

Phenomenology with trans-Planckian asymptotic safety

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in collaboration with

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E. M. Sessolo, Y. Yamamoto

Eur.Phys.J.C 81 (2021) 4, 272 (arXiv: 2007.03567)
Phys. Rev. D 103, 115032 (2021) (arXiv: 2012.15200)
JHEP 01 (2023) 164 (arXiv: 2209.07971)
Eur.Phys.J.C 83 (2023) 7, 644 (arXiv: 2304.08959)

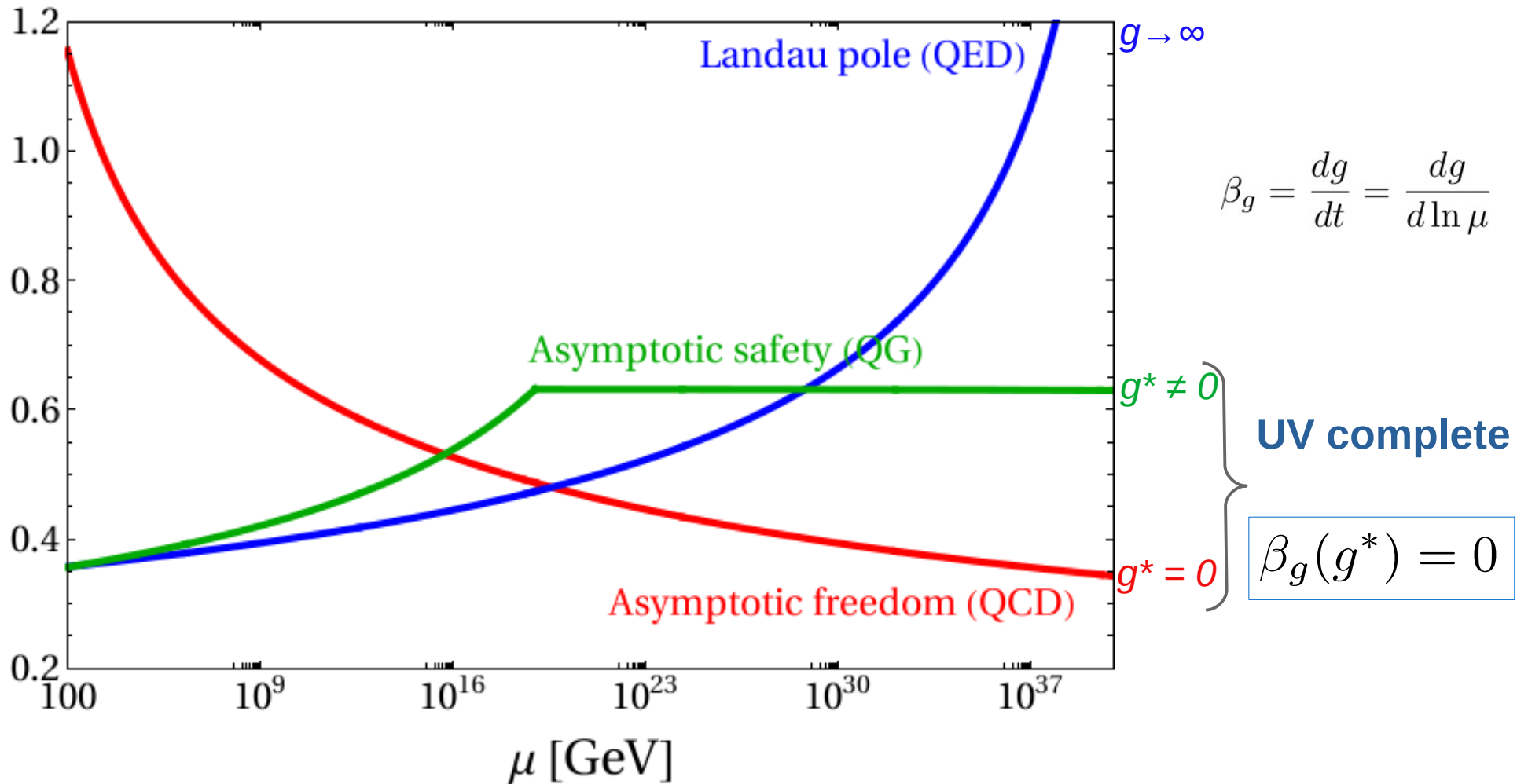
Asymptotic safety in Quantum Field Theory: Grand Unification
Lyon, 5.06.2024

Outline

- **Asymptotic safety (AS)** – what is it all about?
- Evidence for a **trans-Planckian fixed point**
- **Phenomenological predictions** from trans-Planckian AS
- **Uncertainties** of the predictions
- **Conclusions**

Trans-Planckian asymptotic safety

Asymptotic safety

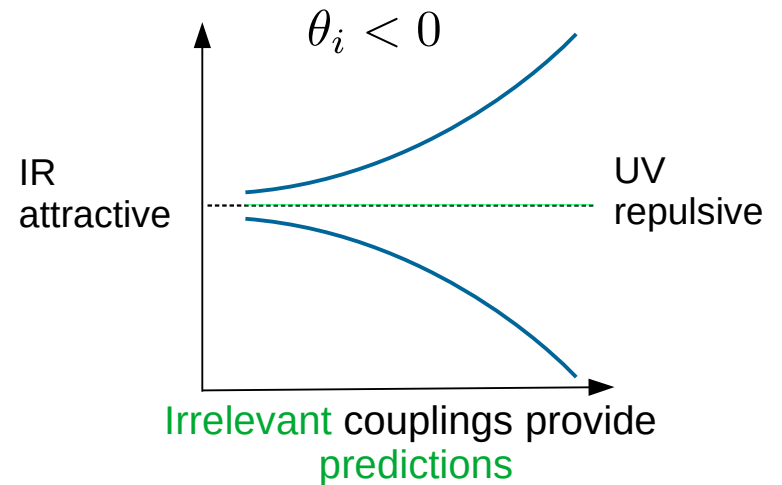
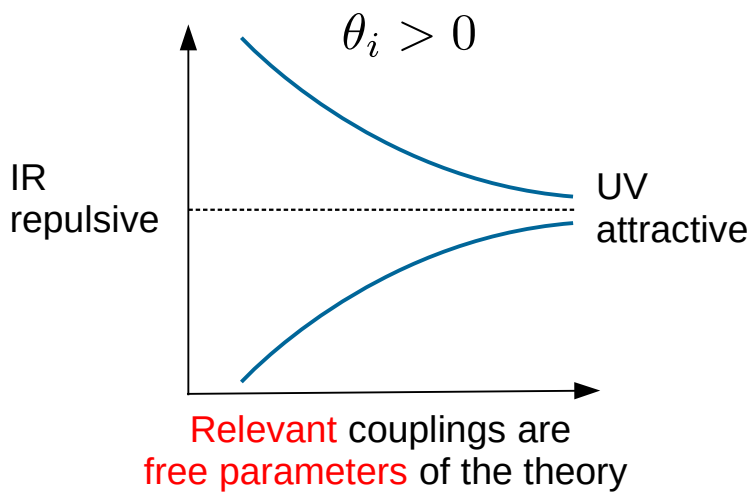


- AS originally proposed by Weinberg to improve the UV behavior of G_N
- Advocated in QFT as solution to $U(1)_Y$ triviality problem
- Allows non-perturbative renormalization

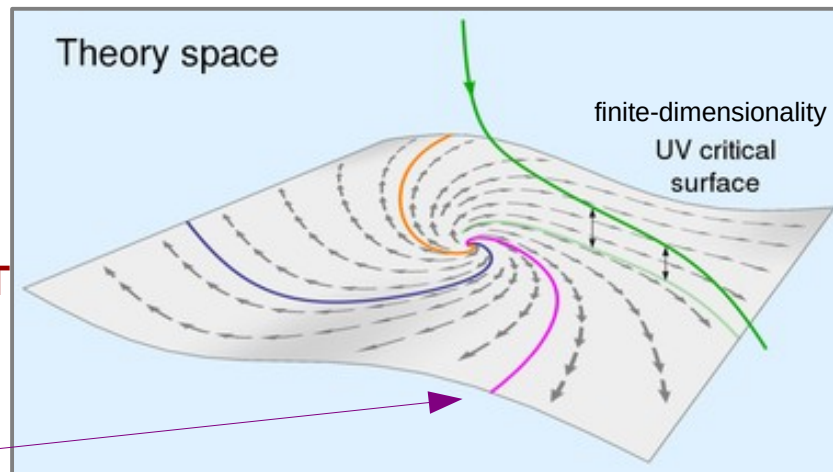
Fixed point and critical surface

$$\beta_i(\{\alpha_j^*\}) = 0 \longrightarrow M_{ij} = \left. \frac{\partial \beta_i}{\partial \alpha_j} \right|_{\{\alpha_i^*\}} \longrightarrow \{-\theta_i\} \text{ critical exponents}$$

stability matrix



span the UV critical surface
defines a fundamental QFT
 once determined by the experiment ...



from Wikipedia

can only deviate from the FP along the critical surface

... can be uniquely fixed
PREDICTIVITY

Wetterich flow equation

C.Wetterich, PLB 301, 90 (1993)

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \text{Tr} \left[\partial_t R_k \left(\Gamma_k^{(2)}[\phi] + R_k \right)^{-1} \right]$$

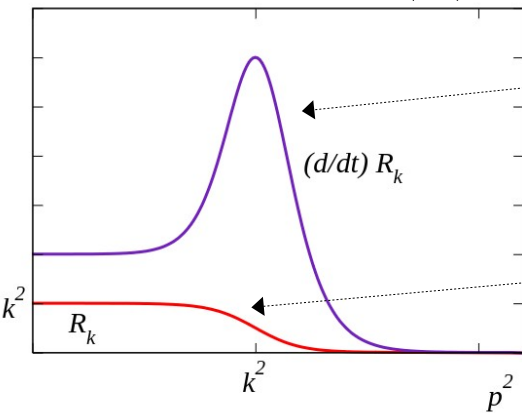
$$\partial_t = k \partial_k$$

an exact differential equation

$$\Gamma_k^{(2)}[\phi] = \frac{\delta^2 \Gamma_k[\phi]}{\delta \phi \delta \phi}$$

H. Gies, Lect.Notes Phys. 852 (2012) 287-348

Regulator function $R_k(p^2)$

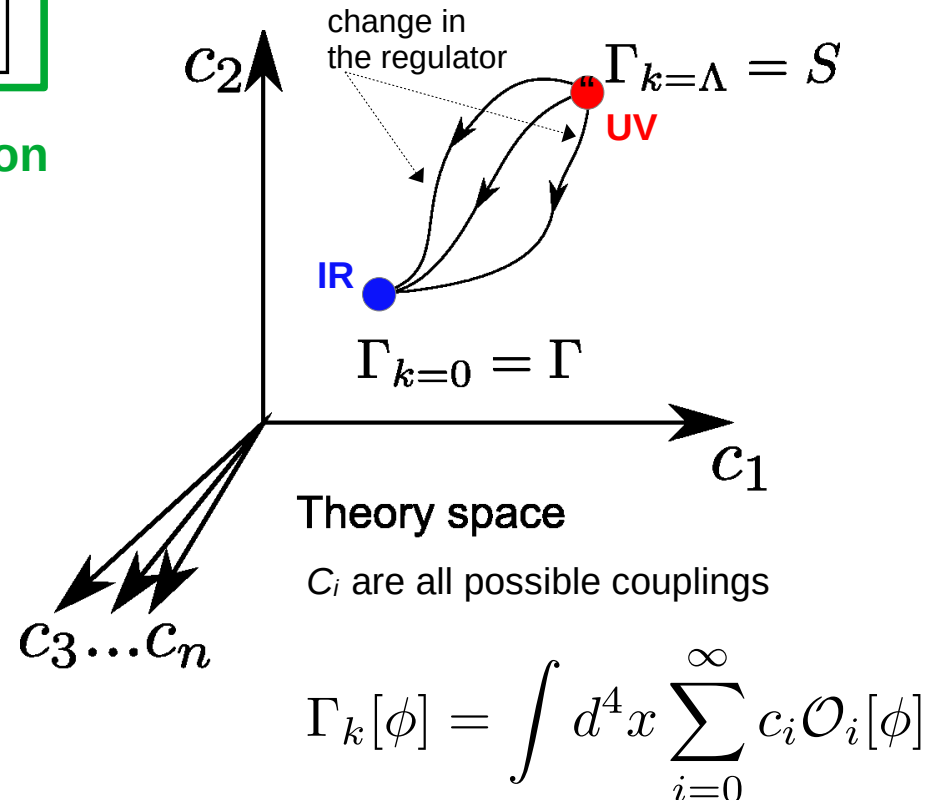


UV regularization for $p^2 \approx k^2$

IR regularization for $p^2 < k^2$

Wilsonian momentum-shell integration

H. Gies, Lect.Notes Phys. 852 (2012) 287-348



non-pert. RGEs $\partial_t \Gamma_k \rightarrow \partial_t c_i$

Asymptotic safety in quantum gravity

M. Reuter, PRD 57, 971 (1998)

Prototype example: **Einstein-Hilbert gravity**

$$S_{\text{EH}}[g_{\mu\nu}] = \frac{1}{16\pi G_N} \int d^4x \sqrt{g} (2\Lambda - R)$$

Dimensionless couplings:

$$g = G_N k^2 \quad \lambda = \Lambda k^{-2}$$

Beta functions for the gravity couplings:

$$k\partial_k g = [2 + \eta_g(g, \lambda)] g$$

$$k\partial_k \lambda = -2\lambda + g\eta_\lambda(g, \lambda)$$

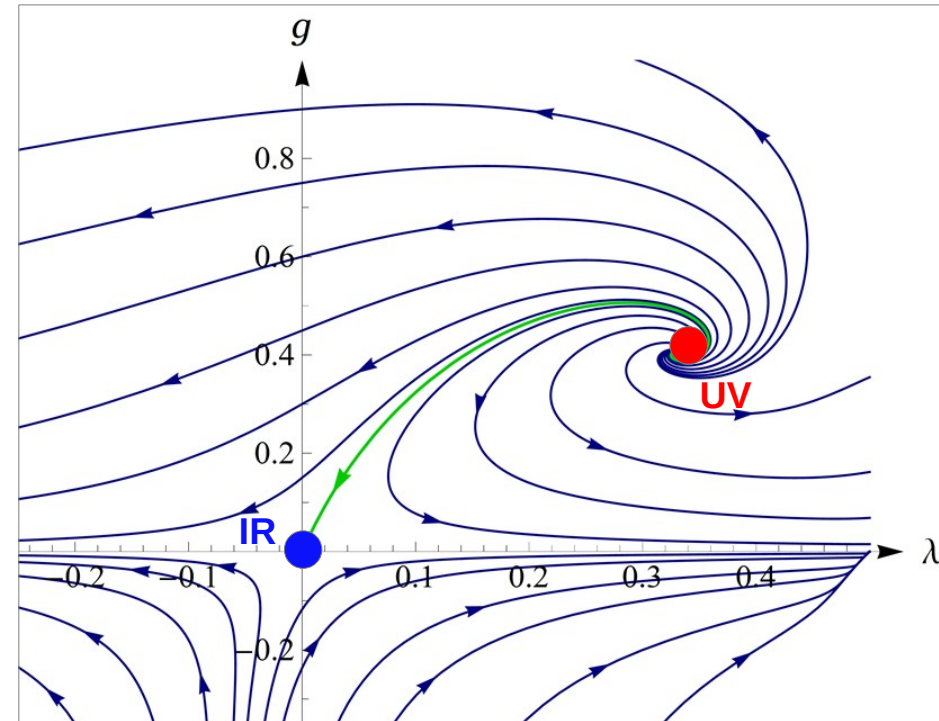
2 fixed points:

$$\text{Gaussian: } g^* = 0 \quad \lambda^* = 0$$

$$\text{Interactive: } g^* \neq 0 \quad \lambda^* \neq 0$$

Trans-Planckian fixed point

M. Reuter, F. Saueressig, PRD 65, 065016 (2002)



physical trajectory

$$G_N(\text{IR}) = 1.22 \times 10^{-22} \text{ GeV}^{-2}$$

$$G_N \Lambda(\text{IR}) = 2.89 \times 10^{-122}$$

Asymptotic safety in quantum gravity

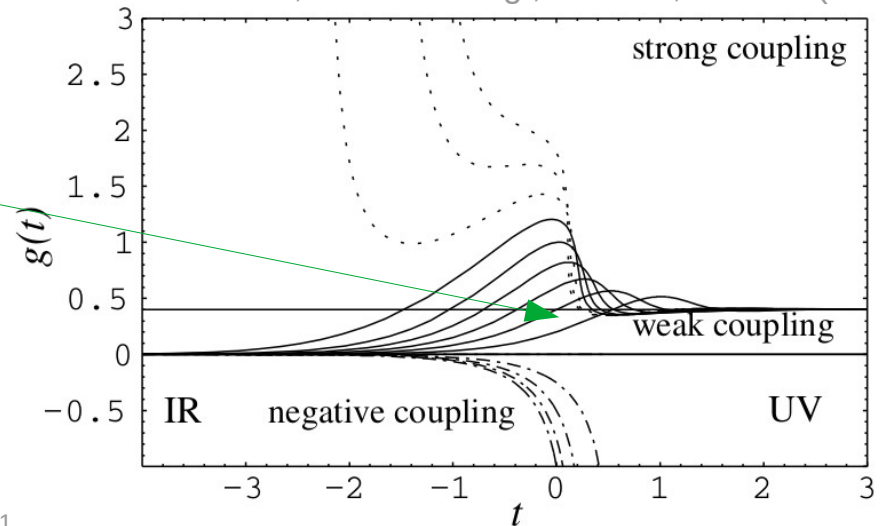
Planck scale generated dynamically at the cross-over

FP persists when adding new interactions

[Litim '04, Codello, Percacci, Rahmede '06, Benedetti, Machado, Saueressig '09, Narain, Percacci '09, Manrique, Rechenberger, Saueressig '11, Falls, Litim, Nikolakopoulos '13, Dona', Eichhorn, Percacci '13, Daum, Harst, Reuter '09, Folkerst, Litim, Pawłowski '11, Harst, Reuter '11, Christiansen, Eichhorn '17, Eichhorn, Versteegen '17, Zanusso *et al.* '09, Oda, Yamada '15, Eichhorn, Held, Pawłowski '16, Pawłowski *et al.* '18 ... many more]

also in Lorentzian QG Fehre, Litim, Pawłowski, Reichert '21

M. Reuter, F. Saueressig, PRD 65, 065016 (2002)



Gravity contributions to matter RGEs

$$\beta_g = \beta_g^{\text{SM+NP}} - g f_g$$

$$\beta_y = \beta_y^{\text{SM+NP}} - y f_y$$

$$\beta_\lambda = \beta_\lambda^{\text{SM+NP}} - \lambda f_\lambda$$

contributions are universal

from the FRG calculations...

EXAMPLE : U(1) + Φ + E-H:

A.Eichhorn, F.Versteegen
JHEP 01 (2018) 030

$$f_g = G \frac{1 - 4\Lambda}{4\pi(1 - 2\Lambda)^2}$$

... but large uncertainties

(truncation in number of operators, cut-off scheme dependence, higher-order loop corrections in matter, etc...)

[Lauscher, Reuter '02, Codello, Percacci, Rahmede '07-'08, Benedetti, Machado, Saueressig '09, Narain, Percacci '09, Dona', Eichhorn, Percacci '13, Falls, Litim, Schroeder '18, ...]

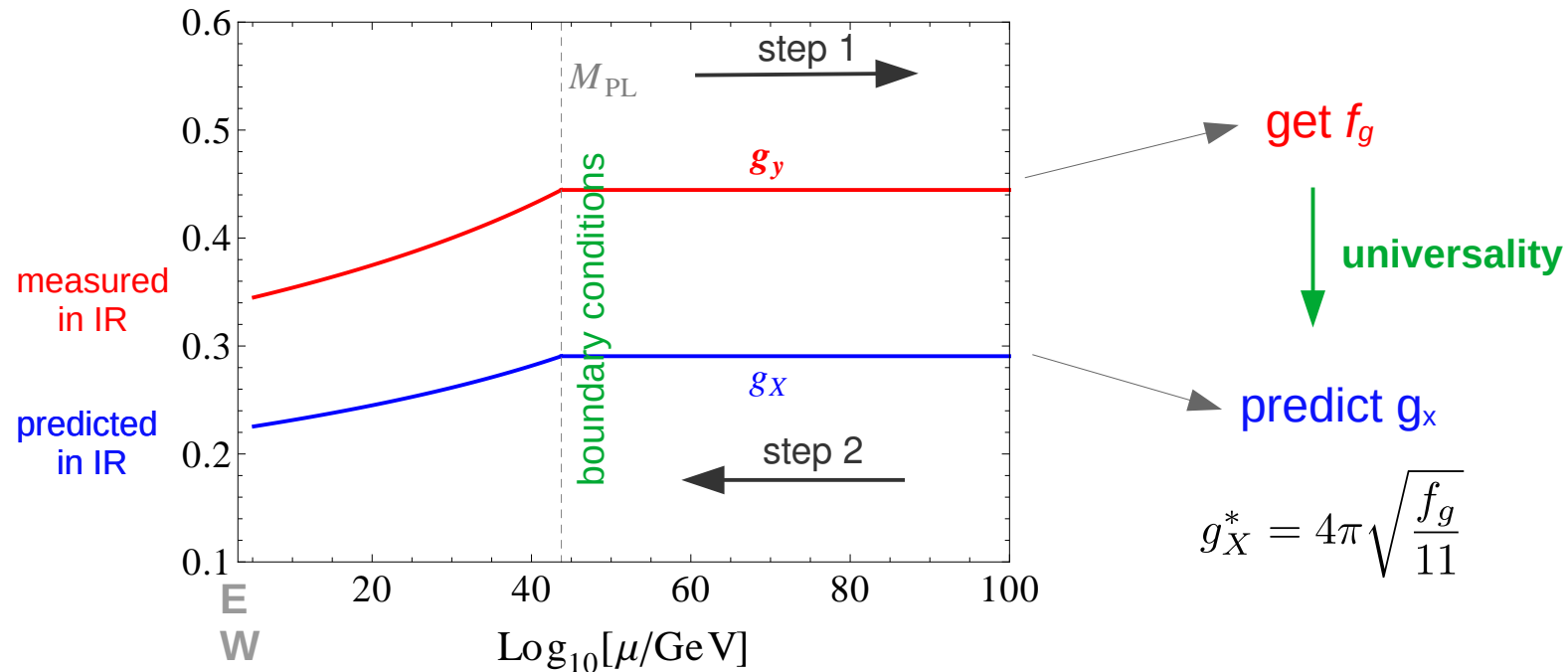
Interacting UV FP in the matter sector!

Predictions from trans-Planckian AS

FRG calculation is not required to get predictions...

Wetterich, Shaposhnikov '09, Eichhorn, Held '18, Reichert, Smirnov '19; Alkofer *et al.* '20, KK, Sessolo, Yamamoto '20, KK, Sessolo '21, Chikkaballi, Kotlarski, KK, Rizzo, Sessolo '22 ...

... as the set of *irrelevant* couplings is overconstrained: 3 (or more) eqs (g_Y, y_t, y_X, \dots)
2 unknowns (f_g, f_y)



Effective approach - assumptions: (to be discussed later)

- 1-loop matter RGEs
- Planck scale set at 10^{19} GeV
- Gravity parameters f are constant
- Gravity decouples instantaneously

Predictions from trans-Planckian AS

Particularly useful to constrain NP models

NP operator (eg. Weinberg operator)

$$\frac{C_{\text{NP}}}{\Lambda^n} \approx \frac{c_i c_j}{m_{\text{NP}}^n} \times \text{loop factor}$$

AS: irrelevant couplings c_i
fixed by the RG flow



Experimental anomaly

measurement of the operator



bounds on the NP mass derived from
the measurement of the operator

AS leads to specific and testable signatures

- eg. $b \rightarrow s$ anomalies: KK, Sessolo, Yamamoto, Eur.Phys.J.C 81 (2021) 4, 272
Chikkaballi, Kotlarski, KK, Rizzo, Sessolo, JHEP 01 (2023) 164
 $b \rightarrow c$ anomalies: KK, Sessolo, Yamamoto, Eur.Phys.J.C 81 (2021) 4, 272
g - 2: KK, Sessolo, Phys. Rev. D 103, 115032 (2021)

Predictions for NP - muon $g-2$

Predictions from AS: muon (g-2)

Measured value at BNL (2006):

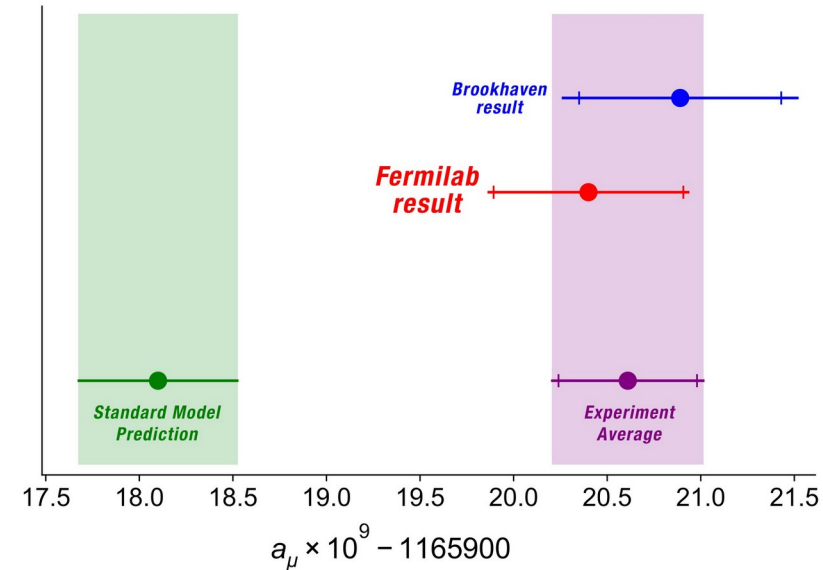
Bennet *et al*, Phys. Rev. D 73 (2006) 072003 (hep-ex/0602035)

$$a_{\mu}^{\text{BNL}} = (116592089 \pm 63) \times 10^{-11}$$

Measured value at FNAL (2021,2023):

Muon g-2 Collaboration, Phys. Rev. Lett. 126 (2021) 141801
D. P. Aguillard *et al.* (Muon g-2) (2023), arXiv:2308.06230

$$a_{\mu}^{\text{FNAL}} = (116592055 \pm 24) \times 10^{-11}$$



$$\Delta a_{\mu} = (24.9 \pm 4.8) \times 10^{-10}$$

discrepancy at $\sim 5.1 \sigma$

Calls for a NP explanation...

... although stay tuned for the lattice results

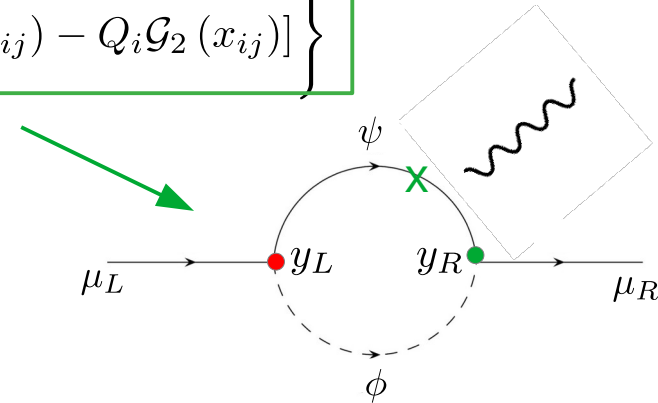
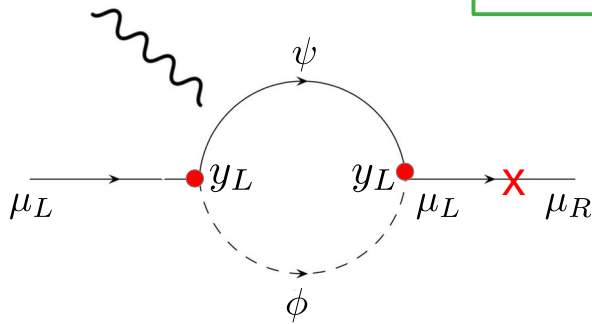
Predictions from AS: muon (g-2)

1-loop contribution from scalar(s) ϕ_i and VL fermions ψ_j

$$\delta(g-2)_\mu = m_\mu \frac{C_5^{\mu\mu\gamma}}{\Lambda} = \sum_{i,j} \left\{ -\frac{m_\mu^2}{16\pi^2 m_{\phi_i}^2} \left(|y_L^{ij\mu}|^2 + |y_R^{ij\mu}|^2 \right) [Q_j \mathcal{F}_1(x_{ij}) - Q_i \mathcal{G}_1(x_{ij})] \right.$$

$$x_{ij} = m_{\psi_j}^2 / m_{\phi_i}^2$$

$$\left. -\frac{m_\mu m_{\psi_j}}{16\pi^2 m_{\phi_i}^2} \text{Re} \left(y_L^{ij\mu} y_R^{ij\mu*} \right) [Q_j \mathcal{F}_2(x_{ij}) - Q_i \mathcal{G}_2(x_{ij})] \right\}$$



- minimal: 1 VL lepton and 1 scalar
- $m_\psi, m_\phi \sim \mathcal{O}(100 \text{ GeV})$
- Yukawa couplings > 1
- **excluded by the LHC**
- **Landau Pole**

see P. Athron et al., 2104.03691
for the most recent results

e.g. KK. E.Sessolo, 1707.00753

- 2 VL + 1 S or 1 VL + 2 S needed
- parametrically enhanced
- LHC bounds easily avoided...



... but PS largely unconstrained



Predictions from AS: muon (g-2)

KK, E.M.Sessolo (PRD '21, arXiv: 2012.15200)

minimal SM extension: two different VL leptons + extra scalar

extra assumption: a DM particle and a symmetry to stabilize it

$$\mathcal{L}_{\text{NP}} \supset (Y_R \mu_R E' S + Y_L F' S^\dagger l_\mu + Y_1 E h^\dagger F + Y_2 F' h E' + \text{H.c.})$$

Minimally coupled to QG above the Planck scale

$$\begin{aligned} \frac{dg_Y}{dt} &= \frac{g_Y^3}{16\pi^2} B_Y - \underline{f_g g_Y} \\ \frac{dy_t}{dt} &= \frac{1}{16\pi^2} \left[\frac{9}{2} y_t^2 + C_1 (Y_1^2 + Y_2^2) - \frac{17}{12} g_Y^2 - \frac{9}{4} g_2^2 - 8g_3^2 \right] y_t - \underline{f_y y_t} \\ \frac{dY_1}{dt} &= \frac{1}{16\pi^2} \left[3y_t^2 + C_3 Y_2^2 + \frac{5}{2} C_1 Y_1^2 + C_6 Y_L^2 + C_7 Y_R^2 - G_Y g_Y^2 - G_2 g_2^2 \right] Y_1 - f_y Y_1 \\ \frac{dY_2}{dt} &= \frac{1}{16\pi^2} \left\{ \left[3y_t^2 + \frac{5}{2} C_1 Y_2^2 + C_3 Y_1^2 + C_4 Y_L^2 + \frac{1}{2} Y_R^2 - G_Y g_Y^2 - G_2 g_2^2 \right] Y_2 + C_5 y_\mu Y_L Y_R \right\} - f_y Y_2 \\ \frac{dY_L}{dt} &= \frac{1}{16\pi^2} \left\{ \left[C_4 Y_2^2 + C_6 Y_1^2 + C_8 Y_L^2 + C_9 Y_R^2 + \frac{1}{2} y_\mu^2 - H_Y g_Y^2 - H_2 g_2^2 \right] Y_L + C_5 y_\mu Y_R Y_2 \right\} - f_y Y_L \\ \frac{dY_R}{dt} &= \frac{1}{16\pi^2} \left\{ \left[Y_2^2 + 2C_7 Y_1^2 + 2C_9 Y_L^2 + C_{10} Y_R^2 + y_\mu^2 - J_Y g_Y^2 - J_2 g_2^2 \right] Y_R + 2C_5 y_\mu Y_L Y_2 \right\} - f_y Y_R \end{aligned}$$

Predictions from AS: muon (g-2)

KK, E.M.Sessolo (PRD '21, arXiv: 2012.15200)

IR predictions



	$Y_L(Q_0)$	$Y_R(Q_0)$	$Y_1(Q_0)$	$Y_2(Q_0)$
M_1	0.21	0.91	0.62	9×10^{-4}
M_2	0.65	0.59	0.03	6×10^{-4}
M_3	0.01	0.77	0.18	3×10^{-5}
M_6	0.04	0.78	0.65	9×10^{-5}
M_{10}	0.98	0.87	0.03	1×10^{-3}

UV fixed point

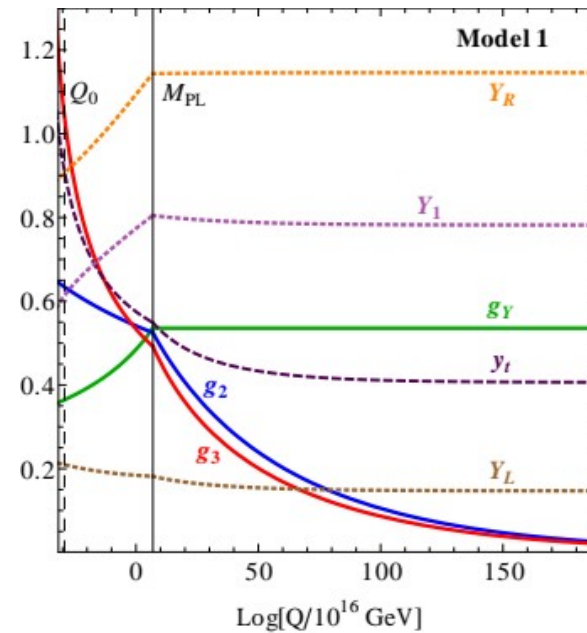
$$Y_R^* \neq 0$$

$$Y_1^* \neq 0$$

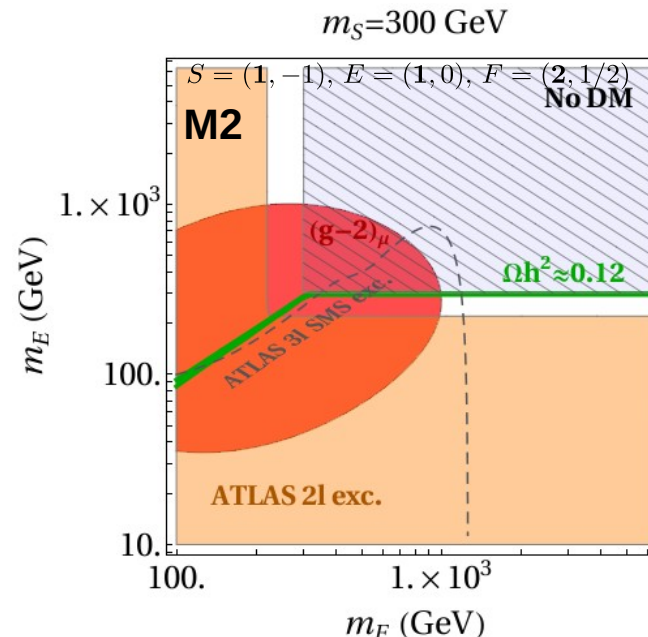
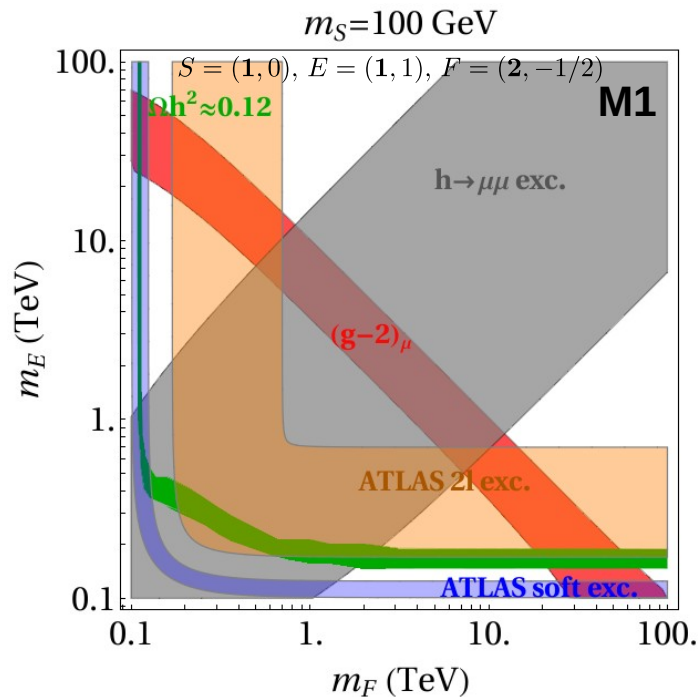
$$g_Y^* = 4\pi \sqrt{\frac{f_g}{B_Y}}$$

$$y_t^* = F(f_g, f_y)$$

$$Y_L^* \neq 0$$



free parameters: m_S, m_E, m_F



1) Fundamentally different and testable signatures.

Entirely consequence of asymptotic safety.

2) Relevant parameters constrained.

Other BSM predictions can be made...

- **anomalies in $b \rightarrow s$**

KK, E.M.Sessolo, Y.Yamamoto,
Eur.Phys.J.C 81 (2021) 4, 272

A.Chikkaballi, W. Kotlarski, KK, D.Rizzo, E.M.Sessolo,
JHEP 01 (2023) 164

- **anomalies in $b \rightarrow c$**

KK, E.M.Sessolo, Y.Yamamoto,
Eur.Phys.J.C 81 (2021) 4, 272

- **neutrino masses**

KK, S.Pramanick, E.M.Sessolo,
JHEP 08 (2022) 262

A.Chikkaballi, KK, E.M.Sessolo,
arXiv: 2308.06114



**Naturally small
parameters**

see Enrico's talk

- **dark matter, baryon number, ALPs, GWs**

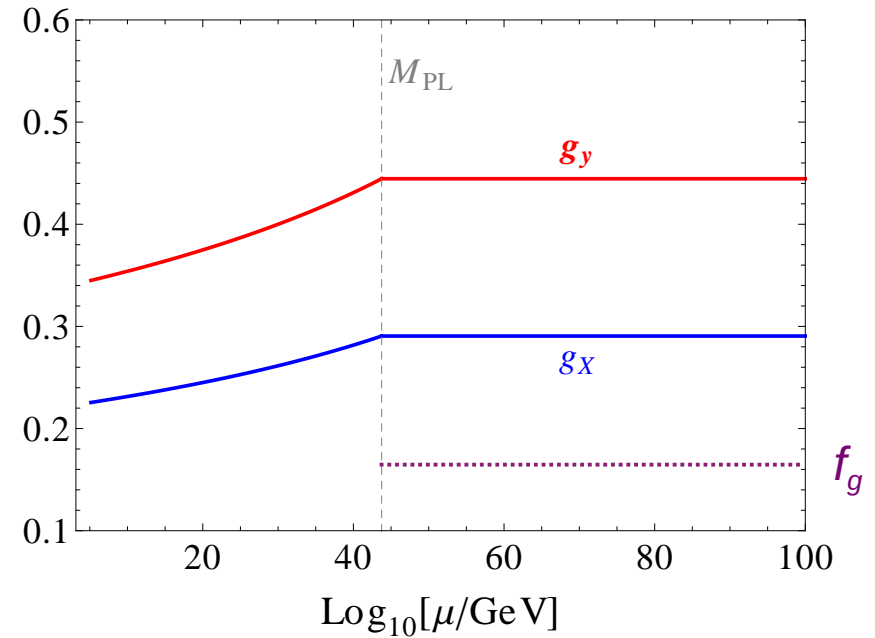
see eg. Reichert, Smirnov, 1803.04027; Grabowski, Kwapisz, Meissner, 1810.08461; Hamada, Tsumura, Yamada, 2002.03666, Eichhorn, Pauly, 2005.03661; de Brito, Eichhorn, Lino dos Santos, 2112.08972, Boos, Carone, Donald, Musser, 2206.02686, 2209.14268, Eichhorn, dos Santos, Miqueleto, 2306.17718,

Uncertainties of the effective approach

Predictions for NP - assumptions

W.Kotlarski, KK, D.Rizzo, E.M.Sessolo
EPJC '23, arXiv: 2304.08959

- 1-loop matter RGEs
- Planck scale set at 10^{19} GeV
- Gravity parameters f are constant
- Gravity decouples instantaneously



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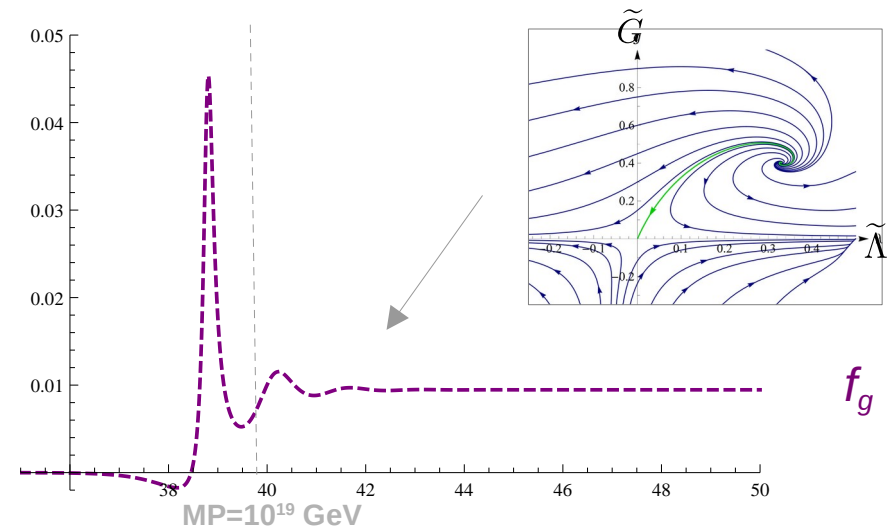
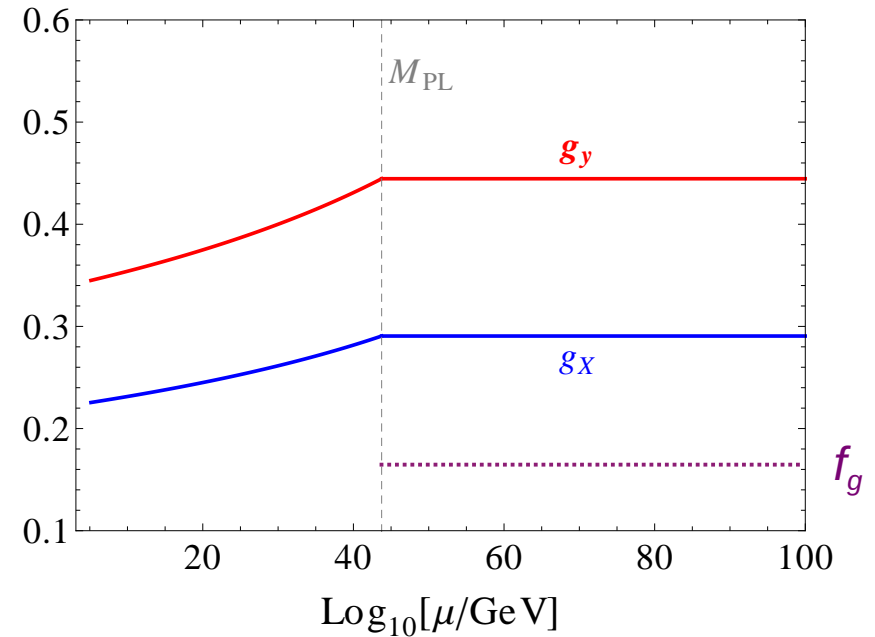
- 1-loop matter RGEs
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But in FRG:

eg. EH truncation, $\alpha=0$, $\beta=1$ g.f
A. Eichhorn, F. Versteegen, JHEP 01 (2018) 030

$$f_g(t) = \tilde{G}(t) \frac{1 - 4\tilde{\Lambda}(t)}{4\pi \left(1 - 2\tilde{\Lambda}(t)\right)^2}$$

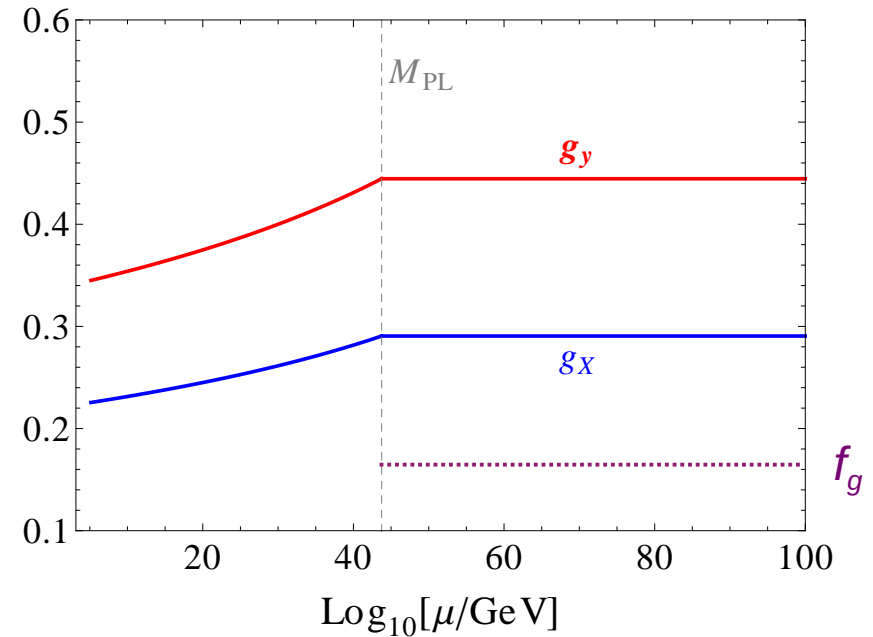
Let's drop the assumptions...



Predictions for NP - assumptions

W.Kotlarski, KK, D.Rizzo, E.M.Sessolo
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uncertainties from FP analysis
should be below
experimental uncertainties of Wilson coefficients

$$\text{ex. for } g-2: \quad \frac{\delta C_{NP}}{C_{NP}} \sim \frac{1}{5(\sigma)} \approx 20\%$$

Uncertainties – gauge sector

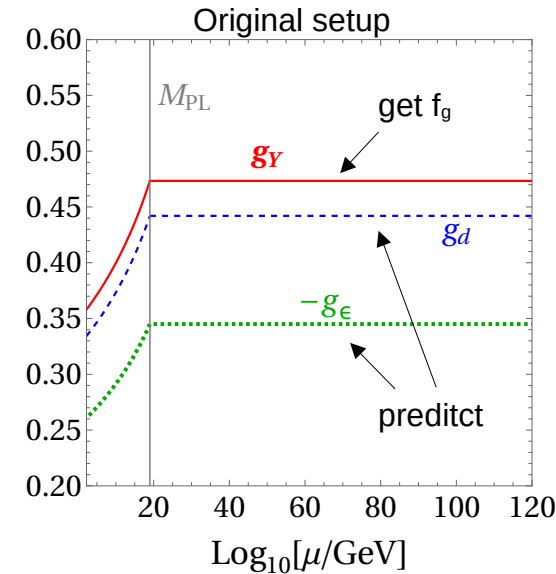
W.Kotlarski, KK, D.Rizzo, E.M.Sessolo
EPJC '23, arXiv: 2304.08959

eg. $U(1)_Y \times U(1)_D$

$$\frac{dg_Y}{dt} = \frac{1}{16\pi^2} \left(\overbrace{b_Y + \Pi_{n \geq 2}^{(Y)}}^{\tilde{b}_Y} \right) g_Y^3 - g_Y f_g(t)$$

$$\frac{dg_d}{dt} = \frac{1}{16\pi^2} \left[\left(b_Y + \Pi_{n \geq 2}^{(Y)} \right) g_d g_\epsilon^2 + \left(b_d + \Pi_{n \geq 2}^{(d)} \right) g_d^3 + \left(b_\epsilon + \Pi_{n \geq 2}^{(\epsilon)} \right) g_d^2 g_\epsilon \right] - g_d f_g(t)$$

$$\frac{dg_\epsilon}{dt} = \frac{1}{16\pi^2} \left[\left(b_Y + \Pi_{n \geq 2}^{(Y)} \right) (g_\epsilon^3 + 2g_Y^2 g_\epsilon) + \left(b_d + \Pi_{n \geq 2}^{(d)} \right) g_d^2 g_\epsilon + \left(b_\epsilon + \Pi_{n \geq 2}^{(\epsilon)} \right) (g_Y^2 g_d + g_d g_\epsilon^2) \right] - g_\epsilon f_g(t)$$



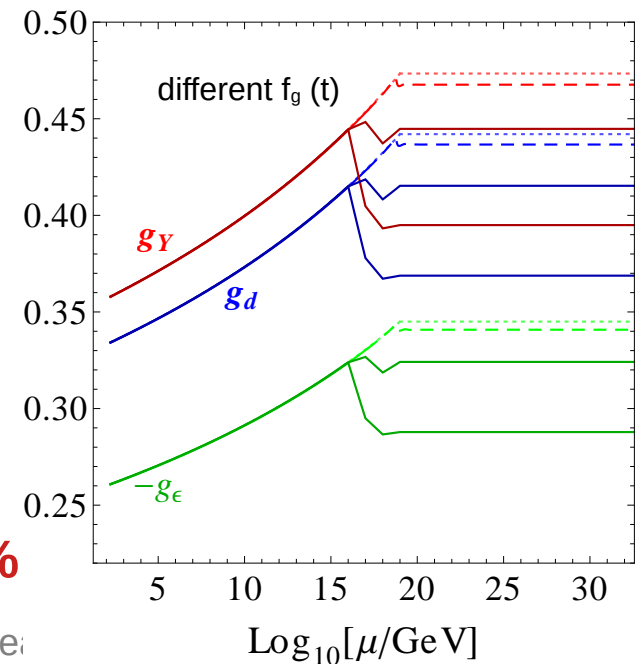
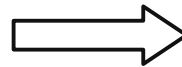
The coupling ratios do not depend on f_g

(due to the universality of QG)

$$\frac{g_d^*}{g_Y^*} (n \text{ loops}) \approx \frac{2\tilde{b}_Y}{\sqrt{4\tilde{b}_Y\tilde{b}_d - \tilde{b}_\epsilon^2}}$$

$$\frac{g_\epsilon^*}{g_Y^*} (n \text{ loops}) \approx -\frac{\tilde{b}_\epsilon}{\sqrt{4\tilde{b}_Y\tilde{b}_d - \tilde{b}_\epsilon^2}}$$

Invariant of the RGE flow



PREDICTIONS VERY STABLE

$\delta g \leq 0.1\%$

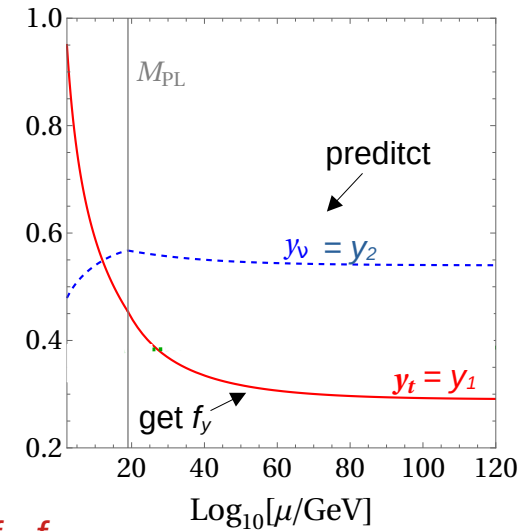
Uncertainties – Yukawa sector

W.Kotlarski, KK, D.Rizzo, E.M.Sessolo
EPJC '23, arXiv: 2304.08959

2-Yukawa system

$$\frac{dy_1}{dt} = \frac{y_1}{16\pi^2} \left(a_1^{(1)} y_1^2 + a_2^{(1)} y_2^2 - a'^{(1)} g_1^2 + \sum_{n \geq 2} \Pi_n^{(1)} \right) - y_1 f_y(t)$$

$$\frac{dy_2}{dt} = \frac{y_2}{16\pi^2} \left(a_1^{(2)} y_1^2 + a_2^{(2)} y_2^2 - a'^{(2)} g_1^2 + \sum_{n \geq 2} \Pi_n^{(2)} \right) - y_2 f_y(t)$$



The FP ratio y_2 to y_1 depends on FP of other couplings

$$\frac{y_2^*}{y_1^*} (1 \text{ loop}) \approx \left[\underbrace{\frac{\left(a_1^{(2)} - a_1^{(1)} \right) + \left(a'^{(1)} - a'^{(2)} \right) g_1^{*2} / y_1^{*2}}{a_2^{(1)} - a_2^{(2)}}}_{\text{fixed } f_g \text{ and } f_y} + \underbrace{\frac{\left(a_1^{(2)} - a_1^{(1)} \right) \delta y_1^{*2} + \left(a'^{(1)} - a'^{(2)} \right) \delta g_1^{*2}}{y_1^{*2} \left(a_2^{(1)} - a_2^{(2)} \right)}}_{\text{shift due to the running } f_g, f_y} \right]^{1/2}$$

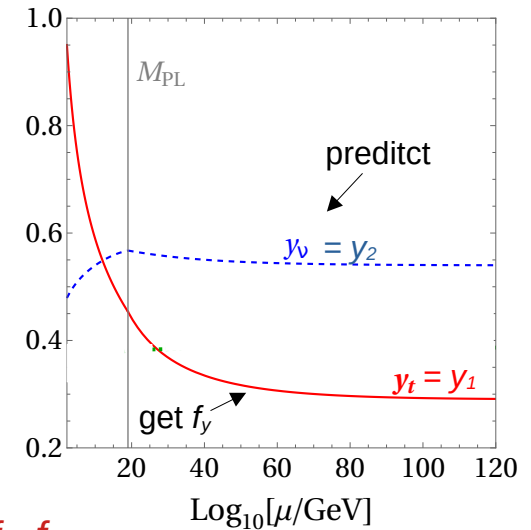
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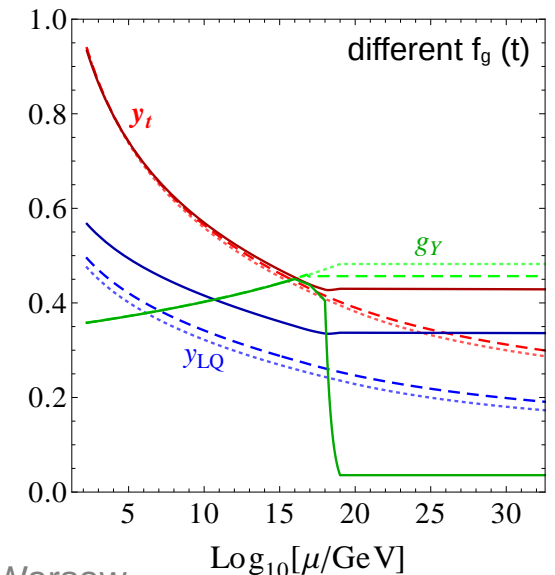
The FP ratio y_2 to y_1 depends on FP of other couplings

$$\frac{y_2^*}{y_1^*} (1 \text{ loop}) \approx \left[\frac{\overbrace{\left(a_1^{(2)} - a_1^{(1)} \right) + \left(a'^{(1)} - a'^{(2)} \right) g_1^{*2} / y_1^{*2}}^{\text{fixed } f_g \text{ and } f_y}}{a_2^{(1)} - a_2^{(2)}} + \frac{\overbrace{\left(a_1^{(2)} - a_1^{(1)} \right) \delta y_1^{*2} + \left(a'^{(1)} - a'^{(2)} \right) \delta g_1^{*2}}^{\text{shift due to the running } f_g, f_y}}{y_1^{*2} (a_2^{(1)} - a_2^{(2)})} \right]^{1/2}$$

eg. LQ S_3 model:

$$\mathcal{L} \supset -Y_{LQ} Q^T \tilde{\epsilon} S_3 L + \text{H.c.}$$

PREDICTION UNSTABLE ...



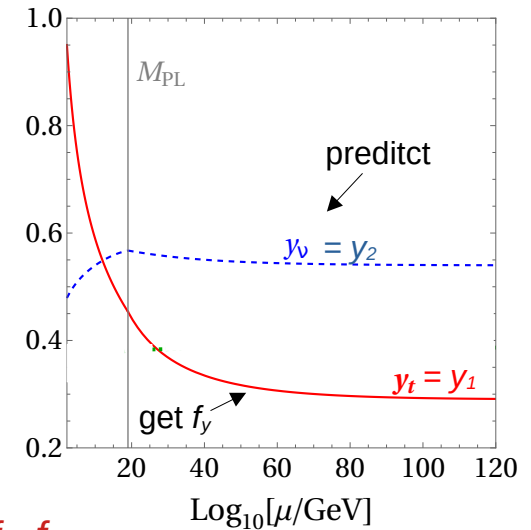
Uncertainties – Yukawa sector

W.Kotlarski, KK, D.Rizzo, E.M.Sessolo
EPJC '23, arXiv: 2304.08959

2-Yukawa system

$$\frac{dy_1}{dt} = \frac{y_1}{16\pi^2} \left(a_1^{(1)} y_1^2 + a_2^{(1)} y_2^2 - a'^{(1)} g_1^2 + \sum_{n \geq 2} \Pi_n^{(1)} \right) - y_1 f_y(t)$$

$$\frac{dy_2}{dt} = \frac{y_2}{16\pi^2} \left(a_1^{(2)} y_1^2 + a_2^{(2)} y_2^2 - a'^{(2)} g_1^2 + \sum_{n \geq 2} \Pi_n^{(2)} \right) - y_2 f_y(t)$$



The FP ratio y_2 to y_1 depends on FP of other couplings

$$\frac{y_2^*}{y_1^*} (1 \text{ loop}) \approx \left[\frac{\overbrace{\left(a_1^{(2)} - a_1^{(1)} \right) + \left(a'^{(1)} - a'^{(2)} \right) g_1^{*2} / y_1^{*2}}^{\text{fixed } f_g \text{ and } f_y}}{a_2^{(1)} - a_2^{(2)}} + \frac{\overbrace{\left(a_1^{(2)} - a_1^{(1)} \right) \delta y_1^{*2} + \left(a'^{(1)} - a'^{(2)} \right) \delta g_1^{*2}}^{\text{shift due to the running } f_g, f_y}}{y_1^{*2} (a_2^{(1)} - a_2^{(2)})} \right]^{1/2}$$

eg. LQ S_3 model:

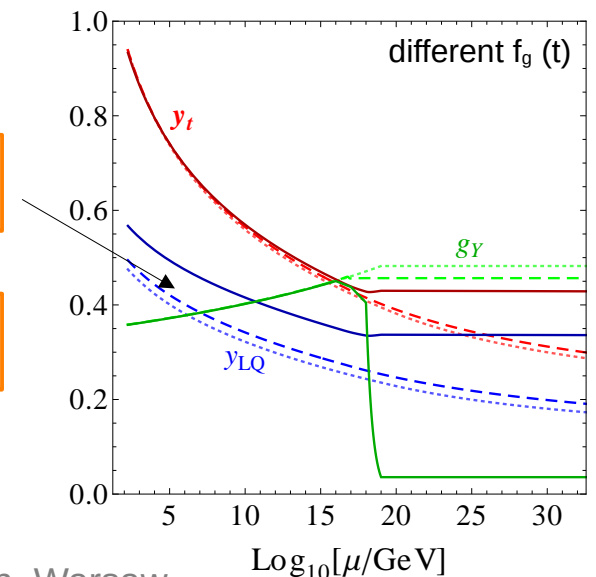
$$\mathcal{L} \supset -Y_{LQ} Q^T \tilde{\epsilon} S_3 L + \text{H.c.}$$

... but not so much in FRG

... IR focusing helps

PREDICTION UNSTABLE ...

$$\delta y \leq 20\%$$



Conclusions

- Trans-Planckian AS is a **very predictive UV framework**. Applications for SM and NP.
- Flavor anomalies, $g-2$ anomaly, dark matter, etc. can lead to very **testable signatures**.
- **AS predictions** in the **gauge sector** are **stable** under higher-order corrections and running of the gravity parameters.
- **Uncertainties** of the AS predictions in **the Yukawa sector** do **not exceed 20%**.