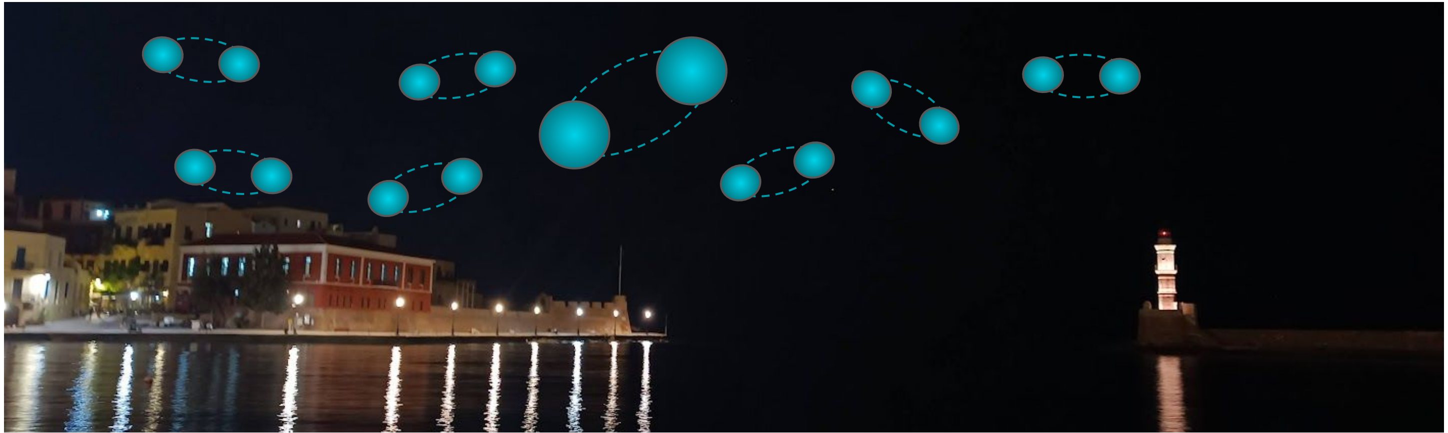
Gravitational-wave cosmology: current results and statistical challenges ahead

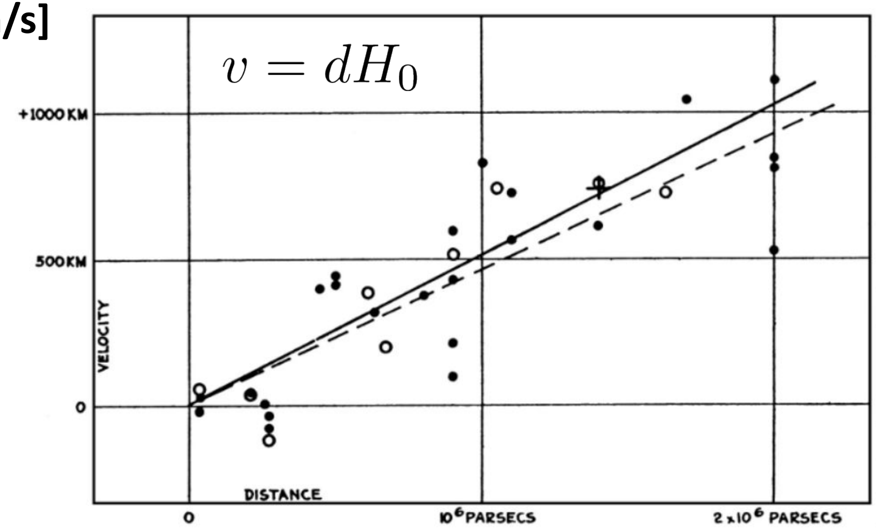
S. Mastrogiovanni



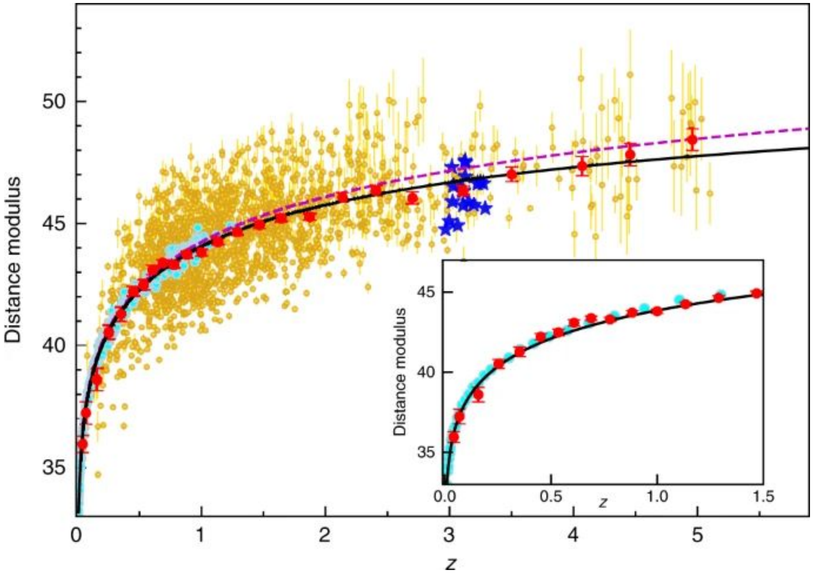
The standard cosmological model?

For almost 100 years, we have been measuring the expansion of the Universe

[E. Hubble 1929]



[G. Risalti et al, Nature 2023]



The standard cosmological model?

According to General Relativity, and confirmed by many observations, the Universe is expanding with a rate described by

$$\boxed{\frac{H(z)}{H_0} = \sqrt{\Omega_{m,0}(1+z)^3 + \Omega_\Lambda + \Omega_r(1+z)^4 + \Omega_k(1+z)^2}}$$

Hubble constant **Dark matter** **Dark energy** **Radiation** **Curvature**

Critical density

$$\rho_c = \frac{3H_0^2}{8\pi G}$$

Energy density

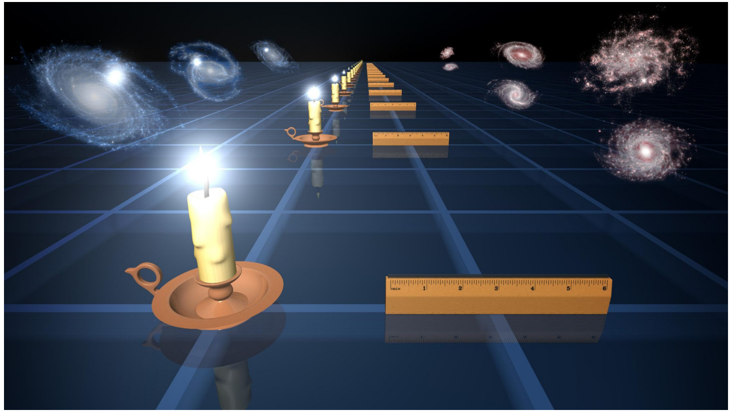
$$\Omega_\mathrm{X} = \frac{\rho_\mathrm{X}}{\rho_c}$$

The cosmic expansion offers us many potential discoveries:

- *What are the energy species living in our Universe?*
- *Is General Relativity valid on cosmological scales?*
- *What are the average and critical densities of the Universe?*

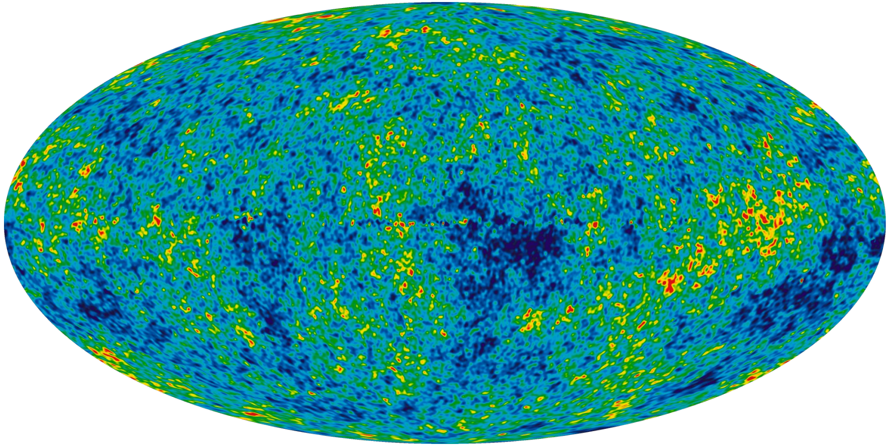
How have been measuring the Universe expansion so far?

Direct (Standard Candles)



- Cepheids, Supernovae Type IA, Active Galactic nuclei ,Kilonovae (?) and short Gamma-ray Burst
- **Issues:** Requires complex astrophysical calibration

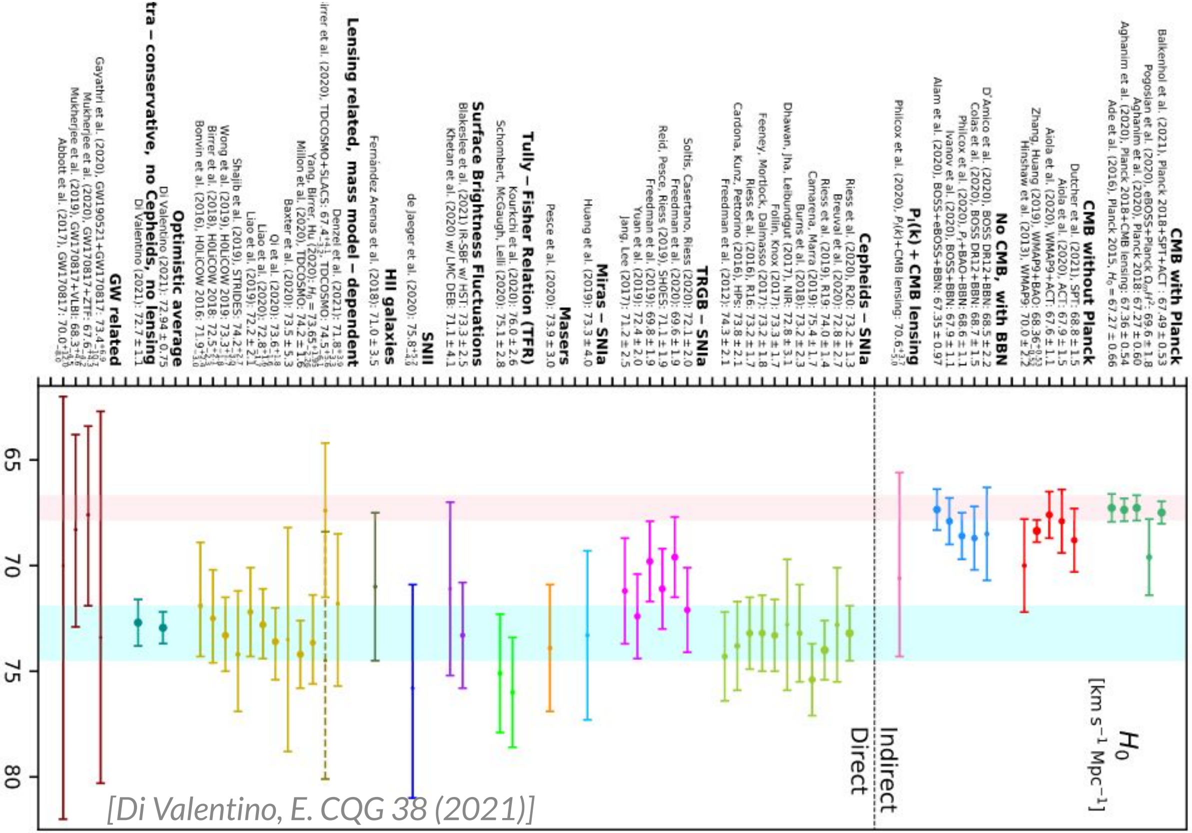
Indirect (Standard Candles)



- Cosmic Microwave Background temperature fluctuations, Baryonic nucleosynthesis
- **Issues:** Cosmic variance (a single Universe)

Measurements of the Hubble constant

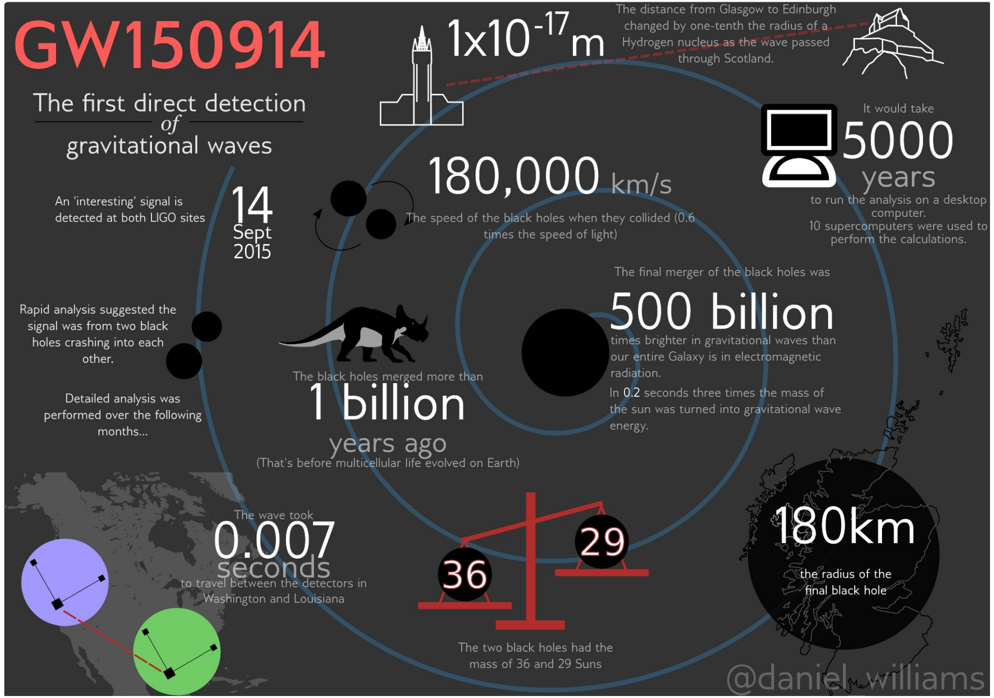
- There is a tension between direct and indirect measurements of the Hubble constant.
- Although in-depth studies for hidden systematics the tension has not been yet alleviated.
- We require to directly measure the Universe expansion in all the observable Universe.



[Di Valentino, E. CQG 38 (2021)]

Gravitational Wave sources at cosmological scales

B. P. Abbott et al. PRL 116, 061102 (2016)



The first direct detection *of* gravitational waves

An 'interesting' signal is detected at both LIGO sites

14 Sept 2015

Rapid analysis suggested the signal was from two black holes crashing into each other.

Detailed analysis was performed over the following months...

1x10^{-17}m

The distance from Glasgow to Edinburgh changed by one-tenth the radius of a Hydrogen nucleus as the wave passed through Scotland.

180,000 km/s

The speed of the black holes when they collided (0.6 times the speed of light)

It would take 5000 years to run the analysis on a desktop computer. 10 supercomputers were used to perform the calculations.

The final merger of the black holes was 500 billion times brighter in gravitational waves than our entire Galaxy is in electromagnetic radiation.

In 0.2 seconds three times the mass of the sun was turned into gravitational wave energy.

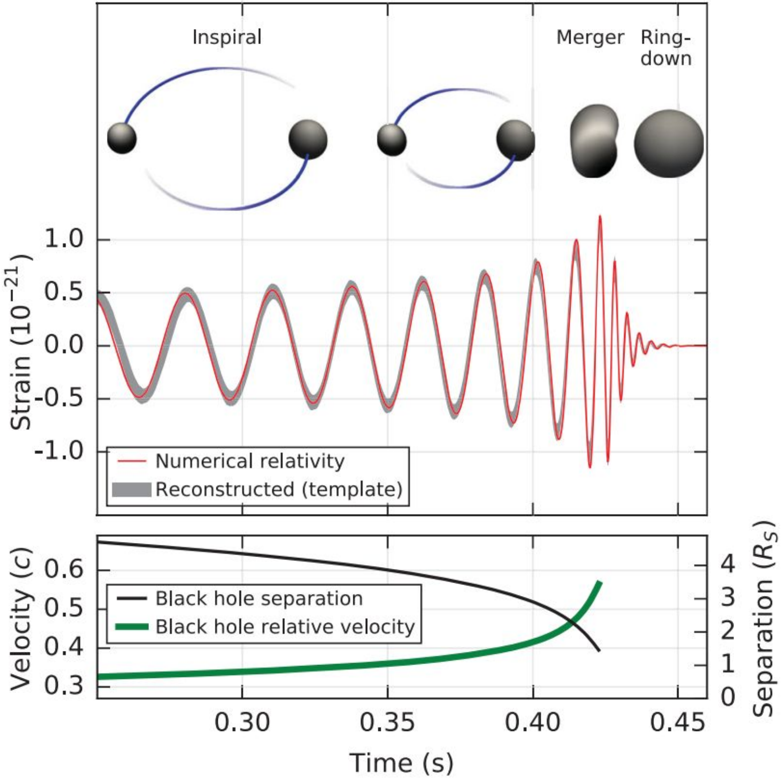
The black holes merged more than 1 billion years ago (That's before multicellular life evolved on Earth)

The wave took 0.007 seconds to travel between the detectors in Washington and Louisiana

The two black holes had the mass of 36 and 29 Suns

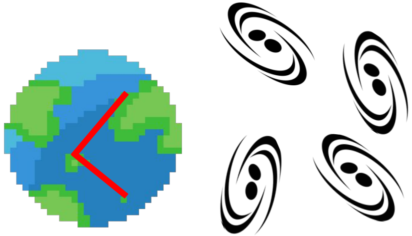
180km the radius of the final black hole

Credit: LVK EPO and Outreach groups



Gravitational Waves from cosmic distances

Source frame

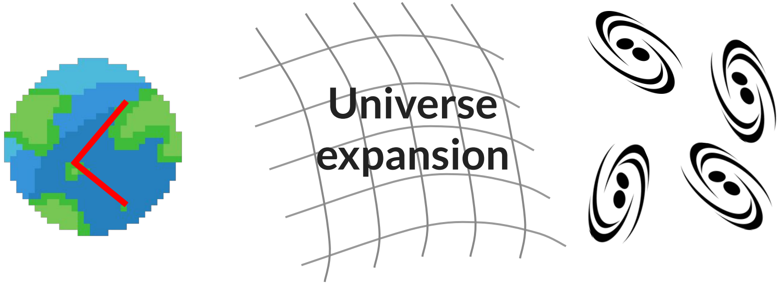


Chirp mass   Source time

$$h(t) \propto \frac{\mathcal{M}^{5/4}(t_c - t)^{-1/4}}{d} e^{\phi(t)}$$

Physical distance

Detector frame



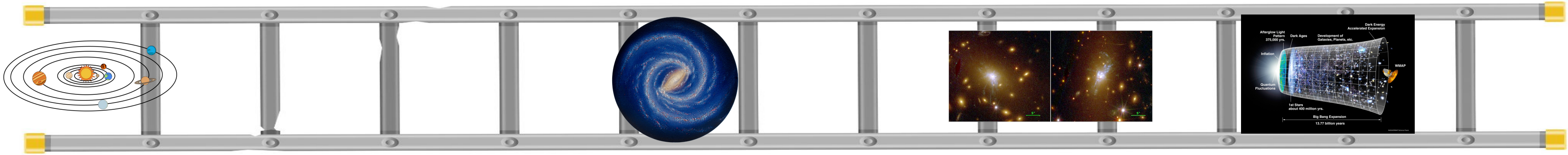
Redshifted chirp mass   Detector time

$$h(t^{\rm det}) \propto \frac{[\mathcal{M}(1+z)]^{5/4}(t_c^{\rm det} - t^{\rm det})^{-1/4}}{d_{\rm L}} e^{\phi(t)}$$

Luminosity distance

Gravitational Wave sources at cosmological scales

In order to measure the expansion of the Universe we need to know the source's distance and recessional velocity



1 AU 1 pc 100 pc 1 kpc 100 kpc 1 Mpc 100 Mpc 1 Gpc 10 Gpc

Distances with Electromagnetic observations

Radar

Cepheids

Parallax

Type Ia Supernovae

Distances with GW observations

Compact binary coalescences

Gravitational Wave sources at cosmological scales

From GWs we can not measure the **source redshift** (escaping velocity)

In recent years, we used several methods to assign a redshift to GW sources

- **Bright sirens:** An associated Electromagnetic (EM) counterpart (GRB, Kilonova etc…) can provide the identification of the host galaxy.
- **Dark sirens:** Galaxy surveys can be used to identify possible hosts in the GW localization volume.
- **Spectral sirens:** Knowledge of the source-frame mass distribution can be used to assign a redshift to GW sources.

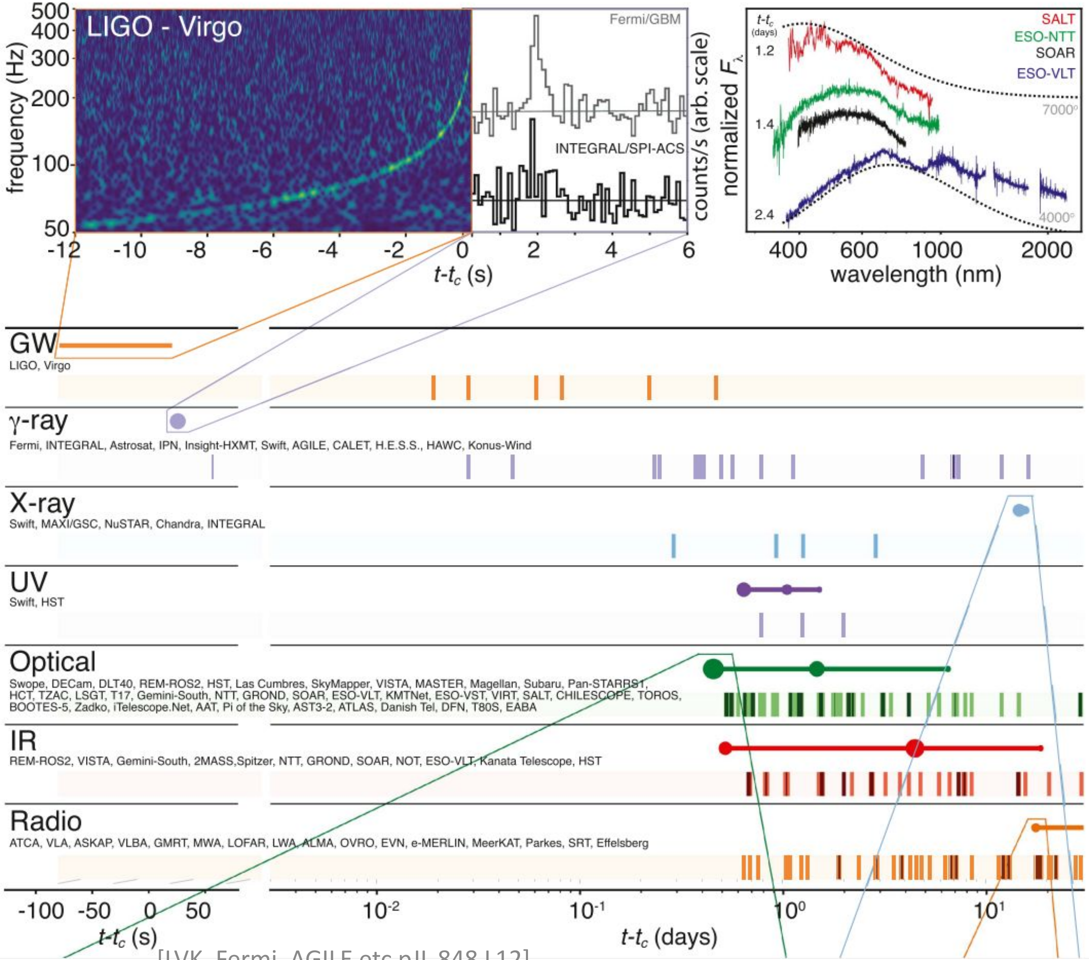
GW170817 + EM transients

At 12:41:04 UTC a GW from the merger of two neutron stars is detected

- +2 seconds later Integral and Fermi detect a GRB.
- ~10 hrs later a kilonova emission from NGC4993 is observed.

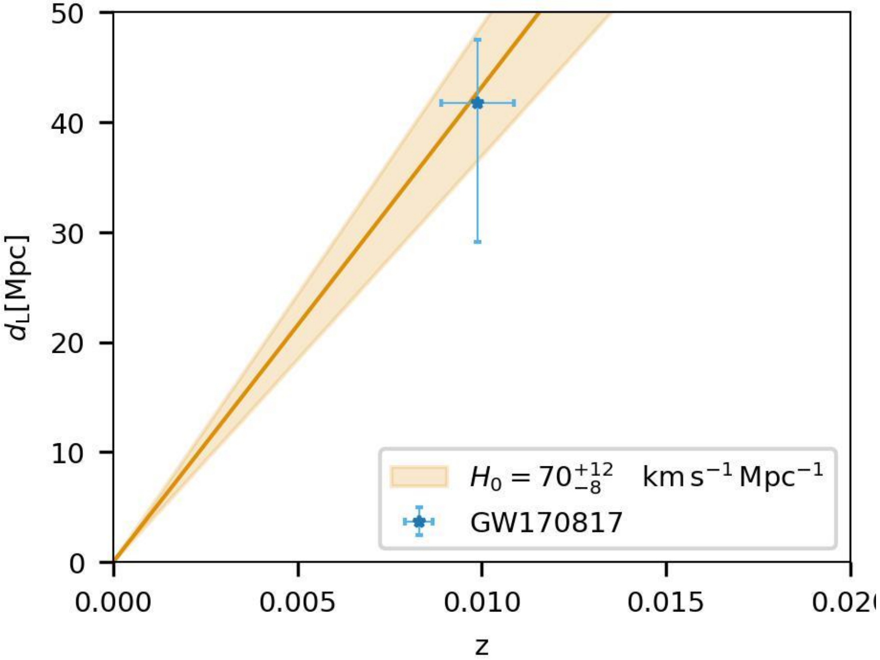
With GW170817 we have been provided:

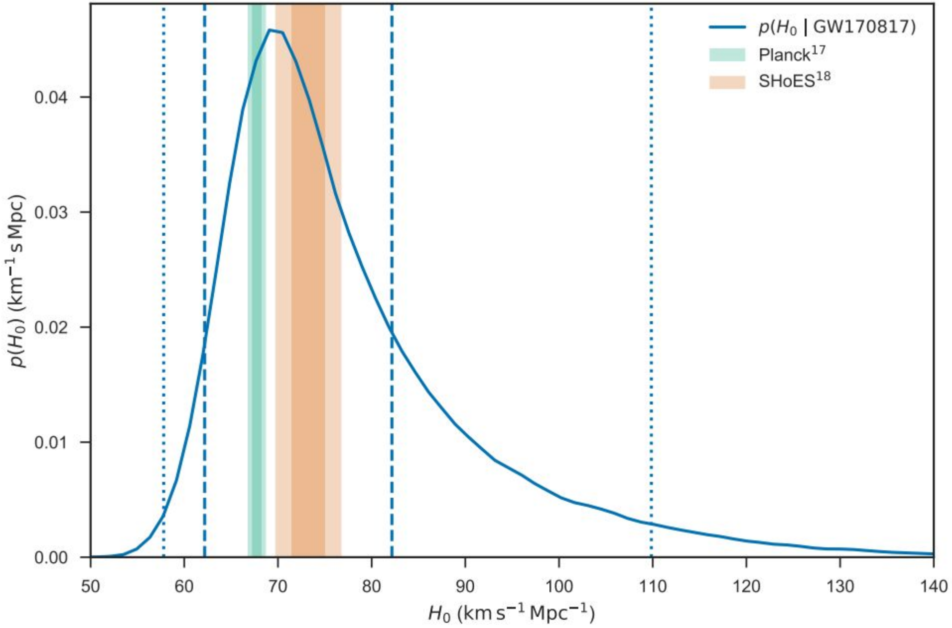
- Luminosity distance from GW170817.
- Redshift identification of the host galaxy from NGC4993.
- Peculiar motion of NGC4993.



[LVK, Fermi, AGILE etc pJL 848 L12]

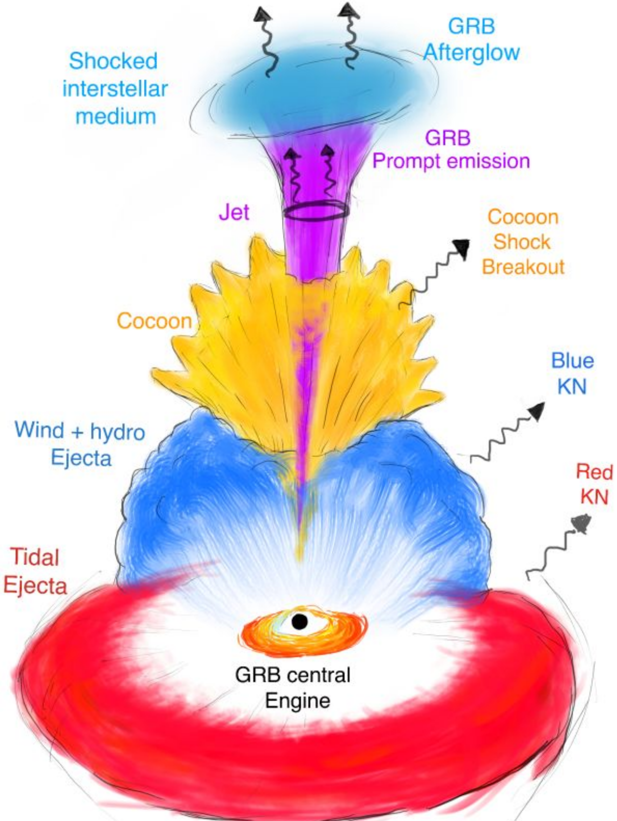
Bright sirens: Cosmology with GW170817



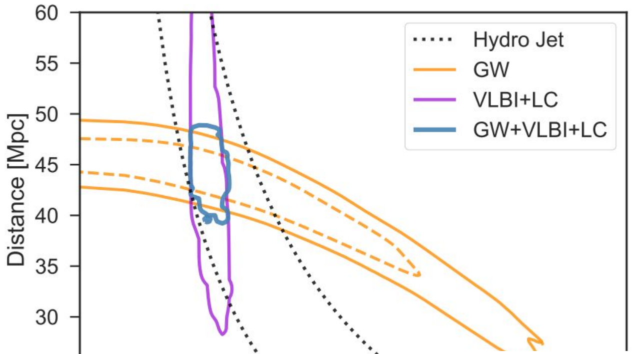


[LVC+, Nature (2017)]

Bright sirens: Cosmology with GW170817



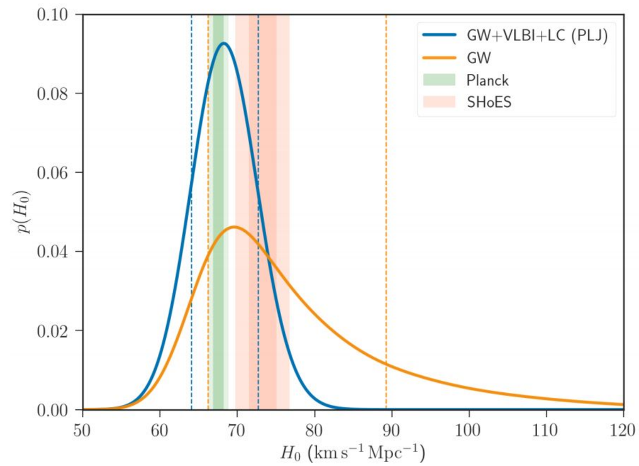
Ascenzi+, Journal of Plasma Physics, 87, 1 2021



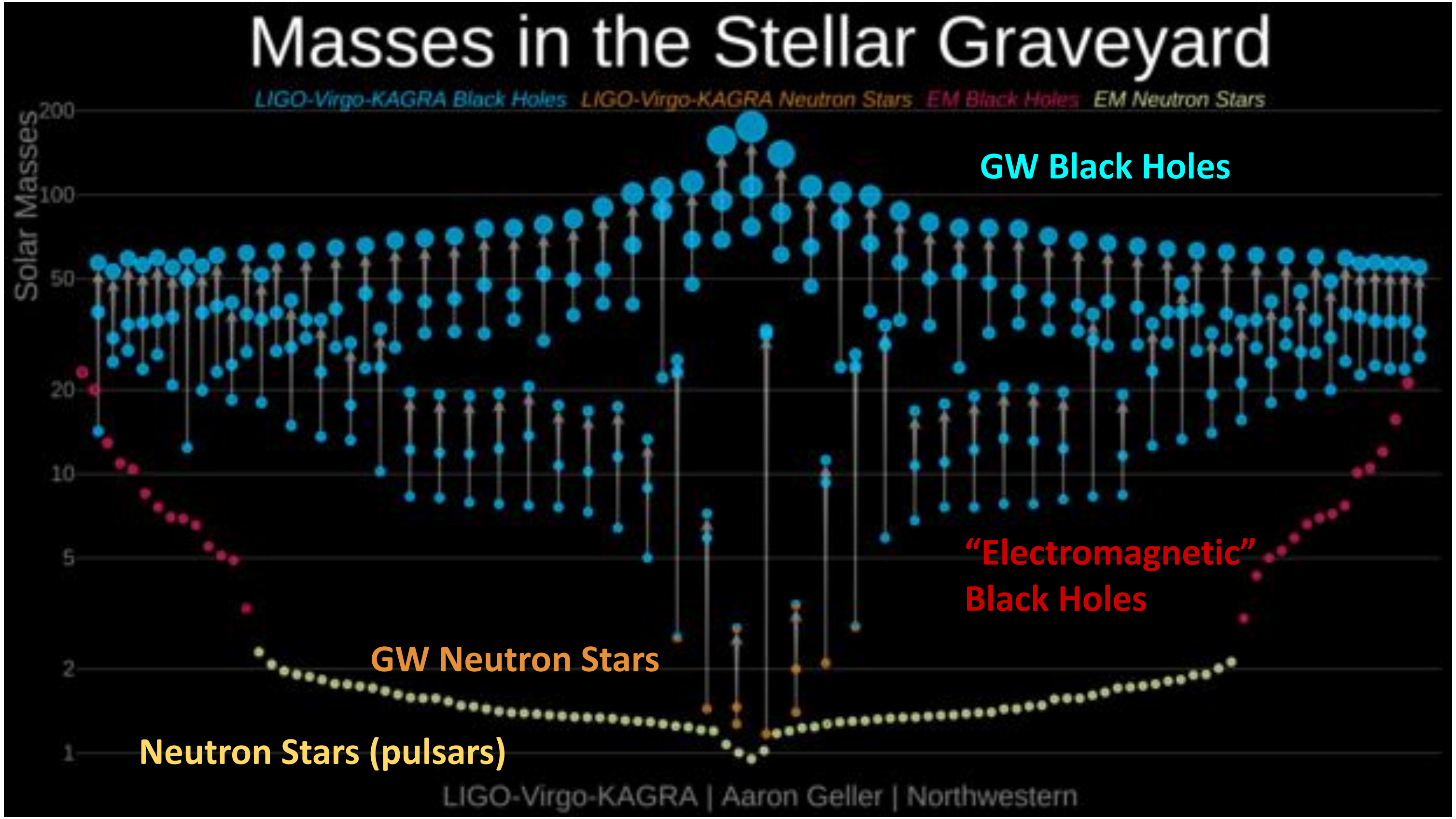
[K. Hotokezaka, Nature Astronomy, 3(2019)]

Afterglow informed cosmology

- ***Open question:*** *Can we standardize the GRB and KN emissions from BNS mergers?*



Dark Sirens cosmology



GW Black Holes

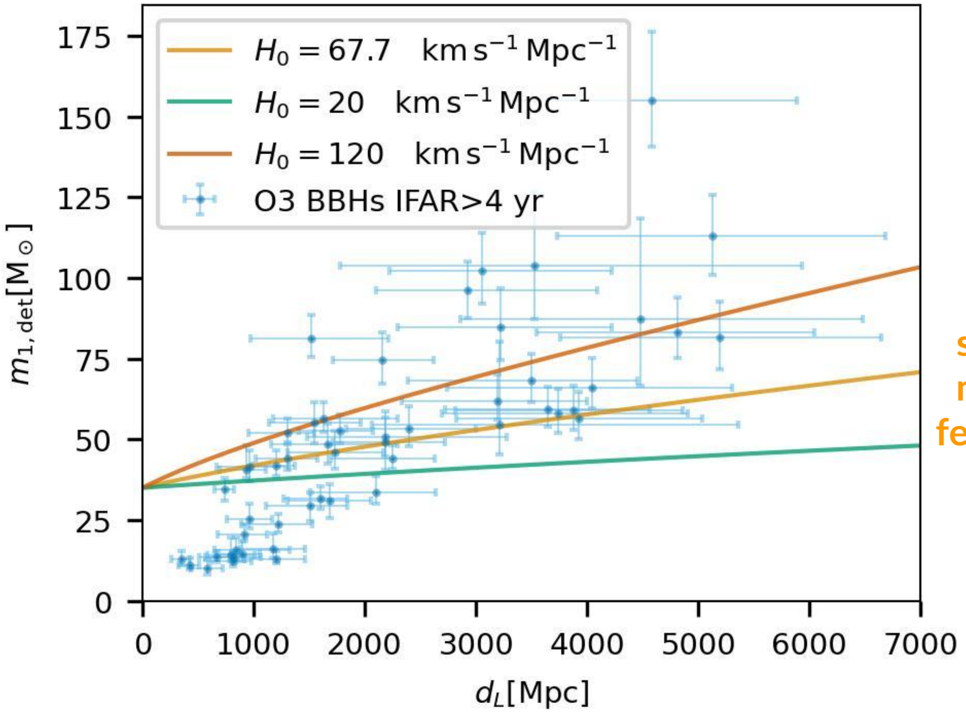
"Electromagnetic" Black Holes

GW Neutron Stars

Neutron Stars (pulsars)

Spectral sirens: GW-only cosmology

- If we assume an overdensity of BBHs produced at 35 solar masses, some “extreme” cosmologies can not fit the overdensity of BBHs.



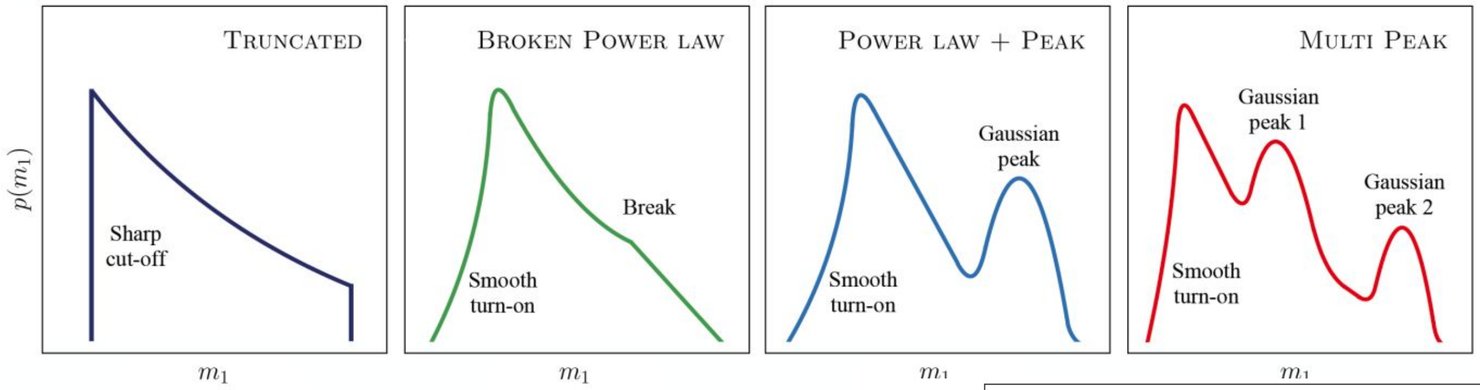
35 solar mass feature

Open questions

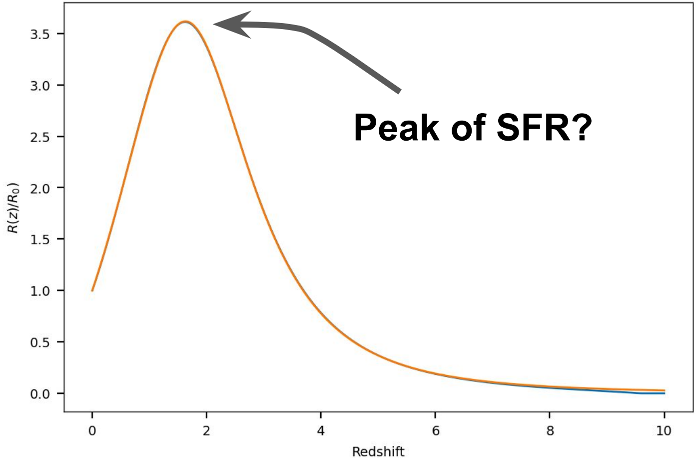
- What is the mass and redshift distribution of BBHs?
- Which are the various formation channels for BBHs?

[M. Mapelli arXiv:2106.00699 for review]

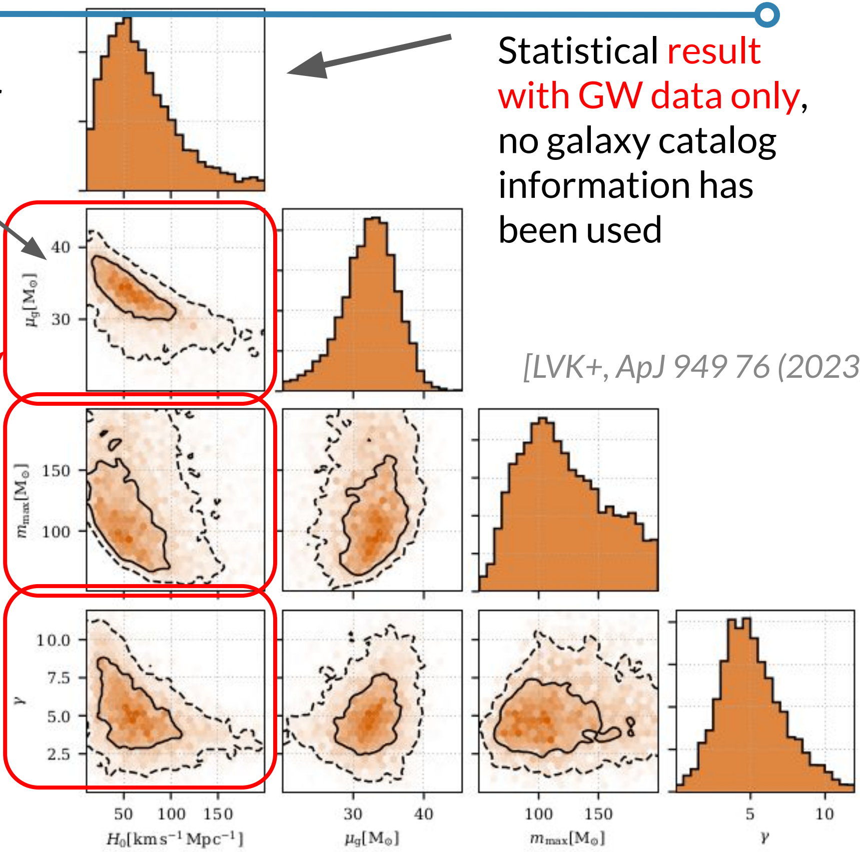
GW cosmology after GWTC-3: Spectral sirens



- Several mass models include various scales to describe stellar processes *[LVK+ 2021 ApJL 913 L7]*.
- As a consequence, the masses of GW observations are redshifted by the expansion of the Universe *[Ye+ PRD 104 2021]*.
- Open question: Are these models able to adapt to complex astrophysical observations? *[Pierra+, PRD 109, 2024]*



GW cosmology after GWTC-3: Spectral sirens



The excess of BBHs around 35 solar masses sets a scale for the redshift and provides constraints on H0.

Statistical result with GW data only, no galaxy catalog information has been used

[LVK+, ApJ 949 76 (2023)]

Population parameters that correlate with H0

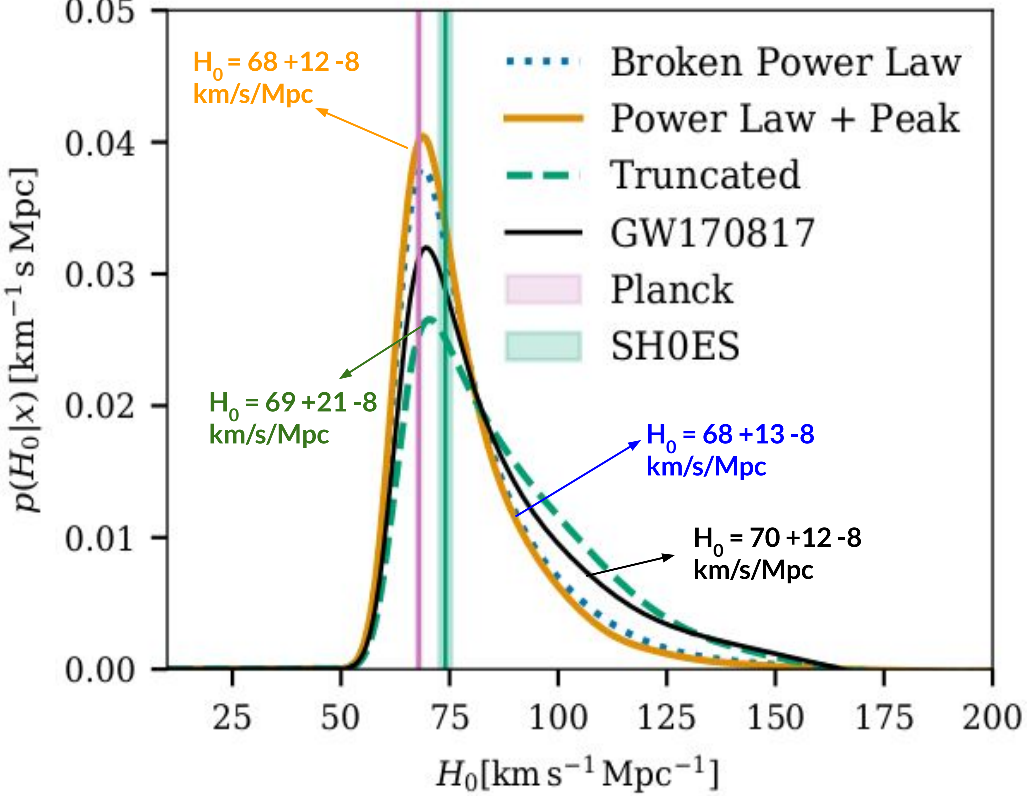
Posteriors of H_0, mu of the Gaussian peak, maximum mass of the BH and the merger rate evolution.

GW cosmology after GWTC-3: Spectral sirens

The only EM information is the counterpart of GW170817

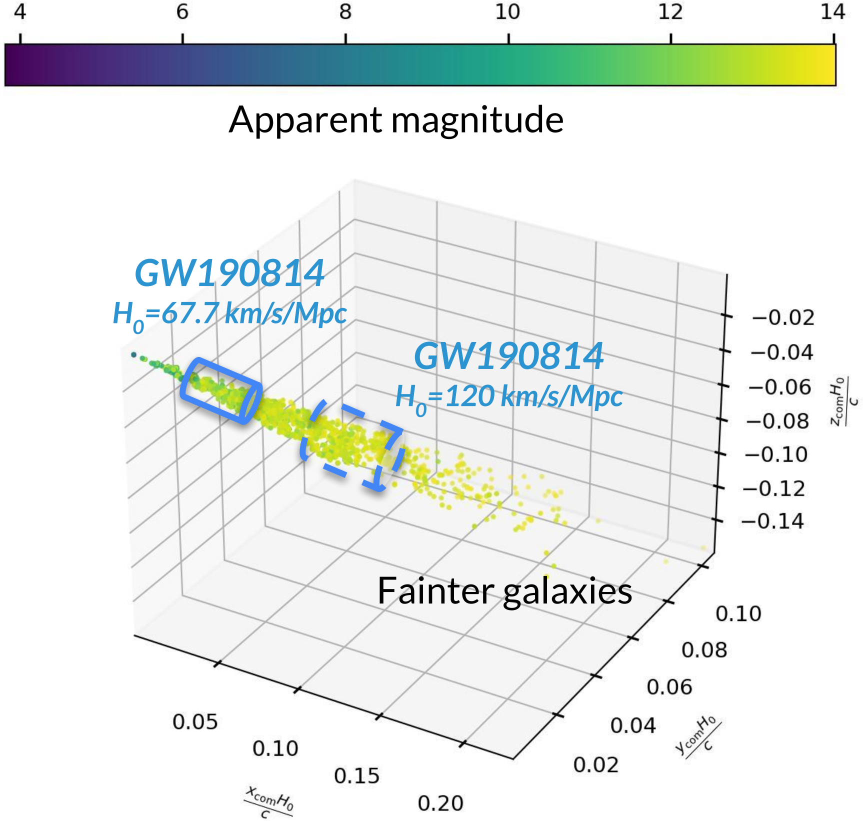
H_0 posterior of the 3 mass models combined with GW170817 posterior

[LVK+, ApJ 949 76 (2023)]

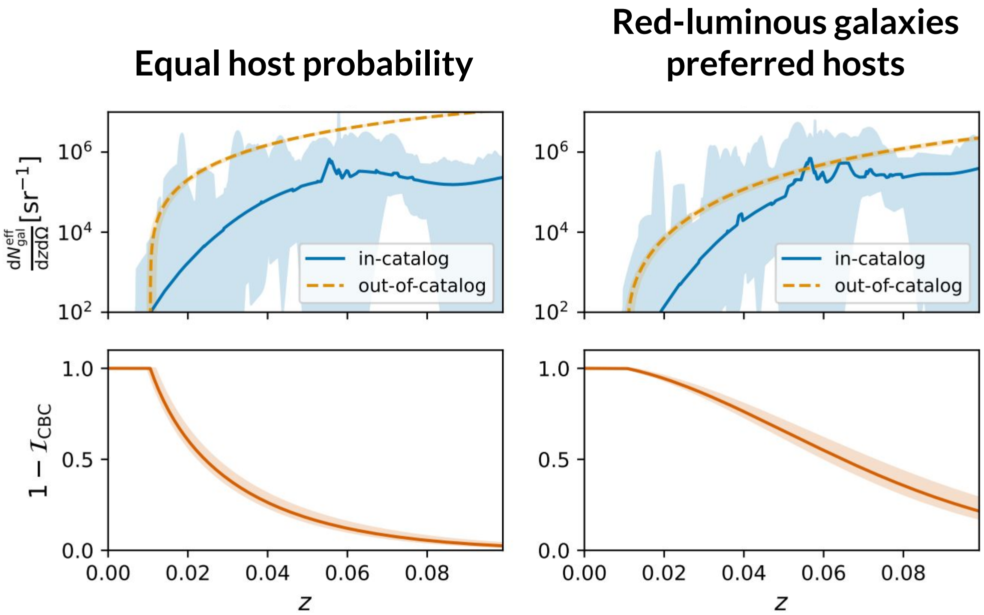


Dark sirens: Cosmology aided by galaxy surveys

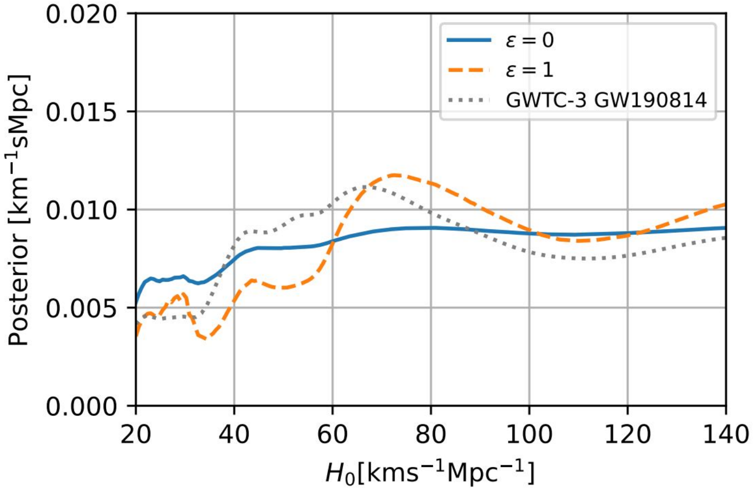
- A cosmological model has statistical support when the GW localization matched an *overdensity* of galaxies.
- Galaxy catalogs are not complete at higher redshifts, we need to apply corrections in order to now bias our analyses *[R. Gray+, PRD (2019)].*
- ***Open question:*** *How does galaxy properties correlate with CBC hosting? [Perna,* ***SM****, 2405.07904]*
    - *Two main actors: Star Formation rate and total stellar mass [M. Artale +, MNRAS (2021)]*



Dark sirens: Cosmology aided by galaxy surveys


Equal host probability

Red-luminous galaxies preferred hosts



[Mastrogiovanni+, PRD 2023]

GW hosting models will have an important impact for cosmology

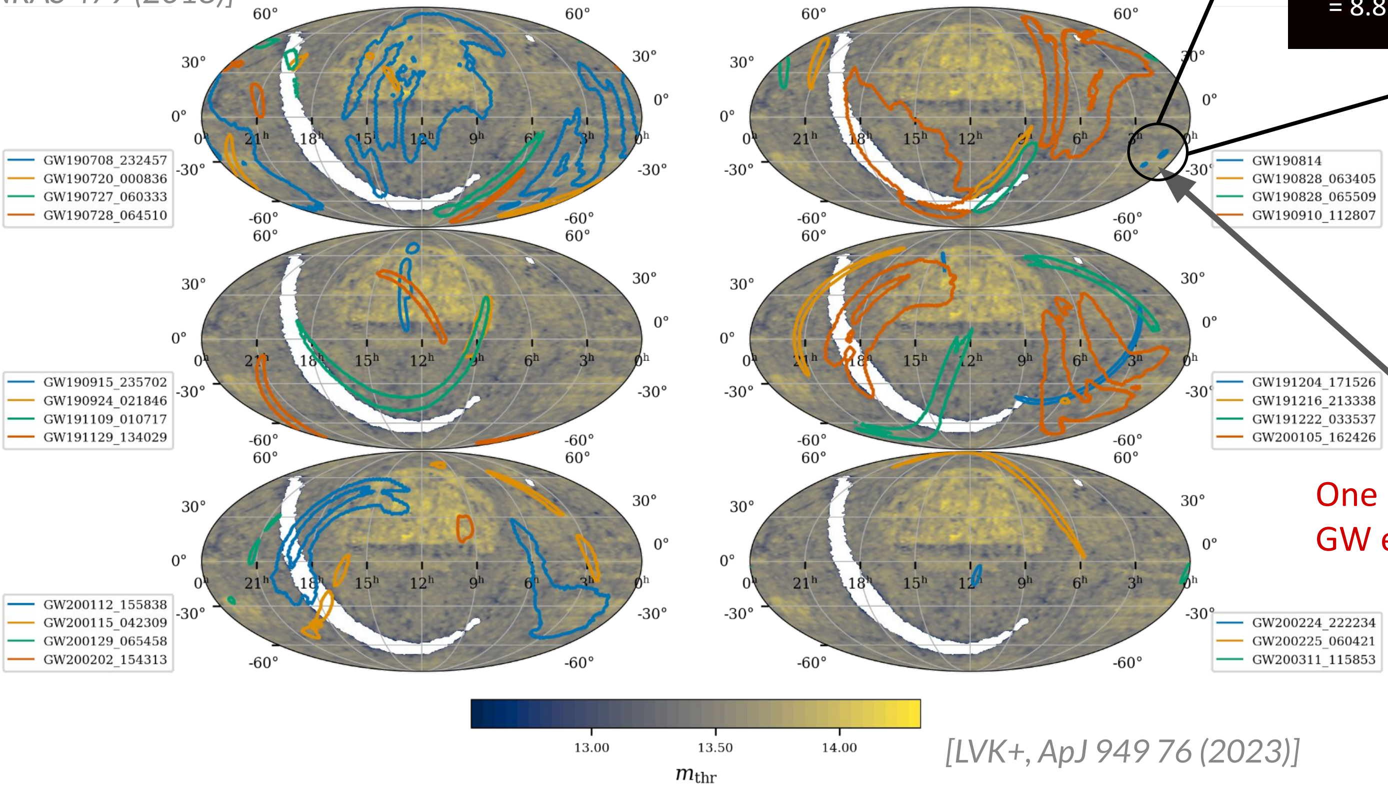
GW cosmology after GWTC-3: Dark sirens

GW190814

Sky area ~ 20 sq. deg.

Localisation volume = 8.8 x 10^{-7} Gpc^3

[Dalya+, MNRAS 479 (2018)]



One of the highly localised GW events

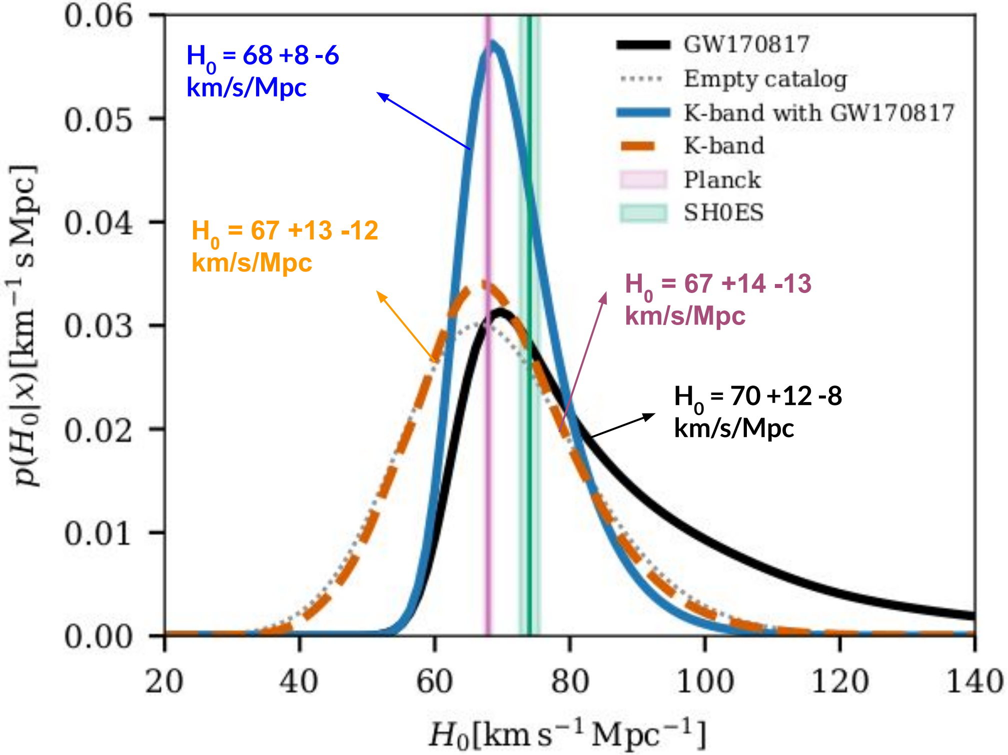
[LVK+, ApJ 949 76 (2023)]

GW cosmology after GWTC-3: Dark sirens

Main result of the paper showing various H0 posteriors.

We select the K-band for the luminosities of galaxies and the preferred mass model (powerlaw+Gaussian peak)

[LVK+, ApJ 949 76 (2023)]



Hierarchical Bayesian inference

Physics	Statistics
Particles energy **E** are generated with a gaussian distribution	$p_{\mathrm{pop}}(E\|\Lambda) = \frac{1}{N}\frac{dN}{dE}$
Particles arrive at our detectors, due to noise I measure **E$_{\mathrm{obs}}$**	$\mathcal{L}_n(E_{\mathrm{obs}}\|E)$
My experiment can detect **E$_{\mathrm{obs}}$** only above a certain threshold	$p_{\mathrm{det}}(E) = \int_{E_{\mathrm{thr}}}^{\infty} \mathcal{L}_n(E_{\mathrm{obs}}\|E) dE_{\mathrm{obs}}$

Hierarchical Bayesian inference

The likelihood for an inhomogeneous Poisson process in presence of selection biases, for a **constant rate in detector time**, is (see Mandel+ 2018 MNRAS, Vitale+ 2020)

Rate

$$\mathcal{L}(x|\Lambda) \propto e^{-N_{\rm exp}} \prod_{i=1}^{N_{\rm obs}} T_{\rm obs} \int \mathcal{L}_n(x|\theta,\Lambda) \frac{dN}{dt d\theta} d\theta$$

Noise process

Expected number of detections

$$N_{\rm exp} = T_{\rm obs} \int p_{\rm det}(\theta,\Lambda) \frac{dN}{dt d\theta} d\theta.$$

Hierarchical Bayesian inference

Integral over the events (numerator): Done summing over posterior samples

$$\int \mathcal{L}_n(x|\theta,\Lambda)\frac{dN}{dtd\theta}d\theta \approx \frac{1}{N_s}\sum_j^{N_s}\frac{1}{\pi_{\rm PE}(\theta_j)}\frac{dN}{dtd\theta}\Big|_j$$

Number of samples

Prior used for Posterior samples

Integral over the events (numerator): Done summing over posterior samples

Number of detected events

$$N_{\rm exp} \approx \frac{T_{\rm obs}}{N_{\rm gen}}\sum_j^{N_{\rm det}}\frac{1}{\pi_{\rm inj}(\theta_j)}\frac{dN}{dtd\theta}\Big|_j$$

Number of injections generated (even not detected)

Prior used for injections

Hierarchical Bayesian inference

The hierarchical likelihood computed numerically is given by the equation below

$$\mathcal{L}(x|\Lambda) \approx e^{-N_{\rm exp}} \prod_{i=1}^{N_{\rm obs}} \frac{T_{\rm obs}}{N_s} \sum_{j=1}^{N_s} \frac{1}{\pi(\theta_{j,i})} \frac{dN}{dtd\theta}\Big|_{j,i}$$

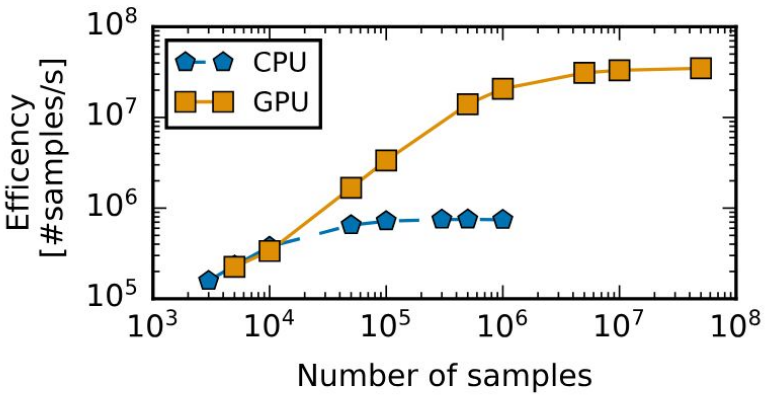
Can we keep sampling this likelihood? Most likely no…

- Problem 1: Timing
- Problem 2: Numerical stability
- Problem 3: Rate modelling (**Physics**) - for another talk :)

Hierarchical Bayesian inference

Solution 1

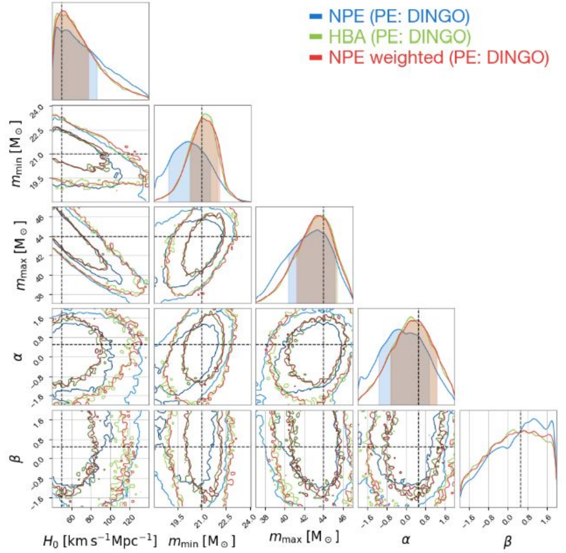
- Graphical processing units offer 2 order of magnitudes of boost



[SM+, A&A 682 (2024)]

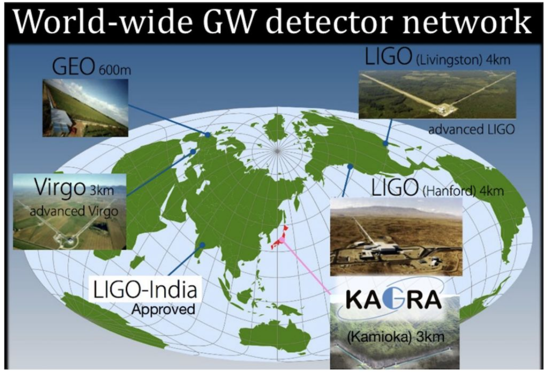
Solution 2

- Likelihood-free approaches (Normalizing flow) requires no likelihood.



[Leyde+, PRD 109 (2024)]

What is next for GW cosmology?



Improving the current detector network < 2035

May 15, 2024

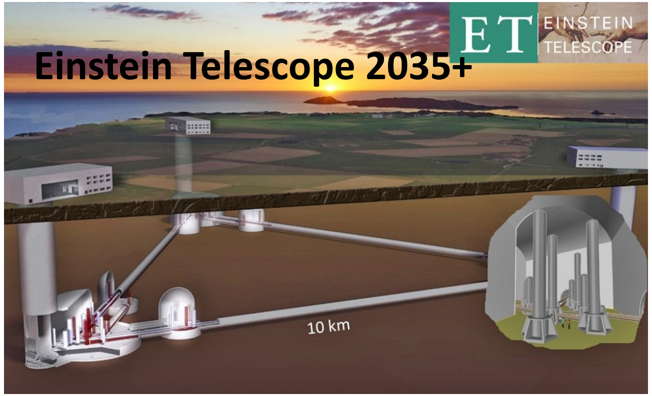
00:53:01 UTC

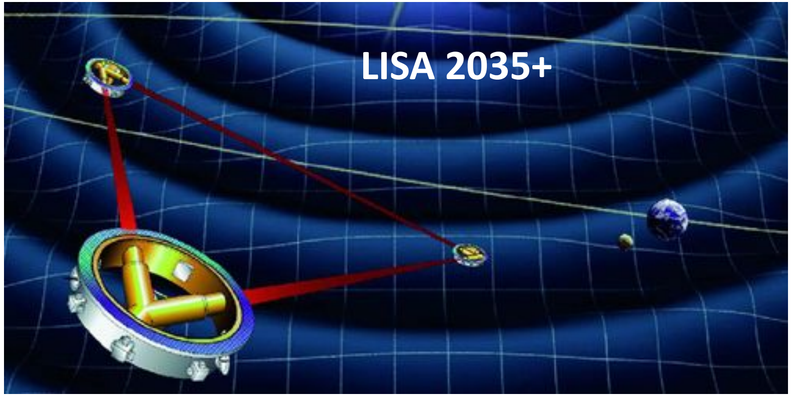
GCN Circular Query

Notices | VOE

1 per 6.0109e+12 years

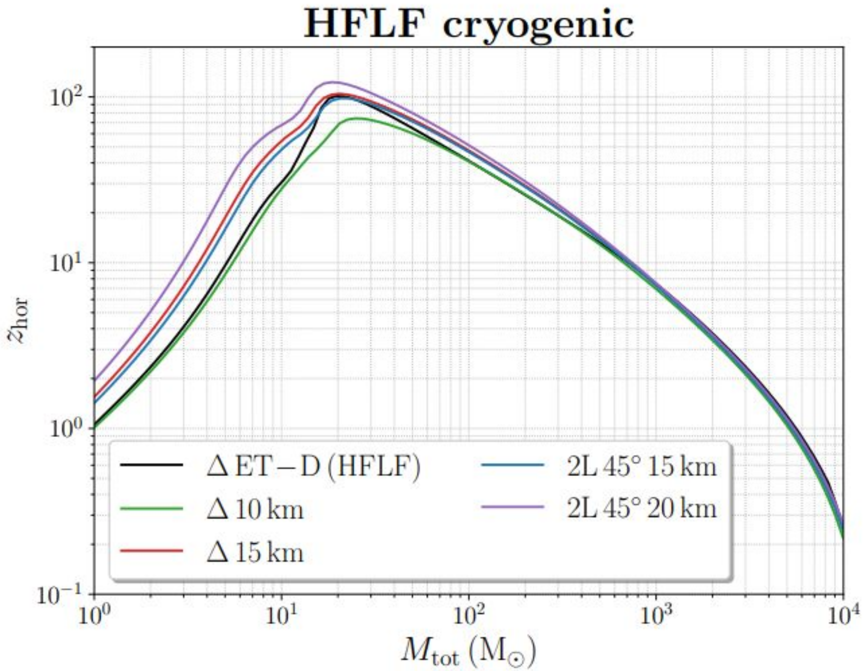
O4 data taking, 97 new candidates





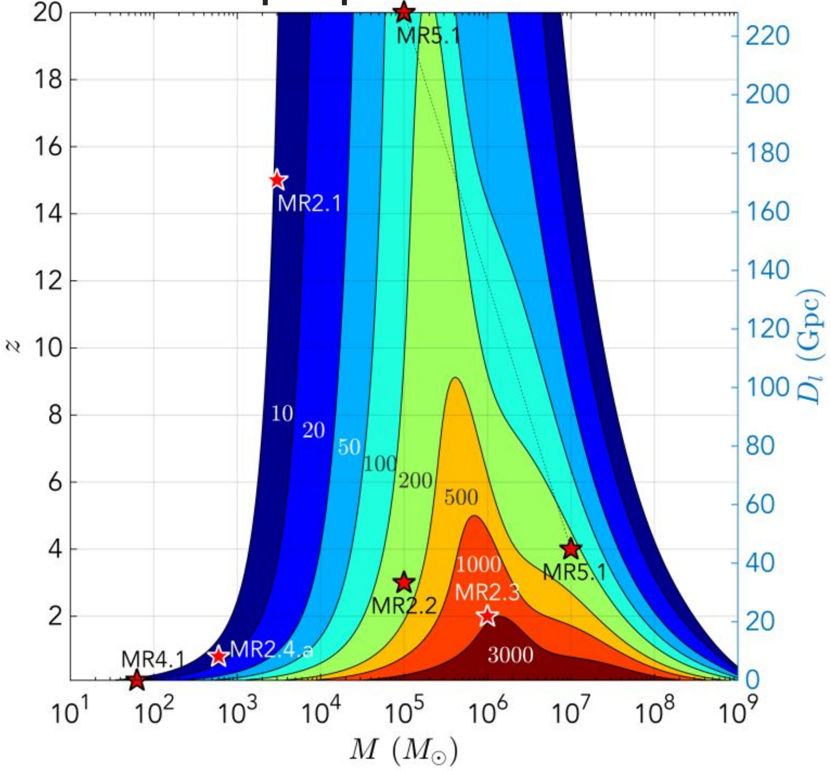
Detection ranges of 3G detectors

Detection prospects for Einstein Telescope



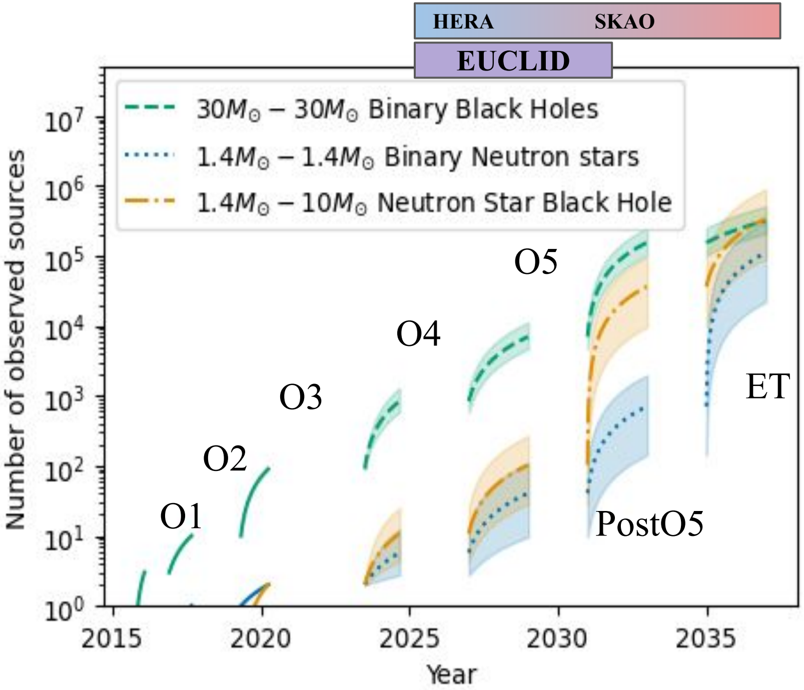
M. Branchesi, JCAP07(2023)068

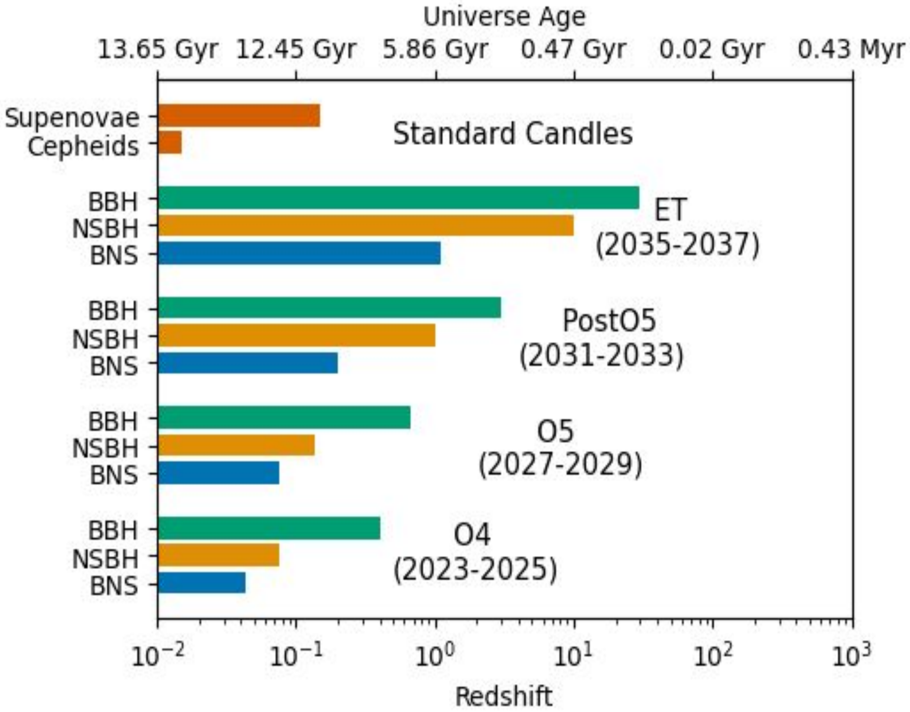
Detection prospects for LISA



P. Amaro-Seoane arXiv 1702.00786

Conclusions: A snapshot for the expected numbers





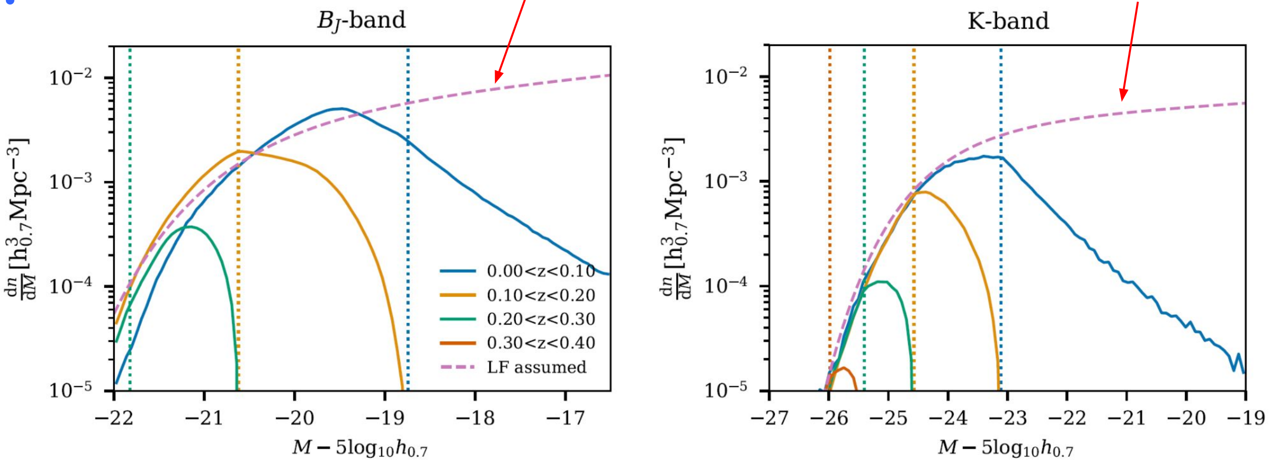
Using expected detection ranges from

Thank you for your attention

Robust assumptions?

Observed vs **predicted** abs mag distributions, binned by redshift

Motivates choice of K-band for our main results

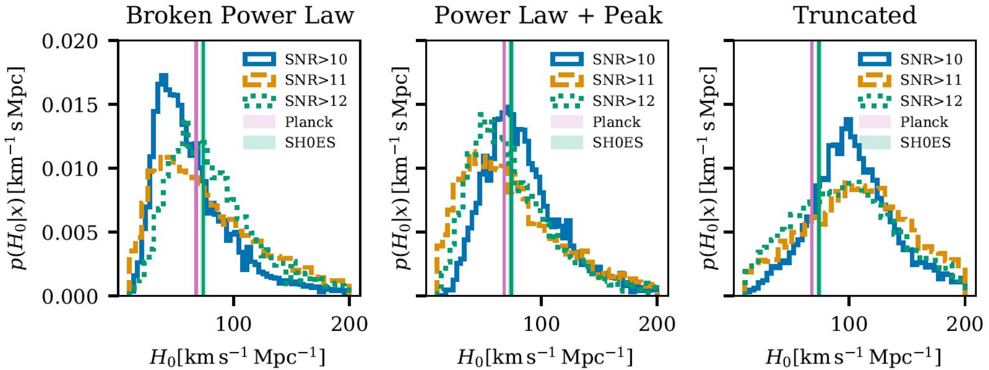


Schechter luminosity function

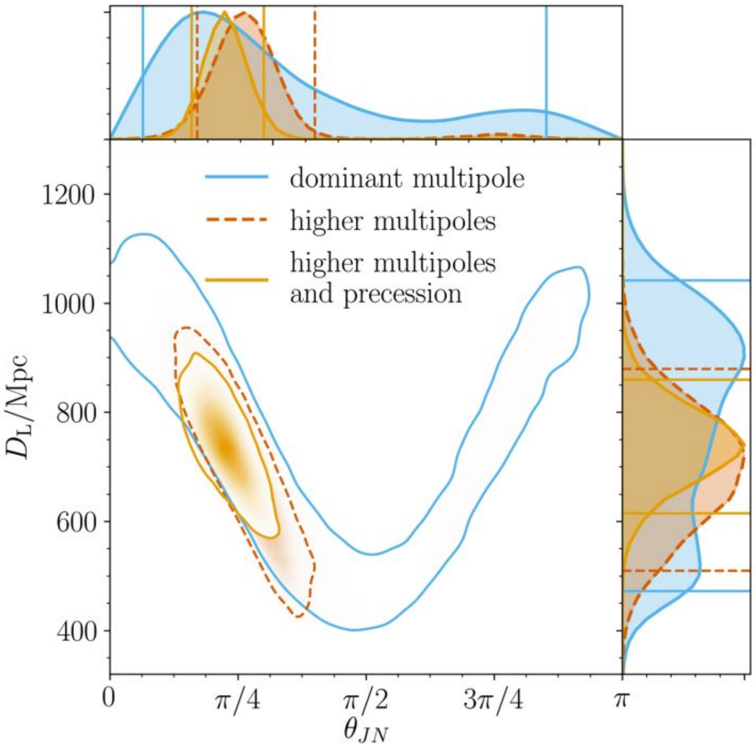
Schechter luminosity function

Changing SNR cut produces **consistent H_0 posteriors.**

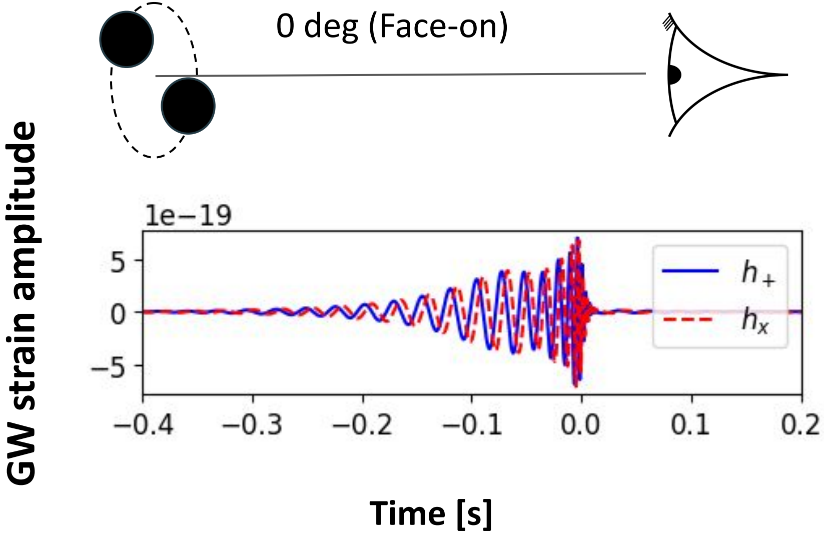
Population excess at ~ **35** $\mathrm{M}_\odot$ observed for each SNR cut



Gravitational Wave sources at cosmological scales



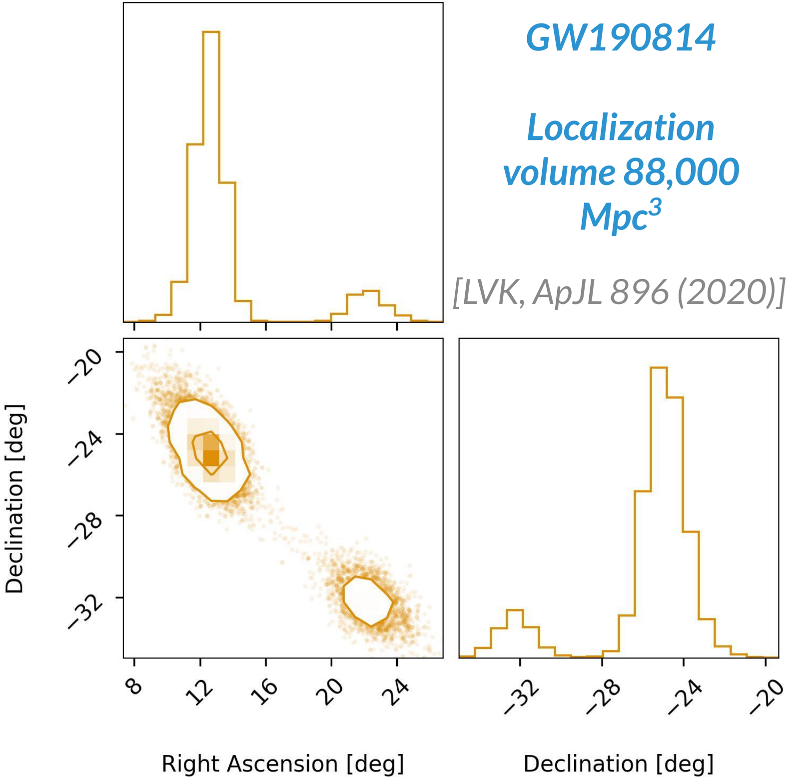
[B. P. Abbott, PRD 102, 043015]



There are large uncertainties on the GW estimation of the luminosity distance. The precision can be improved with (i) extra EM information (ii) precession or higher order modes.

Dark sirens: Cosmology aided by galaxy surveys

- In the case that the GW is not observed with EM counterpart, we can use galaxy catalogs to identify possible galaxy hosts *[Schutz, Nature 1986]*.
- Galaxy surveys will provide possible redshifts.
- GW will provide luminosity distance.
- Best localized events provide better constraints for cosmology.



GW190814

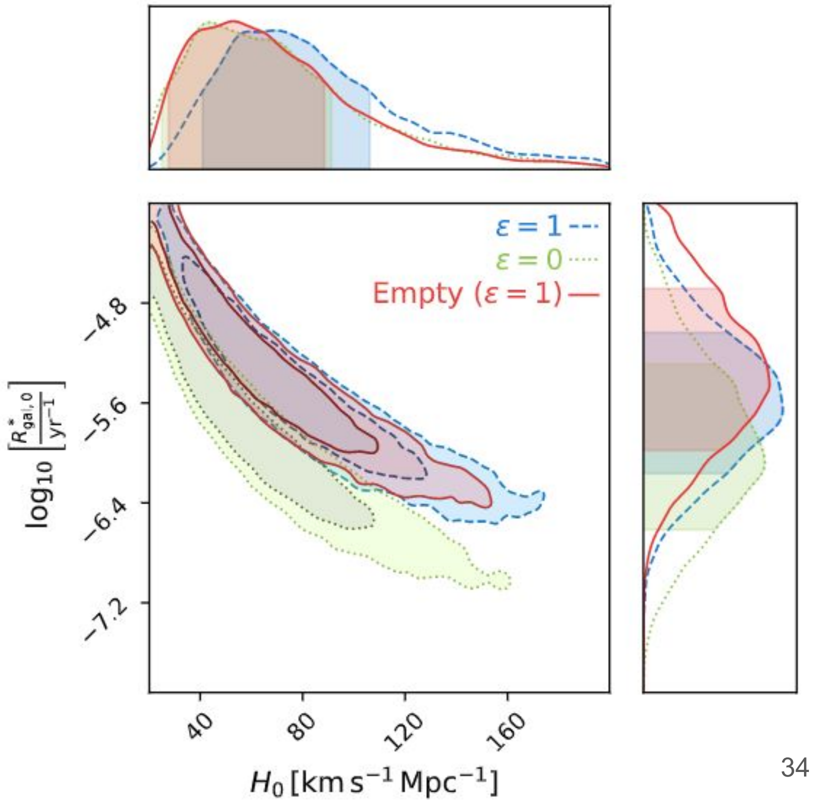
Localization volume 88,000 Mpc^3

[LVK, ApJL 896 (2020)]

GW dark sirens, galaxies and cosmological properties

- CBCs hosted by luminous galaxies:
  - we can exclude lower H0 values
  - The rate of CBC mergers per galaxy should be 1 every 100’000 years.
- CBCs hosted by all the galaxy type:
  - Lower values of H0 can not be easily excluded
  - The rate of CBC merger per galaxy should be 1 every 1,000,000 years.

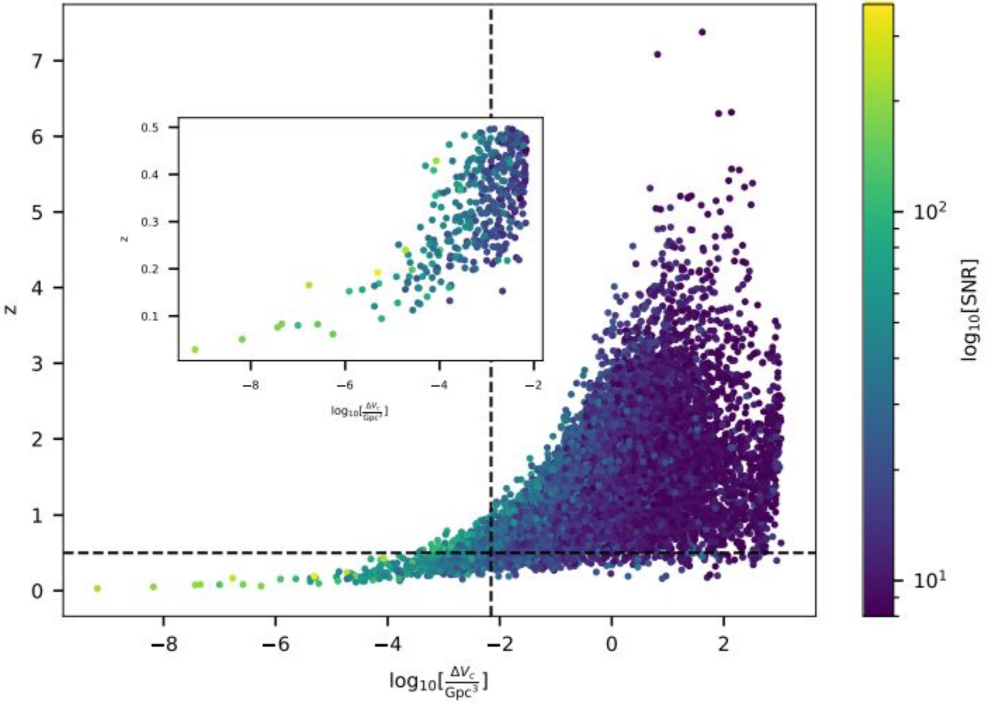
[Mastrogiovanni+, PRD 2023]



Dark sirens with 2.5 Generation GW detectors

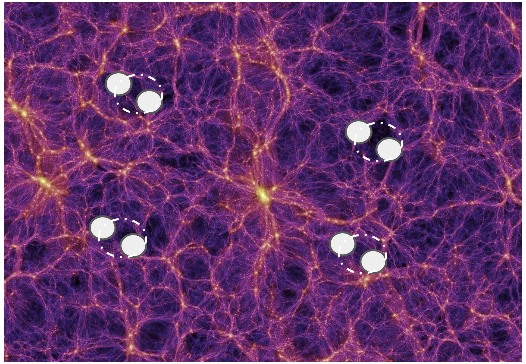
Localization is of crucial importance for the galaxy catalog method.

- About 3000 dark sirens will be localized better than GW190814.
- ~5 dark sirens will be so well localized to have ~1 galaxy in their localization volume.
- ~100 dark sirens will have less than 1000 galaxies in their localization paper.
- With one year of observation, constraint on H0 at the 5% precision

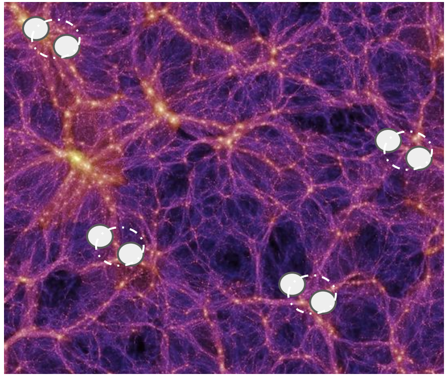


GW and Large-scale structures

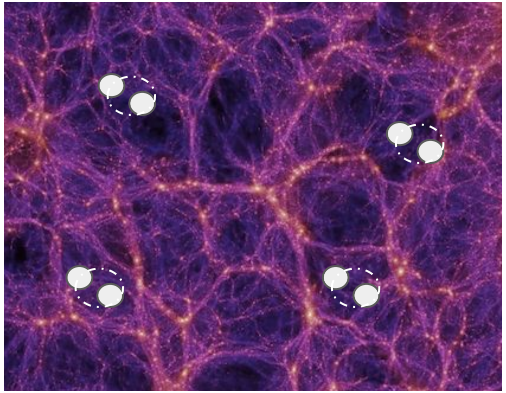
Redshift 0



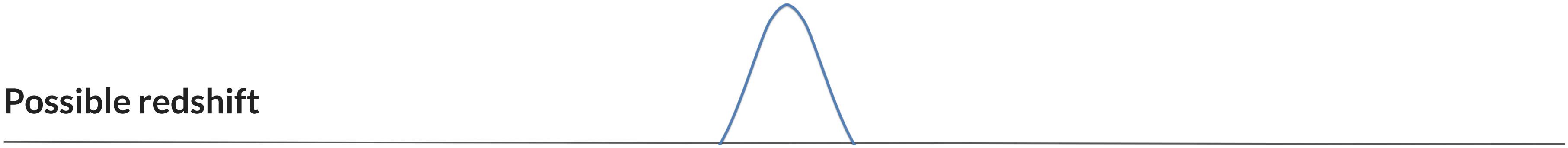
Redshift 1.5



Redshift 2



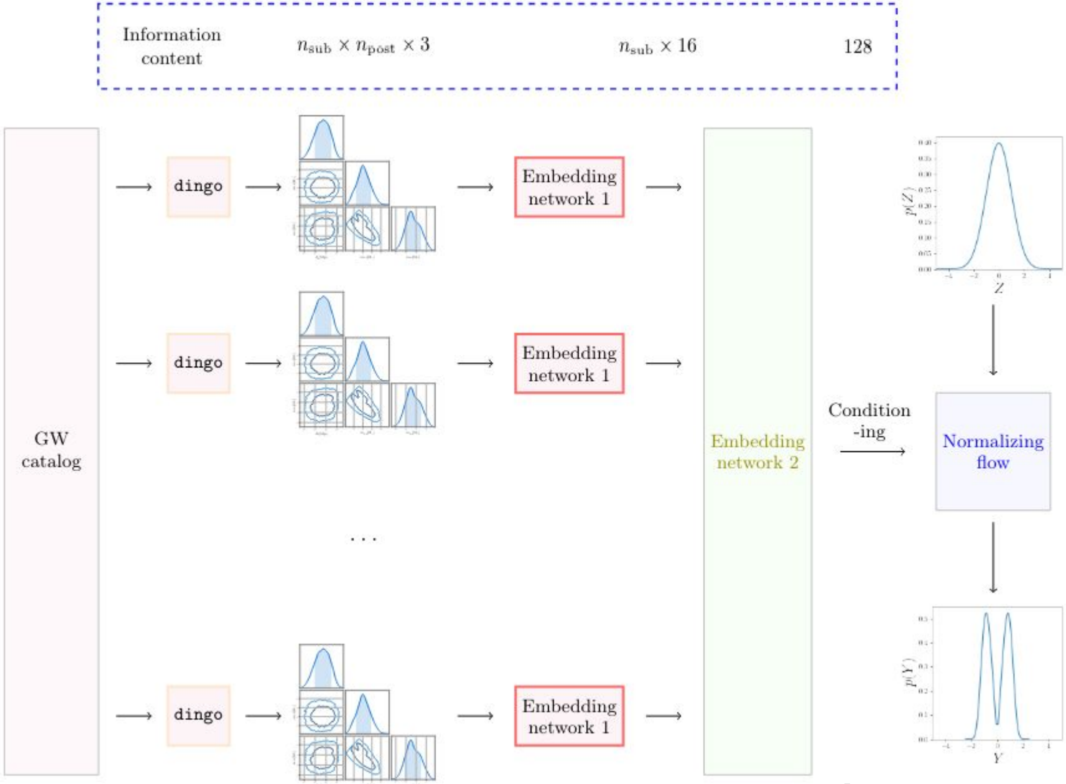
Possible redshift



Open question: How does GW track other Large-scale structure tracers? (Mostly galaxy clusters and HI maps)

[S. Libanore et al JCAP02(2021)035]

Normalizing flows for GW cosmology



Information content $n_{\mathrm{sub}} \times n_{\mathrm{post}} \times 3$ $n_{\mathrm{sub}} \times 16$ 128

GW catalog

dingo

Embedding network 1

Embedding network 2

Condition -ing

Normalizing flow

[Leyde+, PRD 109 (2024)]