

Musings on horizontal gauge symetries



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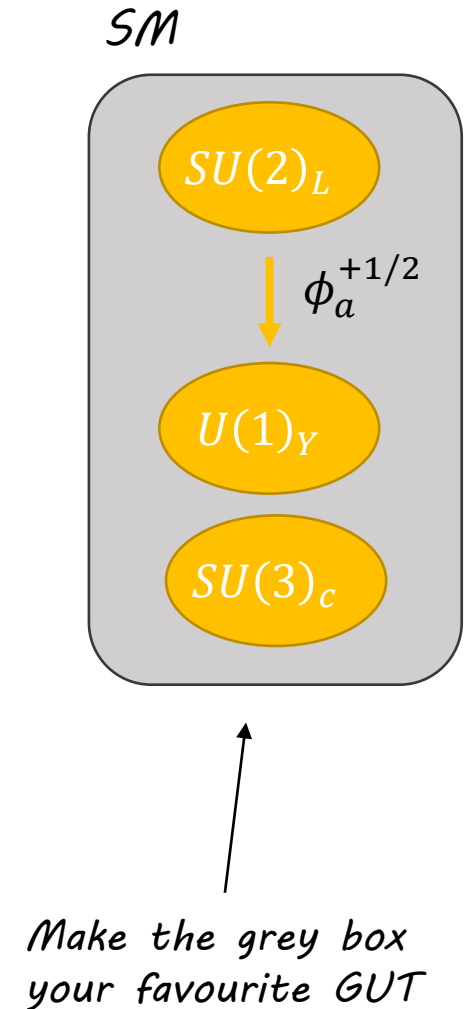
Based on 2307.09595, 2211.05796, 2102.05055 and ongoing works

This work has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 101028626



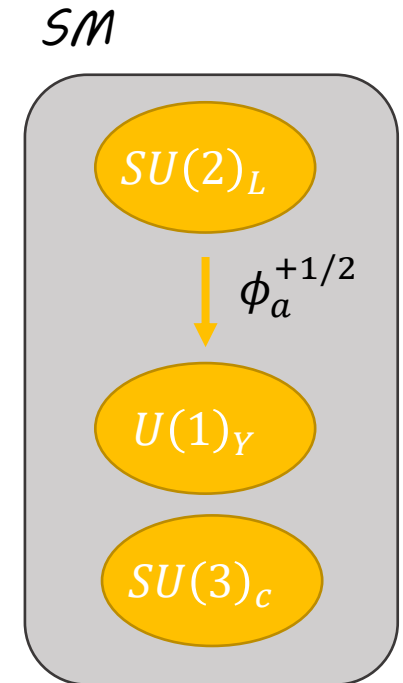
The SM gauge groups and beyond

- The Standard Model gauge content is remarkably anomaly-free and somehow « maximal »
 - It is surprisingly hard to add more gauge structure without adding fermionic matter in the theory



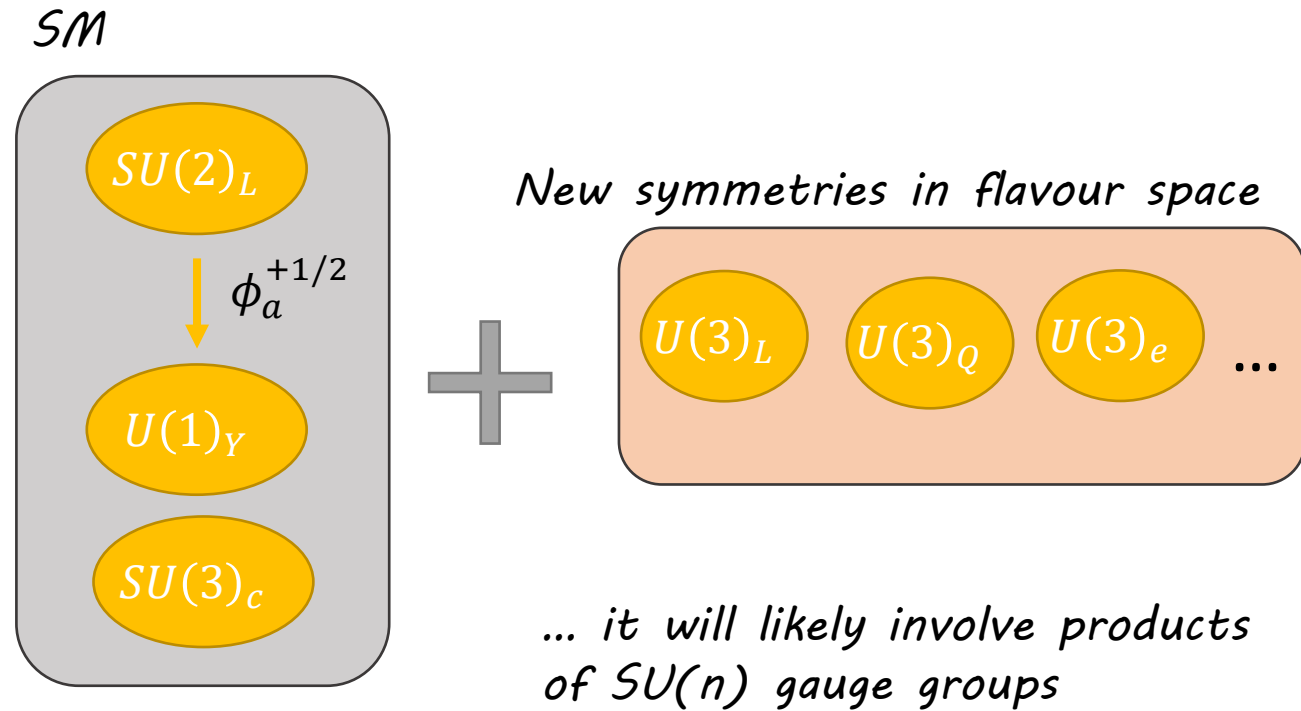
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- The Standard Model gauge content is remarkably anomaly-free and somehow « maximal »
 - It is surprisingly hard to add more gauge structure without adding fermionic matter in the theory
 - The gauge structure is constraining enough to lead to several “accidental” symmetries into the final theory
 - Custodial symmetry
 - Tree-level baryon and lepton number conservation
 - No Majorana mass terms
- How can I add gauge structures without increasing the fermionic content ?



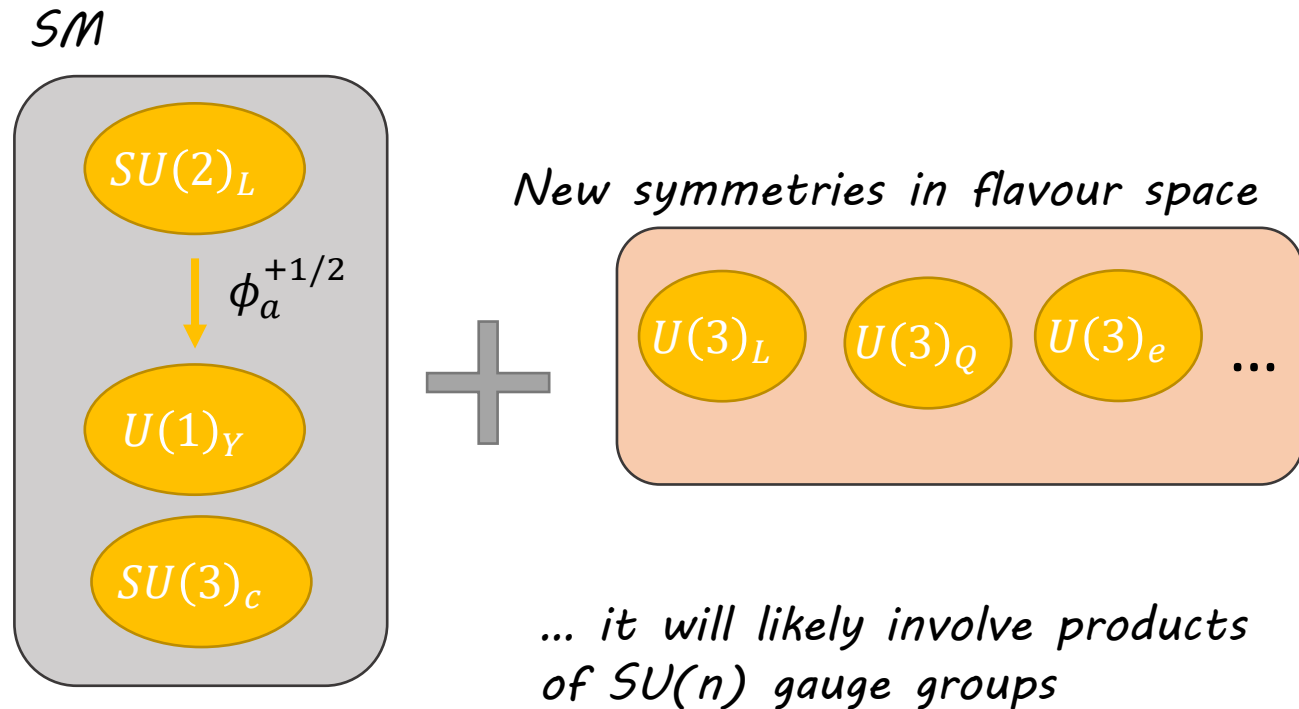
Horizontal gauge symmetries

- The SM has a large global $U(3)^5$ symmetry group
 - broken by the Yukawa interactions
- New « horizontal gauge symmetries », acting mostly in flavour space
 - Will likely adds new structures, both in the fermion and scalar sector of the UV theory



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Generate new accidental symmetries

Structure in the Yukawa interaction (flavour)

Structure for NP processes (flavour transfers)

Horizontal gauge symmetries

- The gauge coupling itself is completely free and we have little guidance on its value
 - Small gauge couplings possible, and thus light new bosons $M_V \propto g_f v_S$
 - Alternative approaches to set this coupling interesting to explore (AS, etc...)

Horizontal gauge symmetries

- The gauge coupling itself is completely free and we have little guidance on its value

→ Small gauge couplings possible, and thus light new bosons $M_V \propto g_f v_S$

→ Alternative approaches to set this coupling interesting to explore (AS, etc...)

- Anomaly cancellation a stringent requirement (may lead to extra required fermions)

$$\sum_{i=1}^3 (6F_{Q_i} + 2F_{L_i} - 3F_{u_i} - 3F_{d_i} - F_{e_i} - F_{\nu_i}) = 0, \quad \sum_{i=1}^3 (F_{Q_i} + 3F_{L_i} - 8F_{u_i} - 2F_{d_i} - 6F_{e_i}) = 0, \quad \sum_{i=1}^3 (3F_{Q_i} + F_{L_i}) = 0,$$

$$\sum_{i=1}^3 (F_{Q_i}^2 - F_{L_i}^2 - 2F_{u_i}^2 + F_{d_i}^2 + F_{e_i}^2) = 0, \quad \sum_{i=1}^3 (2F_{Q_i} - F_{u_i} - F_{d_i}) = 0, \quad \sum_{i=1}^3 (6F_{Q_i}^3 + 2F_{L_i}^3 - 3F_{u_i}^3 - 3F_{d_i}^3 - F_{e_i}^3 - F_{\nu_i}^3) = 0,$$

... for adding a single $U(1)$

- Of course, gauging the flavour space leads to strong constraint from flavour ...

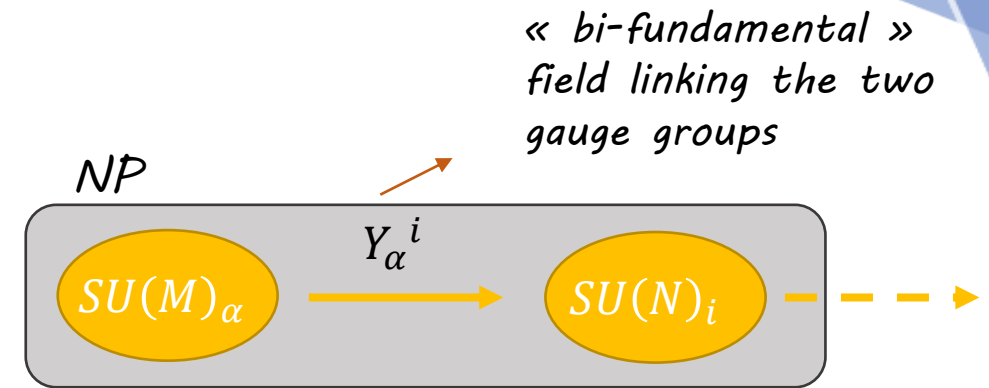
Semi-simple gauge groupes and rectangular symmetries

... or what happens when you gauge a big semi-simple gauge group

Based on 2211.05796, 2102.05055 with E. Nardi and C. Smarra

Rectangular gauge groups

- Semi-simple gauge groups of the form $SU(M) \times SU(N)$, with $M > N$
 - Invariance under such gauge groups is very constraining on effective operators in the scalar sector



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- The scalar fields are rectangular matrices

→ The hermitian terms are quite simple with a structure close to the SM Higgs one

→ Automatically invariants global re-phasing $U(1)$ symmetries

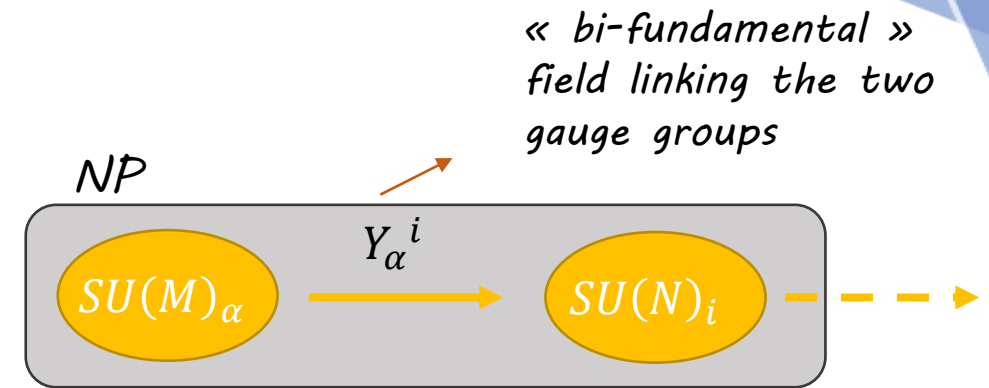
→ Such $U(1)$ are only broken by operators which are non-hermitians

$$T \equiv \text{Tr} (Y^\dagger Y)$$

$$A = \frac{1}{2}(T^2 - T_4)$$

$$T_4 \equiv \text{Tr} (Y^\dagger Y)^2$$

$$V(Y) = \kappa (T - \mu_Y^2)^2 + \lambda A$$



$$Y = \begin{pmatrix} Y_1^1 & \dots & Y_1^N \\ \vdots & \ddots & \vdots \\ Y_{M-1}^1 & \dots & Y_{M-1}^N \\ Y_M^1 & & Y_M^N \end{pmatrix}$$

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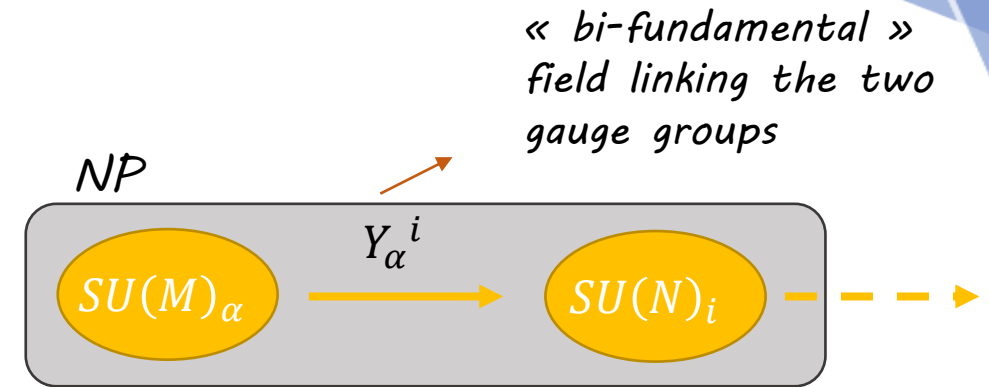
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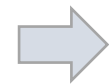


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$$T \equiv \text{Tr}(Y^\dagger Y) \quad \text{In eigenvalues ...}$$

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$$T(\hat{Y}) = \sum_{i=1}^N y_i^2,$$

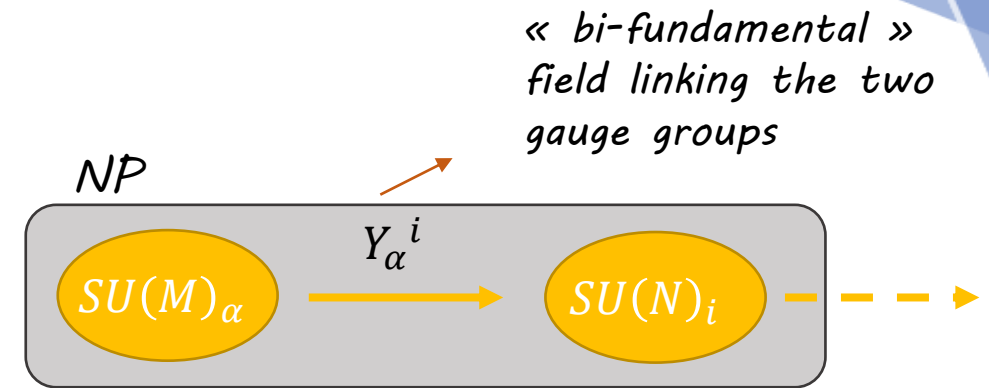
Like a
« radius »

$$A(\hat{Y}) = \sum_{i < j} y_i^2 y_j^2.$$

Vanishes if a
single non-zero y_i

$$V(Y) = \kappa (T - \mu_Y^2)^2 + \lambda A$$

Rectangular gauge groups (2)



- Non-hermitians operators are very constrained

→ Either form « cycles » of the diagrams or constructed from ϵ -tensors, which have a strong tendency to vanish

$$\epsilon^{\alpha_1 \dots \alpha_M} Y_{\alpha_1, i_1} \dots Y_{\alpha_M, i_M} \equiv (\epsilon_M Y^M)_{i_1 \dots i_M}$$

Always vanishes when $M > N$ since it must have two redundant i indices (there are only N possibilities, but we must have $M > N$ indices...)

Similar to $\epsilon^{ab} H_a H_b = 0$ in SM

- Accidental global U(1) symetries (from rephasing of the scalar fields) can easily occur

→ And the non-hermitian U(1) breaking operators can be made naturally small

A first use: shaping the scalar potential

- Rectangular symmetries are powerful tools to « shape » the scalar potential of the theory:
→ A simple $SU(3) \times SU(2)$ example



$$V(Y, Z) = (\kappa_Y (T_Y - \mu_Y^2)^2 + \lambda_Y A_Y) + V(Z) + V(Y, Z) + A_3 \epsilon^{\alpha_1 \alpha_2 \alpha_3} \epsilon_{i_1 i_2} Y_{\alpha_1}^{i_1} Y_{\alpha_1}^{i_2} Z_{\alpha_3} + \dots$$

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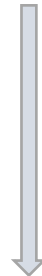
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Negative squared term, Y
will pick up a VEV v_Y

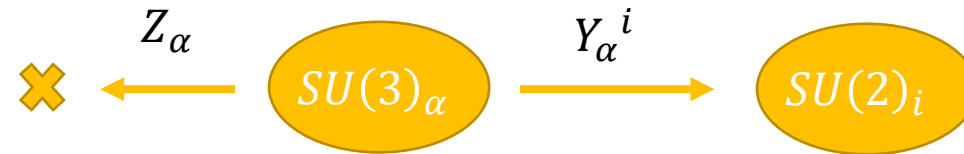


Choosing $\lambda_Y < 0$ align the
VEVs of Y

$$Y \propto \begin{pmatrix} v_Y & 0 \\ 0 & v_Y \\ 0 & 0 \end{pmatrix}$$

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Negative squared term, Y will pick up a VEV v_Y

$\mu_Z^2 > 0$ implies that Z does not pick up a VEV

Non-hermitian contribution $\propto A_3 \cos(\phi_{A_3}) v_Y^2 Z_3$ must always be maximised → triggers a VEV for Z :

Choosing $\lambda_Y < 0$ align the VEVs of Y

$$Y \propto \begin{pmatrix} v_Y & 0 \\ 0 & v_Y \\ 0 & 0 \end{pmatrix}$$

(Known setup to generate tiny V EVs)

$$v_{Z_3} \propto \frac{A_3 v_Y^2}{\mu_Z^2}$$

A second use: flavour symmetries and axions

- Very well protected global U(1) symmetries have a range of applications, and can be useful to generate axion

$$V(a, \pi^a) = -m_\pi^2 f_\pi^2 \cos\left(\frac{\pi}{f_\pi}\right) + (PQ \text{ breaking terms}) \\ + \frac{1}{2} \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f_\pi^2}{f_a^2} a^2 \cos\left(\frac{\pi}{f_\pi}\right) + \mathcal{O}\left(\frac{a^3}{f_a^3}\right)$$

→ Stringent criterium on the Peccei-Quinn symmetry (PQ): it must be endowed with a $U(1)_{PQ} \times SU(3)_C^2$ anomaly, while being protected in effective operators up to dimension ~ 10

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- The PQ « quality problem » thus requires an very-well protected global symmetry
 - We can use a rectangular gauge group to do the job !
 - That means charging quarks under the rectangular gauge groups, leading to two main problems

Avoid anomalies (we must be careful with the quarks representations)

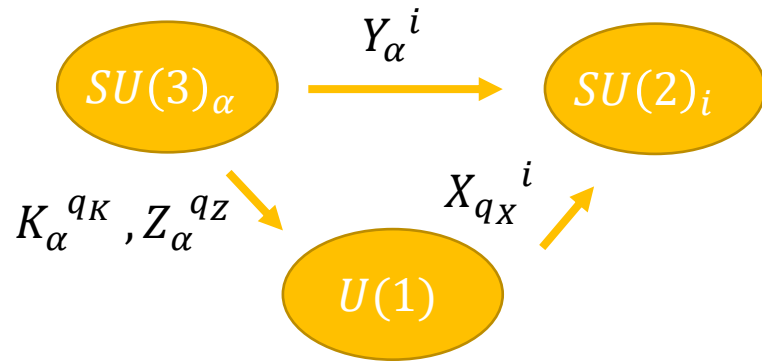
Fully break the horizontal gauge group → must include more scalar fields, thus leading to more possible non-hermitian terms

An explicit example

- Goal : Build an horizontal gauge group model **reproducing the SM fermion mass hierarchies AND preserving a high-quality accidental PQ global symmetry** solving the strong CP problem

→ Extra $U(1)$ needed to ensure simultaneously a QCD anomaly and non-zero quark masses

→ Need new VL pairs for the quark mass generations
→ Standard 2HDM Higgs structure to generate the axion



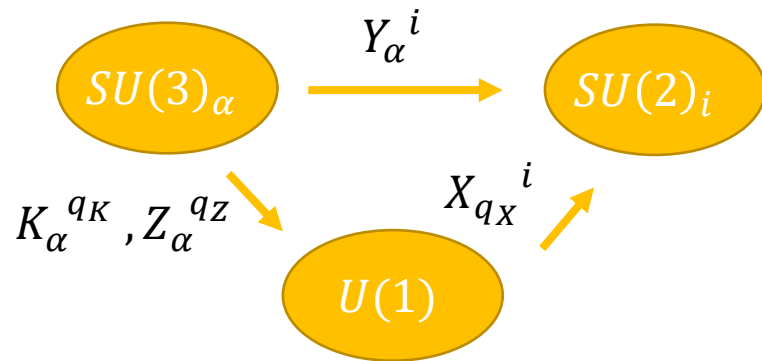
$$\mathcal{M}_u = \begin{pmatrix} u_R & u_R & t_R & U_R & U_R & U_R & Q_R \\ 0 & 0 & 0 & v & 0 & 0 & z_1 \\ 0 & 0 & 0 & 0 & v & 0 & z_2 \\ 0 & 0 & 0 & 0 & 0 & v & z_3 \\ 0 & 0 & v & 0 & 0 & 0 & M \\ \Lambda_u & 0 & x_1^* & y_1^* & 0 & 0 & 0 \\ 0 & \Lambda_u & x_2^* & 0 & y_2^* & 0 & 0 \\ x_1 & x_2 & \Lambda_t & z_1^* & z_2^* & z_3^* & v \end{pmatrix} \begin{matrix} q_L \\ q_L \\ q_L \\ Q_L \\ U_L \\ U_L \\ T_L \end{matrix},$$

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It works !



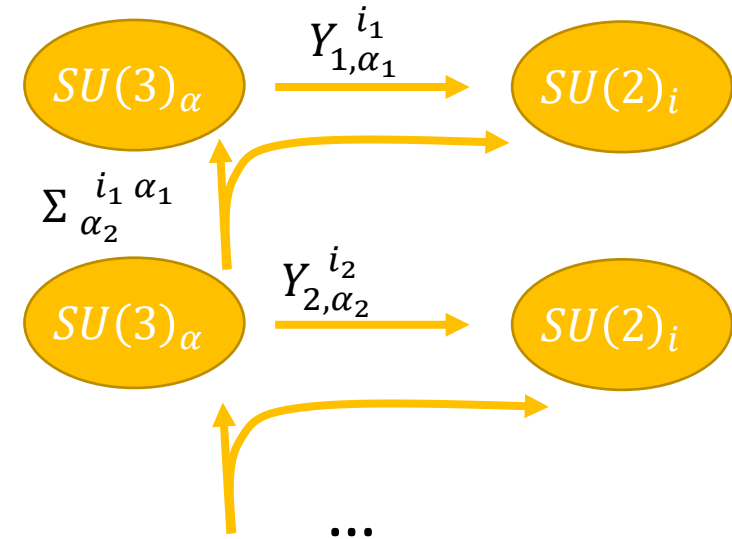
VEVs hierarchies arise naturally from the structure of the potential



Several new fields required, including « redundant » scalar fields

An other funny possibility, creating clockworks

- Start from a theory with long « quiver-like » chains of gauge groups
 - The scalar sector link each gauge groups together
 - The renormalisable non-hermitian part of scalar potential is extremely constrained with only terms of the form :

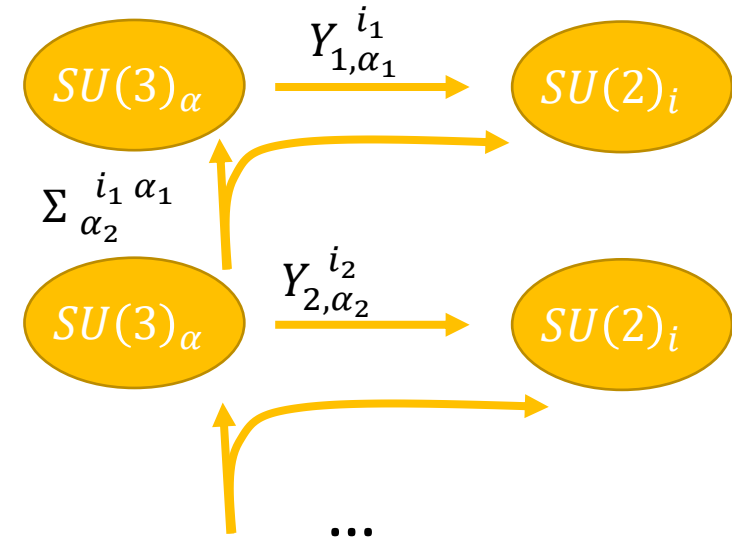


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These are the only allowed non-hermitian terms !

$$V \supset \sum_{p=2}^n (\epsilon_3 \epsilon_2 Y_{p-1}^2 \Sigma_p)^{\alpha_p i_p} Y_{p \alpha_p i_p}$$

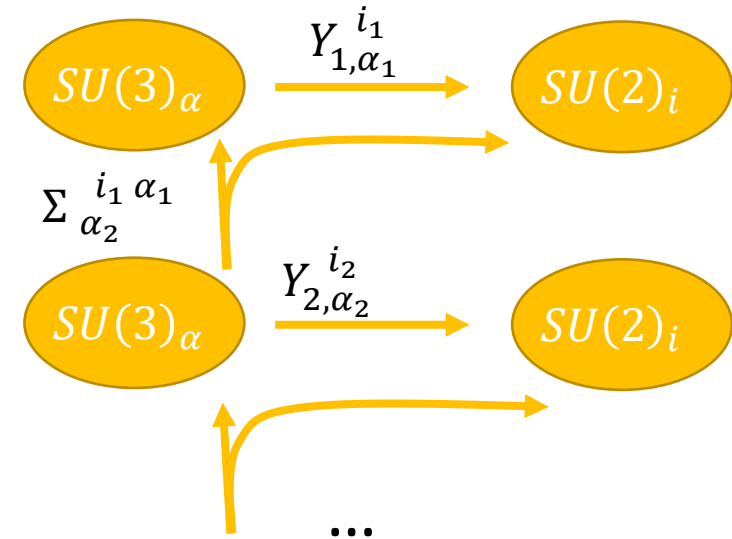


The residual PQ symmetry, presents typical clockwork-like charges

$$\tilde{\chi}_{Y_p} = (-2)^p \quad \tilde{\chi}_\Sigma = 0$$

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The VEVs of each fields can **decrease as a power-law** since each gear in Y_{p-1}^2 induces a linear term for Y_p

$$(\epsilon_3 \epsilon_2 Y_{p-1}^2 \Sigma_p)^{\alpha_p i_p} Y_{p \alpha_p i_p}$$

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Flavoured horizontal symmetries and the flavour problem

SU(2) flavour gauge groups

- Starting point: **add a new SU(2) gauge group in the SM, acting on flavour space**
 - The « charged » SM fermion can be either part of a doublets or a triplet
 - Only the mixed $SU(2)_f^2 \times U(1)_Y$ anomaly is non-zero

$$\mathcal{A} = ([C(Q_i) - C(L_i)] - [2C(u_{R,i}) - C(d_{R,i}) - C(e_{R,i})])$$

- In absence of new low-energy fermions, there is a finite (and quite small) number of possible combination !
 - Left-handed (or right-handed) : Q_i, L_i (or $d_{R,i}, u_{R,i}, e_{R,i}$)
 - Lepton (or baryon) : $Q_i, d_{R,i}, u_{R,i}$ (or $L_i, e_{R,i}$)
 - SU(5)-motivated $Q_i, u_{R,i}, e_{R,i}$ (or $L_i, d_{R,i}$)

Masses and textures (1)

- The presence of $SU(2)_f$ implies that the fermion mass matrices have a structure: let us focus on a left-handed model with Q_i, L_i
 - We introduce δY_i , a $SU(2)_f$ spurion
 - In the most generic case, this does not distinguish first and second generation

$$L \supset y_d^\alpha \delta Y_i \bar{Q}^i \cdot H d_{R,\alpha} + \tilde{y}_d^\alpha \delta Y^{\dagger,i} \epsilon_{ij} \bar{Q}^j \cdot H d_{R,\alpha} + Y_{3,d} \bar{Q}_3 \cdot H b_R$$

α are generation indices but NOT gauge indices

i, j are $SU(2)_f$ gauge indices

We use the $U(3)_f$ global reparametrisation for $d_{R,\alpha}$

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$\delta Y_i = (\delta Y, 0)$

$$L \supset \delta Y (\bar{Q}^1 \cdot H (y_d^\alpha d_{R,\alpha}) - \delta Y (\bar{Q}^2 \cdot H (\tilde{y}_d^\alpha d_{R,\alpha}) + Y_{3,d} \bar{Q}_3 \cdot H b_R$$

↓

Arranging $\delta Y \ll Y_3$ still leads to the same mass scale for first and second generation

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Masses and textures (2)

- How can we generate a hierarchy between 1st and 2nd generation ?

→ Standard approach: add another U(1) factor distinguishing 1st and 2nd

→ We take a step back and realise that y_d^α and \tilde{y}_d^α are not necessarily independent parameters

→ Let's consider a simple model with a $SU(2)_f$ breaking scalar S_i and a VL quark

*and therefore
a new
spurion...*

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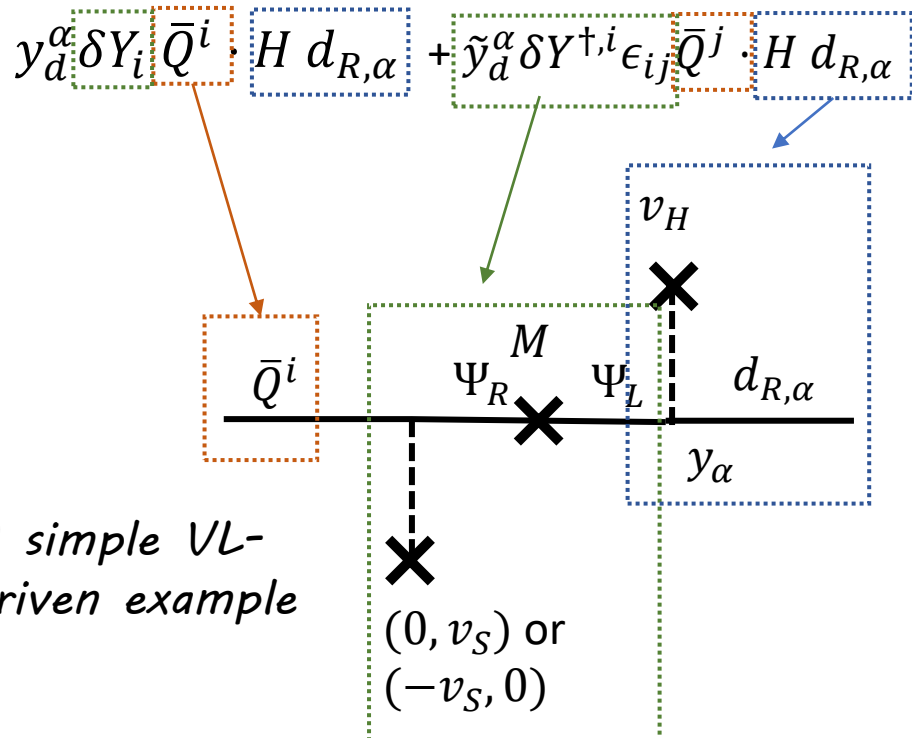
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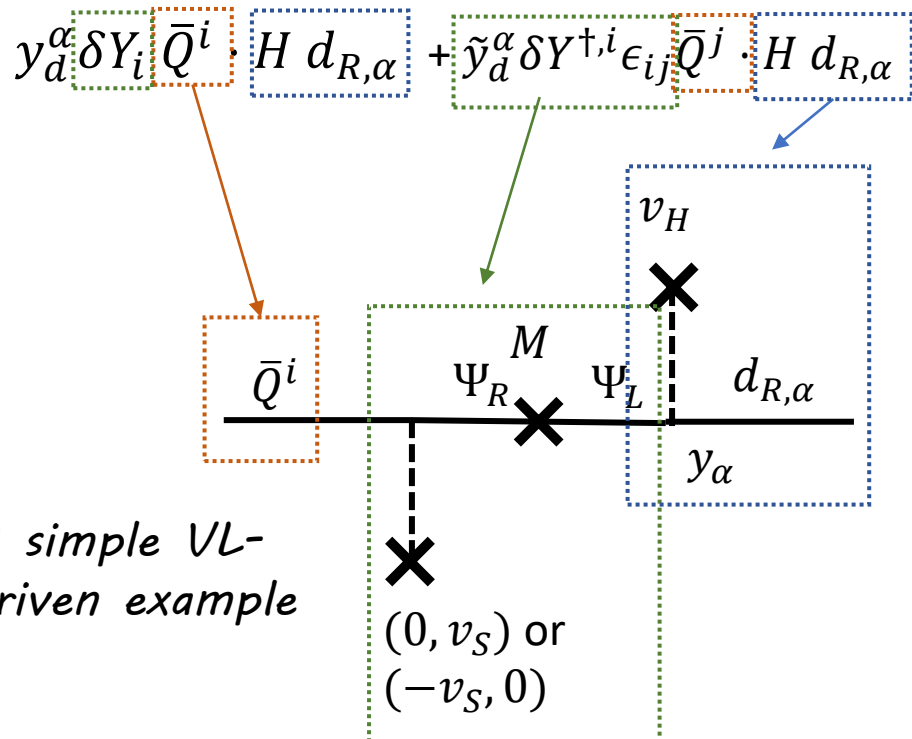
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Leads to $y_d^\alpha \propto \tilde{y}_d^\alpha$

→ The down-quark mass matrix is only rank 2

$$L \supset \delta Y (\bar{Q}^2 \cdot H (y_d^\alpha d_{R,\alpha}) + Y_{3,d} \bar{Q}_3 \cdot H b_R$$

→ Repeat for the third generation

Rising through the ranks

*Coined by Greljo et al.
2309.11547*

- Main idea : generate the spurions « by steps », with two scales to ensure a hierarchy
 - Use the reparametrisation on right-handed particle to put the spectrum in triangular form
 - Generate both spurions using different mechanisms

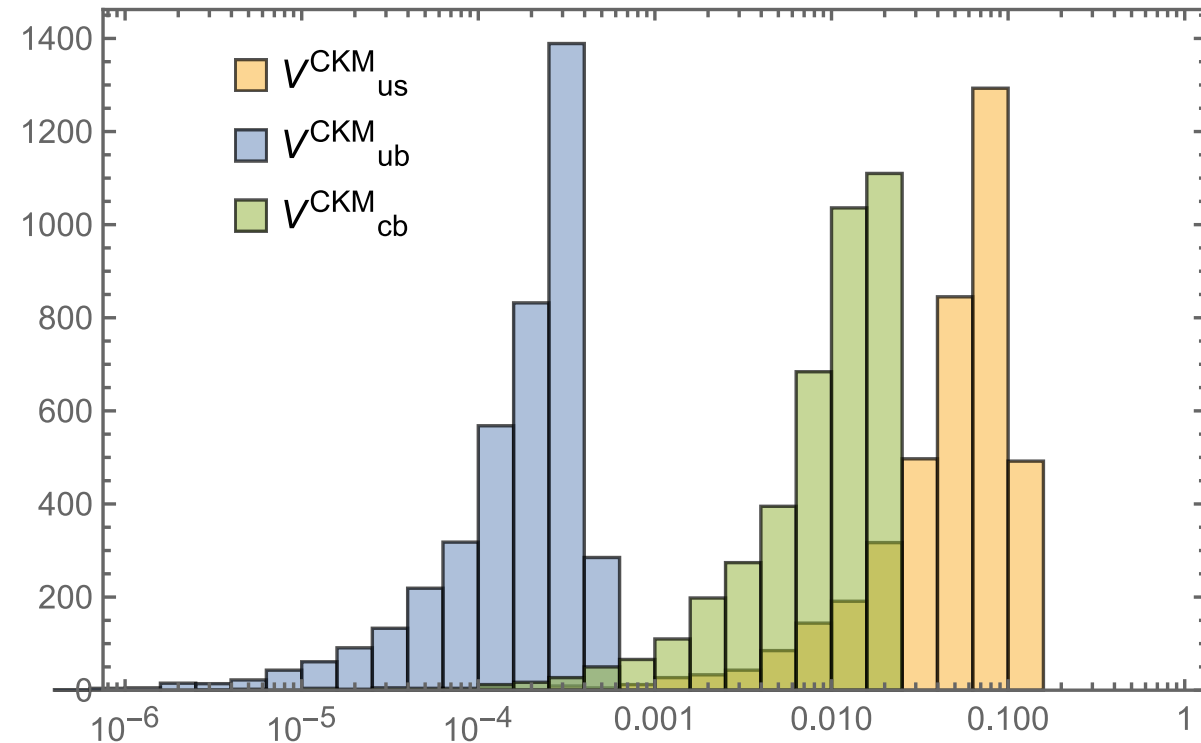
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2309.11547 used a VL for 2nd generation, and loop-induced LQ-driven contribution for the 1st generation, but all standard techniques can be used here

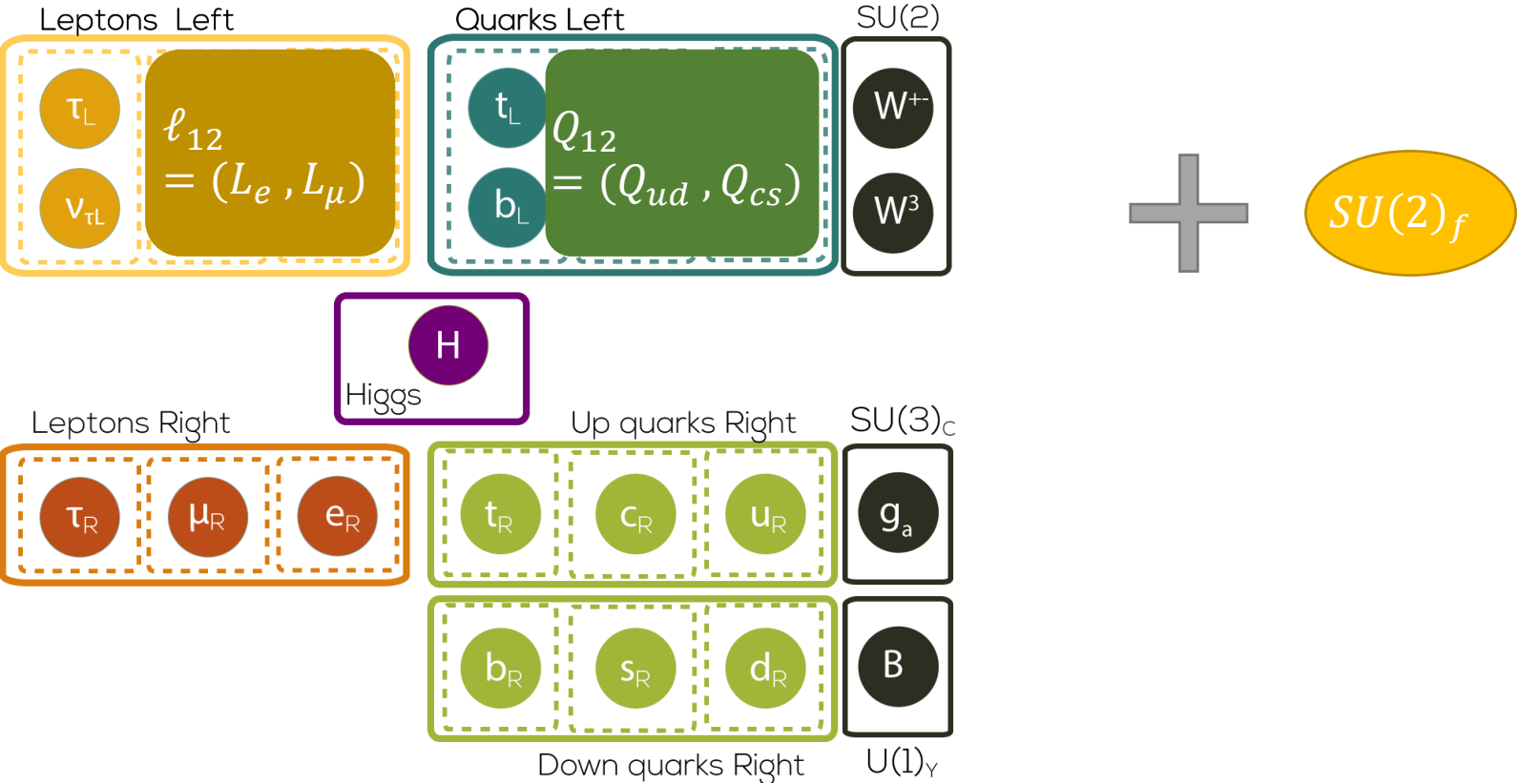
$$Y_d = V_Z \begin{pmatrix} z_{d1b} & z_{d2b} & z_{d3b} \\ & y_{d2a} & y_{d3a} \\ & & x_{d3} \end{pmatrix}, Y_e = V_Z \begin{pmatrix} z_{e1b} \\ z_{e2b} & y_{e2a} \\ z_{e3b} & y_{e3a} & x_{e3} \end{pmatrix}$$



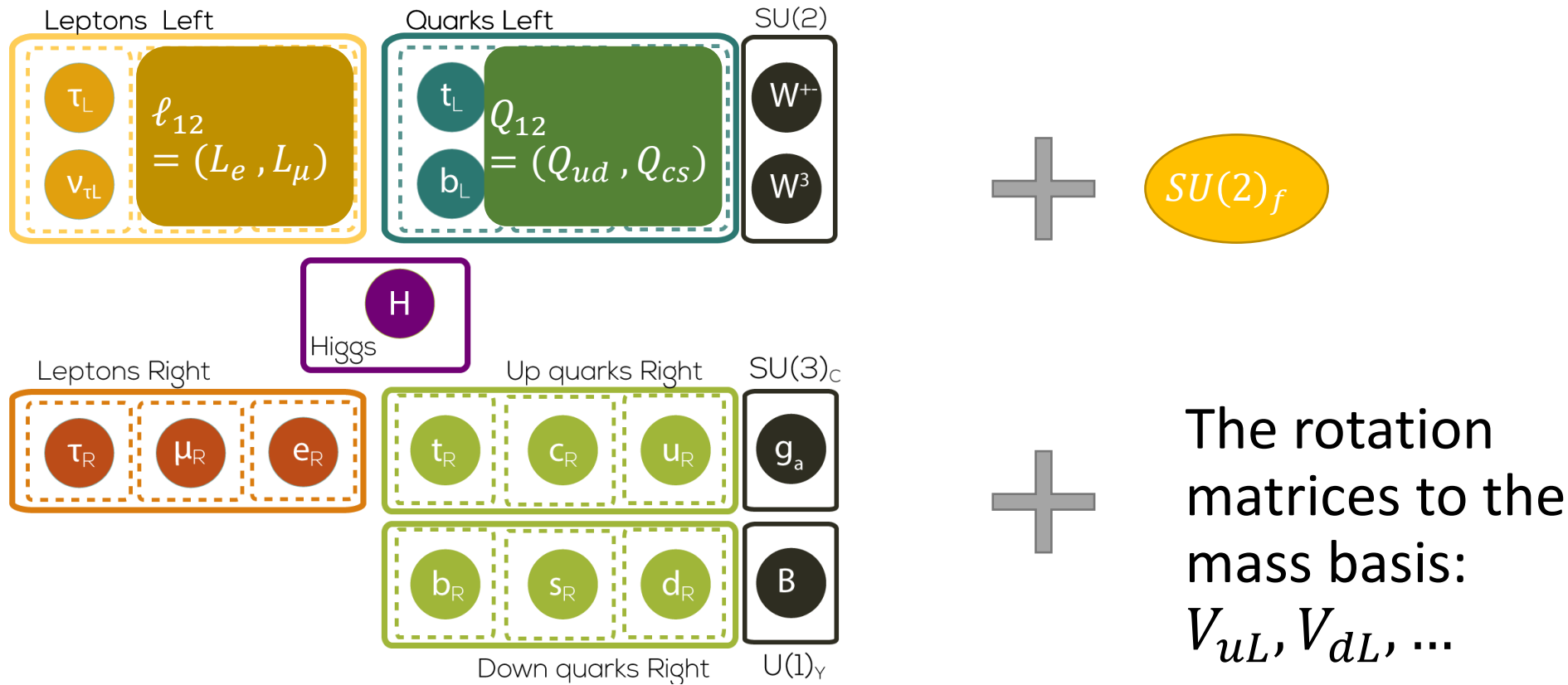
Flavour transfers

Based on 2307.09595 with A. Deandrea and N. Mahmoudi

Beyond textures : low-energy $SU(2)_f$?



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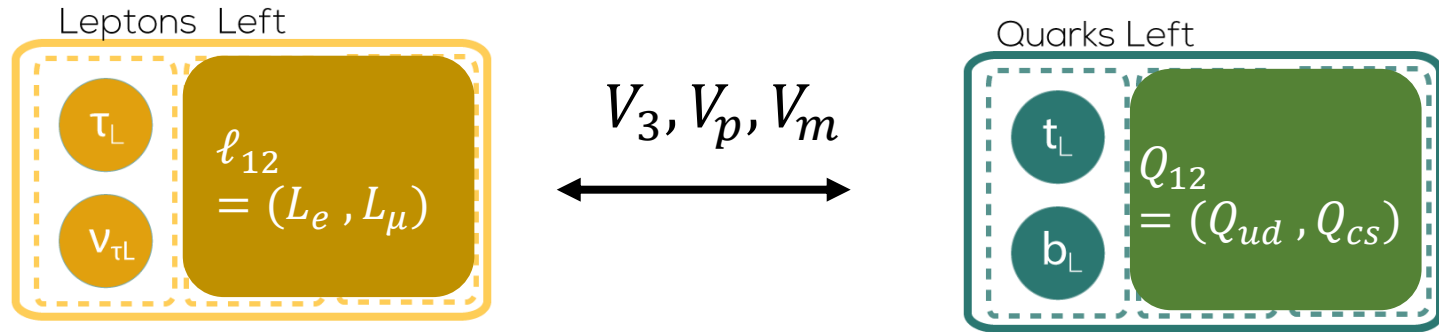


- Three new gauge bosons with mass M_V gauge coupling g_f

$$V_3, V_p, V_m \longleftrightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{The corresponding generators in flavour space}$$

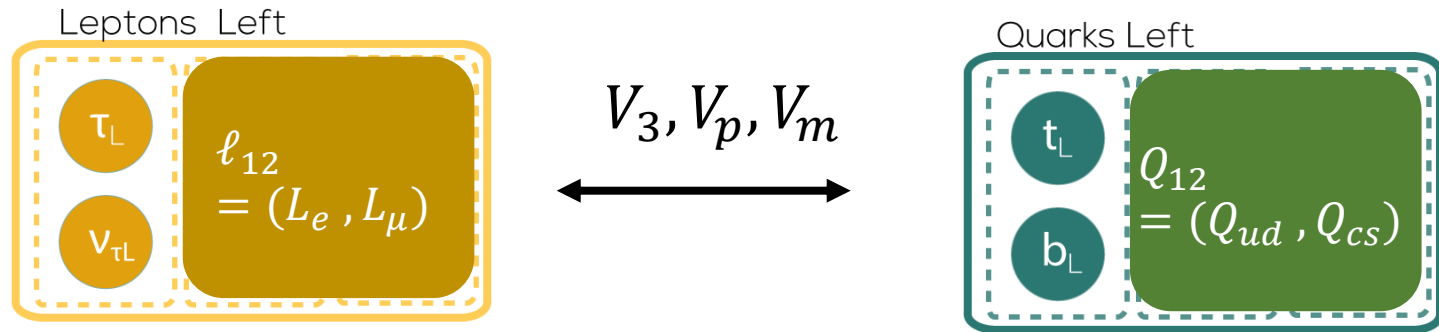
Flavour transfer

- The key point: new flavour gauge boson do not « break » flavour, they only transfer it from one fermionic sector to another



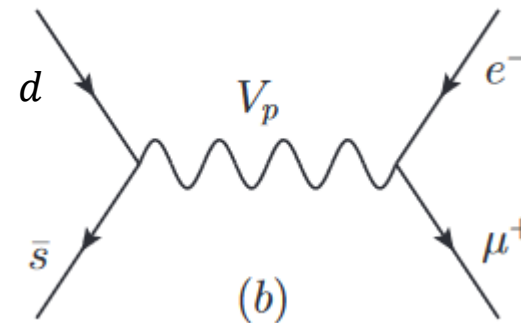
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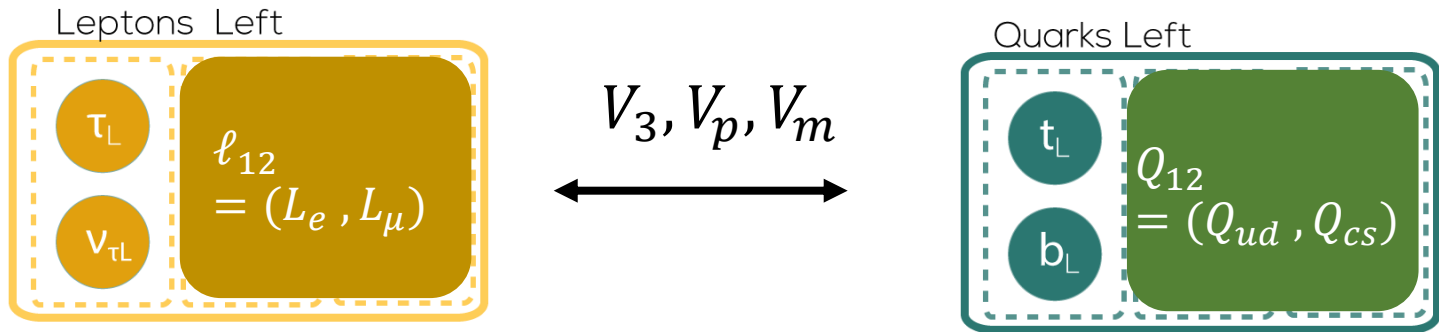
- The «W-like» flavour bosons thus carry a « flavour-charge »

$$V_p^\nu (\bar{\mu} \gamma_\nu e + \bar{s} \gamma_\nu d) + V_m^\nu (\bar{e} \gamma_\nu \mu + \bar{d} \gamma_\nu s)$$



Flavour transfer

- The key point: new flavour gauge bosons do not « break » flavour, they only transfer it from one fermionic sector to another



Different predictions than MFV like patterns

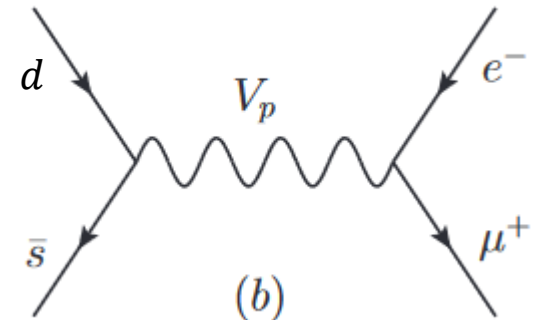
→ Particularly for $M_{V_1} = M_{V_2} = M_{V_3}$, in the gauge basis we have

$$\mathcal{L}_{\text{eff}} \supset - \sum_{a, f, f'} \frac{g_f^2}{8M_V^2} (2\delta^{il}\delta^{jk} - \delta^{ij}\delta^{kl}) (\bar{f}_i \gamma^\mu f_j) (\bar{f}'_k \gamma_\mu f'_l)$$

Symmetry factor

Flavour transfer !

Flavour diagonal



Moving to the mass basis

- Since we did not focus on a particular flavour texture mechanism, the rotation matrices are « a priori » free
 - Of course in most actual models, the rotation matrices **will be hierarchical as a by-product of the hierarchy in the fermion masses**

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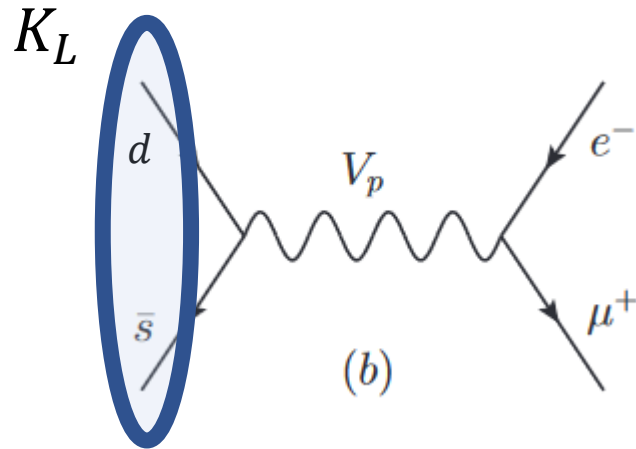
Greljo et al. 2309.11547

$$L_d \simeq \begin{pmatrix} 1 & \frac{m_d}{m_s} \frac{z_{d2}}{z_{d1}} & \frac{m_d}{m_b} \frac{z_{d3}}{z_{d1}} \\ -\frac{m_d}{m_s} \frac{z_{d2}^*}{z_{d1}} & 1 & \frac{m_s}{m_b} \frac{y_{d3}}{y_{d2}} \\ \frac{m_d}{m_b} \left(\frac{y_{d3}^* z_{d2}}{y_{d2} z_{d1}} - \frac{z_{d3}^*}{z_{d1}} \right) & -\frac{m_s}{m_b} \frac{y_{d3}^*}{y_{d2}} & 1 \end{pmatrix} \rightarrow \begin{matrix} V_f = \Phi^z V_x V_z V_y \\ V_{dLx} = \mathbb{1} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\epsilon_{23} \\ 0 & \epsilon_{23} & 0 \end{pmatrix} \\ \dots \end{matrix}$$

- Numerically : **scan full parameter space**
- Analytical result : use a **small spurion approach**, but allowing for different flavour alignment for the $SU(2)$ doublets (e.g $(12)_\ell (12)_{Q_L}$)

An example: kaonic decays

- With the above choice of flavour doublets, V_p, V_m bosons trigger the decays of kaons



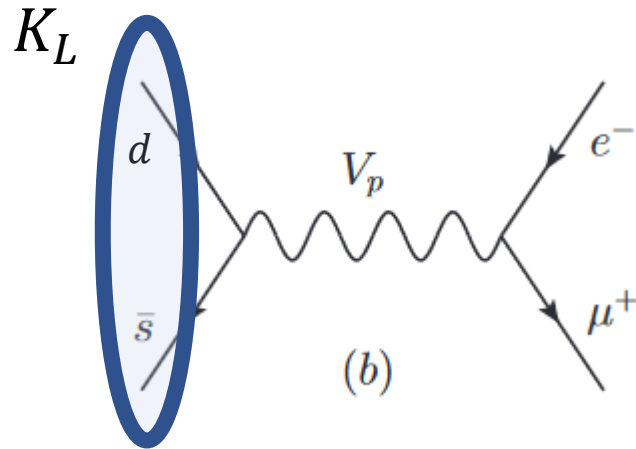
$$\text{BR}(K_L \rightarrow \mu^\pm e^\mp) < 4.7 \times 10^{-12}$$

In particular the process

$K_L \rightarrow e \mu$, but $K_+ \rightarrow \pi_+ e \mu$ is also similarly un-suppressed

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In particular the process

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$$\text{BR}(K_L \rightarrow \mu^+ e^-) = \frac{1}{\Gamma_{K_L}} \frac{M_K f_K^2}{128\pi^3} \alpha_{\text{em}}^2 G_F^2 |V_{td}^* V_{ts}|^2 \left(1 - \frac{m_\mu^2}{M_K^2}\right)^{3/2} \times \left(|C_9^{sd\mu e} + C_9^{sde\mu^*}|^2 + |C_{10}^{sd\mu e} + C_{10}^{sde\mu^*}|^2\right)$$

- The corresponding limit is at the **250 TeV level**

$$\text{BR}(K_L \rightarrow \mu^\pm e^\pm) = 1.2 \cdot 10^{-10} \left(\frac{100 \text{ TeV}}{M_V/g_f}\right)^4 \times \begin{cases} 1 & \text{for (12)}_\ell \\ \theta_{\ell 23}^2 & \text{for (13)}_\ell \end{cases}$$

SuperIso implementation

- Interface between the χ^2 routines of SuperIso and BSMart (using MultiNest)
 -> 212 observables included, (~ 180 of B-physics, ~ 15 of Kaons, ~ 15 of leptons)

Constraints	Refs.	$SU(2)_f$ flavour alignment		
		$(12)_Q(12)_\ell$	$(23)_Q(23)_\ell$	$(12)_Q(13)_\ell$
$B \rightarrow Kee$ (C_9)	/	$-\theta_{Q23}$	$+\theta_{\ell 12}\theta_{\ell 13}$	$-\theta_{Q23}$
$B \rightarrow K\mu\mu$ (C_9)	/	$+\theta_{Q23}$	$-\theta_{\ell 23}$	0
$K \rightarrow \pi ee$ (C_9)	/	$+\theta_{\ell 12}$	0	$+\theta_{\ell 13}$
$K \rightarrow \pi\mu\mu$ (C_9)	/	$-\theta_{\ell 12}$	$+\theta_{Q12}$	$\theta_{\ell 12}\theta_{\ell 23}$
$\text{BR}_{K^+ \rightarrow \pi^+ \mu^+ e^-}^{(\text{E865})} < 1.3 \times 10^{-11}$	[32, 82]	1	0	$\theta_{\ell 23}^2$
$\text{BR}_{K^+ \rightarrow \pi^+ \mu^- e^+}^{(\text{E865})} < 6.6 \times 10^{-11}$	[32, 82]	0	0	0
$\text{Br}_{K^+ \rightarrow \pi^+ \nu \bar{\nu}}^{(\text{NA62})} = 1.06_{-0.35}^{+0.41} \times 10^{-10}$	[22]	1	θ_{Q12}^2	1
$\text{BR}_{K_L \rightarrow \mu^+ e^-}^{(\text{BNL})} < 4.7 \times 10^{-12}$	[20]	1	0	$\theta_{\ell 23}^2$
$\text{BR}_{B^+ \rightarrow K^+ \nu \bar{\nu}}^{(\text{BaBar})} < 1.6 \times 10^{-5}$	[95]	$2\theta_{Q13}^2 + \theta_{Q23}^2$	1	$2\theta_{Q13}^2 + \theta_{Q23}^2$
$\text{BR}_{B^+ \rightarrow K^+ e^- \mu^+}^{(\text{LHCb})} < 6.4 \times 10^{-9}$	[118]	θ_{Q13}^2	$\theta_{\ell 13}^2$	0
$\text{BR}_{B^+ \rightarrow K^+ \mu^- \tau^+}^{(\text{BaBar})} < 2.8 \times 10^{-5}$	[119]	0	1	0
K oscillations (C_1)	[120]	0	θ_{Q12}^2	0
D oscillations (C_1)	[120]	θ_{Q13}^2	$1 - 8\theta_{Q12}$	θ_{Q13}^2
B_d oscillations (C_1)	[120]	θ_{Q13}^2	θ_{Q13}^2	θ_{Q13}^2
B_s oscillations (C_1)	[120]	θ_{Q23}^2	0	θ_{Q23}^2
$\text{BR}_{\mu \rightarrow e \bar{e} e}^{(\text{SINDRUM})} < 1.0 \cdot 10^{-12}$	[105]	0	0	$\theta_{\ell 23}^2$
$\text{BR}_{\tau \rightarrow 3\mu}^{(\text{BELLE})} < 2.1 \cdot 10^{-8}$	[106]	$\theta_{\ell 23}^2$	0	0
$\text{BR}_{\tau \rightarrow 3e}^{(\text{BELLE})} < 3.3 \cdot 10^{-8}$	[106]	$\theta_{\ell 13}^2$	0	0
$\text{BR}_{\mu \rightarrow e \gamma}^{(\text{MEG})} < 4.2 \cdot 10^{-13}$	[100, 101]	0	$\theta_{\ell 12}^2$	$\theta_{\ell 13}^2$
$\text{BR}_{\tau \rightarrow e \bar{K}^*}^{(\text{Belle})} < 3.2 \cdot 10^{-8}$	[110]	0	0	1
$\text{BR}_{\tau \rightarrow \mu \bar{K}^*}^{(\text{Belle})} < 7.0 \cdot 10^{-8}$	[110]	$\theta_{\ell 13}^2$	θ_{Q13}^2	$\theta_{\ell 12}^2$
$\text{CR}_{Au, \mu \rightarrow e}^{(\text{SINDRUM-II})} < 7 \cdot 10^{-13}$	[21, 103, 112]	$1 + 20\theta_{\ell 12}$	$\theta_{\ell 12}^2$	$\theta_{\ell 12}(2.3\theta_{\ell 12} - \theta_{\ell 23})$
$\mu \bar{e} \rightarrow e \bar{\mu}$ oscillations (C_1)	[117]	0	$\theta_{\ell 12}^2$	$\theta_{\ell 12}^2$

SuperIso implementation

- Interface between the χ^2 routines of SuperIso and BSMart (using MultiNest)
 - > 212 observables included, (~ 180 of B-physics, ~ 15 of Kaons, ~ 15 of leptons)
- Flavour transfer observables lead to strong bounds even for small mixing angles.

$$\Delta F_f + \Delta F_{f'} = 0$$

→ Typical limits on M_V / g_f at the 100 TeV scale

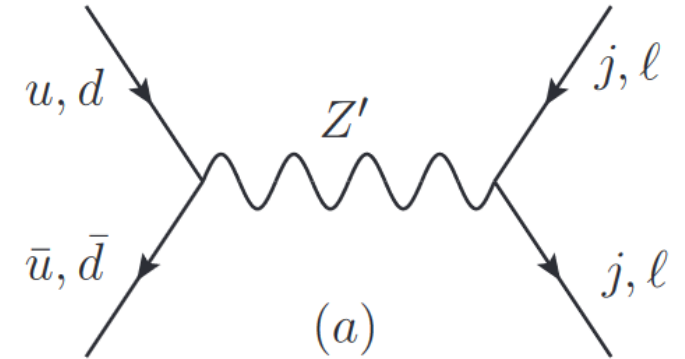
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$B \rightarrow Kee$ (C_9)	Flavour universality violation	$-\theta_{Q23}$	$+\theta_{\ell 12}\theta_{\ell 13}$	$-\theta_{Q23}$
$B \rightarrow K\mu\mu$ (C_9)		$+\theta_{Q23}$	$-\theta_{\ell 23}$	0
$K \rightarrow \pi ee$ (C_9)		$+\theta_{\ell 12}$	0	$+\theta_{\ell 13}$
$K \rightarrow \pi\mu\mu$ (C_9)		$-\theta_{\ell 12}$	$+\theta_{Q12}$	$\theta_{\ell 12}\theta_{\ell 23}$
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On LHC constraints

- When $M_V \lesssim$ few TeV, direct production at LHC becomes possible $M_V \propto g_f v_S$
- LHC is « perfect » for the flavour transfer processes since NP candidate can be produced from quark (or gluon) fusion, but decay leptonically to ensure detection.

$$pp \rightarrow V + X, V \rightarrow \ell\ell$$

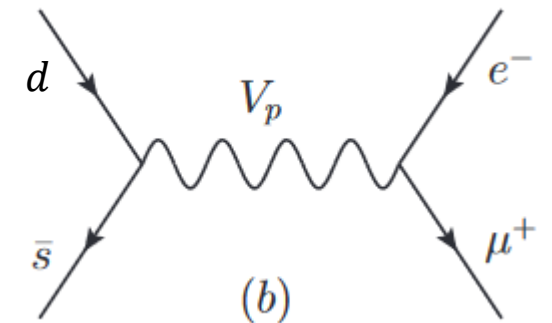
→ Standard searches for Z' : di-leptons and di-jets



- Searches using LFV final states are extremely attractive

→ The proton contains enough sea-quarks to produce the off-diagonal flavour boson

→ Lepton flavour violation in the final states limit the QED background

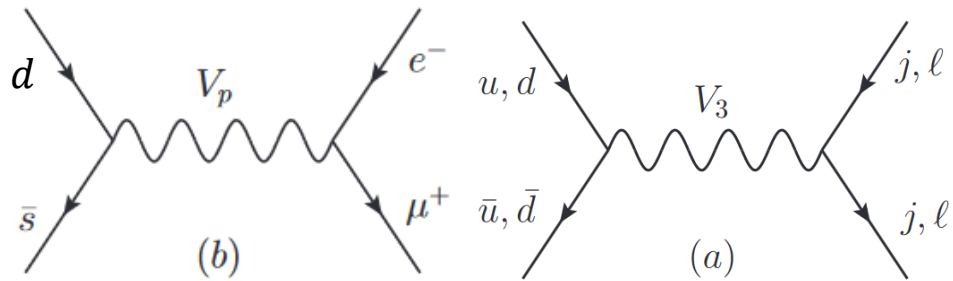


LHC limits and flavour: LH - $(12)_\ell(12)_Q$

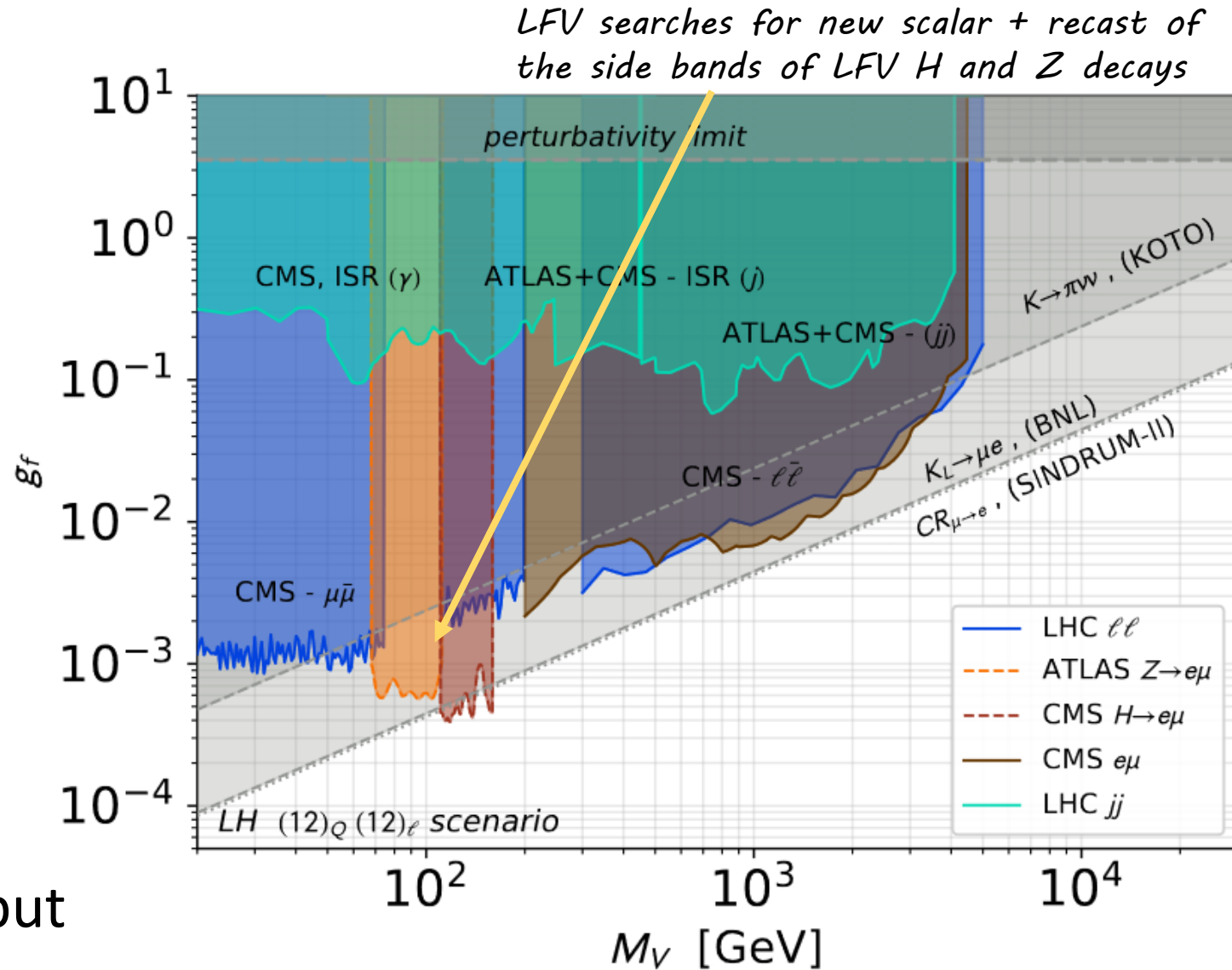
- Use the (LH) scenario

→ Assume that 1st and 2d generations of left-handed fermions are part of a flavour doublets

→ Production at LHC is huge !

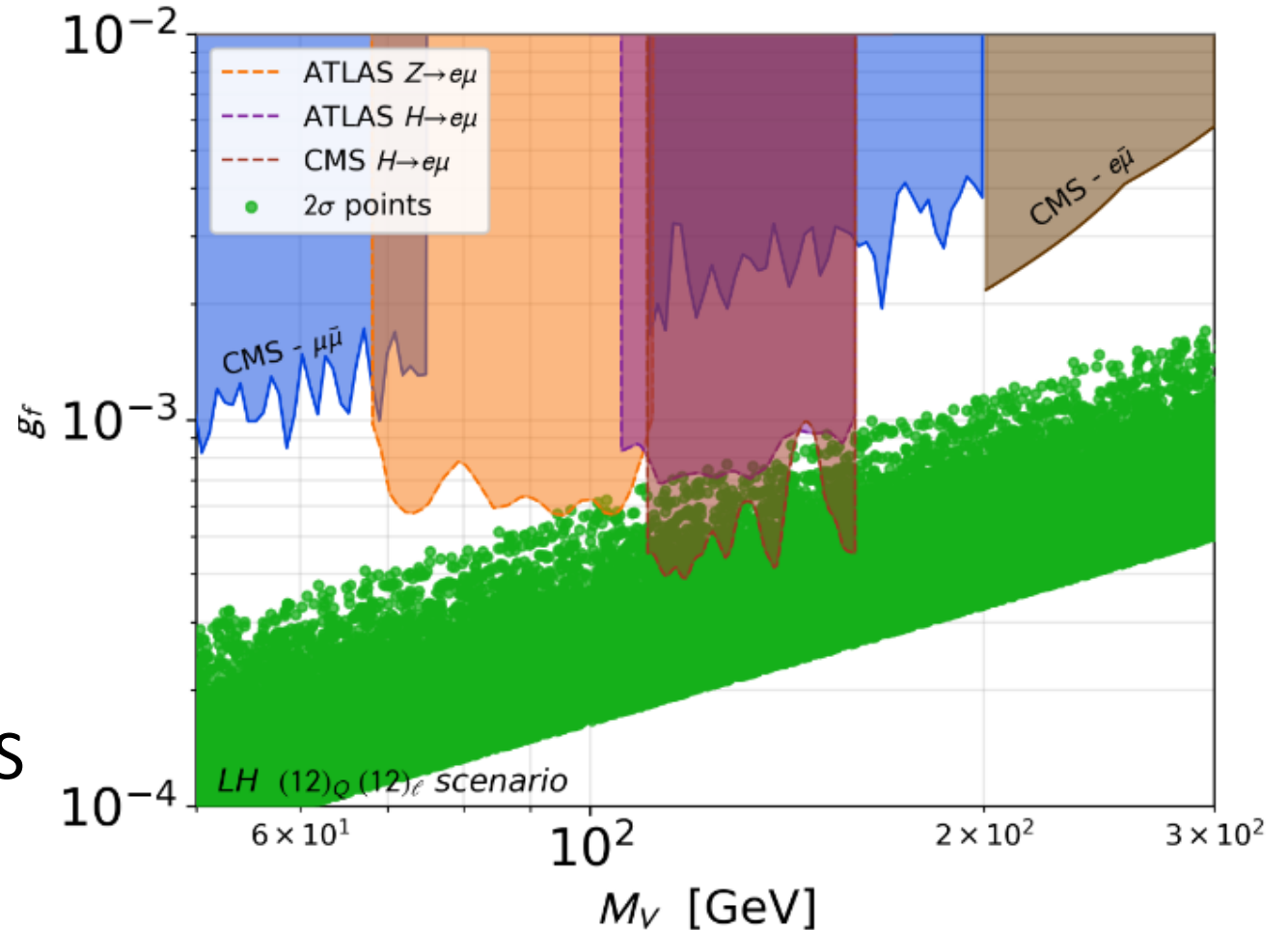


- Limits from Kaonic and muon conversion in nuclei dominate, but LHC constraints are close



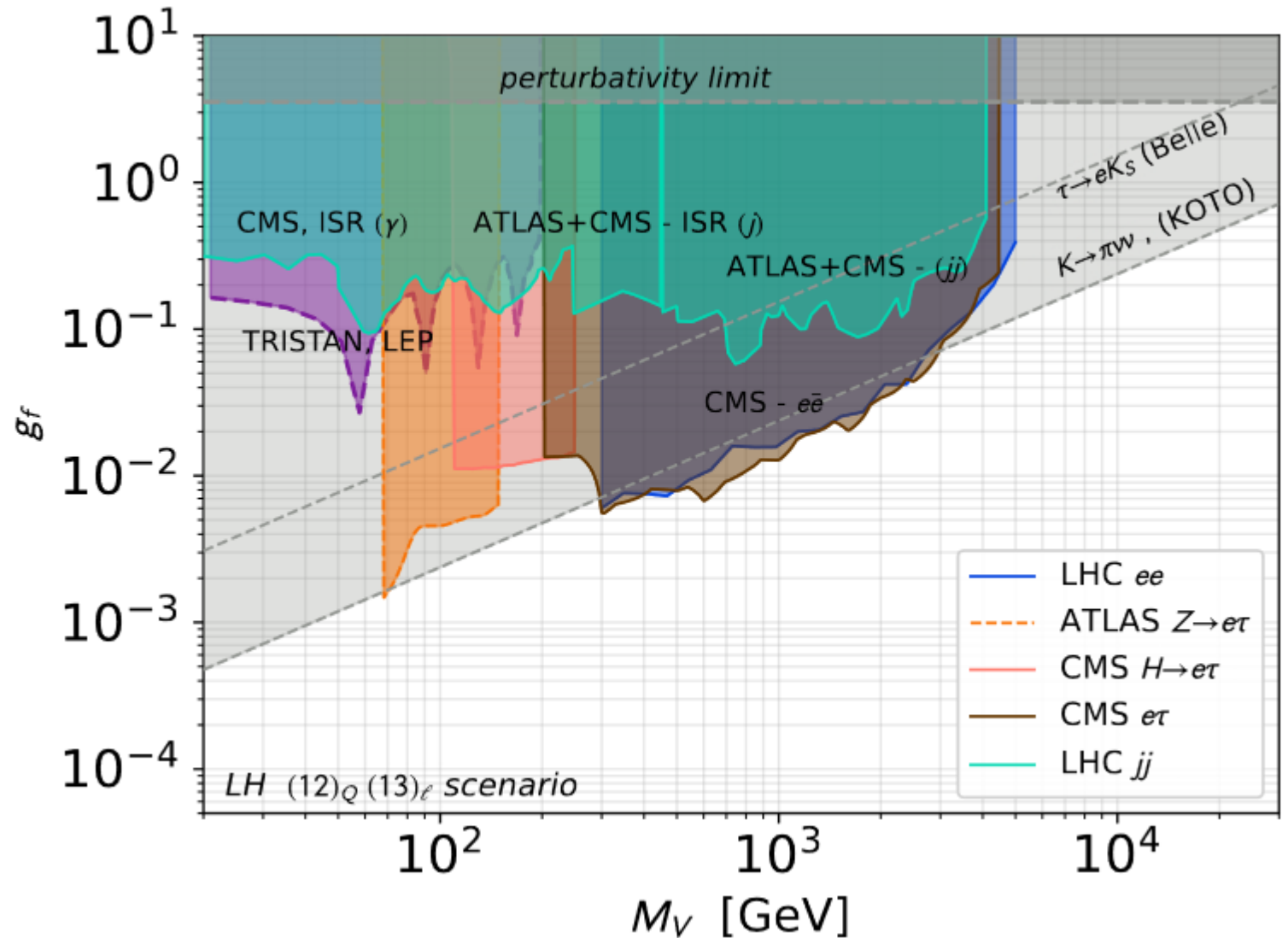
LFV decays of H and Z

- The best constraints arise from the recasting of LFV H and Z decays
 - $Z \rightarrow e\mu, e\tau, \mu\tau$ and $h \rightarrow e\mu, e\tau, \mu\tau$
 - We calibrate the signal on the Z and H one for the efficiency, then uses the side-band data to put a limit
- There is a $\sim 3\sigma$ anomaly in the CMS data set, ATLAS data not precise enough to call... for now



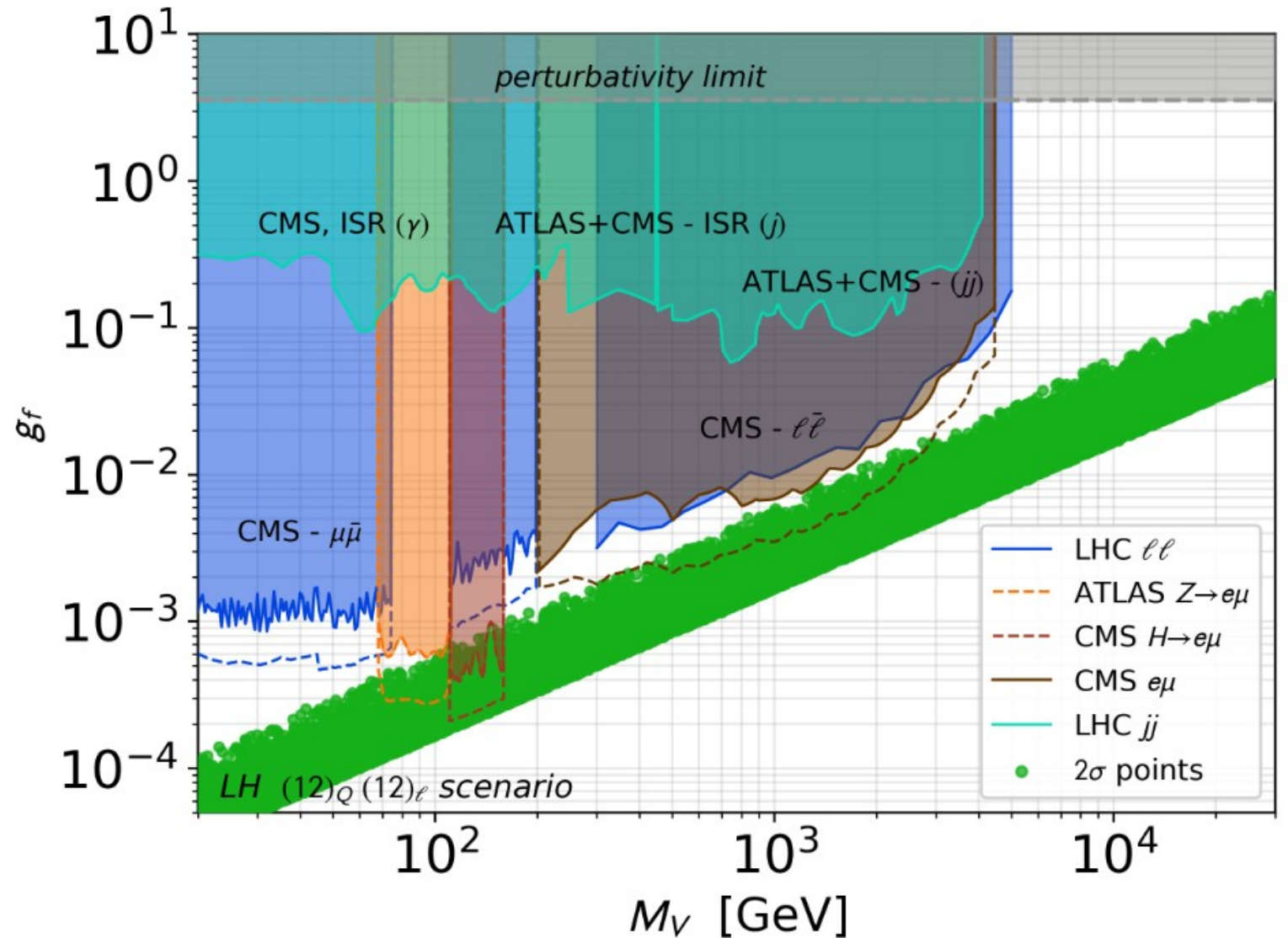
Another flavour alignment LH - $(13)_\ell(12)_Q$

- Corresponds to a « muon as a third generation lepton » scenario
- Now the strongest limits arise from Kaonic neutrino decays (since do not depend on the neutrino flavour)
- LHC constraints are also weakened



Future prospects

- LHC constraints (and most importantly the recasting of $H \rightarrow e \mu$ and $Z \rightarrow e \mu$ limits) are close or overlapping with the flavour constraints
- HL-LHC could probe even deeper, as would dedicated resonance searches around and below the 100 GeV range



Conclusion

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- New horizontal gauge symmetries are a great model building tool ! Which do not necessarily increase the SM complexity
 - Create and protect new accidental symmetries
 - Generate textures
 - Shape the scalar potential (clockwork structures ...) and the vacuum structure of the theory (create hierarchical VEVs)
- They have significant consequences for pheno
 - Non-abelian flavour gauge symmetries can naturally lead to GeV to TeV new vectors for small couplings
 - LHC has an important role to play for new vectors at and below the TeV
- Creating links with GUT / AS approach an interesting issue as it could help it constraining the actual values of the new gauge couplings

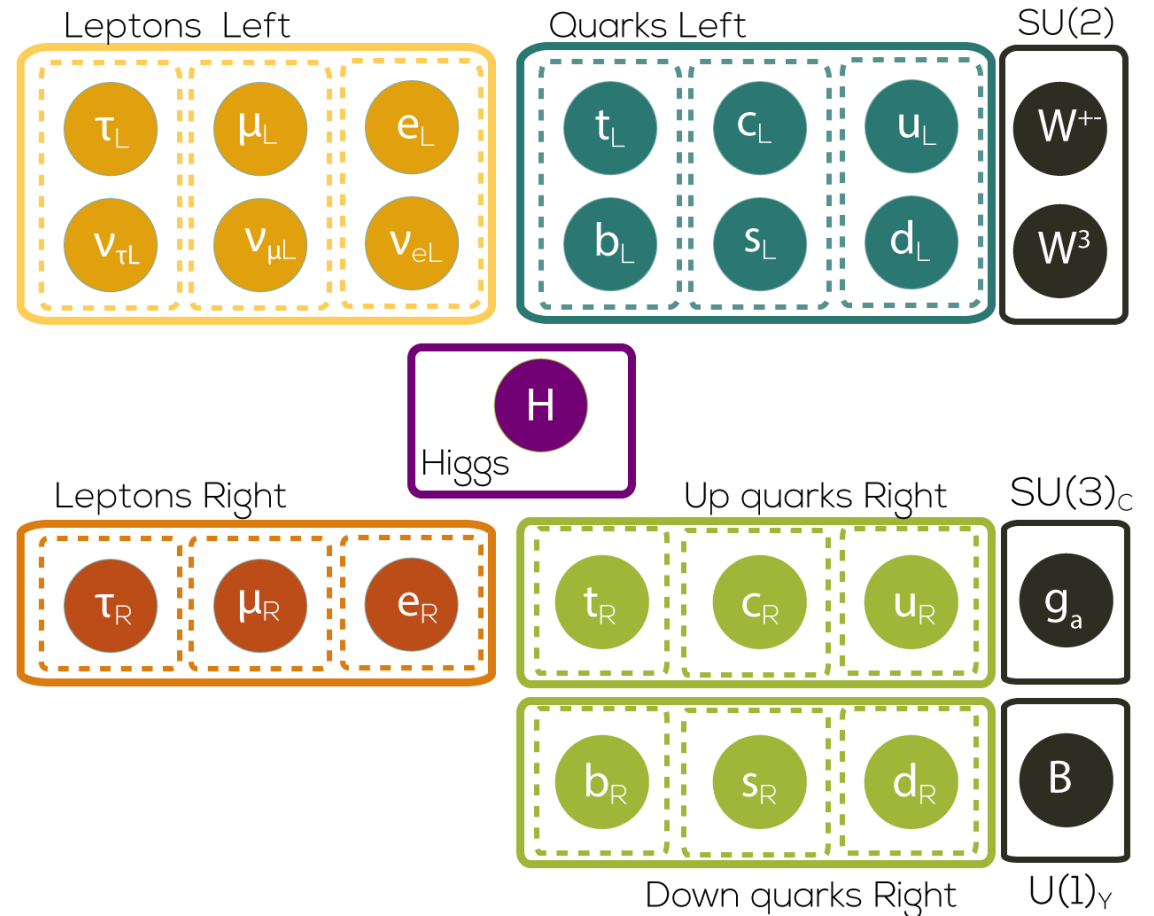
Backup

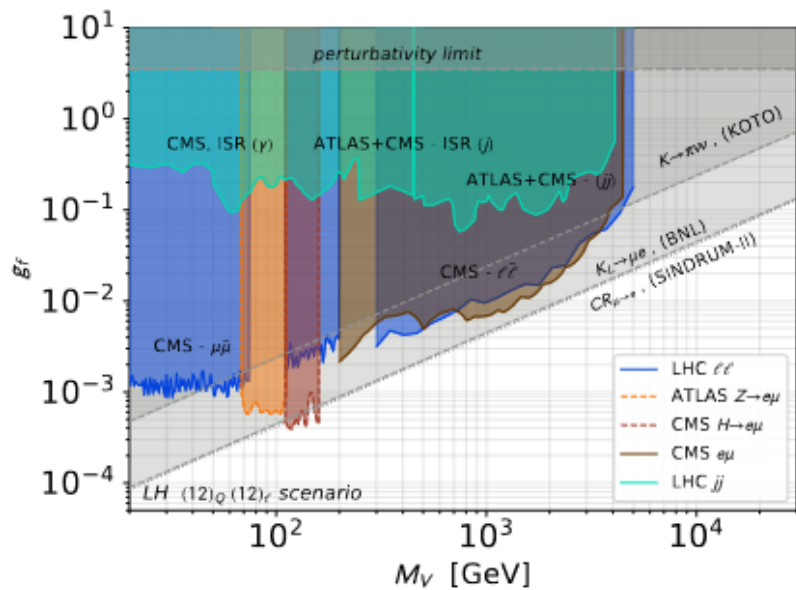
Horizontal flavour gauge groups

- The SM has a large global $U(3)^5$ symmetry group
 → broken by the Yukawa interactions

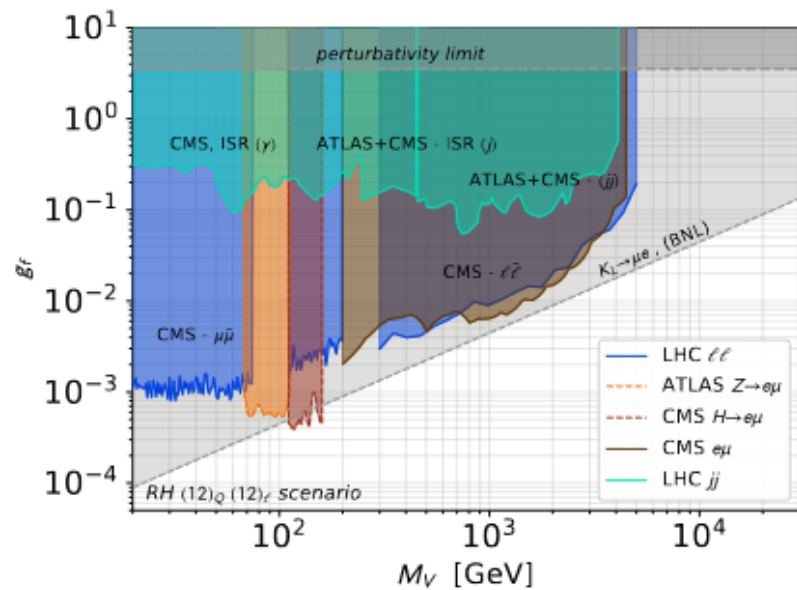
$$\mathcal{L}_Y = -Y_{ij}^d \overline{Q_{Li}^I} \phi d_{Rj}^I - Y_{ij}^u \overline{Q_{Li}^I} \epsilon \phi^* u_{Rj}^I + \text{h.c.},$$

- We can gauge a subset of this group ?
 → U(1) case: Frogatt-Nielsen constructions, $L_\mu - L_\tau$, flavons, etc...
 → The non-abelian case has been sparsely studied.
- We can also consider larger gauge groups by adding fermions

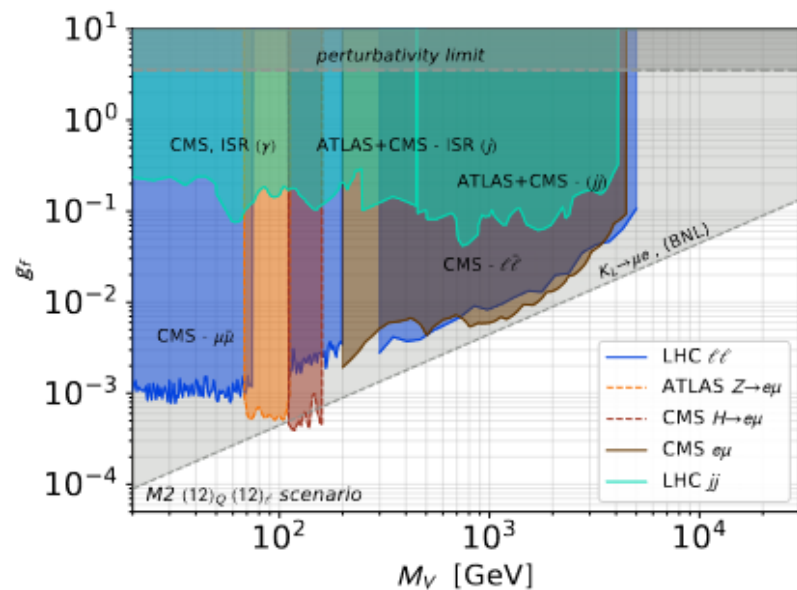
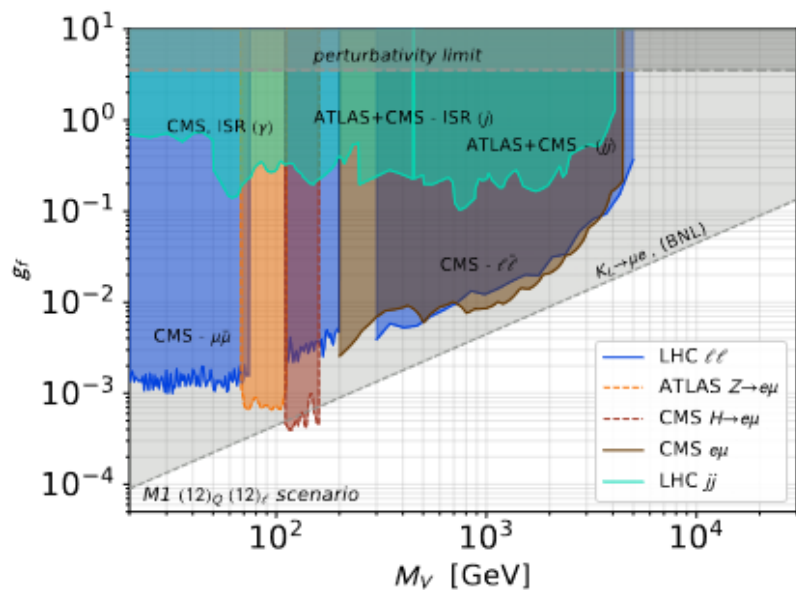




(a)



(b)



Accidental symmetries vs accidentally light scalars

- Comparison with Michele's works
 - Operators will vanish on the vacuum
 - Test larger mixed representations

Two accidental phenomena

(I) Familiar : **accidental symmetries** → **light scalars**

- A given gauge theory → **accidental global symmetries**

Spontaneous symmetry breaking (SSB) $G \rightarrow H$
Nambu Goldstone bosons (NGBs) $\in G/H$

- **NGB masses** controlled by explicit-symmetry-breaking sources

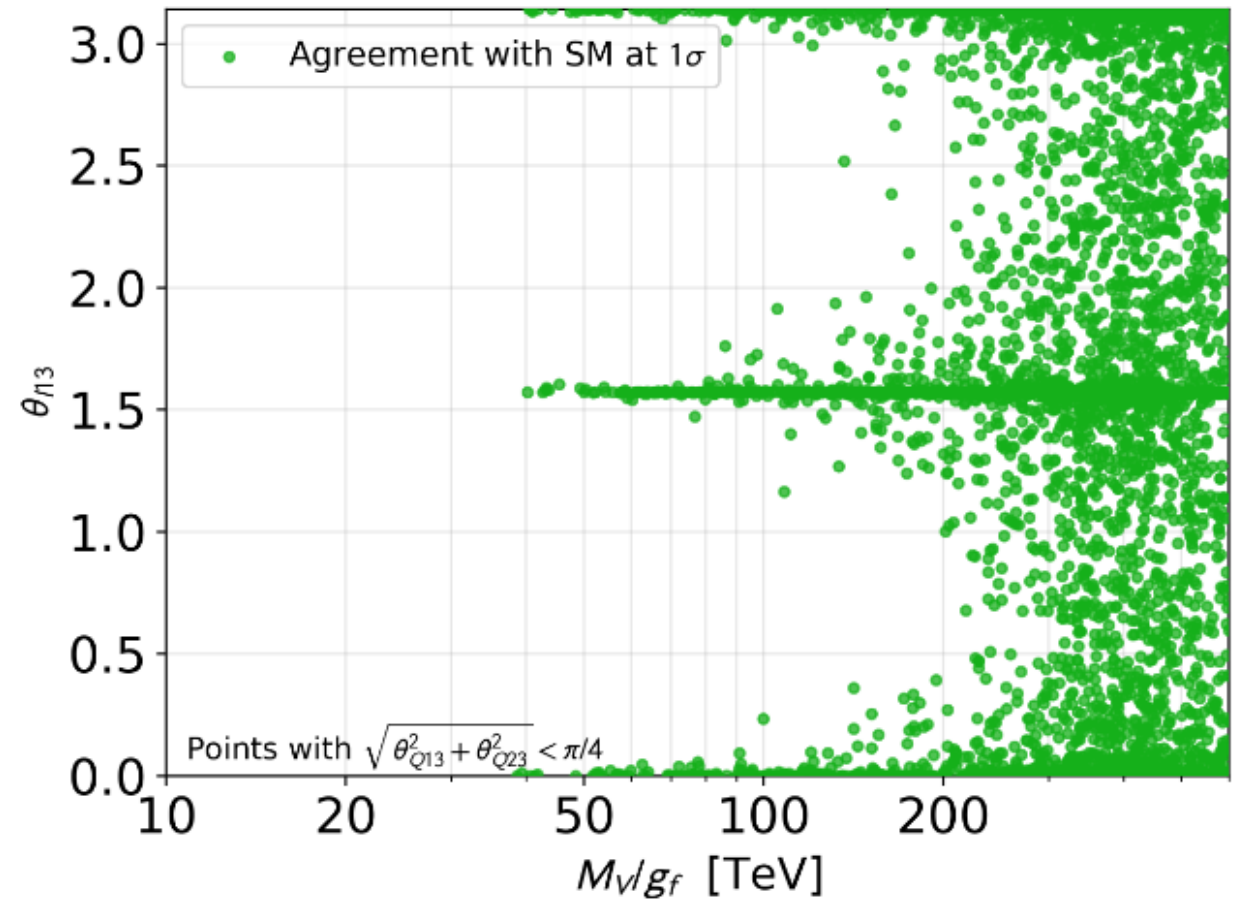
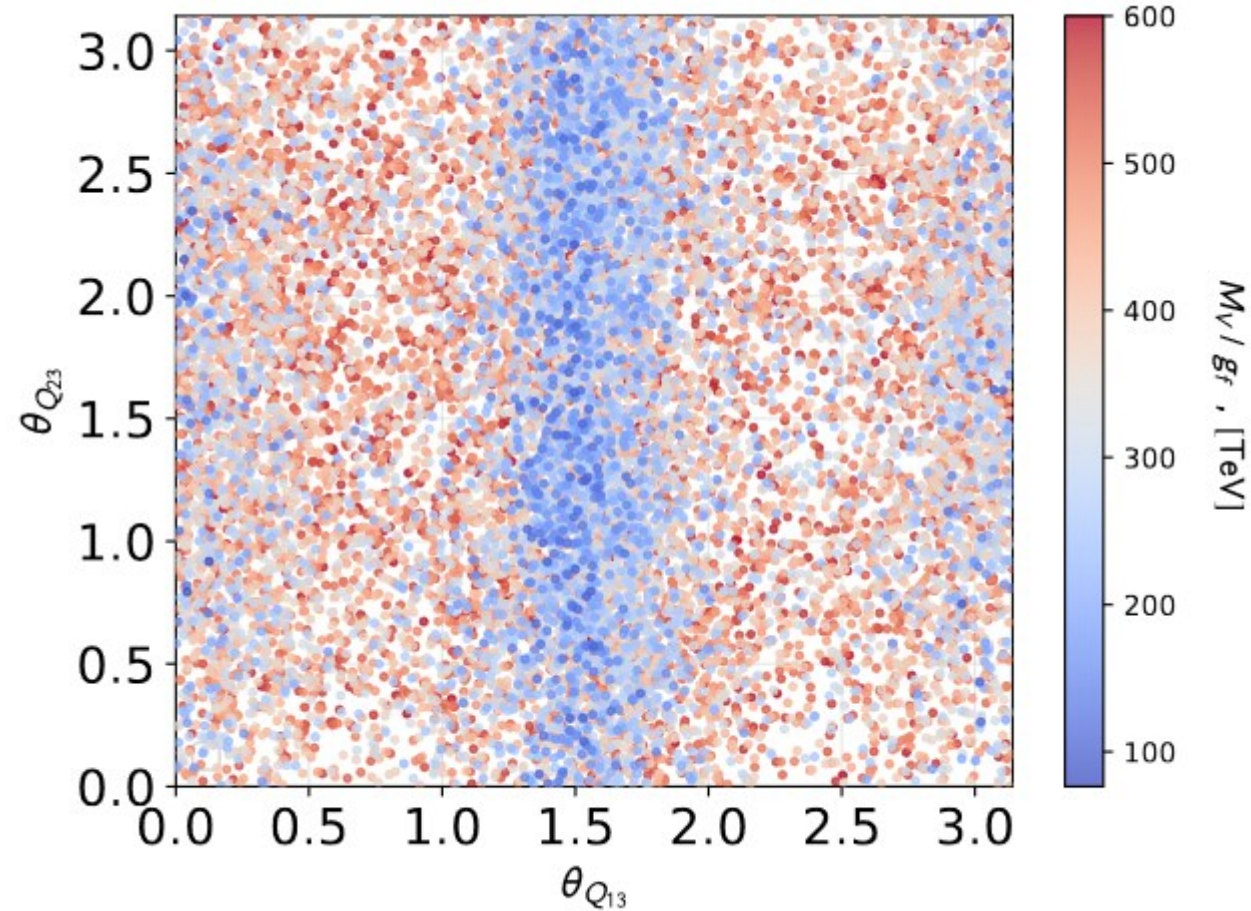
(II) Unfamiliar: **accidentally light scalars, without a symmetry**

- For some special choices of scalar representations

$\langle \phi \rangle : G \rightarrow H$ but vacuum manifold larger than G/H

- Unexpected, still **natural hierarchy** among scalar masses

Some results



- First generations couplings are avoided as much as possible of course ...