



# COMoving Computer Acceleration: N-body simulations in an emulated frame of reference

Colloque national Action Dark Energy 2024

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**anr**®

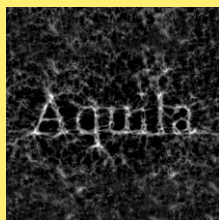
ANR-23-CE46-0006: INFOCW

Institut d'Astrophysique de Paris  
CNRS & Sorbonne Université



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In collaboration with:  
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Ludvig Doerer (Stockholm University)



and the Aquila Consortium

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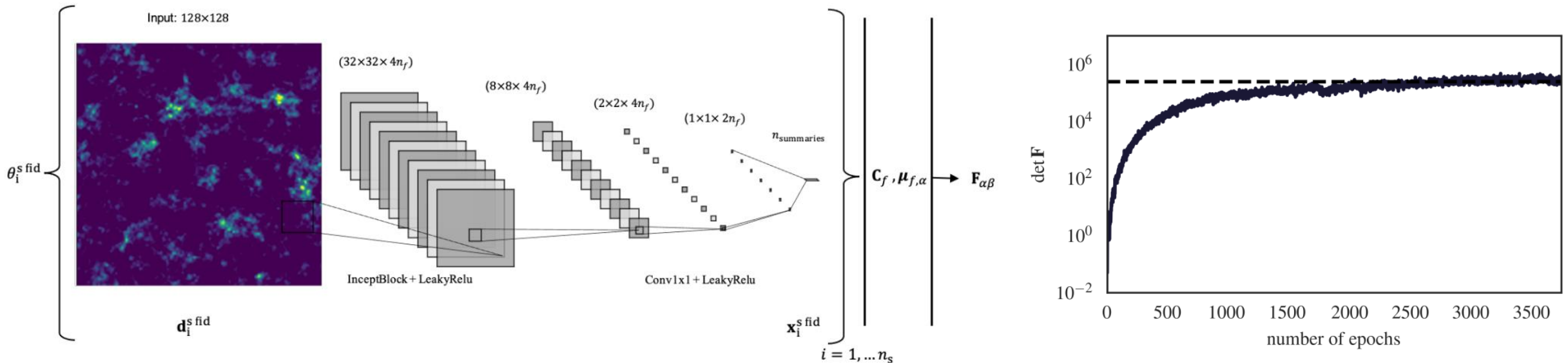
**28 OCTOBER 2024**



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# Safe uses of machine learning

- Safe use: applying machine learning (ML) — in particular neural networks (NNs) — in a way that ensures the results are either correct by construction or, at worst, suboptimal.
- Safe uses of ML include:
  - Ensuring certifiability of the model used for parameter inference and model comparison.
  - Eliminating the requirement for explainability.
- Examples: denoising autoencoders (DAE) to build summaries, information-maximising neural networks (IMNN) for simulation-based inference (SBI).



Charnock *et al.*, 1802.03537, Makinen *et al.*, 2107.07405, Makinen *et al.*, 2410.07548

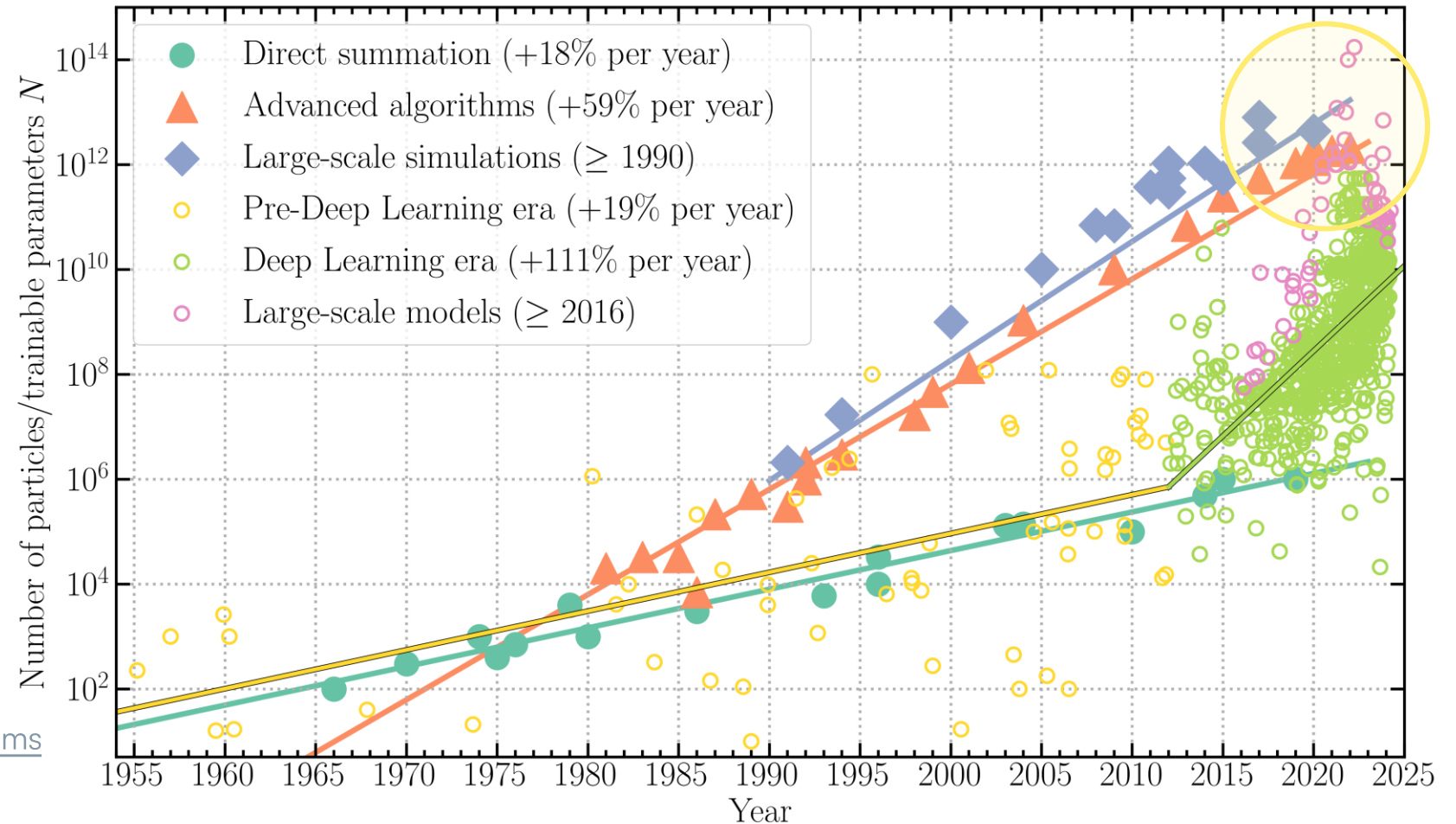


# Comparative growth of models and methods

- Amdahl's law: latency kills the gains of parallelisation.

[Amdahl 1967, doi:10.1145/1465482.1465560](https://doi.org/10.1145/1465482.1465560)

- Machine learning (ML) has caught up with the largest cosmological simulations!



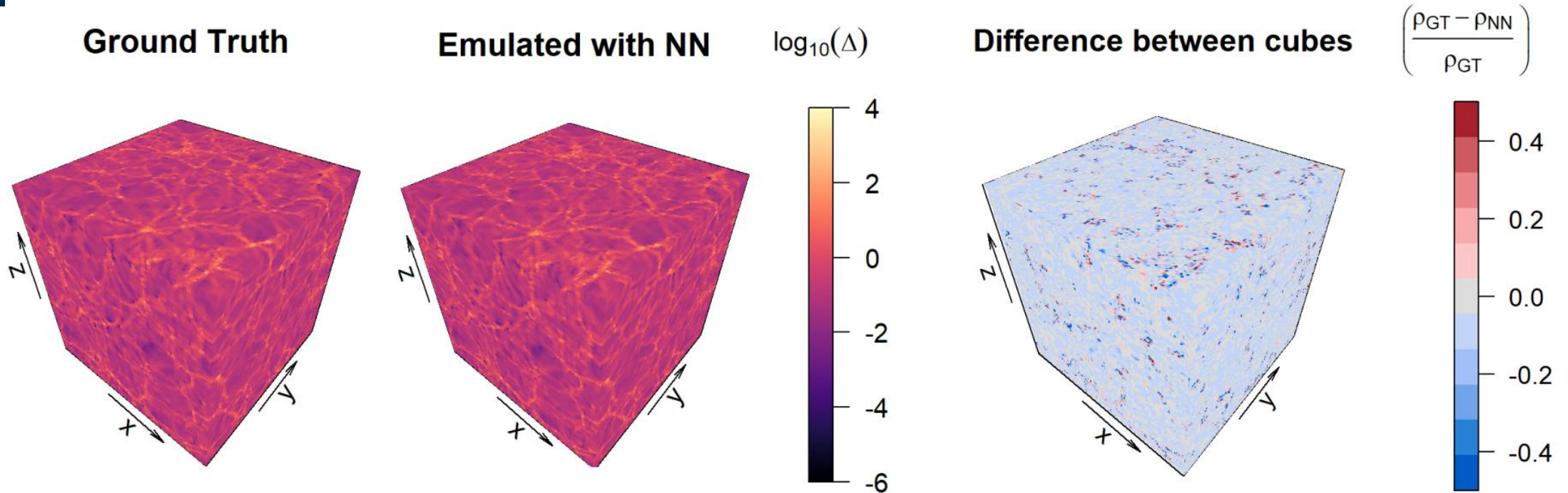
Cosmological simulations:

[Github:florent-leclercq/Moore\\_low\\_cosmosims](https://github.com/florent-leclercq/Moore_low_cosmosims)

IA models: data from [epochai.org](https://epochai.org)



# Emulation of N-body simulations



- Pleasantly fast, but what about the accuracy?
- There remains an emulation error [up to  $\mathcal{O}(10\%)$ ] that we cannot ever correct for.
- Using these emulators as forward models **does not qualify as a safe use** of NNs.

[He et al., 1811.06533](#), [Lucie-Smith et al., 1802.04271](#), [Jamieson et al., 2206.04594](#), [Conceição et al., 2304.06099](#), [Doeser et al., 2312.09271](#), [Jamieson et al., 2408.07699](#)



# The tCOLA framework: (temporal) COmoving Lagrangian Acceleration

- Idea behind tCOLA: we can make use of the analytical solution at large scales and early times: Lagrangian perturbation theory (LPT).

- Write the displacement vector as:

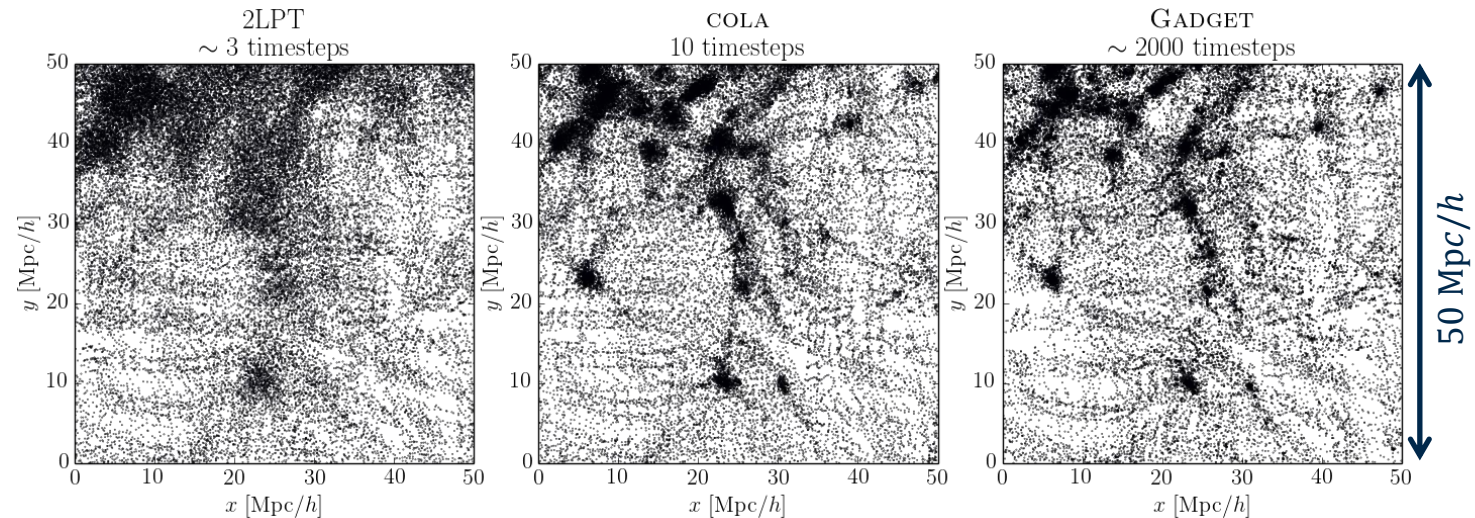
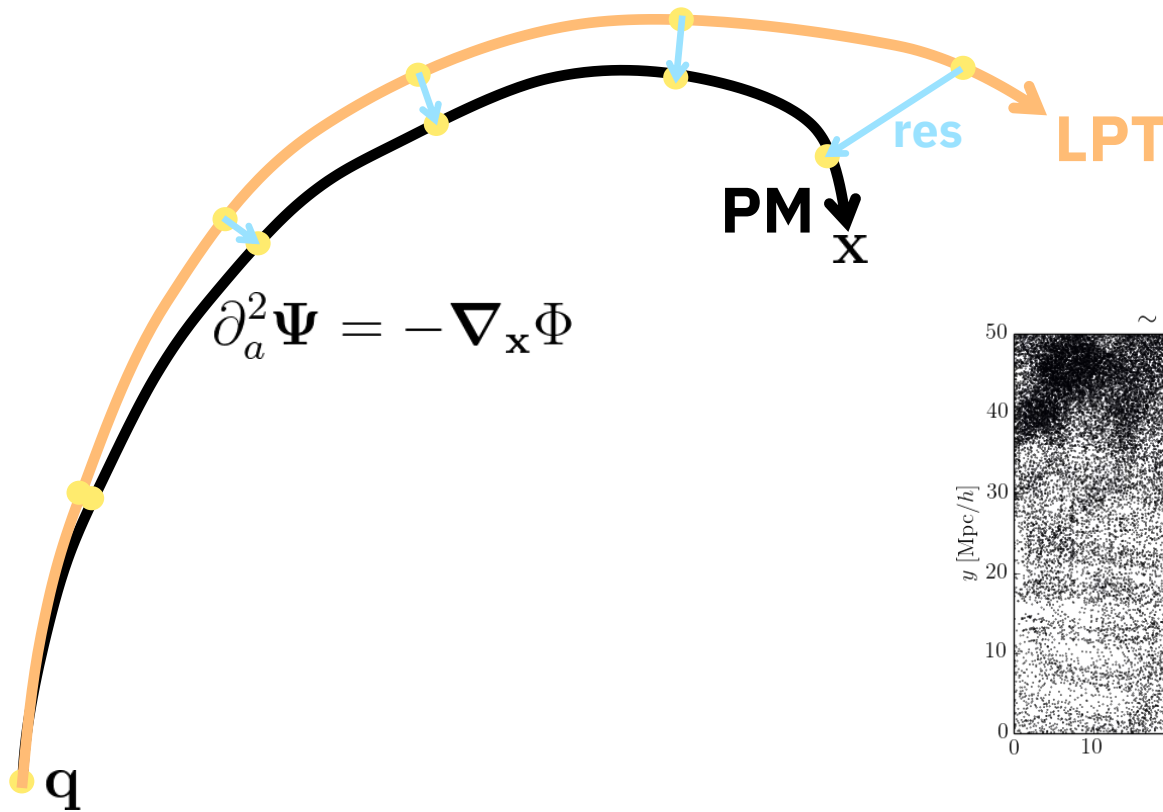
$$\Psi = \Psi_{\text{LPT}} + \Psi_{\text{res}}^{\text{COLA}} \quad (\mathbf{x} = \mathbf{q} + \Psi)$$

Tassev & Zaldarriaga, 1203.5785

- Equation of motion (omitted constants and Hubble expansion):

$$\partial_a^2 \Psi_{\text{res}}^{\text{COLA}} = \partial_a^2 (\Psi - \Psi_{\text{LPT}}) = -\nabla_{\mathbf{x}} \Phi - \partial_a^2 \Psi_{\text{LPT}}$$

Analytical solutions!

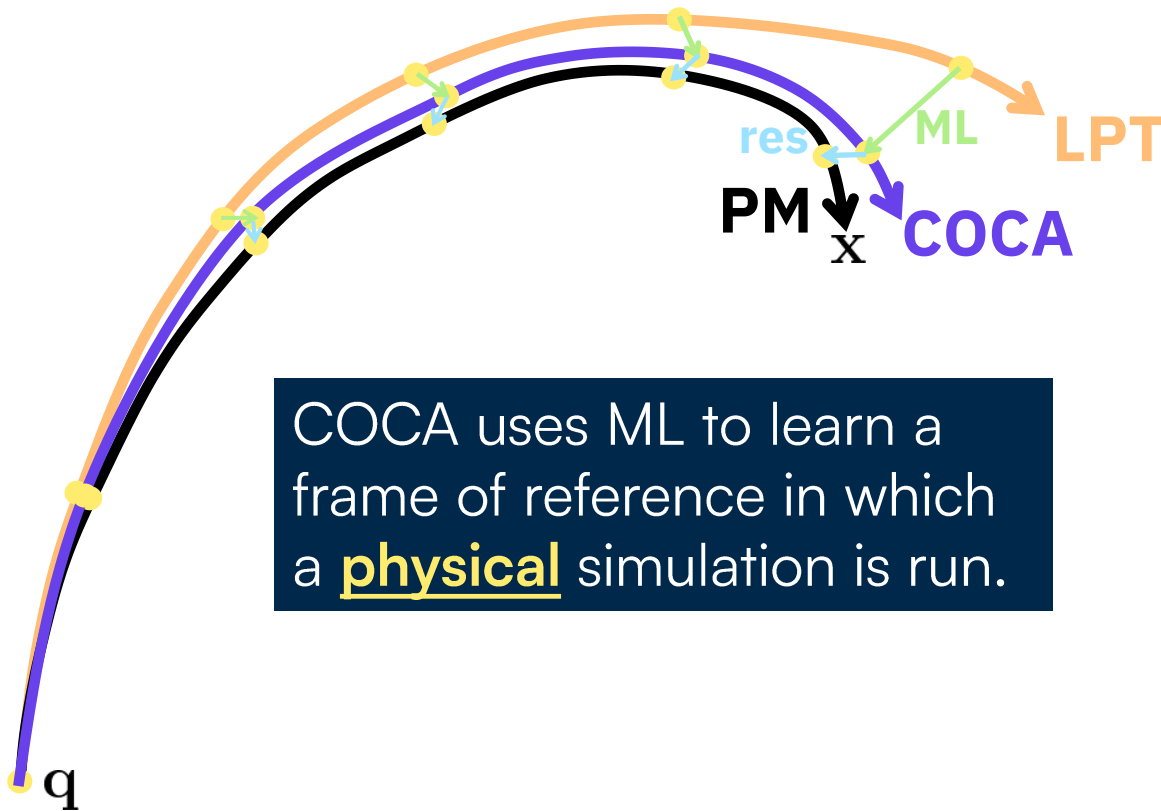


Tassev, Zaldarriaga & Eisenstein, 1301.0322



# The tCOCA framework: (temporal) COmoving Computer Acceleration

- The idea behind tCOCA: the easiest simulation to run is the one where nothing moves!



- Write the displacement vector as:

$$\Psi = \Psi_{\text{LPT}} + \Psi_{\text{ML}} + \Psi_{\text{res}}^{\text{COCA}} \quad (\mathbf{x} = \mathbf{q} + \Psi)$$

- Equation of motion (omitted constants and Hubble expansion):

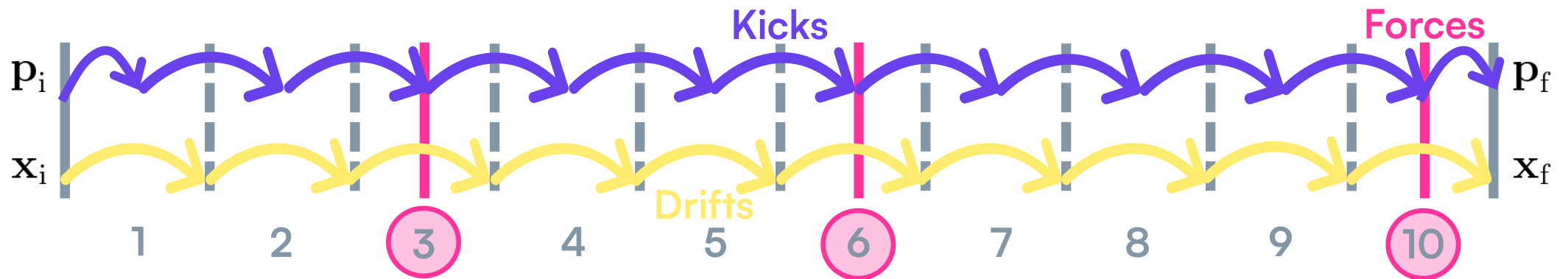
$$\partial_a^2 \Psi_{\text{res}}^{\text{COCA}} = -\nabla_{\mathbf{x}} \Phi - \partial_a^2 \Psi_{\text{LPT}} - \partial_a^2 \Psi_{\text{ML}}$$

$$\Leftrightarrow \partial_a^2 \Psi = -\nabla_{\mathbf{x}} \Phi$$

- With COCA:
  - Any emulation error will be corrected by solving the correct physical equation of motion.
  - Any ML algorithm can do the job!
  - Building a data model is a safe use of ML.

# Time stepping and force calculations in COCA

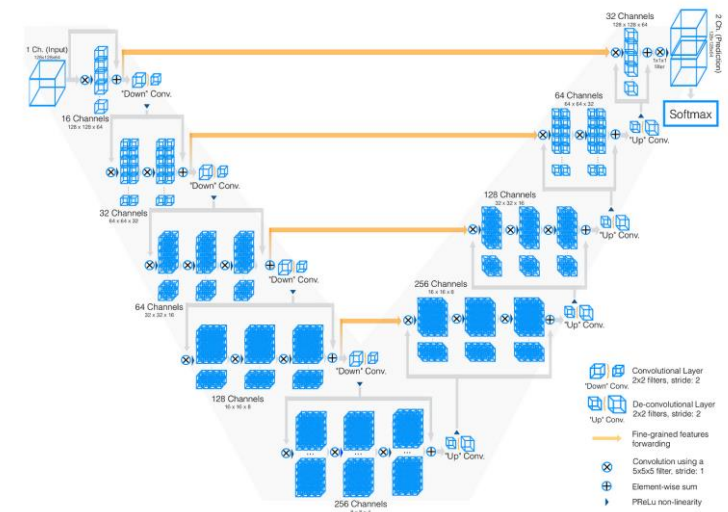
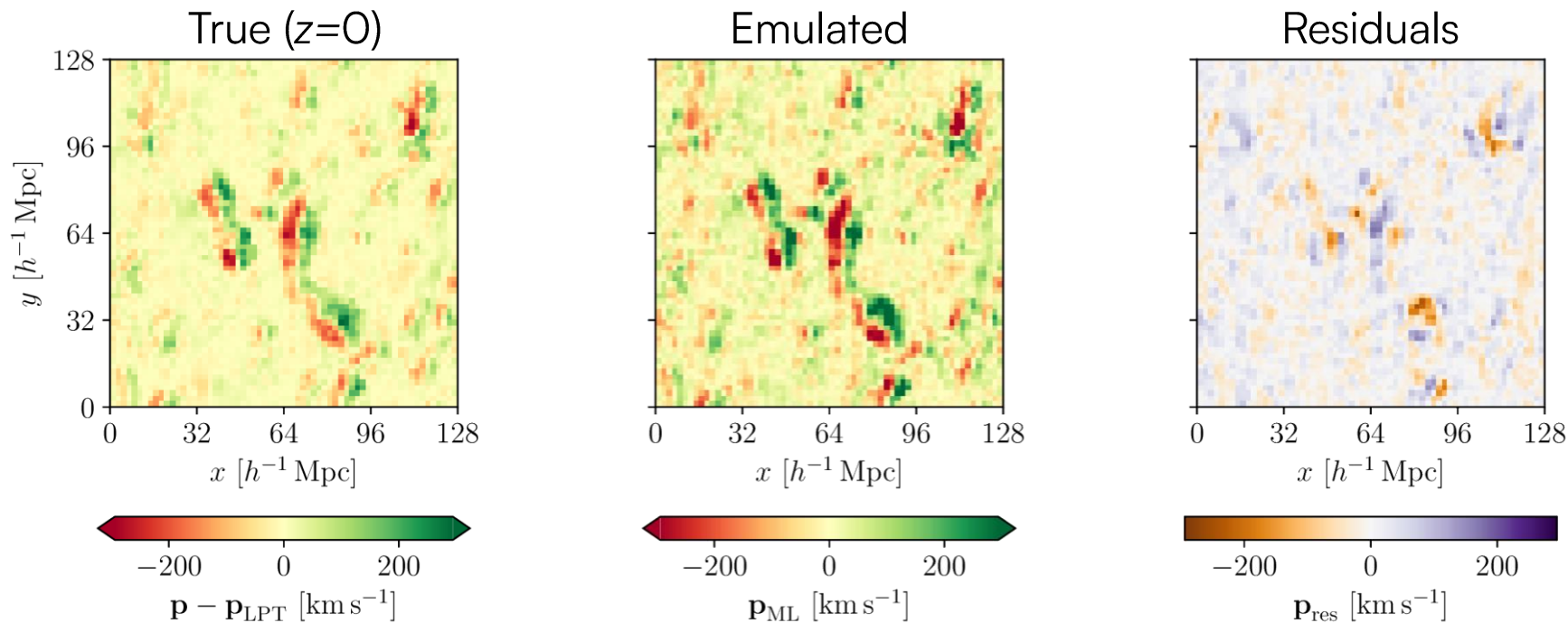
- Our implementation of COCA in the Simbelmyne code uses the standard [Kick-Drift-Kick](#) (leapfrog) discretisation of the equation of motion.  
<https://simbelmyne.florent-leclercq.eu> — [Bitbucket: florent-leclercq/simbelmyne](https://bitbucket.org/florent-leclercq/simbelmyne)
- Learning the new frame of reference means emulating the COLA residual momenta at every time step:  $\mathbf{p}_{\text{res}}^{\text{COLA}} = \mathbf{p} - \mathbf{p}_{\text{LPT}}$ .
- When the emulation error is small ( $\mathbf{p}_{\text{ML}} \approx \mathbf{p}_{\text{res}}^{\text{COLA}}$ ), particles are already at rest in the COCA frame of reference, so it is [unnecessary to compute forces at every step](#).



- A good frame-of-reference emulator therefore makes COCA cheaper than COLA.

# Training a frame-of-reference emulator for COCA

- We trained a **styled V-net** with initial density field and scale factor as inputs; frame of reference (particles' residual momenta) as output.
- We used 100 training COLA simulations with  $L=128 \text{ Mpc}/h$ ,  $N=64^3$  particles, and 277 epochs.
- We can predict the frame of reference to run test COCA simulations:



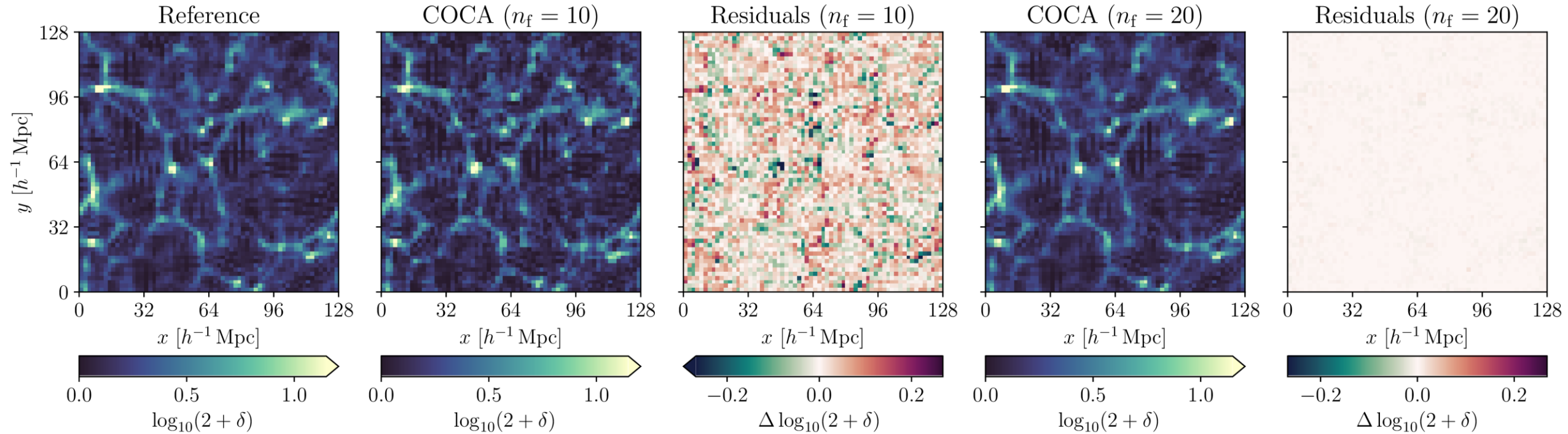
Milletari, Navab & Ahmadi, 1606.04797

Bartlett, Chiarenza, Doeser & FL, 2409.02154





# Results: COCA density field



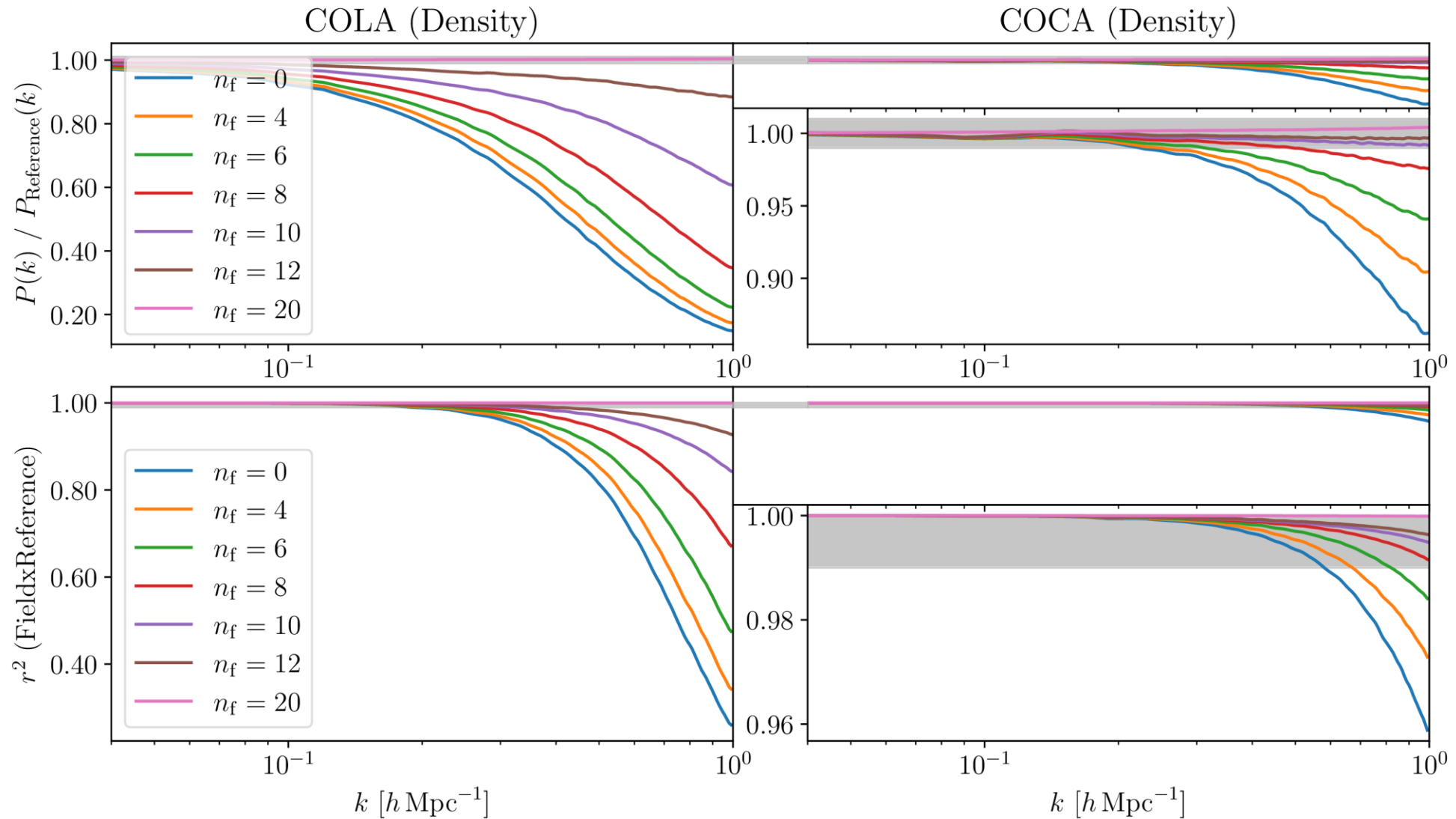
[Bartlett, Chiarenza, Doeser & FL, 2409.02154](#)



Florent Leclercq

COmoving Computer Acceleration (COCA)

# Results: COCA two-point statistics

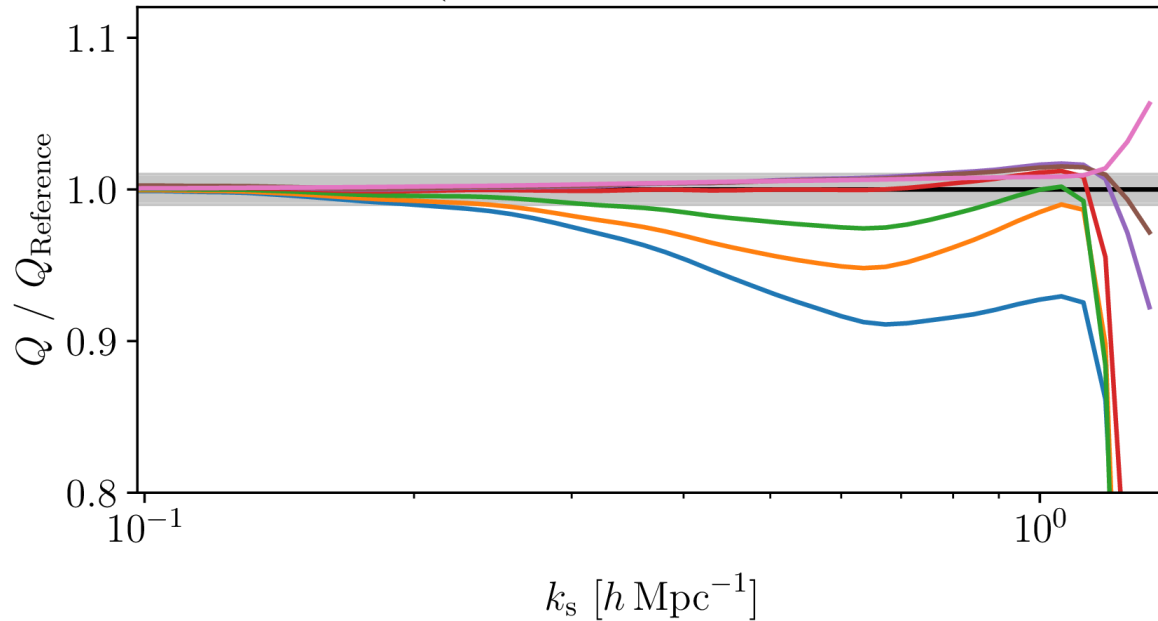


Bartlett, Chiarenza, Doeser & FL, 2409.02154

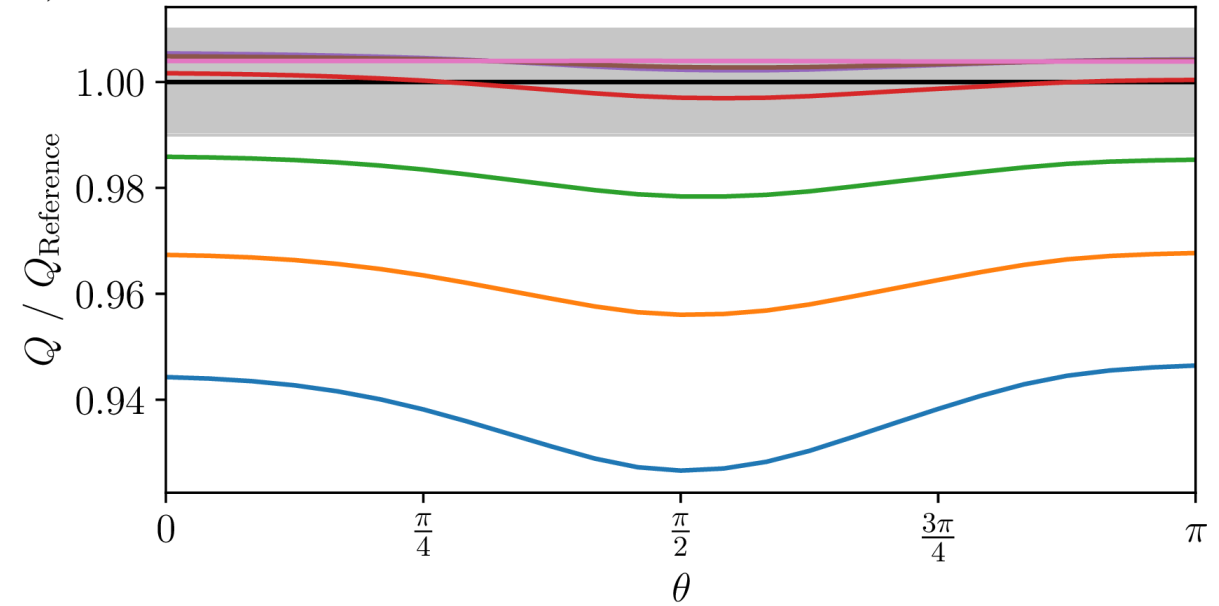


# Results: COCA three-point statistics

COCA: Squeezed ( $k_1 = k_2 = k_s, k_\ell = 9.8 \times 10^{-2} h \text{ Mpc}^{-1}$ )



COCA:  $k_1 = 0.1 h \text{ Mpc}^{-1}, k_2 = 1.0 h \text{ Mpc}^{-1}$



Reference
   $n_f = 0$ 
  $n_f = 4$ 
  $n_f = 6$ 
  $n_f = 8$ 
  $n_f = 10$ 
  $n_f = 12$ 
  $n_f = 20$

[Bartlett, Chiarenza, Doerer & FL, 2409.02154](#)

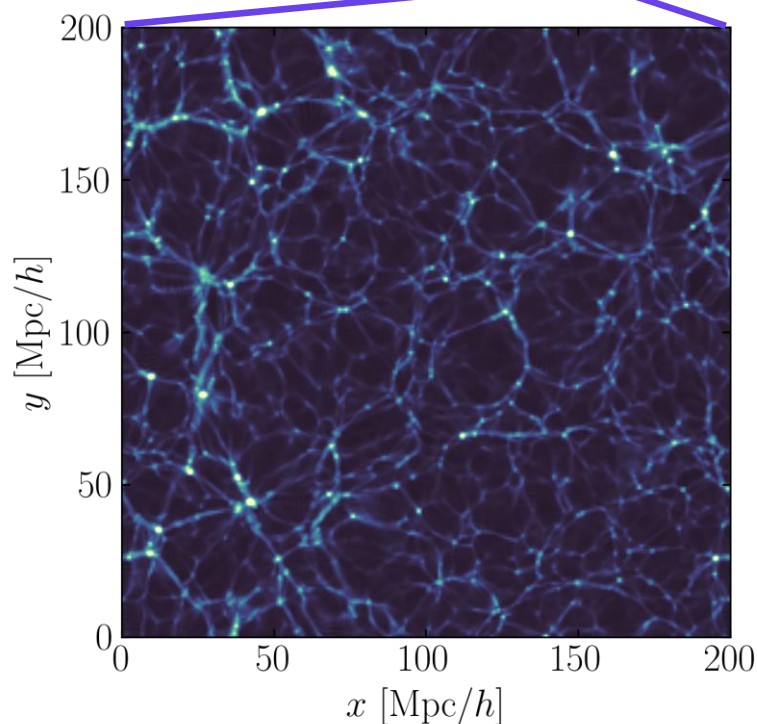


# Perfectly parallel cosmological simulations using **spatial** comoving Lagrangian acceleration (sCOLA)

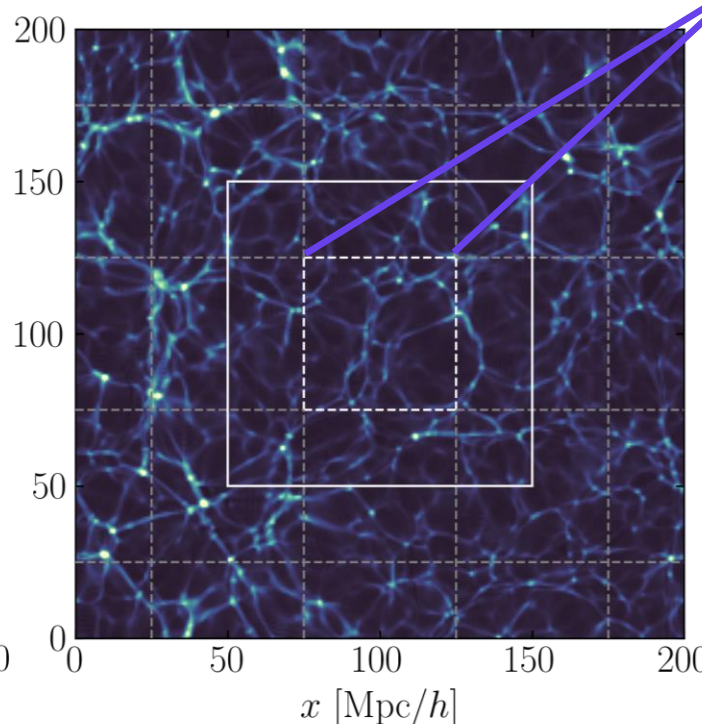
- Can we decouple sub-volumes by using the large-scale solution?

$$\partial_a^2 \Psi = -\nabla_{\mathbf{x}} \left[ \underbrace{\Delta^{-1} \delta}_{\text{non-local}} \right] \iff \partial_a^2 (\Psi - \underbrace{\Psi_{\text{l.s.}}}_{\text{local}}) = -\nabla_{\mathbf{x}} \left[ \underbrace{\Delta^{-1} (\delta - \delta_{\text{l.s.}})}_{\text{local}} \right]$$

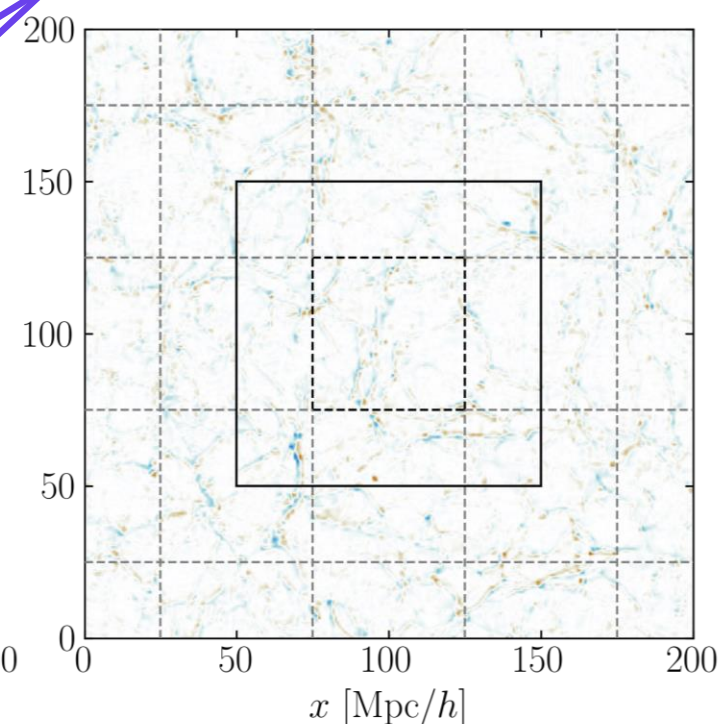
LPT so far  
(analytical solution) → sCOLA;  
soon ML solution → sCOCA



tCOLA (reference)



sCOLA



Difference

[FL, Faure, Lavaux, Wandelt, Jaffe, Heavens, Percival & Noûs, 2003.04925](#)

Publicly available implementation:  
[Bitbucket:florent-leclercq/simbelmyne/](https://bitbucket.org/florent-leclercq/simbelmyne/)





## Conclusions

- Safe uses of neural networks exist, where:
  - The answer is correct by construction or suboptimal,
  - Use for physics (parameter inference, model comparison) is certifiably robust,
  - Explainability is not needed.
- tCOCA reimagines the use of neural networks for emulating  $N$ -body simulations:
  - It generalises the idea of tCOLA: running simulations in a new frame of reference,
  - It solves the correct equations of motion, so it is a safe use of neural networks,
  - It makes simulations cheaper by skipping unnecessary force evaluations.
- The large-scale ML solution can also be used to decouple sub-volumes, in the same spirit as sCOLA: the sCOCA framework!

# Acknowledgements, credits, contacts



## References:

- **Simbelmynë**: Leclercq, Jasche & Wandelt 2014, 1403.1260, *Bayesian analysis of the dynamic cosmic web in the SDSS galaxy survey* — <https://simbelmyne.florent-leclercq.eu>
- **sCOLA**: Leclercq et al. 2020, 2003.04925, *Perfectly parallel cosmological simulations using spatial comoving Lagrangian acceleration*
- **COCA**: Bartlett, Chiarenza, Doerer & Leclercq 2024, 2409.02154, *COMoving Computer Acceleration (COCA): N-body simulations in an emulated frame of reference*

[www.florent-leclercq.eu](http://www.florent-leclercq.eu)

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