Naturally small Yukawa couplings from asymptotic safety

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Based on

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JHEP 11 (2023) 224 (2308.06114)

in collaboration with
Abhishek Chikkaballi, Kamila Kowalska, Soumita Pramanick

National Science Centre
Poland

AS/GUTs workshop, IP2I Lyon

05.06.2024

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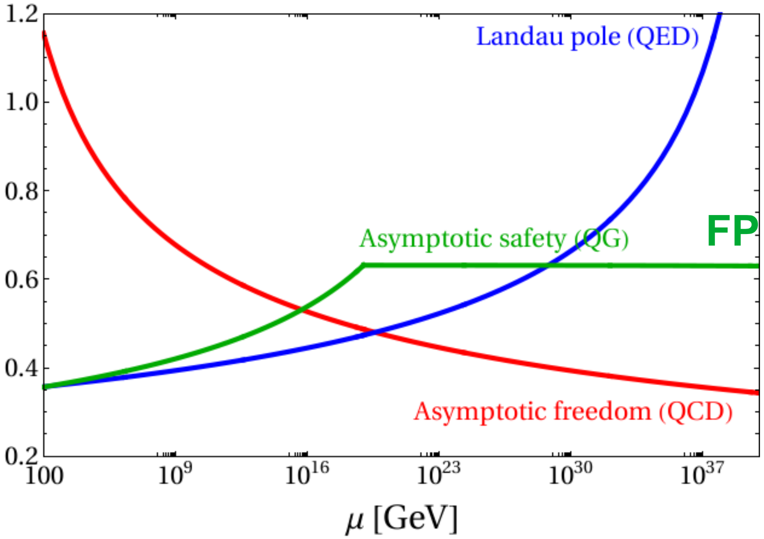
Outline

- Super-brief recall of trans-Planckian AS in matter
- Naturally small dimensionless couplings: the neutrino case
- Fixed points of SM + RHN
- Connections to AS quantum gravity
- Fixed points of gauged *B-L* model
- Gravitational waves from FOPTs vs. AS

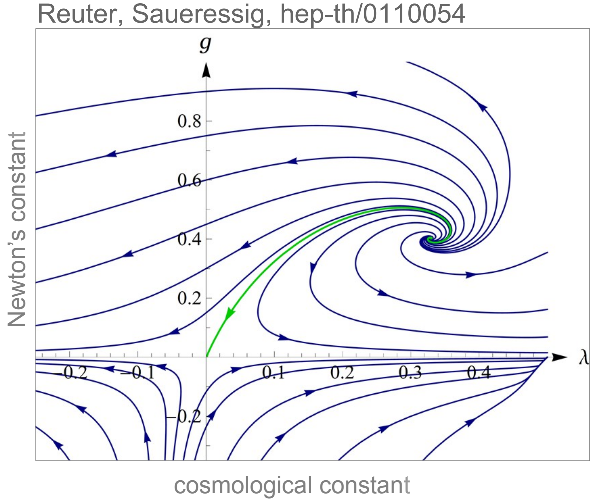
Asymptotic safety in a nutshell

Hints from the UV for IR model building

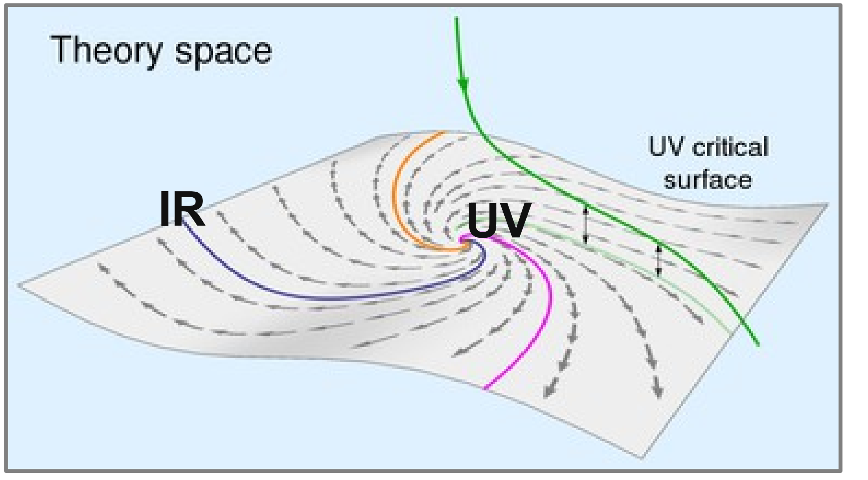


AS defines UV boundary conditions

Reuter, Saueressig, hep-th/0110054



AS inspired by quantum gravity



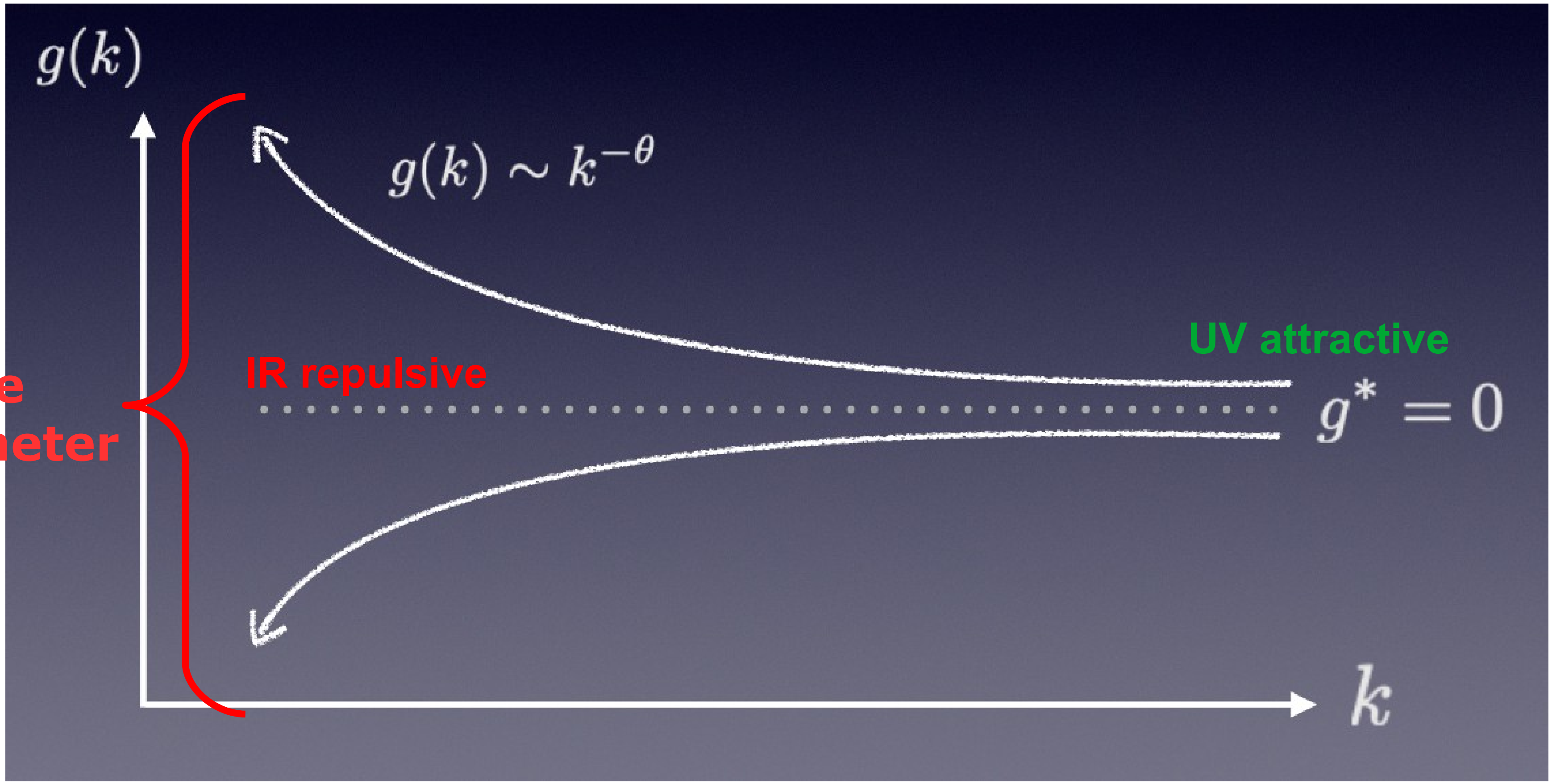
from Wikipedia (created by Andreas Nink)

Transmitted to the IR by the RGE flow

Scaling properties

$$M_{ij} = \partial\beta_i/\partial\alpha_j|_{\{\alpha_i^*\}}$$

(-) eigenvalue (critical exponent): θ > 0



free parameter

M.Yamada, Warsaw 08.10.2019

Relevant couplings are **free parameters**

Scaling properties

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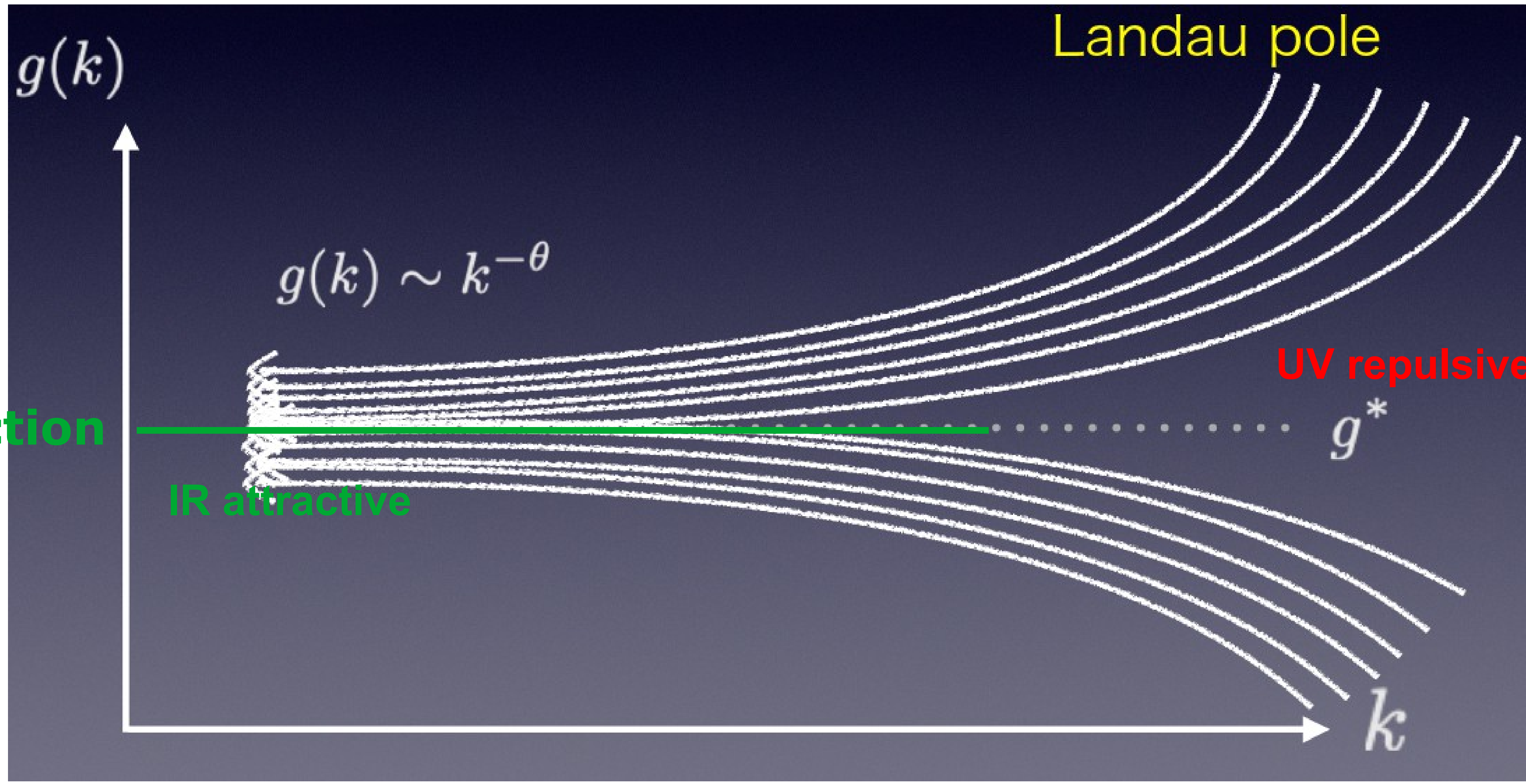
(-) eigenvalue (critical exponent): θ < 0

prediction



M.Yamada, Warsaw 08.10.2019

Irrelevant couplings provide **predictions**

Matter RGEs with quantum gravity

Robinson, Wilczek '06, Pietrykowski '07, Toms '08, Rodigast, Schuster '09, Zanusso et al. '09, Daum et al. '09,'10, Folkers et al. '11, Dona' et al. '13, Eichhorn et al. '16-17...

Trans-Planckian corrections of matter RGEs $k > M_{\mathrm{Pl}}$ (functional renormalization group)

Wetterich, Reuter, Saueressig, Percacci, Litim, Pawlowski, Eichhorn ...

SM gauge couplings

$$\frac{dg_Y}{dt} = \frac{g_Y^3}{16\pi^2}\frac{41}{6} \quad - f_g\, g_Y$$

$$\frac{dg_2}{dt} = -\frac{g_2^3}{16\pi^2}\frac{19}{6} \quad - f_g\, g_2$$

$$\frac{dg_3}{dt} = -\frac{g_3^3}{16\pi^2}7 \quad - f_g\, g_3$$

universal corrections depend on gravity fixed points

$$f_g = \tilde{G}^* \frac{1-4\tilde{\Lambda}^*}{4\pi\left(1-2\tilde{\Lambda}^*\right)^2}, \qquad f_y = -\tilde{G}^* \frac{96+\tilde{\Lambda}^*\left(-235+103\tilde{\Lambda}^*+56\tilde{\Lambda}^{*2}\right)}{12\pi\left(3-10\tilde{\Lambda}^*+8\tilde{\Lambda}^{*2}\right)^2}$$

e.g. A. Eichhorn, A. Held, 1707.01107
A. Eichhorn, F. Versteegen, 1709.07252

SM Yukawa couplings

$$\frac{dy_t}{dt} = \frac{y_t}{16\pi^2}\left(\frac{9}{2}y_t^2 + \frac{3}{2}y_b^2 + 3y_s^2 + 3y_c^2 - \frac{9}{4}g_2^2 - 8g_3^2 - \frac{17}{12}g_Y^2 + y_e^2 + y_\mu^2 + y_\tau^2\right) \quad - f_y\, y_t$$

$$\frac{dy_b}{dt} = \frac{y_b}{16\pi^2}\left(\frac{9}{2}y_b^2 + \frac{3}{2}y_t^2 + 3y_s^2 + 3y_c^2 - \frac{9}{4}g_2^2 - 8g_3^2 - \frac{5}{12}g_Y^2 + y_e^2 + y_\mu^2 + y_\tau^2\right) \quad - f_y\, y_b \quad \ldots$$

... same for other quarks and leptons

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... same for other quarks and leptons

get fixed points

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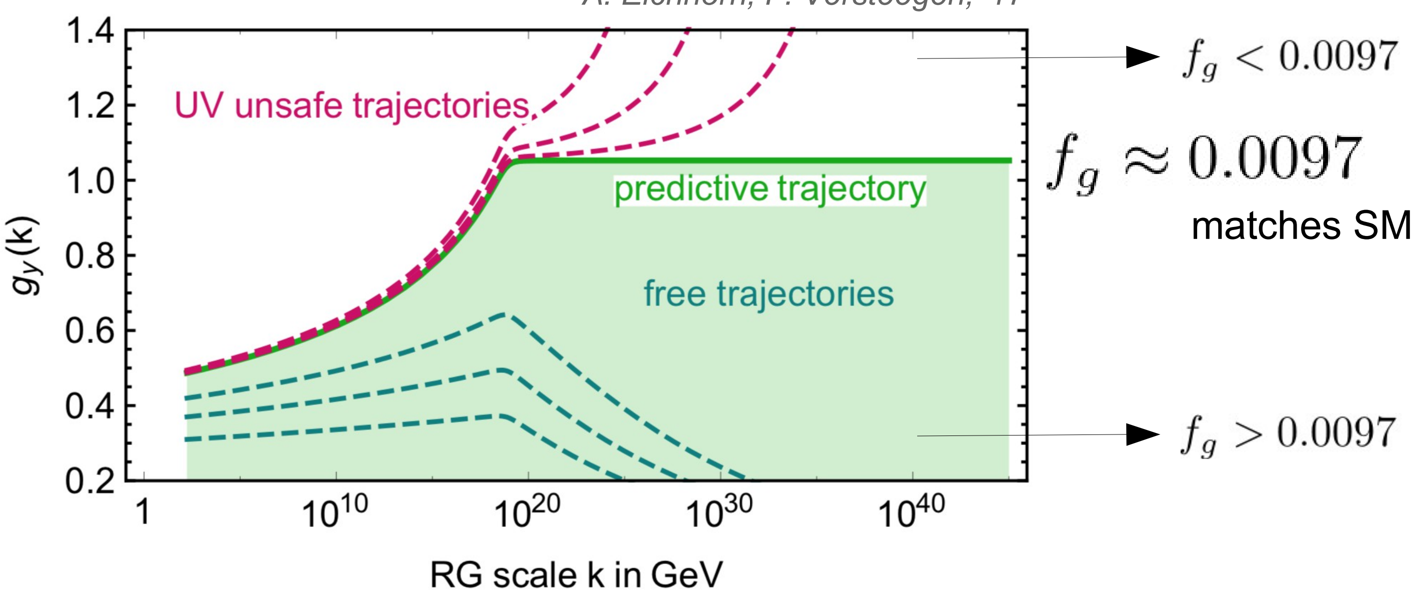
A. Eichhorn, F. Versteegen, '17



euristic determination $f_{g,y}$ → universality of couplings → **predictions BSM**

Kamila's talk

Naturalness
with
asymptotic safety

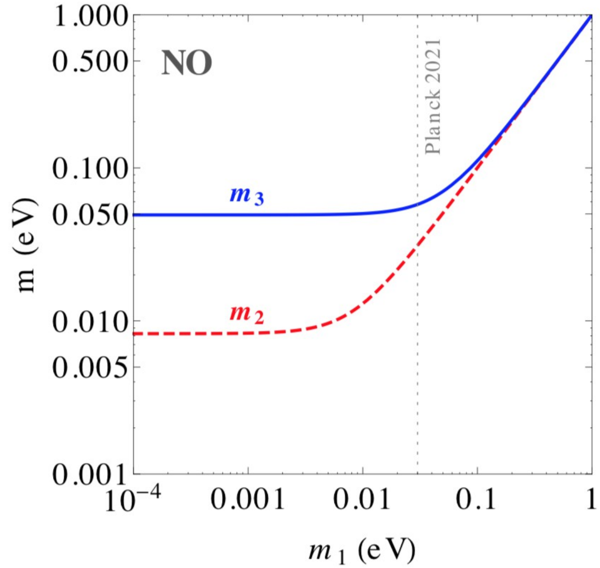
Neutrinos – experimental status

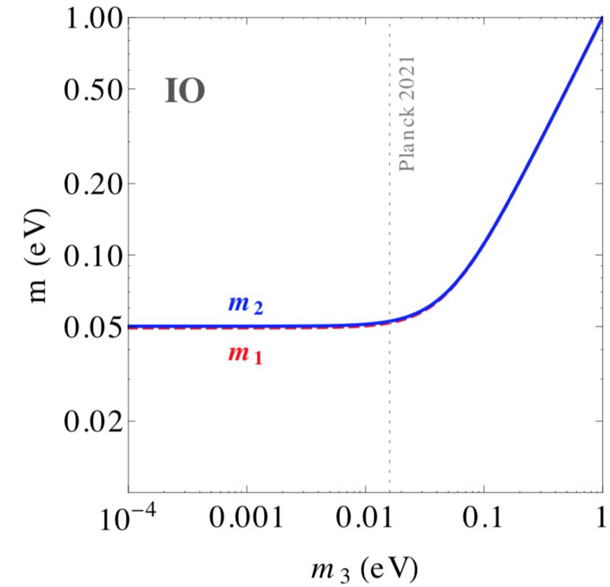
NuFIT5.1 (2021) 2007.14792

$$
\begin{aligned}
&\Delta m^2_{21} = 7.42^{+0.21}_{-0.20} \times 10^{-5}\,\mathrm{eV}^2, \\
\text{NO:}\quad &\Delta m^2_{31} = 2.515^{+0.028}_{-0.028} \times 10^{-3}\,\mathrm{eV}^2, \\
\text{IO:}\quad &\Delta m^2_{32} = -2.498^{+0.028}_{-0.029} \times 10^{-3}\,\mathrm{eV}^2,
\end{aligned}
$$

Planck (2021) 1807.06209

$$\sum_{i=1,2,3} m_i < 0.12\,\mathrm{eV}$$





PMNS pars ...

$$
U = \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\
s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13}
\end{pmatrix}
$$

PMNS fit .... (PDG)

$$
\begin{aligned}
\text{NO}:\quad &\theta_{12} = \left(33.44^{+0.77}_{-0.74}\right)^\circ, \quad \theta_{23} = \left(49.2^{+1.0}_{-1.3}\right)^\circ, \quad \theta_{13} = \left(8.57^{+0.13}_{-0.12}\right)^\circ, \quad \delta_{\rm CP} = \left(194^{+52}_{-25}\right)^\circ \\
\text{IO}:\quad &\theta_{12} = \left(33.45^{+0.77}_{-0.74}\right)^\circ, \quad \theta_{23} = \left(49.5^{+1.0}_{-1.2}\right)^\circ, \quad \theta_{13} = \left(8.60^{+0.12}_{-0.12}\right)^\circ, \quad \delta_{\rm CP} = \left(287^{+27}_{-32}\right)^\circ
\end{aligned}
$$

Neutrino mass

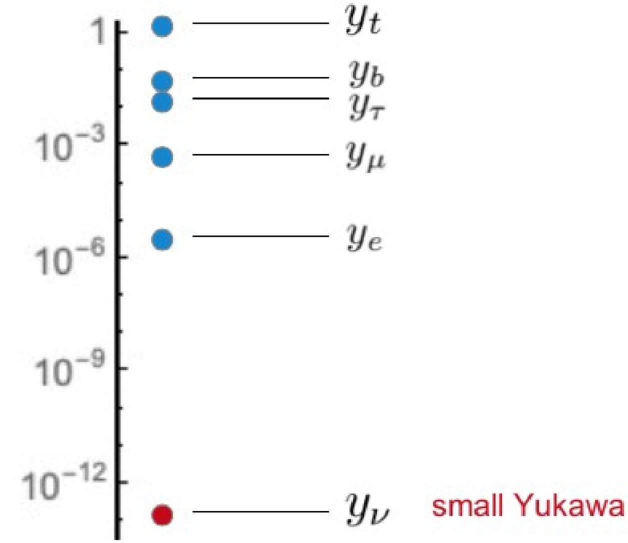
Neutrino masses are very small !

either Dirac neutrino ...

RHN → Higgs mechanism → small Yukawa

$$\mathcal{L}_D = -y_\nu^{ij}\,\nu_{R,i}\,(H^c)^\dagger L_j + \mathrm{H.c.}$$

$$m_\nu = \frac{y_\nu v_H}{\sqrt{2}}$$



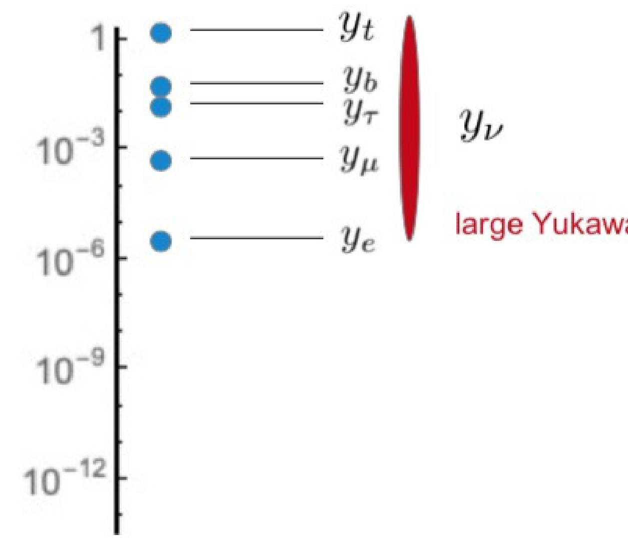
or Majorana neutrino ...

see-saw mechanism → large Yukawa

$$\mathcal{L}_M = \mathcal{L}_D - \frac{1}{2} M_N^{ij}\,\nu_{R,i}\nu_{R,j} + \mathrm{H.c.}$$

$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D & M_N \end{pmatrix} \qquad m_\nu = \frac{y_\nu^2 v_H^2}{\sqrt{2} M_N}$$

1 free parameter M_N



Fixed points SM + RHN

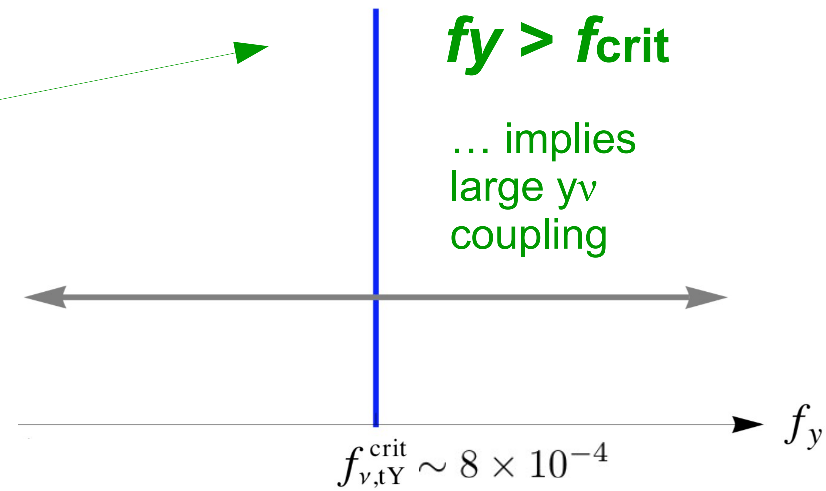
K. Kowalska, S. Pramanick, EMS '22

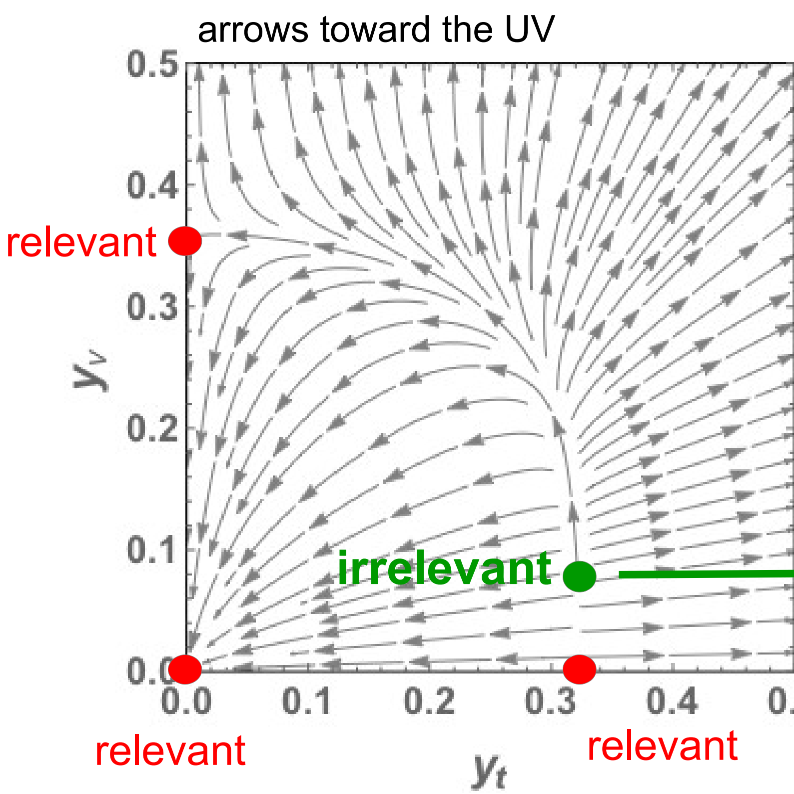
$$\frac{dg_Y}{dt} = \frac{g_Y^3}{16\pi^2}\frac{41}{6} - f_g\, g_Y \;\mathbf{= 0}$$

$$\frac{dy_t}{dt} = \frac{y_t}{16\pi^2}\left[\frac{9}{2}y_t^2 + y_\nu^2 - \frac{17}{12}g_Y^2\right] - \underline{f_y}\, y_t \;\mathbf{= 0}$$

$$\frac{dy_\nu}{dt} = \frac{y_\nu}{16\pi^2}\left[3y_t^2 + \frac{5}{2}y_\nu^2 - \frac{3}{4}g_Y^2\right] - \underline{f_y}\, y_\nu \;\mathbf{= 0}$$

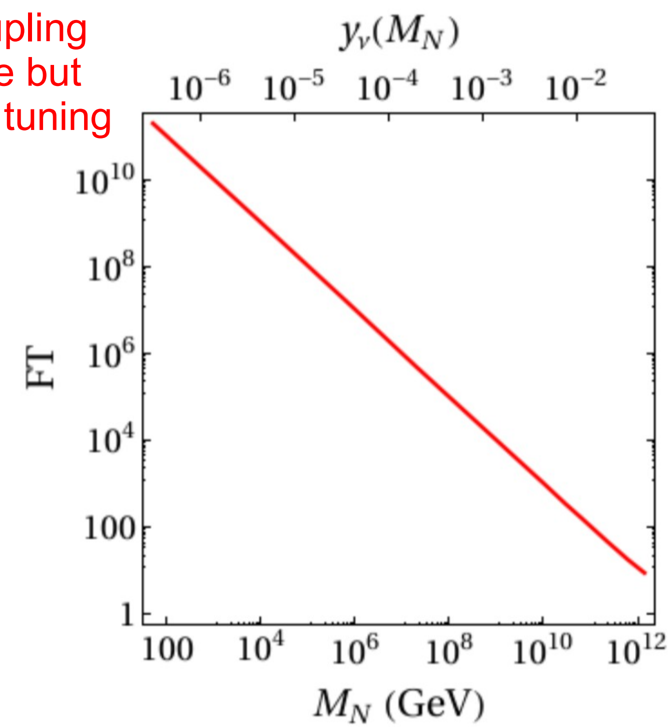
$$-1\times 10^{-4} \lesssim f_y \lesssim 1\times 10^{-3}$$





$y_{\nu*}$ is $O(1)$

Small coupling is possible but large fine tuning



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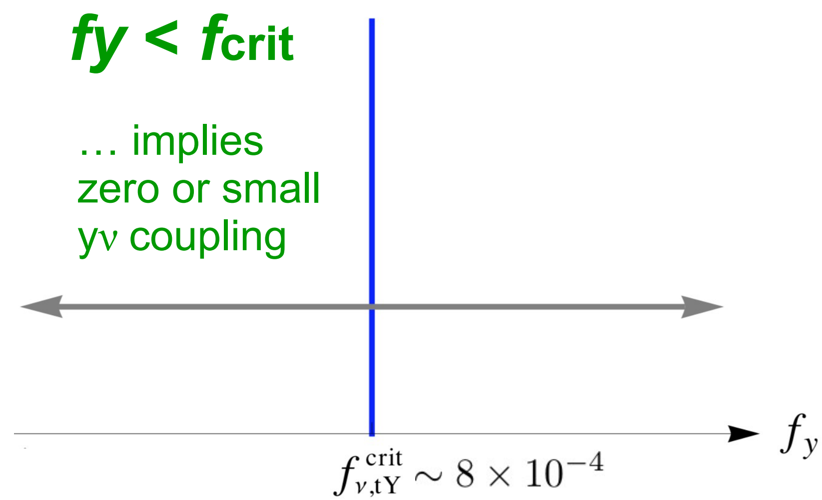
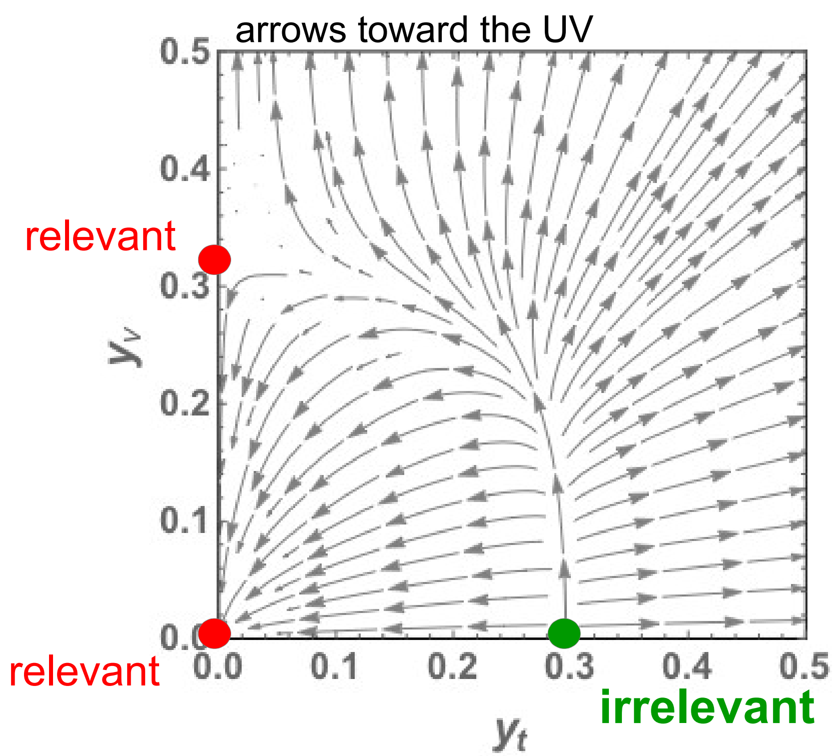




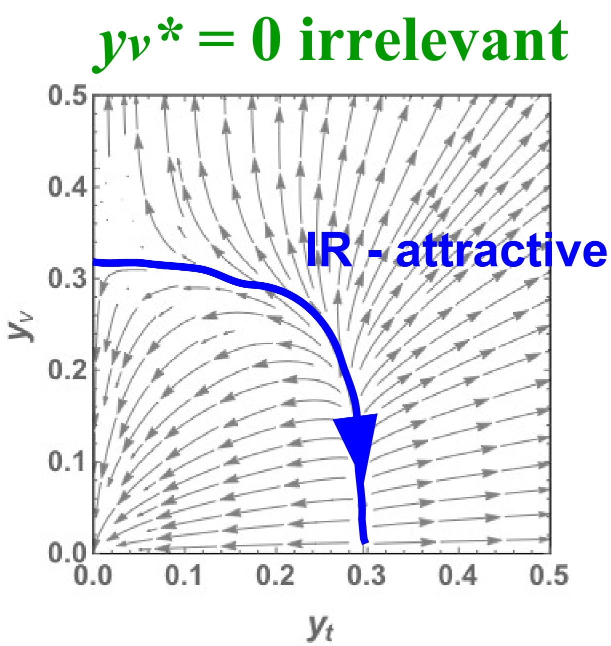
irrelevant $y_{\nu^*} = 0$

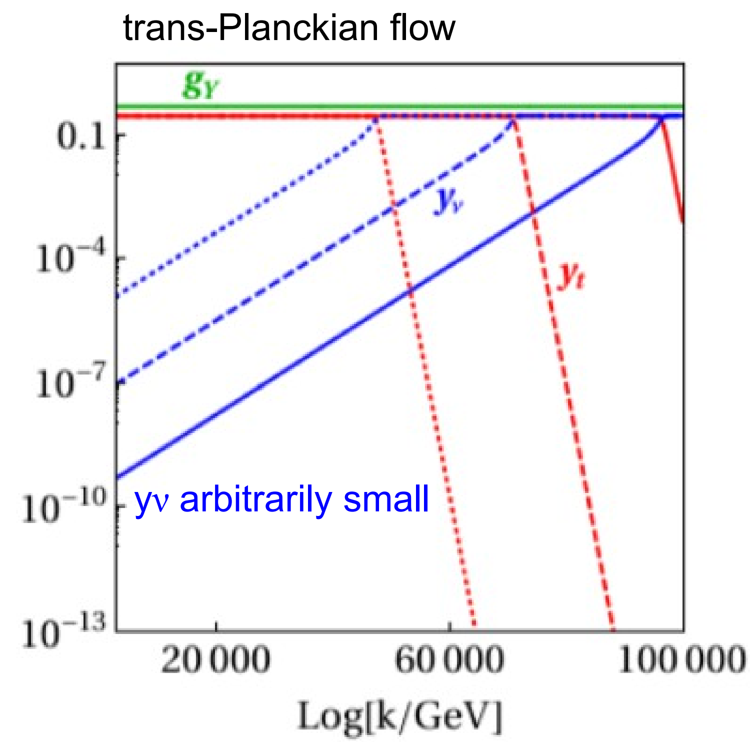
$y_\nu = 0$ …
predicted mass is zero?

let's go deeper ...

Fixed points SM + RHN

K. Kowalska, S. Pramanick, EMS '22





Integrate the curve:

$$y_\nu(t,\kappa) \approx \sqrt{\frac{16\pi^2(f_{\text{crit}} - f_y)}{e^{(f_{\text{crit}}-f_y)(16\pi^2\kappa - t)} + 5/2}}$$

$16\,\pi^2\kappa$ = Planck "distance" (e-folds) (1 free parameter, not worse than see-saw)

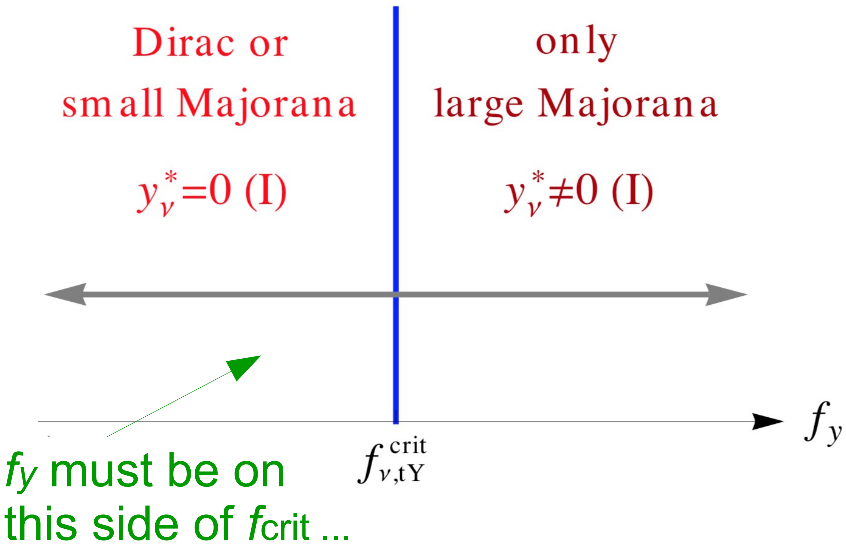
No fine tuning

(Neutrinos could be Dirac **naturally**)

Some comments ...

1. Is the neutrino special?

(Can we make anything else small via **irrelevant** Gaussian FP?)



… and there's an f_{crit} for each fermion ...

$$\frac{dy_X}{dt} = \frac{y_X}{16\pi^2}\left[\alpha_X\, y_X^2 + \alpha_Z y_Z^2 - \alpha_Y\, g_Y^2\right] - f_y\, y_X$$

$$\frac{dy_Z}{dt} = \frac{y_Z}{16\pi^2}\left[\alpha'_X\, y_X^2 + \alpha'_Z y_Z^2 - \alpha'_Y\, g_Y^2\right] - f_y\, y_Z$$

$$f_{Z,XY}^{\mathrm{crit}} = \frac{g_Y^{*2}}{16\pi^2}\,\frac{\alpha'_X\alpha_Y - \alpha'_Y\alpha_X}{\alpha_X - \alpha'_X}$$

… but top mass is good only if... $-1\times 10^{-4} \lesssim f_y \lesssim 1\times 10^{-3}$

Z	$\alpha'_{X=t}$	α'_Y	$f_{Z,tY}^{\mathrm{crit}}$	
u, c	3	$\frac{17}{12}$	-20.0×10^{-4}	TOP BAD
b	$\frac{3}{2}$	$\frac{5}{12}$	1.17×10^{-4}	TOP OKAY
d, s	3	$\frac{5}{12}$	22.3×10^{-4}	TOP GOOD
ν_i	3	$\frac{3}{4}$	8.22×10^{-4}	TOP GOOD
e, μ, τ	3	$\frac{15}{4}$	-119×10^{-4}	TOP BAD

Some comments ...

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Z	$\alpha'_{X=t}$	α'_Y	$f^{\text{crit}}_{Z,tY}$
u, c	3	$\frac{17}{12}$	-20.0×10^{-4}
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d, s	3	$\frac{5}{12}$	22.3×10^{-4}
ν_i	3	$\frac{3}{4}$	8.22×10^{-4}
e, μ, τ	3	$\frac{15}{4}$	-119×10^{-4}

TOP GOOD?

… running CKM pushes *f*crit to the left (spoils the fit)

Fixed CKM:

Running CKM:

$$16\pi^2\theta_{d,s} \approx 16\pi^2 f_y - 3y_t^{*2} + \frac{5}{12}g_Y^{*2} \quad \Rightarrow \quad 16\pi^2\theta_{d,s} \approx 16\pi^2 f_y - \frac{3}{2}\left(1 + |V_{tb}|^2\right) y_t^{*2} + \frac{5}{12}g_Y^{*2}$$

FP = 0

R. Alkofer *et al.* (2003.08401)

… perhaps the neutrino is special after all

More comments...

2. What about the full lepton sector?

$$\begin{aligned}\frac{dy_e}{dt} &= \frac{y_e}{16\pi^2}\left\{\frac{3}{2}y_e^2 - \frac{3}{2}\left[X y_{\nu 1}^2 + Y y_{\nu 2}^2 + (1-X-Y)y_{\nu 3}^2\right] + y_e^2 + y_\mu^2 + y_\tau^2 + y_{\nu 1}^2 + y_{\nu 2}^2 + y_{\nu 3}^2\right.\\ &\quad \left. - \left(\frac{15}{4}g_Y^2 + \frac{9}{4}g_2^2\right) + 3\left(y_t^2 + y_b^2\right)\right\} - f_y\, y_e \end{aligned} \tag{A.9}$$

$$\begin{aligned}\frac{dy_\mu}{dt} &= \frac{y_\mu}{16\pi^2}\left\{\frac{3}{2}y_\mu^2 - \frac{3}{2}\left[Z y_{\nu 1}^2 + W y_{\nu 2}^2 + (1-Z-W)y_{\nu 3}^2\right] + y_e^2 + y_\mu^2 + y_\tau^2 + y_{\nu 1}^2 + y_{\nu 2}^2 + y_{\nu 3}^2\right.\\ &\quad \left. - \left(\frac{15}{4}g_Y^2 + \frac{9}{4}g_2^2\right) + 3\left(y_t^2 + y_b^2\right)\right\} - f_y\, y_\mu \end{aligned} \tag{A.10}$$

$$\begin{aligned}\frac{dy_\tau}{dt} &= \frac{y_\tau}{16\pi^2}\left\{\frac{3}{2}y_\tau^2 - \frac{3}{2}\left[(1-X-Z)y_{\nu 1}^2 + (1-Y-W)y_{\nu 2}^2 + (X+Y+Z+W-1)y_{\nu 3}^2\right]\right.\\ &\quad \left. + y_e^2 + y_\mu^2 + y_\tau^2 + y_{\nu 1}^2 + y_{\nu 2}^2 + y_{\nu 3}^2 - \left(\frac{15}{4}g_Y^2 + \frac{9}{4}g_2^2\right) + 3\left(y_t^2 + y_b^2\right)\right\} - f_y\, y_\tau \end{aligned} \tag{A.11}$$

$$\begin{aligned}\frac{dy_{\nu 1}}{dt} &= \frac{y_{\nu 1}}{16\pi^2}\left\{\frac{3}{2}y_{\nu 1}^2 - \frac{3}{2}\left[X y_e^2 + Z y_\mu^2 + (1-X-Z)y_\tau^2\right] + y_e^2 + y_\mu^2 + y_\tau^2 + y_{\nu 1}^2 + y_{\nu 2}^2 + y_{\nu 3}^2\right.\\ &\quad \left. - \left(\frac{3}{4}g_Y^2 + \frac{9}{4}g_2^2\right) + 3\left(y_t^2 + y_b^2\right)\right\} - f_y\, y_{\nu 1} \end{aligned} \tag{A.12}$$

$$\begin{aligned}\frac{dy_{\nu 2}}{dt} &= \frac{y_{\nu 2}}{16\pi^2}\left\{\frac{3}{2}y_{\nu 2}^2 - \frac{3}{2}\left[Y y_e^2 + W y_\mu^2 + (1-Y-W)y_\tau^2\right] + y_e^2 + y_\mu^2 + y_\tau^2 + y_{\nu 1}^2 + y_{\nu 2}^2 + y_{\nu 3}^2\right.\\ &\quad \left. - \left(\frac{3}{4}g_Y^2 + \frac{9}{4}g_2^2\right) + 3\left(y_t^2 + y_b^2\right)\right\} - f_y\, y_{\nu 2} \end{aligned} \tag{A.13}$$

$$\begin{aligned}\frac{dy_{\nu 3}}{dt} &= \frac{y_{\nu 3}}{16\pi^2}\left\{\frac{3}{2}y_{\nu 3}^2 - \frac{3}{2}\left[(1-X-Y)y_e^2 + (1-Z-W)y_\mu^2 + (X+Y+Z+W-1)y_\tau^2\right]\right.\\ &\quad \left. + y_e^2 + y_\mu^2 + y_\tau^2 + y_{\nu 1}^2 + y_{\nu 2}^2 + y_{\nu 3}^2 - \left(\frac{3}{4}g_Y^2 + \frac{9}{4}g_2^2\right) + 3\left(y_t^2 + y_b^2\right)\right\} - f_y\, y_{\nu 3} \end{aligned} \tag{A.14}$$

$$\begin{aligned}\frac{dX}{dt} &= -\frac{3}{(4\pi)^2}\left[\left(\frac{y_e^2+y_\mu^2}{y_e^2-y_\mu^2}\right)\left\{(y_{\nu 1}^2-y_{\nu 3}^2)XZ + \frac{(y_{\nu 3}^2-y_{\nu 2}^2)}{2}[W(1-X)+X-(1-Y)(1-Z)]\right\}\right.\\ &\quad + \left(\frac{y_e^2+y_\tau^2}{y_e^2-y_\tau^2}\right)\left\{(y_{\nu 1}^2-y_{\nu 3}^2)X(1-X-Z) + \frac{(y_{\nu 3}^2-y_{\nu 2}^2)}{2}[(1-Y)(1-Z)-X(1-2Y)-W(1-X)]\right\}\\ &\quad + \left(\frac{y_{\nu 1}^2+y_{\nu 2}^2}{y_{\nu 1}^2-y_{\nu 2}^2}\right)\left\{(y_e^2-y_\tau^2)XY + \frac{(y_\tau^2-y_\mu^2)}{2}[W(1-X)+X-(1-Y)(1-Z)]\right\}\\ &\quad \left. + \left(\frac{y_{\nu 1}^2+y_{\nu 3}^2}{y_{\nu 1}^2-y_{\nu 3}^2}\right)\left\{(y_e^2-y_\tau^2)X(1-X-Y) + \frac{(y_\tau^2-y_\mu^2)}{2}[(1-Y)(1-Z)-X(1-2Z)-W(1-X)]\right\}\right] \end{aligned} \tag{A.15}$$

$$\begin{aligned}\frac{dY}{dt} &= -\frac{3}{(4\pi)^2}\left[\left(\frac{y_e^2+y_\mu^2}{y_e^2-y_\mu^2}\right)\left\{\frac{(y_{\nu 3}^2-y_{\nu 1}^2)}{2}[W(1-X)+X-(1-Y)(1-Z)] + (y_{\nu 2}^2-y_{\nu 3}^2)YW\right\}\right.\\ &\quad + \left(\frac{y_e^2+y_\tau^2}{y_e^2-y_\tau^2}\right)\left\{\frac{(y_{\nu 3}^2-y_{\nu 1}^2)}{2}[(1-Y)(1-Z)-W(1-X)-X(1-2Y)] + (y_{\nu 2}^2-y_{\nu 3}^2)Y(1-Y-W)\right\}\\ &\quad + \left(\frac{y_{\nu 2}^2+y_{\nu 1}^2}{y_{\nu 2}^2-y_{\nu 1}^2}\right)\left\{(y_e^2-y_\tau^2)XY + \frac{(y_\tau^2-y_\mu^2)}{2}[W(1-X)+X-(1-Y)(1-Z)]\right\}\\ &\quad \left. + \left(\frac{y_{\nu 2}^2+y_{\nu 3}^2}{y_{\nu 2}^2-y_{\nu 3}^2}\right)\left\{(y_e^2-y_\tau^2)Y(1-X-Y) + \frac{(y_\mu^2-y_\tau^2)}{2}[W(1-X-2Y)+X-(1-Z)(1-Y)]\right\}\right] \end{aligned} \tag{A.16}$$

$$\begin{aligned}\frac{dZ}{dt} &= -\frac{3}{(4\pi)^2}\left[\left(\frac{y_\mu^2+y_e^2}{y_\mu^2-y_e^2}\right)\left\{(y_{\nu 1}^2-y_{\nu 3}^2)XZ + \frac{(y_{\nu 3}^2-y_{\nu 2}^2)}{2}[W(1-X)+X-(1-Y)(1-Z)]\right\}\right.\\ &\quad + \left(\frac{y_\mu^2+y_\tau^2}{y_\mu^2-y_\tau^2}\right)\left\{(y_{\nu 1}^2-y_{\nu 3}^2)Z(1-X-Z) + \frac{(y_{\nu 2}^2-y_{\nu 3}^2)}{2}[W(1-X-2Z)+X-(1-Y)(1-Z)]\right\}\\ &\quad + \left(\frac{y_{\nu 1}^2+y_{\nu 2}^2}{y_{\nu 1}^2-y_{\nu 2}^2}\right)\left\{\frac{(y_e^2-y_\tau^2)}{2}[(1-Y)(1-Z)-X-W(1-X)] + (y_\mu^2-y_\tau^2)ZW\right\}\\ &\quad \left. + \left(\frac{y_{\nu 1}^2+y_{\nu 3}^2}{y_{\nu 1}^2-y_{\nu 3}^2}\right)\left\{\frac{(y_\tau^2-y_e^2)}{2}[(1-Z)(1-Y)-W(1-X)-X(1-2Z)] + (y_\mu^2-y_\tau^2)Z(1-Z-W)\right\}\right] \end{aligned} \tag{A.17}$$

$$\begin{aligned}\frac{dW}{dt} &= -\frac{3}{(4\pi)^2}\left[\left(\frac{y_\mu^2+y_e^2}{y_\mu^2-y_e^2}\right)\left\{(y_{\nu 2}^2-y_{\nu 3}^2)WY + \frac{(y_{\nu 3}^2-y_{\nu 1}^2)}{2}[W(1-X)+X-(1-Y)(1-Z)]\right\}\right.\\ &\quad + \left(\frac{y_\mu^2+y_\tau^2}{y_\mu^2-y_\tau^2}\right)\left\{(y_{\nu 2}^2-y_{\nu 3}^2)W(1-Y-W) + \frac{(y_{\nu 3}^2-y_{\nu 1}^2)}{2}[(1-Y)(1-Z)-X-W(1-X-2Z)]\right\}\\ &\quad + \left(\frac{y_{\nu 2}^2+y_{\nu 1}^2}{y_{\nu 2}^2-y_{\nu 1}^2}\right)\left\{(y_\mu^2-y_\tau^2)WZ + \frac{(y_\tau^2-y_e^2)}{2}[(1-X)W+X-(1-Y)(1-Z)]\right\}\\ &\quad \left. + \left(\frac{y_{\nu 2}^2+y_{\nu 3}^2}{y_{\nu 2}^2-y_{\nu 3}^2}\right)\left\{(y_\mu^2-y_\tau^2)W(1-Z-W) + \frac{(y_\tau^2-y_e^2)}{2}[(1-Y)(1-Z)-X-W(1-X-2Y)]\right\}\right]. \end{aligned} \tag{A.18}$$

More comments...

2. Asym. safe SM full fit works (with normal ordering)

PMNS parametrization

$$U_2 = |U_{\alpha i}|^2 = \begin{bmatrix} X & Y & 1-X-Y \\ Z & W & 1-Z-W \\ 1-X-Z & 1-Y-W & X+Y+Z+W-1 \end{bmatrix}$$

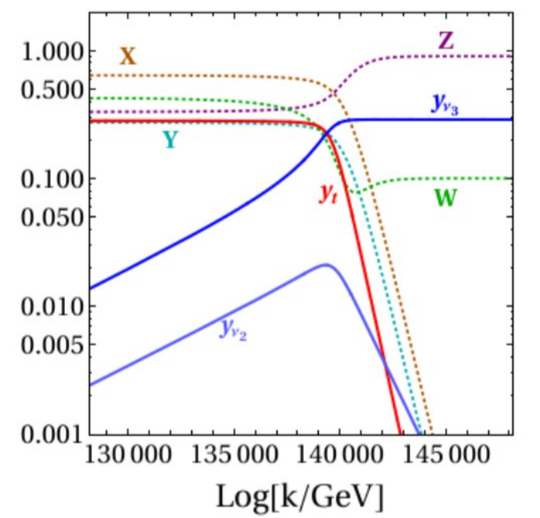
$$\theta_{12} = \arctan\sqrt{\frac{Y}{X}}$$
$$\theta_{13} = \arccos\sqrt{X+Y}$$
$$\theta_{23} = \arcsin\sqrt{\frac{1-W-Z}{X+Y}}$$

$$\delta = \arccos\frac{(X+Y)^2 Z - Y(X+Y+Z+W-1) - X(1-W-Z)(1-X-Y)}{2\sqrt{XY(1-X-Y)(1-Z-W)(X+Y+Z+W-1)}}$$

PMNS fit

$$X \in [0.64-0.71] \quad Y \in [0.26-0.34] \quad Z \in [0.05-0.26] \quad W \in [0.21-0.48]$$

K. Kowalska, S. Pramanick, EMS '22



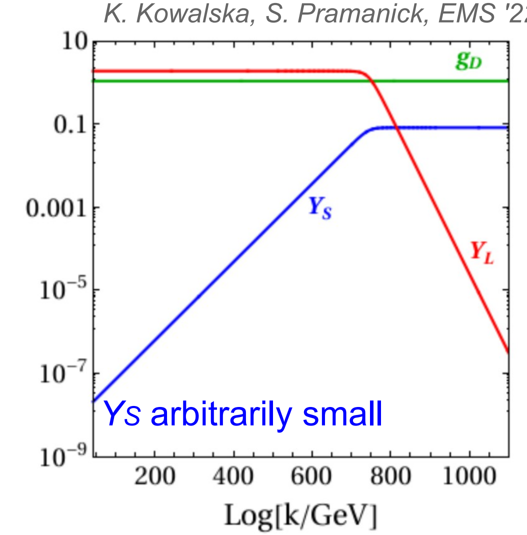
3. The mechanism is more generic than SM

e.g. dark gauge coupling g_D + Yukawa interactions

$$\mathcal{L} \supset Y_S\,\chi_R \Phi \chi_L + Y_L\,\psi_R \Phi \psi_L + \mathrm{H.c.}$$

$$Q_\psi \gg Q_\chi$$ (dark abelian charge)

K. Kowalska, S. Pramanick, EMS '22



Can use it to justify freeze-in, feebly interacting models, etc...

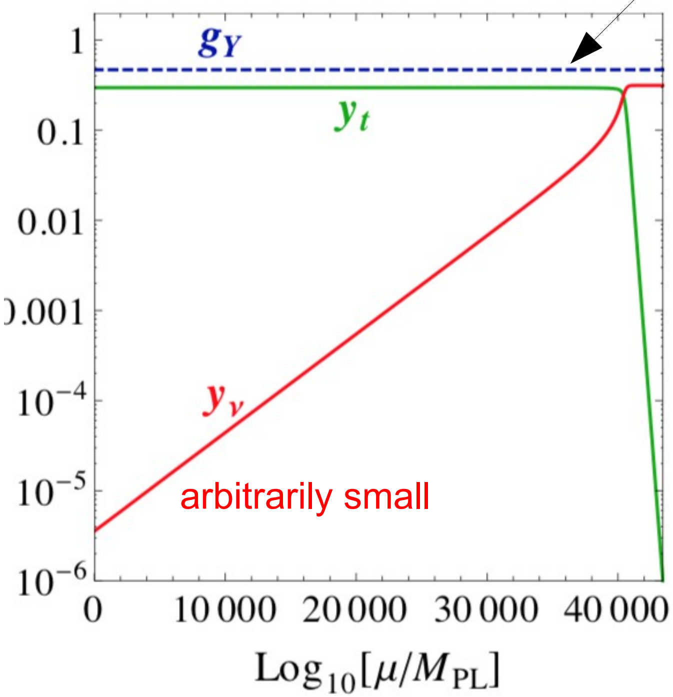
Connections to quantum gravity

In SMRHN + Gravity

f_g value linked to hypercharge gauge coupling

$$16\pi^2\theta_\nu \approx -\frac{2}{3}g_Y^{*2} + \frac{3}{2}y_t^{*2} < 0$$

trans-Planckian running



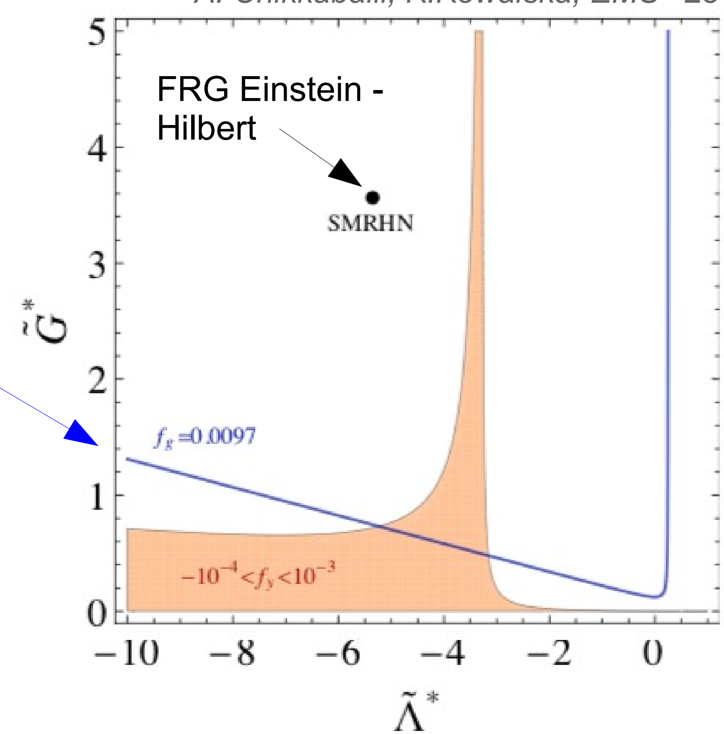
$$f_g \approx 0.0097$$

to match SM value

... It is a line in space of gravity fixed points

$$f_g = \tilde{G}^* \frac{1-4\tilde{\Lambda}^*}{4\pi\left(1-2\tilde{\Lambda}^*\right)^2}$$

A. Chikkaballi, K.Kowalska, EMS '23



Quantum gravity calculation should eventually match the blue line (difficult)

FRG calculation following

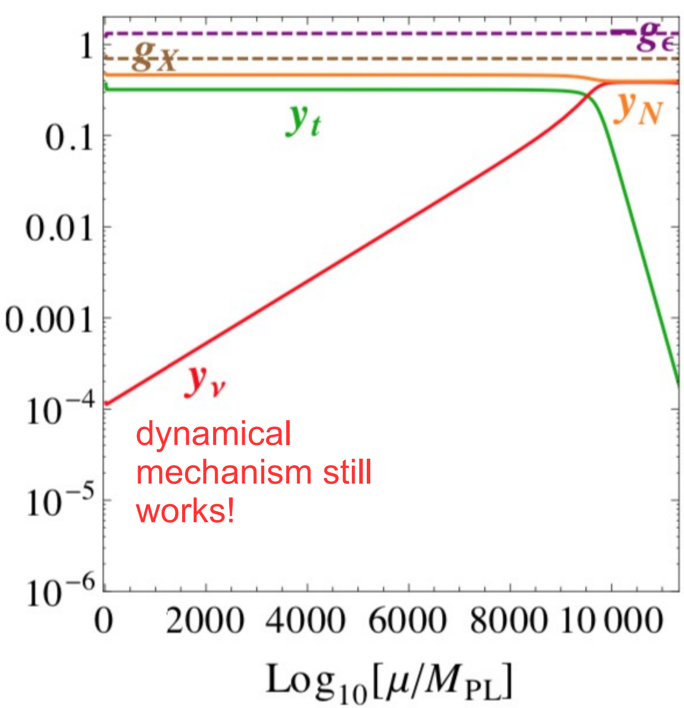
A. Eichhorn, F.Versteegen, 1709.07252

Connections to quantum gravity

In U(1)$_{B-L}$ + Gravity

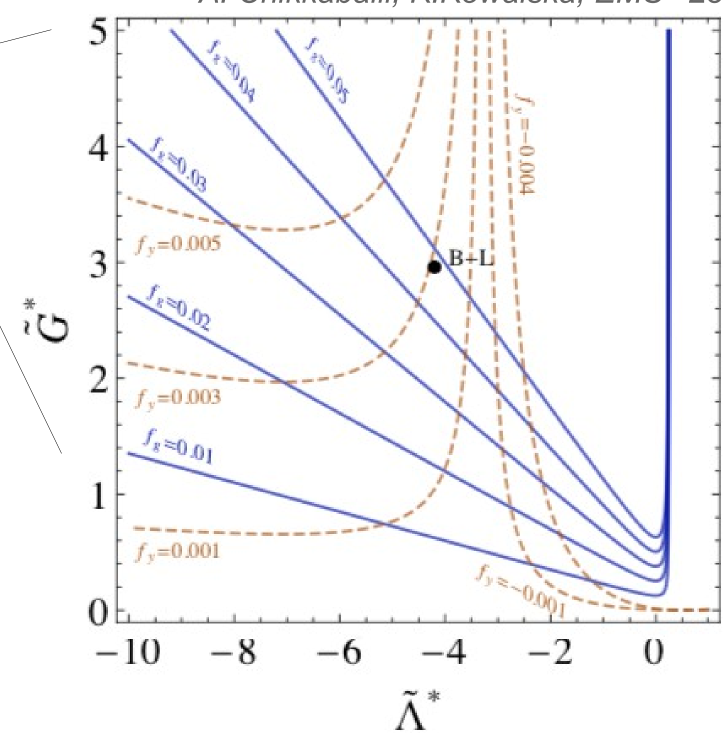
g_Y is *relevant* (free) … f_g value linked to $g_X = \frac{g_{B-L}}{\sqrt{1-\epsilon^2}}\,, \quad g_\epsilon = -\frac{\epsilon\, g_Y}{\sqrt{1-\epsilon^2}}$

trans-Planckian running



$f_g =$ any

A. Chikkaballi, K.Kowalska, EMS '23



Quantum gravity calculation provides **predictions** for g_X, g_ε, and new Yukawa

FRG calculation following

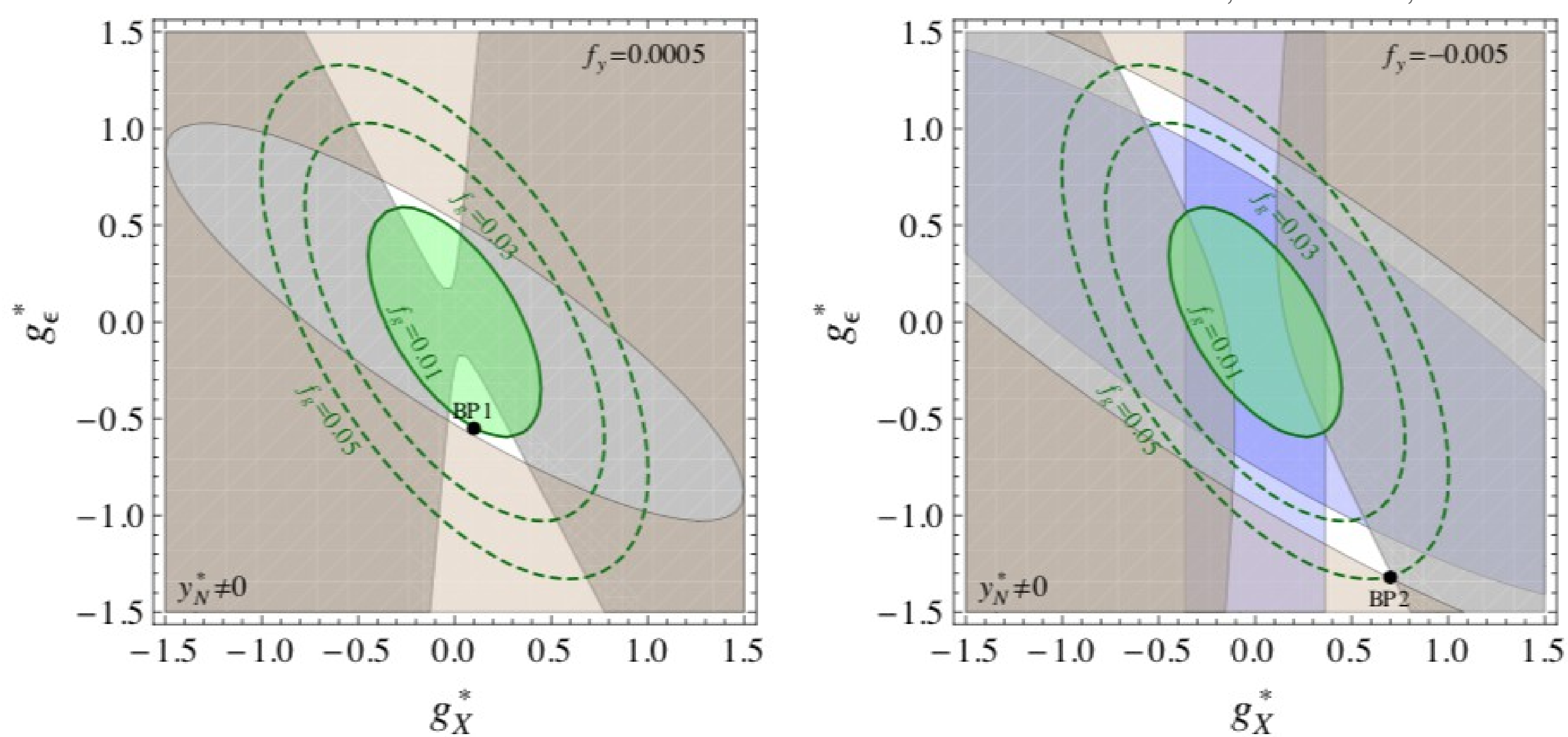
A. Eichhorn, F.Versteegen, 1709.07252

Predictions *B-L*

Conditions on the choice of benchmark points

1. Feasible dynamical mechanism: $y_\nu^* = 0$ irrelevant (gray)
2. $g_Y^* = 0$ relevant (green)
3. Matching top quark mass (brown)

A. Chikkaballi, K.Kowalska, EMS '23



Predictions *B-L*

- New gauge sector ***gX, gε*** (I)
- New Yukawa coupling ***yN*** $\mathcal{L}_M = -y_N^{ij} S\, \nu_{R,i}\, \nu_{R,j} + \text{H.c.}$ (Majorana mass term) (I)
- New scalar vev ***vS*** *breaking* U(1)*B-L* (R)

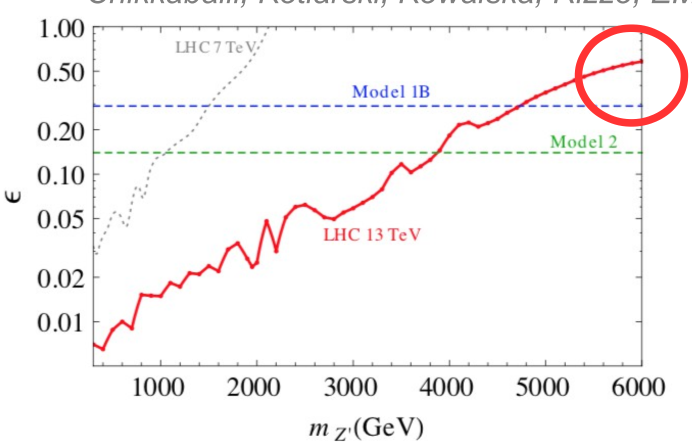
Different *fg, fy* lead to **predictive fixed points** for *gX, gε, yN* :

	f_g	f_y	g_X^*	g_ϵ^*	y_N^*	g_X $(10^{5,7,9}$ GeV)	g_ϵ $(10^{5,7,9}$ GeV)	y_N $(10^{5,7,9}$ GeV)
BP1	0.01	0.0005	0.10	−0.55	0.12	0.29, 0.29, 0.30	−0.26, −0.27, −0.28	0.16, 0.16, 0.16
BP2	0.05	−0.005	0.70	−1.32	0.47	0.40, 0.41, 0.44	−0.52, −0.56, −0.61	0.42, 0.44, 0.45
BP3	0.02	−0.0015	0.10	−0.75	0.0	0.12, 0.12, 0.12	−0.33, −0.35, −0.37	0.0
BP4	0.03	−0.004	0.10	0.75	0.0	0.09, 0.09, 0.09	0.23, 0.25, 0.28	0.0

Majorana

Dirac

Chikkaballi, Kotlarski, Kowalska, Rizzo, EMS '22



large kinetic mixing implies *vS* >> *vH* :

Predictions *B-L*

Possible stochastic GWs from FOPTs?

Coleman-Weinberg $$V_{\rm CW} = \frac{1}{4}\lambda_S \phi^4 + \frac{1}{128\pi^2}\left(20\lambda_S^2 + 96 g_X^4 - 48 y_N^4\right)\phi^4 \left(-\frac{25}{6} + \ln\frac{\phi^2}{k^2}\right)$$

Thermal corrections $$V(\phi,T) = V_0(\phi) + V_{1-\rm loop}(\phi) + V_{\rm thermal}(\phi,T) + V_{\rm daisy}(\phi,T)$$

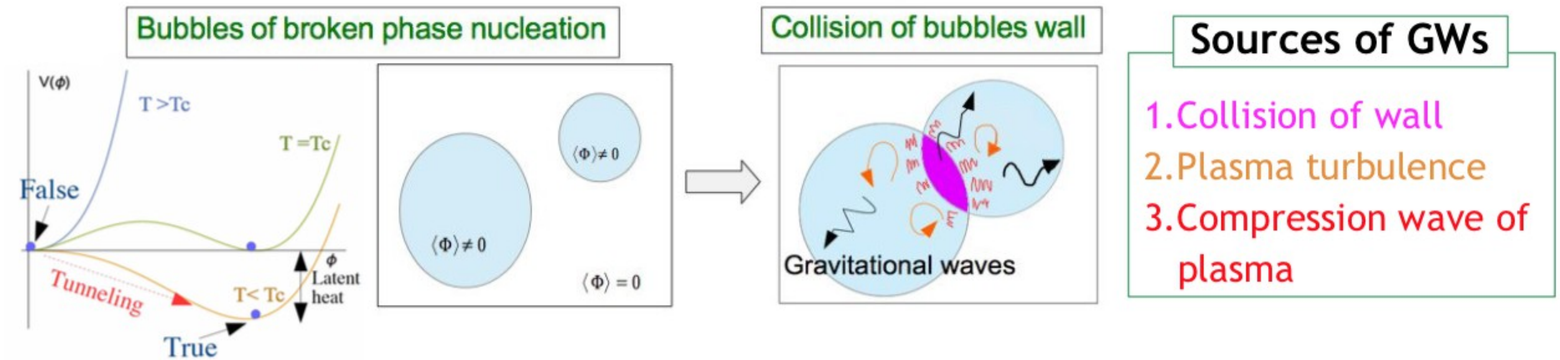
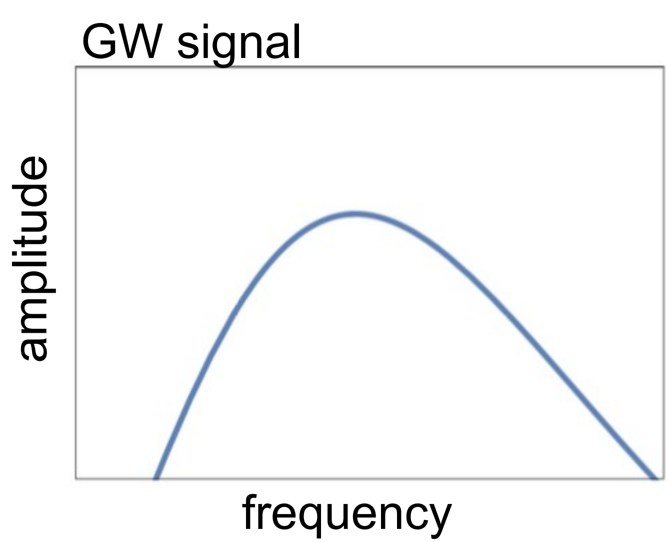


illustration
Hashino, Toyama 2018

Latent heat α

Nucleation speed β/H*



← < ϕ vev > →

Linde '81, '83, Turner *et al.* '92-'94; Guth *et al.* '80, '81, Caprini *et al.* '07-'09, Hindmarsh *et al.* '13-'17, Ellis *et al.* '18 … many more

Predictions *B-L*

Our predictions have strong discriminating features... may show up in GW amplitude!

	f_g	f_y	g_X^*	g_ϵ^*	y_N^*	g_X ($10^{5,7,9}$ GeV)	g_ϵ ($10^{5,7,9}$ GeV)	y_N ($10^{5,7,9}$ GeV)
BP1	0.01	0.0005	0.10	−0.55	0.12	0.29, 0.29, 0.30	−0.26, −0.27, −0.28	0.16, 0.16, 0.16
BP2	0.05	−0.005	0.70	−1.32	0.47	0.40, 0.41, 0.44	−0.52, −0.56, −0.61	0.42, 0.44, 0.45
BP3	0.02	−0.0015	0.10	−0.75	0.0	0.12, 0.12, 0.12	−0.33, −0.35, −0.37	0.0
BP4	0.03	−0.004	0.10	0.75	0.0	0.09, 0.09, 0.09	0.23, 0.25, 0.28	0.0

Majorana

Dirac

Well known signals if C-W is "conformal" ...

$$V_{\rm CW} = \cancel{\frac{1}{2}m_S^2\phi^2} + \frac{1}{4}\lambda_S\phi^4 + \frac{1}{128\pi^2}\left(20\lambda_S^2 + 96g_X^4 - 48y_N^4\right)\phi^4\left(-\frac{25}{6} + \ln\frac{\phi^2}{k^2}\right)$$

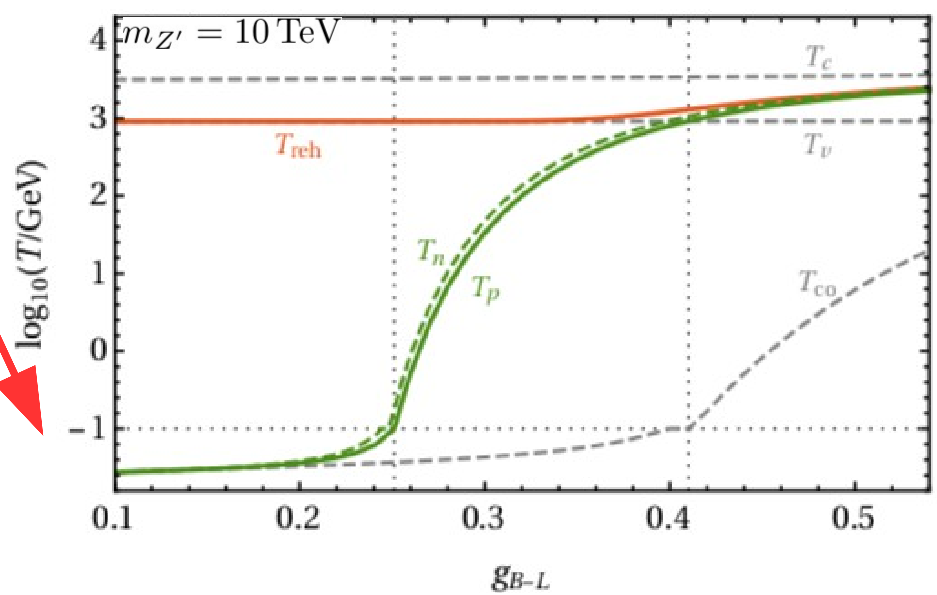
(gives nucleation rate, T_n, T_p, α, β/H* ...)

… but supercooling is too efficient for us

NO GW SIGNAL HERE! ☹

nucleation/percolation temp. **below** QCD
+
FOPT stop condition not satisfied

C.Marzo, L.Marzola, V.Vaskonen, '18



Classical scale invariance vs. asymptotic safety

$$\frac{d\tilde{m}_S^2}{dt} \approx (-2 - f_\lambda)\,\tilde{m}_S^2$$

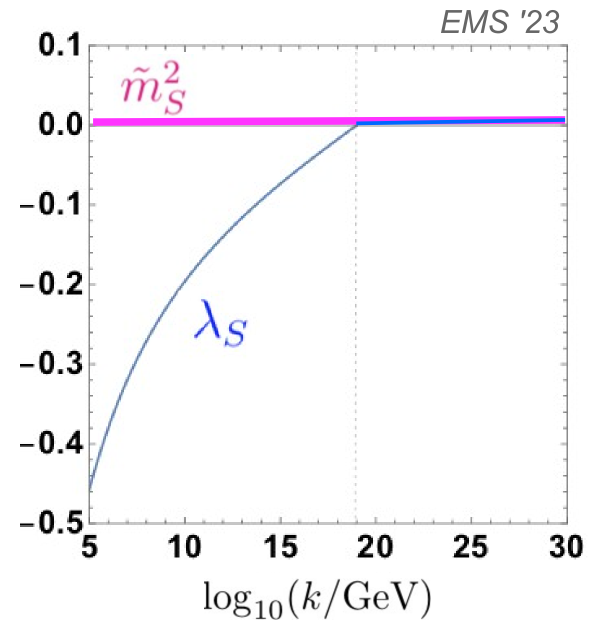
$$\frac{d\lambda_S}{dt} \approx -f_\lambda \lambda_S + \frac{6 g_X^{*4}}{\pi^2} + \ldots$$

$f_\lambda \ll -2$

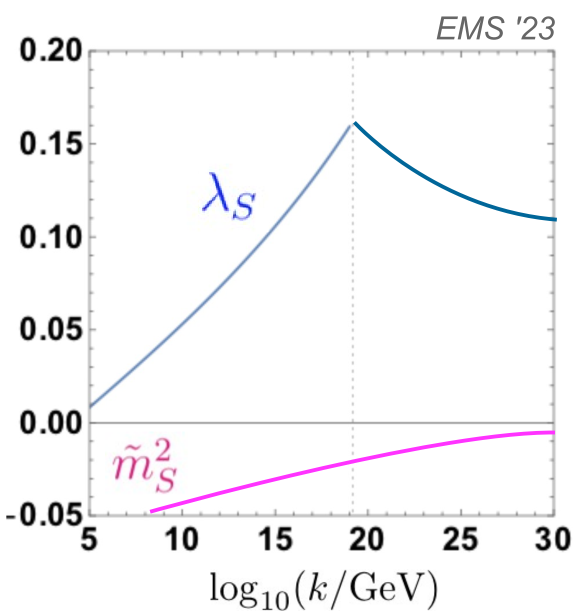
$\tilde{m}_S^{2*} = 0$ irrelevant

implies predictive $\lambda_S(t)$

potential destabilized!



viceversa...



$f_\lambda > 0$

$\lambda_S(t)$ consistent with C-W

implies $\tilde{m}_S^{2*} = 0$ relevant

tree-level mass is allowed

hence

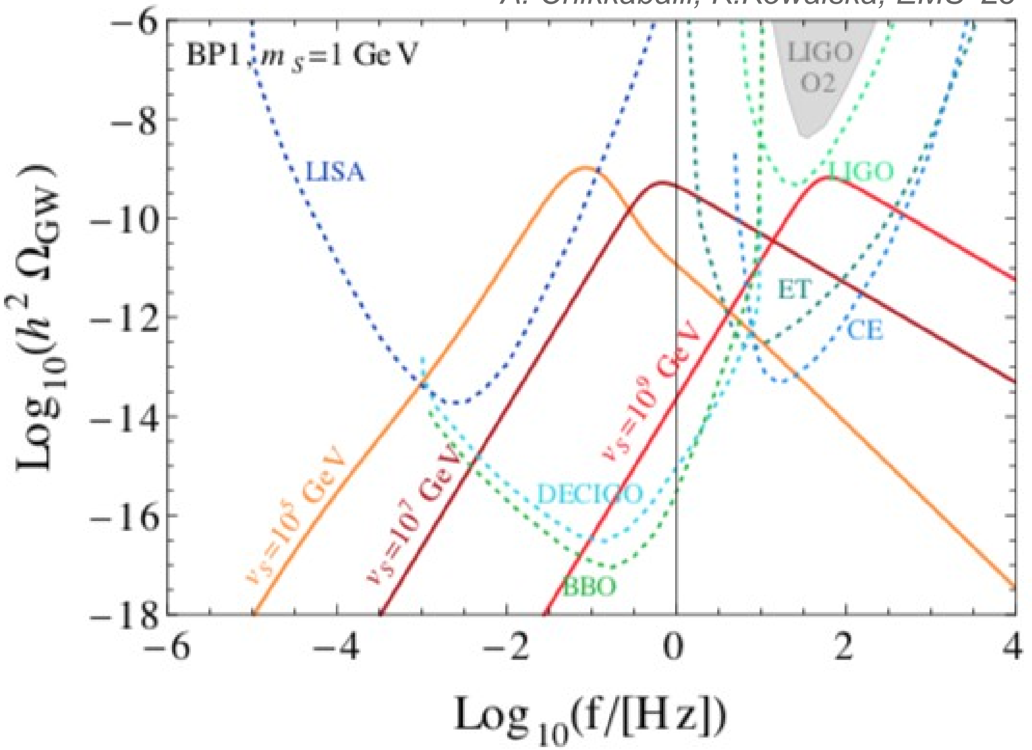
No conformal potential

Gravitational waves

$m_S^2 < 0$ $\rightarrow$ reduced supercooling, lower barrier

We have observable GW signals!

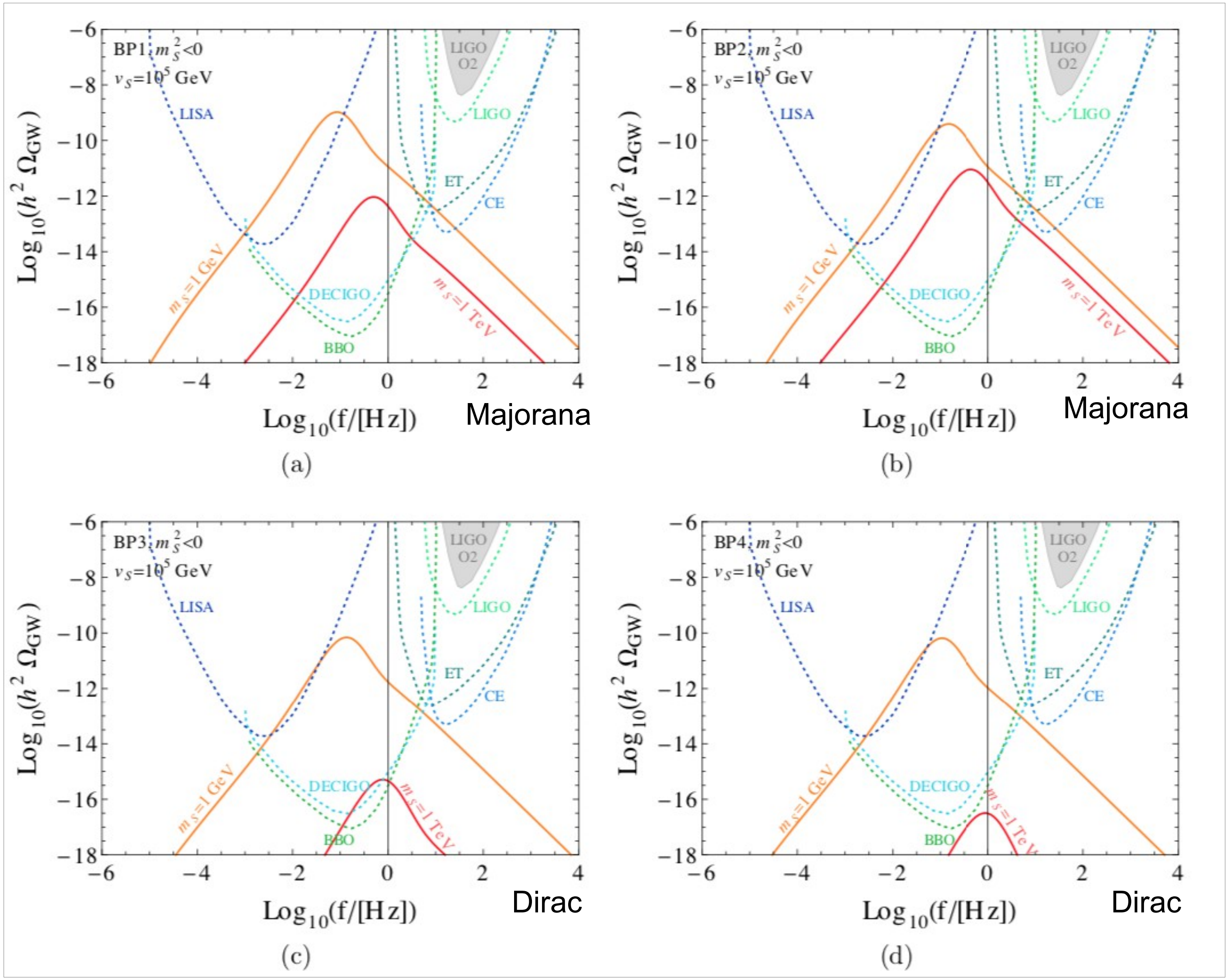
A. Chikkaballi, K.Kowalska, EMS '23



Gravitational waves

Signal with mass is interesting ...

A. Chikkaballi, K.Kowalska, EMS '23



… but discriminating features are washed out

To take home...

AS was used to make the neutrino (or other) Yukawa coupling **arbitrarily small dynamically**

Mechanism relies on an **irrelevant Gaussian fixed point** of the trans-Planckian RG flow of Yukawa coupling

In the SM + QG **some tension** between the FRG results and phenomenology, but perhaps not so in gauged *B-L*

Gravitational wave signatures from FOPTs

Majorana/Dirac discrimination via gravitational waves from FOPTs **not possible** in this scenario

Backup

Asymptotically safe gravity

Quantum gravity might feature interactive UV fixed points (functional renormalization group)

Reuter '96, Reuter, Saueressig '01, Litim '04, Codello, Percacci, Rahmede '06, Benedetti, Machado, Saueressig '09, Narain, Percacci '09, Manrique, Rechenberger, Saueressig '11, Falls, Litim, Nikolakopoulos '13, Dona', Eichhorn, Percacci '13, Daum, Harst, Reuter '09, Folkerst, Litim, Pawlowski '11, Harst, Reuter '11, Zanusso *et al.* '09 … many more

EAA *e.g.* Einstein-Hilbert action

$$\Gamma_k = \frac{1}{16\pi G}\int d^4x\sqrt{g}\,[-R(g)+2\Lambda]$$

FRG (Wetterich equation)

$$\partial_t\Gamma_k = k\,\partial_k\Gamma_k = \frac{1}{2}\mathrm{Tr}\left(\frac{1}{\Gamma_k^{(2)}+\mathcal{R}_k}\partial_t\mathcal{R}_k\right)$$

Beta functions of grav. couplings

$\tilde{G} = G(k)k^2$

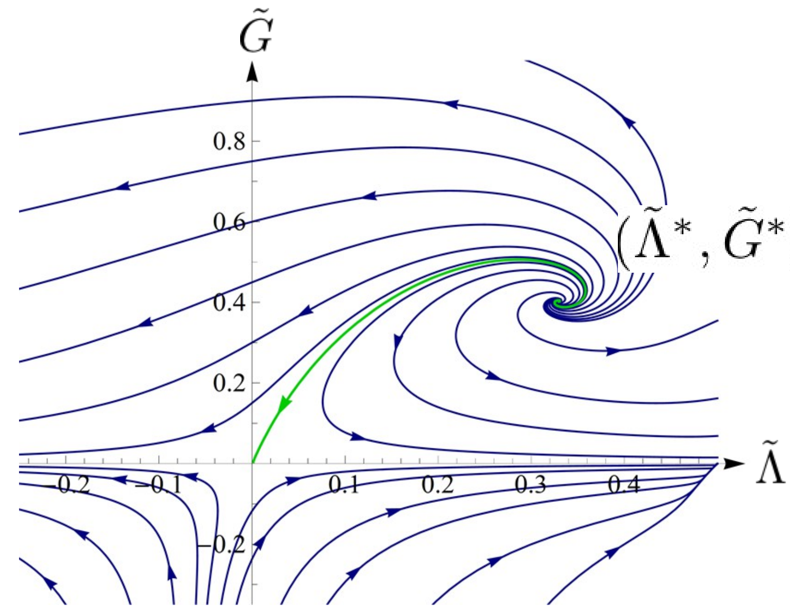
$\tilde{\Lambda} = \Lambda(k)k^{-2}$

$t = \ln k$

$$\frac{d\tilde{G}}{dt} = \left[2+\tilde{G}\,\eta_1(\tilde{G},\tilde{\Lambda})\right]\tilde{G} \quad = 0$$

$$\frac{d\tilde{\Lambda}}{dt} = -2\tilde{\Lambda}+\tilde{G}\,\eta_2(\tilde{G},\tilde{\Lambda}) \quad = 0$$

Reuter, Saueressig, hep-th/0110054



2 relevant fixed points

... fixed points persist under the addition of gravity and matter interactions

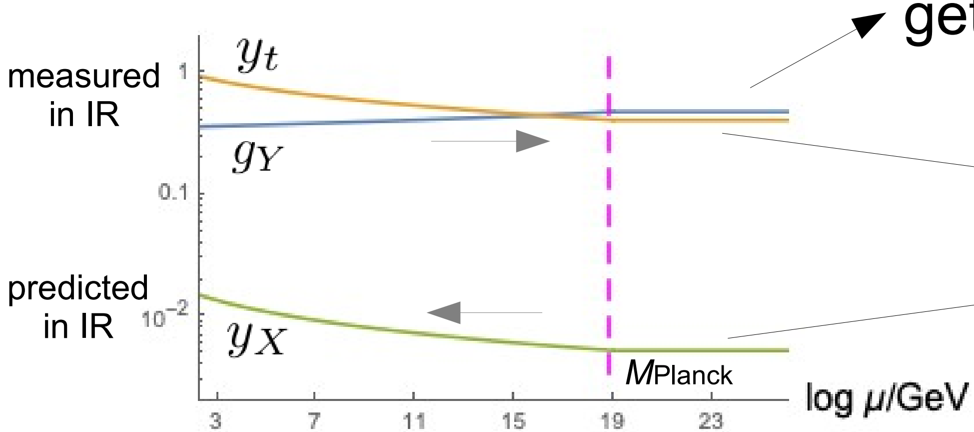
Predictions from trans-Planckian AS

- **FRG calculation of f_g, f_y has very large uncertainties...**
  (truncation in number of operators, cut-off scheme dependence, etc.)
  Lauscher, Reuter '02, Codello, Percacci, Rahmede '07-'08, Benedetti, Machado, Saueressig '09, Narain, Percacci '09, Dona', Eichhorn, Percacci '13, Falls, Litim, Schroeder '18 ...

- **FRG calculation is *<u>not required</u>* to get predictions...**
  Wetterich, Shaposhnikov '09, Eichhorn, Held '18, Reichert, Smirnov '19; Alkofer *et al.* '20, Kowalska, EMS, Yamamoto '20, Kowalska, EMS '21, Chikkaballi, Kotlarski, Kowalska, Rizzo, EMS '22 ...

… as the set of *irrelevant* couplings is overconstrained:

3 (or more) eqs (g_Y, y_t, y_x, ...)
2 unknowns (f_g, f_y)



get fg

get f_y

Predict y_x
arbitrary gauge or Yukawa coupling (in SM or BSM)

AS leads to testable signatures ...

e.g. in flavor anomalies: *Kowalska, EMS, Yamamoto,* Eur.Phys.J.C 81 (2021) 4, 272
Chikkaballi, Kotlarski, Kowalska, Rizzo, EMS, JHEP 01 (2023) 164

in g-2 and DM: *Kowalska, EMS,* Phys. Rev. D 103, 115032 (2021)

… and neutrinos!
(this talk)

Gravitational wave signal

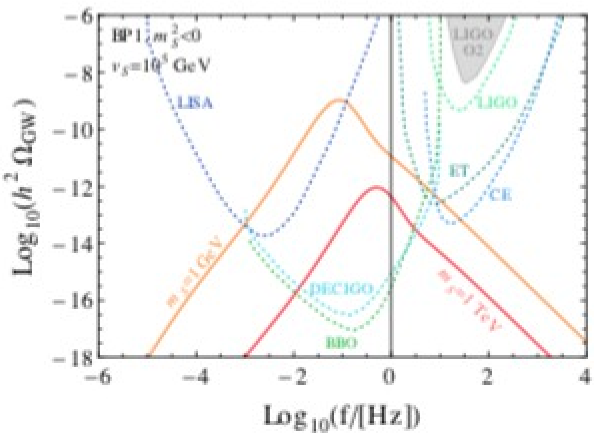
$$\alpha = \frac{\Delta V(T) - T\frac{\partial \Delta V(T)}{\partial T}}{\rho_R(T)}\bigg|_{T_p} \qquad \frac{\beta}{H_*} = T_p \frac{\mathrm{d}(S_3/T)}{\mathrm{d}T}\bigg|_{T_p} \qquad T_{\rm rh} = T_p[1+\alpha(T_p)]^{1/4}$$

$$h^2\Omega^{\rm peak}_{\rm coll} = 1.67\times 10^{-5}\,\kappa^2_{\rm coll}\left(\frac{\alpha}{1+\alpha}\right)^2\left(\frac{v_w}{\beta/H_*}\right)^2\left(\frac{100}{g_*}\right)^{1/3}\left(\frac{0.11 v_w}{0.42+v_w^2}\right)$$
$$h^2\Omega^{\rm peak}_{\rm sw} = 2.65\times 10^{-6}\,\kappa^2_{\rm sw}\left(\frac{\alpha}{1+\alpha}\right)^2\left(\frac{v_w}{\beta/H_*}\right)\left(\frac{100}{g_*}\right)^{1/3}$$
$$h^2\Omega^{\rm peak}_{\rm turb} = 3.35\times 10^{-4}\,\kappa^{3/2}_{\rm turb}\left(\frac{\alpha}{1+\alpha}\right)^{3/2}\left(\frac{v_w}{\beta/H_*}\right)\left(\frac{100}{g_*}\right)^{1/3},$$

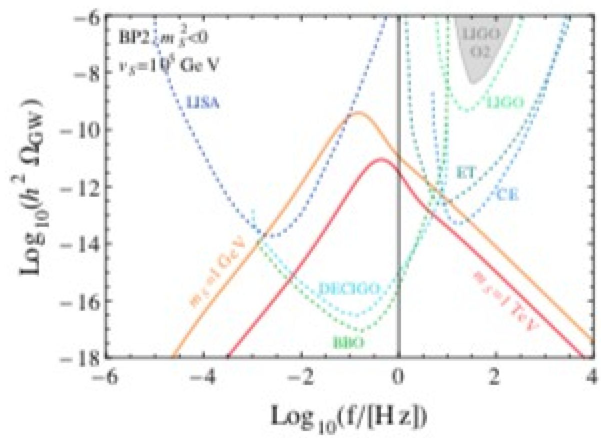
$$f^{\rm peak}_{\rm coll} = 1.65\times 10^{-5}\,{\rm Hz}\left(\frac{v_w}{\beta/H_*}\right)^{-1}\left(\frac{100}{g_*}\right)^{-1/6}\left(\frac{T_*}{100\ {\rm GeV}}\right)\left(\frac{0.62 v_w}{1.81-0.1v_w+v_w^2}\right)$$
$$f^{\rm peak}_{\rm sw} = 1.90\times 10^{-5}\,{\rm Hz}\left(\frac{v_w}{\beta/H_*}\right)^{-1}\left(\frac{100}{g_*}\right)^{-1/6}\left(\frac{T_*}{100\ {\rm GeV}}\right)$$
$$f^{\rm peak}_{\rm turb} = 2.70\times 10^{-5}\,{\rm Hz}\left(\frac{v_w}{\beta/H_*}\right)^{-1}\left(\frac{100}{g_*}\right)^{-1/6}\left(\frac{T_*}{100\ {\rm GeV}}\right). \qquad \text{(C.13)}$$

Details of BP1 and BP2

BP1



BP2



$m_S = 1\ GeV : \alpha = 10^{10}\,,\ \beta = 49.8$

$T_p = 14.6\ GeV$

$m_S = 1\ GeV : \alpha = 10^{11}\,,\ \beta = 78.9$

$T_p = 8\ GeV$

$m_S = 1\ TeV : \alpha = 0.27\,,\ \beta = 185$

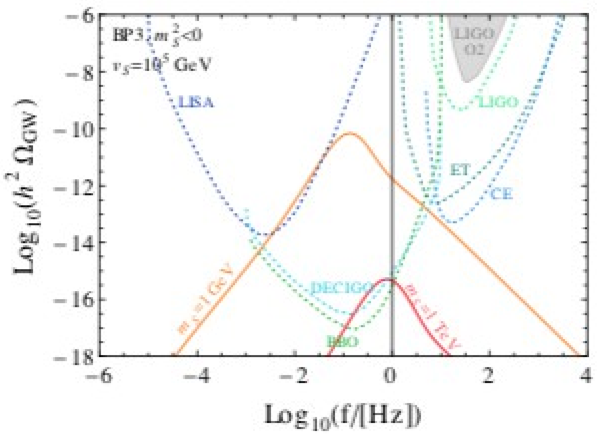
T_p ~ 10 TeV

$m_S = 1\ TeV : \alpha = 0.88\,,\ \beta = 187$

T_p ~ 10 TeV

Details of BP3 and BP4

BP3



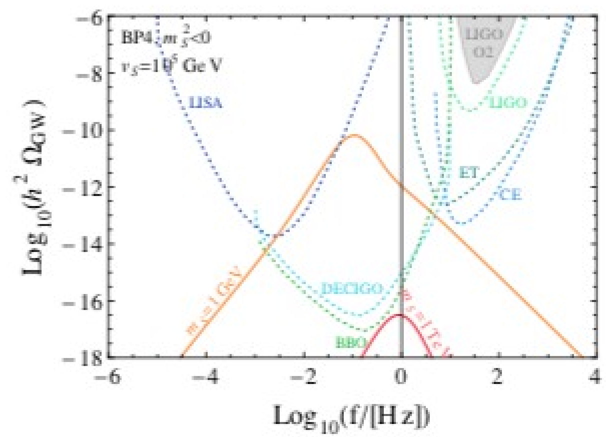
$m_S = 1\ GeV : \alpha = 10^9\ ,\ \beta = 189$

$T_p = 10.04\ GeV$

$m_S = 1\ TeV : \alpha = 0.02\ ,\ \beta = 227$

T_p ~ 10 TeV

BP4



$m_S = 1\ GeV : \alpha = 10^8\ ,\ \beta = 201$

$T_p = 11.5\ GeV$

$m_S = 1\ TeV : \alpha = 0.01\ ,\ \beta = 229$

$T_p =$ ~ 10 TeV