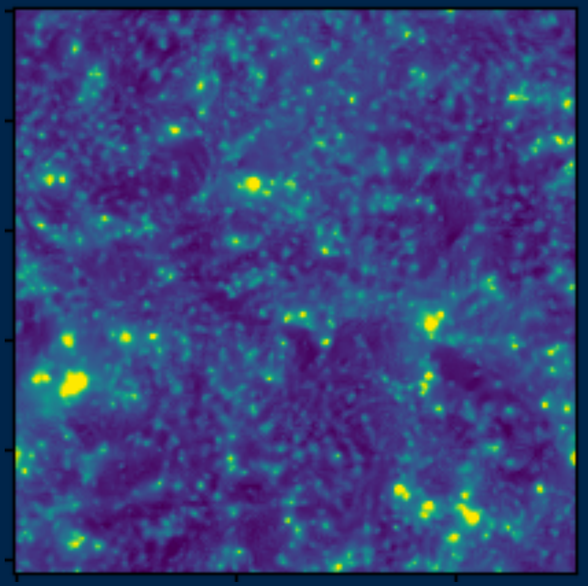




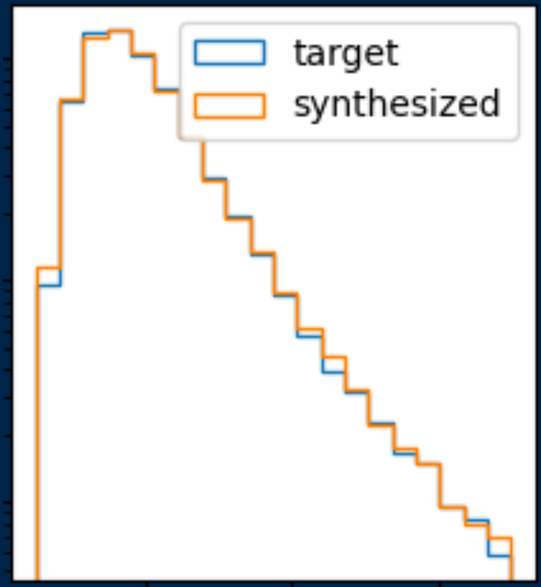
lensing field



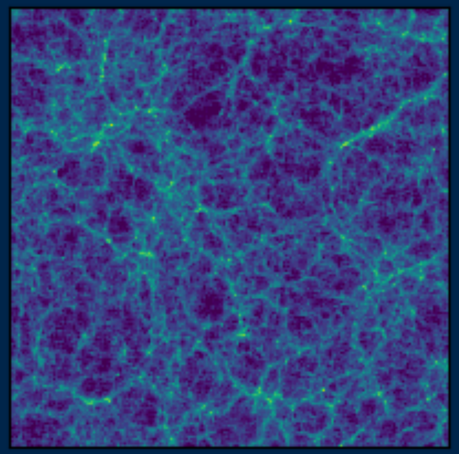
synthesized



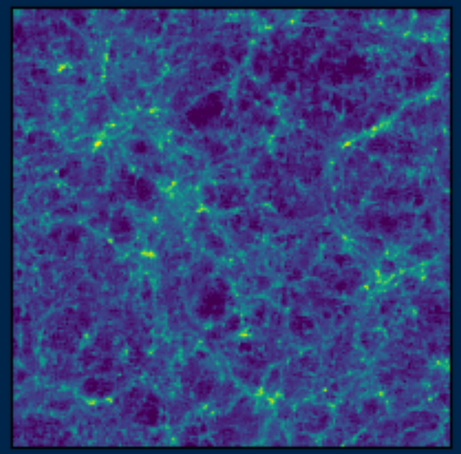
histogram



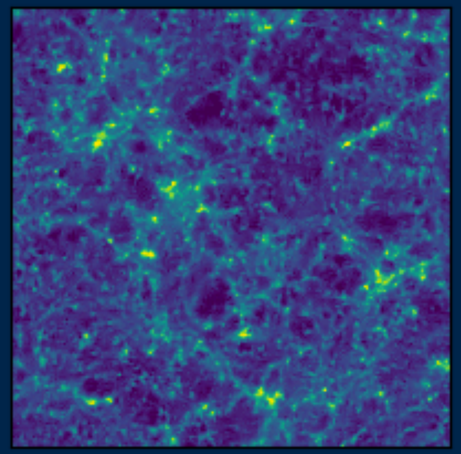
cosmic web



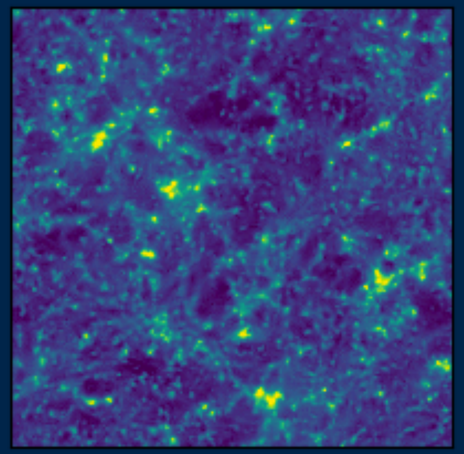
25%



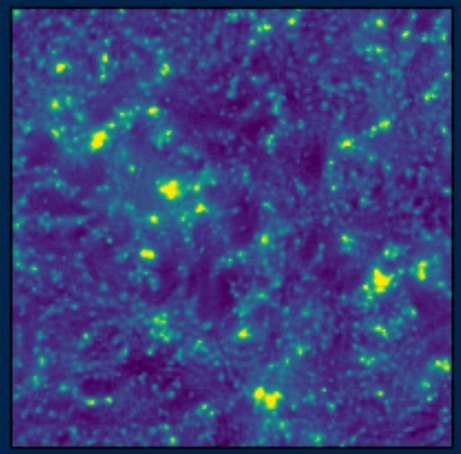
50%

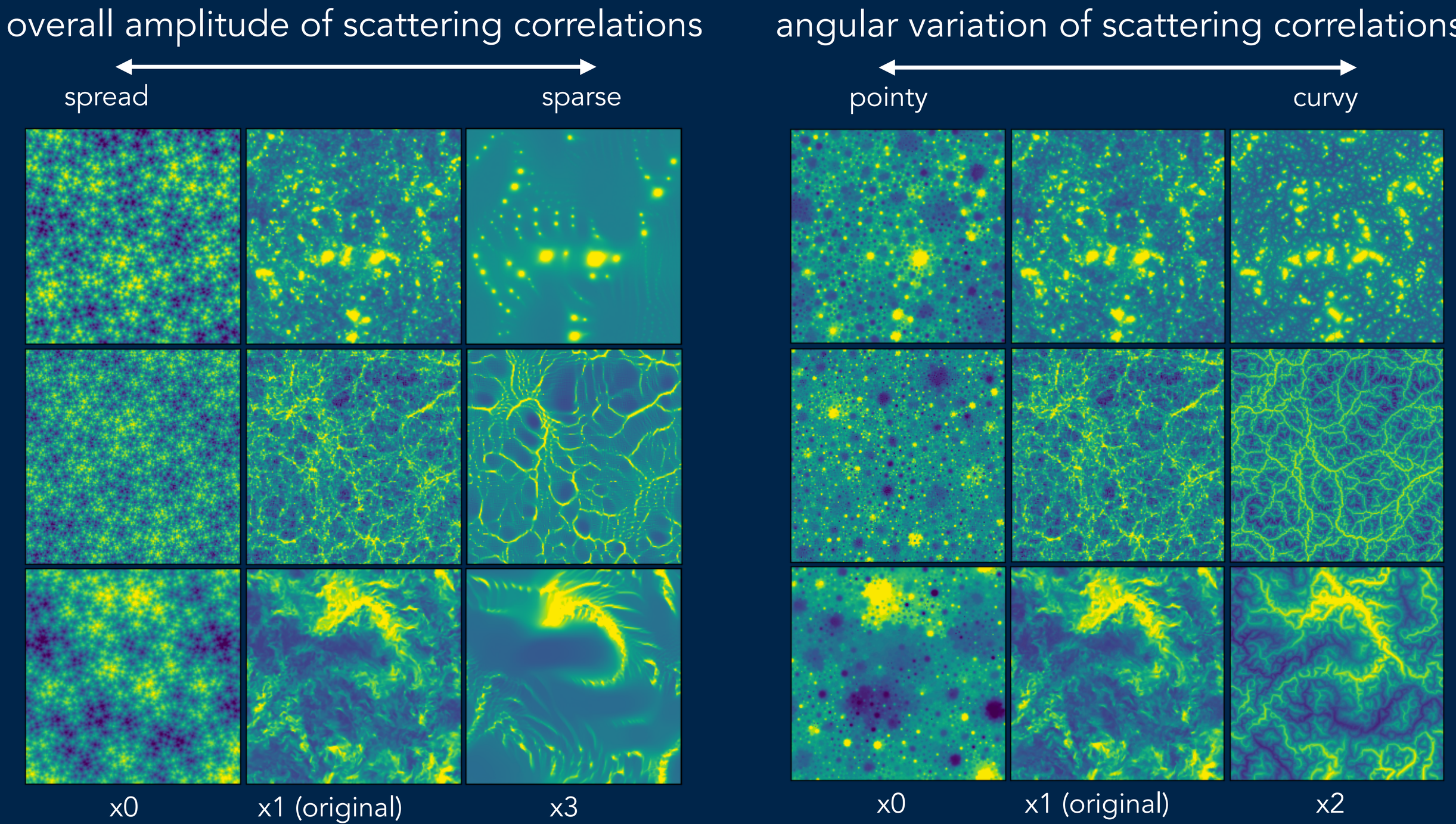


75%

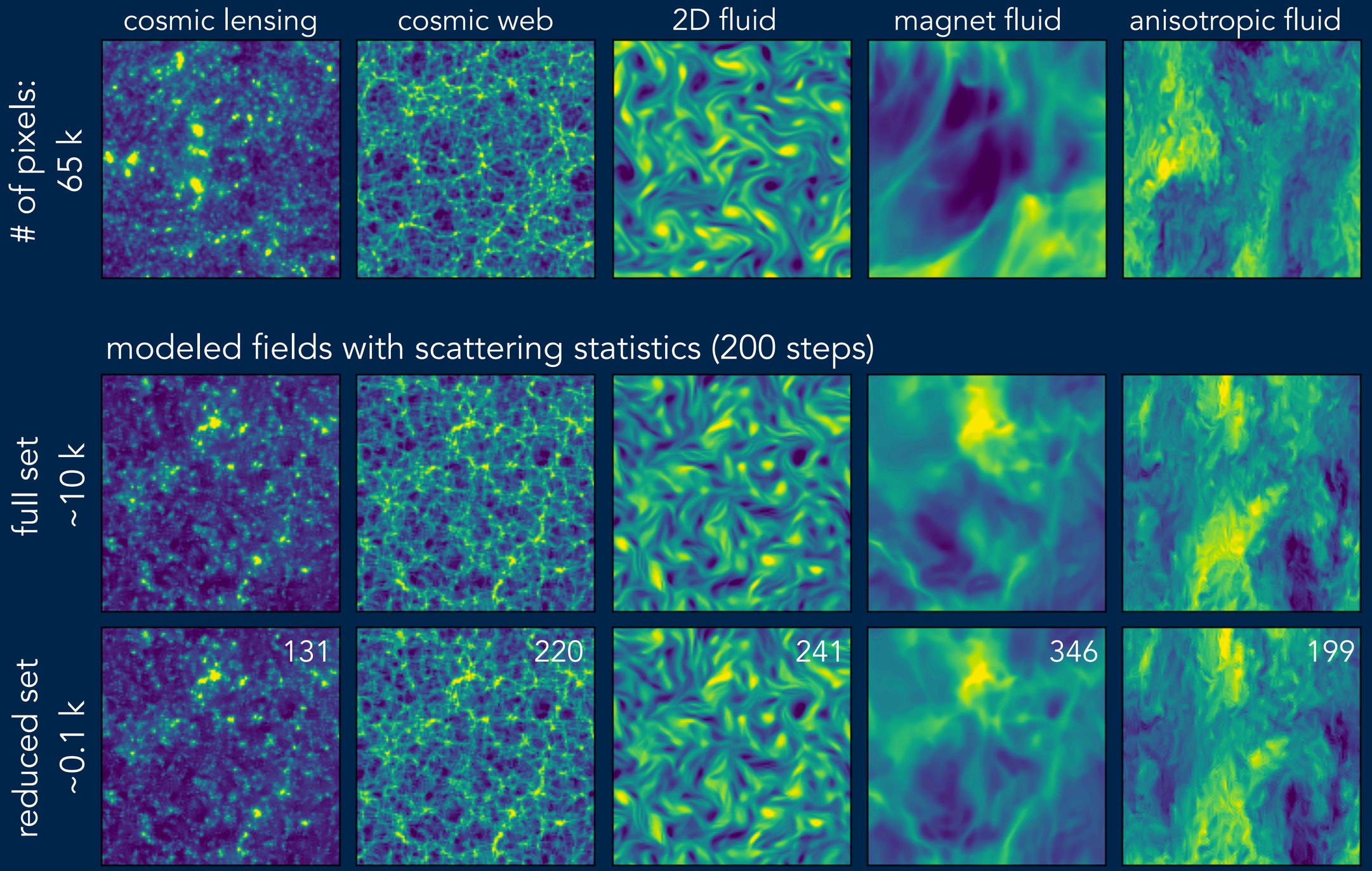


lensing



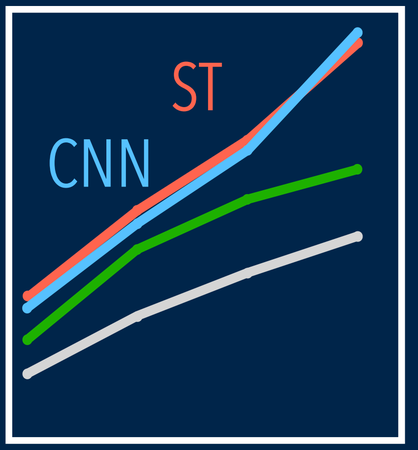


Cheng, Morel, Allys, Ménard, Mallat, 2024, PNAS Nexus



Cheng, Morel, Allys, Ménard, Mallat, 2024, PNAS Nexus

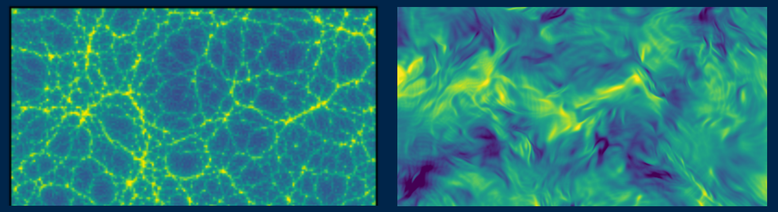
How do we characterize a field?



apparent "complexity"

power spectrum

scattering transform



CNN

information

- number of statistics
- relation to physics