HALO MASS FUNCTION RESCALING AND CLUSTER COUNTS COSMOLOGY

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Action Dark Energy 2024

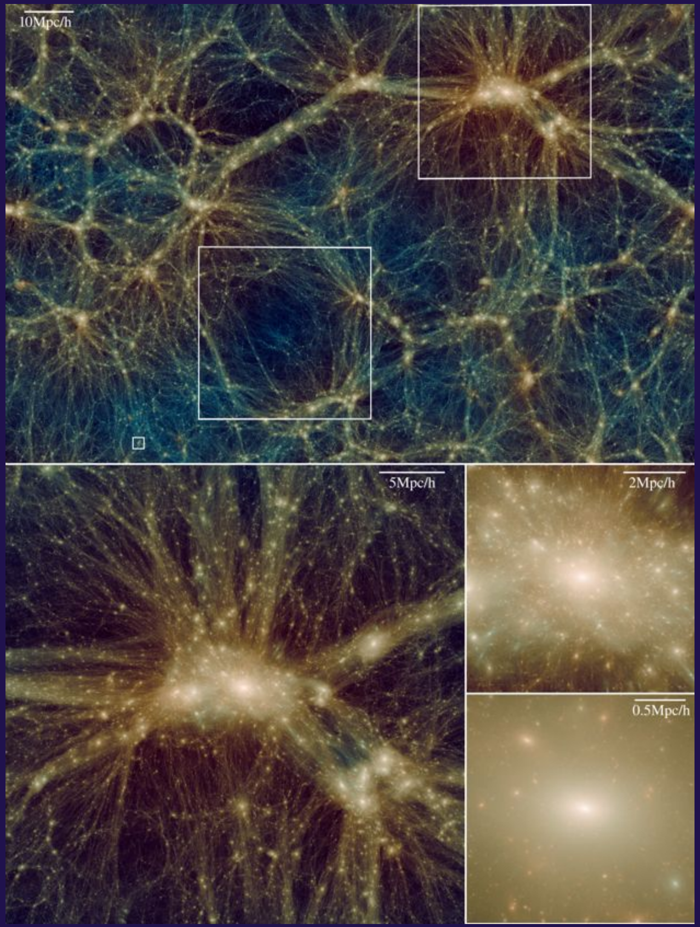
CONTEXT
GALAXY CLUSTERS

Properties :

→ Largest structures gravitationally virialized in the Universe

→ Multicomponent systems : Dark Matter (~85%) & Baryons

→ Abundance very sensitive to cosmological parameters (e.g. σ_8, Ω_m)



https://skiesanduniverses.org/Simulations/Uchuu/Visualize/

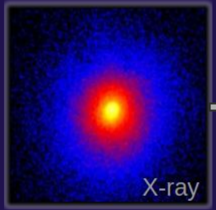
CONTEXT
GALAXY CLUSTERS

Observational side

Richness
→ M_{200c}

Scaling relations

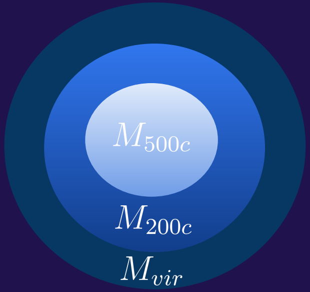
$$M_{\Delta c} = \frac{4}{3}\pi R_{\Delta}^{3}\Delta\rho_c$$



X-ray luminosity
→ M_{500c}

Images of Abell 1835 (z = 0.25)

CONTEXT
GALAXY CLUSTERS



SO masses

Observational **side**

Richness
→ M200c

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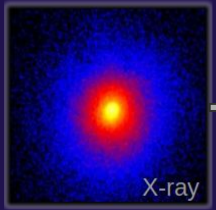

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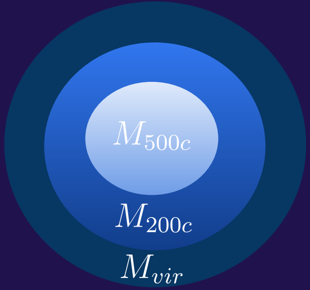
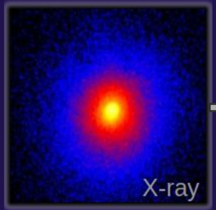


Observational side

Richness
$\longrightarrow$ M200c



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Scaling relations

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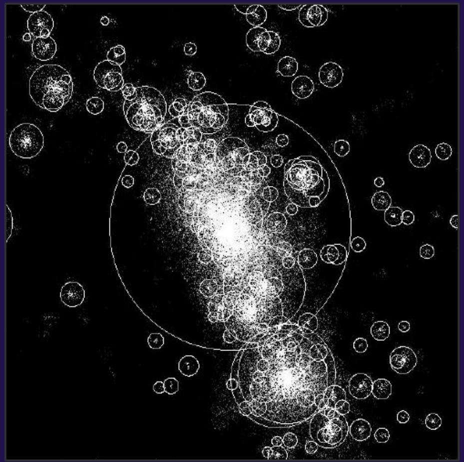
Halo finders

Theoretical side



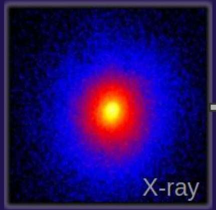
Kravtsov et Al. 2003

N-body simulations - Dark matter halos

CONTEXT
GALAXY CLUSTERS

***Observational* side**

Richness
$\longrightarrow$ M200c



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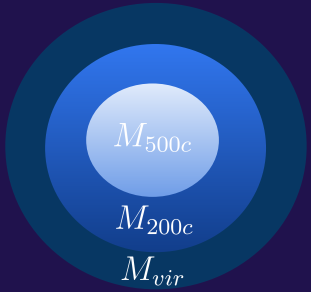


SO masses

Scaling relations

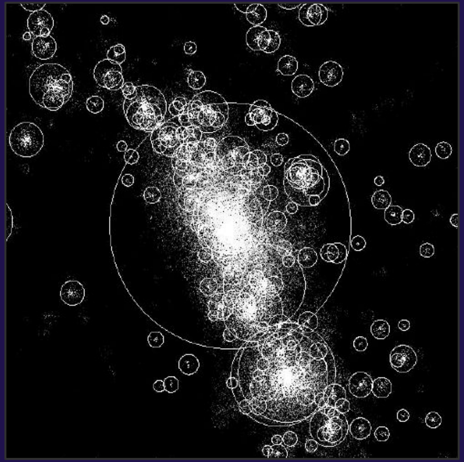
$$M_{\Delta c} = \frac{4}{3}\pi R_{\Delta}^{3}\Delta\rho_{c}$$

Halo finders

Fitting halo catalogues

$$\frac{dn}{dM_{\Delta}}$$ Halo mass function (HMF)

***Theoretical* side**



Kravtsov et Al. 2003

N-body simulations - Dark matter halos

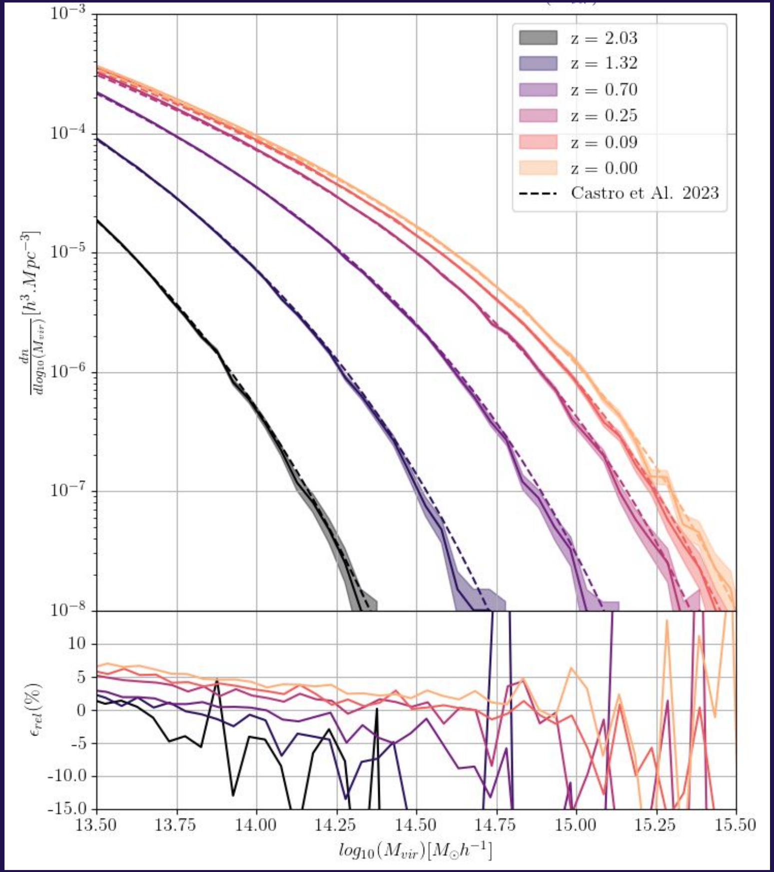
CONTEXT
HALO MASS FUNCTION

$$\frac{dn}{dM_\Delta}$$ Number of haloes per unit volume per mass bin

Euclid HMF (Castro et al. 2023)

→Calibrated at M_{vir}

→Captures deviation from non-universality.



CONTEXT
NUMBER COUNT

Expected number count (NC): Number of clusters of a given observable X and z within the survey area

$$\frac{dN(X;z)}{dX\,dz} = \frac{dV}{dz} f(X,z) \int_0^\infty \frac{dn(M,z)}{dM_\Delta} \frac{dp(X|M,z)}{dX} dM$$

Survey's volume

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HMF

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Survey's volume

Selection function

HMF

Probability of X given M and z

e.g. we have $\frac{dn(M,z)}{dM_{vir}}$ (e.g. Euclid HMF) and we have $X = L_X$

OBJECTIF AND METHODS

RESCALING

Objective : Rescale the HMF (e.g. Euclid HMF Castro et al 23.) to match observables using accurate method to reduce the systematics on the NC

Rescaling

$$X = L_X$$

$$\frac{dn}{dM_{vir}} \qquad \frac{dn}{dM_{500c}}$$

$$\Delta = \Delta_{vir} \qquad \Delta_c = 500$$

OBJECTIF AND METHODS

METHODS - DETERMINISTIC (C-M)

What we need

Chain rule: $$\frac{dn}{dM_\Delta} = \frac{dn}{dM_{vir}} \boxed{\frac{dM_{vir}}{dM_\Delta}}$$

$M_\Delta = f(M_{vir})$?

OBJECTIF AND METHODS

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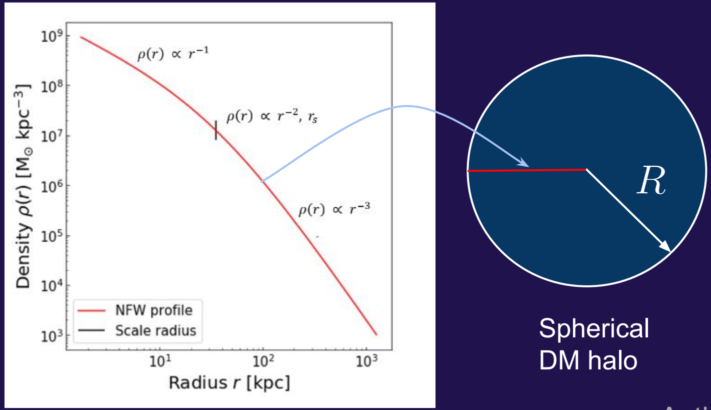
NFW profile (Navarro, Frenk & White 97)

$$\rho_{NFW}(r) = \frac{M_{vir}}{4\pi[ln(1+c_{vir}) - c_{vir}/(1+c_{vir})]} \times \frac{1}{r\left(\frac{R_{vir}}{c_{vir}} + r\right)^2}$$



Spherical DM halo

OBJECTIF AND METHODS

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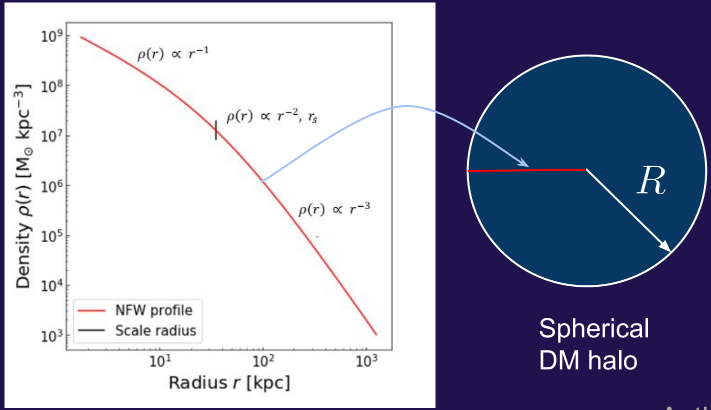
c-M relation (Ishiyama et al. 21) $c_{vir} = f(M_{vir}, z, cosmo)$



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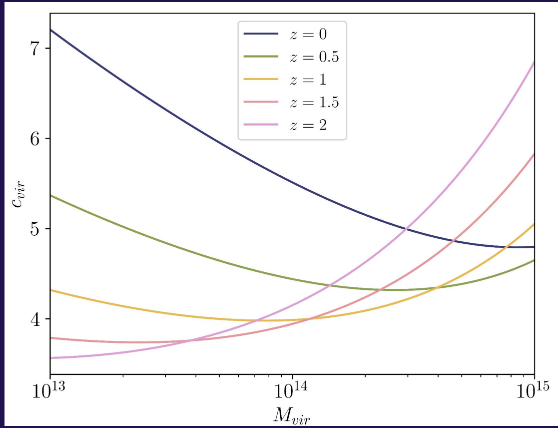

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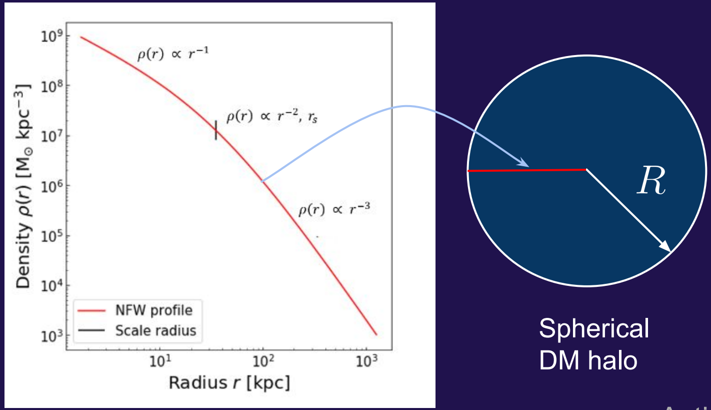

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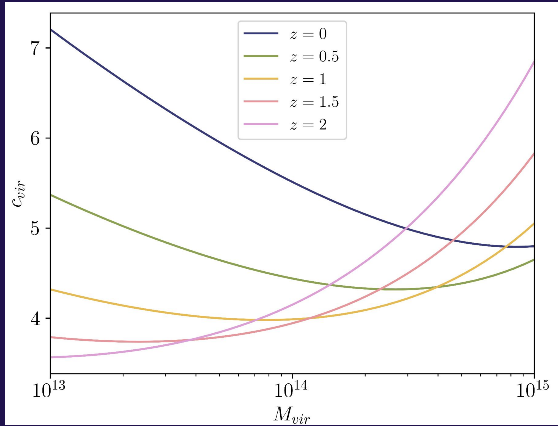

We can deduce M_Δ knowing ρ_{NFW} (Hu & Kratsov 18)

OBJECTIF AND METHODS

METHODS - SEMI-DETERMINISTIC (C-M + PDF)

M : random variable drawn from the HMF

$$M_\Delta \sim \frac{dn}{dM_\Delta}$$

$$\frac{dn_\Delta}{dM_\Delta} = \int \frac{dn_{vir}}{dM_{vir}} \rho(M_\Delta | M_{vir}) dM_{vir}$$

Despali et al. 2016

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Internal structure

What we need

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Internal structure: NFW profile + intrinsic scatter of c_{vir} (formation history of DM haloes)

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Internal structure: NFW profile + intrinsic scatter of c_{vir} (formation history of DM haloes)

$$\rho(M_\Delta | M_{vir}) dM_{vir} = \rho(c_{vir} | M_{vir}) \left| \frac{dM_{vir}}{dc_{vir}} \right| dc_{vir}$$

log-normal distribution

c-M relation

OBJECTIF AND METHODS

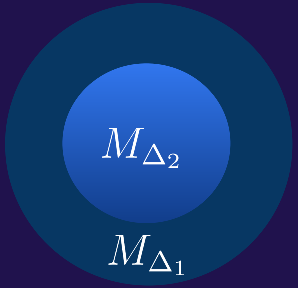
METHODS - STOCHASTIC (SPARSITIES)

→ **Non-parametric approach** using sparsities (Richardson & Corasaniti 2023)

Sparsities

$$s_{\Delta_1,\Delta_2} = \frac{M_{\Delta_1}}{M_{\Delta_2}}$$

$$\Delta_1 < \Delta_2$$



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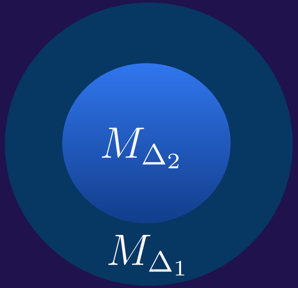

s : random variable

$$s \sim \rho_s(s|M_{vir})$$

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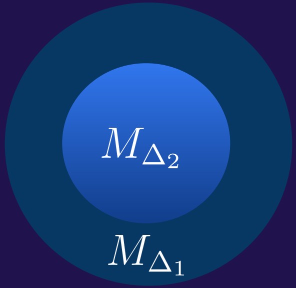

s : random variable

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Internal structure: Sparsities p.d.f.s

$$\rho(M_\Delta|M_{vir})dM_{vir} = s\rho(s|sM_\Delta)ds$$

Calibrated with N-body sim

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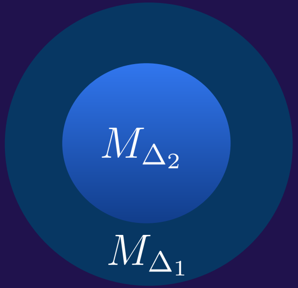

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Calibrated with N-body sim

Inward transformation

$$\frac{dn}{dM_\Delta}(M_\Delta) = \int_1^\infty s\rho_s(s|sM_\Delta)\frac{dn}{dM_{\Delta_{vir}}}(sM_\Delta)ds$$

OBJECTIF AND METHODS
METHODS - STOCHASTIC PDFS

$\rho_s(s|sM_{\Delta_2})$ is calibrated using N-body simulation

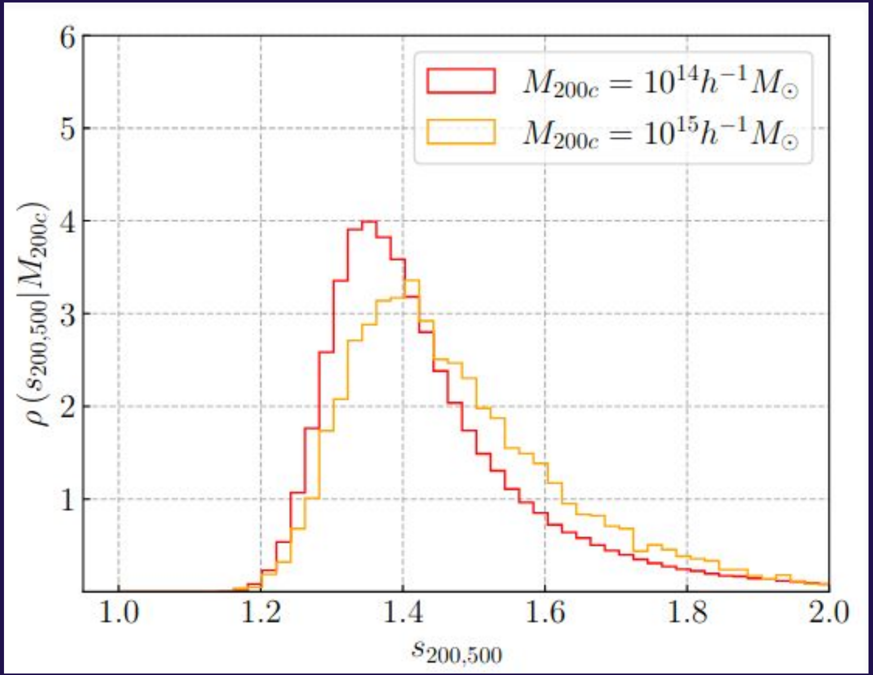
→ **Cosmology dependent**

RayGal LCDM (Rasera et al., 2021)

- **Nbody DM only simulation**
- WMAP-7 best-fit cosmological model
- Np = 4096^3 particles
- Halos detected with SO halo finder
- L = 2625 Mpc/h^3

OBJECTIF AND METHODS
METHODS - STOCHASTIC PDFS

Casarez et al. in prep



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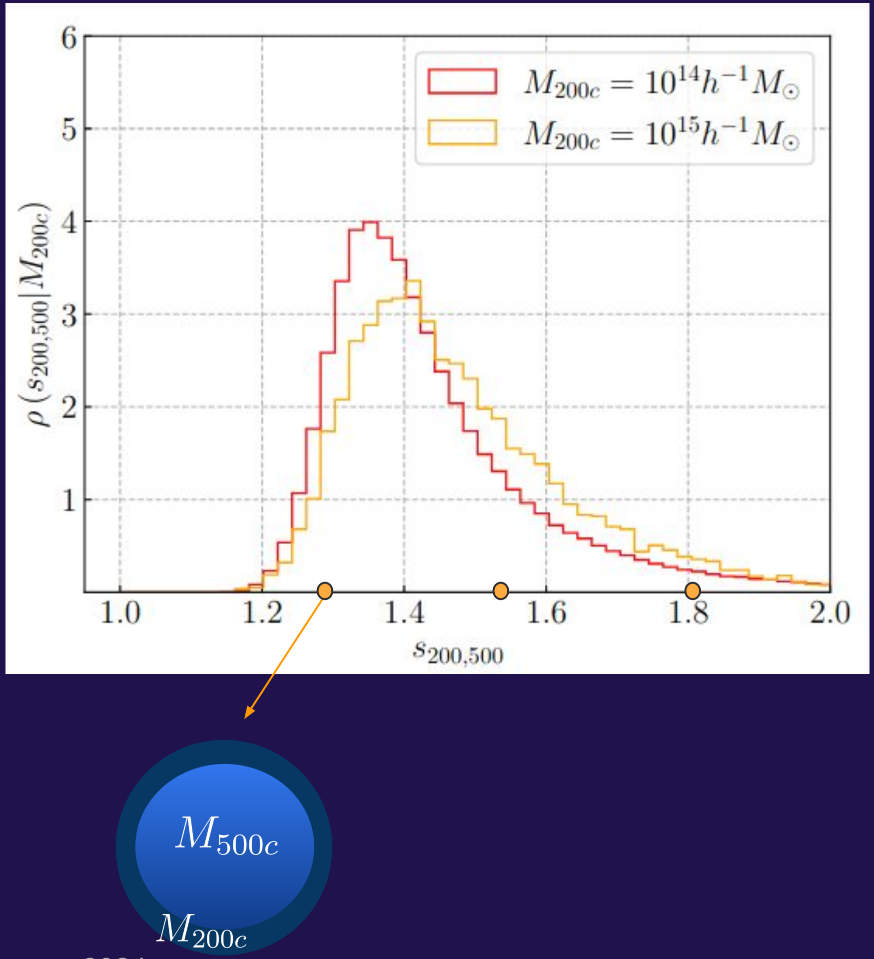

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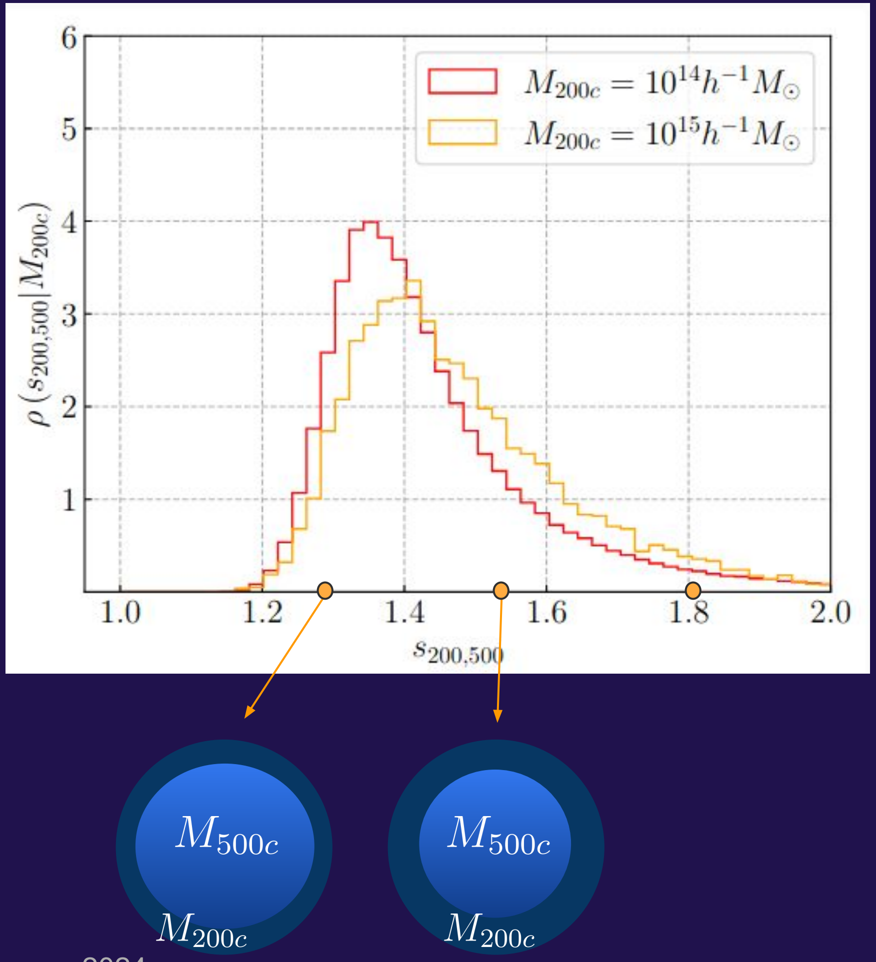

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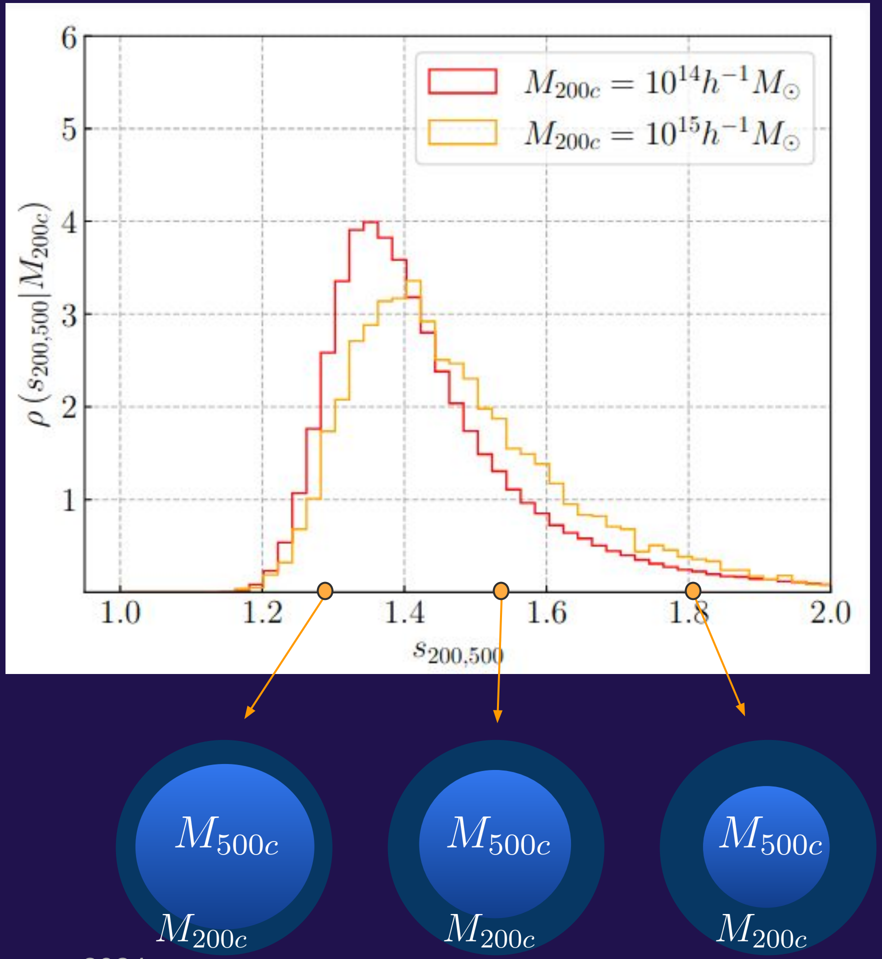

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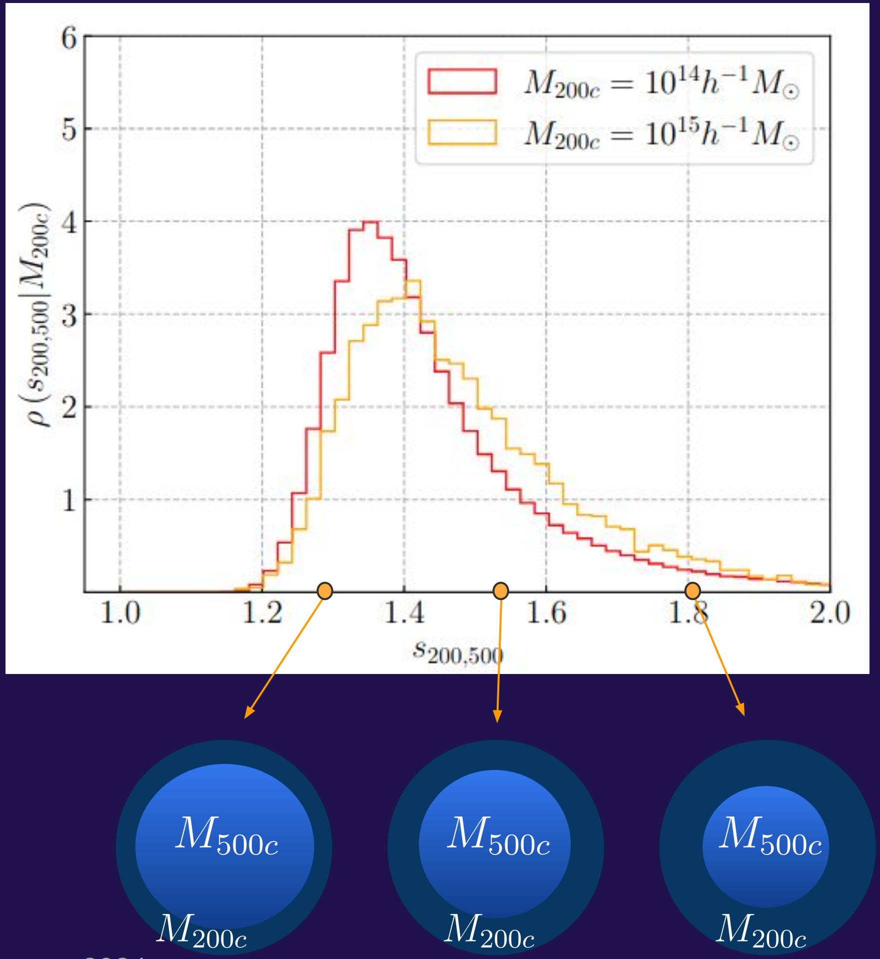


→ Give information on matter distribution within haloes with a statistical approach

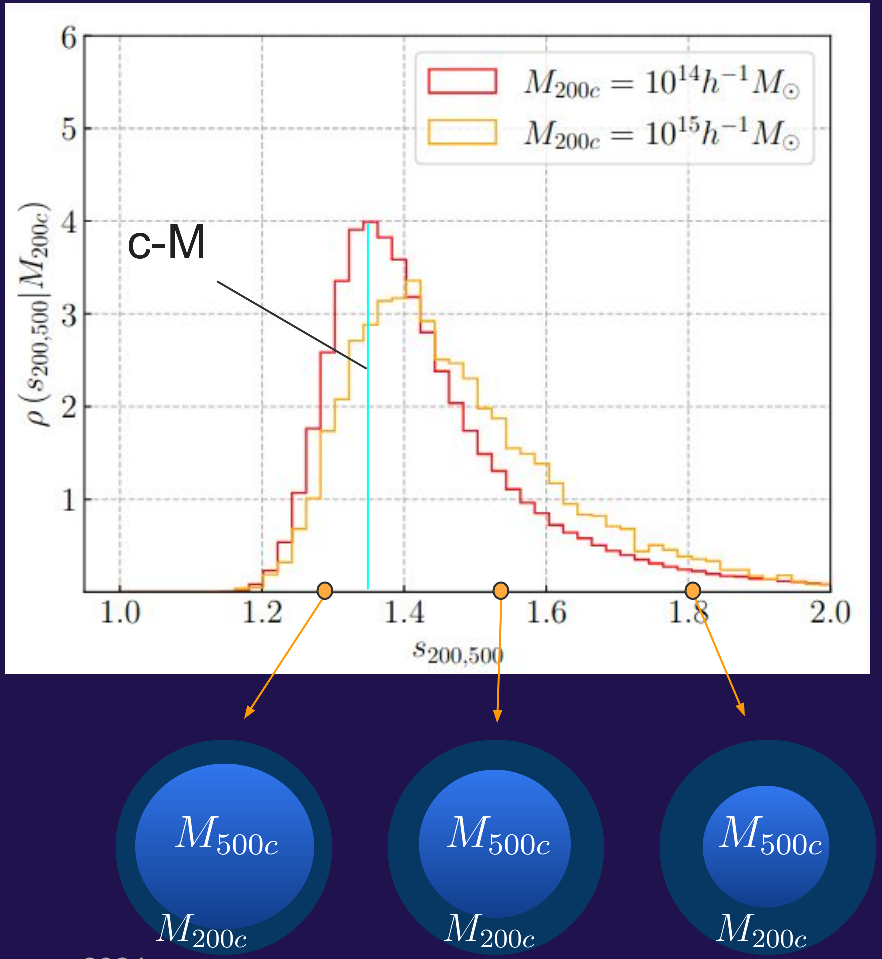
OBJECTIF AND METHODS
METHODS - STOCHASTIC PDFS

→ Give information on matter distribution within haloes with a statistical approach

→ In the case of c-M :

$$\rho_s(s|sM_\Delta) = \delta(s - s^{NFW}(c))$$

Casarez et al. in prep



OBJECTIF AND METHODS

SUMMARY - 3 METHODS

$$\frac{dn}{dM_\Delta} = \frac{dn}{dM_{vir}} \frac{dM_{vir}}{dM_\Delta}$$

c-M

$$\frac{dn_\Delta}{dM_\Delta} = \int \frac{dn_{vir}}{dM_{vir}} \rho(c_{vir}|M_{vir}) \left| \frac{dM_{vir}}{dc_{vir}} \right| dc_{vir}$$

c-M + PDF

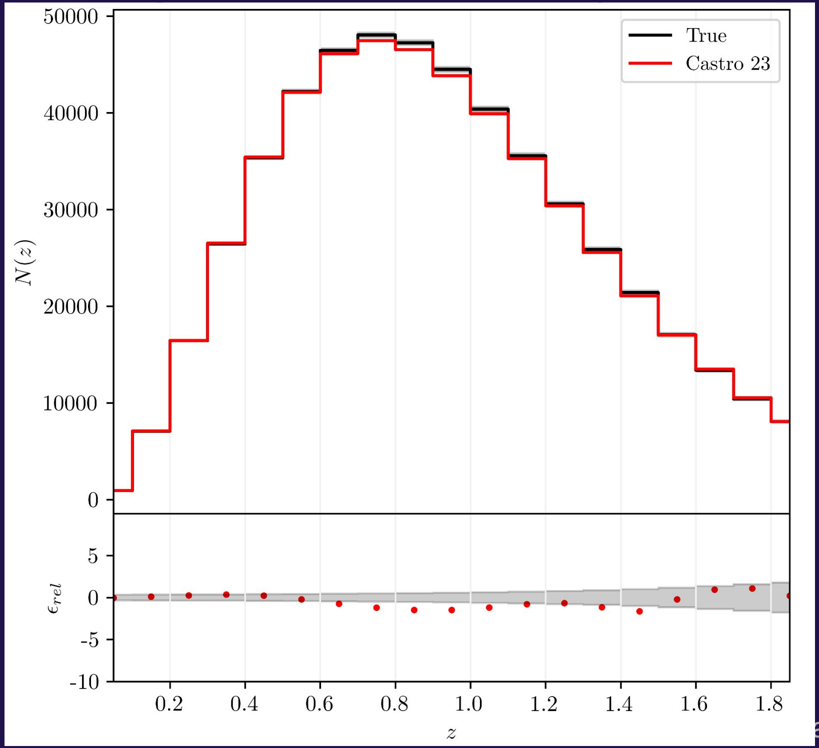
$$\frac{dn}{dM_\Delta}(M_\Delta) = \int_1^\infty s\rho_s(s|sM_\Delta) \frac{dn}{dM_{\Delta_{vir}}}(sM_\Delta) ds$$

Sparsities

RESULTS (PRELIMINARY)
CLUSTER COUNTS - MVIR

$$N_{vir}(z) = \Delta\Omega\Delta z \frac{d^2V}{d\Omega dz} \int_{M_{min}}^{M_{max}} dM \frac{dn}{dM_{vir}}(M, z)$$

$$M_{min} = 3.10^{13} M_\odot/h$$



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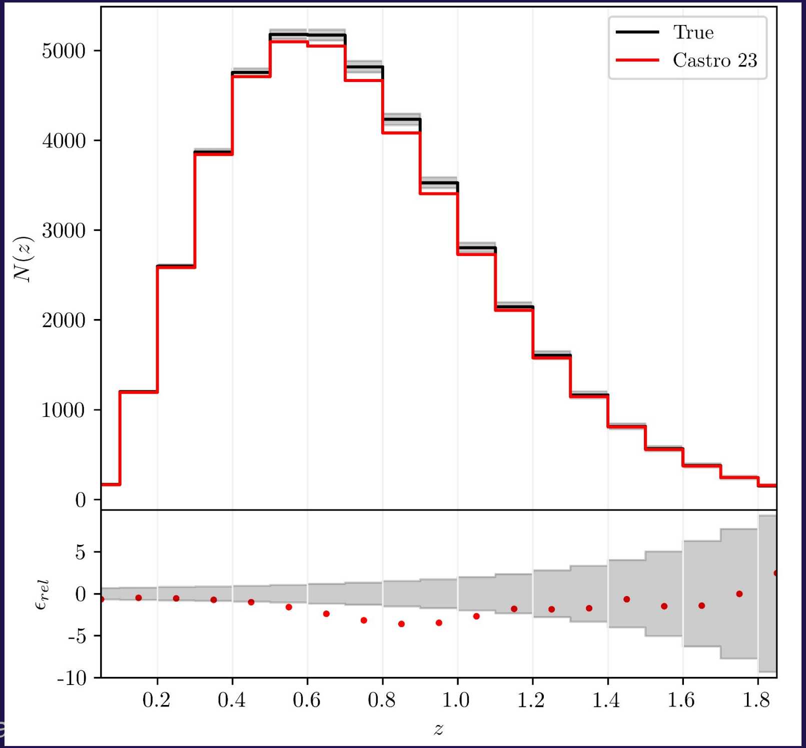
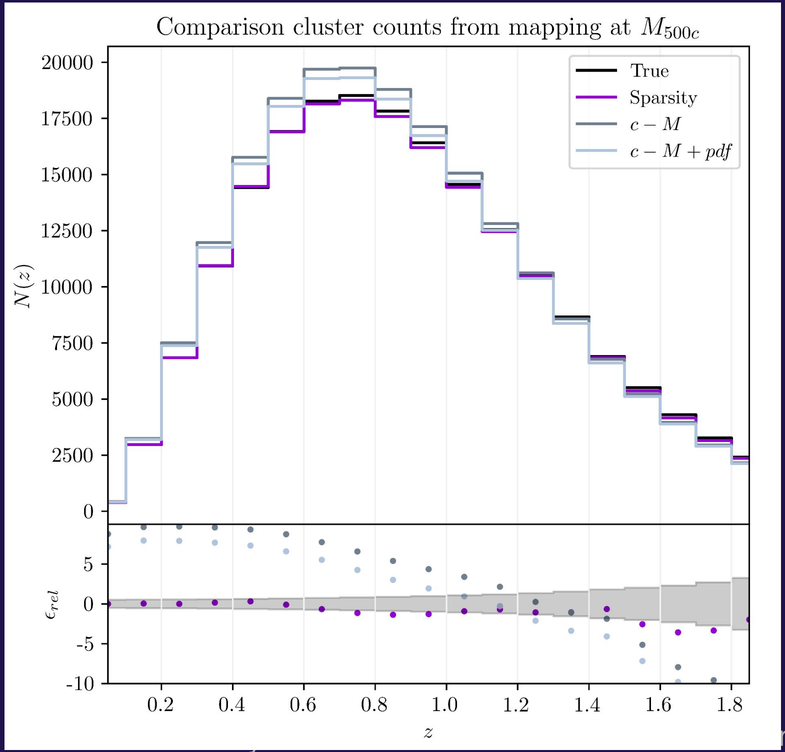


RESULTS (PRELIMINARY)
CLUSTER COUNTS - M500C

$$N_{500c}(z) = \Delta\Omega\Delta z \frac{d^2V}{d\Omega dz} \int_{M_{min}}^{M_{max}} dM \frac{dn}{dM_{500c}}(M,z)$$

$M_{min} = 3.10^{13} M_\odot/h$



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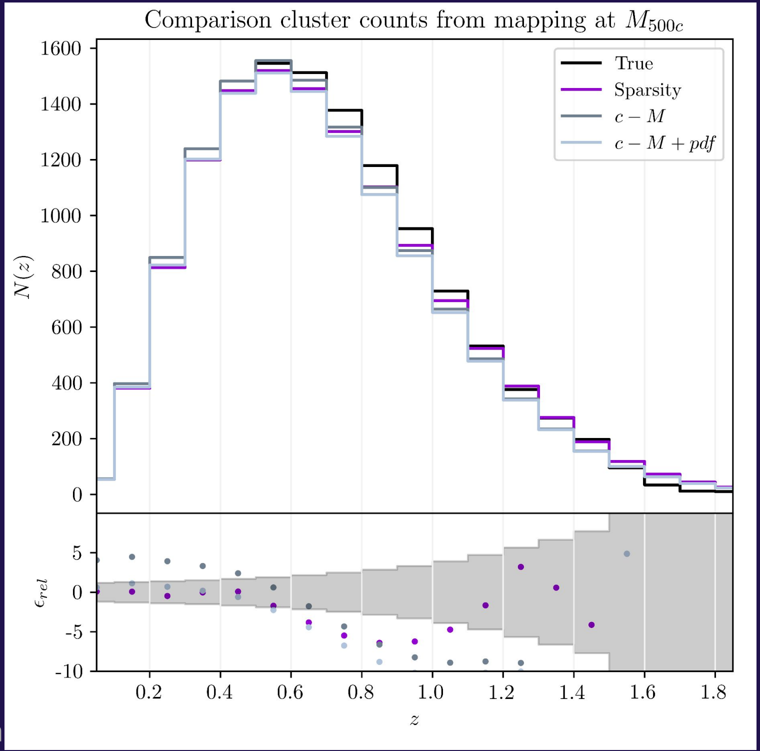


Stochastic approach:

Maintain a good level of accuracy

c-M & pdf relations:

Gives rise to a ~7-8% discrepancy

RESULTS (PRELIMINARY)
LIKELIHOOD

$$\mathcal{L}(N_{\mathrm{obs}}|N_{\mathrm{theory}}) = \prod_{i=1}^{N_{\mathrm{bins}}} \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left(-\frac{1}{2}\frac{(N_{\mathrm{obs},i} - N_{\mathrm{theory},i})^2}{\sigma_i^2}\right)$$

Gaussian Likelihood

RayGal data

Shot noise

Priors:

$$\sigma_8 \sim \mathrm{Uniform}(c, d)$$

$$\Omega_m \sim \mathrm{Uniform}(a, b)$$

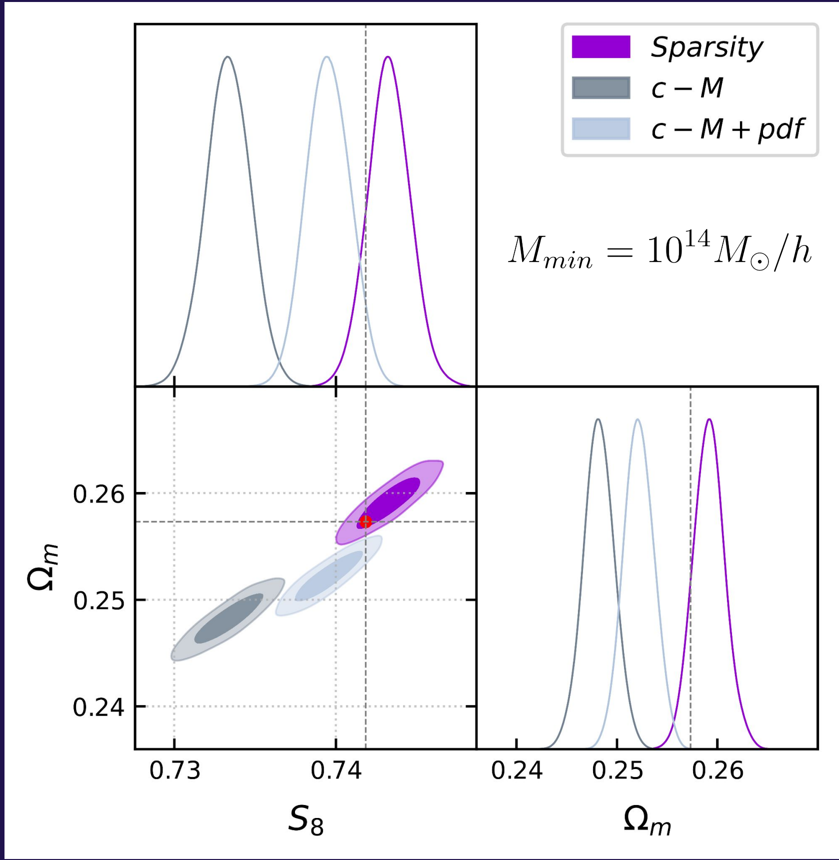
$$H_0 \sim \mathcal{N}(\mu_{H_0}, \sigma_{H_0}^2)$$

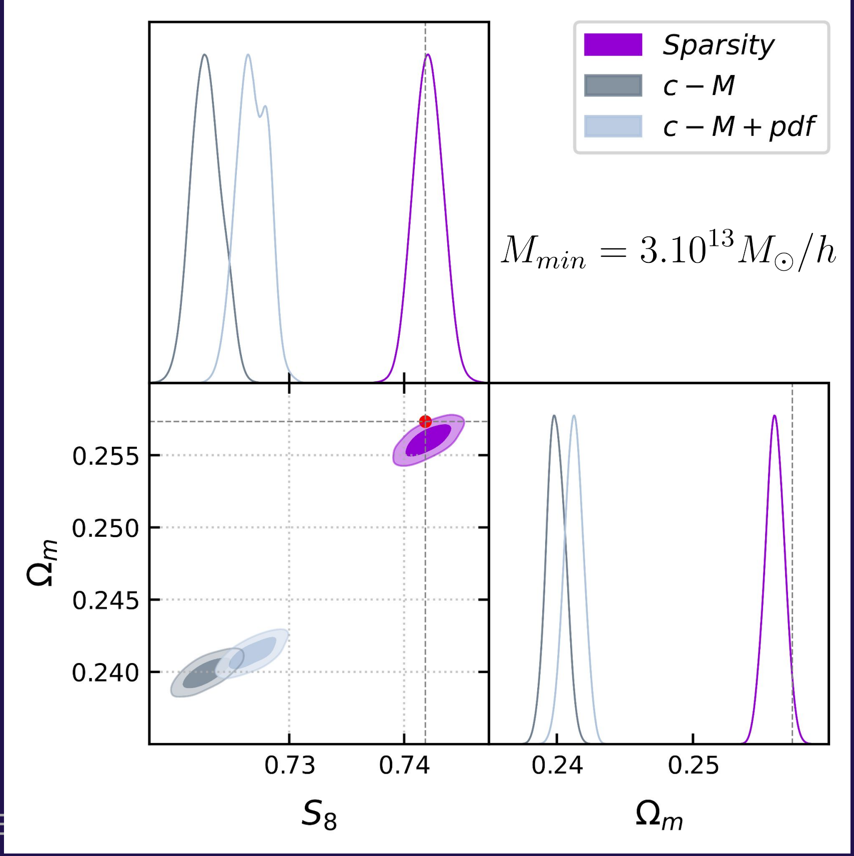
$$\Omega_b \sim \mathcal{N}(\mu_{\Omega_b}, \sigma_{\Omega_b}^2)$$

N.B.
Cosmology for c-M
and p.d.fs of ***s*** are fixed to RayGal
cosmology

RESULTS (PRELIMINARY)
COSMOLOGICAL PARAMS

$$\Delta_c = 500$$





CONCLUSION

Deterministic (c-M)	**Semi-deterministic** (c-M+pdf)	**Stochastic** (sparsities)
Strong disagreement with the fiducial cosmology (for both M_{min}) **Systematic bias** toward lower values of S8 and Omega_m	**2σ disagreement** with RayGal cosmology but a slight improvement	**Very good agreement** for both M_{min} (< 1σ) **Non-parametric approach** gives robust cosmological predictions in this case

PERSPECTIVES

- Use more **realistic synthetic data** set (e.g. proper light cone) → **Flagship** octant of the sky (WIDE survey)

- Add more complexity in the cluster counts (e.g. **selection function**, **mass-observable relation** etc.)

- Handle the cosmology dependence of sparsities → ***e-mantis*** emulator (Saez Casares et al. in prep)

THANK YOU FOR YOUR ATTENTION :)

BACKUP SLIDES

C-M + HMF

c-M relation:

$$c = C(\alpha_{\text{eff}}) \times \tilde{G}\left(\frac{A(\alpha_{\text{eff}})}{\nu}\left[1 + \frac{\nu^2}{B(\alpha_{\text{eff}})}\right]\right)$$

6 free parameters

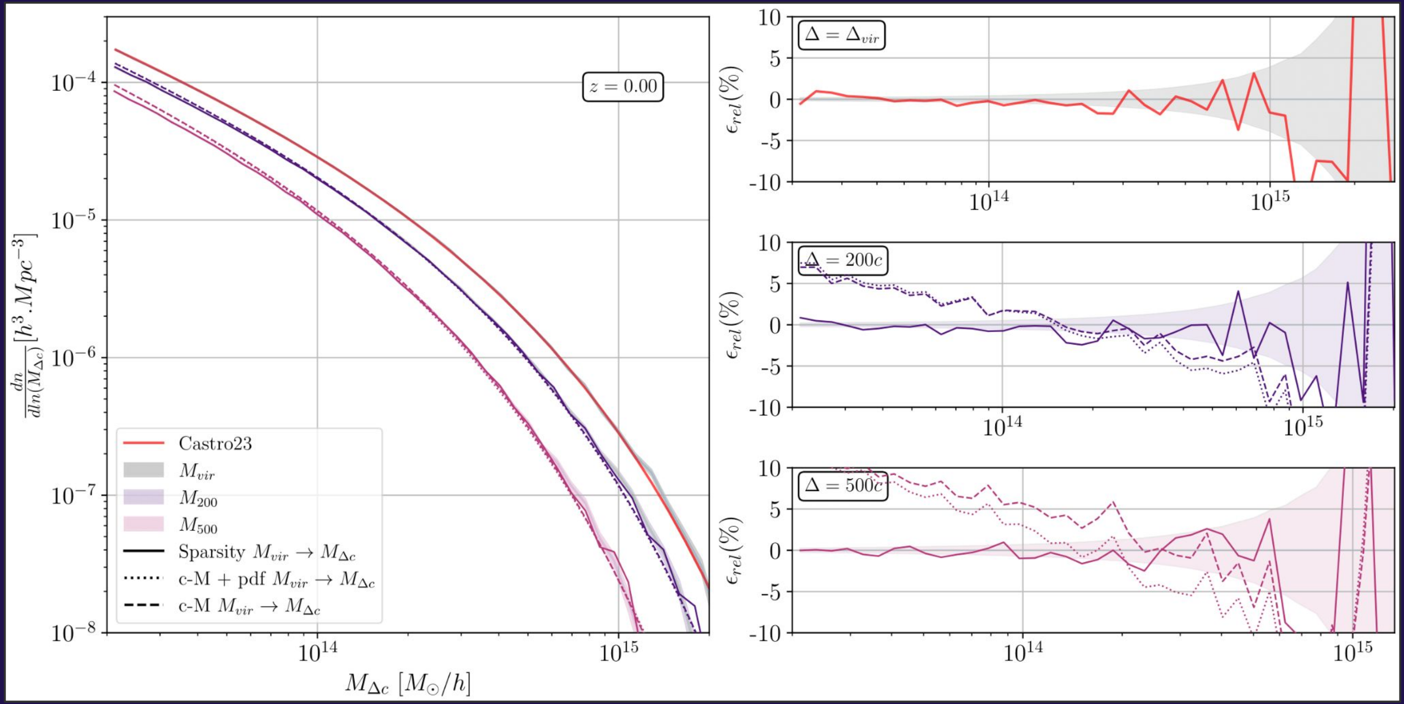
HMF:

$$\frac{dn}{dln(M)} = \frac{\bar{\rho}_m(0)}{M}\nu f(\nu) dln(\nu)$$

$$\nu f(\nu) = A(p,q)\sqrt{\frac{2a\nu^2}{\pi}} e^{-a\nu^2/2}\left(1 + \frac{1}{(a\nu^2)^p}\right)(\nu\sqrt{a})^{q-1}$$

RESCALED HMF

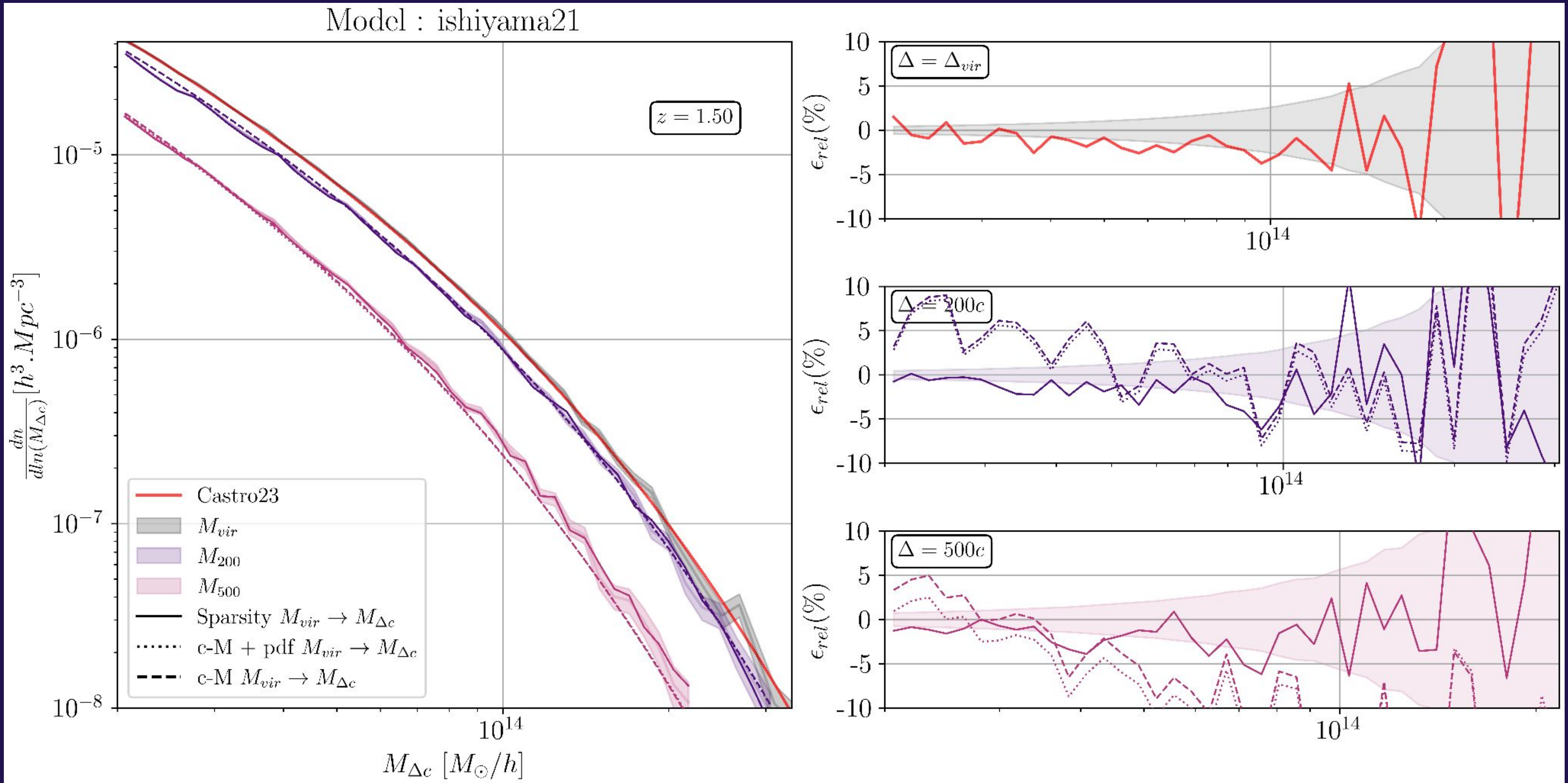
COMPARISON WITH RAYGAL Z=0



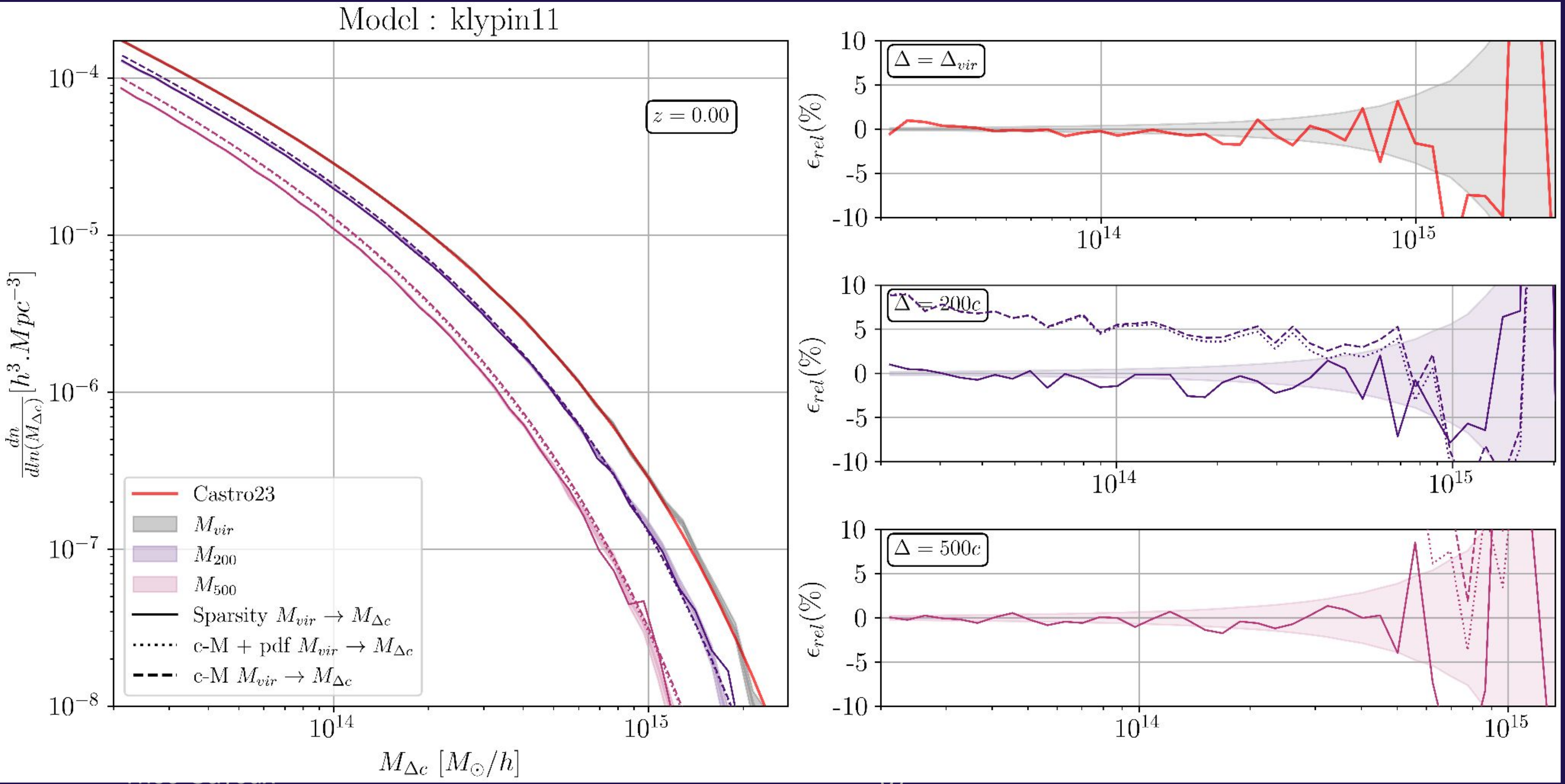
Stochastic approach:
Maintain same level of agreement at Δ_{vir} for 200c and 500c.

c-M & pdf relations:
Introduce discrepancy above 5% at high masses and ~8% at low masses even accounting for the p.d.f of *c*

RESCALED HMF
COMPARISON WITH RAYGAL Z=1.5



RESCALED HMF
KLYPIN 11



BACKUP SLIDES
TINKER 08 + DESPALI 16

$$\Delta_c = 500$$

$$M_{min} = 10^{14} M_\odot / h$$

M20