Reevaluating the cosmological redshift: insights into inhomogeneities and irreversible processes

MdlS, AIM: P. Tremblin
ENS-Lyon, Exeter: G. Chabrier

TC2024: https://www.arxiv.org/pdf/2407.10622
Accepted in A&A

- Outline of the talk

- The redshift calculation is not covariant in the standard cosmological model

- With a covariant formulation, entropy production during virialization of the large scale structure can replace dark energy

- Lack of covariance of the redshift calculation

Metric in expansion $ds^2 = -c^2dt^2 + a(t)(dr^2 + r^2 d\Omega^2)$ $H = \dot{a}/a$

Redshift: $1 + z = \exp \int_{t_{src}}^{t_{dst}} H dt \left(1 + \frac{u_g}{c}\right)$

Cosmological redshift

Caused by expansion of the coordinate system along the line of sight (« space » expansion)

Doppler shift

Caused by peculiar motions at velocities $u_g = a(t_{src}) dr/dt$ only at emission

- Lack of covariance of the redshift calculation

Metric in expansion $ds^2 = -c^2dt^2 + a(t)(dr^2 + r^2d\Omega^2)$ $H = \dot{a}/a$

Redshift: $1 + z = \exp \int_{t_{\rm src}}^{t_{\rm dst}} Hdt \left(1 + \frac{u_g}{c}\right)$

Cosmological redshift
Caused by expansion of the coordinate system along the line of sight (« space » expansion)

Doppler shift
Caused by peculiar motions at velocities $u_g = a(t_{\rm src})dr/dt$ only at emission

But, because of general relativity, the coordinate system is arbitrary, what happens if we take another expanding coordinate system?

- Lack of covariance of the redshift calculation

A simple test case: a FLRW solution in non-co-moving coordinates

$$1 + z = \exp \int_{t_{\rm src}}^{t_{\rm dst}} H dt$$

- Lack of covariance of the redshift calculation

A simple test case: a FLRW solution in non-co-moving coordinates

$$1 + z = \exp \int_{t_{\mathrm{src}}}^{t_{\mathrm{dst}}} H dt$$

$$r_d = a_d(t) r \longrightarrow \begin{gathered} a(t) = a_s(t) a_d(t) \\ H_s = \dot{a}_s / a_s, H_d = \dot{a}_d / a_d \\ H = H_s + H_d \end{gathered} \longrightarrow \begin{gathered} ds^2 \approx -c^2 dt_d^2 + a_s(t)(dr_d^2 + r_d^2 d\Omega^2) \\ u_g = a_s H_d r_d \end{gathered}$$

- Lack of covariance of the redshift calculation

A simple test case: a FLRW solution in non-co-moving coordinates

$$1 + z = \exp \int_{t_{\rm src}}^{t_{\rm dst}} H dt$$

$$r_d = a_d(t) r \longrightarrow \begin{array}{c} a(t) = a_s(t) a_d(t) \\ H_s = \dot{a}_s / a_s, H_d = \dot{a}_d / a_d \\ H = H_s + H_d \end{array} \longrightarrow \begin{array}{c} ds^2 \approx -c^2 dt_d^2 + a_s(t)(dr_d^2 + r_d^2 d\Omega^2) \\ u_g = a_s H_d r_d \end{array}$$

Redshift: $$1 + z \approx \exp \int_{t_{\rm src}}^{t_{\rm dst}} H_s dt \left(1 + \frac{a_s H_d r_d}{c} \right)$$

« Cosmological » redshift
Caused by expansion of the coordinate system along the line of sight (« space » expansion)

« Doppler » shift
Caused by non-comoving motions at velocities $u_g = a_s H_d r_d$ only at emission

The cosmological redshift is transformed into a Doppler shift through a coordinate transformation

- Lack of covariance of the redshift calculation

An even simpler test case : empty space with no expansion in an arbitrary expanding coordinate system

$$1 + z = 1$$

$$r_d = a_d(t)r \longrightarrow \begin{array}{c} a_d(t) = 1/a_s(t) \\ H_d = -H_s \\ H = H_s + H_d = 0 \end{array} \longrightarrow \begin{array}{c} ds^2 \approx -c^2dt_d^2 + a_s(t)(dr_d^2 + r_d^2 d\Omega^2) \\ u_g = -H_s r \end{array}$$

Redshift: $1 + z \approx 1 + \int_{t_{\rm src}}^{t_{\rm dst}} (H_s(t) - H_s(t_{\rm src}))dt \neq 1$

« Cosmological » redshift
Caused by expansion of the coordinate system along the line of sight (« space » expansion)

« Doppler » shift
Caused by non-comoving motions at velocities $u_g = a_s H_d r_d$ only at emission

a shift at emission cannot compensate the expansion of the coordinate system along the line of sight, this redshift calculation is not covariant

- Covariant calculation of the redshift

But we can use a covariant calculation of the redshift (Rasanen 2009) of photons following null geodesics:

Redshift: $$1 + z = \exp \int_{t_{\rm src}}^{t_{\rm dst}} \theta dt/3$$

With theta the local expansion rate along time-like geodesics involving the expansion of the coordinate system and the velocity field:

$$\theta = 3\tilde{H} + \vec{\nabla} \cdot \vec{\tilde{u}_g}$$

With $\tilde{H} = H$, $\vec{\nabla} \cdot \vec{\tilde{u}_g} = 0$ in comoving coordinates

With $\tilde{H} = H_s$, $\vec{\nabla} \cdot \vec{\tilde{u}_g} = 3H_d$ in non-comoving coordinates

$\longrightarrow$ $$1 + z = \exp \int_{t_{\rm src}}^{t_{\rm dst}} H dt$$

- Covariant calculation of the redshift

But we can use a covariant calculation of the redshift (Rasanen 2009) of photons following null geodesics:

Redshift: $$1 + z = \exp \int_{t_{\rm src}}^{t_{\rm dst}} \theta dt/3$$

With theta the local expansion rate along time-like geodesics involving the expansion of the coordinate system and the velocity field:

$$\theta = 3\tilde{H} + \vec{\nabla} \cdot \vec{\tilde{u}_g}$$

With $\tilde{H} = H$, $\vec{\nabla} \cdot \vec{\tilde{u}_g} = 0$ in comoving coordinates

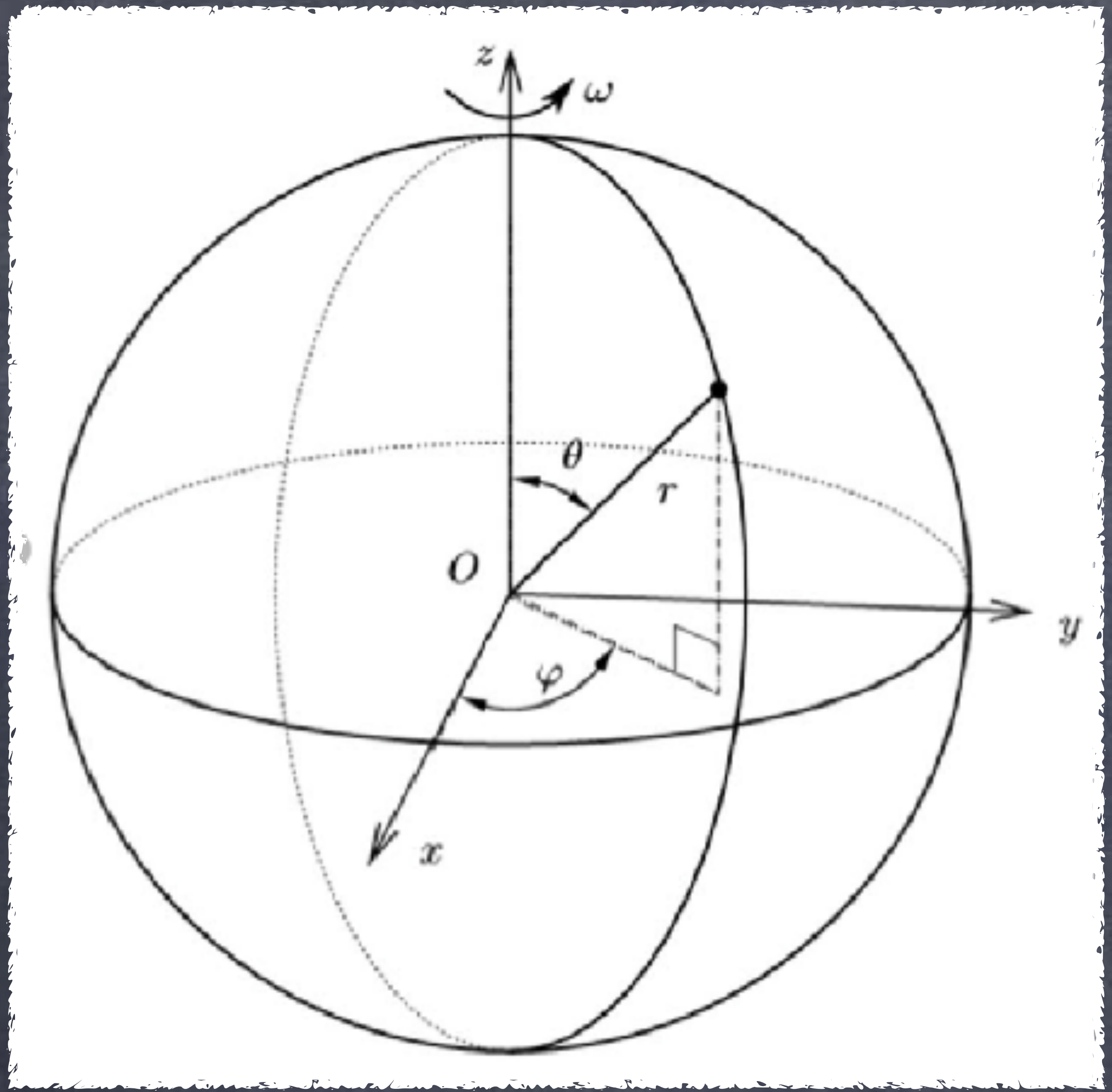
With $\tilde{H} = H_s$, $\vec{\nabla} \cdot \vec{\tilde{u}_g} = 3H_d$ in non-comoving coordinates

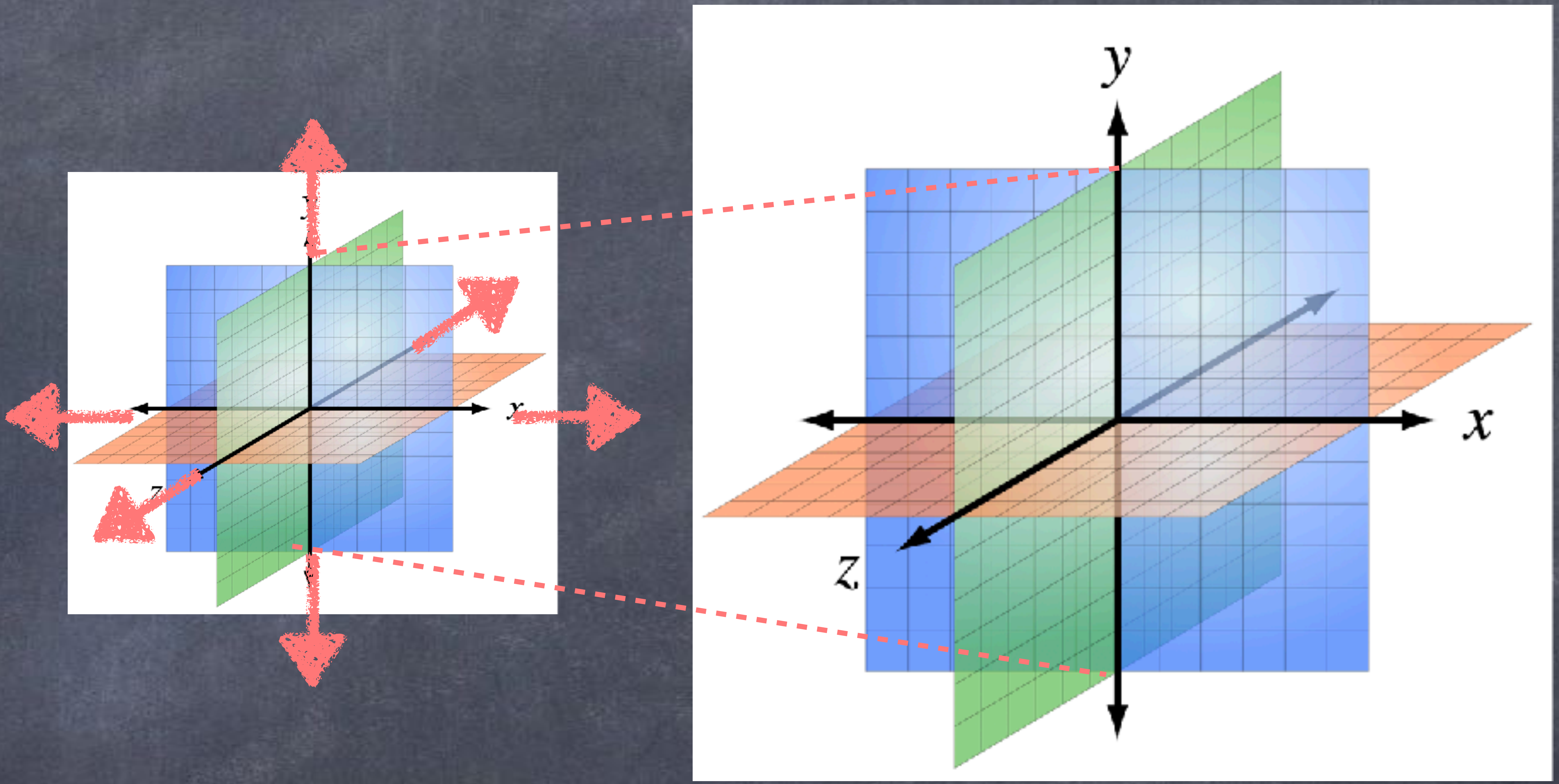
$\longrightarrow$ $$1 + z = \exp \int_{t_{\rm src}}^{t_{\rm dst}} H dt$$

Timelike geodesics, i.e. the true physical entity in GR

We advocate to stop using the expression « expansion of space » and replace it with expansion along timelike geodesics

- Is space rotating because one uses a rotating coordinate system?





$$ds^2 = (-c^2 + r_c^2\omega^2)dt'^2 + dr'^2 + r'^2 d\varphi^2 + dz'^2 - 2r_c^2\omega d\varphi'^2 dt'$$

$$ds^2 = -c^2 dt'^2 + a(t)^2 dx'^2 + a(t)^2 dy'^2 + a(t)^2 dz'^2$$

There is nothing a-priori related to gravity in an expanding coordinate system

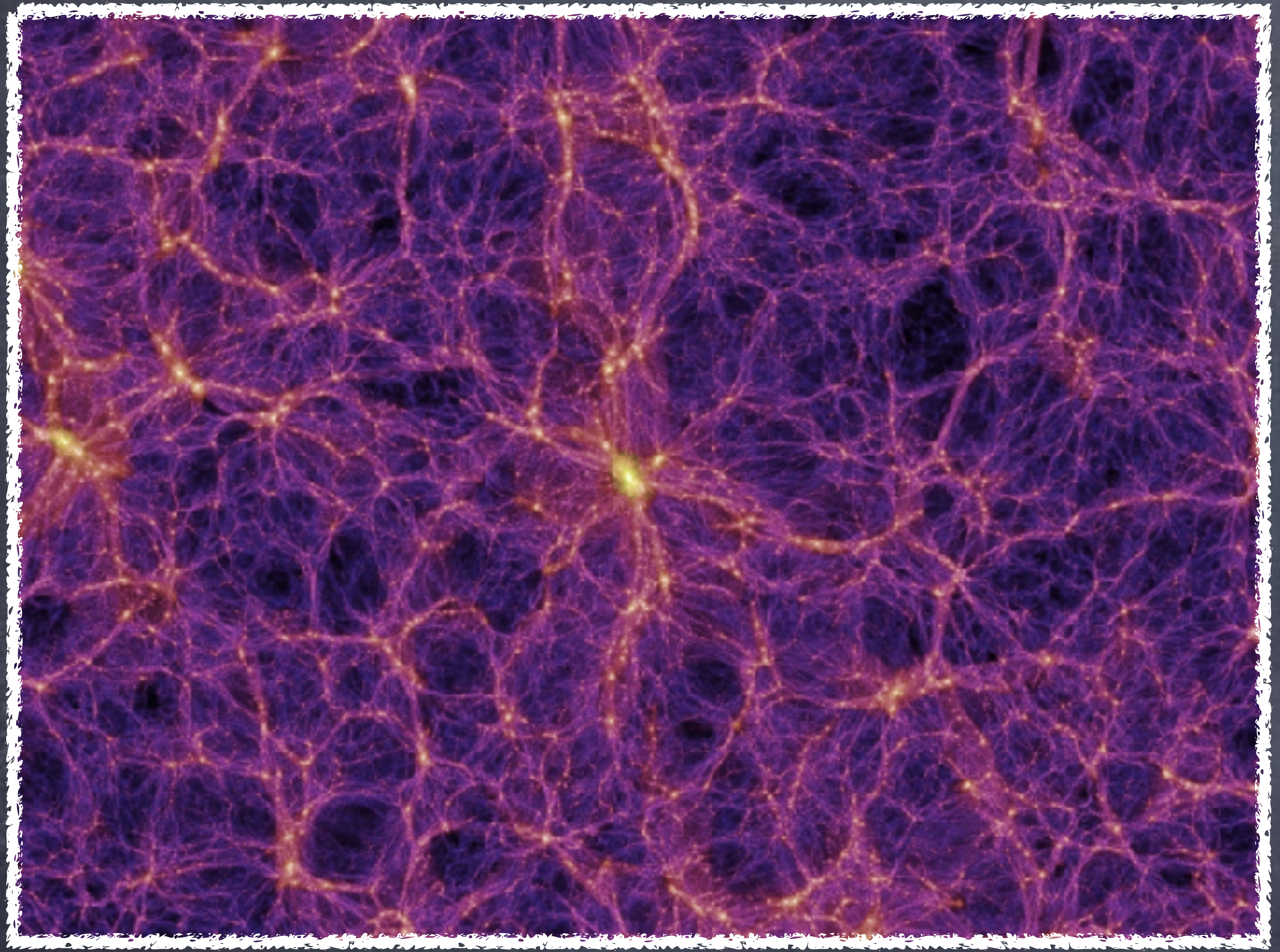
- Outline of the talk

- The redshift calculation is not covariant in the standard cosmological model

- With a covariant formulation, entropy production during virialization of the large scale structure can replace dark energy

- Covariant calculation of the redshift

Let us take a tracer of time-like geodesics: dark matter and a background coordinate system without dark energy



Redshift: $1 + z = \exp \int_{t_{\rm src}}^{t_{\rm dst}} \theta dt/3$

$\theta = 3H + \vec{\nabla} \cdot \vec{u}_g$ with $\frac{\dot{a}^2}{a^2} \equiv H^2 = \frac{8\pi G}{3}\bar{\rho}$

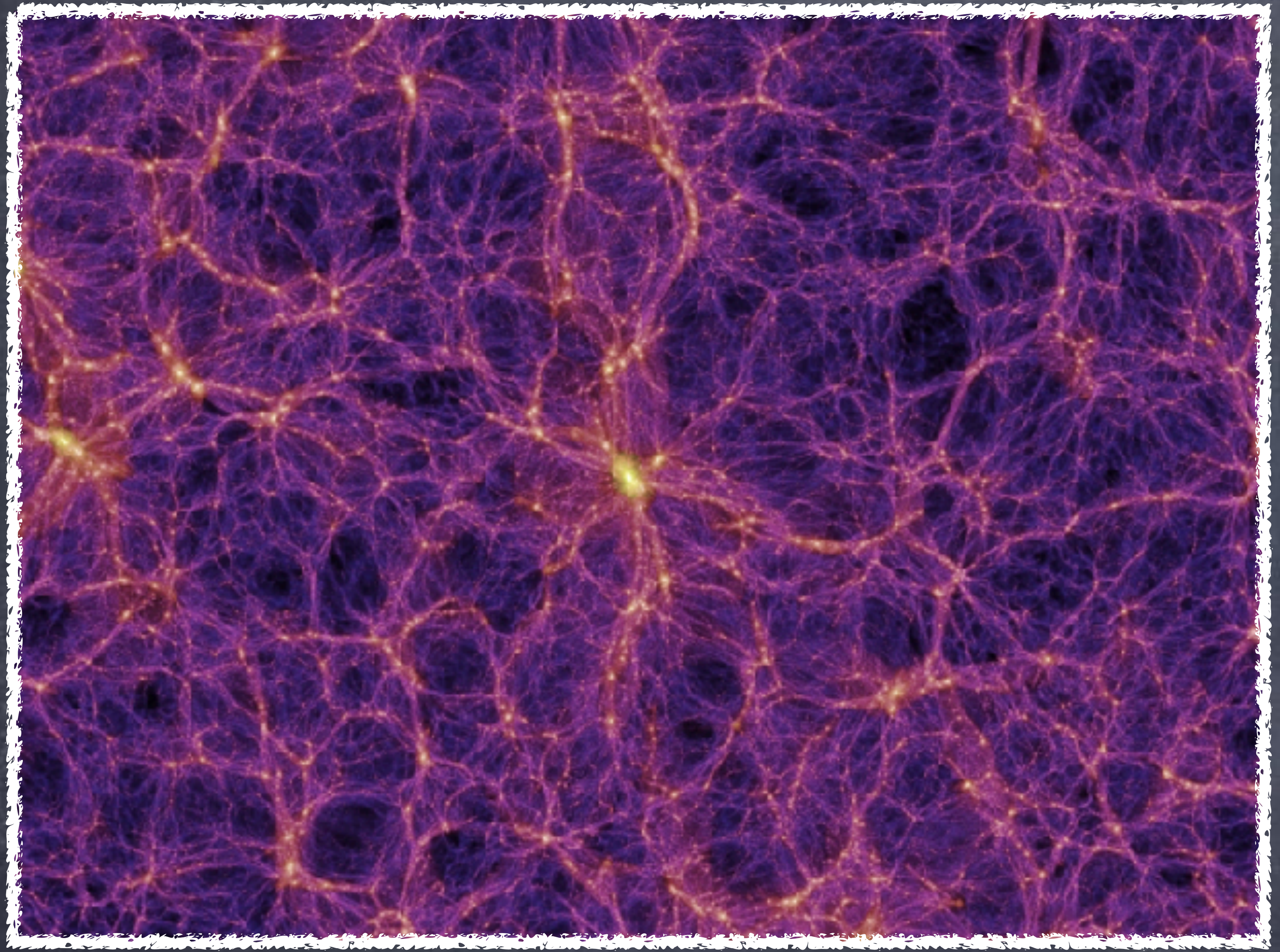
Backreaction of inhomogeneities is negligible and dark energy needed if

$\langle \theta \rangle = 3H \qquad \langle \vec{\nabla} \cdot \vec{u}_g \rangle \approx 0$

And not just. $u_g \ll c$ Even if perturbation theory works for the dynamics

- Covariant calculation of the redshift

Let us take a tracer of time-like geodesics: dark matter and a background coordinate system without dark energy



4 demonstrations showing that virialization and entropy production can replace dark energy

- Qualitative
- Brute-force
- Elegant
- Theoretical (-> TC2024)

Redshift: $1 + z = \exp \int_{t_{\rm src}}^{t_{\rm dst}} \theta dt/3$

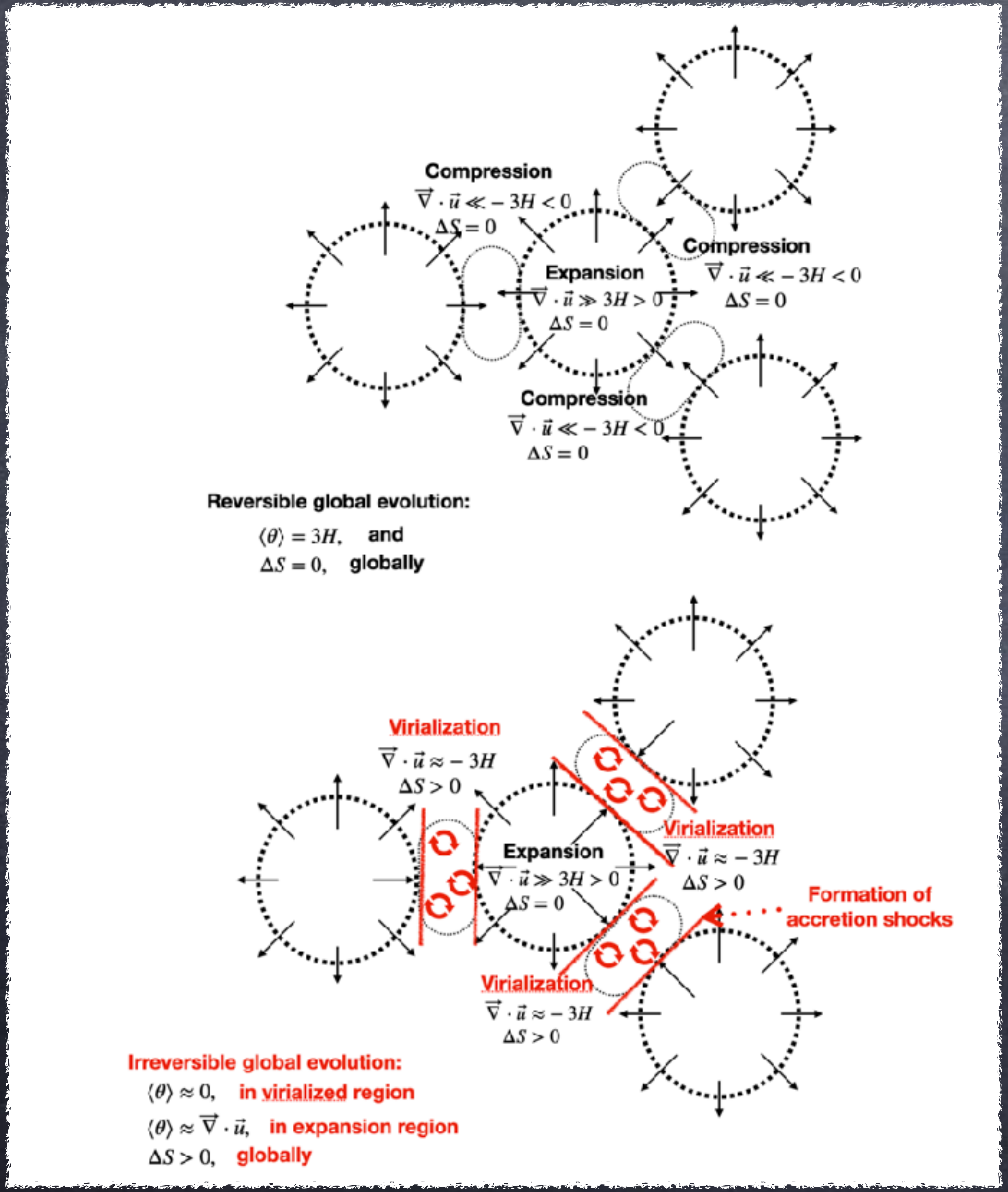
$\theta = 3H + \vec{\nabla} \cdot \vec{u}_g$ With $\frac{\dot{a}^2}{a^2} \equiv H^2 = \frac{8\pi G}{3}\bar{\rho}$

Backreaction of inhomogeneities is negligible and dark energy needed if

$\langle \theta \rangle = 3H \qquad \langle \vec{\nabla} \cdot \vec{u}_g \rangle \approx 0$

And not just. $u_g \ll c$ Even if perturbation theory works for the dynamics

- The role of irreversible processes: Qualitative demonstration



$$\langle \vec{\nabla} \cdot \vec{u_g} \rangle \approx 0$$

if compression mode in structures with divU<0 compensate the expansion of the voids with divU>0, a reversible system.

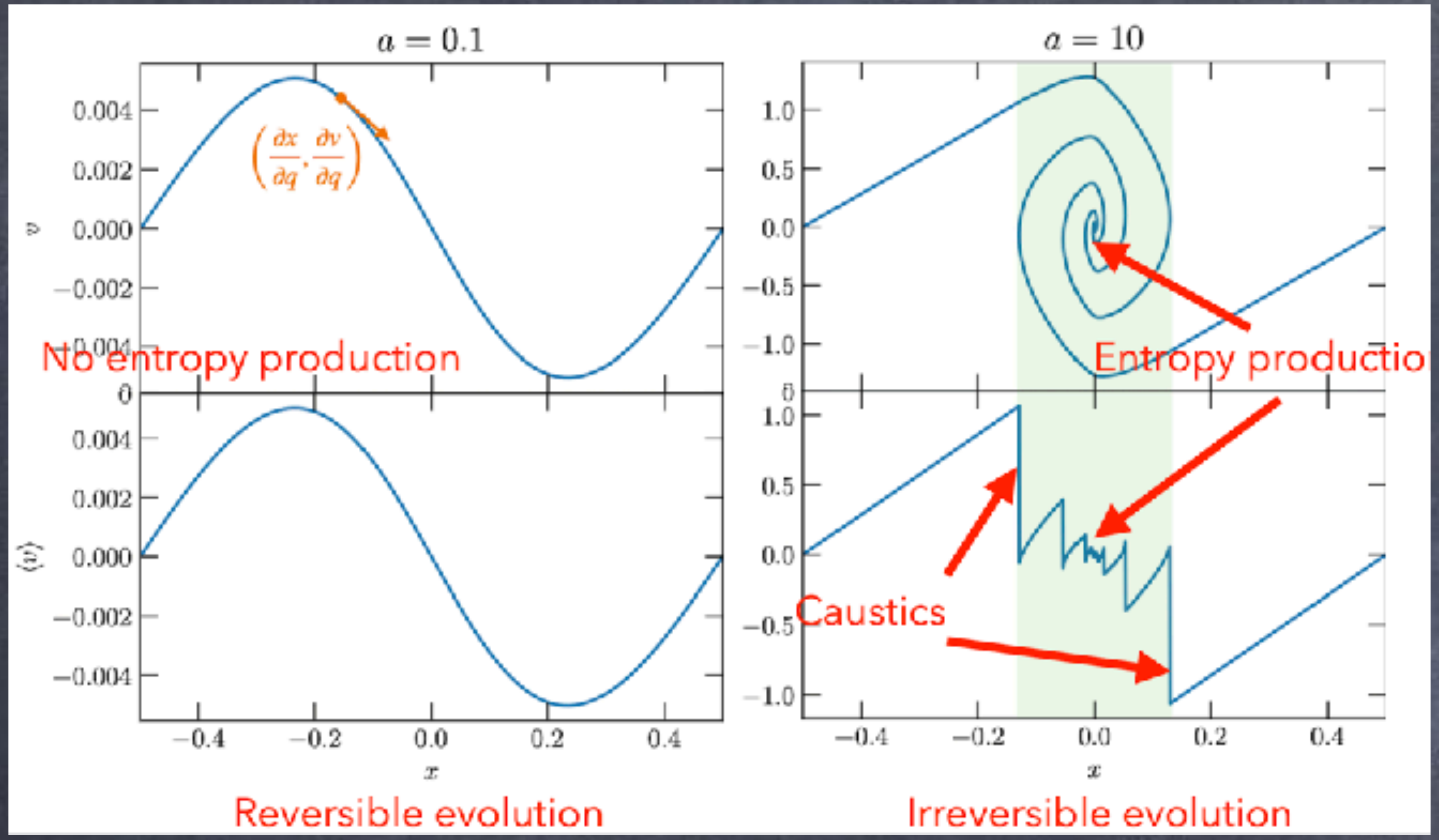
$$\langle \vec{\nabla} \cdot \vec{u_g} \rangle \neq 0$$

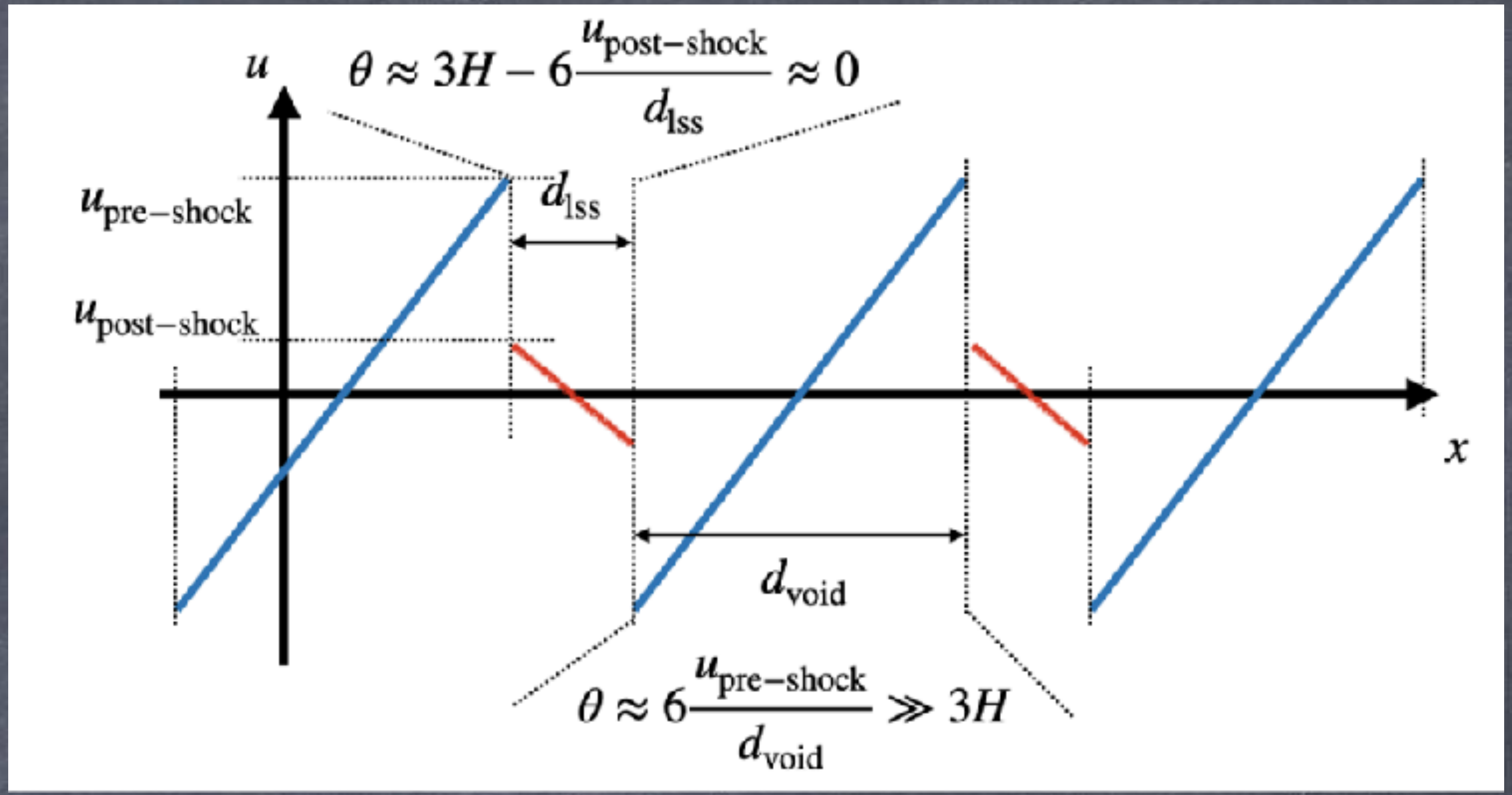
once virialization happens, compression mode in the structures is dissipated and cannot compensate void expansion in $\langle\theta\rangle$. This is associated to entropy production caused by violent relaxation (Lynden-Bell 1967) and irreversible systems.

As virialization of dark matter stabilizes the large-scale collapse, it plays the role of « anti-gravity » needed for dark energy, while remaining in a regime of pure gravitational dynamics

- The role of irreversible processes: brute-force demonstration

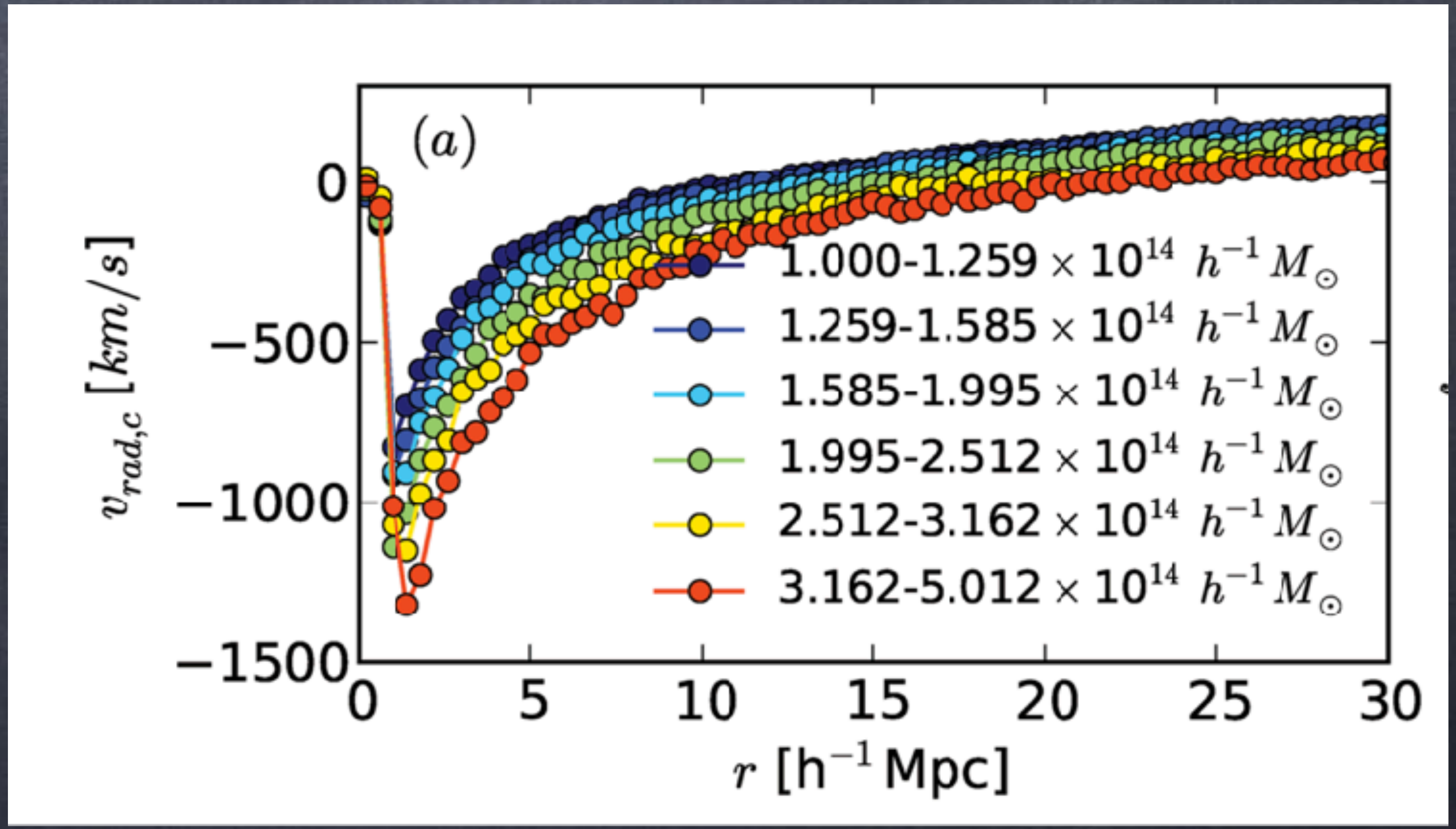
divU is not a conserved quantity, its production is linked to velocity discontinuity -> caustics (or non-collisional shocks) after shell crossing





θ in virialized structures is close to zero.

θ in voids measured from numerical simulations of LSS formation (e.g. Zu & Weinberg 2013).



$\langle\theta\rangle$ is of the the order of 70 km/s/Mpc

- The role of irreversible processes: Elegant demonstration

Use the second principle of thermodynamics, assuming a virialized system in quasi-equilibrium (e.g. with a Jeans stress tensor)

Take the evolution of a reversible and irreversible system reaching the same internal energy, temperature and compressibility factor (Z)

$$\begin{aligned} T\Delta s &= \Delta e + Zk_bT\log(V_{\rm irrev}/V_{\rm ini}), \\ 0 &= \Delta e + Zk_bT\log(V_{\rm rev}/V_{\rm ini}), \end{aligned}$$

Implies

$$\frac{V_{\rm irrev}}{V_{\rm ini}} = \frac{V_{\rm rev}}{V_{\rm ini}}\exp\left(\frac{\Delta s}{k_bZ}\right).$$

With mass conservation

$$\frac{d\log V}{dt} = \theta,$$

$$1+z = \exp\int_{t_{\rm src}}^{t_{\rm dst}} \theta dt/3$$

We get a redshift expression explicitly linked to entropy production that can never average to zero since $\Delta s > 0$

$$\begin{aligned} 1+z &= \left(\frac{V_{\rm irrev}(t_{\rm dst})}{V_{\rm ini}(t_{\rm src})}\right)^{1/3}, \\ &= \left(\frac{V_{\rm rev}}{V_{\rm ini}}\right)^{1/3}\exp\left(\frac{\Delta s}{3k_bZ}\right), \\ &= \frac{a(t_{\rm dst})}{a(t_{\rm src})}\exp\left(\frac{\Delta s}{3k_bZ}\right) \end{aligned}$$

$$1+z = \frac{a(t_{\rm dst})}{a(t_{\rm src})}\exp\int_{t_{\rm src}}^{t_{\rm dst}} \vec{\nabla}\cdot\vec{u_g}\,dt/3$$

- Link between entropy production and an effective cosmological constant

Because of relativity, we can choose any coordinate system in expansion that we want: the value of the cosmological constant is per-se arbitrary. The implicit choice in LCDM is to introduce a cosmological constant such that $\langle \vec{\nabla} \cdot \vec{u}_g \rangle = 0$

$$1 + z = \exp \int_{t_{\rm src}}^{t_{\rm dst}} \theta dt/3 \;=\; \frac{a(t_{\rm dst})}{a(t_{\rm src})} \exp \int_{t_{\rm src}}^{t_{\rm dst}} \vec{\nabla} \cdot \vec{u}_g dt/3$$
$$= \frac{a(t_{\rm dst})}{a(t_{\rm src})} \exp \frac{\Delta s}{3Zk_b}$$
$$= \frac{a_{\rm eff}(t_{\rm dst})}{a_{\rm eff}(t_{\rm src})}$$

→

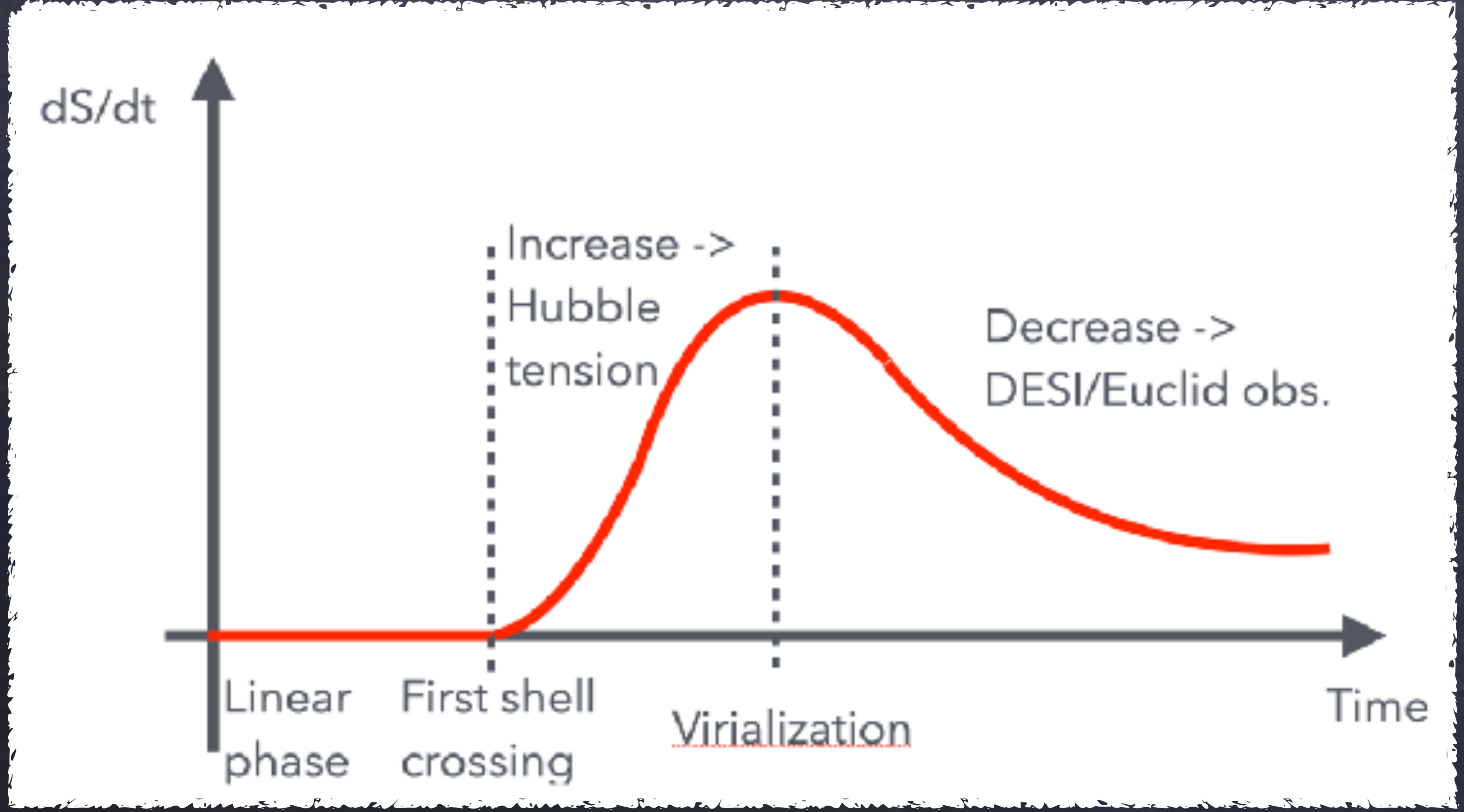
$$H_{\rm eff}(t)^2 = \frac{8\pi G\bar{\rho}}{3} + \frac{\Lambda_{\rm eff}(t)c^2}{3},$$
$$\Lambda_{\rm eff}(t) = \frac{2H}{c^2 Zk_b}\frac{ds}{dt} + \frac{1}{3c^2Z^2k_b^2}\left(\frac{ds}{dt}\right)^2.$$

Similar to a dynamical dark energy in a quintessence model

↓

DESI and Euclid observations can be used to constrain the history of entropy production during the virialization of the LSS

←



- Conclusions

- The redshift calculation is not covariant in the standard cosmological model

We must do something about it, this is not compatible with relativity!

- With a covariant formulation, entropy production during virialization of the large scale structure can replace dark energy

Use DESI/Euclid observations to constrain the evolution of entropy production through the history of the Universe

- Thank you! Questions welcome

- Covariant calculation of the redshift

A covariant definition of the cosmological redshift and Doppler shift

Redshift: $1 + z = \exp \int_{t_{\rm src}}^{t_{\rm dst}} \theta dt/3 \left(1 + u_d/c\right)$

Cosmological redshift
Caused by expansion of time-like geodesics along the line of sight $\theta = 3\tilde{H} + \overrightarrow{\nabla} \cdot \overrightarrow{\tilde{u}_g}$

Doppler shift
Caused by non-geodesic motions $u_d = u_{\rm fluid} - \tilde{u}_g$ only at emission

Both definitions are now covariant !

- The role of irreversible processes: Theoretical demonstration

Discontinuities in GR (from lagrangian hydrodynamics see TC2024)

By using locally comoving (i.e. lagrangian) solution in spherical coordinates (Lemaitre Tollman Bondi solutions) with a local curvature E

$$ds^2 = -N(r,t)^2dt^2 + \frac{1}{1+2E(r,t)}R'^2dr^2 + R(r,t)^2d\Omega^2,$$

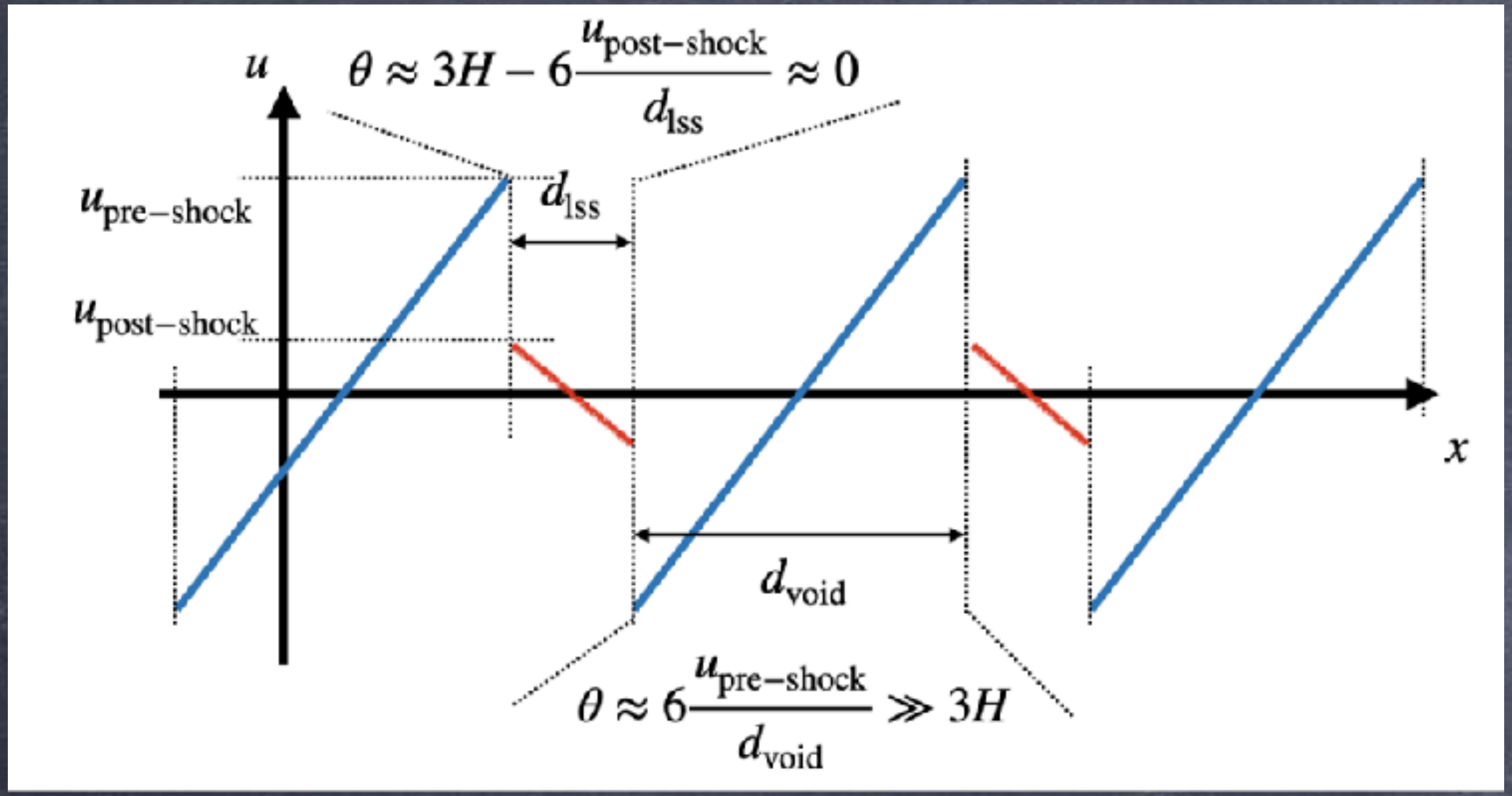
$$\begin{aligned} \frac{\dot{E}}{N} &= -\frac{1+2E}{\rho+\sigma}\frac{1}{R'}\frac{\partial\sigma}{\partial r}u, \\ \frac{\dot{\rho}}{N} &= -N(\rho+\sigma)\frac{1}{R^2R'}\frac{\partial(R^2u)}{\partial r}, \\ \frac{\dot{u}}{N} &= -\frac{GM}{R^2} - 4\pi\sigma R^2 - \frac{1}{R'}\frac{1+2E}{\rho+\sigma}\frac{\partial\sigma}{\partial r}. \end{aligned}$$

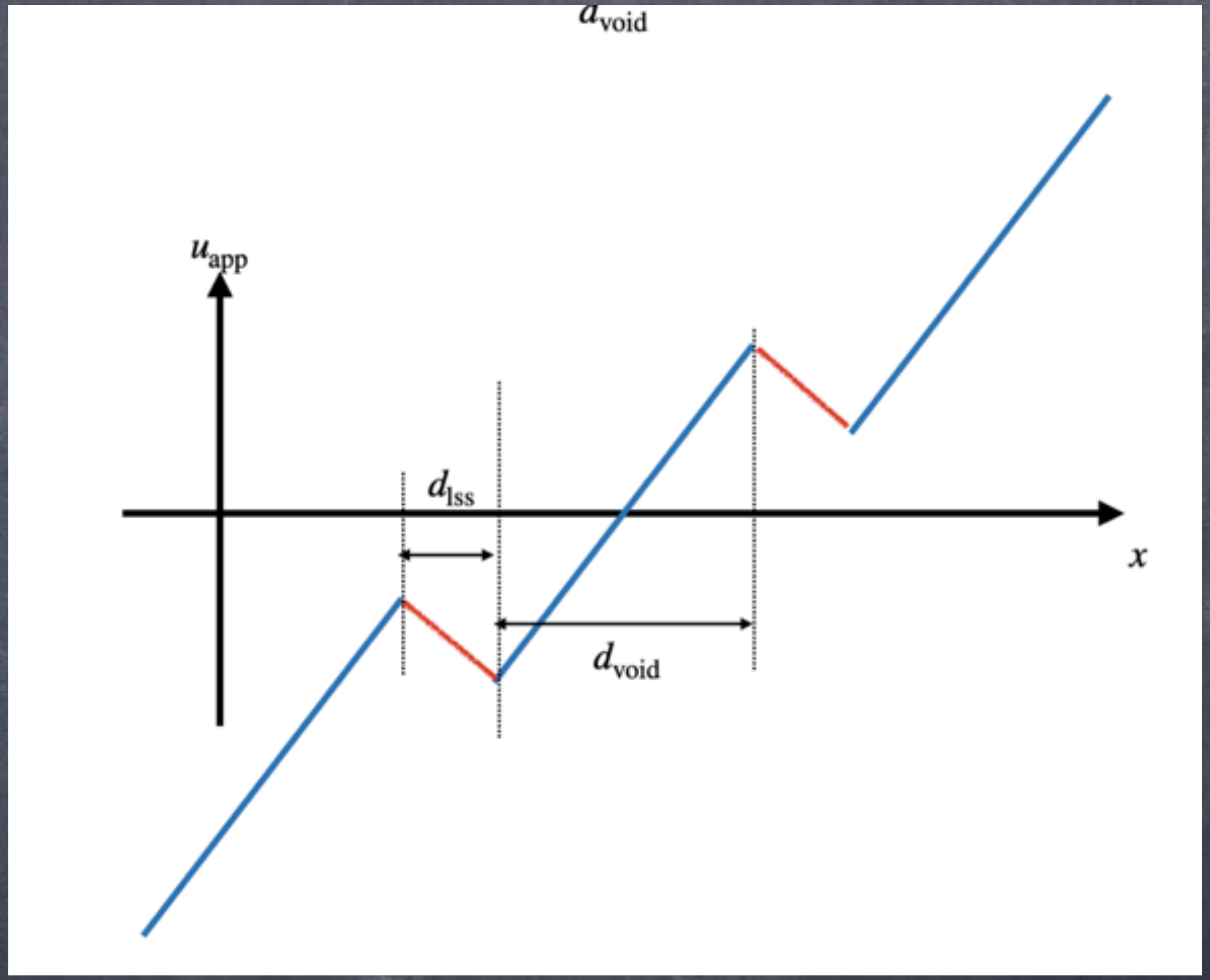
One can derive a conservative equation that links internal energy and the local curvature

$$\dot{E} + \dot{e} = -\frac{\partial}{\partial m}\left(\sigma u R^2\right).$$

The geometric version of entropy production, is the creation of negative curvature through discontinuities (a bound region of space), and again it cannot average to zero because of the 2nd principle of thermodynamics

- The role of irreversible processes: brute-force demonstration





- Covariance of the Hamiltonian constraint

Hamiltonian constraint involves (H,Phi)

$$\tilde{H}^2 + \frac{2}{3}\Delta\tilde{\Phi} = \frac{8\pi G}{3}\rho$$

Includes fictitious force (see centrifugal force)

Redshift involves (H, divU)

$$1 + z = \exp\int_{t_{\rm src}}^{t_{\rm dst}} \theta dt/3$$

$$\theta = 3\tilde{H} + \vec{\nabla}\cdot\vec{\tilde{u}_g}$$

The Hamiltonian constraint is a-priori not relevant for the redshift