Interplay of Chiral Transitions in the Standard Model

LIO International Conference on Asymptotic safety in Quantum Field Theory: Grand Unification

Richard Schmieden

Supervisor: Holger Gies and Luca Zambelli

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Theoretisch-Physikalisches Institut

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Introduction - Chiral Transitions in the Standard Model

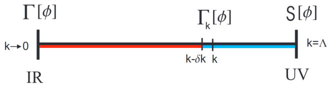
- Fundamental fermionic degrees of freedom in the Standard Model are massless while all matter particles possess mass

  Two symmetry breaking mechanisms:

  - Chiral symmetry breaking due to the strong interaction
  - Higgs mechanism, spontaneously breaks gauge and chiral symmetry

- Investigate interplay of these two mechanisms in a suitable toy model
- Construct Asymptotically free scaling solutions within the toy model

The Functional Renormalization Group



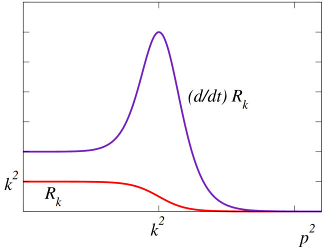
Modified generating functional

$$e^{W_k[J]} = \int \mathcal{D}\varphi e^{-S[\varphi]-\Delta S_k[\varphi]+\int_x J\varphi}$$

where:

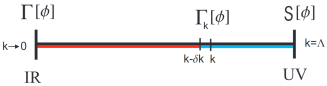
$S[\varphi]$: action of the theory

$\Delta S_k[\varphi] = \int_x \frac{1}{2}\varphi(x)R_k(x)\varphi(x)$:
momentum dependent mass term



Typical form of the regulator $R_k(p^2)$ and its derivative [Gies '06].

The Functional Renormalization Group

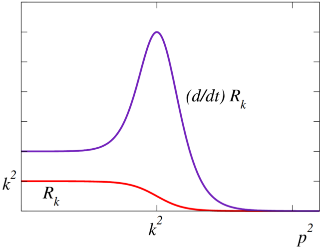


Effective average action

$$\Gamma_k[\phi] = \sup_J \left(\int d^d x J\phi - W_k[J] \right) - \Delta S_k[\phi]$$

$\Rightarrow$ Wetterich equation [Wetterich '93]

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \mathrm{STr} \left[\partial_t R_k \left(\Gamma_k^{(2)}[\Phi] + R_k \right)^{-1} \right]$$



Typical form of the regulator $R_k(p^2)$ and its derivative [Gies '06].

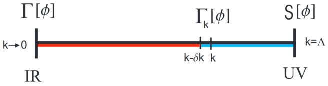
Flow Equations

Truncate the effective average action

$\rightarrow$restrict to manageable amount of operators

Extract flow equations of operators by suitable projection

Compare different truncation schemes as convergence checks



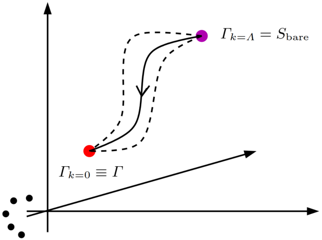


Figure: RG flow of the effective action in theory space [Gies '06].

QCD sector

QCD sector in the Standard Model is asymptotically free

Radiative corrections induce non-trivial IR physics through dimensional transmutation [Coleman, Weinberg '73]

$\rightarrow$ strong gauge coupling g grows towards the IR, diverges at scale Λ_{QCD}

Λ_{QCD} sets the scale of the infrared physics, while degrees of freedom undergo huge change:

- UV: quarks and gluons describe the physics at high energies
- IR: mesons and bound states describe the low energy physics

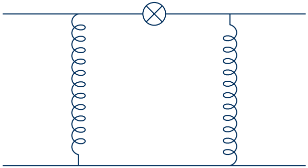
Capturing chiral symmetry breaking from QCD

To capture the dynamical chiral symmetry breaking in QCD we use dynamical bosonization:

Fundamental QCD action

$$S_{\mathrm{QCD}} = \int_x \bar{\psi}_i^a (\mathrm{i}\partial\!\!\!/ \delta_{ij} + \bar{g} A\!\!\!/_{ij}) \psi_j^a + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{(\partial_\mu A^\mu)^2}{2\xi}$$

induces effective four-fermion vertices through quantum fluctuations



$\Rightarrow$ At intermediate scales: NJL-type interactions included

Capturing chiral symmetry breaking from QCD

$$\Gamma_k = \int_x \bar{\psi}_i^a (\mathrm{i} Z_\psi \partial\!\!\!/ \delta_{ij} + \bar{g} A\!\!\!/_{ij}) \psi_j^a + \frac{Z_F}{4} F_{\mu\nu} F^{\mu\nu} + \frac{(\partial_\mu A^\mu)^2}{2\xi} + \frac{1}{2} \bar{\lambda}_\sigma (\mathrm{S} - \mathrm{P}).$$

where $(\mathrm{S} - \mathrm{P}) = \left(\bar{\psi}_i^a \psi_i^b\right)^2 - \left(\bar{\psi}_i^a \gamma_5 \psi_i^b\right)^2$

Flow of four fermion coupling $\partial_t \lambda_\sigma \sim g^4$

Towards IR: $g \nearrow$ due to asymptotic freedom, hence $\lambda_\sigma \nearrow$

$\rightarrow$ Non-perturbative effects like χSB require effective low energy description

Dynamical Bosonization

Translate microscopic degrees of freedom to macroscopic ones (quarks, gluons $\rightarrow$ mesons)

$$\bar{\psi}_i^a \psi_i^b \quad \rightarrow \quad \varphi^{ab}$$

Encode four-fermion interaction in Yukawa interaction

$$\lambda_\sigma \left(\left(\bar{\psi}_i^a \psi_i^b\right)^2 - \left(\bar{\psi}_i^a \gamma_5 \psi_i^b\right) \right) \qquad h \bar{\psi}_i^a \left(P_\mathrm{R} \varphi^{ab} + P_\mathrm{L} \varphi^{\dagger ab} \right) \psi_i^b$$

On all scales k: Additional contributions to the flow eqs. of the Yukawa model to compensate the 4-Fermi coupling [Gies, Wetterich '03]

“re-parametrized” QCD

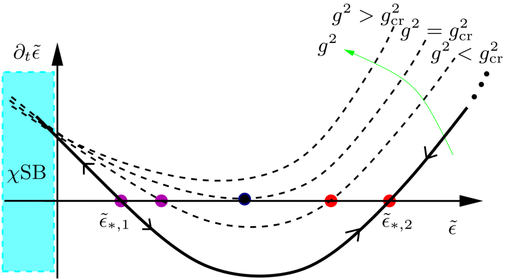
With this dynamical bosonization we can rewrite QCD into a Yukawa-QCD model

$$\Gamma_k = \int_x \; Z_\varphi \left|\partial_\mu \varphi^{ab}\right|^2 + \bar{\psi}_i^a \mathrm{i} \not{D}_{ij} \psi_j^a + \frac{Z_\mathrm{F}}{4} F^z_{\mu\nu} F_z^{\mu\nu} + \frac{(\partial_\mu A^\mu)^2}{2\xi}$$
$$+ (\text{Ghosts}) + U(\varphi) + h \bar{\psi}_i^a \left(P_\mathrm{R} \varphi^{ab} + P_\mathrm{L} \varphi^{\dagger ab}\right) \psi_i^b$$

Initial values of the newly introduced parameters can be chosen to reproduce QCD results.

Chiral symmetry breaking in QCD signaled by non-zero vacuum expectation value in the scalar potential
$\rightarrow$ dynamical generation of quark masses

χSB in QCD



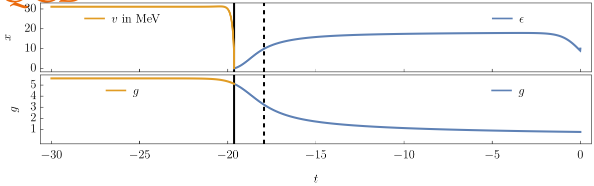
Flow equation for $\tilde{\epsilon} \sim m^2$ of the auxiliary field φ [Gies '06].

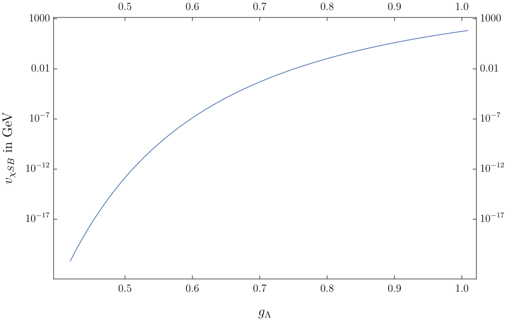
As long as $g < g_{\rm cr}$: $\tilde{\epsilon}$ controlled by fixed point structure

If $g > g_{\rm cr}$: Fixed points annihilate, $\tilde{\epsilon}$ runs fast towards negative values ($m^2 < 0$: χSB)

$\rightarrow$ Scale at which χSB is triggered ($\sim \Lambda_{\rm QCD}$) is set by the strong gauge coupling.

χSB in QCD





Phase transition in QCD described as a crossover

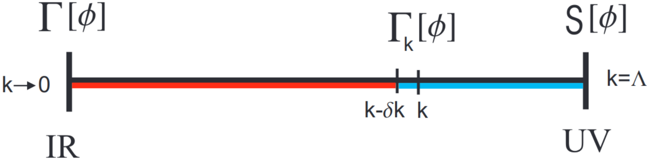
$\rightarrow$ No fine tuning associated with χSB in QCD.

Higgs mechanism

The Higgs mechanism induces masses for all Fermions as well as the weak bosons through the scalar potential acquiring a non-zero vacuum expectation value (vev)

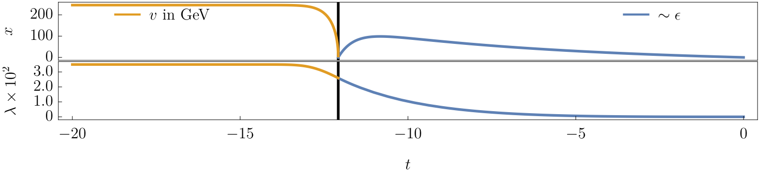
E.g. look at the flow of a scalar potential in a Higgs-Yukawa toy model

$$\Gamma_k = \int_x (\partial_\mu \phi)^2 + U(\phi^2) + \bar{\psi} i \partial\!\!\!/ \psi + i h \bar{\psi} \phi \psi$$

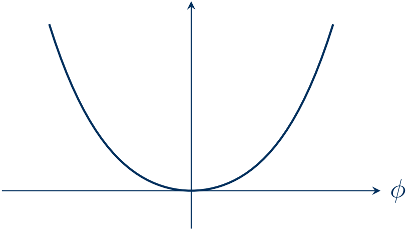


vev $\neq 0$ $\Leftarrow$ vev $= 0$

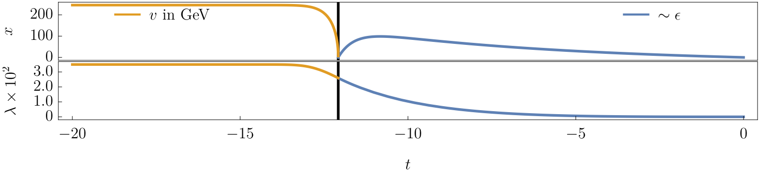
Mass Generation



$$U = \frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4, \quad U(\phi) = U(-\phi)$$



Mass Generation



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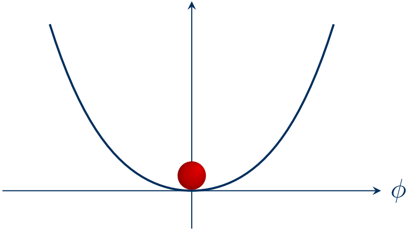
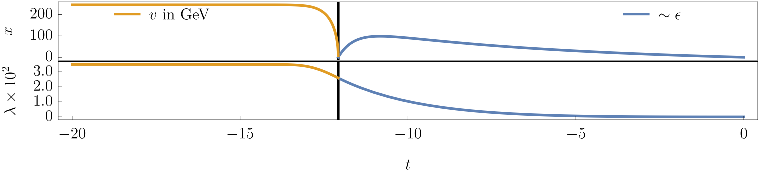
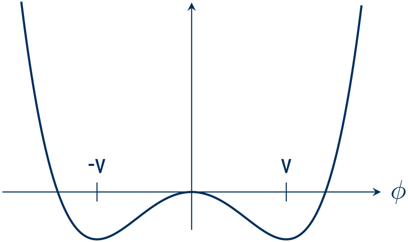


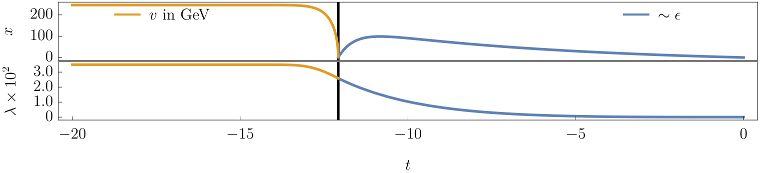
Mass Generation



$$U = \frac{\lambda}{4!}(\phi^2 - v^2)^2, \quad U(\phi) = U(-\phi)$$



Mass Generation



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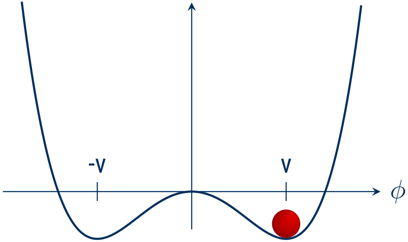
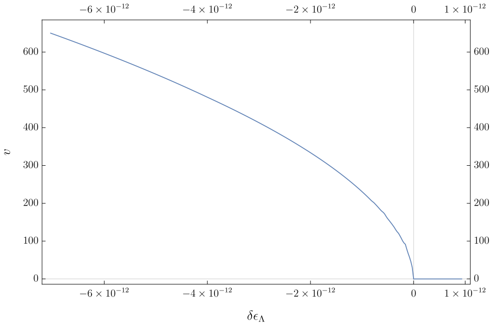


Higgs mechanism - second order phase transition

Varying the couplings in the UV changes the obtained vev



$$\epsilon_\Lambda = \frac{m^2}{\Lambda^2}$$

Larger values of ϵ_Λ: Potential stays symmetric

Smaller values of ϵ_Λ: Potential develops minimum at non-vanishing field.

The vev as a function of the control parameter is described by a second order quantum phase transition (tied to the fine tuning problem)

Reparametrization of the scalar field

Reparametrization of the auxiliary scalar field φ as

$$\Phi^{ab} = \frac{1}{\sqrt{2}}\left(\varphi^{ab} + \epsilon^{ac}\epsilon^{bd}\varphi^{*\,cd}\right)$$

$$\tilde{\Phi}^{ab} = \frac{1}{\sqrt{2}}\left(\varphi^{*\,ab} - \epsilon^{ac}\epsilon^{bd}\varphi^{cd}\right)$$

and setting

$$\phi^{a} \equiv \Phi^{a2}$$

$$\tilde{\phi}^{a} \equiv \tilde{\Phi}^{2a}$$

allows us to identify the $\mathrm{SU}(2)_{\mathrm{L}}$ doublet (ϕ), which acquires its own dynamics through standard model interactions

$$\mathscr{L}_{\mathrm{Yuk}} = \frac{\mathrm{i}h_b}{\sqrt{2}}\left(\bar{\psi}^a_{\mathrm{L},i}\phi^a b_{\mathrm{R},i} + \mathrm{h.c.}\right) + \frac{\mathrm{i}h_t}{\sqrt{2}}\left(\bar{\psi}^a_{\mathrm{L},i}\phi^a_{\mathcal{C}} t_{\mathrm{R},i} + \mathrm{h.c.}\right)$$
$$+ \frac{\mathrm{i}h}{\sqrt{2}}\left(\bar{\psi}^a_{\mathrm{R},i}\tilde{\phi}^a b_{\mathrm{L},i} + \bar{\psi}^a_{\mathrm{R},i}\tilde{\phi}^a_{\mathcal{C}} t_{\mathrm{L},i} + \mathrm{h.c.}\right)$$

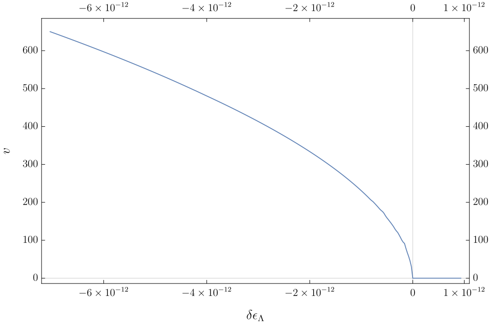
The Full Model

Splitting of the meson field φ into left- and right-doublets yields

$$\Gamma_k = \int_x \; Z_\phi |\partial_\mu \phi|^2 + Z_{\tilde{\phi}} \left|\partial_\mu \tilde{\phi}\right|^2 + \bar{\psi}^a_i \mathrm{i} \not{D}_{ij} \psi^a_j + \frac{Z_\mathrm{F}}{4} F^z_{\mu\nu} F^{\mu\nu}_z$$
$$+ \frac{(\partial_\mu A^\mu)^2}{2\xi} + (\text{Ghosts}) + U(\rho, \tilde{\rho})$$
$$+ \frac{\mathrm{i} h_t}{\sqrt{2}} \left(\bar{\psi}^a_{\mathrm{L},i} \phi^a_{\mathcal{C}} t_{\mathrm{R},i} + \text{h.c.} \right) + \frac{\mathrm{i} h_b}{\sqrt{2}} \left(\bar{\psi}^a_{\mathrm{L},i} \phi^a b_{\mathrm{R},i} + \text{h.c.} \right)$$
$$+ \frac{\mathrm{i} h}{\sqrt{2}} \left(\bar{\psi}^a_{\mathrm{R},i} \tilde{\phi}^a b_{\mathrm{L},i} + \bar{\psi}^a_{\mathrm{R},i} \tilde{\phi}^a_{\mathcal{C}} t_{\mathrm{L},i} + \text{h.c.} \right)$$

$\Rightarrow$ analyze parametric dependence of phase transitions in this model

Phase Transitions

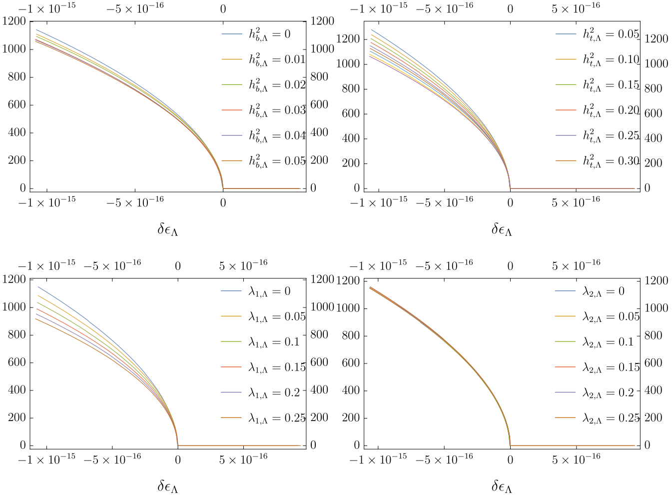


Scaling near quantum phase transition described by critical exponents

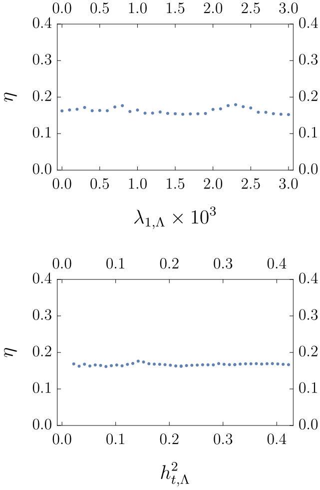
For the order parameter we have $v \propto |\delta\epsilon_\Lambda|^\beta$

β can be related to the critical exponent η which measures the deviation of the RG scaling exponent from its canonical value $\Theta = 2 - \eta$

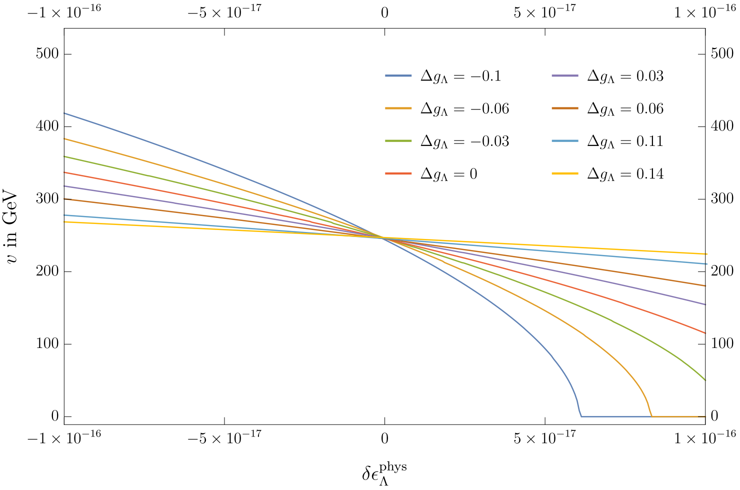
Phase Transitions for different Model Parameters



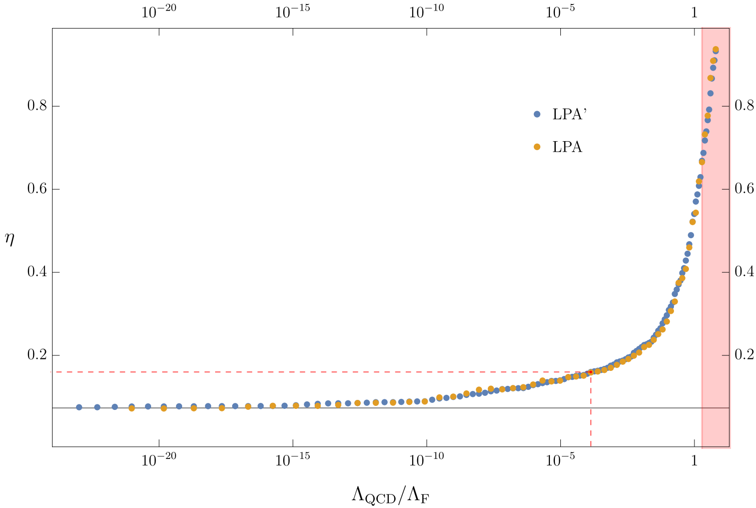
Critical Exponents for different Model Parameters



Phase transitions for increasing QCD coupling



Critical Exponents for increasing QCD coupling



Scaling Solutions in Higgs-QCD Models

Spontaneous symmetry breaking in asymptotically free gauge theories admit Cheng-Eichten-Lee scaling solutions [Cheng,Eichten,Lee '74].

In these models, couplings are controlled by the running of the strong gauge coupling g.

Signaled by a quasi fixed-point in composite couplings $\frac{h^2}{g^2}$, $\frac{\lambda}{g^2}$, etc,
i.e. $h^2 \to 0, \lambda \to 0,$ as $g^2 \to 0.$

Scaling solutions in Higgs-QCD models

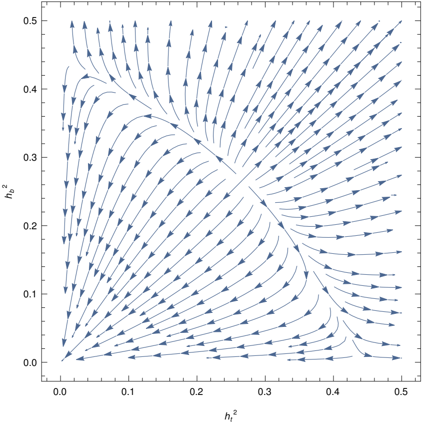
For this, assume validity of the DER, where $p^2 \gg m^2$ and mass threshold effects can be neglected.

Extract one loop polynomial beta functions from the nonpeturbative FRG results.

Check for fixed points in the beta functions: $\partial_t \frac{h^2}{g^2} = \frac{\beta_{h^2}}{g^2} - \eta_{\mathrm{F}} \frac{h^2}{g^2}, \cdots$

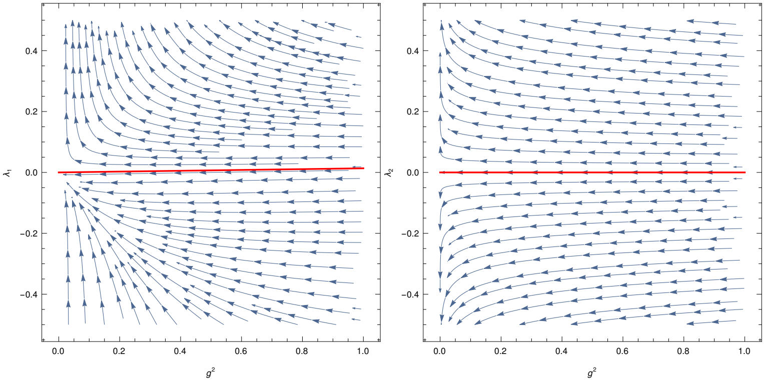
Scaling solutions in the Yukawa sector

For $h_*^2 = 0$ we find 4 quasi fixed points



Scaling solutions in the scalar sector

Assume quartic potential $U = \frac{\lambda_1}{2}(\rho^2 + \tilde{\rho}^2) + \lambda_2 \rho\tilde{\rho}$ where $\rho \sim \phi^2$



Red lines correspond to the QFP solutions, $\lambda_1 \propto g^2, \lambda_2 = 0$

Conclusions

Able to quantify interplay of the two breaking mechanisms through (pseudo-)critical exponents:

- Stronger gauge interactions lessen the fine-tuning necessary
- Other parameters have little influence on the phase transition
- Model admits CEL scaling solutions

Further points of interest:

$\Rightarrow$ Study influence of the electroweak sector on the phase transition

$\Rightarrow$ Study precise details of the rebosonized model's quasi fixed-points

$\Rightarrow$ Fate of the rebosonization in the deep UV

Thank you!

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