Orbifold stability of asymptotic GUTs

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G. Cacciapaglia et al. [in preparation]

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LIO International Conference on
"Asymptotic safety in Quantum Field
Theory: Grand Unification"

Outline

Asymptotic Grand Unified Theories (aGUTs)

- Grand Unified Theories (GUTs) formulated in 5 or more space-time dimensions. [1]

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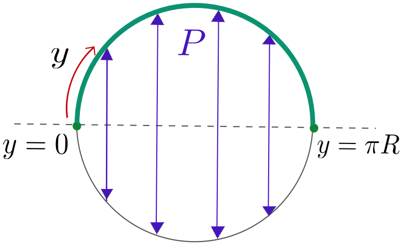
defined on $\mathbb{R}^4 \times K$, where $\mathbb{R}^4$ is the usual 4-dimensional Minkowski space and K defines δ *compact* extra dimensions.

- Gauge symmetry is broken using boundary conditions which violate the GUT symmetry.

[1] A. Hebecker, J. March-Russell, Nuclear Phys. B 625 (2002)

Example: 5D case

- One extra dimension ($\delta = 1$) compactified on $K = \mathbb{S}^1/\mathbb{Z}_2 \times \mathbb{Z}'_2$.

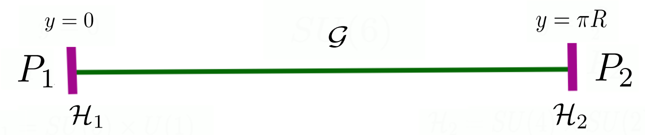


- The inverse radius R^{-1} sets the scale of compactification.

Example: 5D case

- Each intrinsic $\mathbb{Z}_2$ transformation is specified by a parity matrix P acting on the fields

$$\Phi(x^\mu, -y) = P\Phi(x^\mu, y) = \pm\Phi(x^\mu, y).$$



- Each P_i will break $\mathcal{G} \to \mathcal{H}_i$ on one boundary, such that

$$\mathcal{G}_{\rm 4D} \equiv \mathcal{H}_i \cap \mathcal{H}_j$$

- Viable model must contain the Standard Model

$$\mathcal{G}_{\rm 4D} \supset \mathcal{G}_{\rm SM}$$

Example: 5D case

A model can be fully defined in terms of [2]

Gauge group $\mathcal{G}$

Parity P

Parity assignments

[2] G. Cacciapaglia, arXiv:2309.10098 (2023)

Example: 5D case

For a given field $\Phi(x^\mu, y)$ we can do a **KK decomposition**

Decomposition

$$\Phi(x^\mu, y) = \underbrace{\sum_{n=0}^{\infty} \phi_+^{(n)}(x^\mu)\cos\left(\frac{ny}{R}\right)}_{\text{parity-even}} + \underbrace{\sum_{n=1}^{\infty} \phi_-^{(n)}(x^\mu)\sin\left(\frac{ny}{R}\right)}_{\text{parity-odd}}$$

- The 4D fields $\phi_\pm^{(n)} \equiv$ Kaluza-Klein (KK) modes with mass of n/R.
- The Standard Model fields are the massless zero modes of ϕ_+.
- For $E \ll 1/R$, the heavy Kaluza-Klein towers are integrated out.

4D effective field theory

Gauge-Higgs Unification[3] [4]

Assume a 5D gauge theory and A_M ($M = 1, \ldots, 5$) a gauge field

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$$\underbrace{A_\mu(\mu = 1 \ldots 4)}_{\text{4D}} \quad \text{and} \quad \underbrace{A_5}_{\text{extra dimension}}$$

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A_5 behaves as a scalar field in 4D

$\equiv$ **Higgs field**

[3] Y. Hosotani, Phys. Lett. B 126 (1983)

[4] R. Contino,et al, Nucl. Phys. B 671 (2003)

Gauge-Higgs Unification

- Gauge-Higgs scalar A_5 will generate masses for bulk fields when getting a VEV

$-\frac{1}{2}F_{5\mu}F^{5\mu}$	$(D_M\Phi)^\dagger(D^M\Phi)$	$\bar{\Psi}iD_M\Gamma^M\Psi$
gauge	scalar	fermions

- Gauge symmetry forbids a tree-level potential for A_5

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one loop effective potential

One loop effective potential

The effective potential for a scalar field in 4D is given by[5]

$$V_{\text{eff}} = \frac{1}{2}\sum_I (-1)^{F_I} \int \frac{d^4p}{(2\pi)^4} \log[p^2 + m^2]\,, \;\; F_I = 0,\, 1$$
$$= -\frac{1}{32\pi^2}\sum_I (-1)^{F_I} \int_0^\infty dl\; l\; e^{-m^2/l}$$

States appear as towers of KK modes with mass

$$m_n^2 = \frac{(n + c\,\alpha)^2}{R^2}, \qquad c \text{ depends on the representation}$$

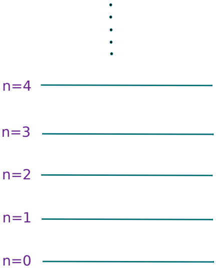
The field $\alpha \sim \langle A_5 \rangle$.

[5] I. Antoniadis, et al, New Journal of Physics 3 (2001)

One loop effective potential

Summing over KK modes we find

$$V_{\text{eff}} = -\sum_I \sum_n (-1)^{F_I} \frac{1}{32\pi^2} \int_0^\infty dl\ l\ e^{-(n+c\alpha)^2/R^2 l}$$



Performing the integral leads to the finite result

$$V_{\text{eff}}(\alpha) = \frac{\mp 1}{32\pi^2} \frac{1}{(\pi R)^4} \mathcal{F}(c\alpha)$$

where

$$\mathcal{F}(\alpha) = \frac{3}{2} \sum_{n=1}^{\infty} \frac{\cos(2\pi n\alpha)}{n^5}.$$

One loop effective potential

$$V_{\text{eff}}^{\text{total}}(\alpha) = V_{\text{eff}}^{\text{gauge}}(\alpha) + V_{\text{eff}}^{\text{scalar}}(\alpha) + V_{\text{eff}}^{\text{fermionic}}(\alpha)$$

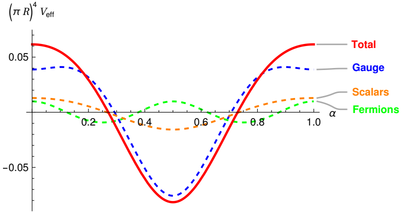


Figure: Potential of an $SU(6)$ model.

- gauge dominated
- minimum at $\alpha = 1/2$

One loop effective potential

$$V_{\text{eff}}^{\text{total}}(\alpha) = V_{\text{eff}}^{\text{gauge}}(\alpha) + V_{\text{eff}}^{\text{scalar}}(\alpha) + V_{\text{eff}}^{\text{fermionic}}(\alpha)$$

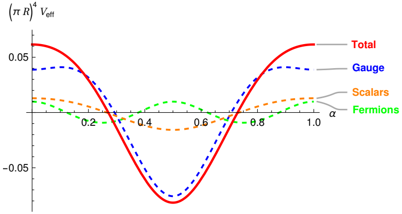


Figure: Potential of an $SU(6)$ model.

- gauge dominated
- minimum at $\alpha = 1/2$

Not a viable model

One loop effective potential

Minimum of $V_{\text{eff}}^{\text{gauge}}(\alpha)$ must be at $\alpha = 0$.

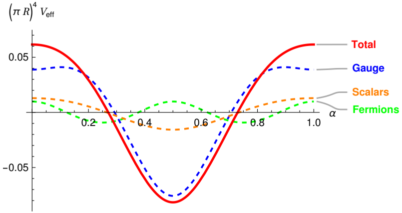


Figure: Potential of an $SU(6)$ model.

Gauge transformation to remove the VEV

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change in the parity on one boundary

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different breaking pattern

One loop effective potential

Assume a model with gauge group $\mathcal{G}$ and parities (P_1, P_2)

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Gauge transformation Ω : $A'_M \to \Omega^\dagger A_M \Omega - \frac{i}{g}\Omega^\dagger \partial_M \Omega$ **adjoint**

Boundary conditions imposed on the adjoint become [6]

$$A'_\mu = P'_i \cdot A'_\mu \cdot P'_i - \frac{i}{g} P'_i \cdot \partial_\mu P'_i$$
$$A'_5 = -P'_i \cdot A'_5 \cdot P'_i + \frac{i}{g} P'_i \cdot \partial_5 P'_i$$

where $P'_i = \Omega^\dagger P_i \Omega$.

Freedom to choose the gauge $\Rightarrow \partial_M P'_i = 0$.

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$(P'_1, P'_2) \sim (P_1, P_2)$ **equivalent!**

[6] N. Haba, et al, Prog.Theor.Phys., 111(2004)

Orbifold stability: $SU(N)$

- aGUT in 5D based on the $SU(N)$ gauge group[7]
- Extra dimension compactified on the orbifold $\mathbb{S}^1/\mathbb{Z}_2 \times \mathbb{Z}_2'$
- Most general parities

$$P_1 = (+1 \cdots, +1, +1, \cdots, +1, -1, \cdots, -1, -1 \cdots, -1),$$
$$P_2 = (\underbrace{+1, \cdots, +1}_{p}, \underbrace{-1, \cdots, -1}_{q}, \underbrace{+1, \cdots, +1}_{r}, \underbrace{-1, \cdots, -1}_{s}).$$

[7] N. Haba, T. Yamashita, JHEP 2004 (2004).

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$$P_2 = (\underbrace{+1, \cdots, +1}_{p}, \underbrace{-1, \cdots, -1}_{q}, \underbrace{+1, \cdots, +1}_{r}, \underbrace{-1, \cdots, -1}_{s}).$$

Breaking pattern

$$SU(N) \to SU(p) \times SU(q) \times SU(r) \times SU(s) \times U(1)^3 \,,$$

where $p + q + r + s = N$.

[7] N. Haba, T. Yamashita, JHEP 2004 (2004).

Orbifold stability: $SU(N)$

- Fields will have parity assignments under (P_1, P_2), which can be

$$(+,+) \quad (+,-) \quad (-,+) \quad (-,-)$$

- Parities of adjoint

$$A_\mu \to P_i \cdot A_\mu \cdot P_i \text{ and } A_5 \to -P_i \cdot A_5 \cdot P_i$$

Orbifold stability: $SU(N)$

- Fields will have parity assignments under (P_1, P_2), which can be

$$(+,+) \quad (+,-) \quad (-,+) \quad (-,-)$$

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⇓

4D vector zero modes with parity $(+,+)$

Orbifold stability: $SU(N)$

- Fields will have parity assignments under (P_1, P_2), which can be

$$(+,+) \quad (+,-) \quad (-,+) \quad (-,-)$$

- Parities of adjoint

$$A_\mu \to P_i \cdot A_\mu \cdot P_i \text{ and } A_5 \to -P_i \cdot A_5 \cdot P_i$$

4D vector zero modes with parity $(+,+)$

gauge-scalar zero-modes with parity $(-,-)$

Orbifold stability: $SU(N)$

- Fields will have parity assignments under (P_1, P_2), which can be

$$(+,+) \quad (+,-) \quad (-,+) \quad (-,-)$$

- Parities of adjoint

$$A_\mu \to P_i \cdot A_\mu \cdot P_i \text{ and } A_5 \to -P_i \cdot A_5 \cdot P_i$$

⇓	⇓
4D vector zero modes with parity $(+,+)$	gauge-scalar zero-modes with parity $(-,-)$

- Similar approach for scalars and fermions, depending on the representation

Orbifold stability: $SU(N)$

- Parity assignments on the fields are

$$A_\mu \to \begin{matrix} p \\ q \\ r \\ s \end{matrix} \begin{pmatrix} (+,+) & (+,-) & (-,+) & (-,-) \\ (+,-) & (+,+) & (-,-) & (-,+) \\ (-,+) & (-,-) & (+,+) & (+,-) \\ (-,-) & (-,+) & (+,-) & (+,+) \end{pmatrix} .$$

- Gauge-scalar zero modes present in the (p, s) and (q, r) blocks
- Scalars in the bi-fundamental representation of $SU(p) \times SU(s)$ and $SU(q) \times SU(r)$

Orbifold stability: $SU(N)$

$$P_1 = (+1\cdots,+1,+1,\cdots,+1,-1,\cdots,-1,-1\cdots,-1),$$
$$P_2 = (\underbrace{+1,\cdots,+1}_{p},\underbrace{-1,\cdots,-1}_{q},\underbrace{+1,\cdots,+1}_{r},\underbrace{-1,\cdots,-1}_{s}).$$

Depending on the values of (p, q, r, s), we can have [8]

- **Two-block** case: two of (p, q, r, s) non-zero, others zero
- **Three-block** case: three of (p, q, r, s) non-zero, others zero
- **Four-block** case: all four (p, q, r, s) non-zero

[8] A.Deandrea et al. [in preparation]

Orbifold stability: $SU(N)$

Two-block case

- Parities present only in two different blocks
- Breaking pattern is

$$SU(N) \to SU(p) \times SU(N-p) \times U(1)$$

- Two distinct cases
  1. The two blocks have $(p, s) \neq 0$

$$A_\mu \to \begin{pmatrix} (+,+) & (-,-) \\ (-,-) & (+,+) \end{pmatrix} \Rightarrow \textbf{stable}$$

  2. The two blocks have $(p, q/r) \neq 0$

$$A_\mu \to \begin{pmatrix} (+,+) & (+,-)/(-,+) \\ (+,-)/(-,+) & (+,+) \end{pmatrix} \Rightarrow \textbf{stable}$$

Orbifold stability: $SU(N)$

Three-block case

- Regardless of choice there will be one parity twist
- We can choose

$$P_1 = (+1 \cdots, +1, +1, \cdots, +1, -1 \cdots, -1),$$
$$P_2 = (\underbrace{+1, \cdots, +1}_{p}, \underbrace{-1, \cdots, -1}_{q}, \underbrace{-1, \cdots, -1}_{s}), \quad p \geq s$$

- All other configurations are equivalent
- Breaking pattern is

$$SU(N) \to SU(p) \times SU(q) \times SU(s) \times U(1)^2$$

Orbifold stability: $SU(N)$

$$P_1 = (+1\cdots,+1,+1,\cdots,+1,-1\cdots,-1),$$
$$P_2 = (\underbrace{+1,\cdots,+1}_{p},\underbrace{-1,\cdots,-1}_{q},\underbrace{-1,\cdots,-1}_{s})$$

If the VEV is $\alpha = 1/2$ (maximal) $\Rightarrow P_2$ will be changed to

$$P_1 = (+1\cdots,+1,+1,\cdots,+1,-1\cdots,-1),$$
$$P_2 = (\underbrace{-1,\cdots,-1}_{q+s},\underbrace{+1,\cdots,+1}_{p-s},\underbrace{+1,\cdots,+1}_{s})$$

⇓

$$SU(N) \to SU(q+s)\times SU(p-s)\times SU(s)\times U(1)^2$$

Orbifold stability: $SU(N)$

Two distinct breaking patterns

$$\text{(I)} \qquad SU(p) \times SU(q) \times SU(s) \times U(1)^2$$
$$\updownarrow$$
$$\text{(II)} \qquad SU(q+s) \times SU(p-s) \times SU(s) \times U(1)^2$$

connected by a gauge transformation

Orbifold stability: $SU(N)$

- **Global minimum check**: compute the potential and evaluate it at the two minima

Breaking pattern (I)

$$V_{\text{eff}}(0) \propto -s^2 - 2(p-s)s + 2qs\,,$$
$$V_{\text{eff}}(1/2) \propto -s^2 + 2(p-s)s - 2qs$$

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$$\Delta V_{\text{eff}} = V_{\text{eff}}(1/2) - V_{\text{eff}}(0) = 4s(p-s-q) = 4s(2p-N)$$

Orbifold stability: $SU(N)$

Breaking pattern (I)

$$\Delta V_{\text{eff}} = V_{\text{eff}}(1/2) - V_{\text{eff}}(0) = 4s(p - s - q) = 4s(2p - N)$$

We can distinguish two cases

$$p \geq N/2 \quad \Rightarrow \quad \text{stable vacuum}$$
$$p < N/2 \quad \Rightarrow \quad \text{unstable vacuum}$$

examples: $SU(6) \to SU(3) \times SU(2) \times U(1)^2$

$SU(7) \to SU(4) \times SU(2) \times U(1)^2$

Orbifold stability: $SU(N)$

Remember the two patterns!

$$\text{(I)} \qquad SU(p) \times SU(q) \times SU(s) \times U(1)^2$$
$$\updownarrow$$
$$\text{(II)} \qquad SU(q+s) \times SU(p-s) \times SU(s) \times U(1)^2$$

For breaking pattern (II) we have $p' = q + s$, $q' = p - s$ and $s' = s$

Breaking pattern (II)

$$\Delta V_{\text{eff}} = V_{\text{eff}}(1/2) - V_{\text{eff}}(0) = 4s(p' - s - q') = 4s(2p' - N)$$

Orbifold stability: $SU(N)$

Breaking pattern (II)

$$\Delta V_{\text{eff}} = V_{\text{eff}}(1/2) - V_{\text{eff}}(0) = 4s(p' - s - q') = 4s(2p' - N)$$

where $p' = q + s = N - p > N/2 \quad \text{for} \quad p < N/2$

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three-blocks are automatically stable

Orbifold stability: $SU(N)$

Four-blocks

$$P_1 = (+1 \cdots, +1, +1, \cdots, +1, -1, \cdots, -1, -1 \cdots, -1),$$
$$P_2 = (\underbrace{+1, \cdots, +1}_{p}, \underbrace{-1, \cdots, -1}_{q}, \underbrace{+1, \cdots, +1}_{r}, \underbrace{-1, \cdots, -1}_{s}).$$

$$SU(N) \to SU(p) \times SU(q) \times SU(r) \times SU(s) \times U(1)^3$$

$V(\alpha, \beta)$ has two VEVs α and β corresponding to (p, s) and (q, r) blocks

- $\alpha = 1/2$ and $\beta = 0$
- $\alpha = 0$ and $\beta = 1/2$
- $\alpha = 1/2$ and $\beta = 1/2$

Orbifold stability: $SU(N)$

- $\alpha = 1/2$ and $\beta = 0$ corresponds to

$$SU(N) \to SU(q+s) \times SU(p-s) \times SU(r+s) \times U(1)^2$$ **three-block**

Orbifold stability: $SU(N)$

- $\alpha = 1/2$ and $\beta = 0$ corresponds to

$$SU(N) \to SU(q+s) \times SU(p-s) \times SU(r+s) \times U(1)^2 \quad \textbf{three-block}$$

- $\alpha = 0$ and $\beta = 1/2$ corresponds to

$$SU(N) \to SU(p+r) \times SU(q-r) \times SU(r+s) \times U(1)^2 \quad \textbf{three-block}$$

Orbifold stability: $SU(N)$

- $\alpha = 1/2$ and $\beta = 0$ corresponds to

$$SU(N) \to SU(q+s) \times SU(p-s) \times SU(r+s) \times U(1)^2 \quad \textbf{three-block}$$

- $\alpha = 0$ and $\beta = 1/2$ corresponds to

$$SU(N) \to SU(p+r) \times SU(q-r) \times SU(r+s) \times U(1)^2 \quad \textbf{three-block}$$

- $\alpha = 1/2$ and $\beta = 1/2$ corresponds to

$$SU(N) \to SU(p-s+r) \times SU(q-r+s) \times SU(s) \times SU(r) \times U(1)^3$$

Orbifold stability: $SU(N)$

Same approach as before: compute the potential and compare the zero-VEV case to the other different minima

$$\Delta V_{\text{eff}}^{(1/2,0)} = V_{\text{eff}}(1/2,0) - V_{\text{eff}}(0,0) = 4s(2p-N)$$
$$\Delta V_{\text{eff}}^{(0,1/2)} = V_{\text{eff}}(0,1/2) - V_{\text{eff}}(0,0) = 4r(2q-N)$$
$$\Delta V_{\text{eff}}^{(1/2,1/2)} = V_{\text{eff}}(1/2,1/2) - V_{\text{eff}}(0,0) = 4(s-r)(p+r-q-s)$$

At least one of $\Delta V_{\text{eff}}^{(1/2,0)}$ or $\Delta V_{\text{eff}}^{(0,1/2)}$ is **smaller than zero**

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instability of four-blocks

Orbifold stability: $SU(N)$

Interpretation

- Four-blocks are unstable: the vacuum prefers the stable three-blocks
- The three blocks corresponding to $(\alpha, \beta) = (1/2, 0)/(0, 1/2)$ have

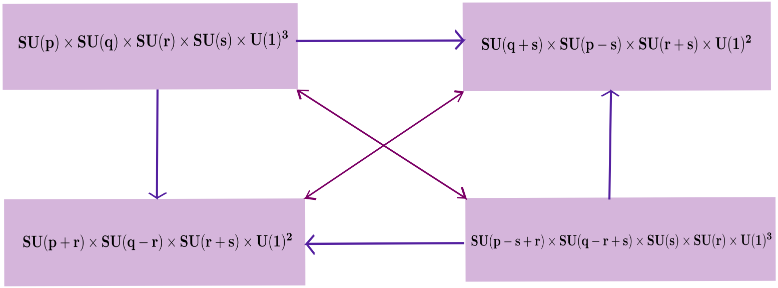
$$p' = q + s \qquad \text{and} \qquad p'' = p + r$$

at least one is $> N/2$ since $p + q + r + s = N$

stable orbifold

Orbifold stability: $SU(N)$

Summary of stability results



Orbifold stability: $Sp(2N)$, $SO(N)$

- Analysis can be extended to other groups as well: $Sp(2N)$, $SO(N)$
- Same strategy as before for identifying stable configurations
- **New parity definitions** are needed $\leftrightarrow$ group theory

examples:

$$Sp(10) \to SU(3) \times SU(2) \times U(1) \times U(1)$$

$$SO(10) \to SU(3) \times SU(2) \times U(1) \times U(1)$$ [9]

$$SO(11) \to SO(3) \times SO(2) \times SO(6) \sim SU(2) \times U(1) \times SU(4)$$

$$SO(12) \to SU(4) \times SU(2) \times U(1) \times U(1)$$

[9] M. Khojali, et al, PACP2022 (2022)

Case example: $SU(6)$

- $SU(6)$ aGUT in 5D [10] with y compactified on $S_1/(\mathbb{Z}_2 \times \mathbb{Z}_2')$
- We choose the parities

$$P_1 = (+1 \cdots, +1, +1, \cdots, +1, -1, \cdots, -1)$$
$$P_2 = (\underbrace{+1, \cdots, +1}_{p=3}, \underbrace{-1, \cdots, -1}_{q=2}, \underbrace{-1, \cdots, -1}_{s=1})$$

such that $SU(6) \to SU(3) \times SU(2) \times U(1)^2$.

Stable?

$\Rightarrow p = 3 \geq N/2 = 6/2$ is satisfied

This model has a global minimum at $\alpha = 0$.

[10] C.S. Lim, N. Maru, Phys.Lett.B, 653(2007)

Case example: $SU(6)$

$$P_1 = (+1 \cdots, +1, +1, \cdots, +1, -1, \cdots, -1)$$
$$P_2 = (\underbrace{+1, \cdots, +1}_{p=3}, \underbrace{-1, \cdots, -1}_{q=2}, \underbrace{-1, \cdots, -1}_{s=1})$$

- The gauge-scalar is in the (p, s) block

$$\varphi = (3, 1)_{-1/3}$$

bi-fundamental of $SU(p) \times SU(s) \equiv SU(3) \times U(1)$.

⇩

VEV necessarily needs to be **zero**

Case example: $SU(6)$

- Not the only choice of parities that breaks $SU(6)$ to SM! [2]

  $\Rightarrow$ we can also have

$$P_1 = (+1 \cdots, +1, +1, \cdots, +1, -1, \cdots, -1)$$
$$P_2' = (\underbrace{+1, \cdots, +1}_{p=3}, \underbrace{-1, \cdots, -1}_{q=2}, \underbrace{+1, \cdots, +1}_{r=1})$$

- The gauge-scalar is in the (q, r) block

$$\varphi = (1, 2)_{-1/2} \qquad \text{SM Higgs-like}$$

bi-fundamental of $SU(q) \times SU(r) \equiv SU(2) \times U(1)$.

[2] G. Cacciapaglia, arXiv:2309.10098 (2023)

Case example: $SU(6)$

However...

$$P_1 = (+1 \cdots, +1, +1, \cdots, +1, -1, \cdots, -1)$$
$$P_2' = (\underbrace{+1, \cdots, +1}_{p=3}, \underbrace{-1, \cdots, -1}_{q=2}, \underbrace{+1, \cdots, +1}_{r=1})$$

equivalent to

$$P_1 = (+1 \cdots, +1, +1, \cdots, +1, -1, \cdots, -1)$$
$$P_2' = (\underbrace{-1, \cdots, -1}_{p=2}, \underbrace{+1, \cdots, +1}_{q=3}, \underbrace{-1, \cdots, -1}_{s=1})$$

not stable! $p < N/2$

Case example: $SU(6)$

$$P_1 = (+1 \cdots, +1, +1, \cdots, +1, -1, \cdots, -1) \qquad \textbf{not}$$
$$P_2' = (\underbrace{-1, \cdots, -1}_{p=2}, \underbrace{+1, \cdots, +1}_{q=3}, \underbrace{-1, \cdots, -1}_{s=1}) \qquad \textbf{stable!}$$

According to our three-block results

$$P_1 = (+1 \cdots, +1, +1, \cdots, +1, -1, \cdots, -1)$$
$$P_2' = (\underbrace{+1, \cdots, +1}_{q+s=4}, \underbrace{-1, \cdots, -1}_{p-s=1}, \underbrace{-1, \cdots, -1}_{s=1})$$

leads to a **stable vacuum**, which gives

$$SU(6) \to SU(4) \times U(1) \times U(1) \qquad \text{does not contain SM}$$

Case example: $SU(6)$

- $P_2 \times P_2'$ also a possibility $\Rightarrow$ vector-like zero modes $\Rightarrow$ ruled out [2]
- All other parity combinations do not provide viable breaking patterns
- Not the only criteria: UV fixed point, SM fermions..

$\Downarrow$

orbifold stability $\equiv$ one of the consistency checks

but others needed as well

[2] G. Cacciapaglia, arXiv:2309.10098 (2023)

Conclusions

- aGUTs as an alternative to standard GUTs: 5D $SU(N)$ models
- Viable models have to pass certain criteria $\Rightarrow$ **orbifold stability**
- Global minimum of the gauge contribution entering the one-loop effective potential must be at $\alpha = 0$
- For $SU(N)$: two-blocks and three-blocks with $p \geq N/2$ are stable, while four-blocks are not
- The criteria of **orbifold stability** helps identify potentially interesting models and discards phenomenologically unrealistic scenarios
- Approach can be extended to $Sp(2N)$ and $SO(N)$ groups, where the results follow a similar logic

Back-up slides

Removing the VEV

The $SU(6)$ model with

$$P_1 = (+1 \cdots, +1, +1, \cdots, +1, -1, \cdots, -1)$$
$$P_2' = (\underbrace{+1, \cdots, +1}_{p=3}, \underbrace{-1, \cdots, -1}_{q=2}, \underbrace{+1, \cdots, +1}_{r=1})$$

has a global minimum at $\alpha = 1/2$.

$\Rightarrow$ remove the value of α using a gauge transformation

$$\Omega(\alpha) = \exp\left(i\frac{g}{R}y\, A_5\right) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos\frac{\alpha y}{R} & \sin\frac{\alpha y}{R} \\ 0 & 0 & 0 & 0 & \sin\frac{\alpha y}{R} & \cos\frac{\alpha y}{R} \end{pmatrix}$$

Removing the VEV

For $y = \pi R$, the gauge transformation flips the last two signs of P_2', such that

$$P_1 = (+1 \cdots, +1, +1, \cdots, +1, -1, \cdots, -1)$$
$$P_2' = (\underbrace{+1, \cdots, +1}_{p=4}, \underbrace{-1, \cdots, -1}_{q=1}, \underbrace{-1, \cdots, -1}_{s=1})$$

The 4D unbroken group will then be $SU(4) \times U(1) \times U(1)$.

Equivalence classes

$$P_1 = (+1 \cdots, +1, +1, \cdots, +1, -1, \cdots, -1, -1 \cdots, -1),$$
$$P_2 = (\underbrace{+1, \cdots, +1}_{p}, \underbrace{-1, \cdots, -1}_{q}, \underbrace{+1, \cdots, +1}_{r}, \underbrace{-1, \cdots, -1}_{s}).$$

Certain parity configurations are equivalent:

$$\begin{aligned} P_1 \to -P_1 &\quad\Rightarrow\quad p \leftrightarrow r, \quad q \leftrightarrow s \\ P_2 \to -P_2 &\quad\Rightarrow\quad p \leftrightarrow q, \quad r \leftrightarrow s \\ P_i \to -P_i &\quad\Rightarrow\quad p \leftrightarrow s, \quad q \leftrightarrow r \\ P_1 \leftrightarrow P_2 &\quad\Rightarrow\quad q \leftrightarrow r \end{aligned}$$

Equivalence classes given by gauge transformations [6]

$$[p, q, r, s] \sim [p+1, q-1, r-1, s+1] \sim [p-1, q+1, r+1, s-1]$$

[6] N. Haba, et al, Prog.Theor.Phys., 111(2004)

Stability of $Sp(2N)$ and $SO(N)$

- For $Sp(2N)$ groups the Cartan generators are given by

$$X_i = \underbrace{\text{diag}(0, \ldots 0, 1, 0, \ldots 0)}_{SU(N) \text{ Cartan generator}} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

⇩

Cartan subalgebra

$$\Omega(\theta_i) = \prod \exp(i\theta_i X_i^C) = \text{diag}(e^{i\theta_1}, \ldots e^{i\theta_N}, e^{-i\theta_1}, \ldots e^{-i\theta_N})$$

Possible parities $\Rightarrow$

$$P^I_{Sp(2N)} = P_{SU(N)} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \theta_i = 0, \pi$$

$$P^{II} Sp(2N) = P_{SU(N)} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \theta_i = \pm\pi/2$$

Stability of $Sp(2N)$ and $SO(N)$

- For $SO(2N)$ groups the Cartan generators are given by

$$X_i = \underbrace{\text{diag}(0, \dots 0, 1, 0 \dots 0)}_{SU(N)\text{ Cartan generator}} \otimes \sigma_2$$

$$\Downarrow$$

Cartan subalgebra

$$\Omega(\theta_i) = \prod \exp(i\theta_i X_i^C) = \sum_i \text{diag}(0, \dots 0, 1, 0 \dots 0) \otimes e^{i\theta_i \sigma_2}$$

Possible parities $\Rightarrow$

$$P^{I}_{SO(2N)} = P_{SU(N)} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \theta_i = 0, \pi$$

$$P^{II}_{SO(2N)} = P_{SU(N)} \otimes \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \theta_i = \pm\pi/2$$