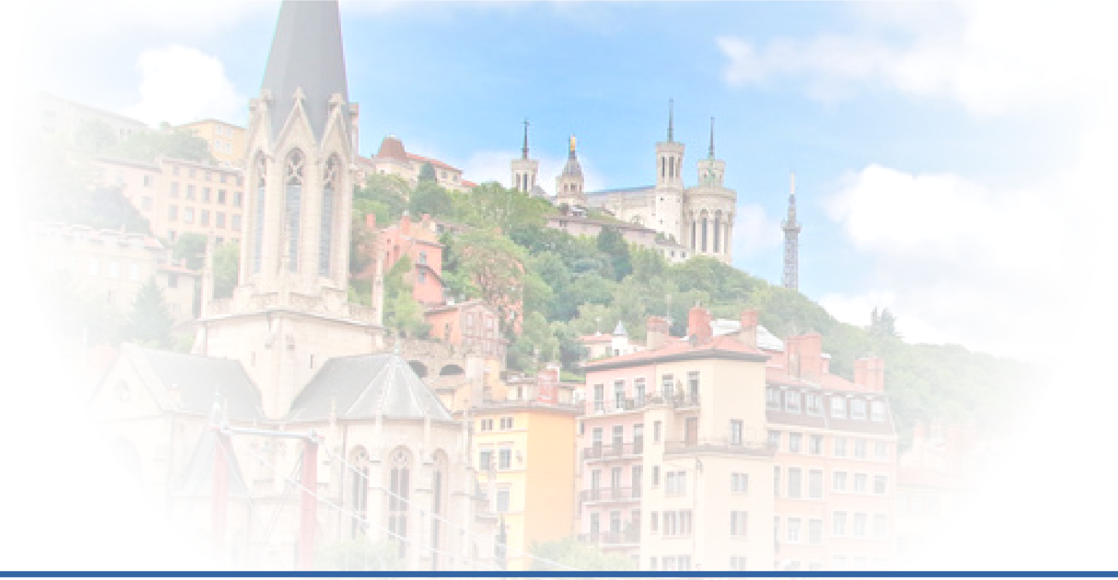
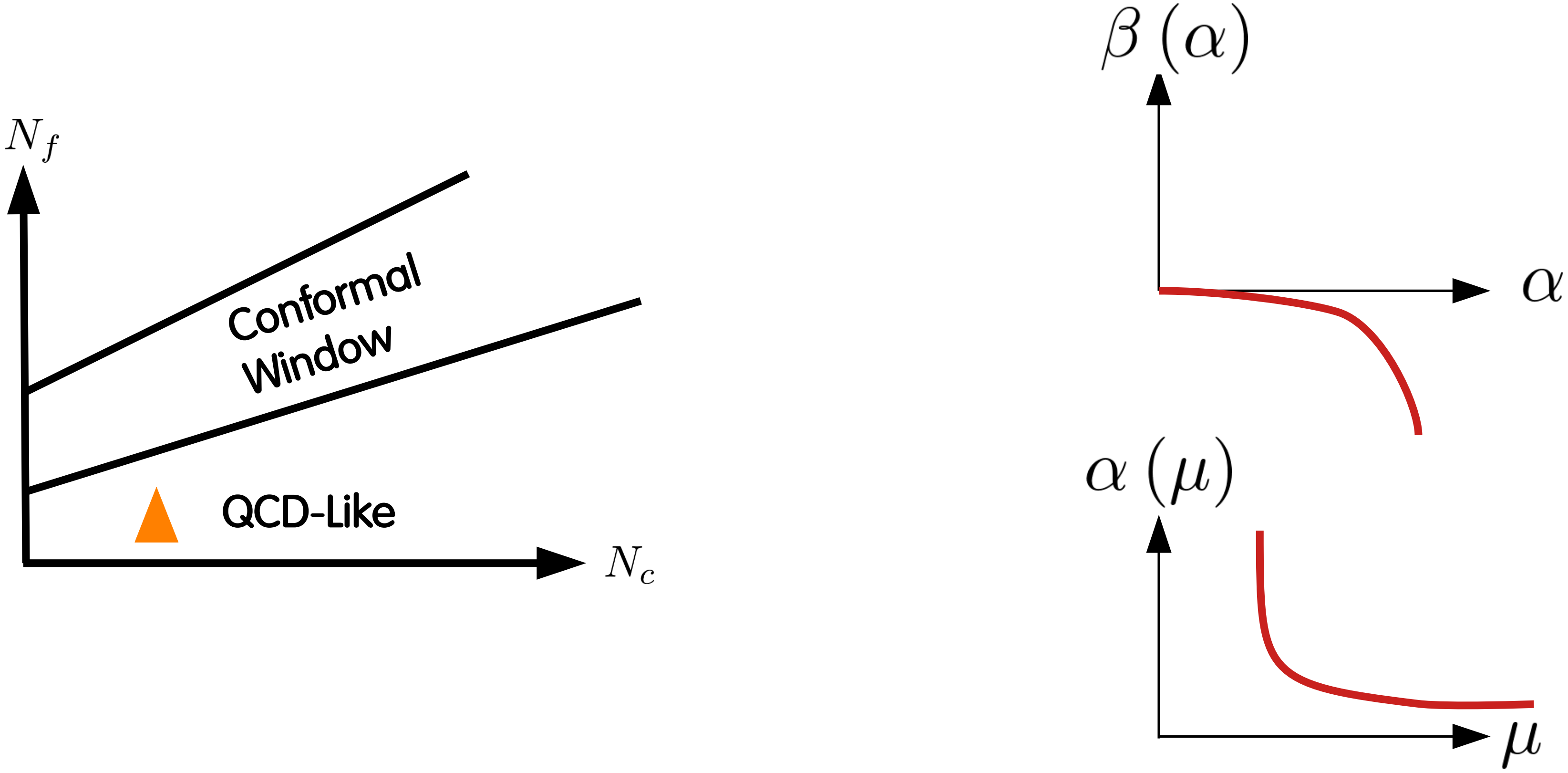
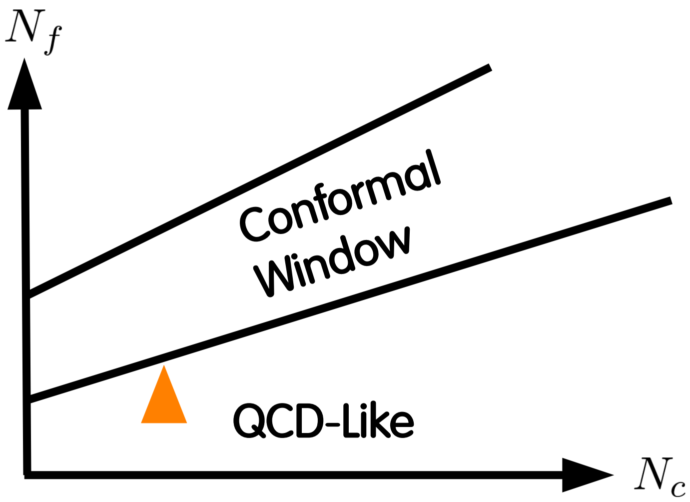


Safety, Criticality, Large N

Shahram Vatani
with Oleg Antipin and Francesco Sannino
Coming soon...


N_f
Conformal Window
QCD-Like
N_c
$\beta(\alpha)$
α
$\alpha(\mu)$
μ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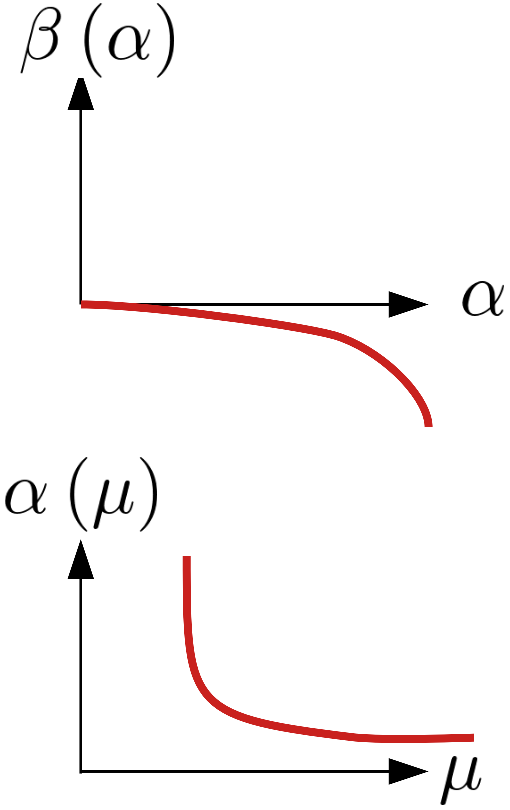
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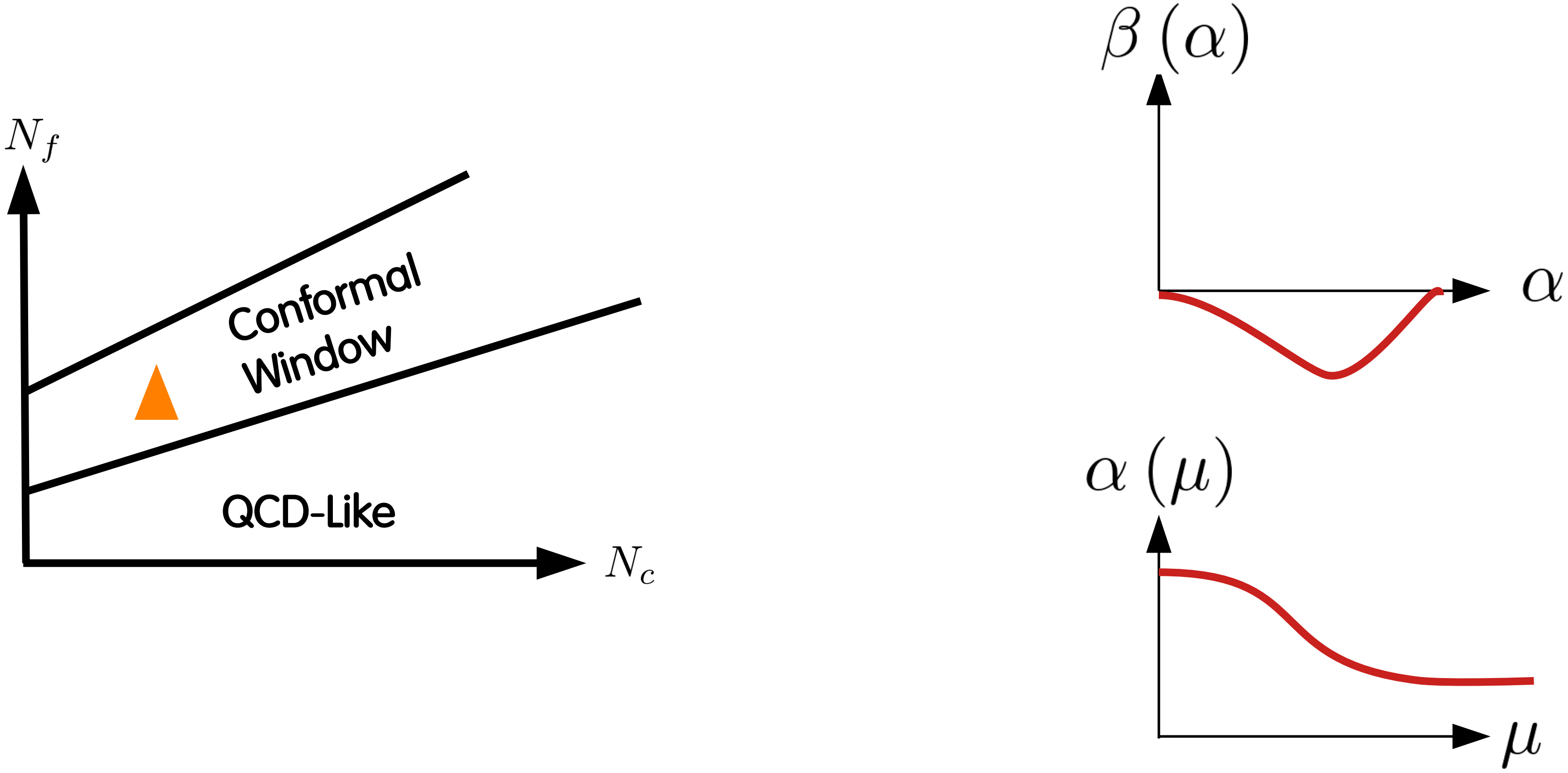
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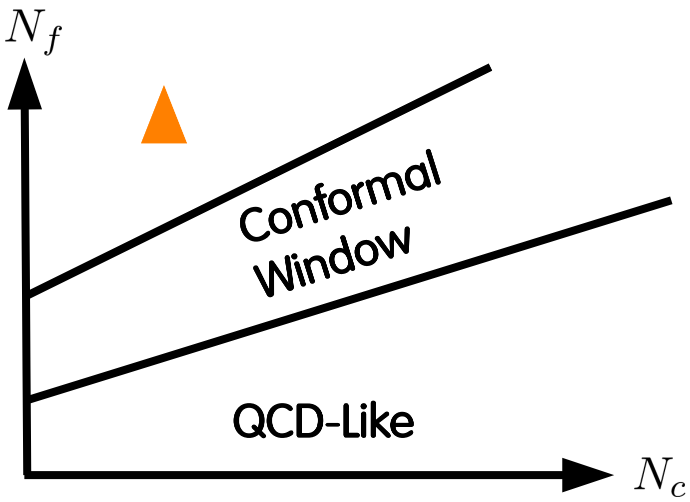
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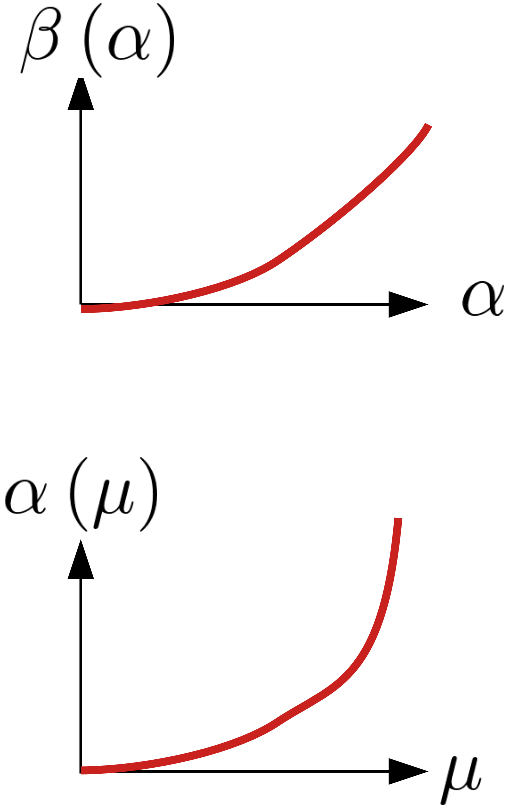
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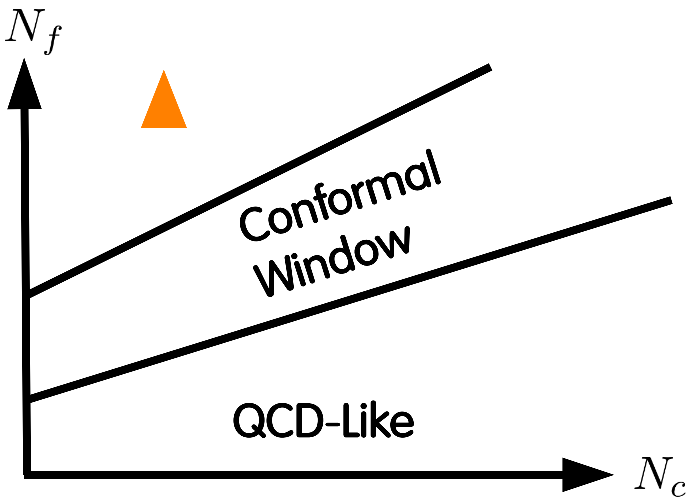
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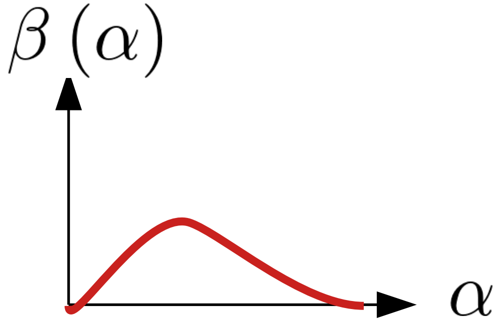
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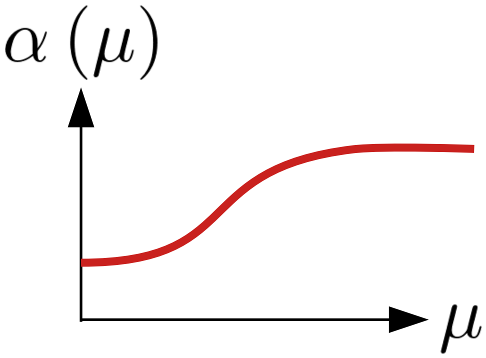
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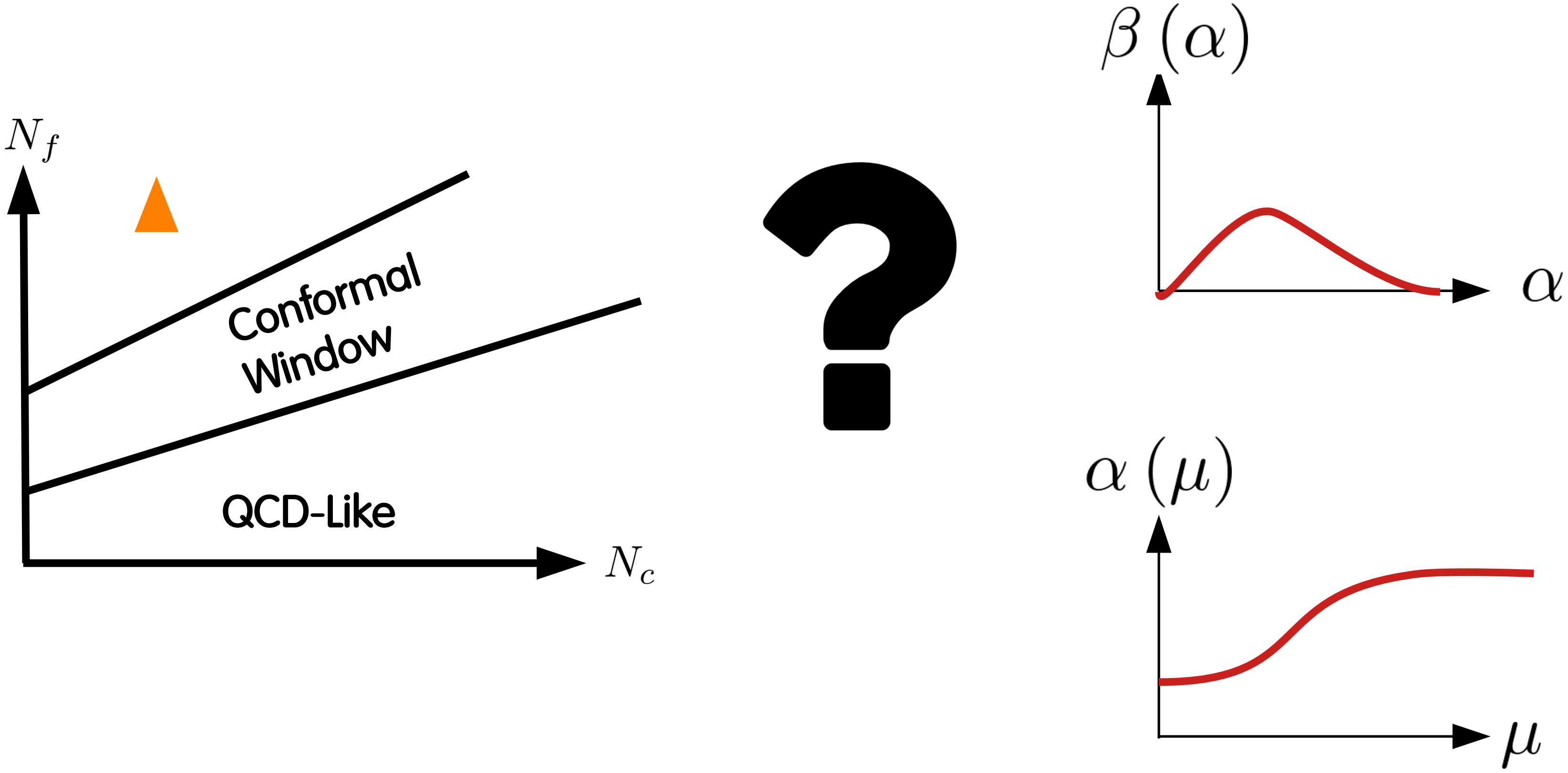

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Motivations

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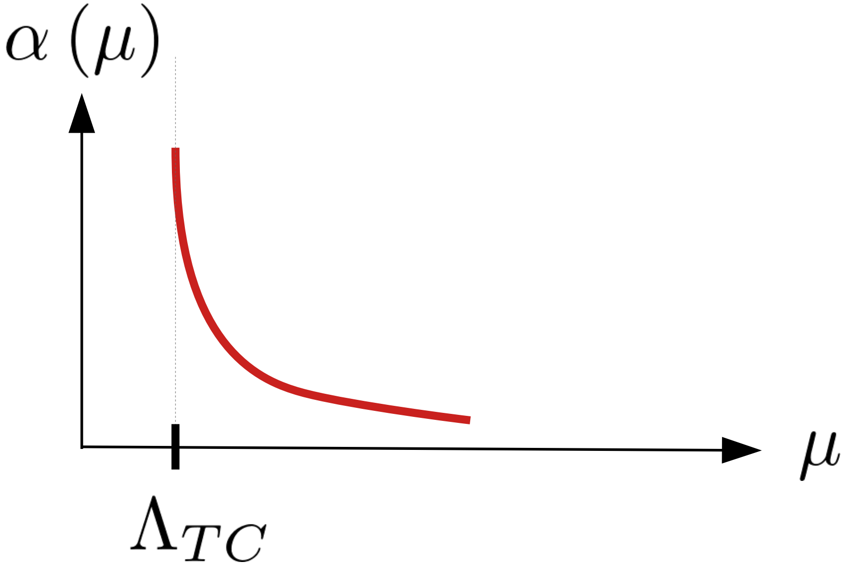

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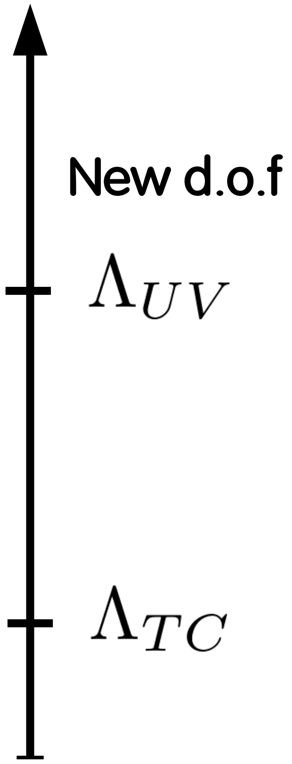

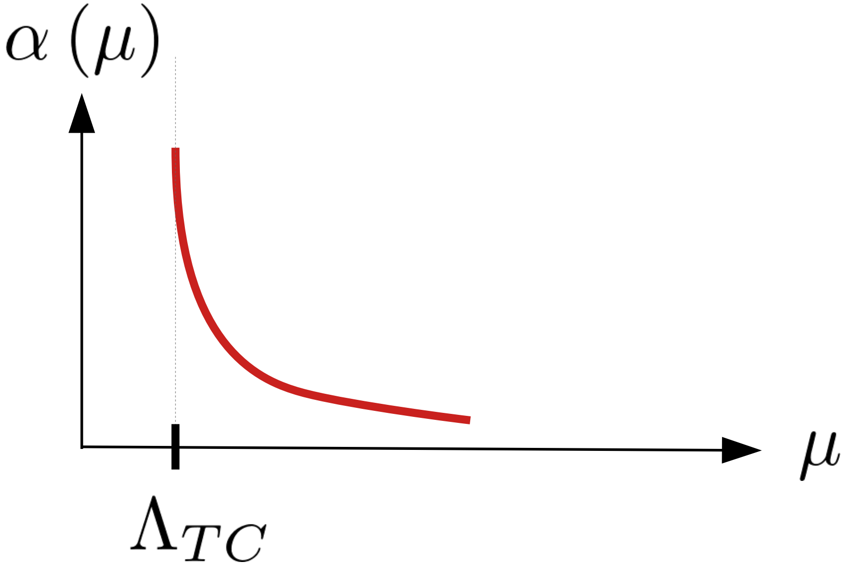

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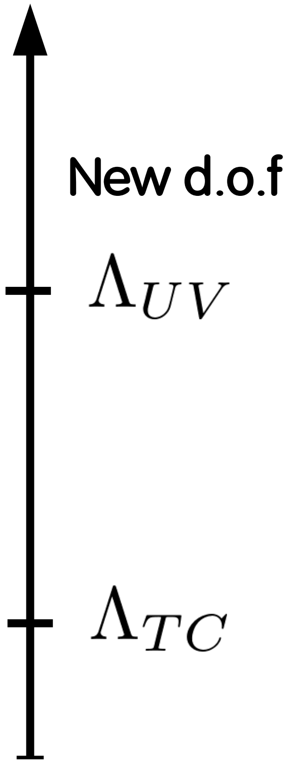

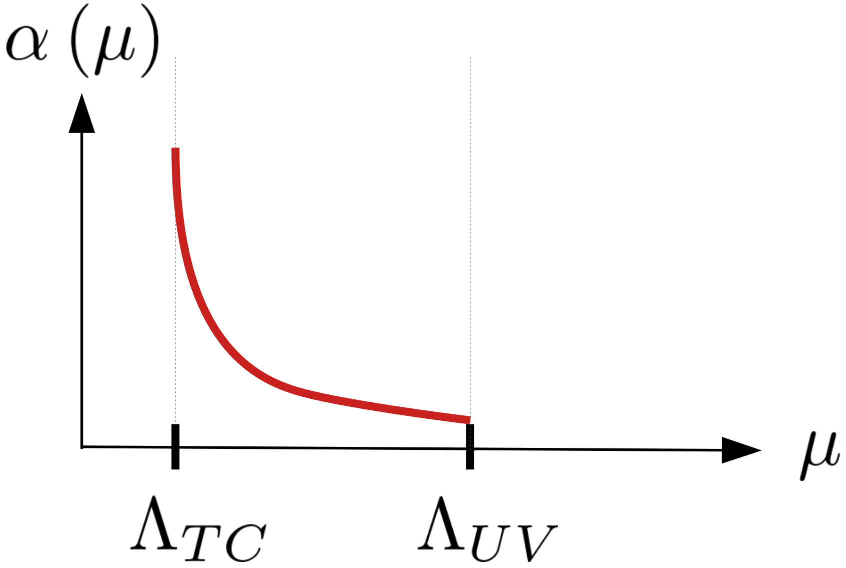

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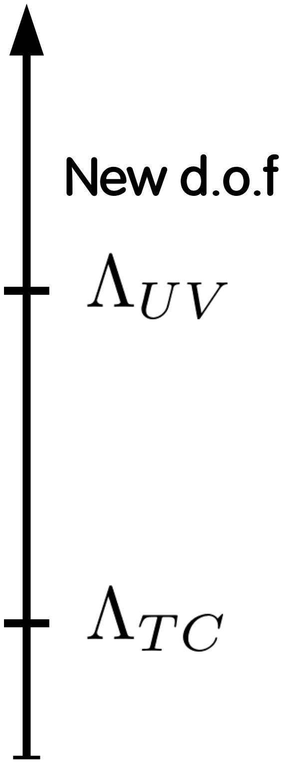

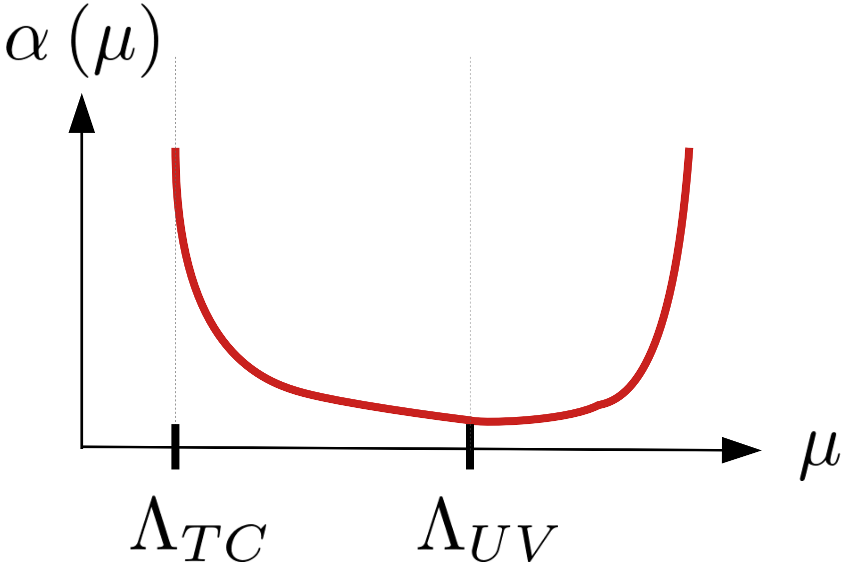

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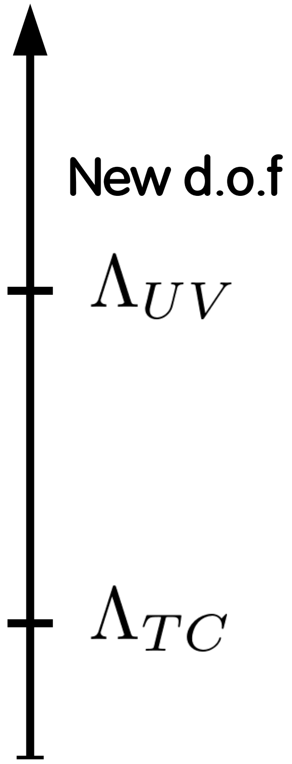

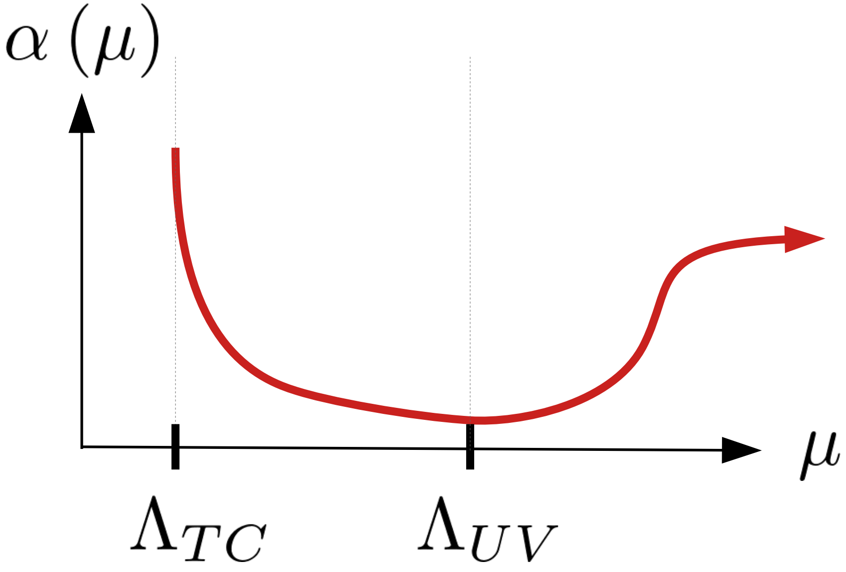

- 1) Large N
- 2) Critical Exponent
- 3) Truncation in 1/N

- 1) Large N

Large N Expansion

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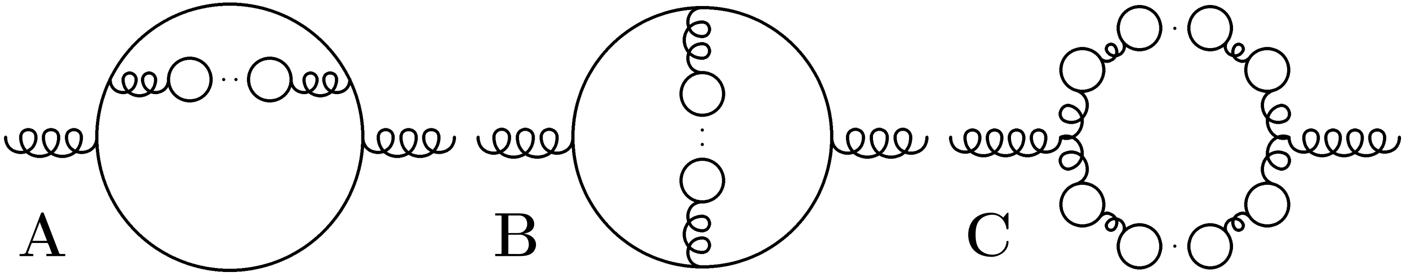

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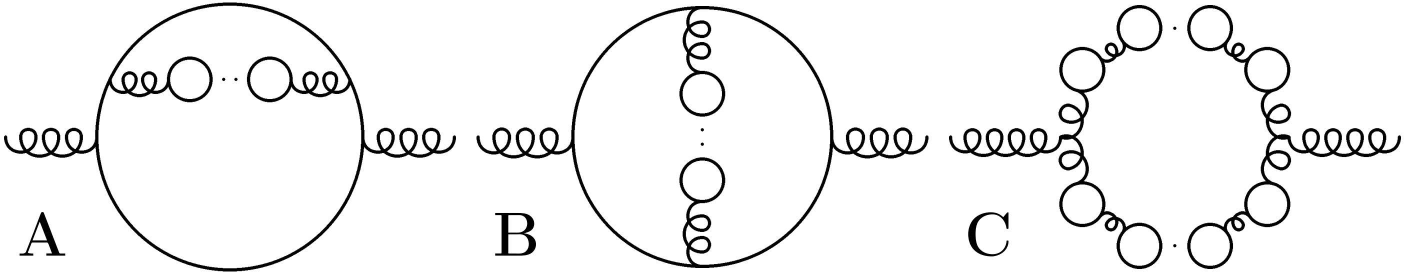

- $F_1\,(K)$ can be computed and exhibits a pole in $K^* = 3$!

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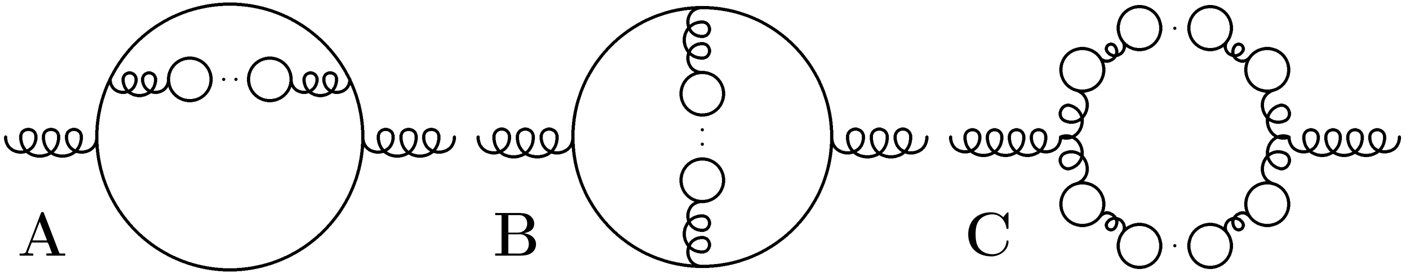

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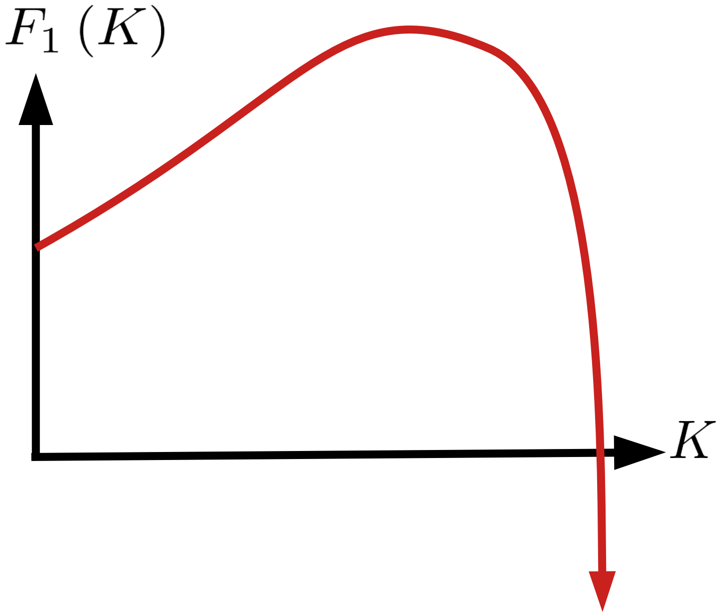

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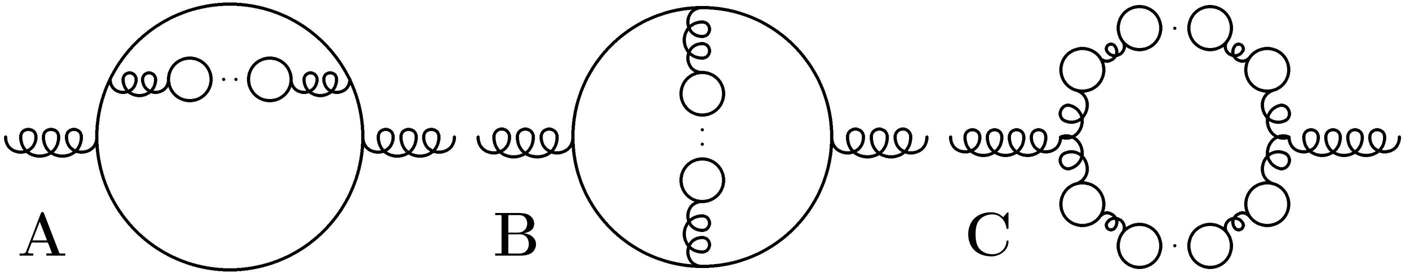
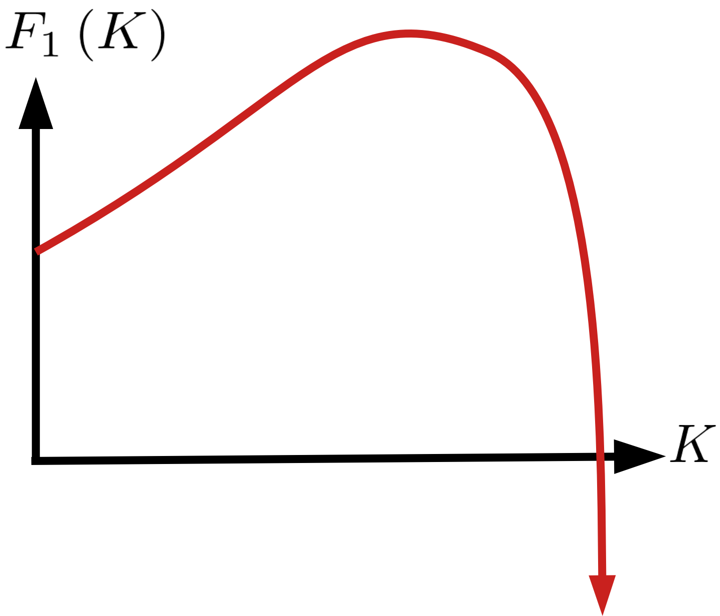


- $F_1\,(K)$ can be computed and exhibits a pole in $K^* = 3$!

- It implies that for any N there is a UV fixed-point



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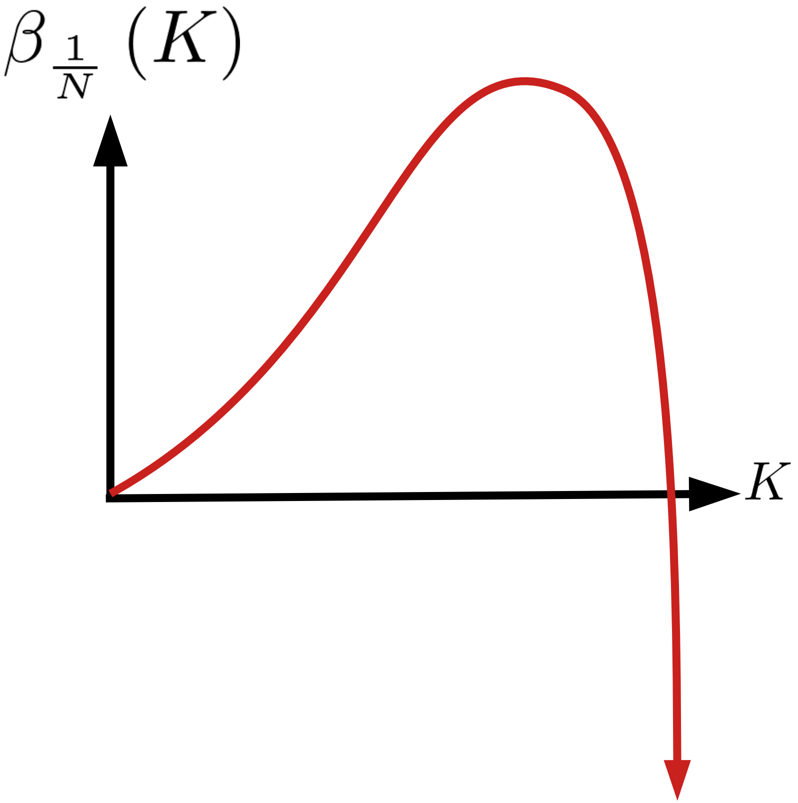

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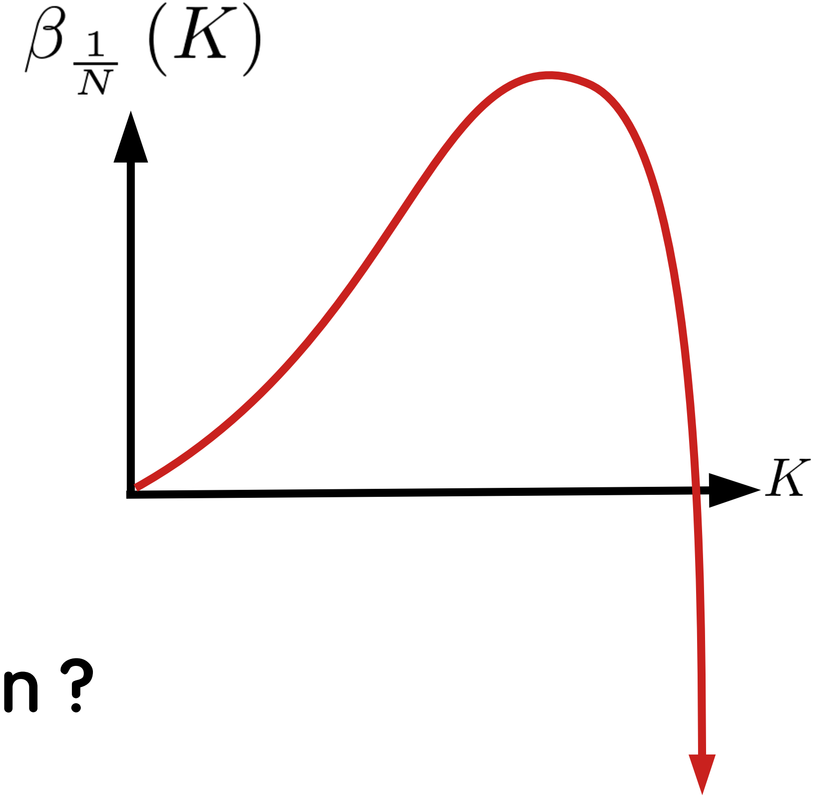

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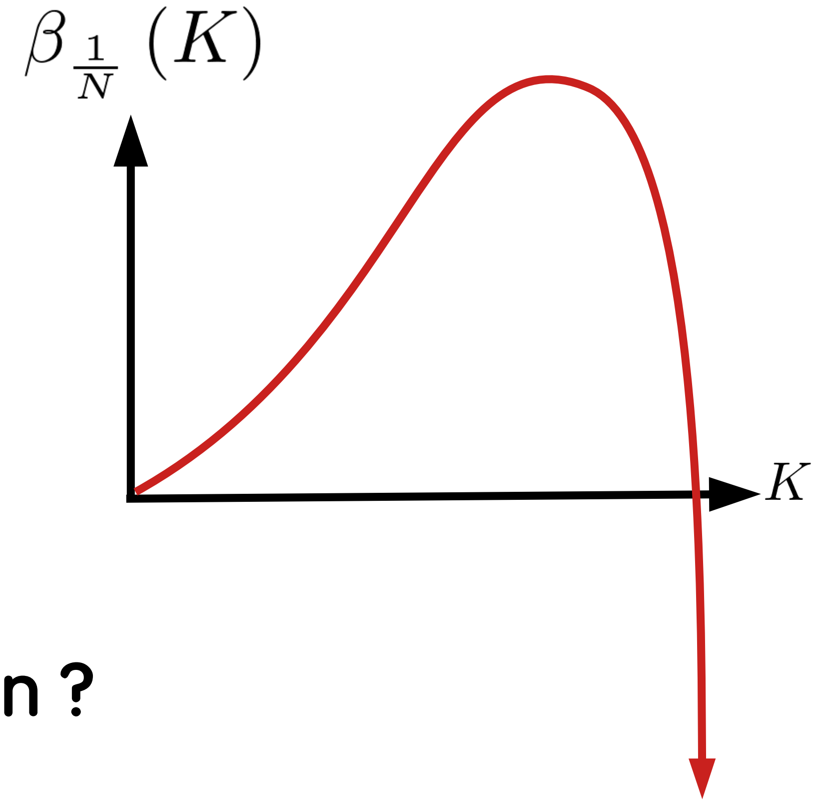

- Just an artefact from the truncation ?
- Is the pole physical ?

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- Yet the cancellation is still here, that’s interesting…

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- Scheme Transformations as a « TRICK » to scope for the pole structure at higher orders.

- 2) Critical Exponent

Application to Large N
& Beyond

- $\Delta(\alpha,\epsilon) = \alpha\epsilon + \beta(\alpha)$ is the $\beta-$ function in $4-\epsilon$ dimension

- $\omega(\epsilon) =$ is a known function for any ϵ
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The Master Equation

$$\frac{\partial\left(\frac{\beta(\alpha)}{\alpha}\right)}{\partial\alpha} = \frac{\omega\left[-\frac{\beta(\alpha)}{\alpha}\right]}{\alpha}$$

Let's choose α,

Then there exist ϵ such as $\Delta(\alpha, \epsilon) = 0$,

Hence α is a zero in $4 - \epsilon$ dimension and by definition we have:

$$\omega(\epsilon) = \frac{\partial \Delta(\alpha, \epsilon)}{\partial \alpha} = \epsilon + \beta'(\alpha)$$

However by construction we have: $\epsilon = -\dfrac{\beta(\alpha)}{\alpha}$

Which yields to $\omega\left(-\dfrac{\beta(\alpha)}{\alpha}\right) = -\dfrac{\beta(\alpha)}{\alpha} + \beta'(\alpha)$,

Finally we deduce that:

$$\frac{\partial \frac{\beta(\alpha)}{\alpha}}{\partial \alpha} = \frac{\alpha\beta'(\alpha) - \beta(\alpha)}{\alpha^2} = \frac{\alpha\omega\left(-\frac{\beta(\alpha)}{\alpha}\right) + \beta(\alpha) - \beta(\alpha)}{\alpha^2} = \frac{\omega\left(-\frac{\beta(\alpha)}{\alpha}\right)}{\alpha}$$

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In practice : $\beta\left(\alpha\right) = \alpha Y\left(\alpha\right) \quad \Longrightarrow \quad Y'\left(\alpha\right) = \frac{\omega\left[-Y\left(\alpha\right)\right]}{\alpha}$

The Master Equation

$$\frac{\partial\left(\frac{\beta(\alpha)}{\alpha}\right)}{\partial\alpha} = \frac{\omega\left[-\frac{\beta(\alpha)}{\alpha}\right]}{\alpha}$$

- Non Pertubative approach
- Hard to solve in general
- There are interesting limits and general behavior

Toy Model

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$$Y' = g\left(Y\right)$$

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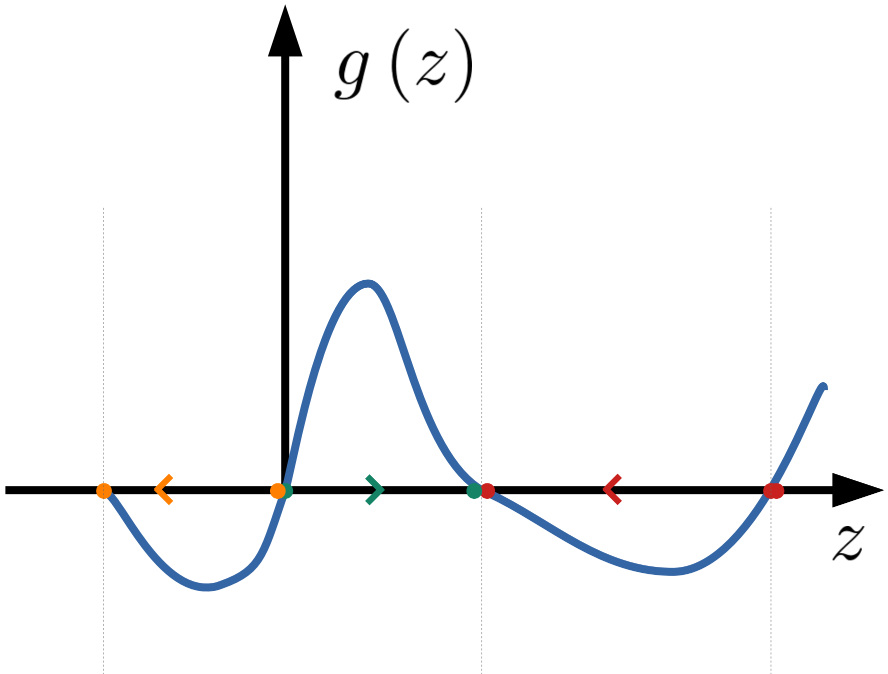

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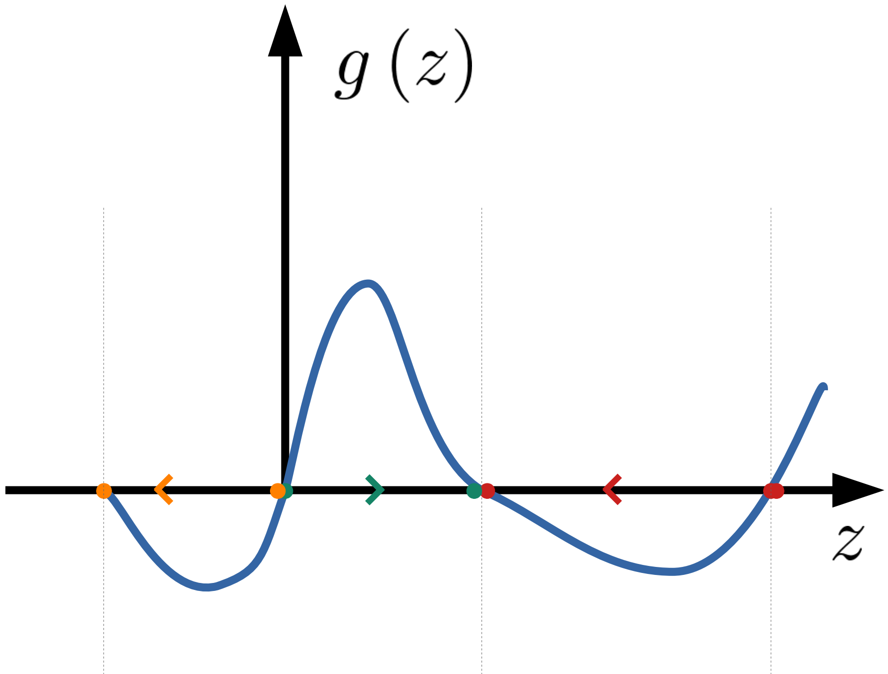

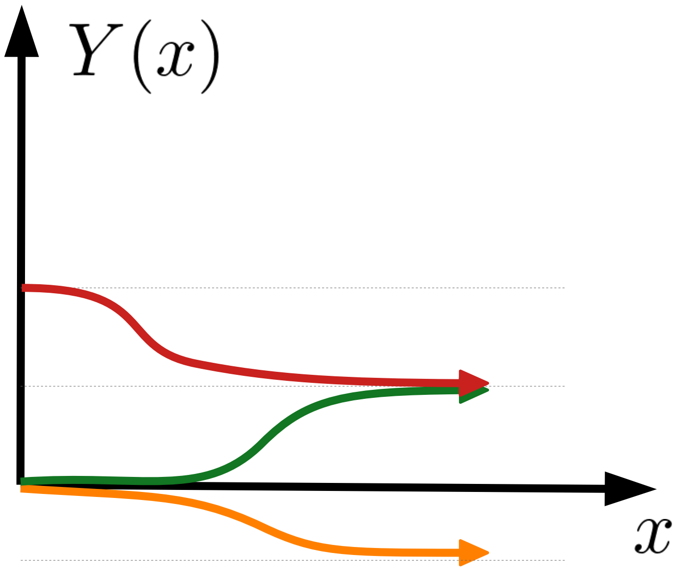

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- If ω has a pole, the solutions will not exhibit one.

Poles in practice

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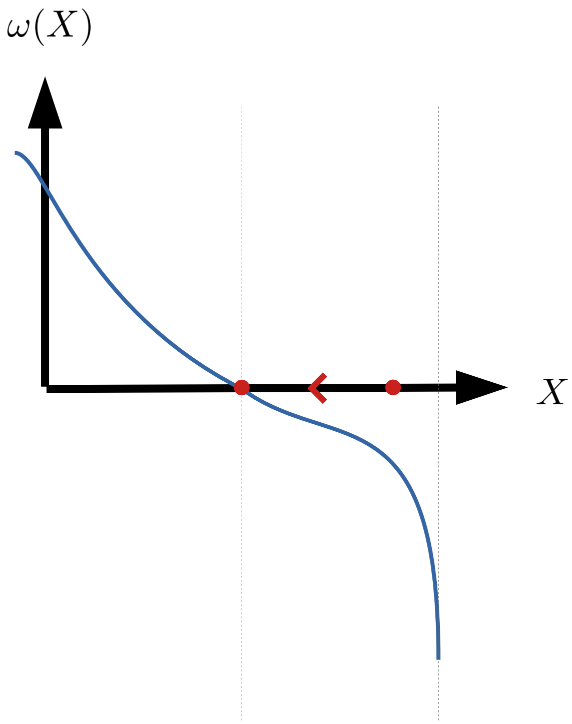

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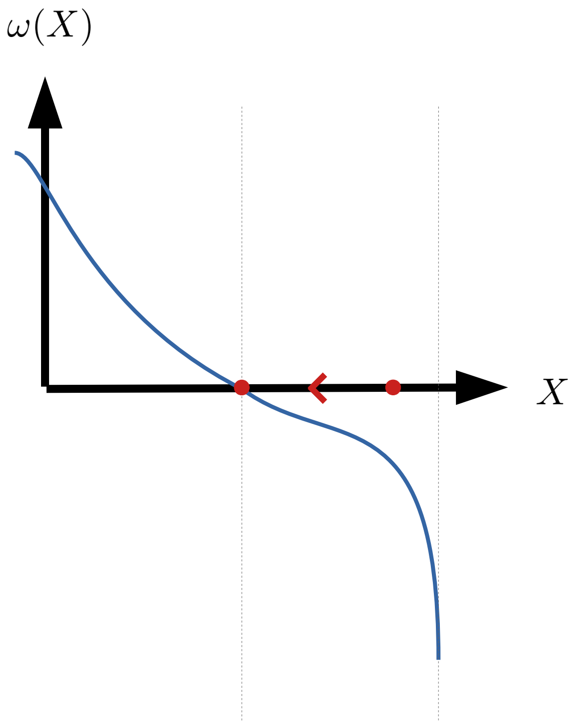

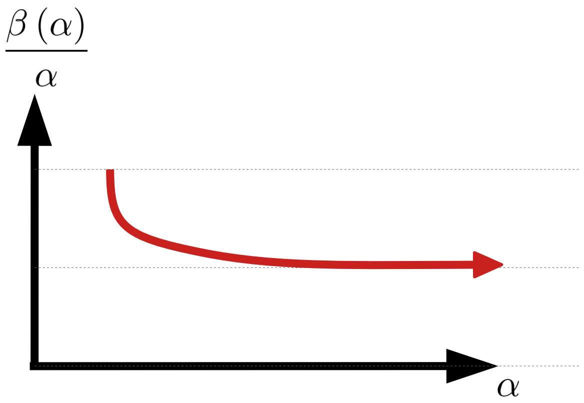

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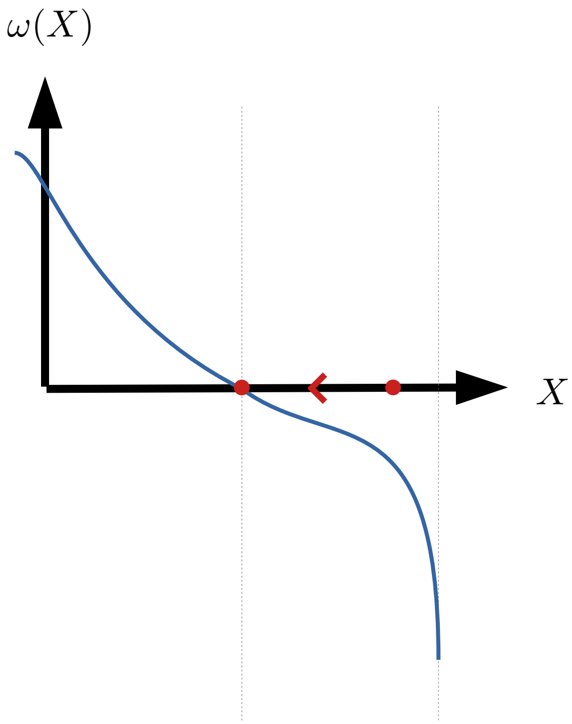

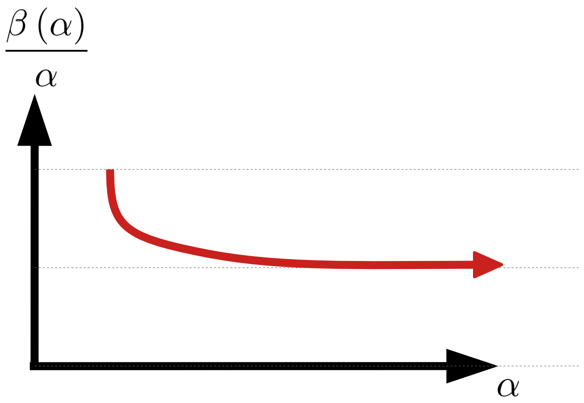

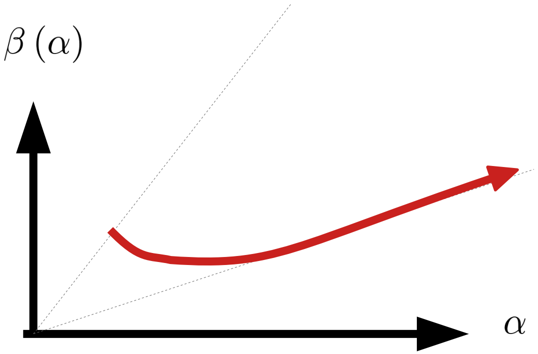

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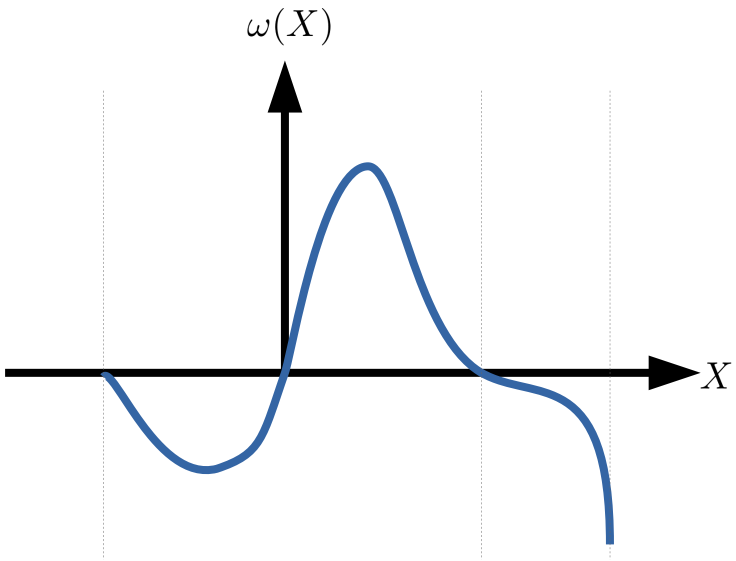

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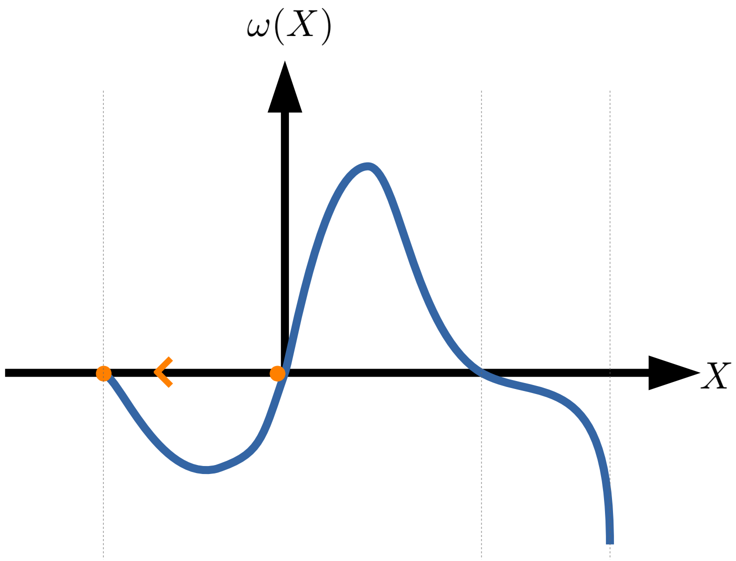

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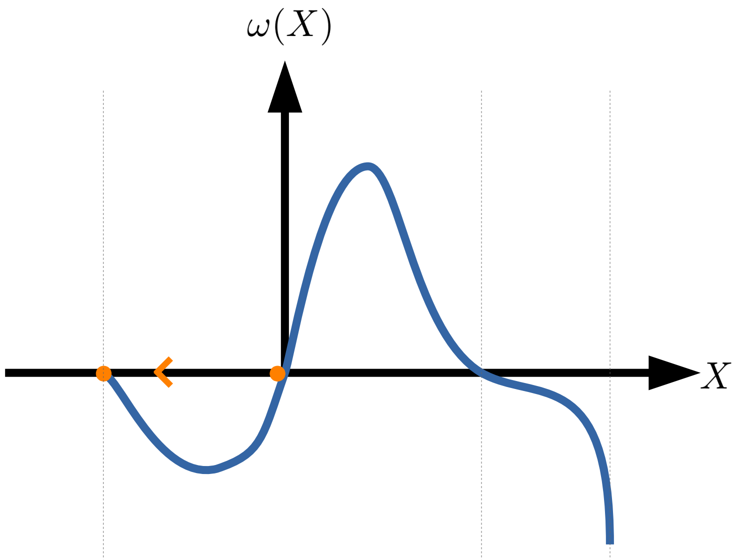

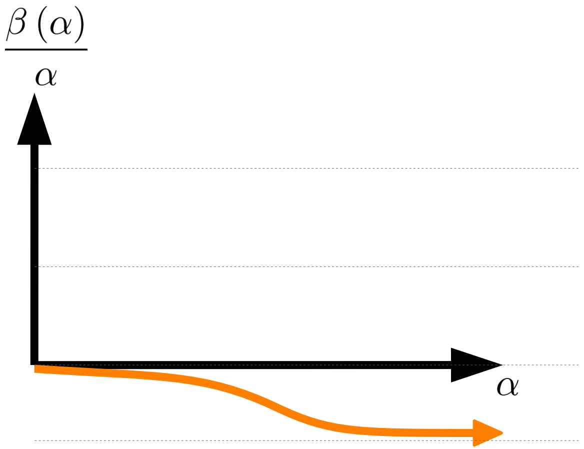

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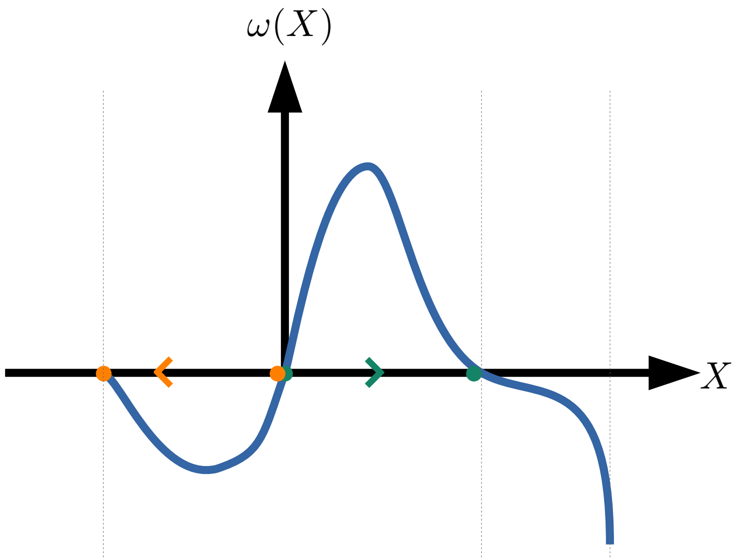

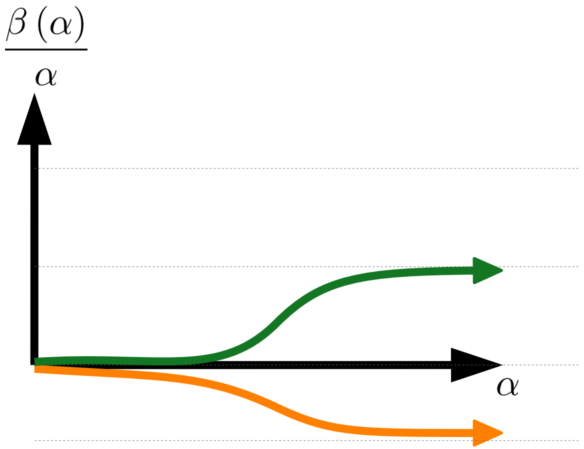

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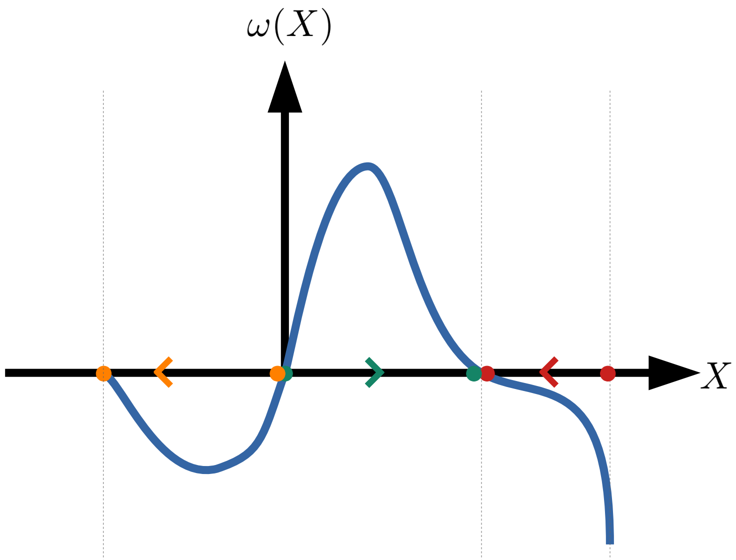

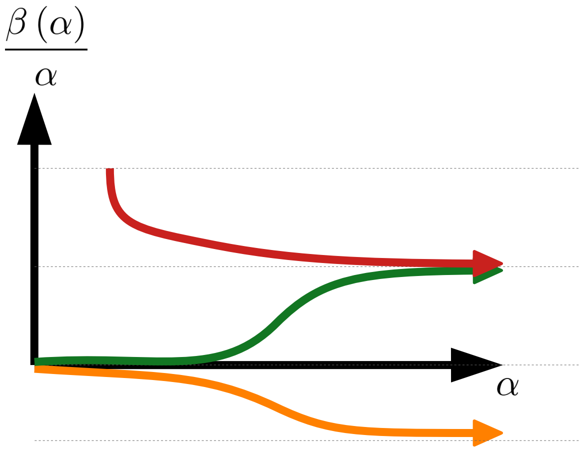

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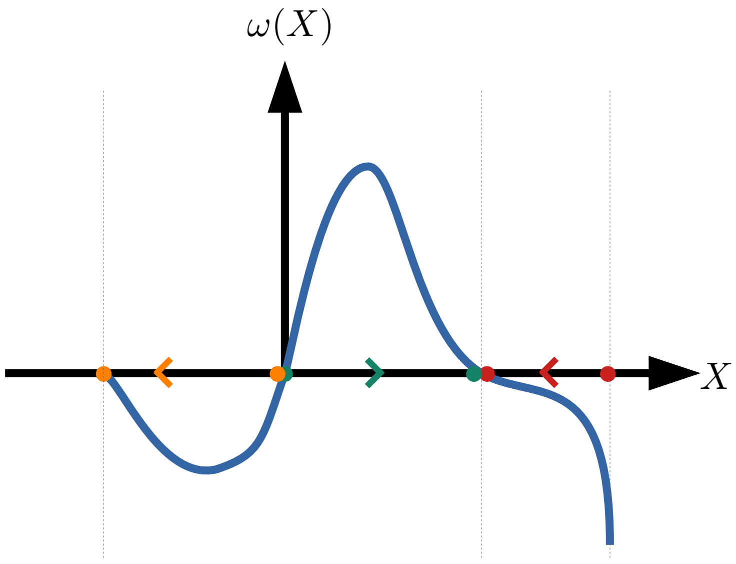
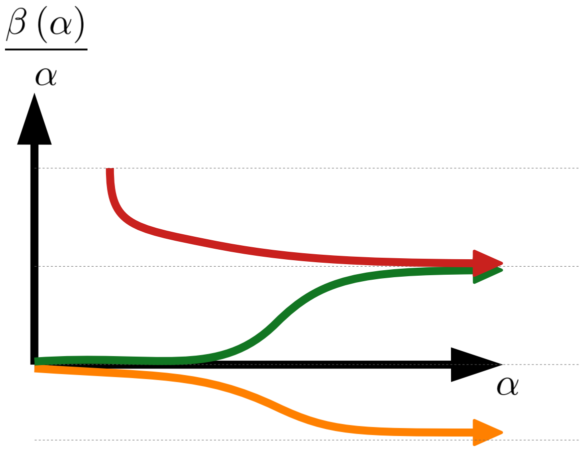
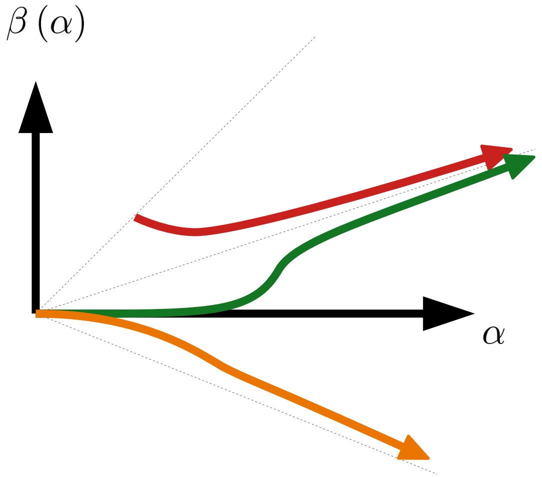
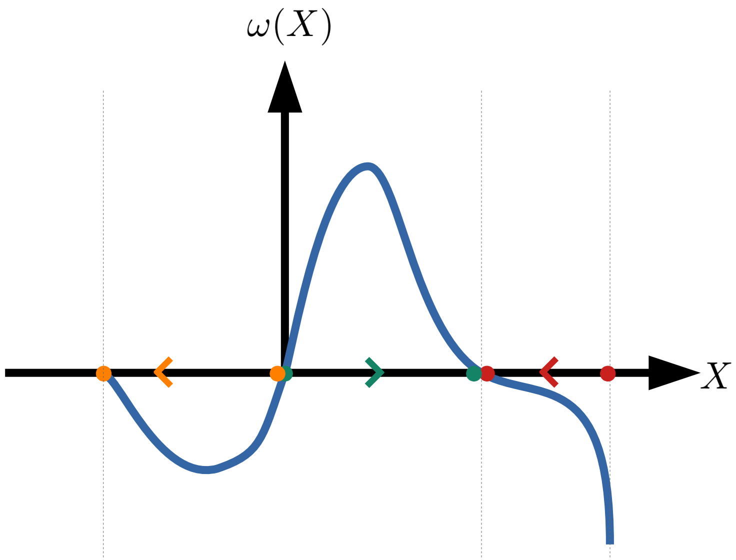
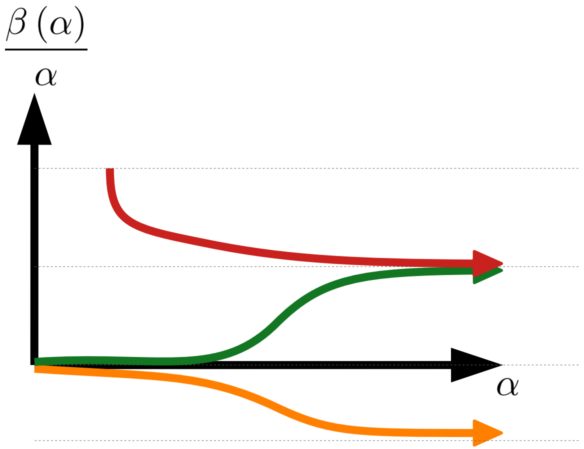


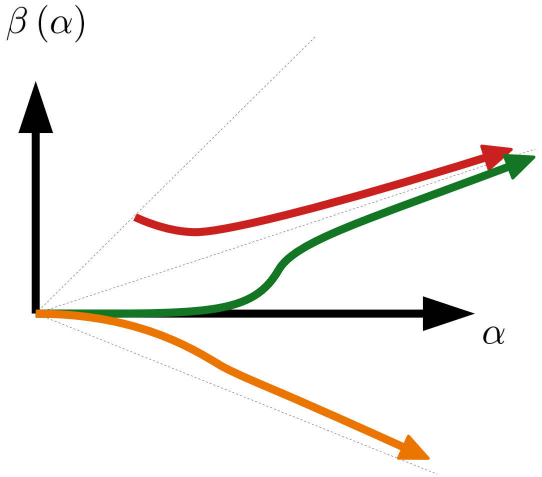




Example







The initial condition is important ⚠

Back To Large N

Back To Large N

$$\beta\left(K\right) = (d - d_c)K + \frac{2K^2}{3}\left[1 + \sum_{n=1} \frac{F_n(K)}{N^n}\right]$$

$$\omega(d) = -(d - d_c) + \sum_{n=1} \frac{\omega^{(n)}(d)}{N^n}$$

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$$K_c = -\frac{d - d_c}{1 + \sum_{n=1}\frac{F_n(K_c)}{N^n}}$$

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$$\vdots$$

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- How can we compute the complete contribution ?

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- $\omega^{(1)}$ appears to all orders.
- How can we compute the complete contribution ?

→ The Master Equation with $\omega^{(1)}$!

What happen ?

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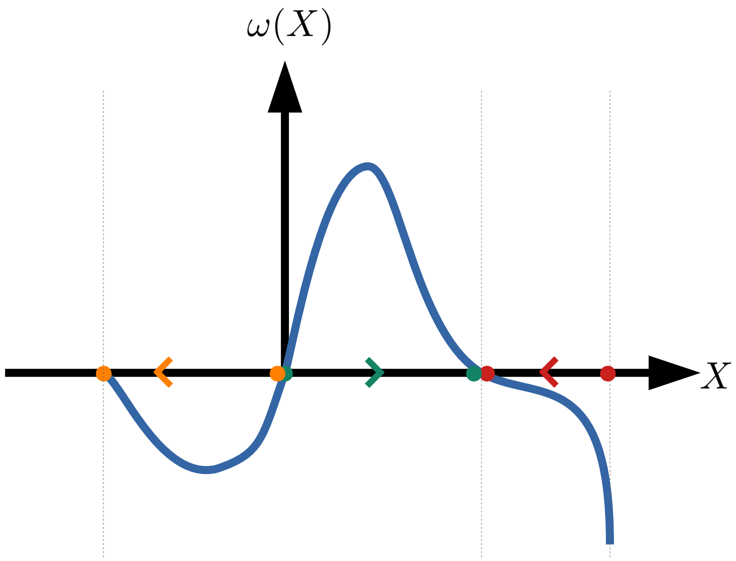

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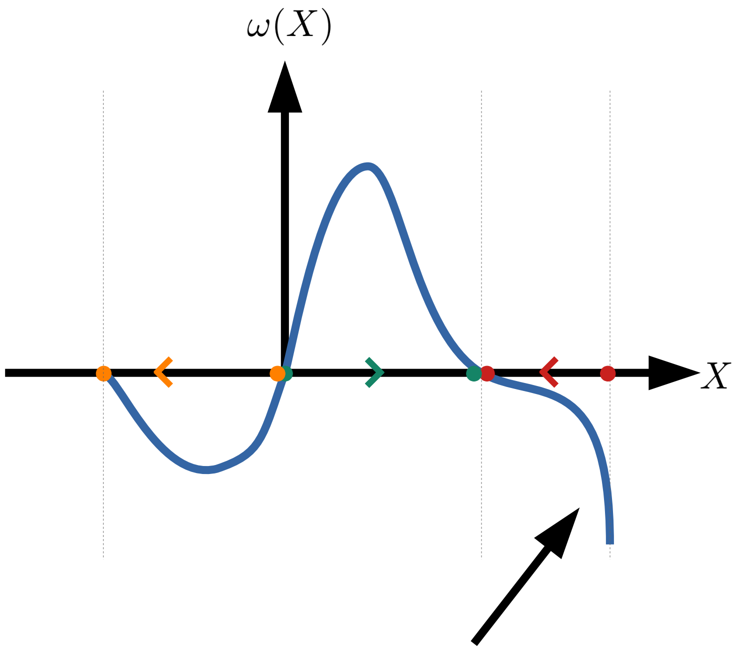
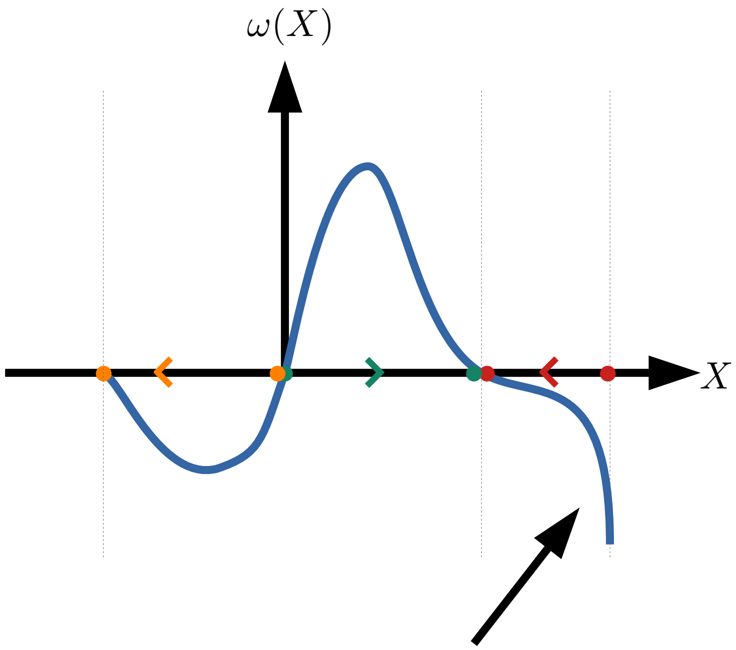
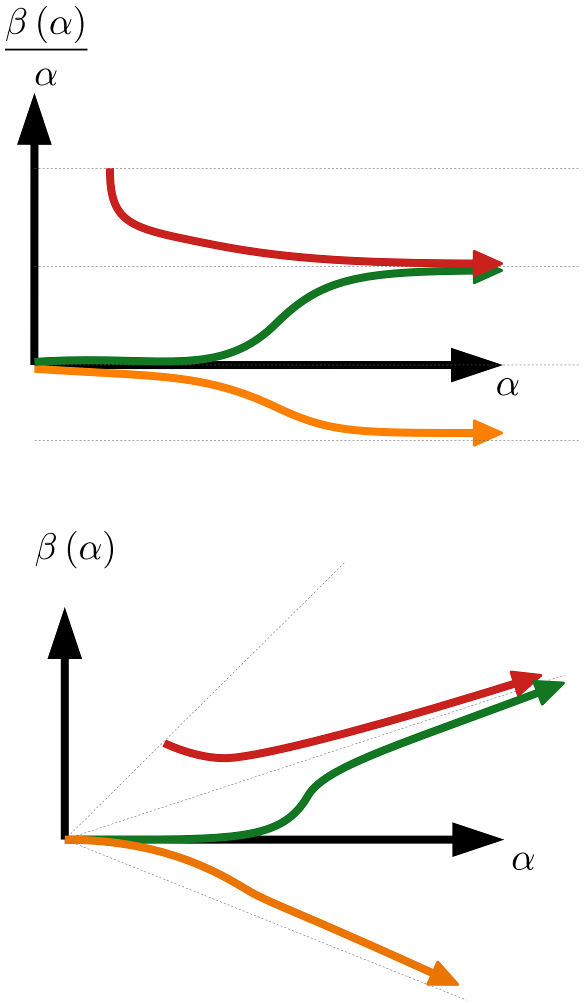


The Pole !

What happen ?



The Pole !



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Let's keep looking at the Master Equation

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- Non Pertubative approach !

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- Non Pertubative approach !
- What does it tell about beta-functions ?

Let's keep looking at the Master Equation

- Non Pertubative approach !

- Can we use it from IR to UV ?

IR Fixed-Point Model

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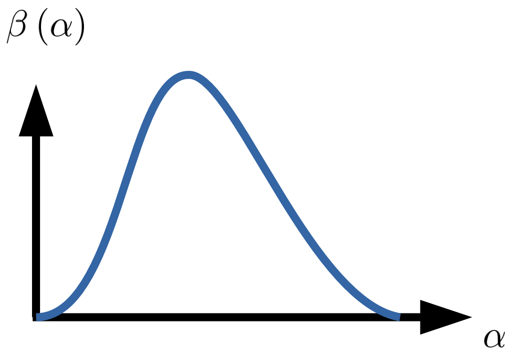

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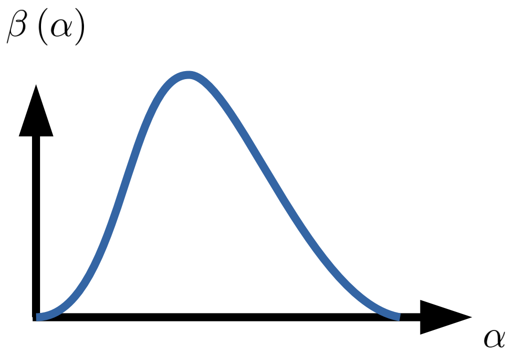
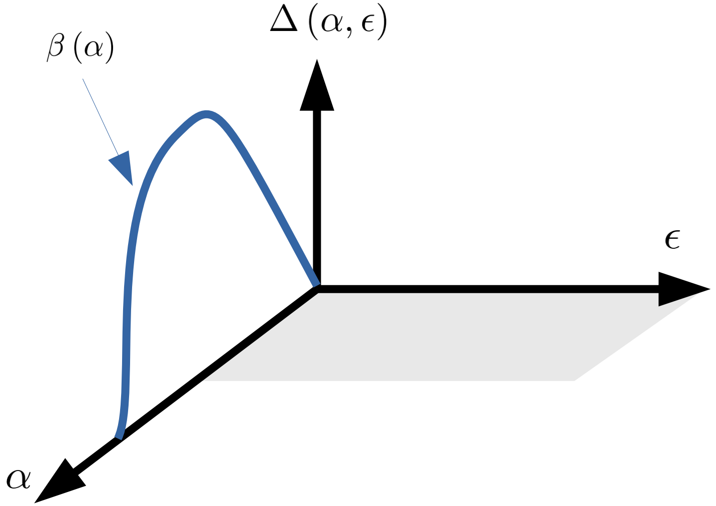
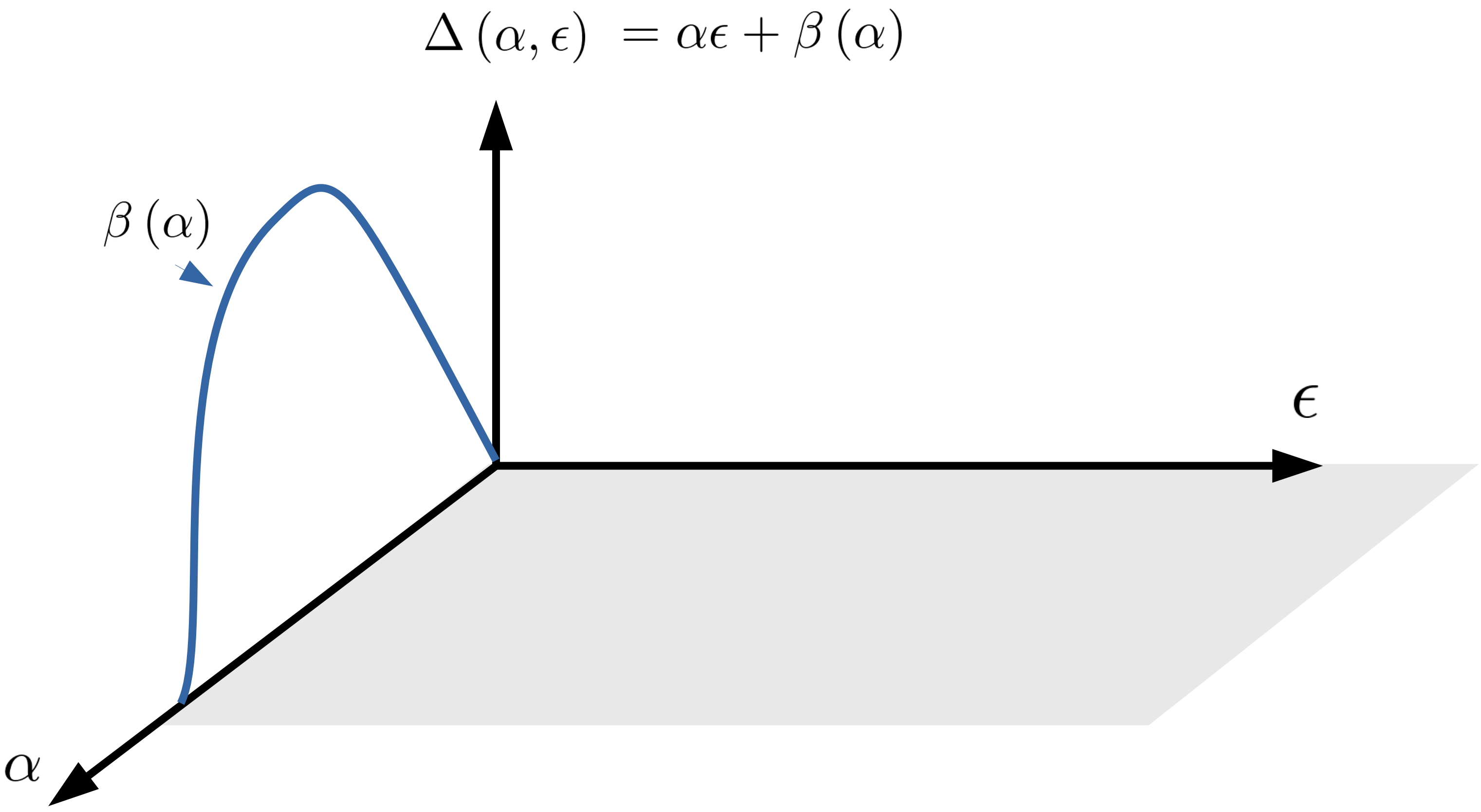
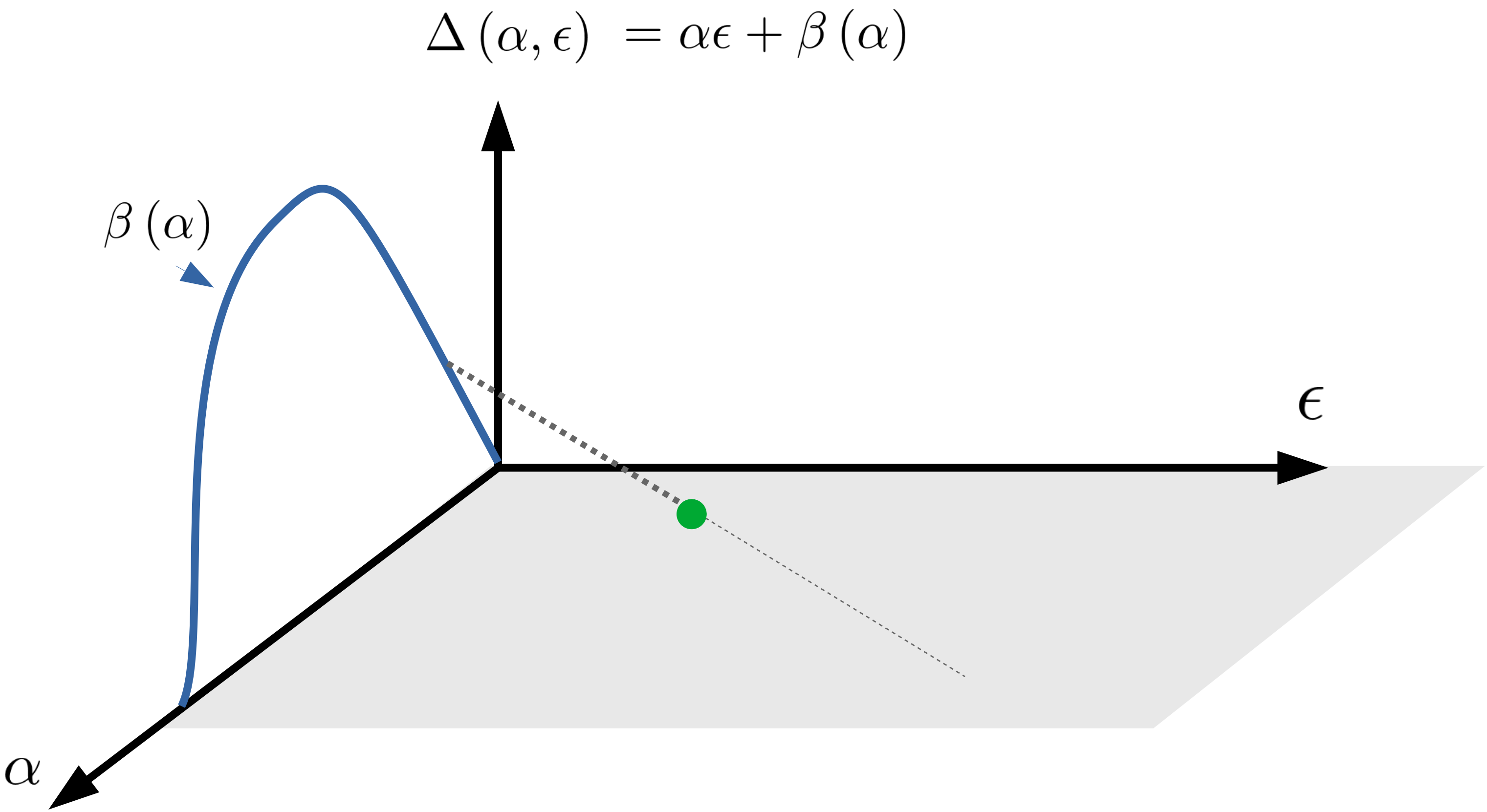





$\Delta(\alpha, \epsilon) = \alpha\epsilon + \beta(\alpha)$
$\beta(\alpha)$
ϵ
α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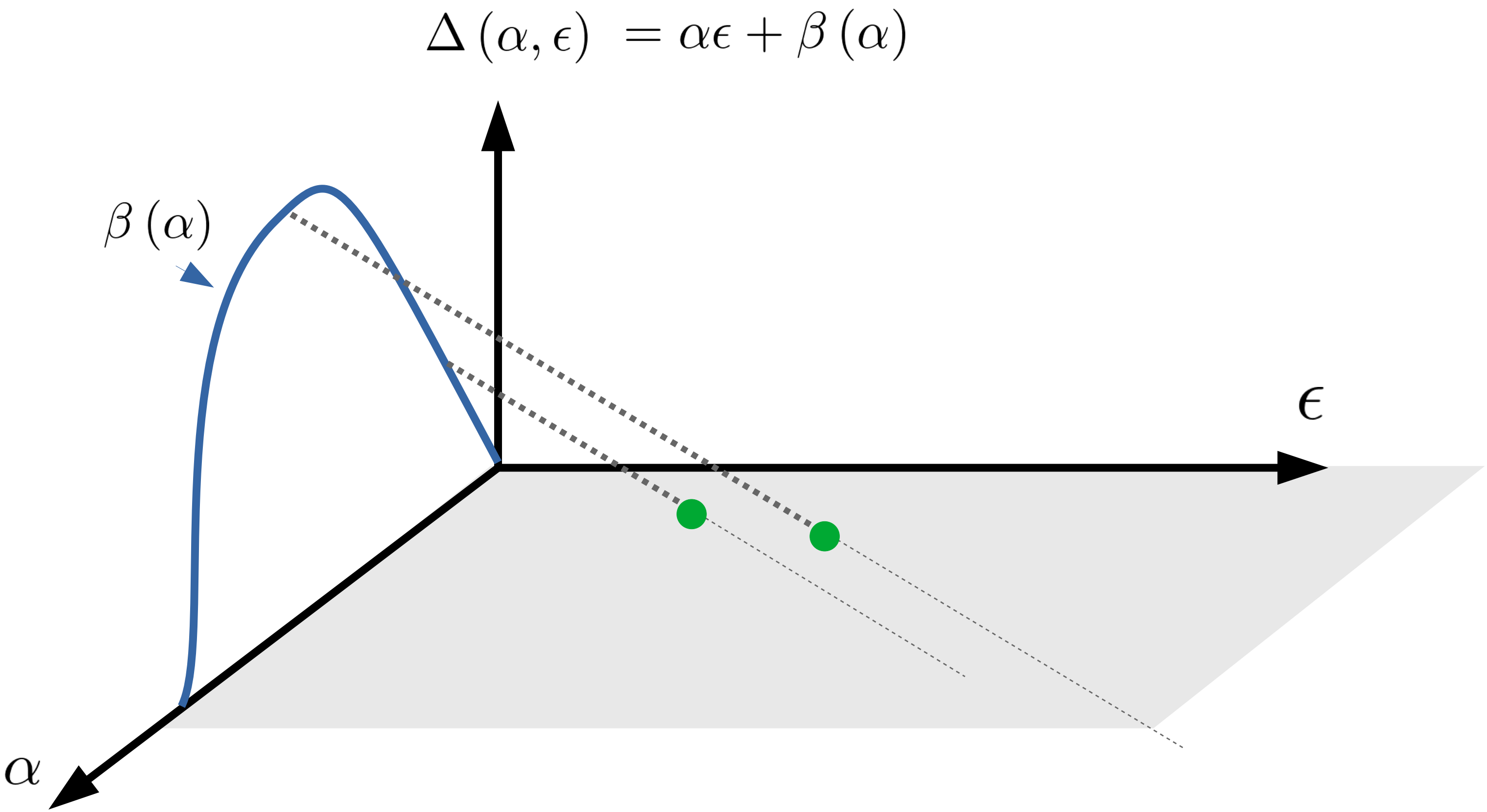

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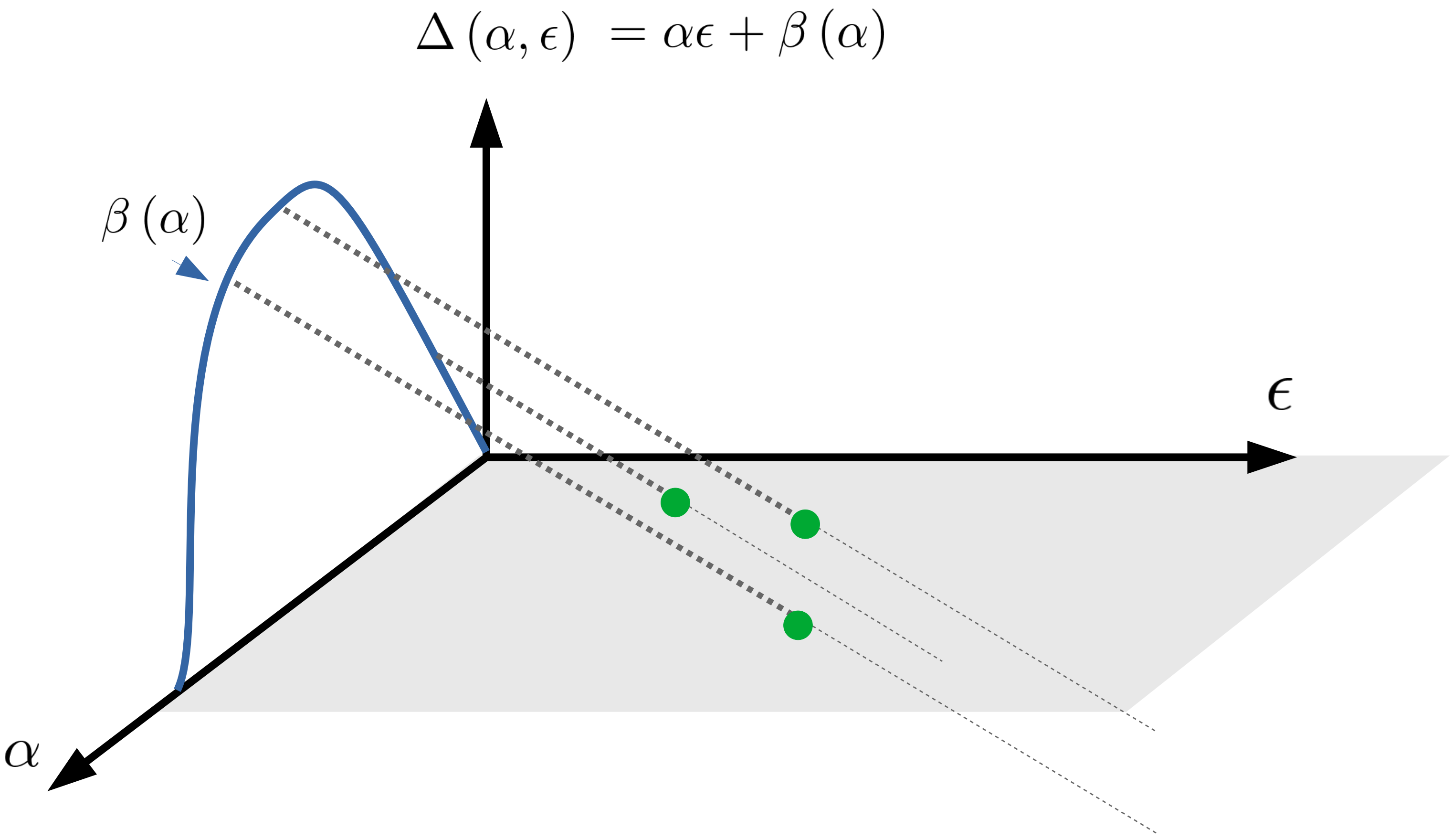
$\Delta(\alpha, \epsilon) = \alpha\epsilon + \beta(\alpha)$
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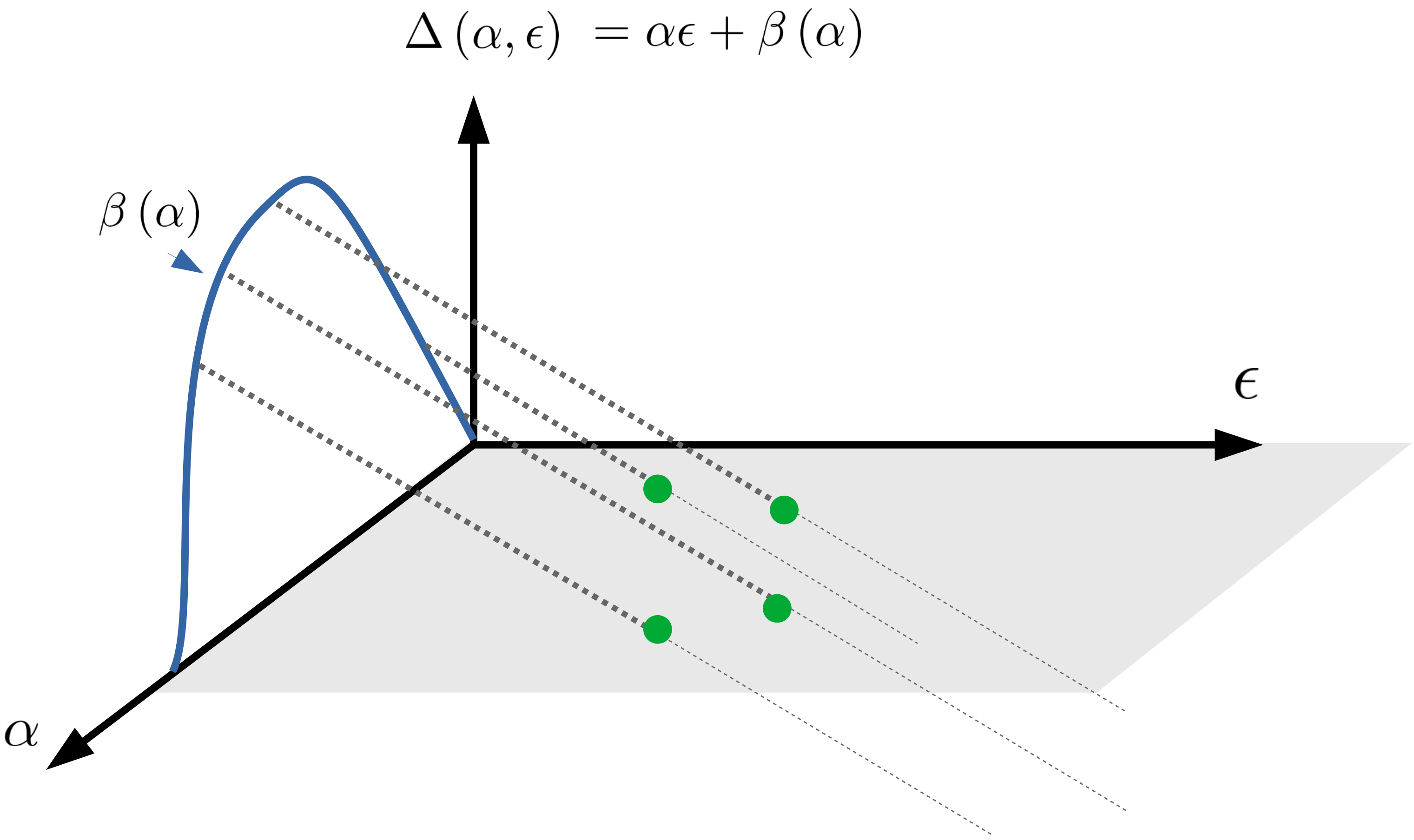
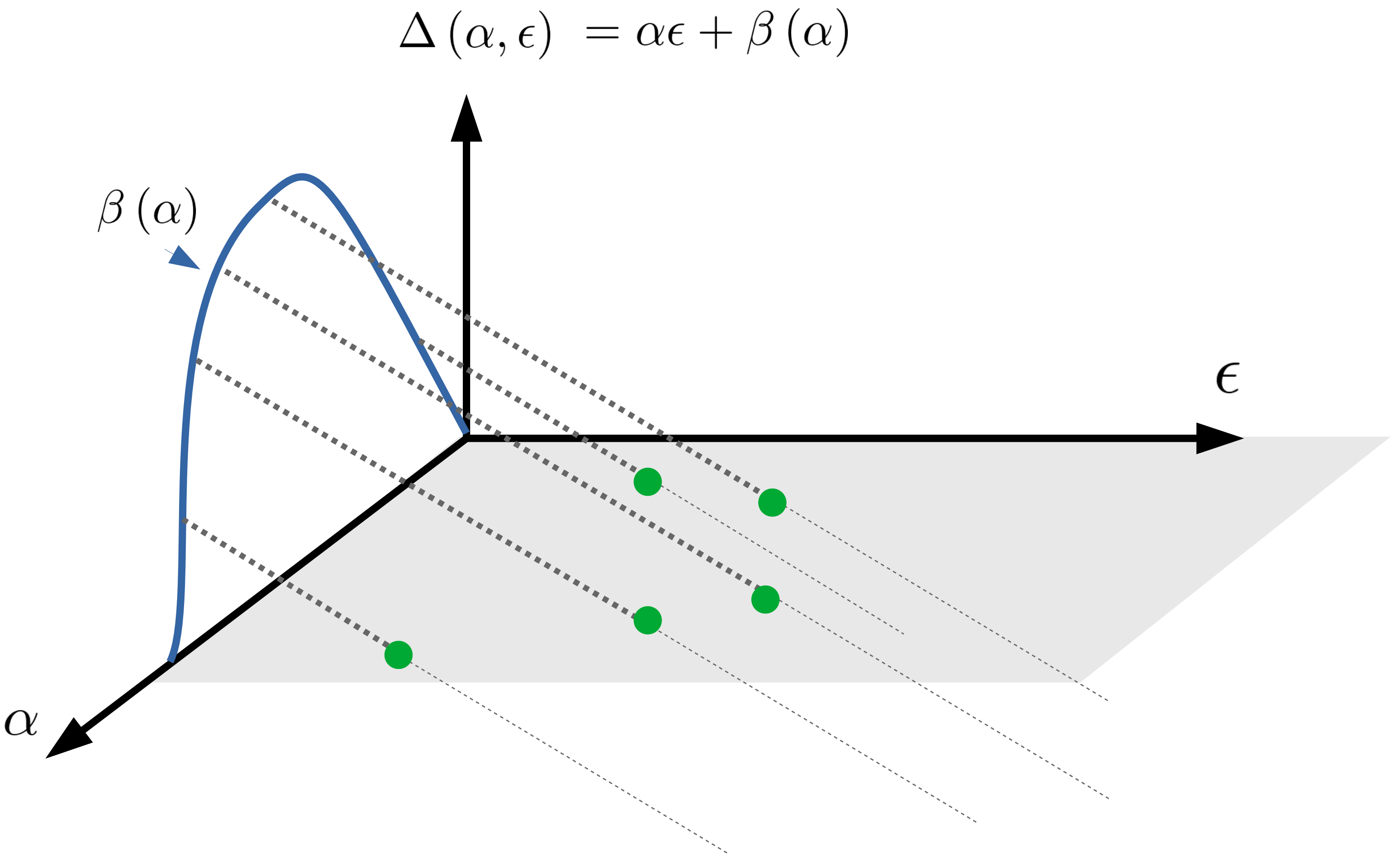


Δ (α, ϵ) = αϵ + β (α)
β (α)
ϵ
α



$\Delta(\alpha,\epsilon) = \alpha\epsilon + \beta(\alpha)$
$\beta(\alpha)$
ϵ
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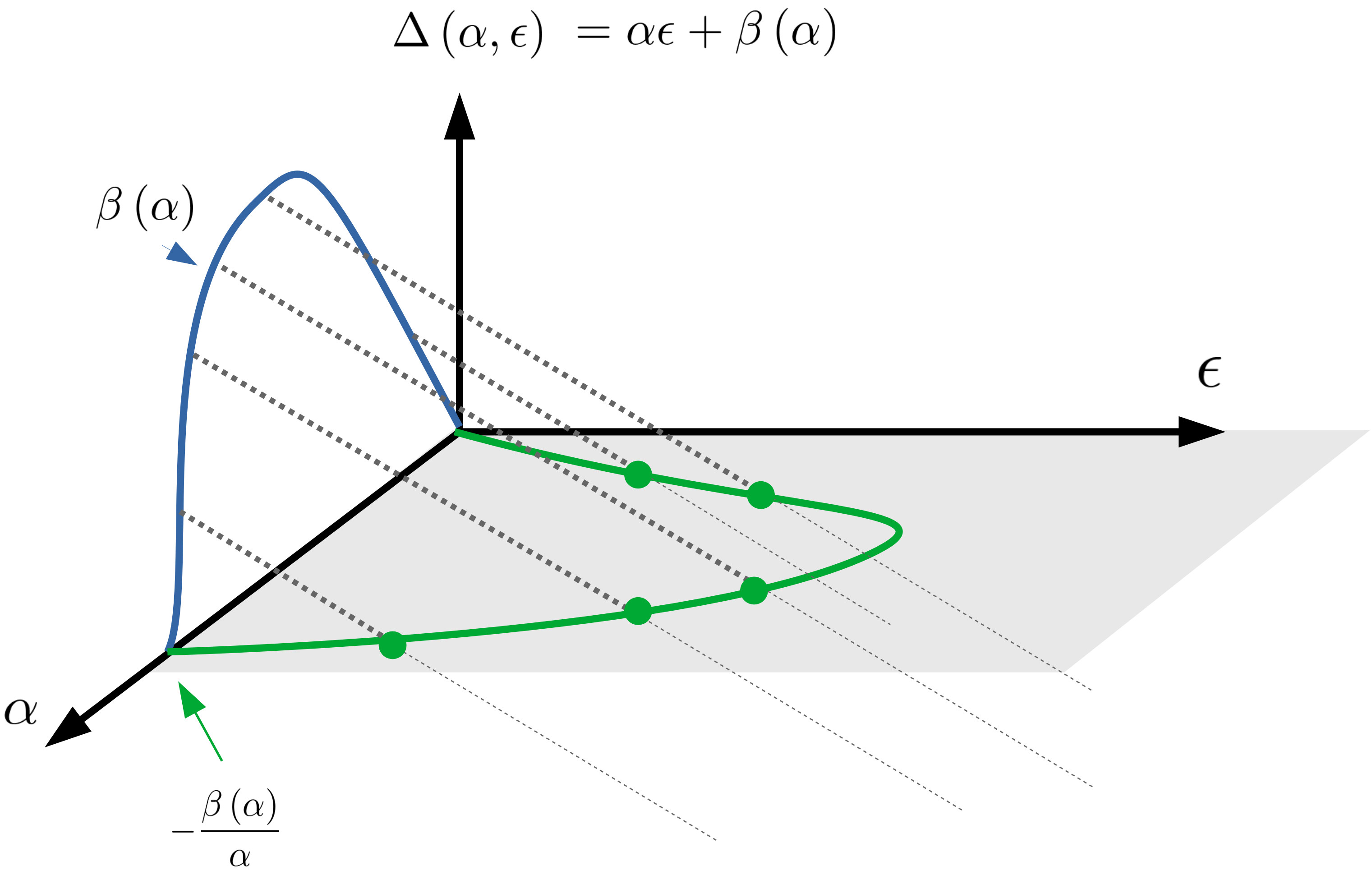
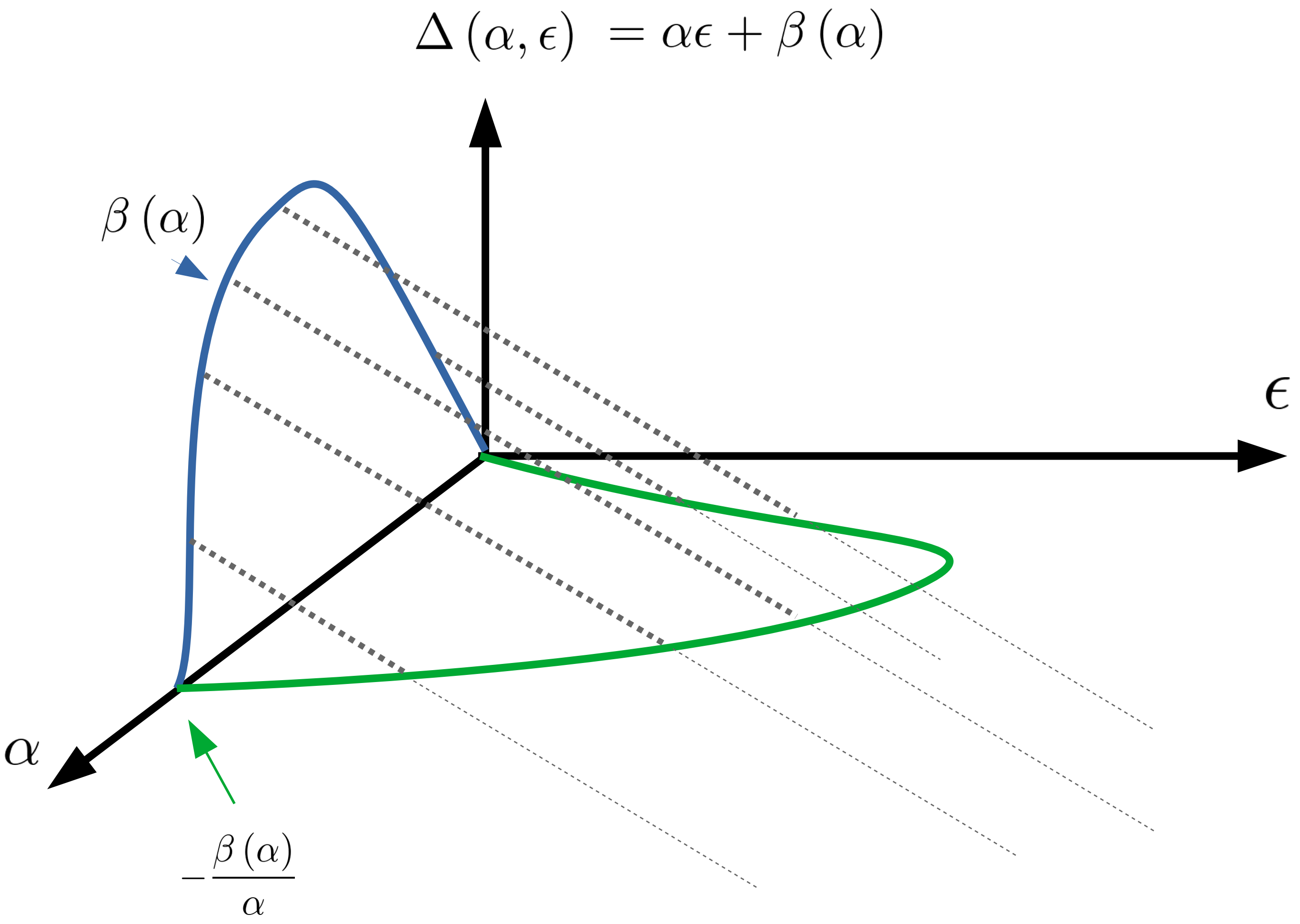


Δ (α, ϵ) = αϵ + β (α)
β (α)
ϵ
α
− β (α) / α


$\Delta(\alpha, \epsilon) = \alpha\epsilon + \beta(\alpha)$
$\beta(\alpha)$
ϵ
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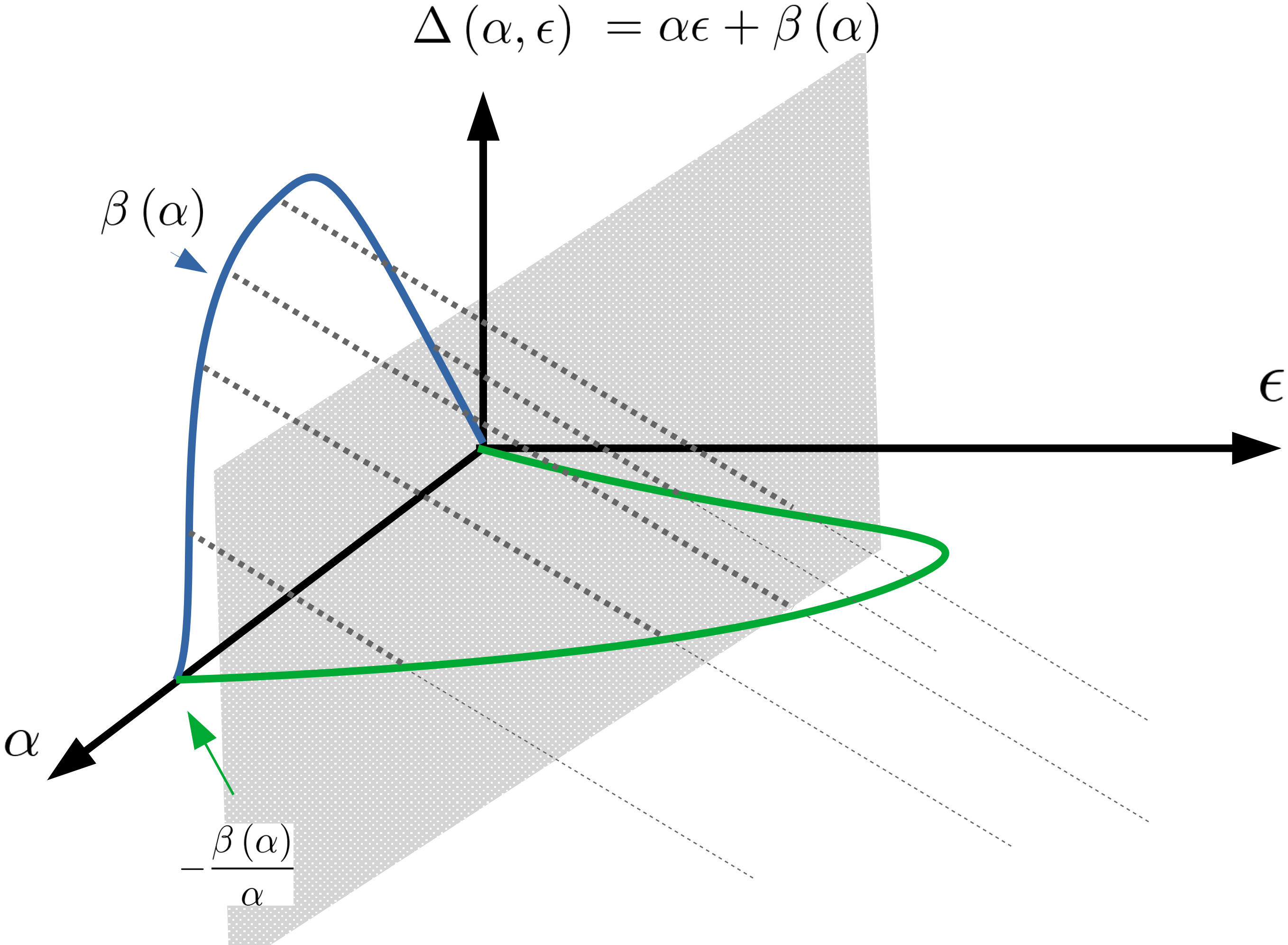
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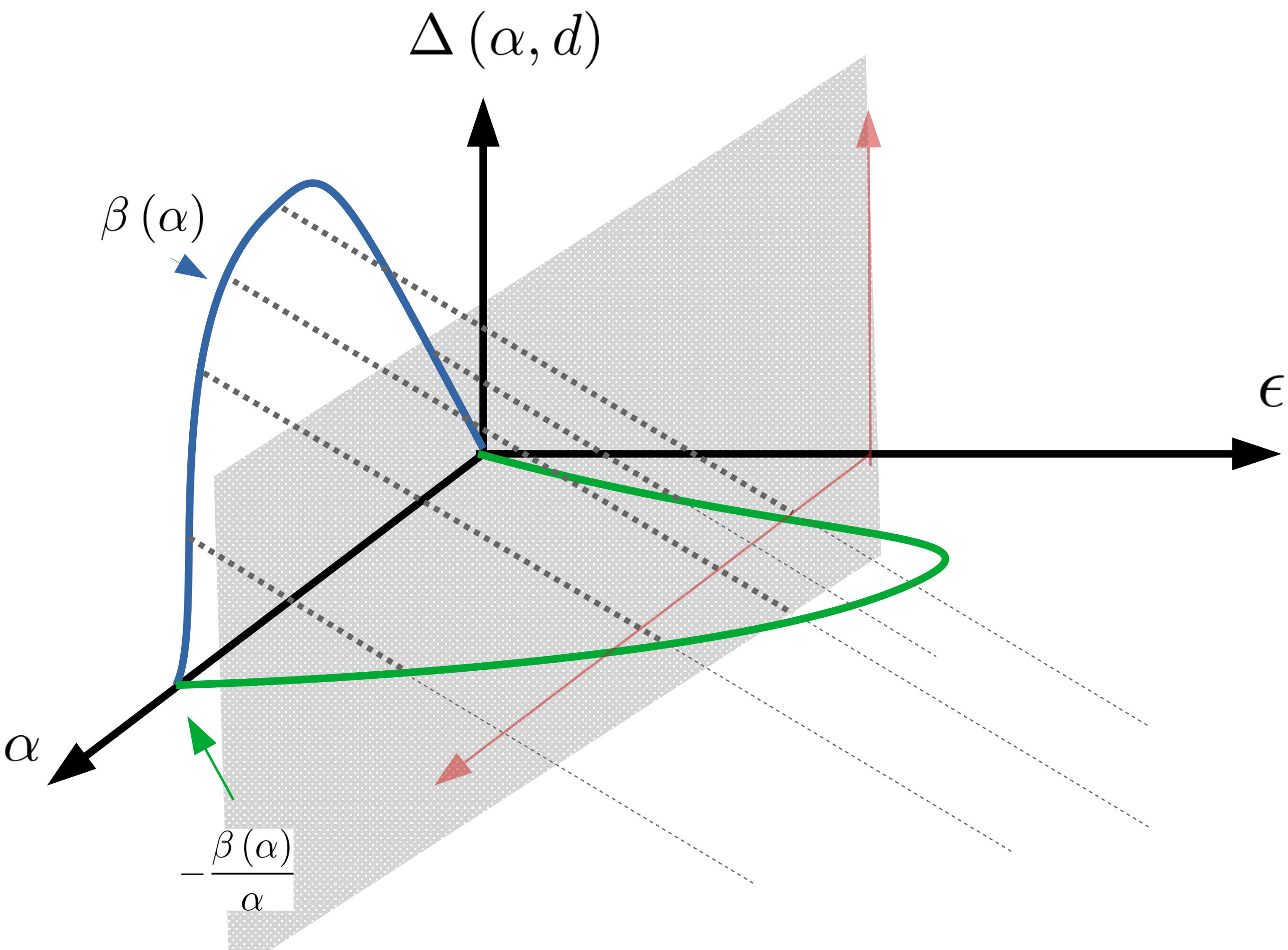
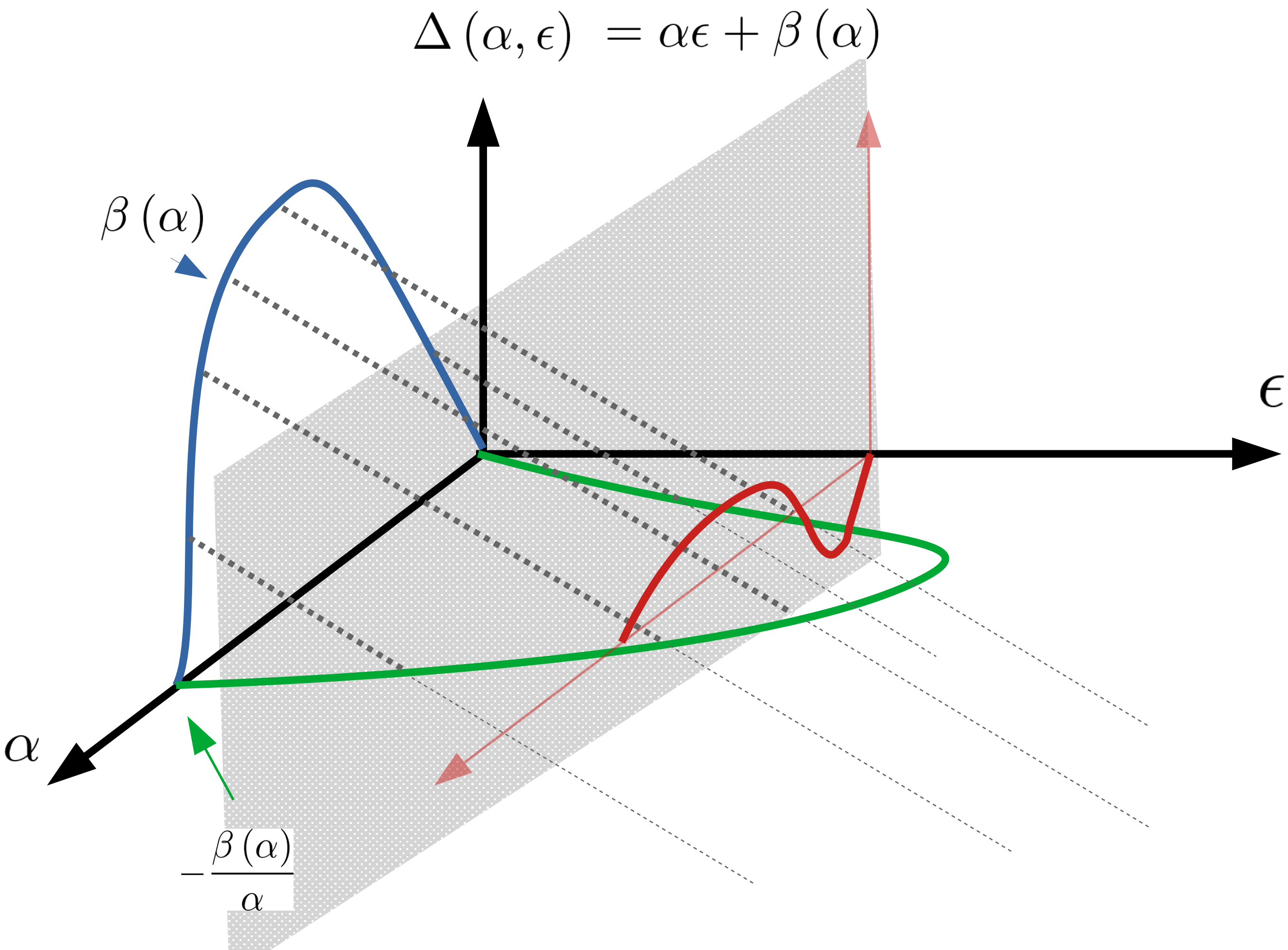

$\Delta(\alpha, d)$
$\beta(\alpha)$
ϵ
α
$-\frac{\beta(\alpha)}{\alpha}$



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$\beta(\alpha)$
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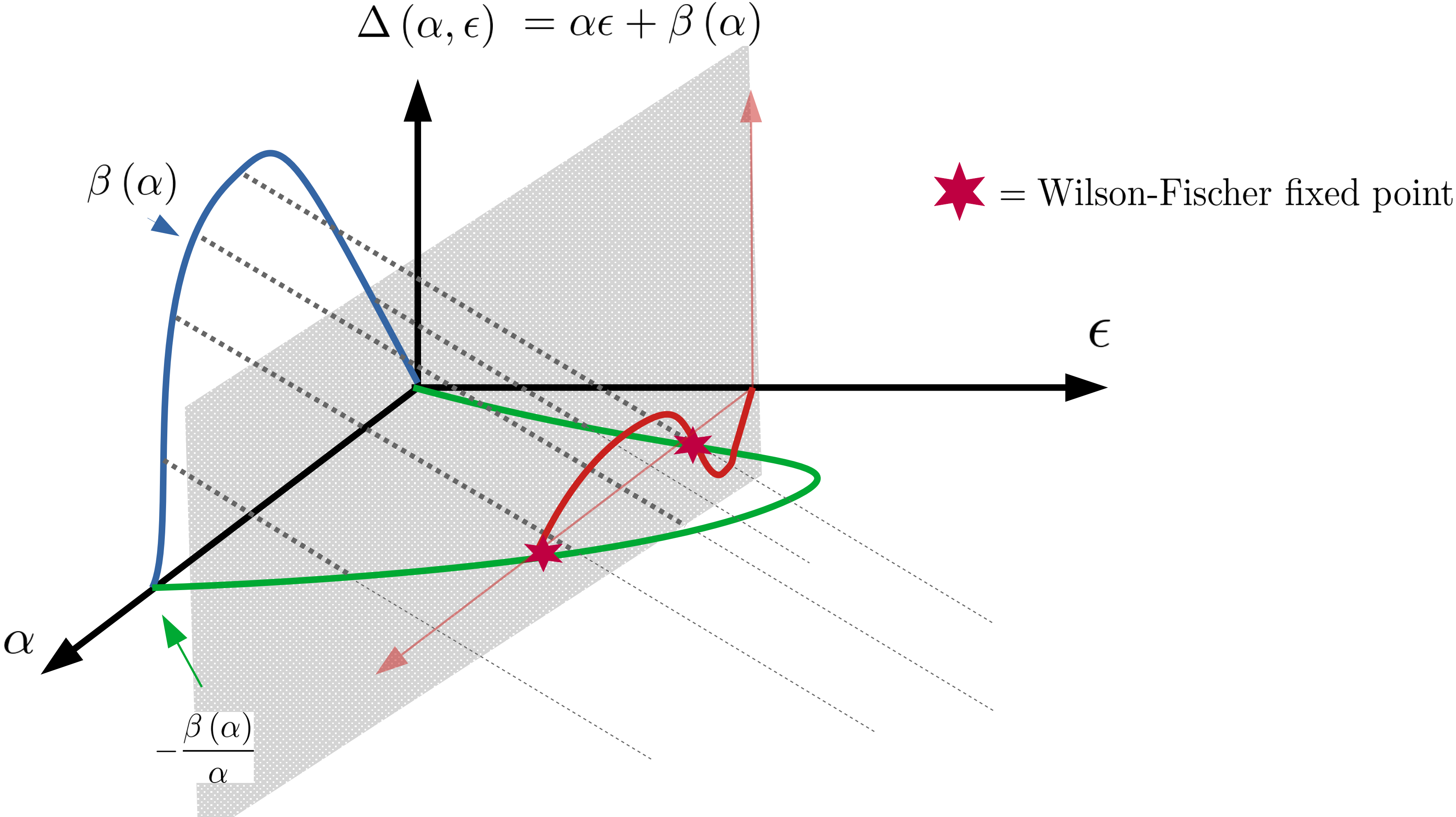

$\Delta(\alpha,\epsilon) = \alpha\epsilon + \beta(\alpha)$
$\beta(\alpha)$
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α
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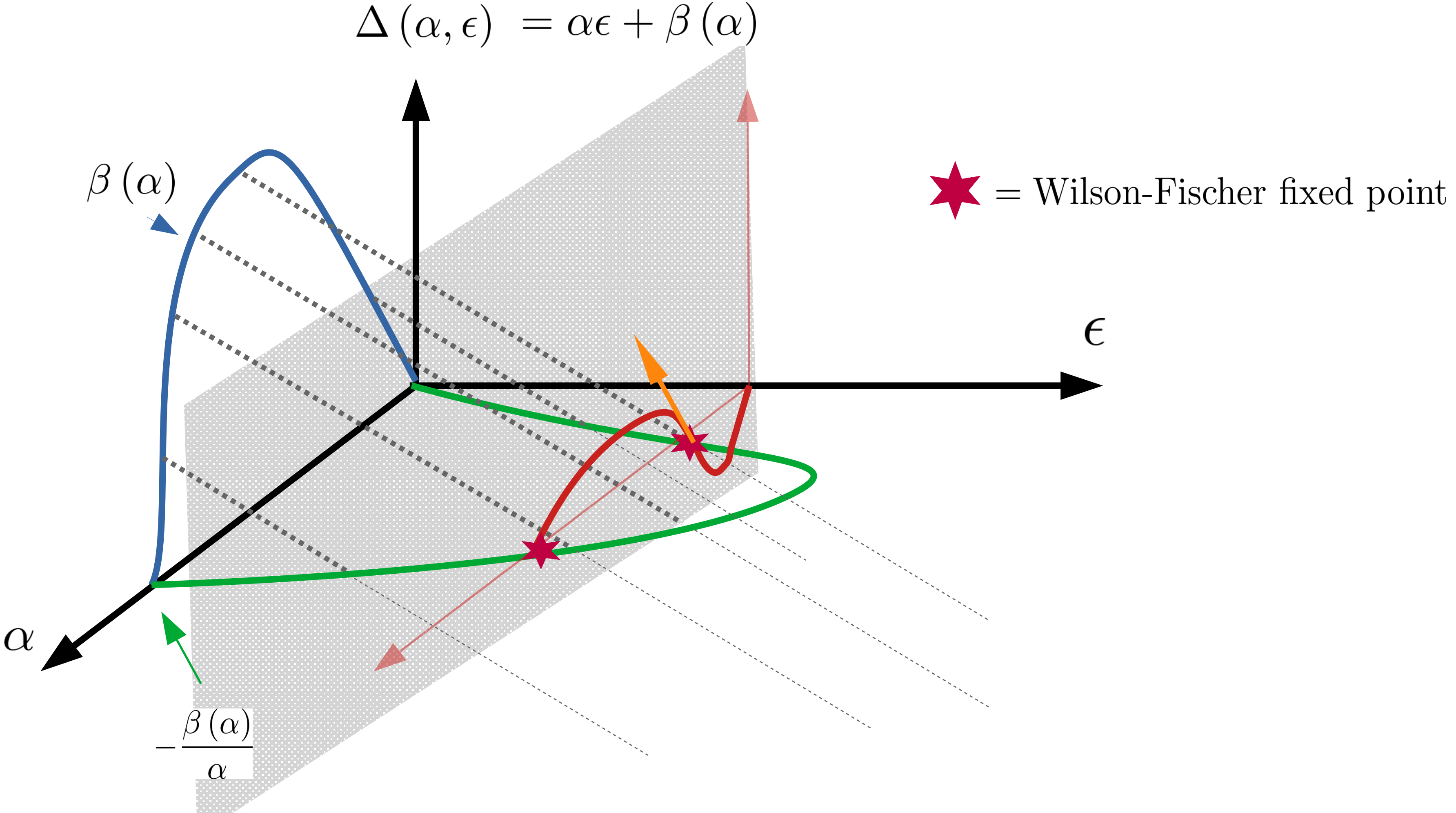
= Wilson-Fischer fixed point



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α
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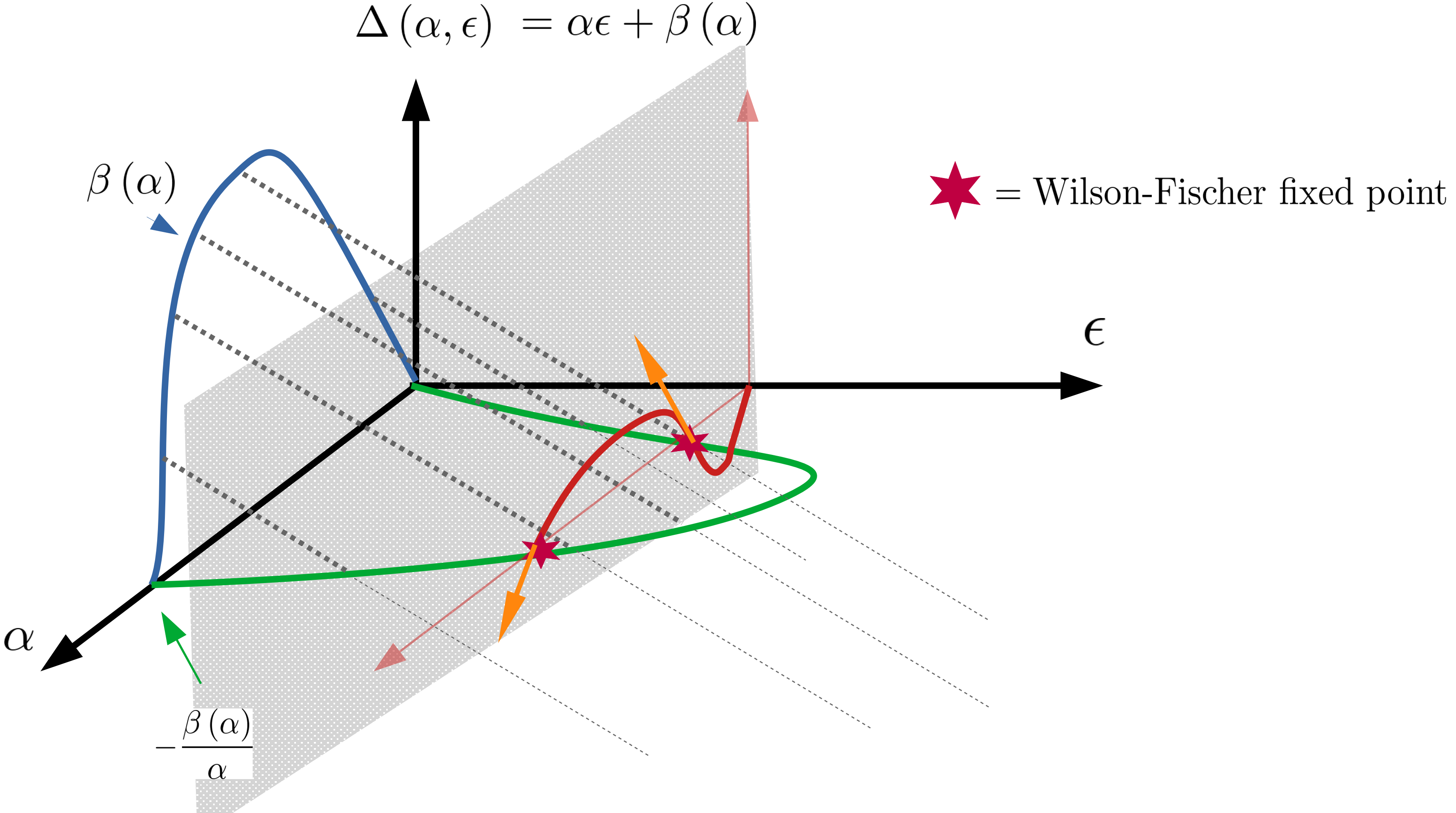
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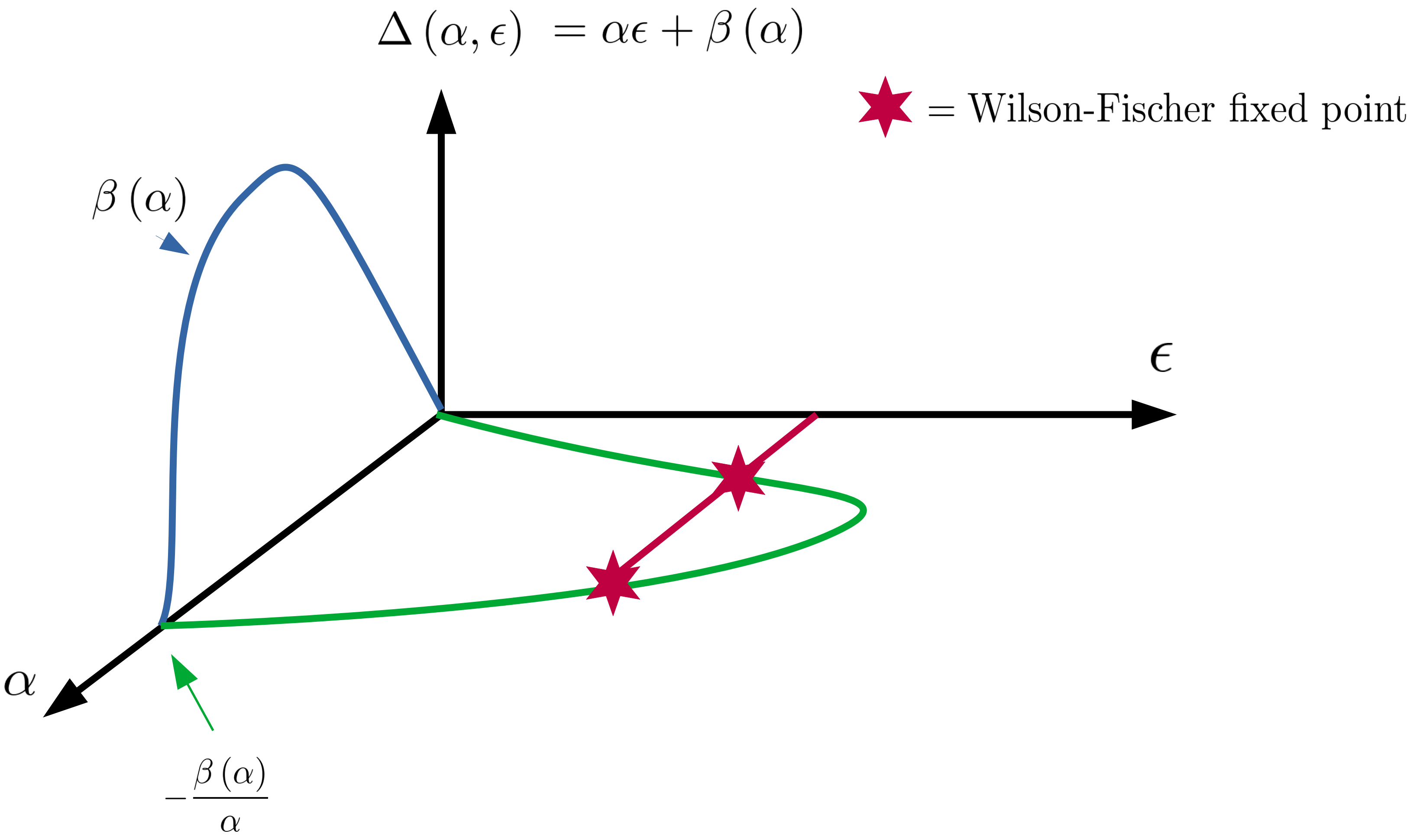

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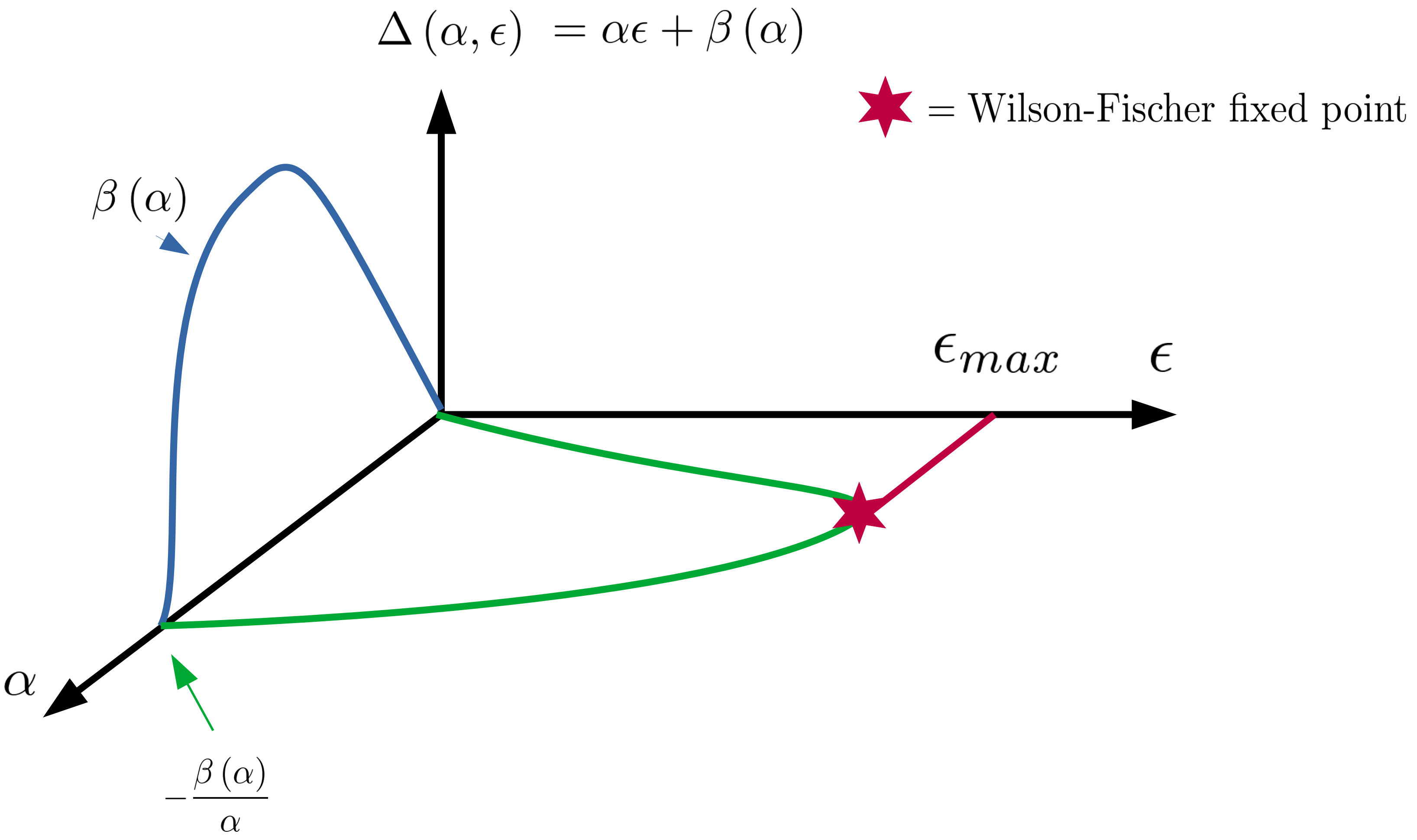
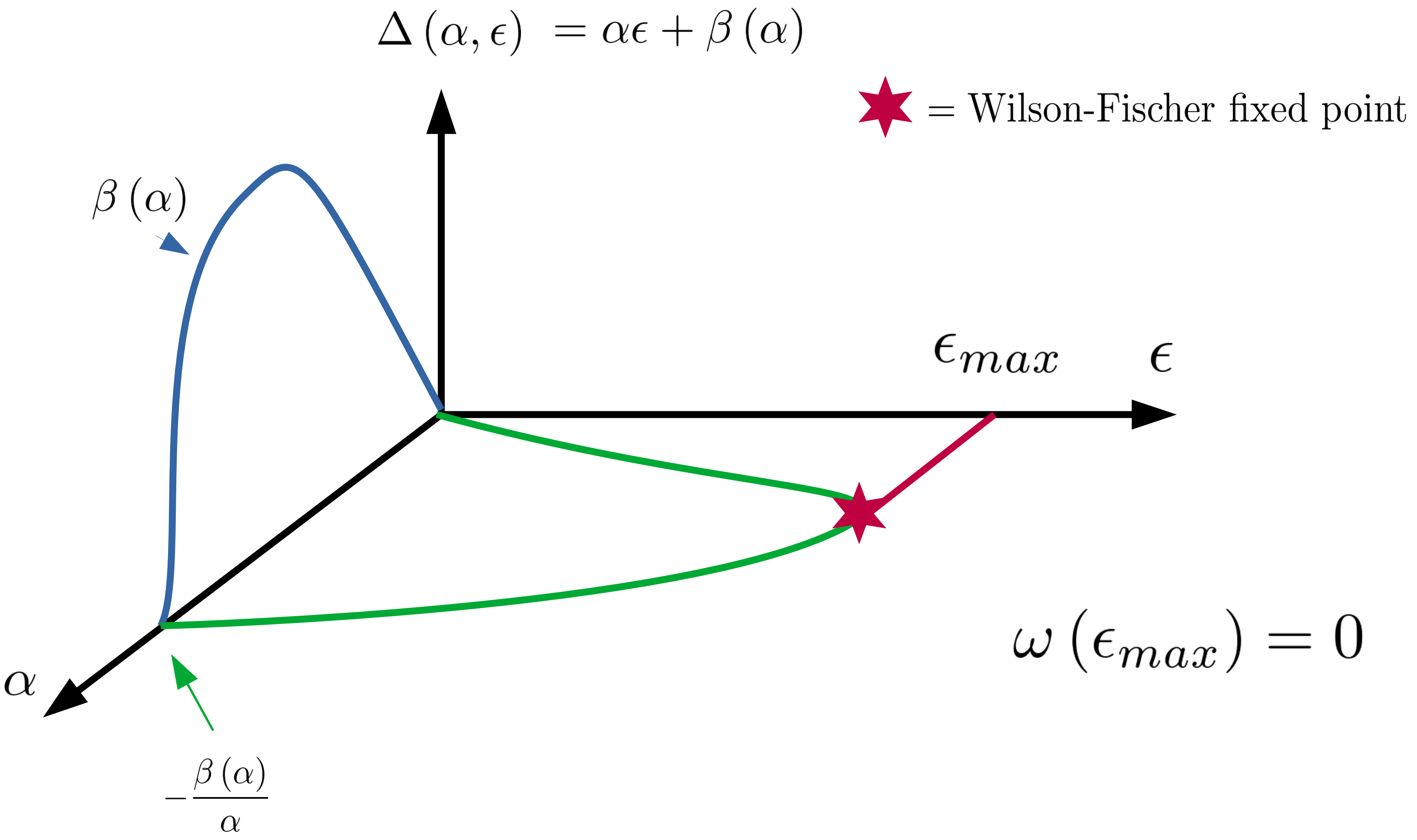

$\Delta(\alpha, \epsilon) = \alpha\epsilon + \beta(\alpha)$
= Wilson-Fischer fixed point
$\beta(\alpha)$
ϵ_{max}
ϵ
α
$-\frac{\beta(\alpha)}{\alpha}$


$\Delta(\alpha, \epsilon) = \alpha\epsilon + \beta(\alpha)$
= Wilson-Fischer fixed point
$\beta(\alpha)$
ϵ_{max}
ϵ
α
$\omega(\epsilon_{max}) = 0$
$-\frac{\beta(\alpha)}{\alpha}$

How can it be realized ?

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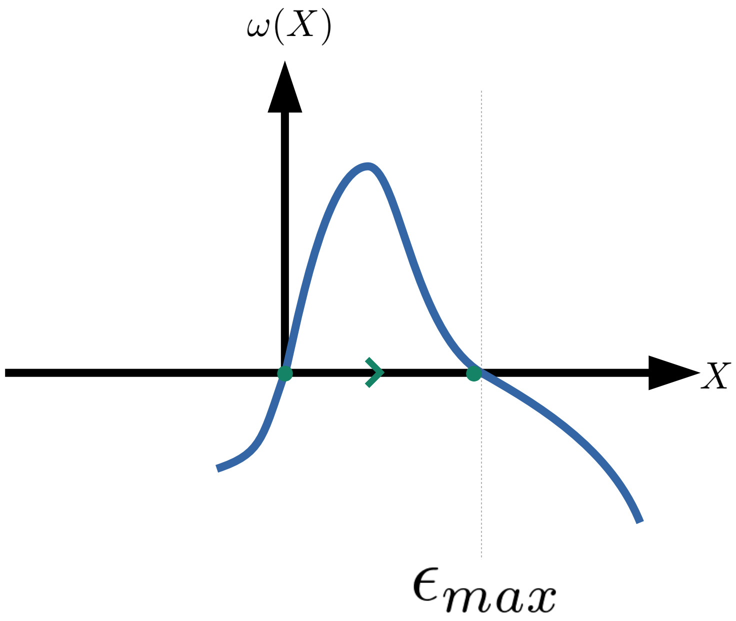

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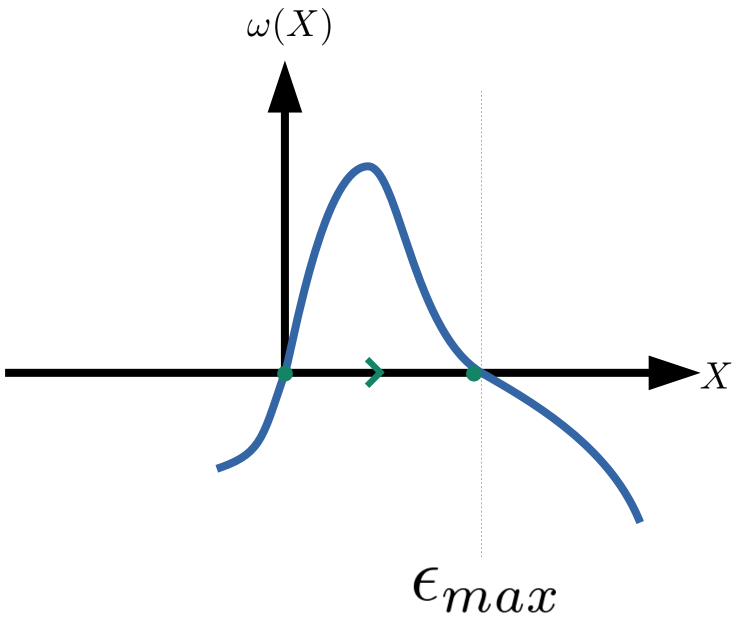

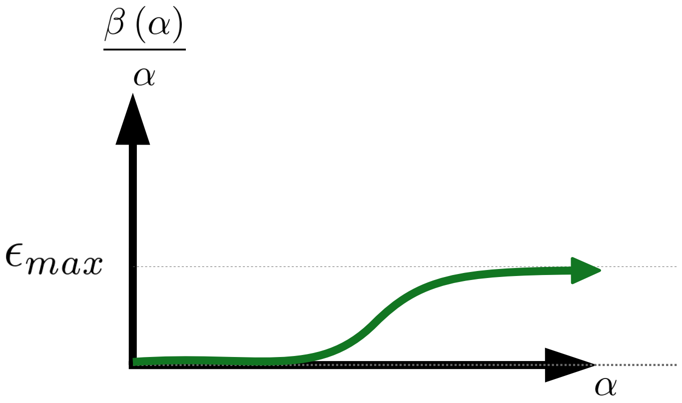

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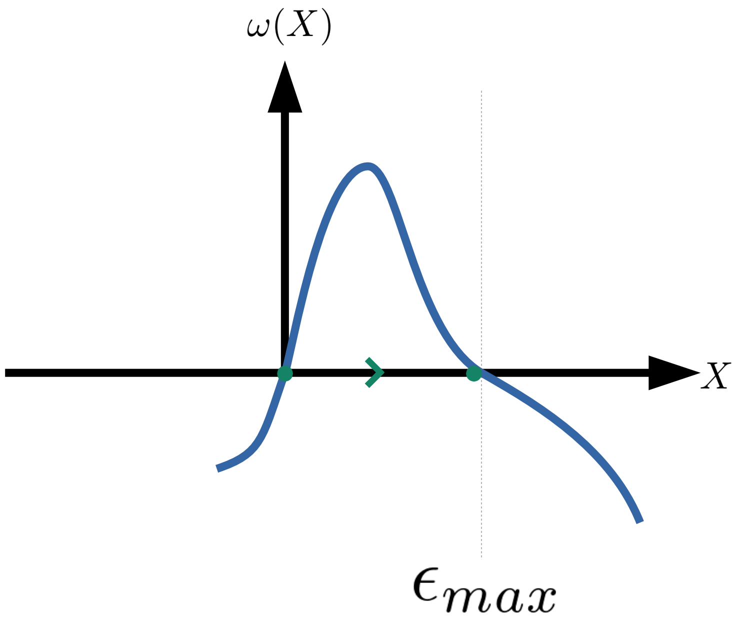

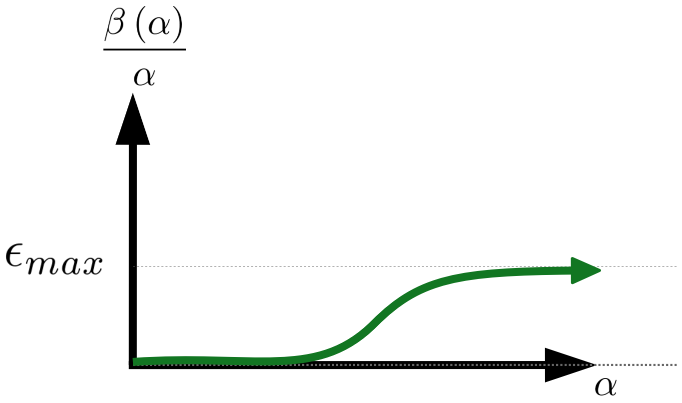

Reached in INFINITE time !

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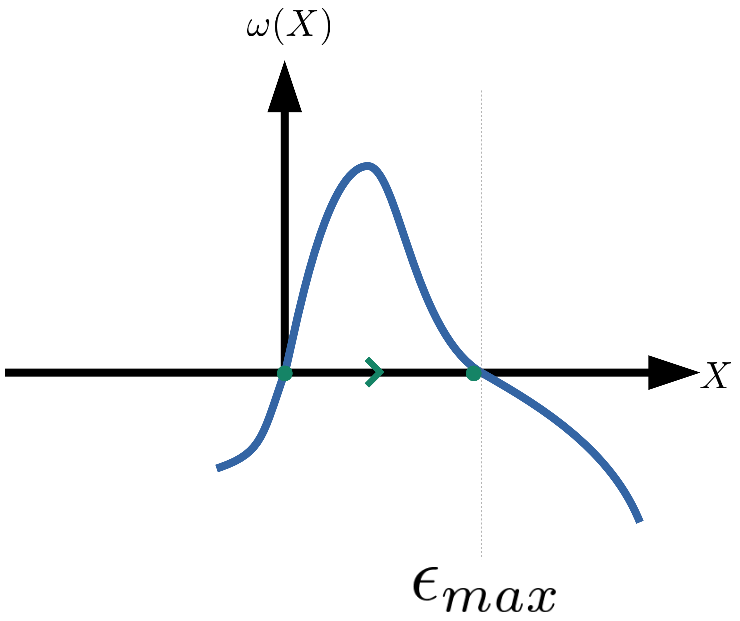

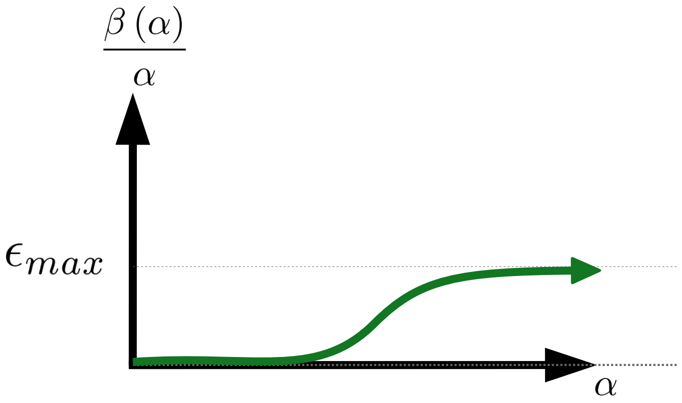

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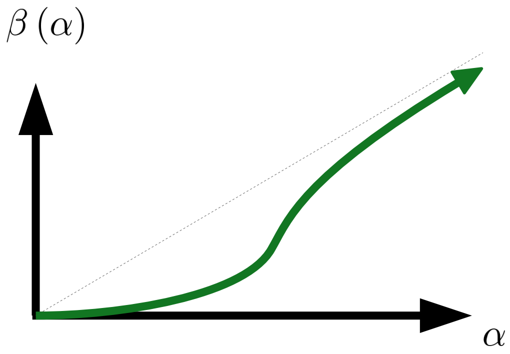

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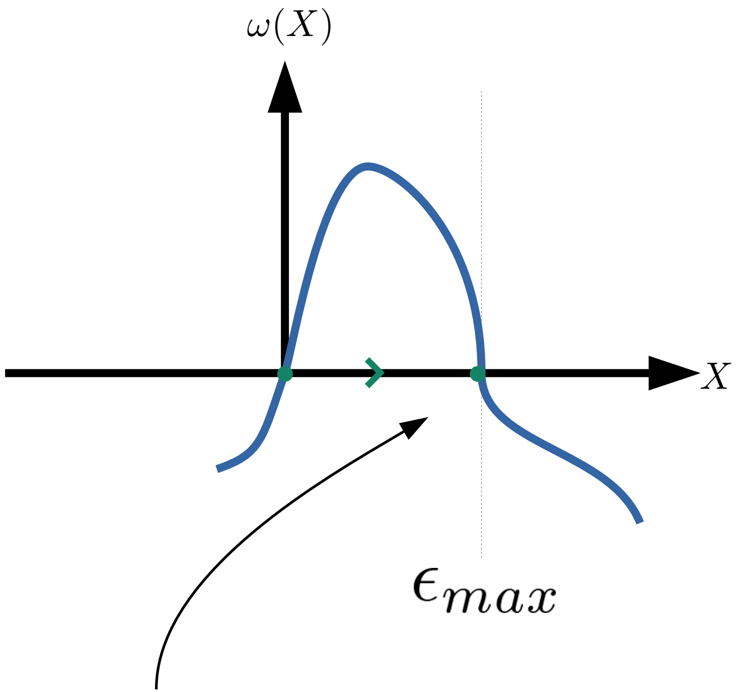
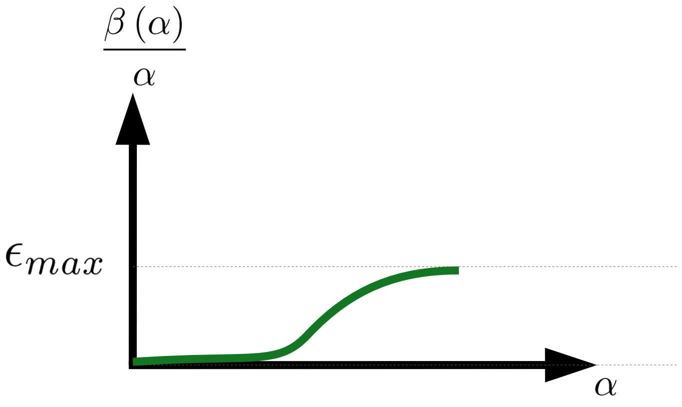
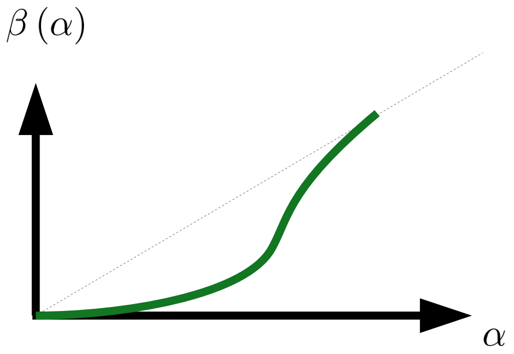


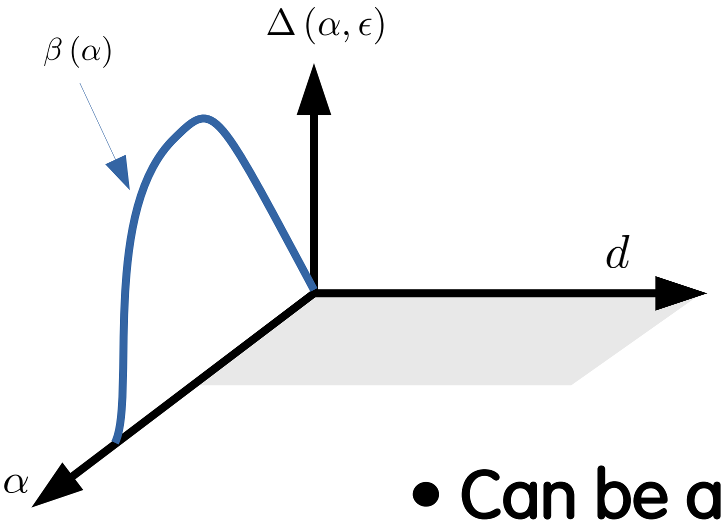
Non-Lifshitz Point



Reached in **FINITE** time !

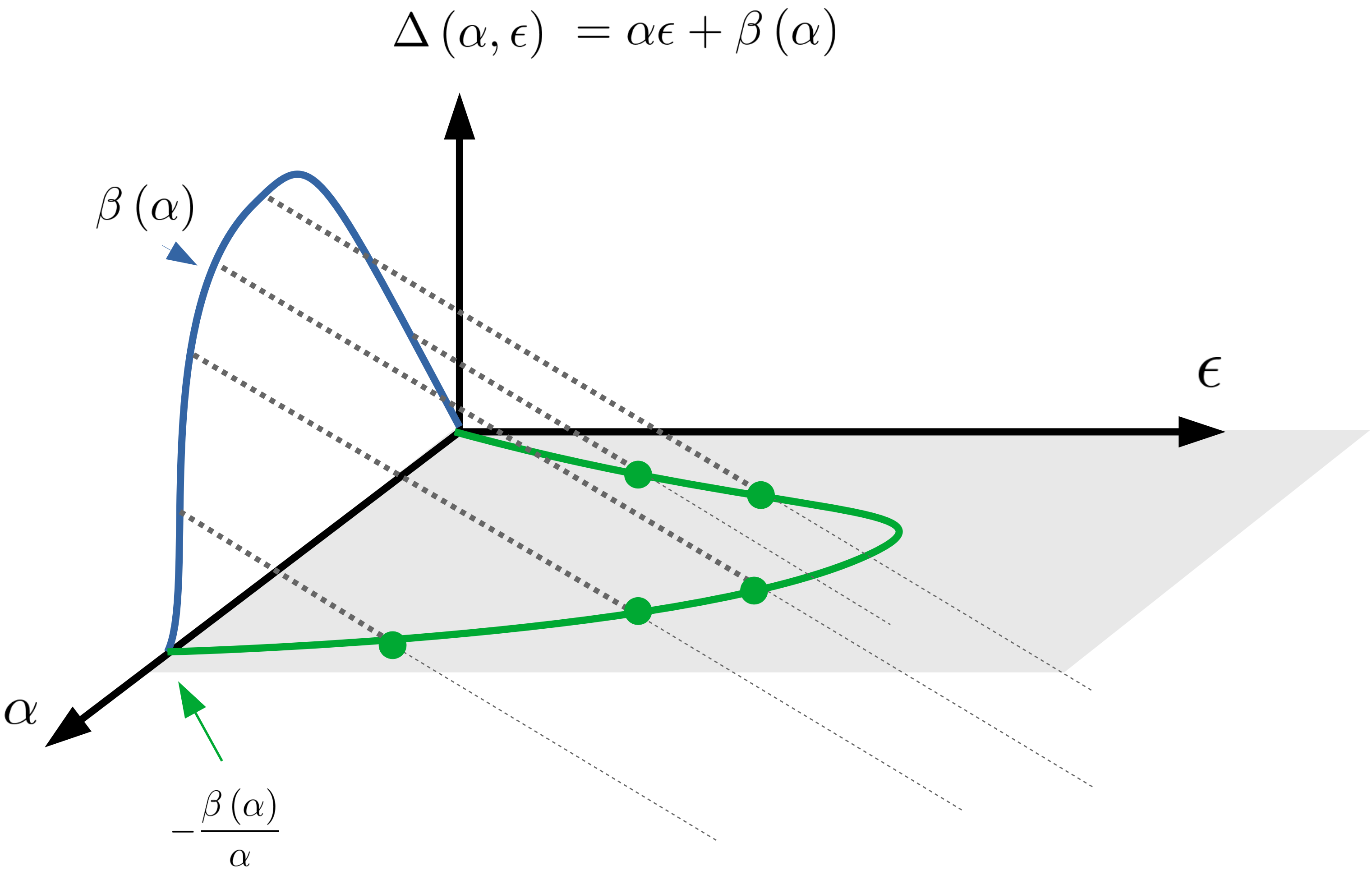


IR Fixed-Point Model

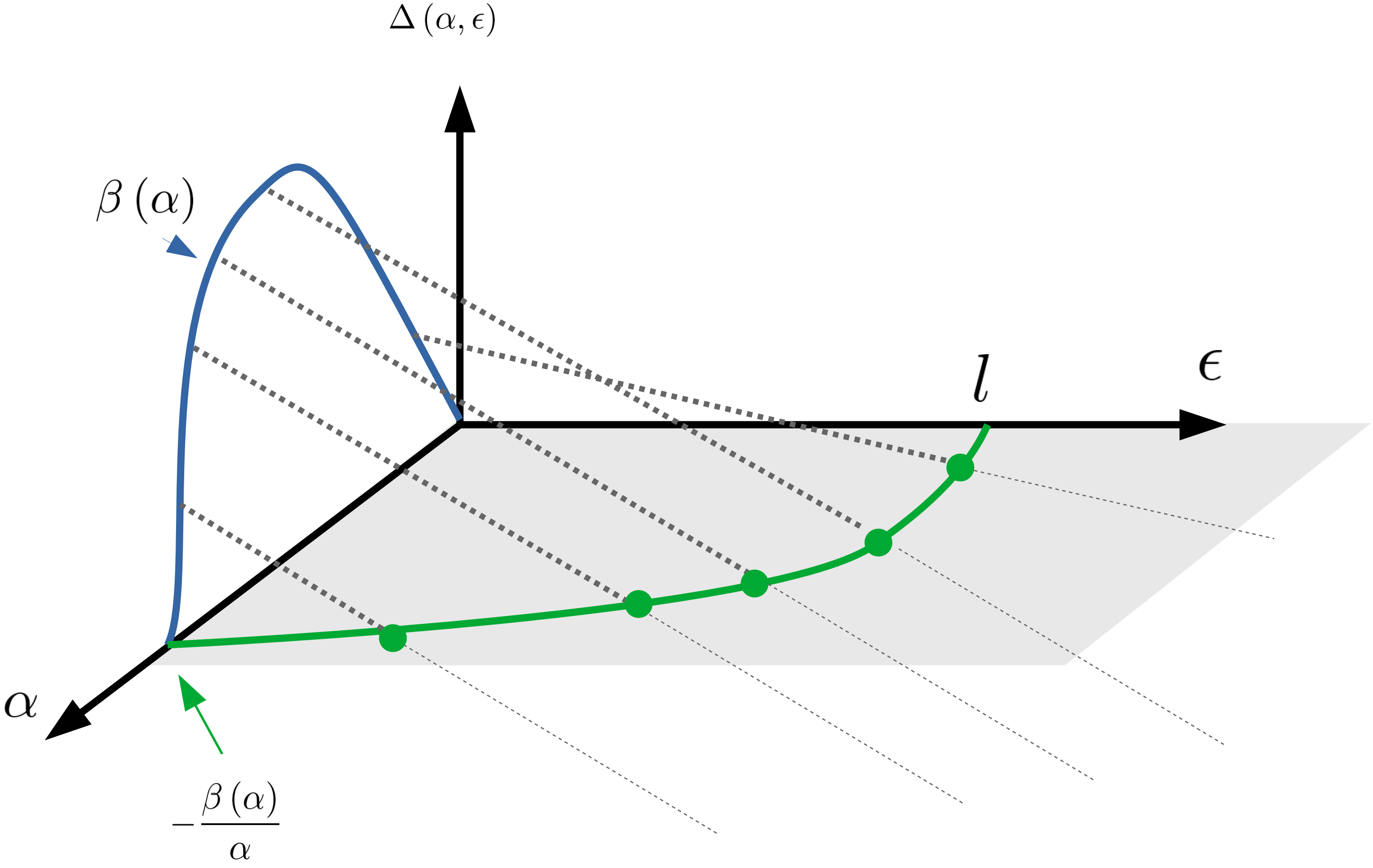


- Can be achieved if ω has a Non-Lifshitz point.
  Ex : It could be naturally generated by
  the cumulations of higher orders poles.

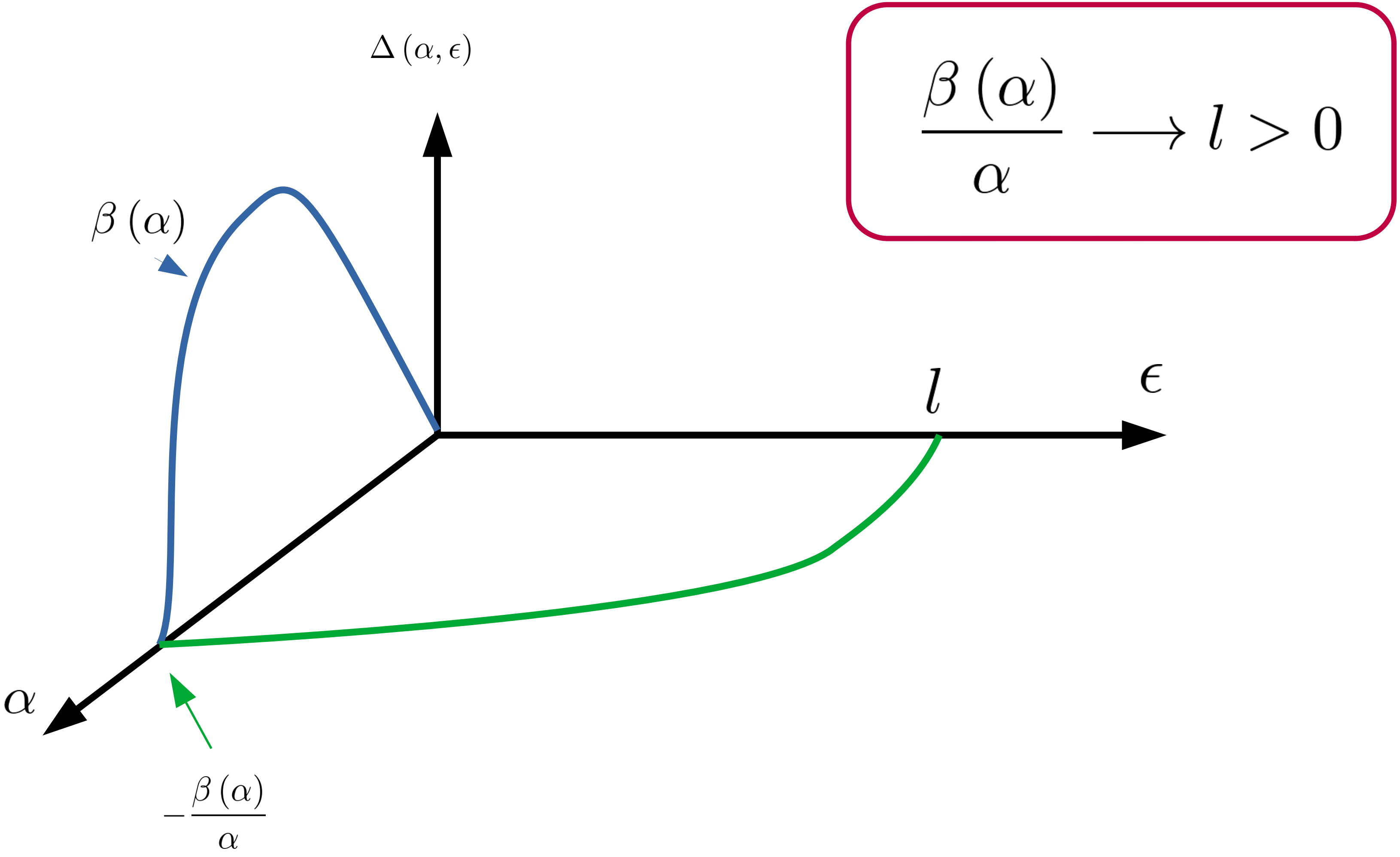
- Is it the only way ?



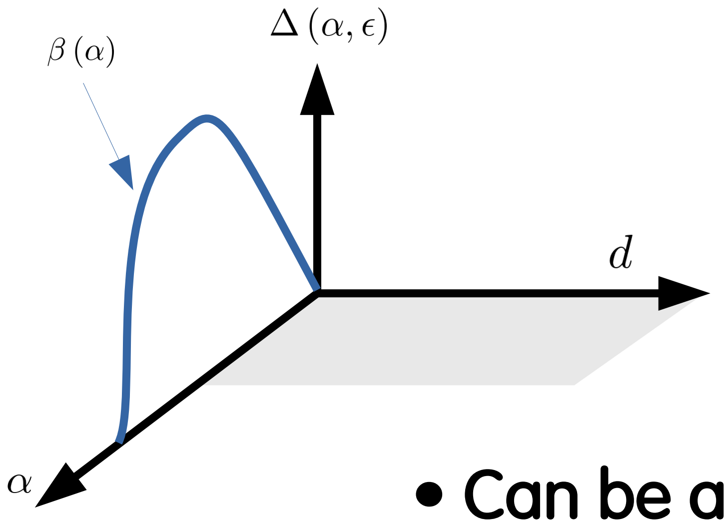
$\Delta(\alpha, \epsilon) = \alpha\epsilon + \beta(\alpha)$
$\beta(\alpha)$
ϵ
α
$-\frac{\beta(\alpha)}{\alpha}$



$\Delta(\alpha, \epsilon)$
$\beta(\alpha)$
l
ϵ
α
$-\frac{\beta(\alpha)}{\alpha}$


$\Delta(\alpha, \epsilon)$
$\frac{\beta(\alpha)}{\alpha} \longrightarrow l > 0$
$\beta(\alpha)$
l
ϵ
α
$-\frac{\beta(\alpha)}{\alpha}$

IR Fixed-Point Model



- Can be achieved if ω has a Non-Lifshitz point.

  Ex : It could be naturally generated by
  the cumulations of higher orders poles.

- If non perturbative contributions generate a « linear term »

Summary

- There is still room for Large N !
- Safety cannot be study from the Master Eq.
- Is there anything more we can say about Large N ?

- 3) Scheme Transformation

A probe to understand Large N

Scheme Transformations

Scheme Transformations

$$K \rightleftarrows \tilde{K}$$

The physics is invariant !

Scheme Transformations

$$\frac{\mathrm{d}K}{\mathrm{d}\mu} = \beta\left(K\right) = \frac{2K^2}{3}\left[1 + \frac{F_1\left(K\right)}{N} + \frac{F_2\left(K\right)}{N^2} + \dots\right]$$

Scheme Transformations

$$\frac{\mathrm{d}K}{\mathrm{d}\mu} = \beta\left(K\right) = \frac{2K^2}{3}\left[1 + \frac{F_1\left(K\right)}{N} + \frac{F_2\left(K\right)}{N^2} + \ldots\right]$$

$$\frac{\mathrm{d}\tilde{K}}{\mathrm{d}\mu} = \tilde{\beta}\left(\tilde{K}\right) = \frac{2\tilde{K}^2}{3}\left[1 + \frac{\tilde{F}_1\left(\tilde{K}\right)}{N} + \frac{\tilde{F}_2\left(\tilde{K}\right)}{N^2} + \ldots\right]$$

Scheme Transformations

$$\frac{\mathrm{d}K}{\mathrm{d}\mu} = \beta\left(K\right) = \frac{2K^2}{3}\left[1 + \frac{F_1\left(K\right)}{N} + \frac{F_2\left(K\right)}{N^2} + \ldots\right]$$

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Scheme Transformations

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What is the relation between F_i and $\tilde{F}_i$?

$$\frac{\mathrm{d}\tilde{K}}{\mathrm{d}\mu} = \tilde{\beta}\left(\tilde{K}\right) = \frac{2\tilde{K}^2}{3}\left[1 + \frac{\tilde{F}_1\left(\tilde{K}\right)}{N} + \frac{\tilde{F}_2\left(\tilde{K}\right)}{N^2} + \dots\right]$$

Scheme Transformations

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What is the relation between F_i and $\tilde{F}_i$?

- $F_1 = \tilde{F}_1$
- $F_i \neq \tilde{F}_i$

$$\frac{\mathrm{d}\tilde{K}}{\mathrm{d}\mu} = \tilde{\beta}\left(\tilde{K}\right) = \frac{2\tilde{K}^2}{3}\left[1 + \frac{\tilde{F}_1\left(\tilde{K}\right)}{N} + \frac{\tilde{F}_2\left(\tilde{K}\right)}{N^2} + \dots\right]$$

Scheme Transformations In Practice:

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$$\alpha = \tilde{\alpha}\mathcal{F}\left(\tilde{\alpha}\right) = \tilde{\alpha}\left(1 + t_k \tilde{\alpha}^k\right)$$

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Scheme Transformations In Practice:

$$\tilde{\beta}\left(\tilde{K}\right) = \left(\frac{\mathrm{d}\tilde{K}}{\mathrm{d}K}\right)\frac{2}{3}\left(\tilde{K}\left(1+t_k\left(\frac{\tilde{K}}{N}\right)^k\right)\right)^2\left[1+\sum_i \frac{F_i}{N^i}\left(\tilde{K}\left(1+t_k\left(\frac{\tilde{K}}{N}\right)^k\right)\right)\right]$$

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Where $s_k = f\left(t_j\right)$, because $K = \tilde{K}\left(1+t_k\left(\frac{\tilde{K}}{N}\right)^k\right)$

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$$F_i\left(\tilde{K}\right) + \frac{\left[\tilde{K}t_k\left(\frac{\tilde{K}}{N}\right)^k\right]}{1!}F'_i\left(\tilde{K}\right) + \frac{\left[\tilde{K}t_k\left(\frac{\tilde{K}}{N}\right)^k\right]^2}{2!}F''_i\left(\tilde{K}\right) + \dots$$

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If we follow the
1/N expansion :

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$$F_i \rightsquigarrow \frac{1}{N^i}, \frac{1}{N^{i+1}}, \frac{1}{N^{i+2}} \cdots$$

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$$\tilde{F}_n\left(X\right) = F_n\left(X\right) + \sum_{k<n} \sim F_k{}^{(i),i\leq n-k}\left(X\right)$$

Scheme Transformations In Practice:

$$\tilde{\beta}\left(\tilde{K}\right) = \frac{2\tilde{K}^2}{3}\left[1 + \frac{\tilde{F}_1\left(\tilde{K}\right)}{N} + \frac{\tilde{F}_2\left(\tilde{K}\right)}{N^2} + \ldots\right]$$

$$\tilde{F}_n\left(X\right) = \left[\ldots\right] + \left(\ldots\right) F_1{}^{(n-1)}$$

$$F_1{}^{(i)} \gg F_1{}^{(j)}\,, \quad i > j$$

- It might compromise the 1/N truncation...

In practice:

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- Suppose the existence of a scheme where all the $F_j\,,\ j > 1$ are dominated by F_1 .

In practice:

- Suppose the existence of a scheme where all the $F_j\,,\ j > 1$ are dominated by F_1 .
- Can there be another scheme where this is the case ?

In practice:

- At $1/N^2$:

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- At $1/N^2$: $\tilde{F}_2(X) = F_2(X) + X^2\left({t_1}^2 - t_2\right) + t_1 X^2 {F'}_1(X)$

In practice:

- At $1/N^2$: $\tilde{F}_2(X) = F_2(X) + X^2\left({t_1}^2 - t_2\right) + t_1 X^2 {F'}_1(X)$

  Thus $t_1 = 0$. Else $\tilde{F}_2$ could not be neglected relatively to $\tilde{F}_1 = F_1$

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In practice:

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In practice:

- At 1/N²: $\tilde{F}_2(X) = F_2(X) + X^2\left({t_1}^2 - t_2\right) + t_1 X^2 {F'}_1(X)$

  Thus $t_1 = 0$. Else $\tilde{F}_2$ could not be neglected relatively to $\tilde{F}_1 = F_1$

- At 1/N³: $\tilde{F}_3(X) = F_3(X) + t_2 X^2\left(X {F'}_1(X) - F_1(X)\right)$

  Thus $t_2 = 0$. Else $\tilde{F}_3$ could not be neglected relatively to $\tilde{F}_1 = F_1$

In practice:

- At each order n there will be only one way to neglect $\tilde{F}_n$:

Imposing $t_n = 0$.

If it EXISTS...
there can be only one scheme with the Large N approximation

Summary

- There is still room for Large N !
- Safety cannot be study from the Master Eq.
- Is there anything more we can say about Large N ?

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Summary

- There is still room for Large N !
- Safety cannot be study from the Master Eq.
- The larger N approx is unlikely to be trusted

Back Up Slides

Back To Large N

$$\left.\frac{\partial\beta}{\partial_K}\right|_{K=K_c} = \omega(d)$$

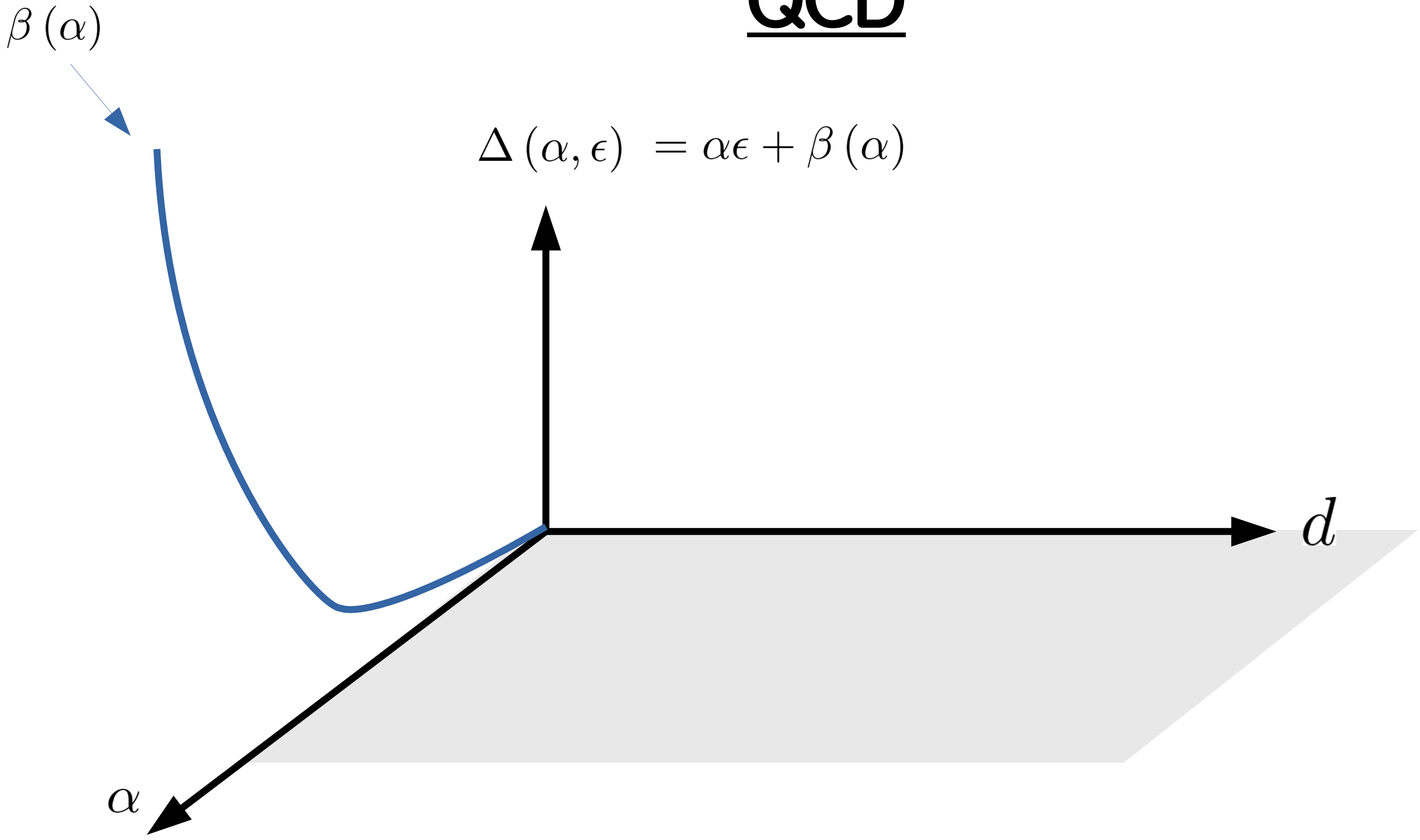
$$-(d-d_c)+\sum_{n=1}\frac{\omega^{(n)}(d)}{N^n} = -(d-d_c)+{K_c}^2\left(1+\sum_{n=1}\frac{\partial F_n(K_c)/\partial_K}{N^n}\right)$$

$$= -(d-d_c)+(d-d_c)^2\left(\frac{1}{\left(1+\sum_{n=1}\frac{F_n(K_c)}{N^n}\right)^2}\right)\times\left(\sum_{n=1}\frac{{F'}_n(K_c)}{N^n}\right)$$

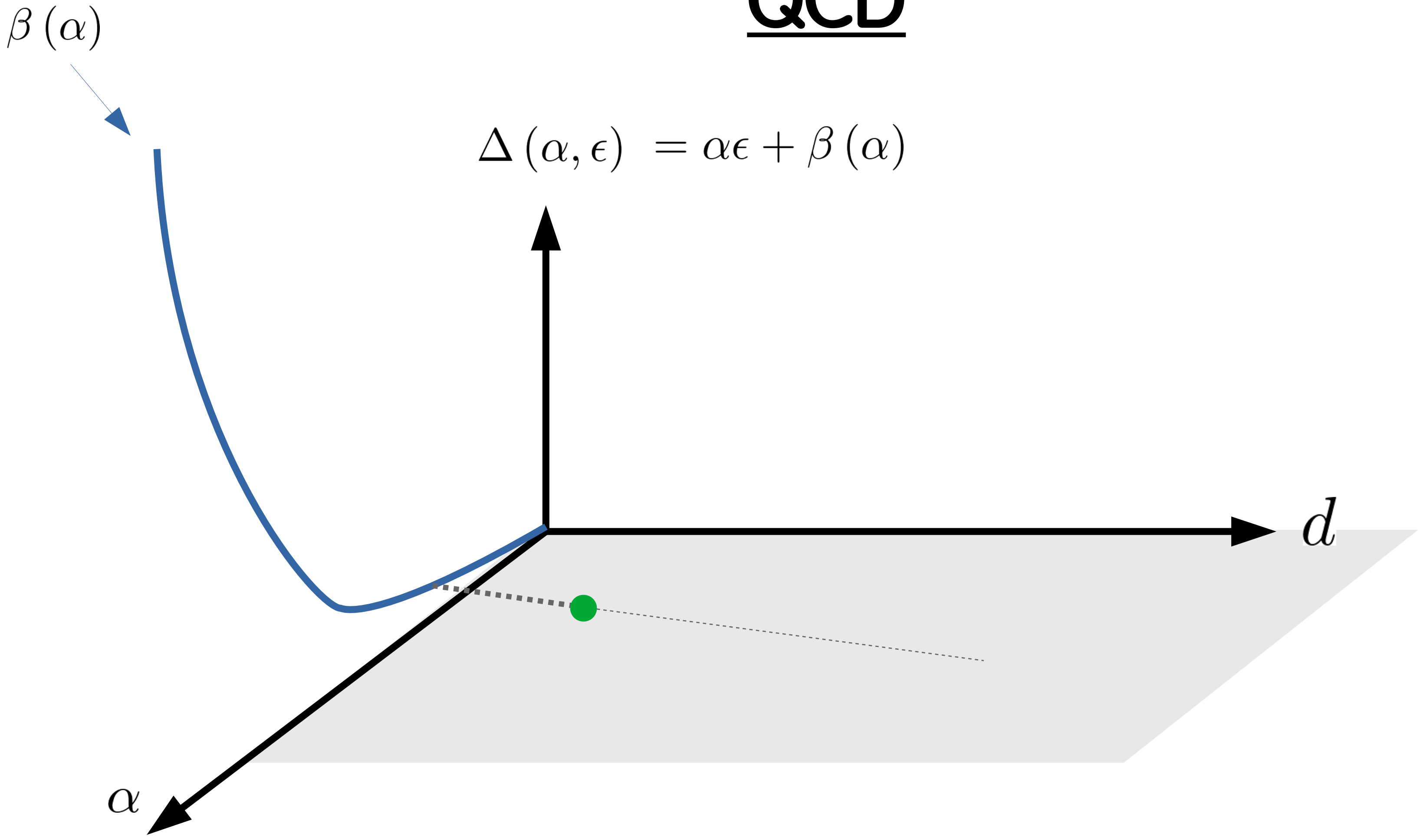
$$= -(d-d_c)+(d-d_c)^2\left(\sum_{k=0}(-1)^k(k+1)\left(\sum_{n=1}\frac{F_n(K_c)}{N^n}\right)^k\right)\times\left(\sum_{n=1}\frac{{F'}_n(K_c)}{N^n}\right)$$

QCD

QCD


$\beta(\alpha)$
$\Delta(\alpha, \epsilon) = \alpha\epsilon + \beta(\alpha)$
d
α

QCD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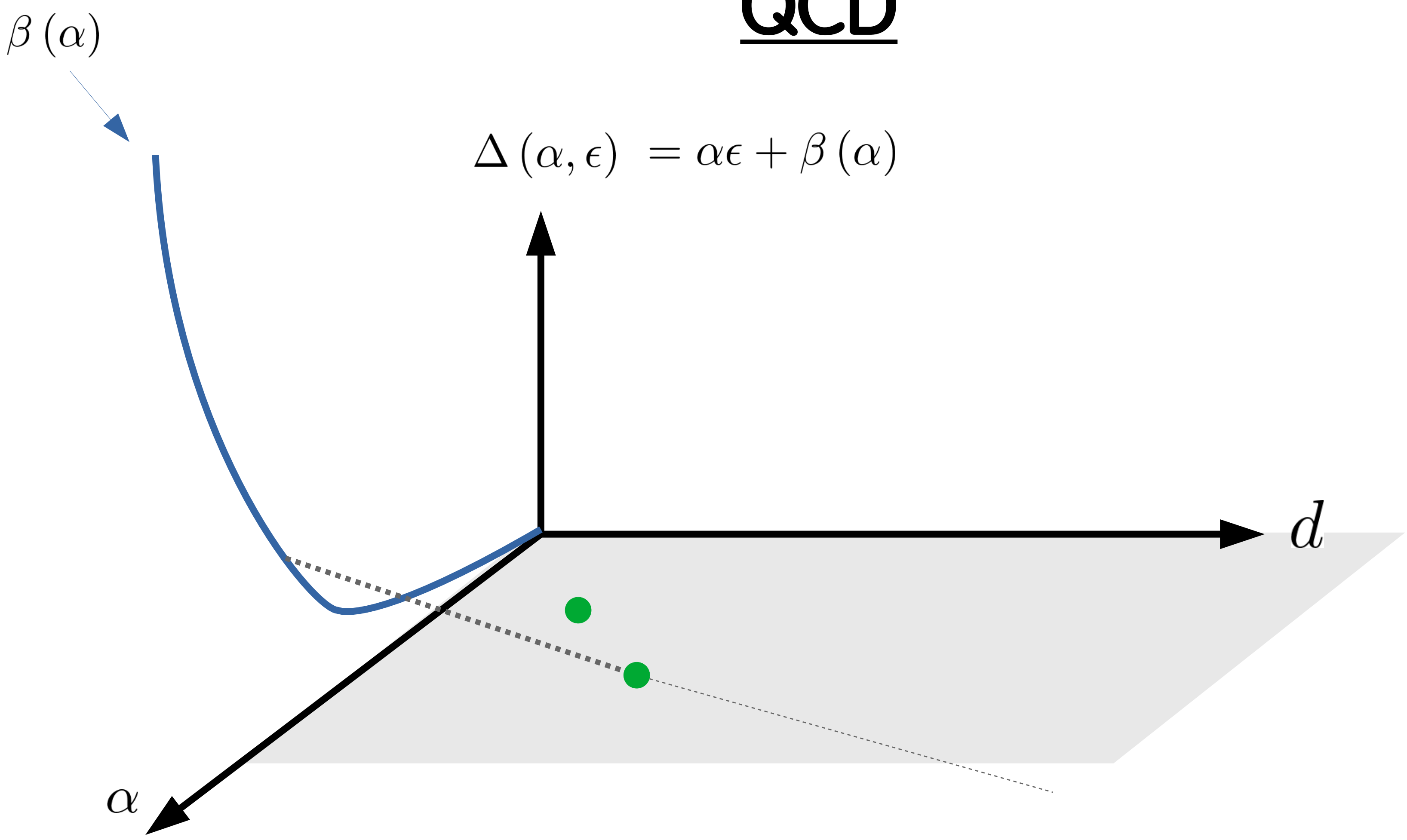

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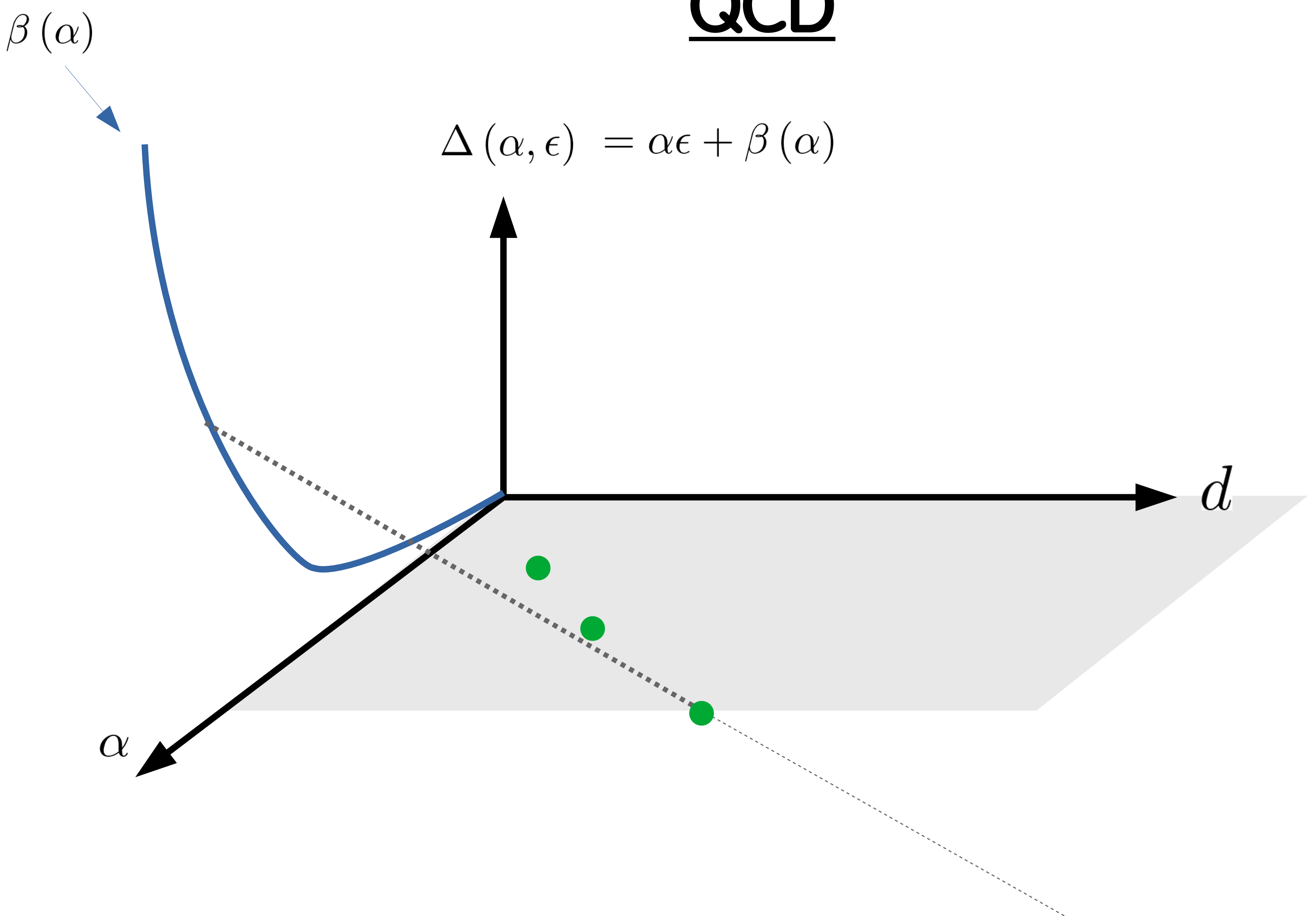

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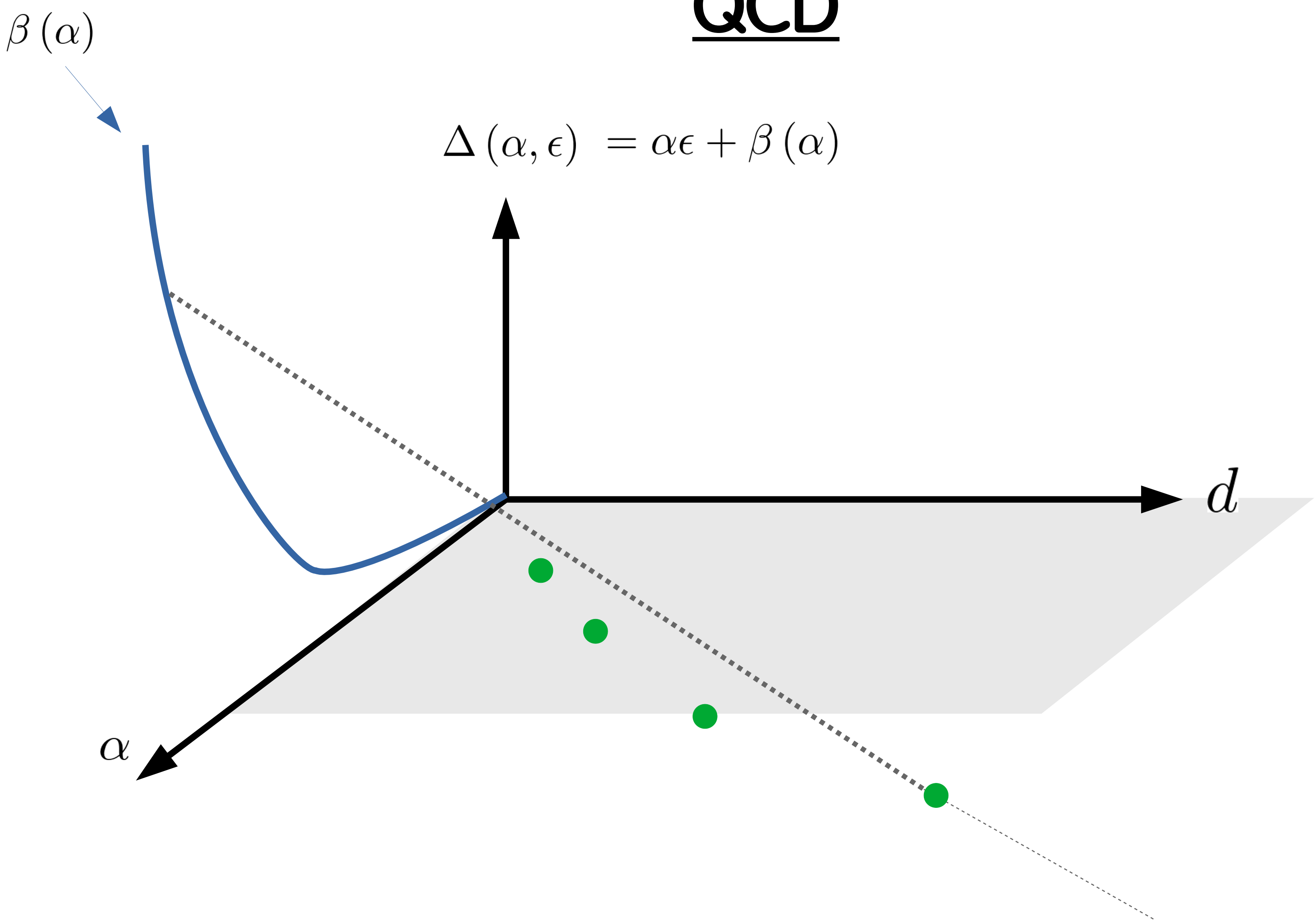

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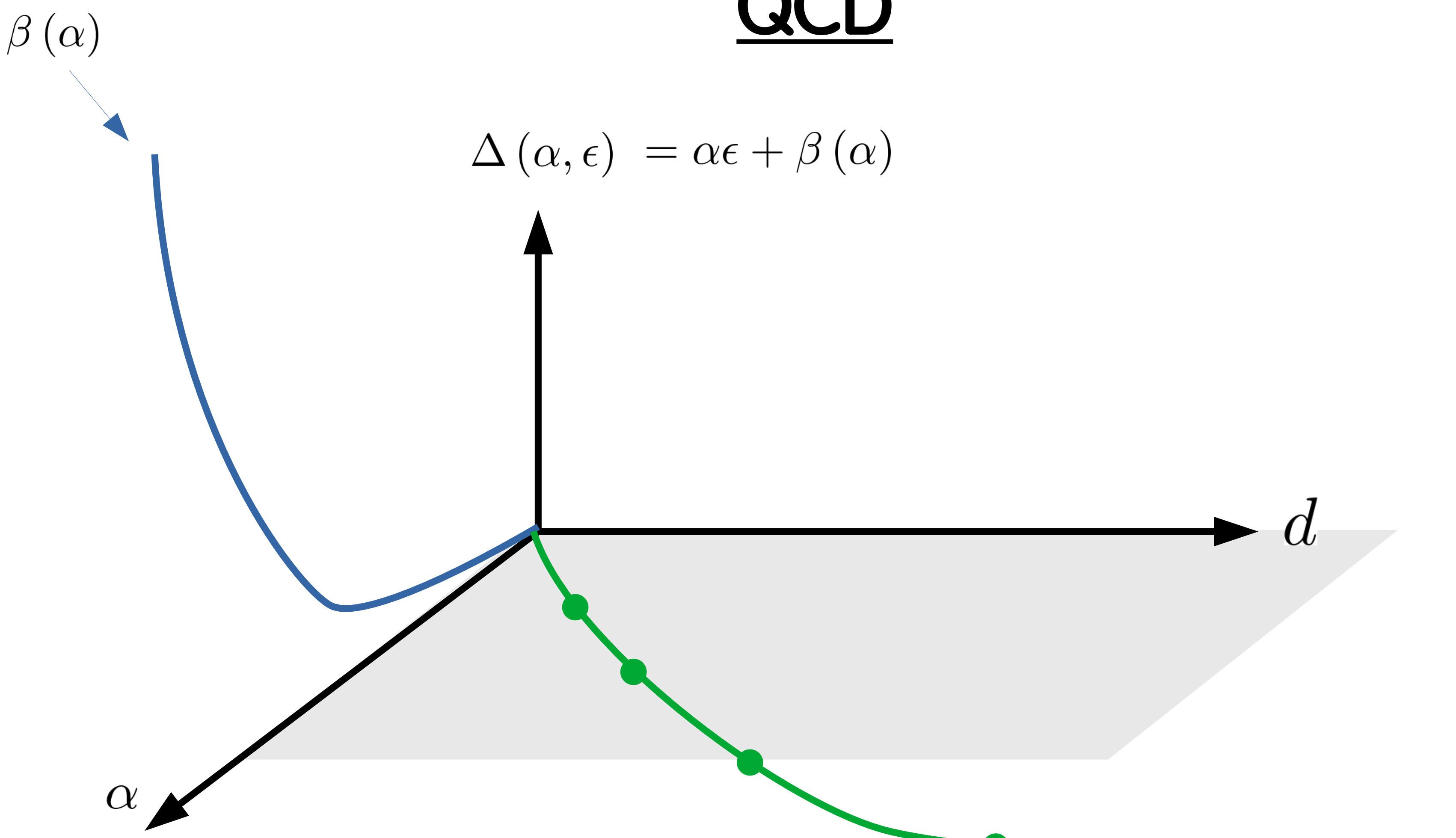
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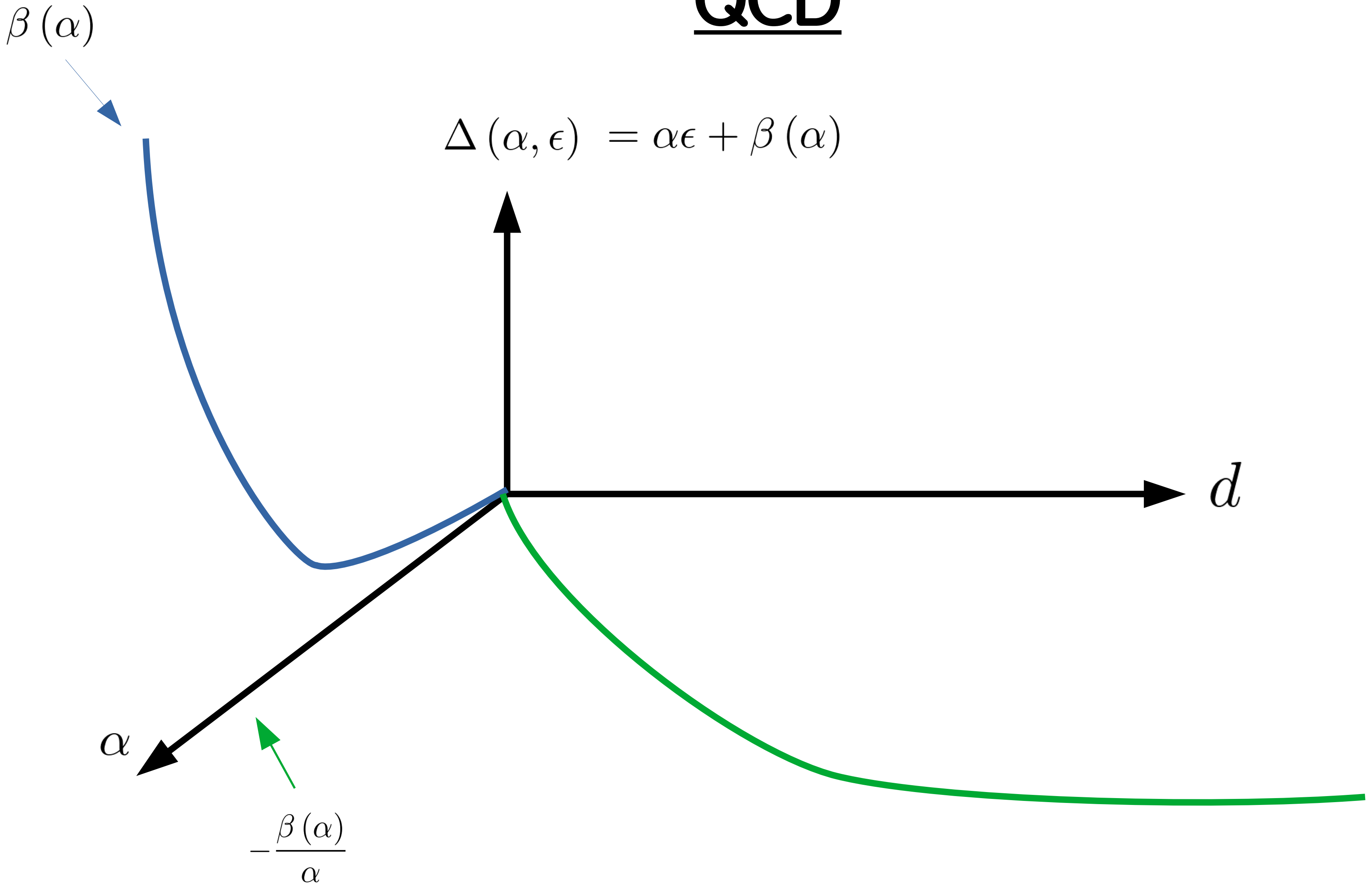
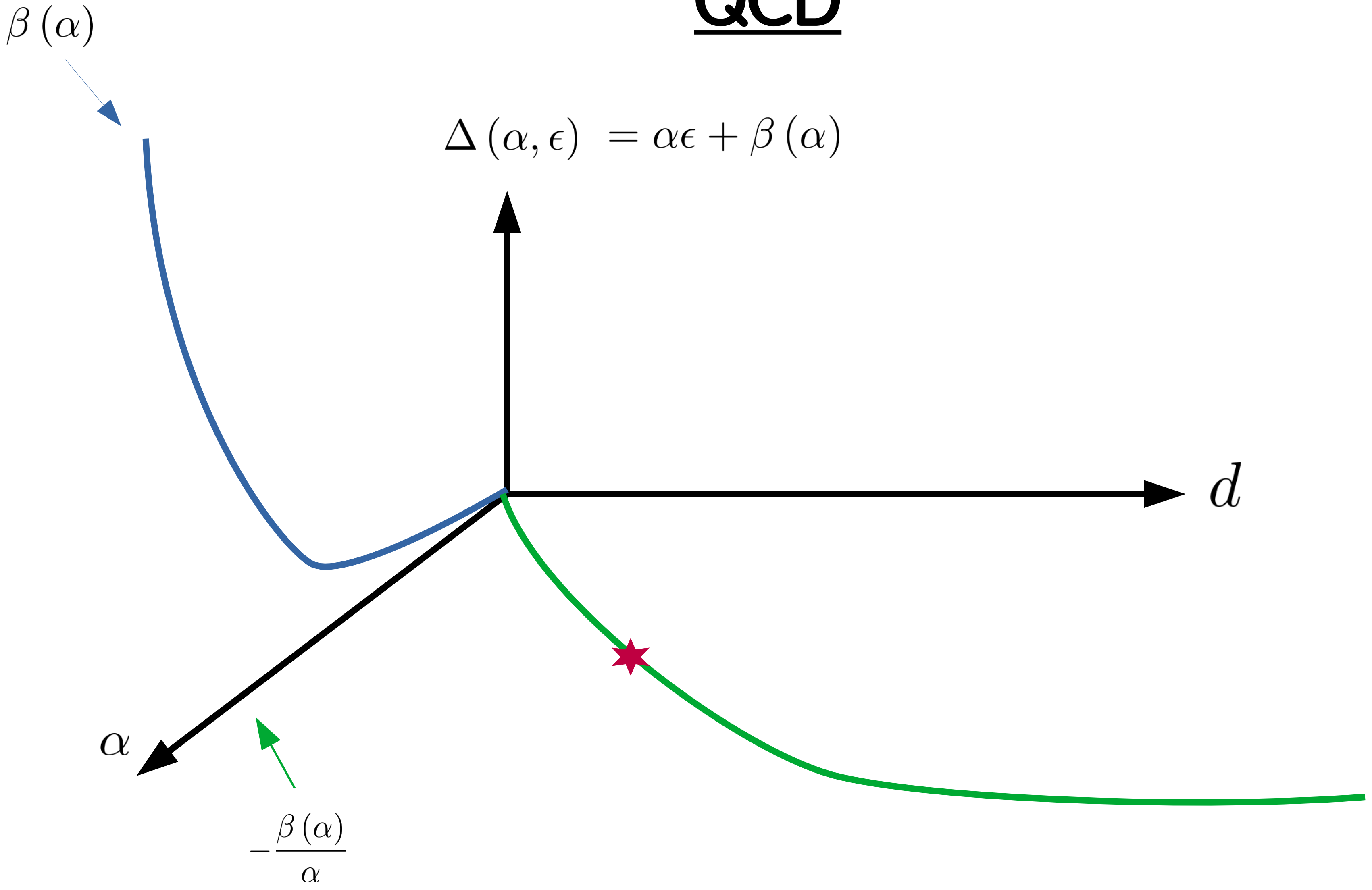


$\beta(\alpha)$
$\Delta(\alpha, \epsilon) = \alpha\epsilon + \beta(\alpha)$
d
α
$-\frac{\beta(\alpha)}{\alpha}$

QCD



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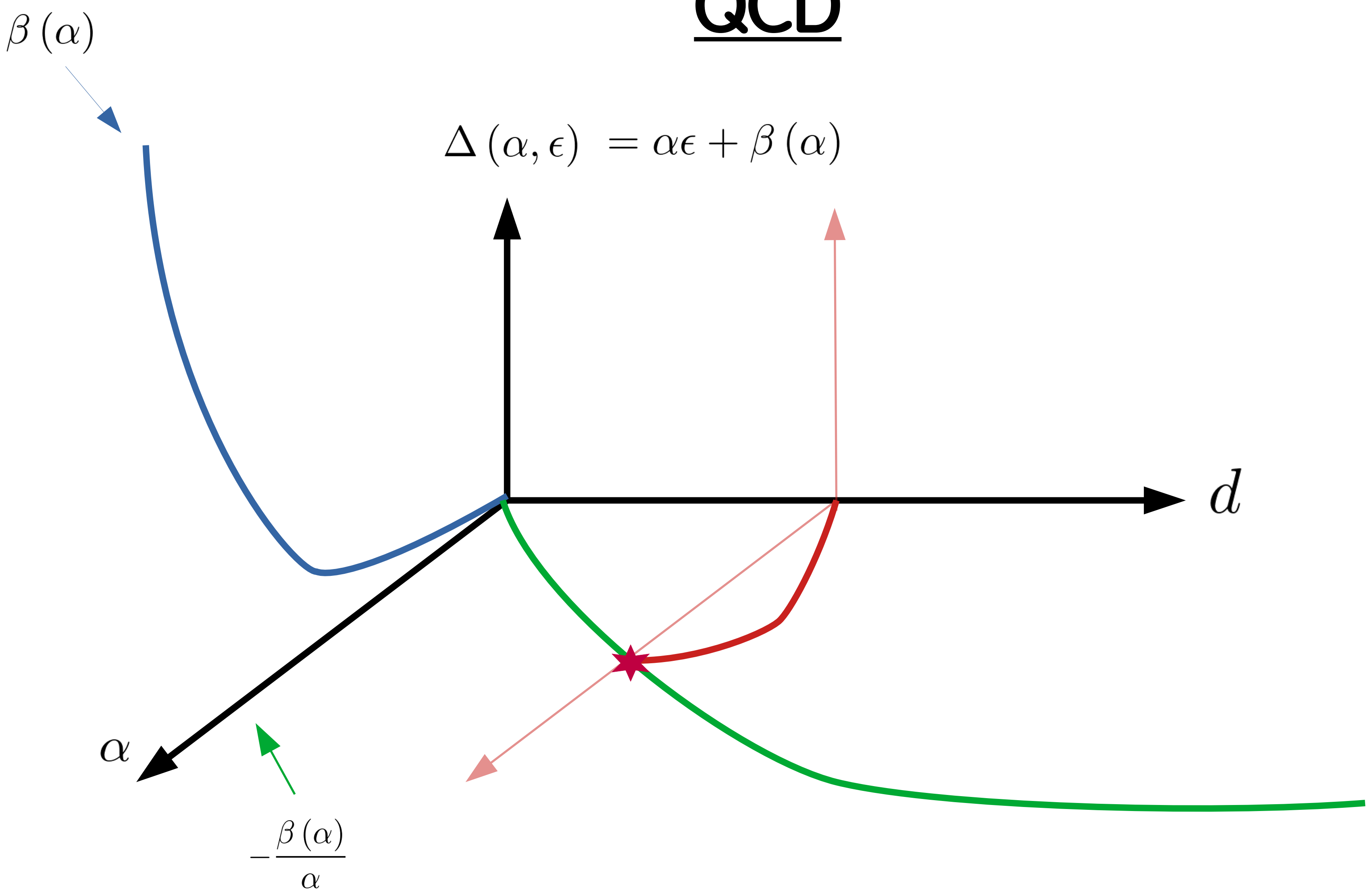

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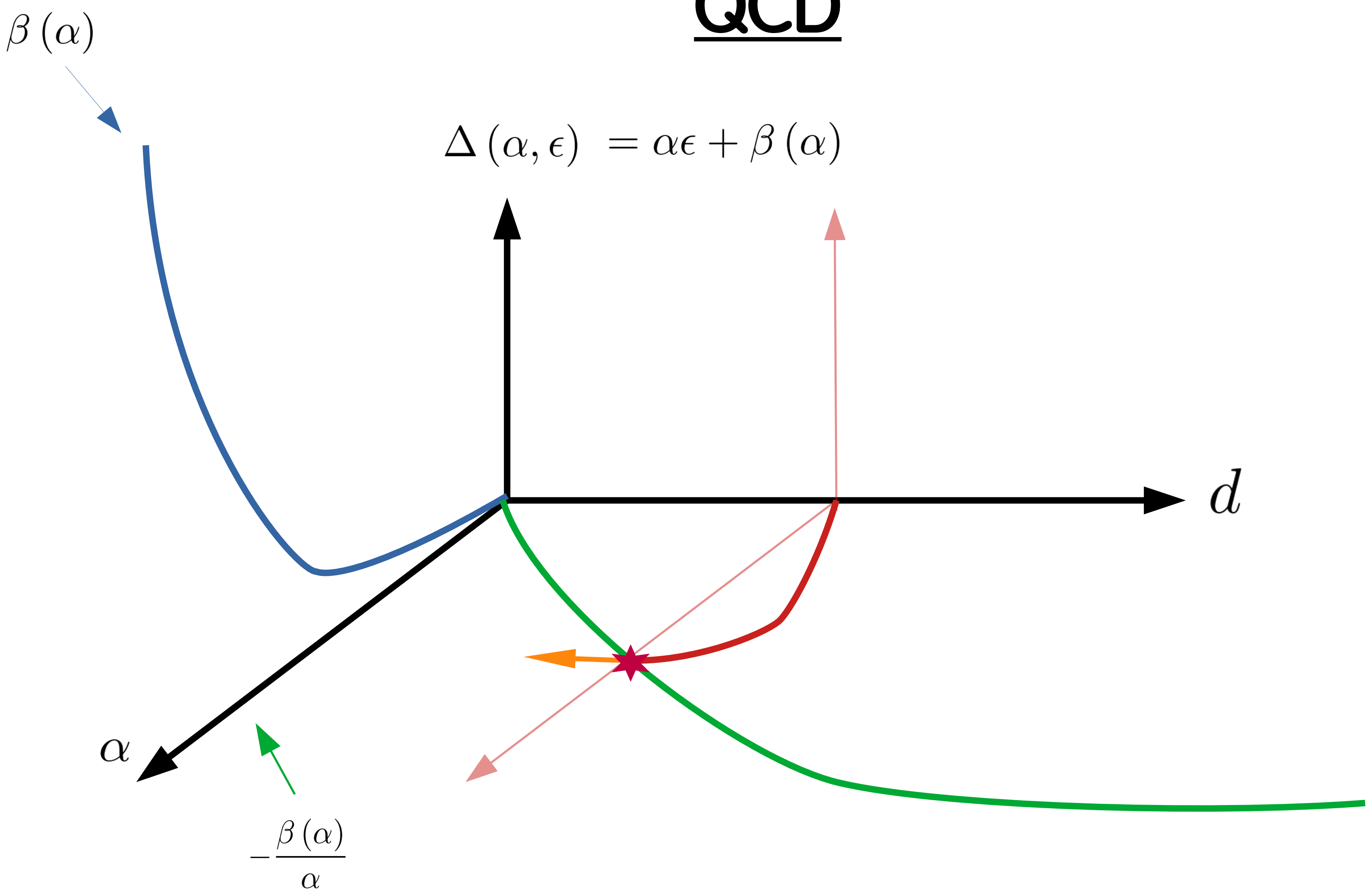

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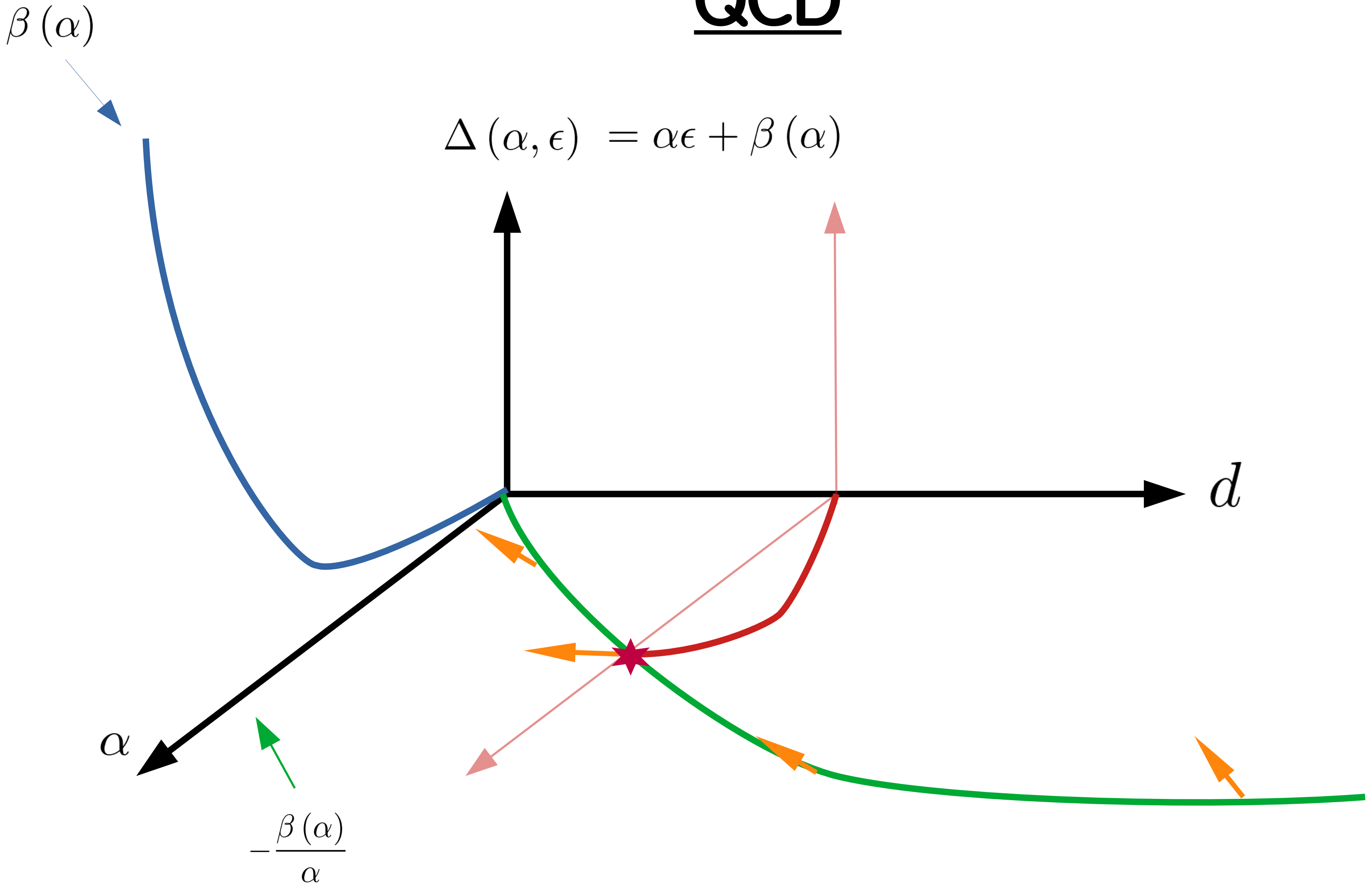

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α
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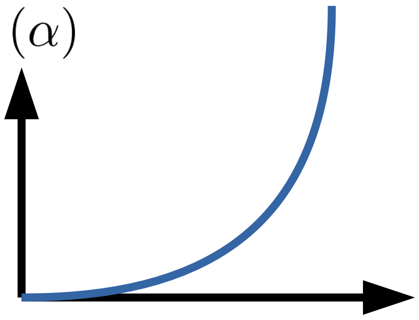
Pure Super Yang Mills

$$\beta(g) = -\frac{g^3}{16\pi^2}\frac{\beta_0}{1-\frac{g^2}{8\pi^2}C_2(G)}$$

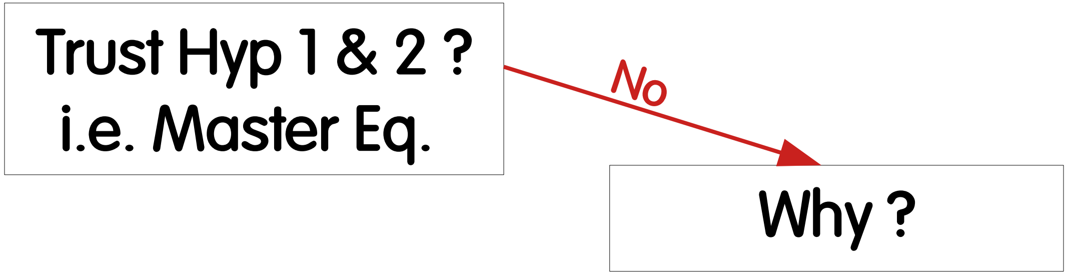


- Pole in the beta function now

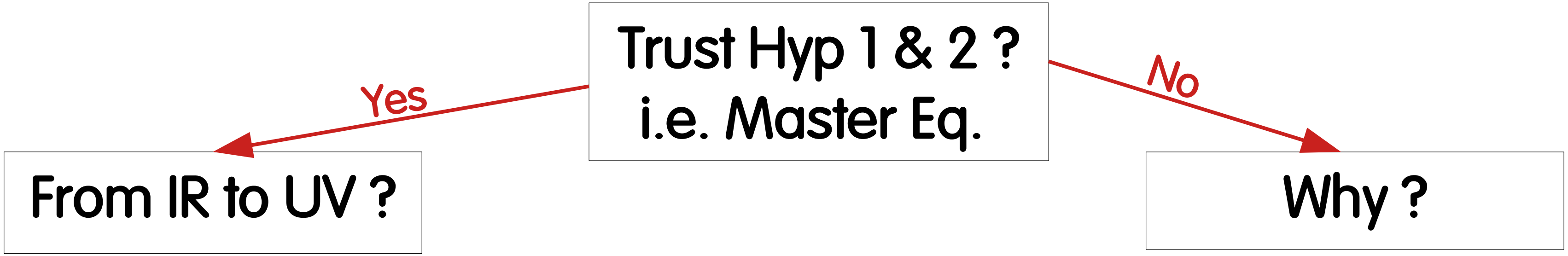
- Still similar to QCD
  i.e. ω of constant sign

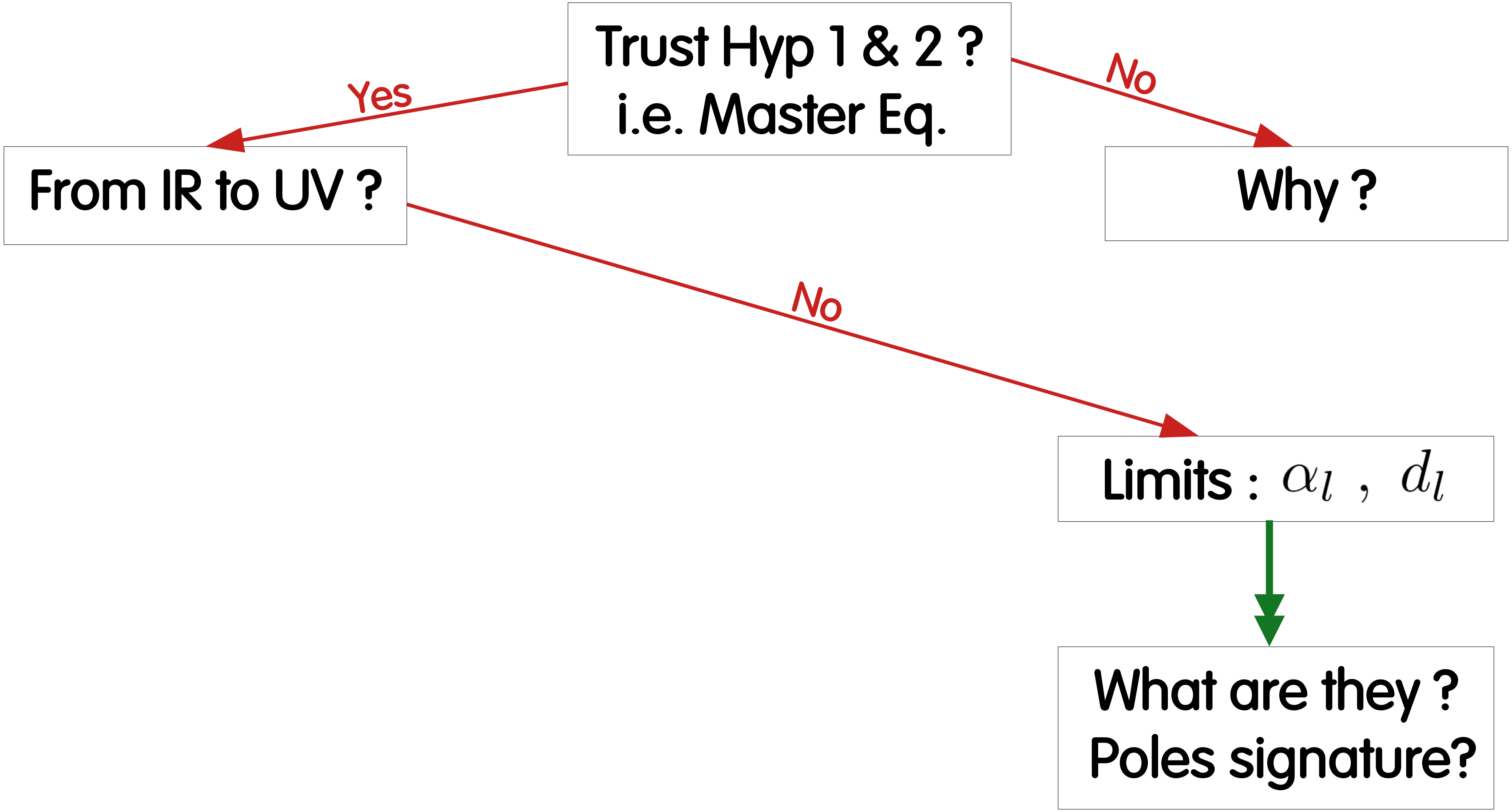
- But now computable ! $\Rightarrow \quad \omega(d) = -\frac{C_2(G)\,d+\beta_0}{\beta_0} = -\frac{d+3}{3}$

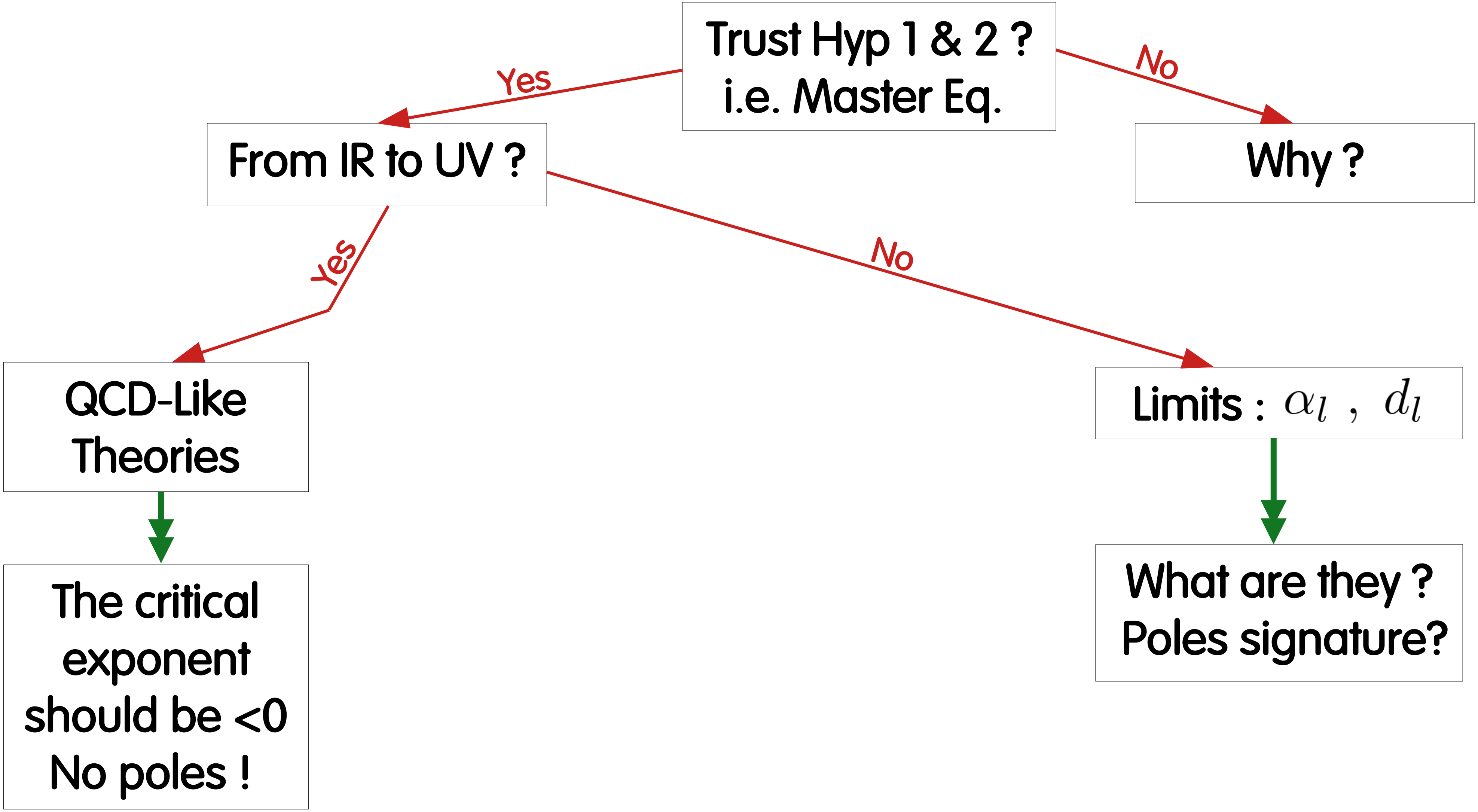
Trust Hyp 1 & 2 ?
i.e. Master Eq.

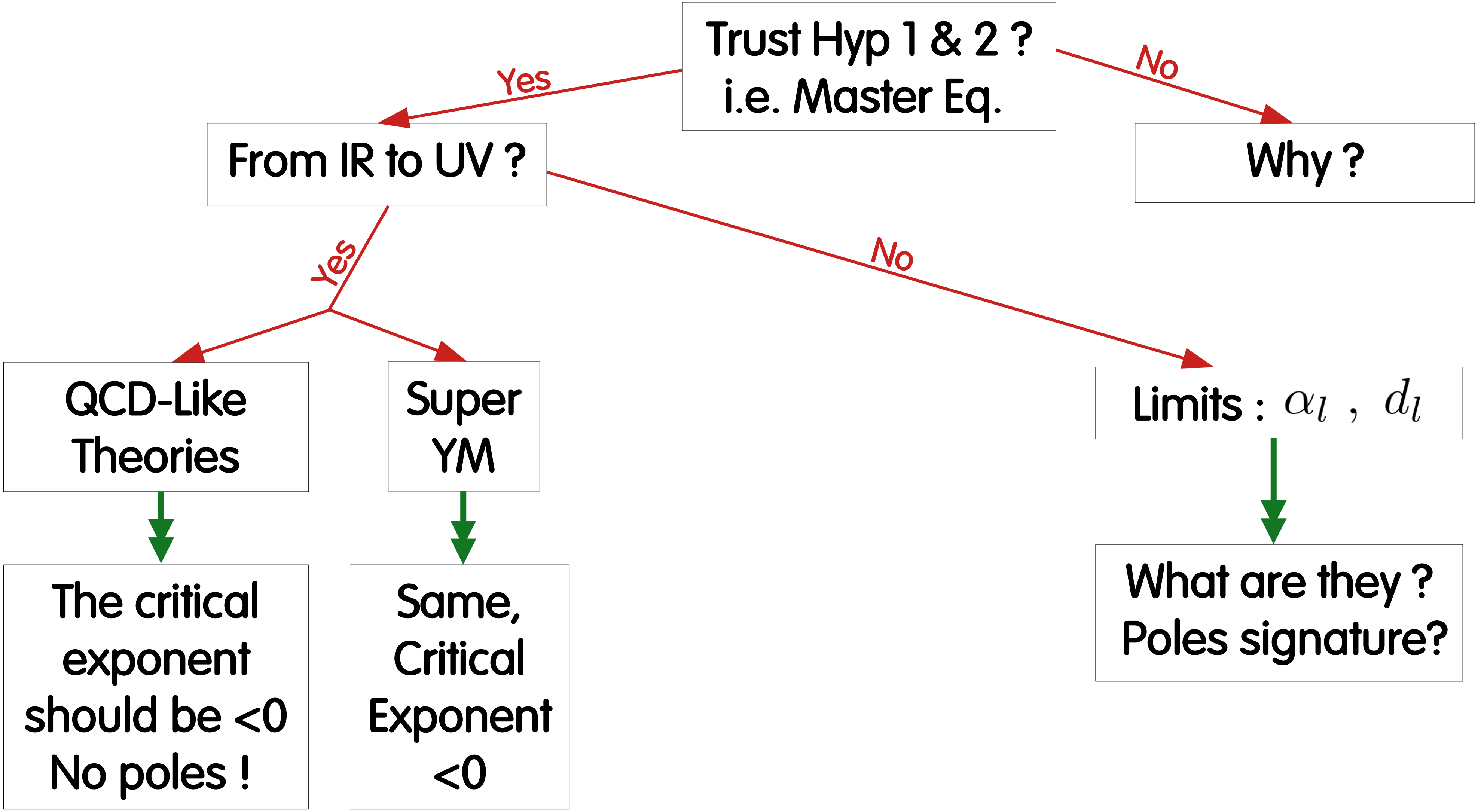


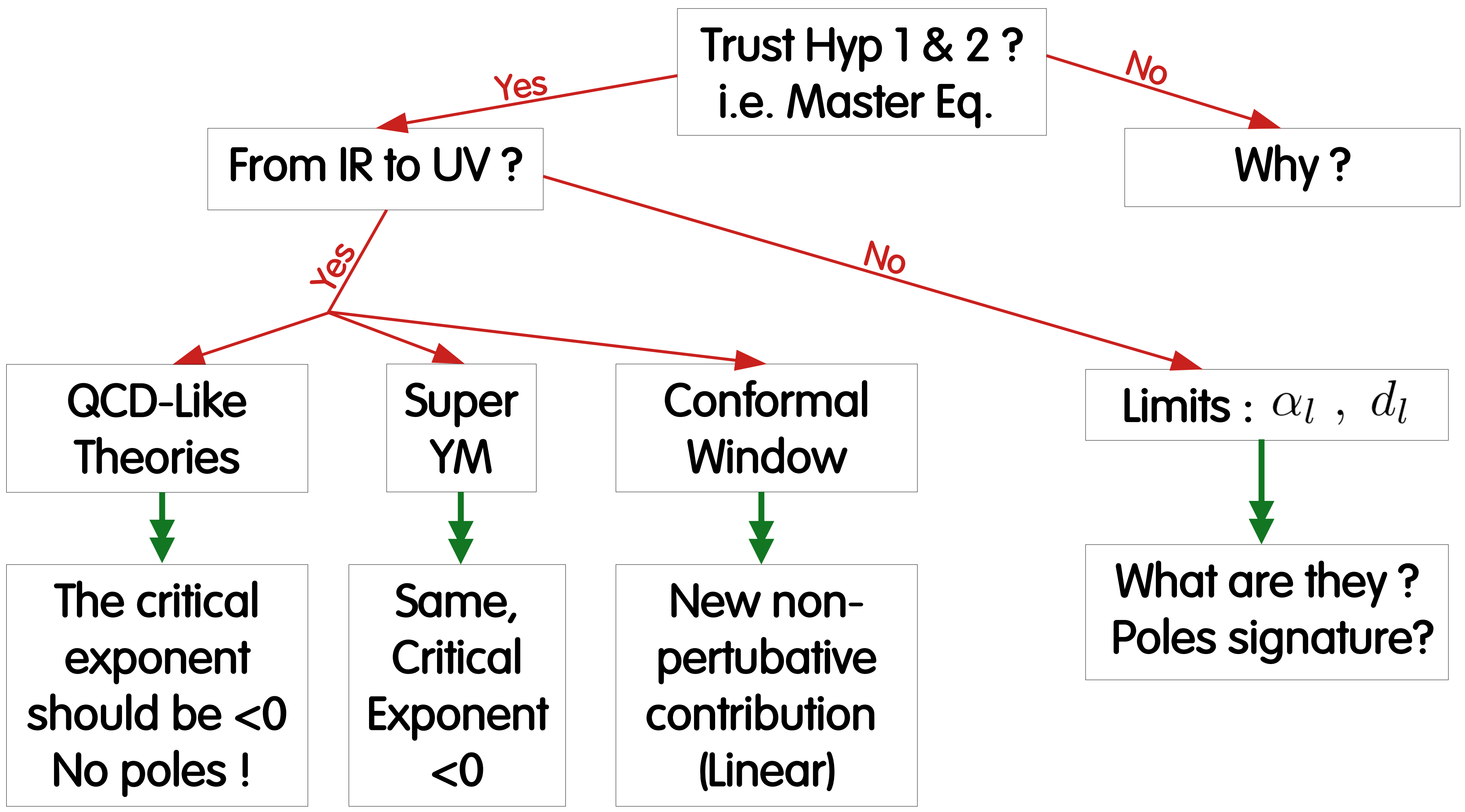
Trust Hyp 1 & 2 ?
i.e. Master Eq.
No
Why ?


Trust Hyp 1 & 2 ?
i.e. Master Eq.
Yes
No
From IR to UV ?
Why ?


Trust Hyp 1 & 2 ?
i.e. Master Eq.
Yes
No
From IR to UV ?
Why ?
No
Limits : $\alpha_l \ , \ d_l$
What are they ?
Poles signature?


Trust Hyp 1 & 2 ?
i.e. Master Eq.
Yes
No
From IR to UV ?
Why ?
Yes
No
QCD-Like
Theories
Limits : α_l , d_l
The critical
exponent
should be <0
No poles !
What are they ?
Poles signature?


Trust Hyp 1 & 2 ?
i.e. Master Eq.
Yes
No
From IR to UV ?
Why ?
Yes
No
QCD-Like Theories
Super YM
Limits : α_l , d_l
The critical exponent should be <0
No poles !
Same, Critical Exponent <0
What are they ?
Poles signature?


Trust Hyp 1 & 2 ?
i.e. Master Eq.
Yes
No
From IR to UV ?
Why ?
Yes
No
QCD-Like Theories
Super YM
Conformal Window
Limits : α_l , d_l
The critical exponent should be <0
No poles !
Same, Critical Exponent <0
New non-pertubative contribution (Linear)
What are they ?
Poles signature?