Precision cosmology with LSST : Development of an unbiased cosmic shear estimator

Enya Van den Abeele, Pierre Astier, Anna Niemiec

Action Dark Energy - October 30th 2024

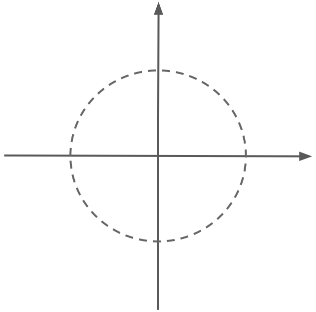
Cosmic shear

Distortion applied to image coordinates (linear approximation) :

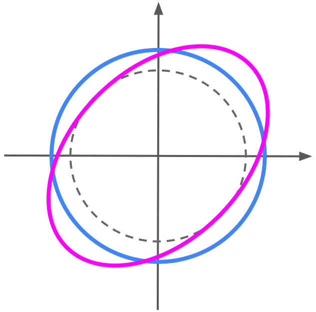
$$\boldsymbol{A} = \begin{pmatrix} 1 - \boxed{\kappa} - \boxed{\gamma_1} & -\gamma_2 \\ -\boxed{\gamma_2} & 1 - \kappa + \gamma_1 \end{pmatrix}$$

convergence

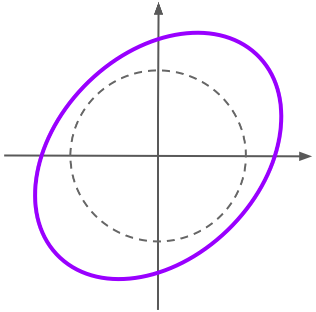
Source before distortion



Shear + Convergence



Source after distortion



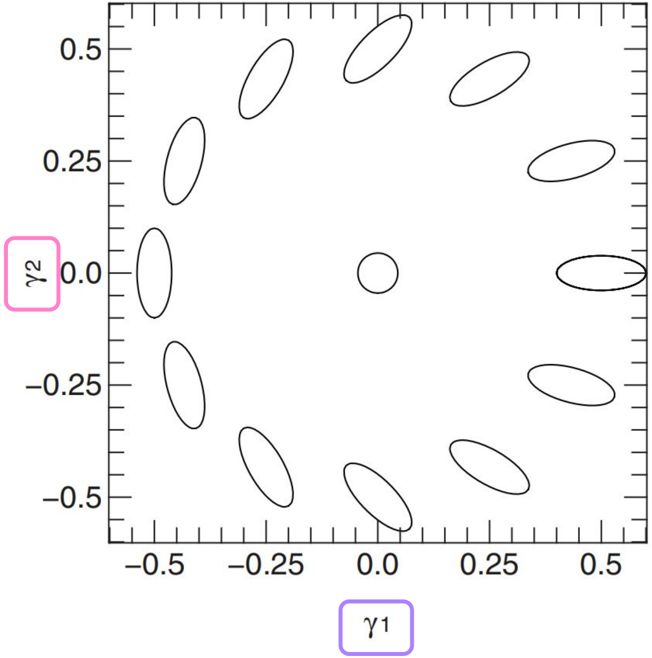


Figure : Martin Kilbinger

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convergence

Weak lensing limit → ***reduced shear***

$$|\gamma| \ll 1 \longrightarrow \boxed{g_i = \frac{\gamma_i}{1-\kappa}}$$

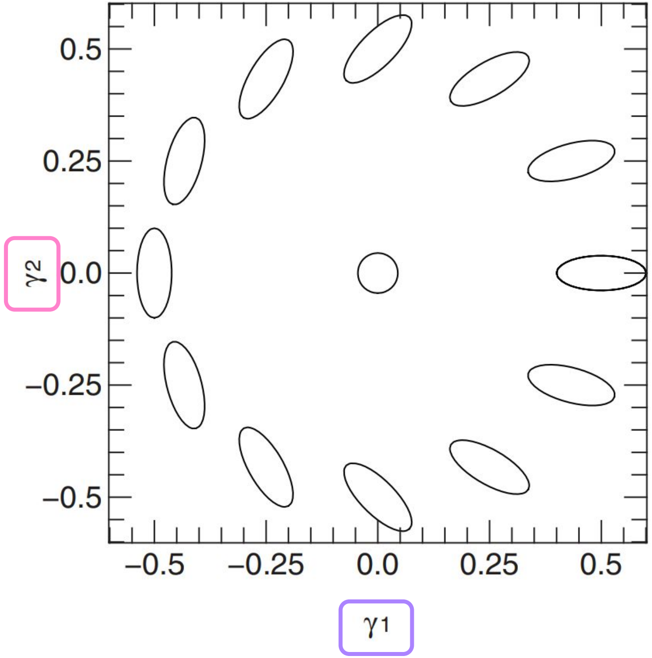
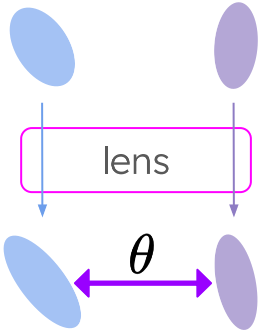


Figure : Martin Kilbinger

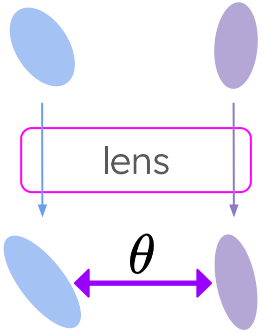
Cosmic shear and cosmological parameters

Two-point correlation function (2PCF) :

$$\xi_\pm(\theta) = \langle\gamma_t\gamma_t\rangle(\theta) \pm \langle\gamma_\times\gamma_\times\rangle(\theta) \quad \begin{cases} \gamma_t = -\gamma_1 \\ \gamma_\times = -\gamma_2 \end{cases}$$



Cosmic shear and cosmological parameters



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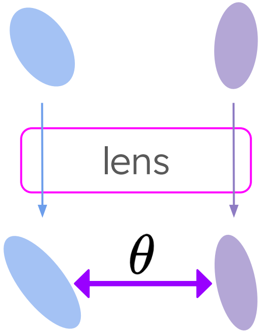
$$\xi_{\pm}(\theta) = \frac{1}{2\pi} \int d\ell \ell J_{0/4}(\ell\theta) \boxed{P_\kappa(\ell)} \longrightarrow \boldsymbol{\kappa}$$

Cosmic shear and cosmological parameters



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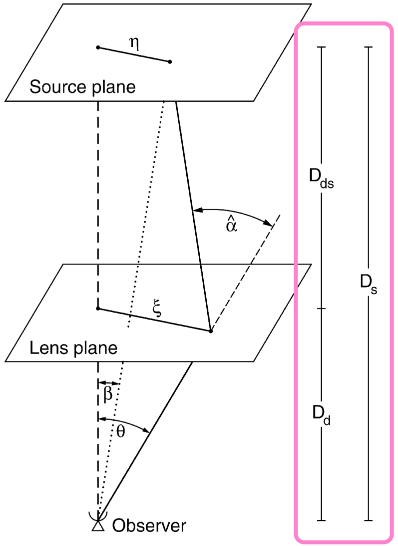
$$\xi_\pm(\theta) = \frac{1}{2\pi}\int d\ell\ell J_{0/4}(\ell\theta)\boxed{P_\kappa(\ell)} \xrightarrow{\quad} \boldsymbol{\kappa}$$

$$\kappa(z) = \frac{3}{2}\left(\frac{H_0}{c}\right)^2\Omega_m\int\frac{\boxed{\delta}}{a}\boxed{\frac{D_D D_{DS}}{D_S}}dz$$

Density contrast :

- Matter density fluctuations
- Evolution with redshift

Distances



LSST : *Large Survey of Space and Time*

Figure : rubinobservatory.org

First ground-based telescope designed for **weak lensing**

→ Primary mirror : 8.4m

→ 3200 megapixels camera

→ Scale : 0.2 arcsec/pixel

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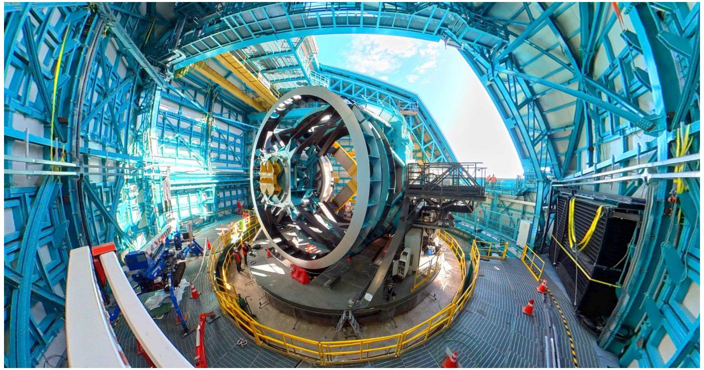

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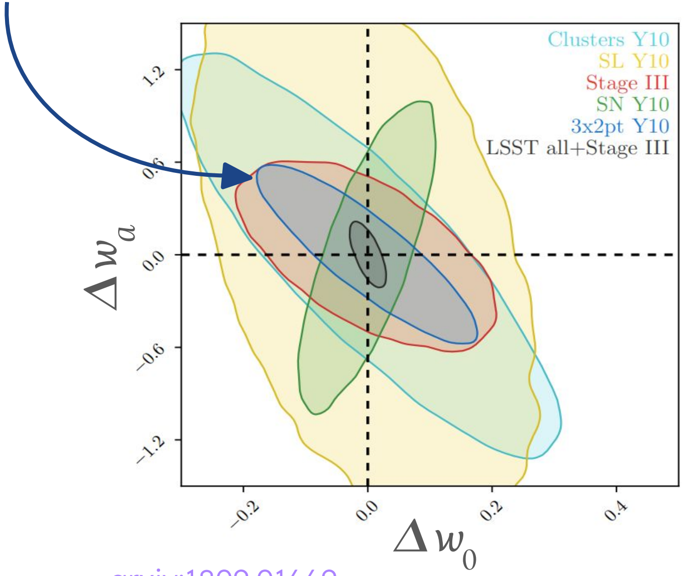
Most sensitive probe for constraining **dark energy**



arxiv:1809.01669

LSST : *Large Survey of Space and Time*

Figure : rubinobservatory.org

Unprecedented density of galaxies in images :

- Density : **~40 galaxies/arcmin²** (~**26** after blending and masking)
- Total lensing : **~1.7 billion** galaxies

DES : ~5.6 galaxies/arcmin²

HSC : ~15 galaxies/arcmin²

→ **Precision cosmology !**

Shear bias

Cosmological estimations with cosmic shear possible with LSST, but complex measurement associated with biases :

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$$g^{obs} = (1 + m)g^{true} + c$$

Shear bias

Cosmological estimations with cosmic shear possible with LSST, but complex measurement associated with biases :

$$g^{obs} = (1 + \boxed{m})g^{true} + \boxed{c}$$

$m < 1.5 \times 10^{-3}$
$c < 1.3 \times 10^{-4}$

(Cropper 2013)

Many sources :

- Poor estimator calibration
- PSF size
- Shot noise
- PSF anisotropy
- Astrometry
- Galaxy selection bias

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$$g^{obs} \longrightarrow \xi^{ij}_{+/-}(\theta) \longrightarrow P_\kappa(l) = f(\Omega_M, \sigma_8)$$

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bias on shear
=
bias on cosmology

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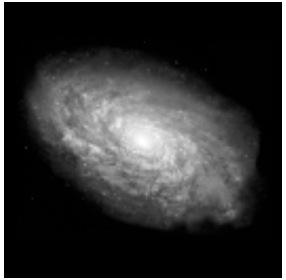
What shape do we measure ?



Galaxy with intrinsic ellipticity

e^i

Figures : Bridle et al. 2009

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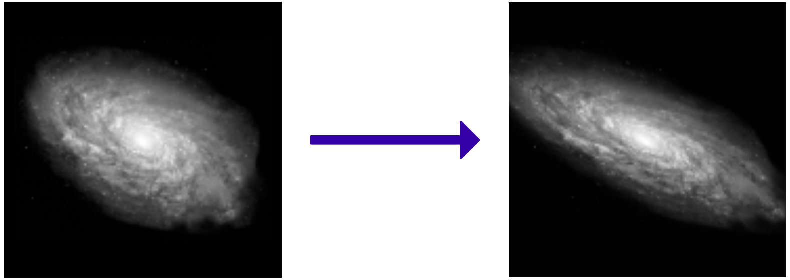

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Cosmic shear distortion

e^i + shear

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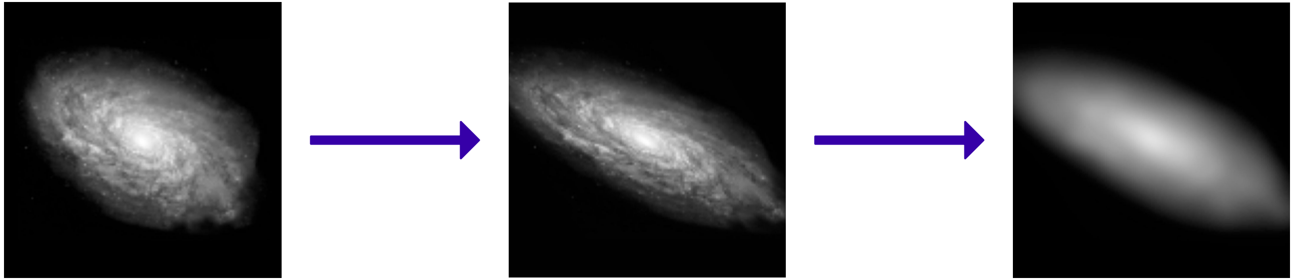

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PSF : atmosphere + instrumental response

e^i + shear + PSF

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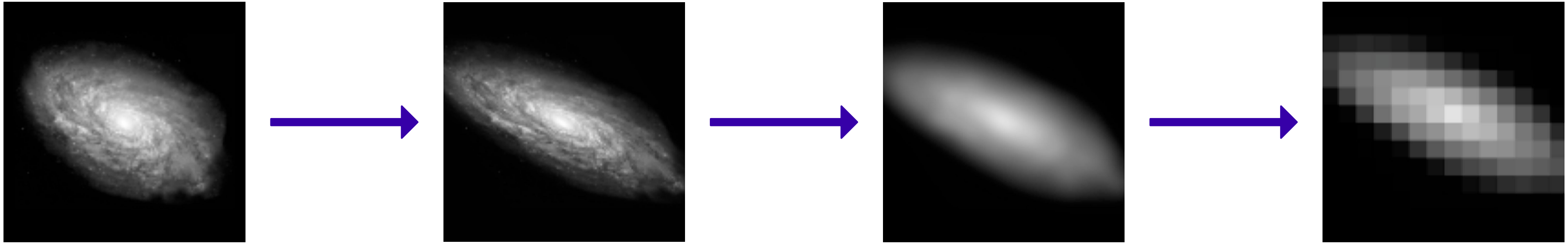

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Pixellisation

e^i + shear + PSF + pixels

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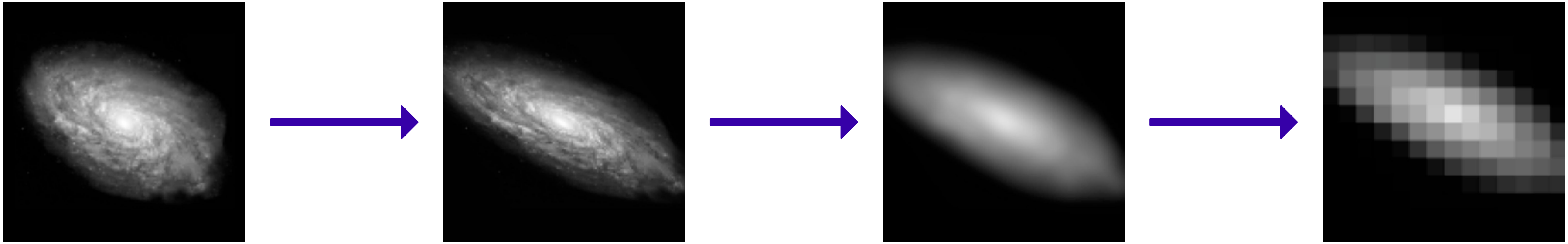

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e^i

Cosmic shear distortion

e^i + shear

PSF : atmosphere + instrumental response

e^i + shear + PSF

Pixellisation

e^i + $\boxed{\text{shear}}$ + PSF + pixels

Our observable (**e**)

Signal we want to measure (**g**)

Figures : Bridle et al. 2009

Estimator : Ellipticity Taylor's expansion

$$e = e|_{g=0} + g\frac{\partial e}{\partial g}|_{g=0} + \ldots$$

Averaging over a large number of galaxies :

$$\langle e\rangle = \underbrace{\langle e|_{g=0}\rangle}_{= 0} + \langle g\frac{\partial e}{\partial g}|_{g=0}\rangle + \ldots \boxed{\simeq \langle \mathbf{R}g\rangle}$$

Method

$$g^{obs} = (1 + \boxed{m})g^{true} + \boxed{c}$$

$m < 1.5\times10^{-3}$
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(Cropper 2013)

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$$\langle g \rangle = \langle R \rangle^{-1} \langle e \rangle$$

Self-calibrated shear estimator

Method : Second moments and calibration

Shape measurement :

Second moments

Method : Second moments and calibration

Shape measurement :

Second moments

$$M = \int \overbrace{X X^T}^{(X - X_0)(X - X_0)^T} W(X) I(X) \, dX^2$$

Pixel coordinates

Weight function

$I = I_0 \circledast \psi$

Method : Second moments and calibration

Shape measurement :

Second moments

$$M = \int \overbrace{XX^T}^{(X-X_0)(X-X_0)^T} W(X) I(X)\, dX^2 \Longrightarrow \mathbf{e} = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \boxed{\frac{1}{F}} \begin{pmatrix} M_{xx} - M_{yy} \\ 2M_{xy} \end{pmatrix}$$

Pixel coordinates

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Ellipticity

Normalized by the flux

Method : Second moments and calibration

Shape measurement :

Second moments

$$M = \int \overbrace{XX^T}^{\left(X-X_0\right)\left(X-X_0\right)^T} W(X) I(X)\, dX^2 \Longrightarrow \mathbf{e} = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \frac{1}{F}\begin{pmatrix} M_{xx} - M_{yy} \\ 2M_{xy} \end{pmatrix}$$

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Calibration :

Second moments derivatives with respect to the shear

Method : Second moments and calibration

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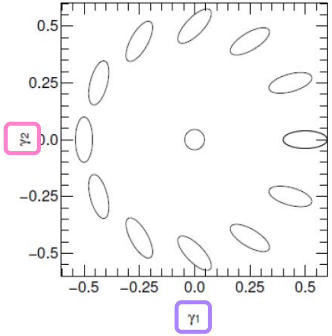
Ellipticity

Calibration :

Second moments derivatives with respect to the shear

$$\frac{dM}{d\gamma} = \int \frac{dG(S(\gamma), X)}{d\gamma} I(X)\, dX^2$$

$$S = \frac{1}{\sqrt{1-g^2}} \begin{pmatrix} 1 + \boxed{g_1} & \boxed{g_2} \\ g_2 & 1 - g_1 \end{pmatrix}$$



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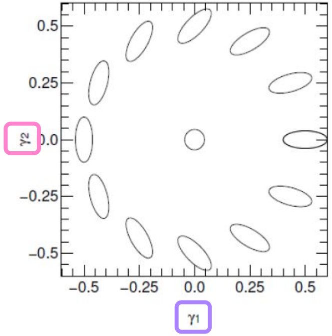

$$= \begin{pmatrix} \frac{\partial M_{xx}}{\partial g_1} - \frac{\partial M_{yy}}{\partial g_1} & \frac{2\partial M_{xy}}{\partial g_1} \\ \frac{\partial M_{xx}}{\partial g_2} - \frac{\partial M_{yy}}{\partial g_2} & \frac{2\partial M_{xy}}{\partial g_2} \end{pmatrix}$$

Auto-calibration factor

Method : Second moments and calibration

Shape measurement :

Second moments

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Pixel coordinates

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Second moments derivatives with respect to the shear

$$\frac{dM}{d\gamma} = \int \frac{d\boxed{G(S(\gamma), X)}}{d\gamma} I(X)\, dX^2 \Longrightarrow \mathbf{R} = \begin{pmatrix} \frac{\partial e_1}{\partial g_1} & \frac{\partial e_2}{\partial g_1} \\ \frac{\partial e_1}{\partial g_2} & \frac{\partial e_2}{\partial g_2} \end{pmatrix}$$

$\uparrow$?

$$= \begin{pmatrix} \frac{\partial M_{xx}}{\partial g_1} - \frac{\partial M_{yy}}{\partial g_1} & \frac{2\partial M_{xy}}{\partial g_1} \\ \frac{\partial M_{xx}}{\partial g_2} - \frac{\partial M_{yy}}{\partial g_2} & \frac{2\partial M_{xy}}{\partial g_2} \end{pmatrix}$$

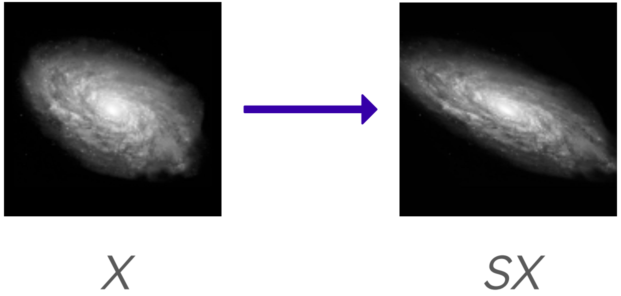
Auto-calibration factor

Method : Second moments and calibration

$$M(S) = \int (XX^T W(X))\psi \circledast I_0(SX)dX^2$$

Metacalibration

arXiv:1702.02601



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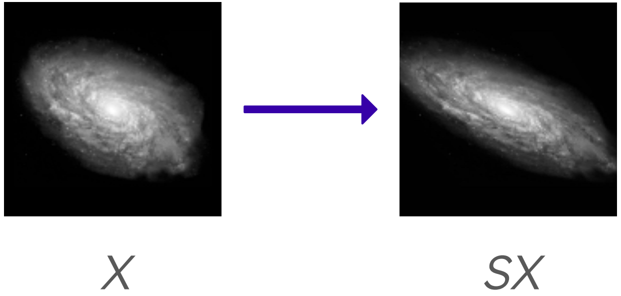

→ Distorting original image introduces **correlated noise** !

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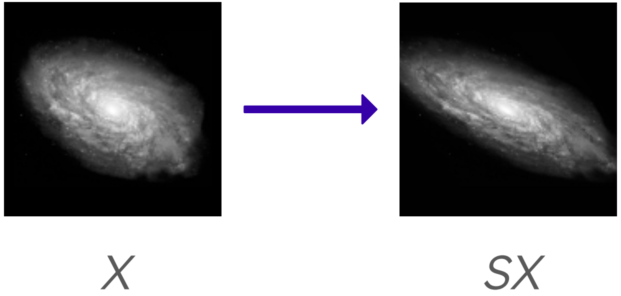

Parseval identity :

$$M(S) = \int \underbrace{(XX^T W(X) \overset{\overset{\circledast}{\curvearrowleft\curvearrowright}}{\circledast \psi_-})}_{F(X)} I_0(SX) dX^2$$

Method : Second moments and calibration

$$M(S) = \int (XX^T W(X)) \psi \circledast I_0(SX) dX^2$$

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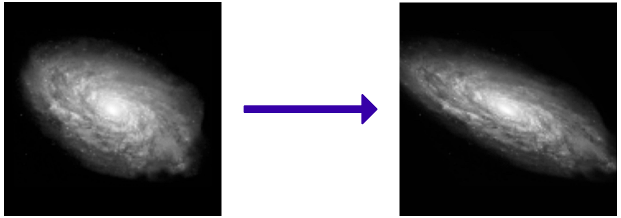

$$\int F(X) I_0(SX) d^2X = \int F(S^{-1}X) I_0(X) d^2X$$

Method : Second moments and calibration

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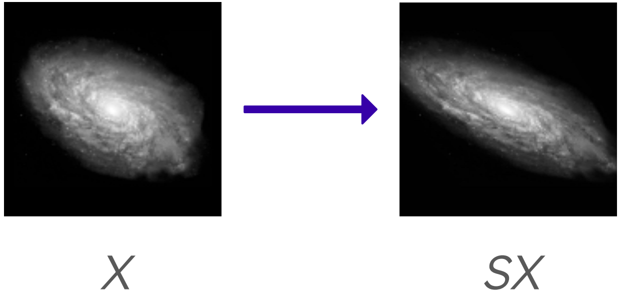


Metacalibration

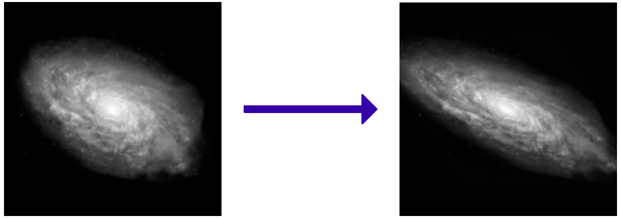
arXiv:1702.02601

$$M(S) = \int F(Sk) I_0(k) d^2k$$

← Fourier space

Method : Second moments and calibration

$$M(S) = \int (XX^T W(X))\psi \circledast I_0(SX) dX^2$$



Metacalibration

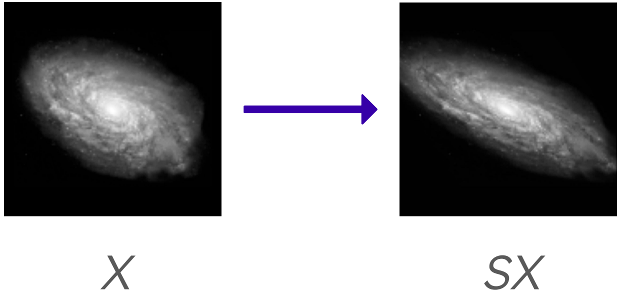
arXiv:1702.02601

$$M(S) = \int F(Sk) I_0(k) d^2k$$
$$= \int \frac{F(Sk)}{\psi^*(k)} \psi^*(k) I_0(k) d^2k$$

← Division + Multiplication by the PSF

Method : Second moments and calibration

$$M(S) = \int (XX^T W(X)) \psi \circledast I_0(SX) dX^2$$



Metacalibration

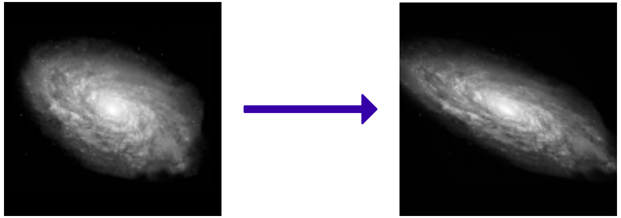
arXiv:1702.02601

$$\begin{aligned} M(S) &= \int F(Sk) I_0(k) d^2k \\ &= \int \frac{F(Sk)}{\psi^*(k)} \psi^*(k) I_0(k) d^2k \\ &= \int G(S,X)[\psi \otimes I_0] d^2X \end{aligned}$$

← = Convolution in real space

Method : Second moments and calibration

$$M(S) = \int (XX^T W(X)) \psi \circledast I_0(SX) dX^2$$



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I(X) : original configuration

Method : Second moments and calibration

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Metacalibration

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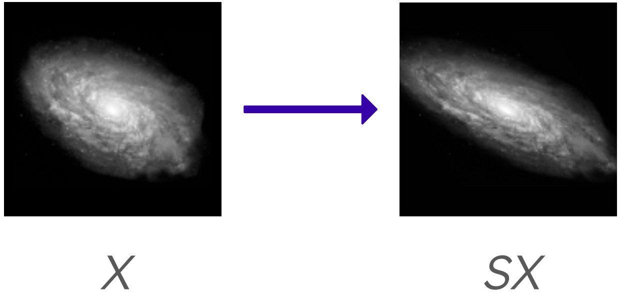

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- No distortion applied on original image !
- Allows shear estimations on under-sampled images

Method : Shear estimation

$$\mathbf{e} = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \frac{1}{F} \begin{pmatrix} M_{xx} - M_{yy} \\ 2M_{xy} \end{pmatrix}$$

$$\mathbf{R} = \begin{pmatrix} \frac{\partial e_1}{\partial g_1} & \frac{\partial e_2}{\partial g_1} \\ \frac{\partial e_1}{\partial g_2} & \frac{\partial e_2}{\partial g_2} \end{pmatrix} = \begin{pmatrix} \frac{\partial M_{xx}}{\partial g_1} - \frac{\partial M_{yy}}{\partial g_1} & \frac{2\partial M_{xy}}{\partial g_1} \\ \frac{\partial M_{xx}}{\partial g_2} - \frac{\partial M_{yy}}{\partial g_2} & \frac{2\partial M_{xy}}{\partial g_2} \end{pmatrix}$$

Shear estimation :

$$\boxed{\langle g \rangle = \langle R \rangle^{-1} \langle e \rangle}$$

(LSST : we aim to have biases less than ‰)

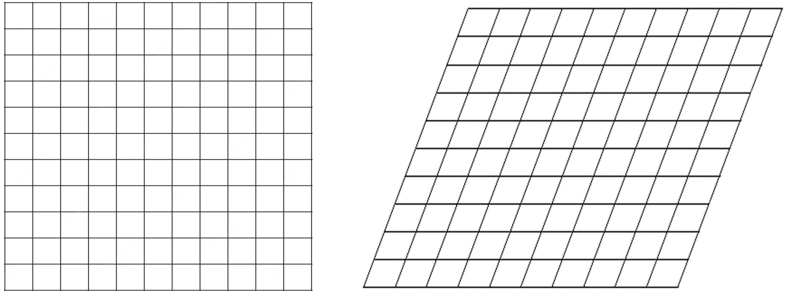
Method : Technical aspects

Galsim convention

$$S = \frac{1}{\sqrt{1-g^2}}\begin{pmatrix}1+g_1 & g_2\\ g_2 & 1-g_1\end{pmatrix} \Longrightarrow \begin{pmatrix}x'\\ y'\end{pmatrix} = S\begin{pmatrix}x\\ y\end{pmatrix}$$

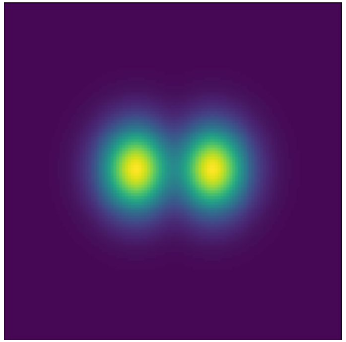
Application of shear variations (±ε) to calculate derivatives : distortion of the coordinate system (with ***S*** matrix), then interpolation of the image (***F*** function) onto the new grid.

$$M(S) = \int [\psi^{-1} \circledast F(SX)] I(X) dX^2$$



Method : Technical aspects

Application of shear variations (±ε) to calculate derivatives : distortion of the coordinate system (with ***S*** matrix), then interpolation of the image (***F*** function) onto the new grid.



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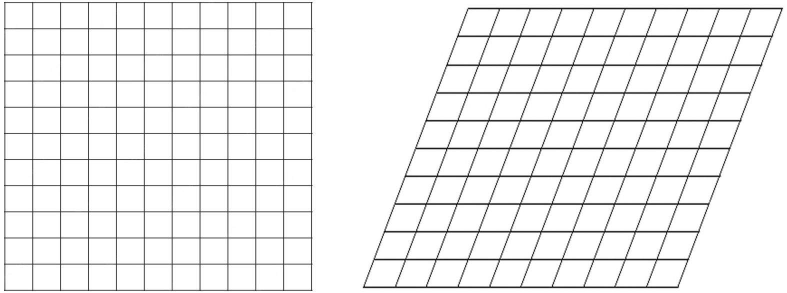

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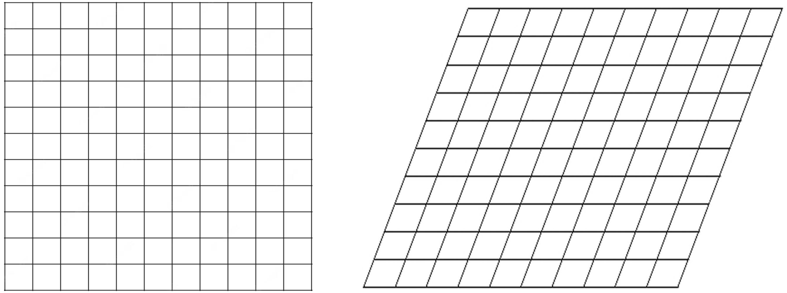
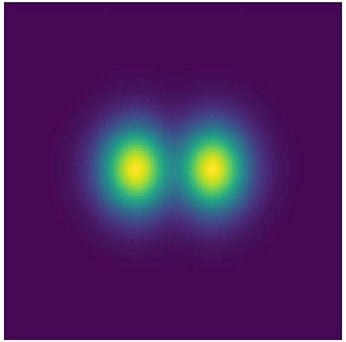
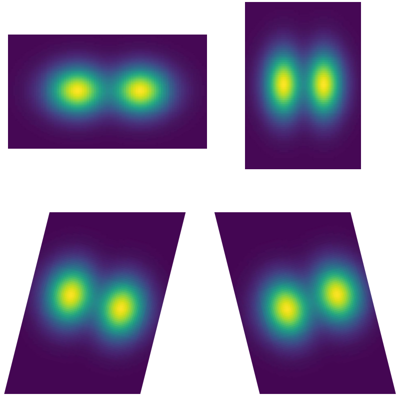




$g_1 = \pm \varepsilon \; ; \; g_2 = 0$

$$\xrightarrow[\text{interpolation}]{\text{grid distortion } +}$$

$g_1 = 0 \; ; \; g_2 = \pm \varepsilon$



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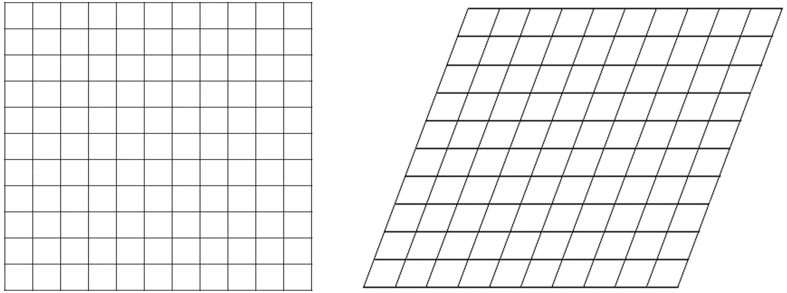

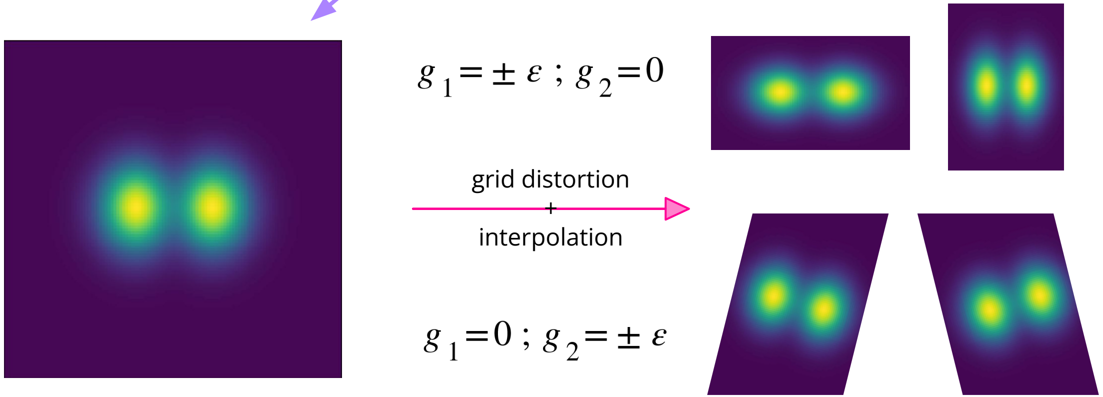

$$\frac{\partial M}{\partial g_1} = \frac{S\,M_{1+} - S\,M_{1-}}{2\epsilon}$$

$$\frac{\partial M}{\partial g_2} = \frac{S\,M_{2+} - S\,M_{2-}}{2\epsilon}$$

Method : Technical aspects

Galsim convention

$$S = \frac{1}{\sqrt{1-g^2}} \begin{pmatrix} 1+g_1 & g_2 \\ g_2 & 1-g_1 \end{pmatrix} \longrightarrow \begin{pmatrix} x' \\ y' \end{pmatrix} = S \begin{pmatrix} x \\ y \end{pmatrix}$$

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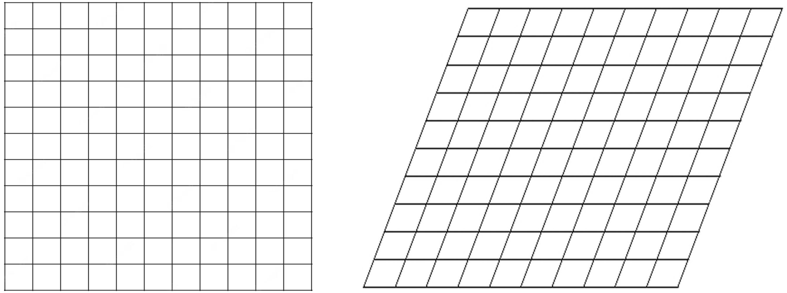

Sampling :

$$I = I_c \otimes \Pi \quad \Leftrightarrow \quad M = \boxed{M_c} + \boxed{M_{pix}}$$

$$M = \boxed{\int XX^T I(X)\, d^2X} + \boxed{\frac{JJ^T}{12}}$$

with :

- J : the Jacobian involved in the affine transformation of coordinates (pixel ↔ physical)

- s : the image pixel scale (arcsec/pixel)

$$J = \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix}$$

Method : Technical aspects - shear & sampling cross-effect

As we measure the *SM* matrices on a distorted pixels grid, **we should subtract a *distorted* pixels second moments matrix *Mpix*** to recover the real object second moments.

New second moments formalism :

$$M(s,\epsilon) \propto \underset{\text{theoretical second moments}}{\boxed{\gamma + \alpha\epsilon + \alpha'\epsilon^2}} + \underset{\text{sampling correction}}{\boxed{\beta s^2 + \beta' s^4}} + \boxed{\delta s^2 \epsilon} + \underset{\text{sampling x shear correction}}{\boxed{\delta' s^4 \epsilon}}$$

Method : Technical aspects - shear & sampling cross-effect

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⇩

No big impact of the galaxy profile on the estimation of δ’, but highly related to the size of the galaxy !

Method : Technical aspects - shear & sampling cross-effect

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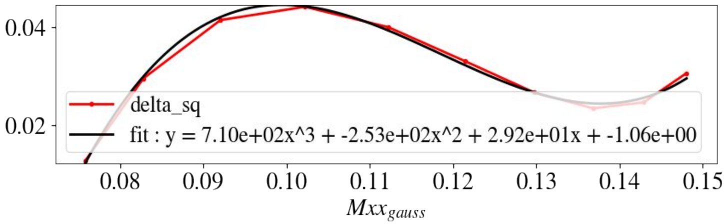
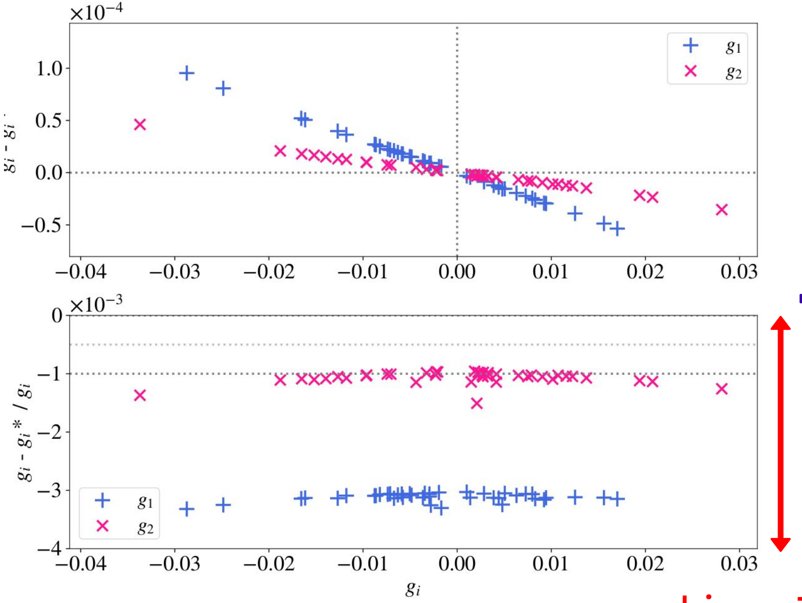


Relation between δ’ and the galaxy 2nd moments

No big impact of the galaxy profile on the estimation of δ’, but highly related to the size of the galaxy !

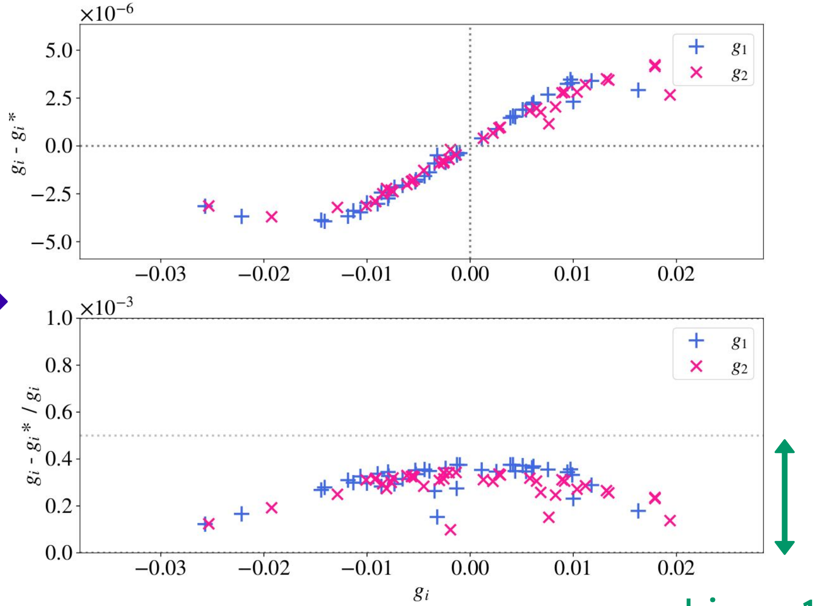
Method : Technical aspects - shear & sampling cross-effect

Before SSB correction



bias ~ 3‰

After SSB correction



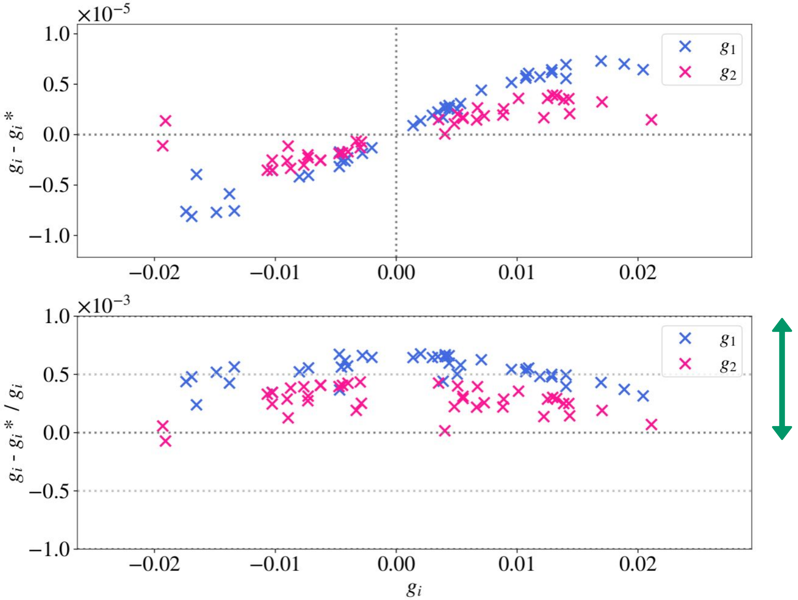
bias < 1‰

Results on noise-free simulations

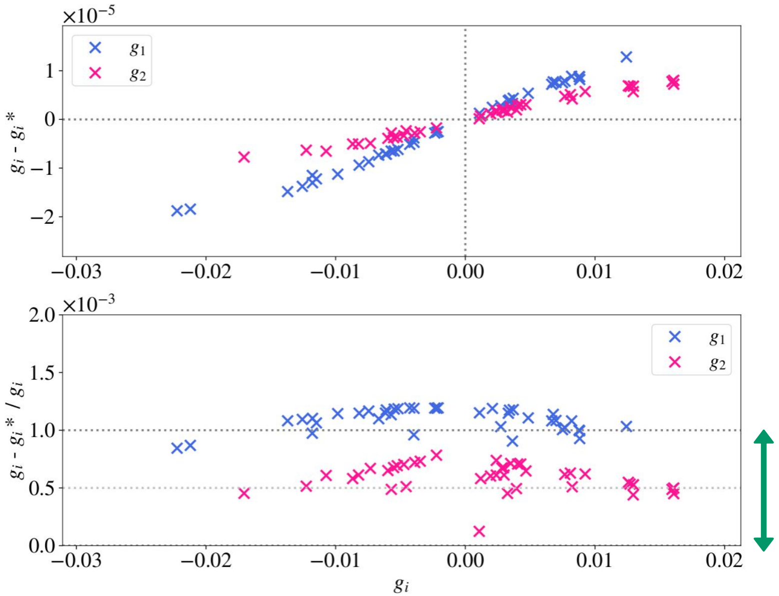
Bulge + Disk galaxies

$$TR = \frac{Tr(M_{image})}{Tr(M_{PSF})}$$

- Moffat PSF
- n = 1.0 (bulge)
- r = 0.8" (disk)
- TR = 1.57

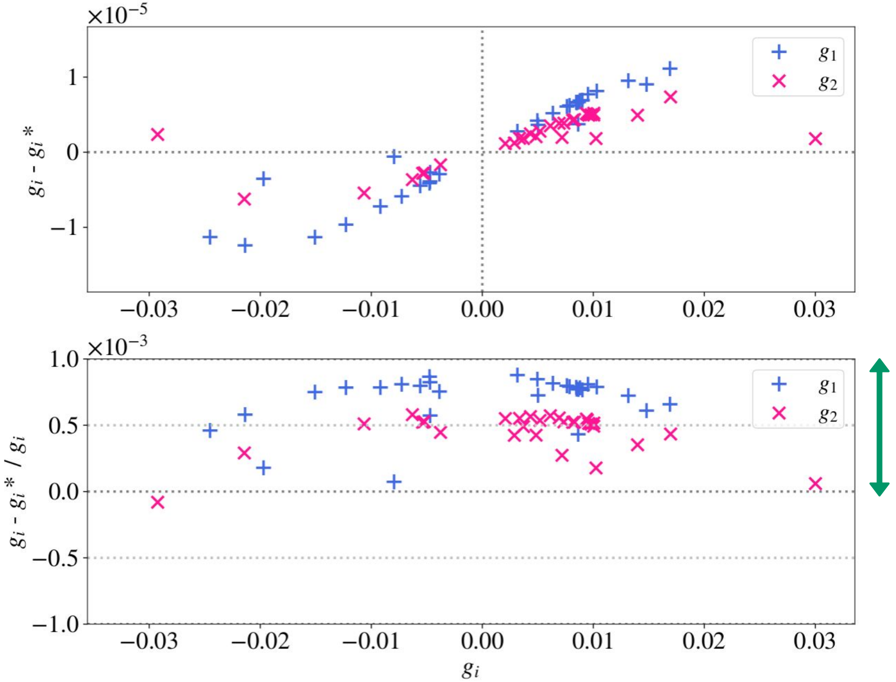


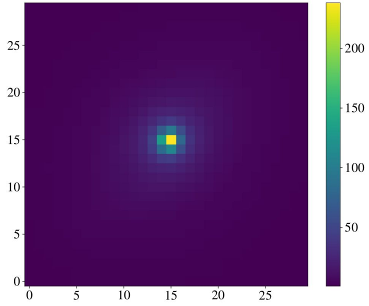
- Moffat PSF
- n = 4.0 (bulge)
- r = 0.8" (disk)
- TR = 1.52

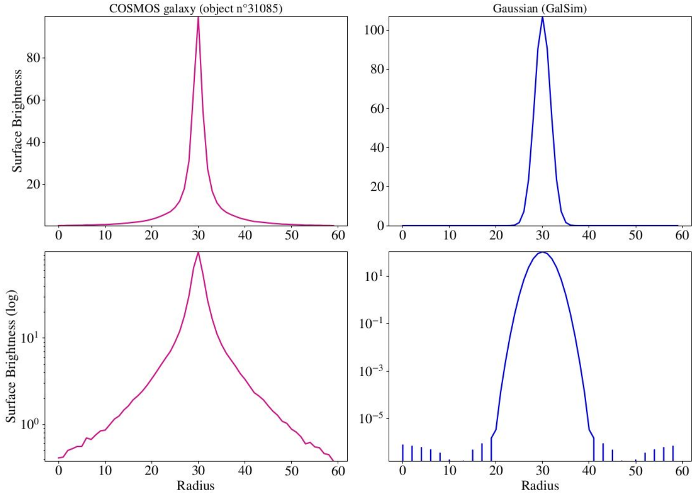


Realistic galaxies (*COSMOS* catalog)

- Moffat PSF
- TR = 1.62

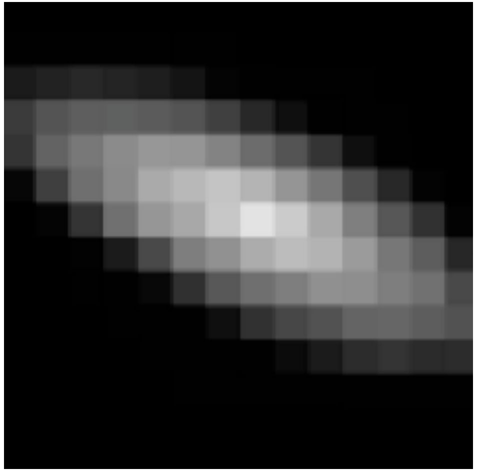




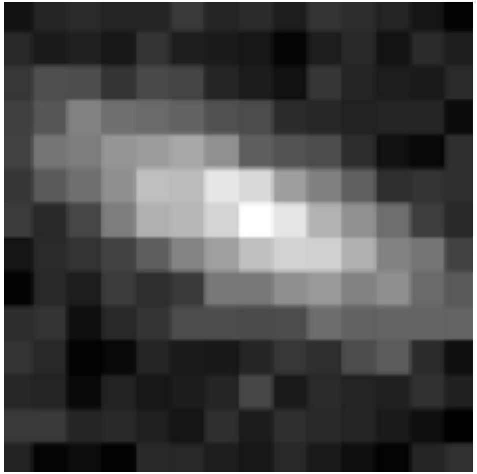


Noisy simulations

Received image



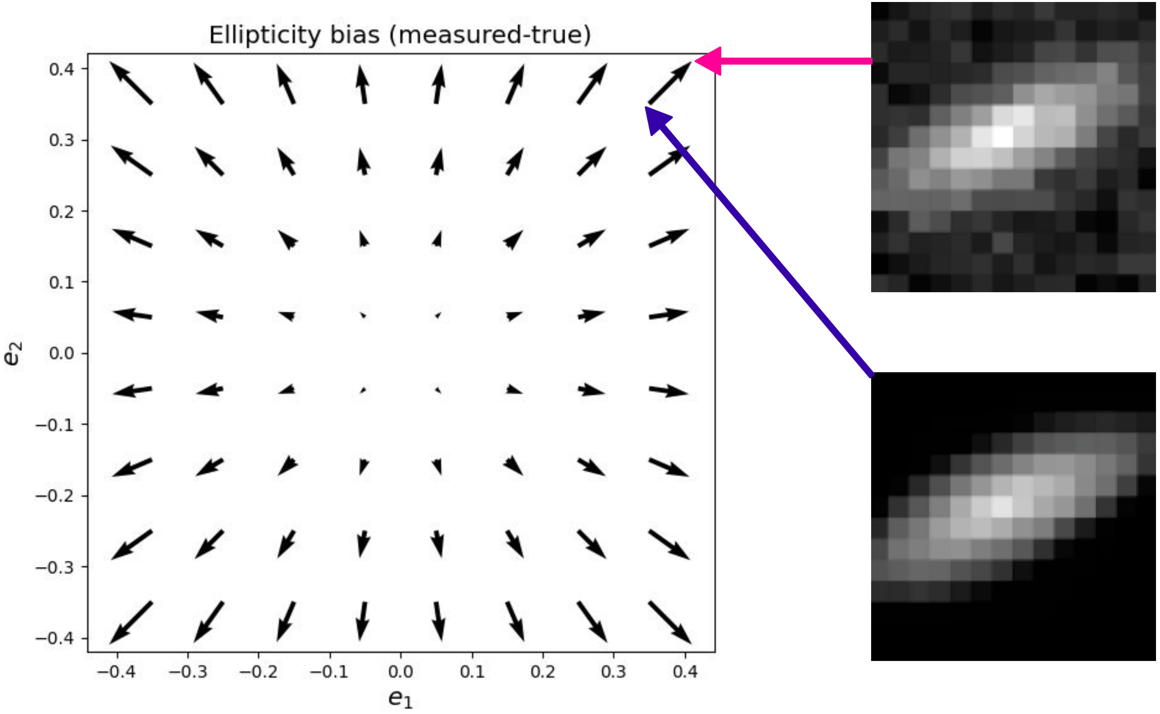
e^i + shear + PSF + pixels

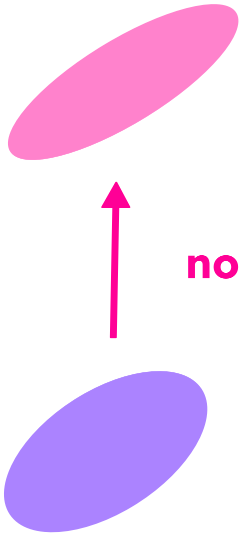


e^i + shear + PSF + pixels + **noise**

Ellipticity bias

Observed ellipticity :


Ellipticity bias (measured-true)



Noise bias correction

$$M = \int \overbrace{XX^T}^{(x - x_0)(x - \boxed{x_0})^T} W(X) I(X)\, dX^2$$

Galaxy position measured on a noisy image → **introduces a bias !**

Noise bias correction

$$M = \int \overbrace{XX^T}^{(x-x_0)(x-\boxed{x_0})^T} W(X) I(X)\, dX^2$$

Galaxy position measured on a noisy image → **introduces a bias !**

Idea : calculate an analytical formula of the second moments noise bias :

$$m_I(I+n) = m_I(I) + \sum_k \frac{dm_I}{dI_k} n_k + \frac{1}{2} \sum_{kl} \boxed{\frac{d^2 m_I}{dI_k dI_l}} n_k n_l$$

Noise bias correction

$$M = \int \overbrace{XX^T}^{(x-x_0)(x-\boxed{x_0})^T} W(X) I(X)\, dX^2$$

Galaxy position measured on a noisy image → **introduces a bias !**

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Same for the **flux** : $F = \sum W(x - x_0) I$

And the **position variance** : $V(x_0)$

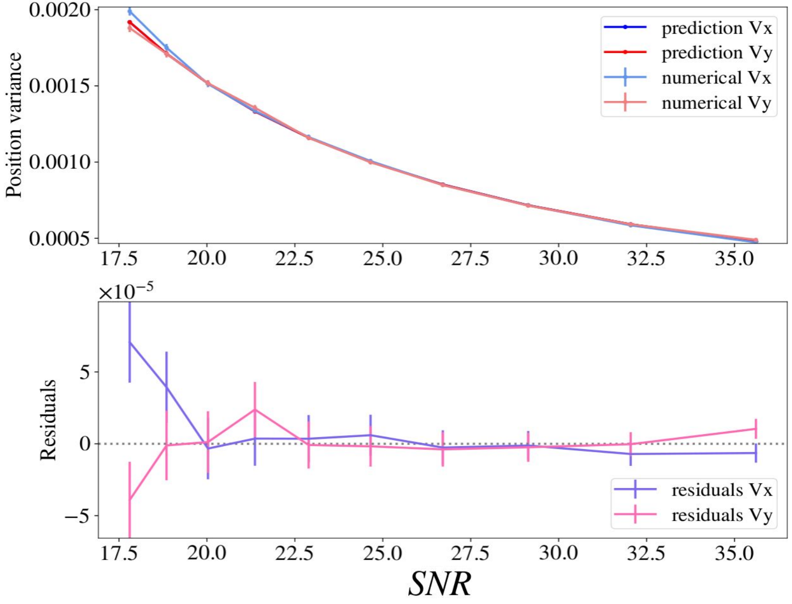
Results : Position variance prediction

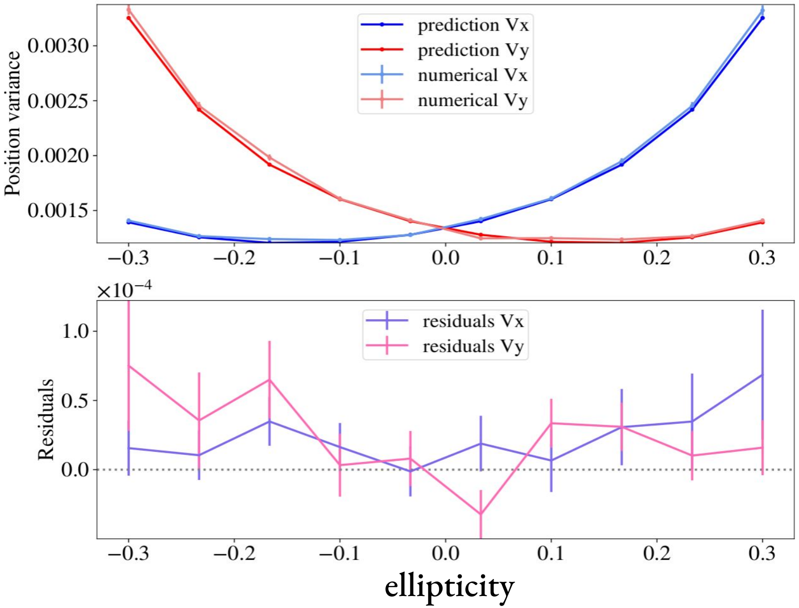
$$V = \sigma^2_{noise} \boxed{K} M_{W^2} K$$

$$\boxed{K} = \frac{1}{\boxed{F}}[\mathbb{1} - M_W^{-1} \boxed{M_P}]^{-1}$$

Flux

P = I*W

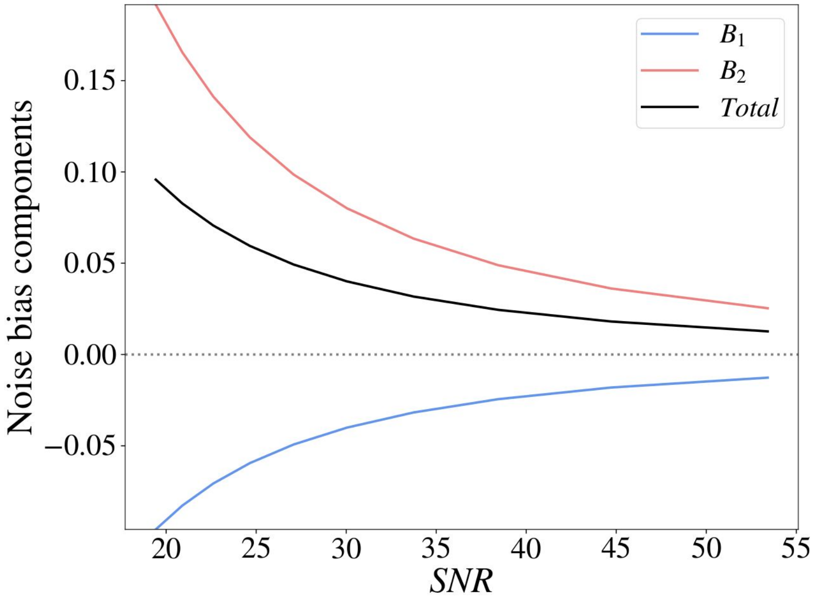


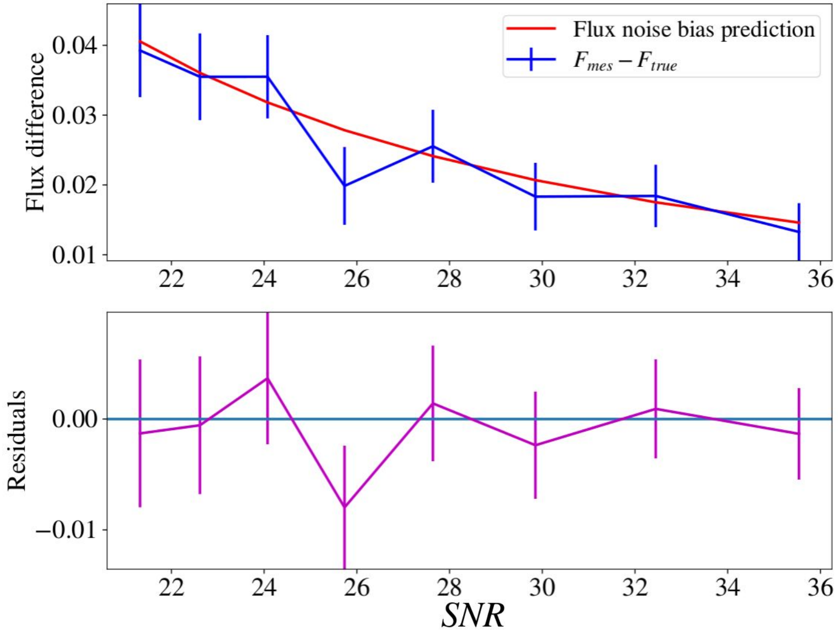


Results : Flux bias prediction

$$F = \sum W(x - x_0) I$$

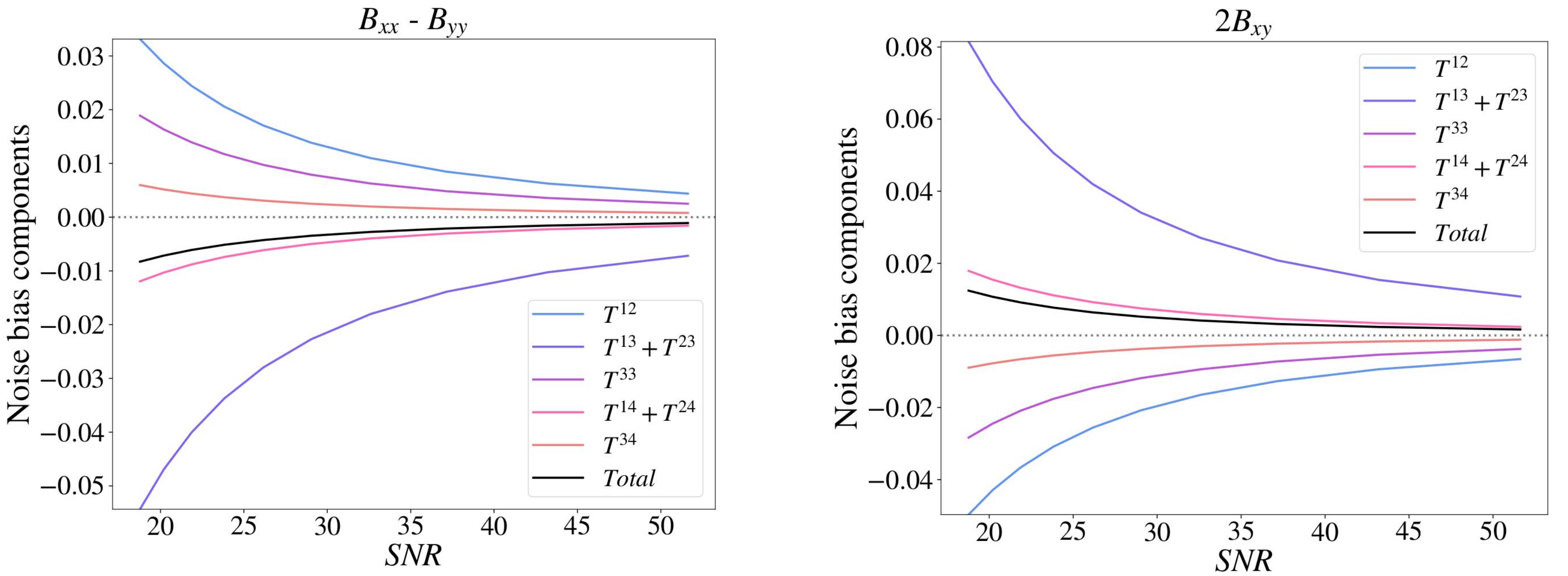
$$B_F = B_1 + B_2$$





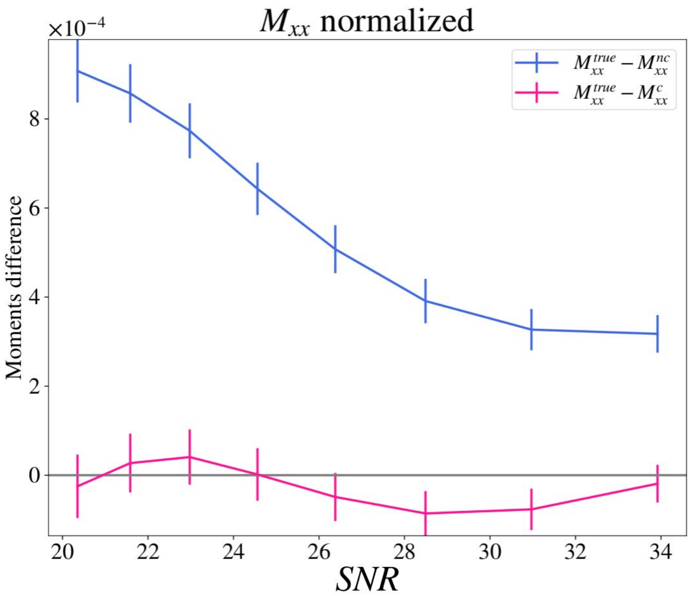
Results : Second moments bias prediction

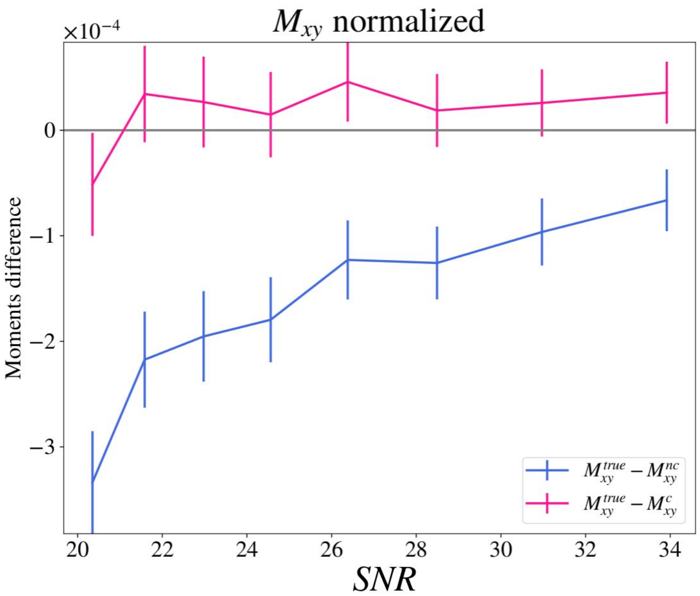
$$m^P_{\alpha\beta} = \sum_k x^k_\alpha x^k_\beta W_k I_k \qquad B_M = \frac{1}{2}(T^{12} + 2T^{13} + T^{33}) + 2T^{14} + T^{34}$$



Results : Second moments bias prediction

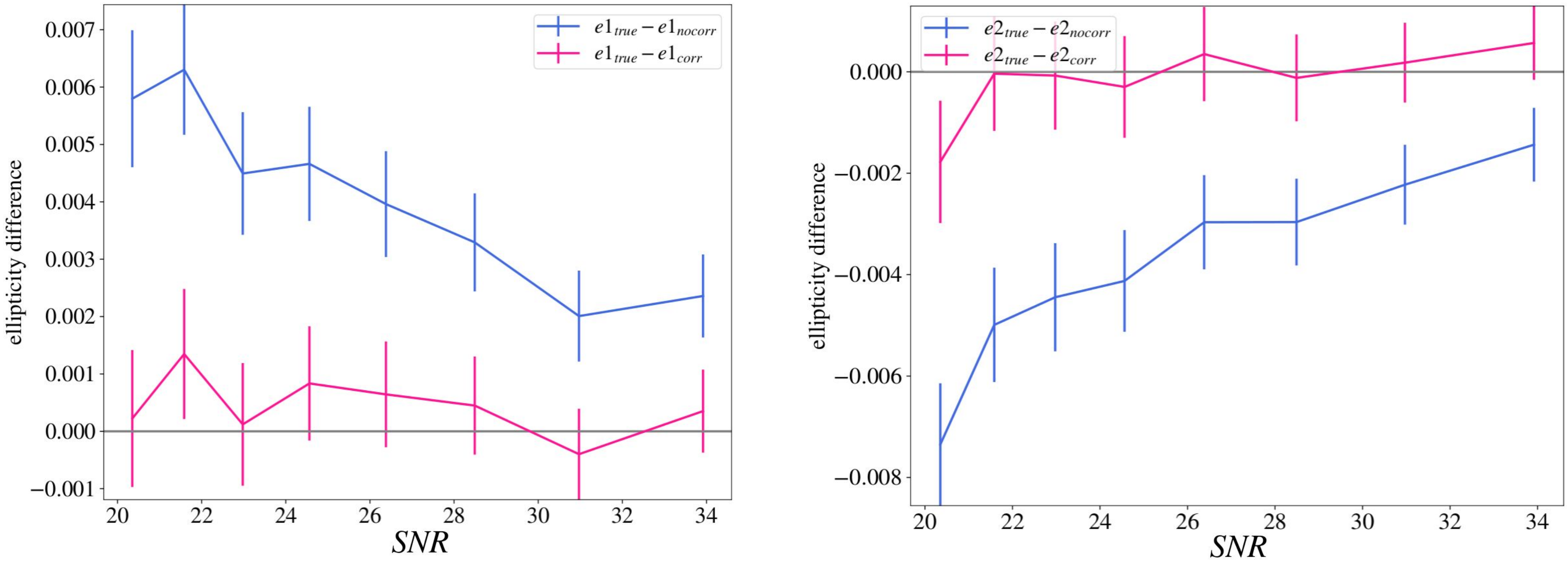
$$M_c = \frac{M_{mes} - B_M}{F_{mes} - B_F}$$





Results : Ellipticity bias correction

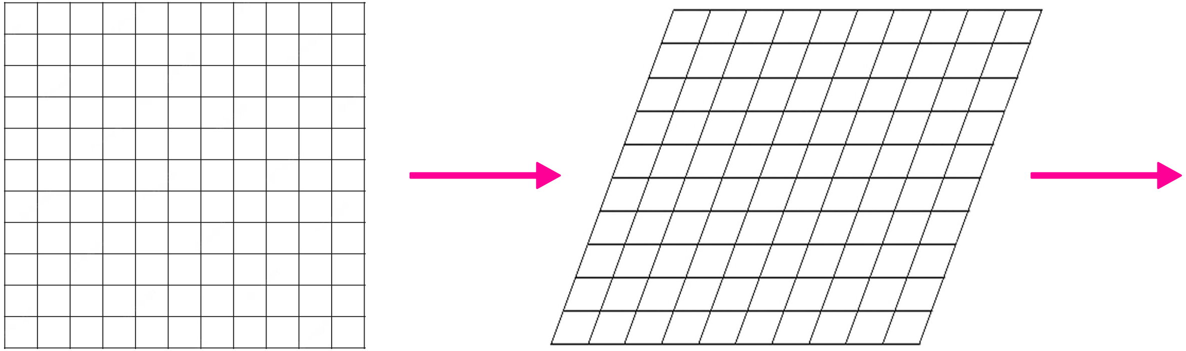
$$\mathbf{e} = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \frac{1}{F} \begin{pmatrix} M_{xx} - M_{yy} \\ 2M_{xy} \end{pmatrix}$$



$$\langle g \rangle = \langle R \rangle^{-1} \overset{\checkmark}{\boxed{\langle e \rangle}}$$

$$\langle g \rangle = {\overset{?}{\boxed{\langle R \rangle}}}^{-1} \langle e \rangle$$

Auto-calibration factor bias prediction

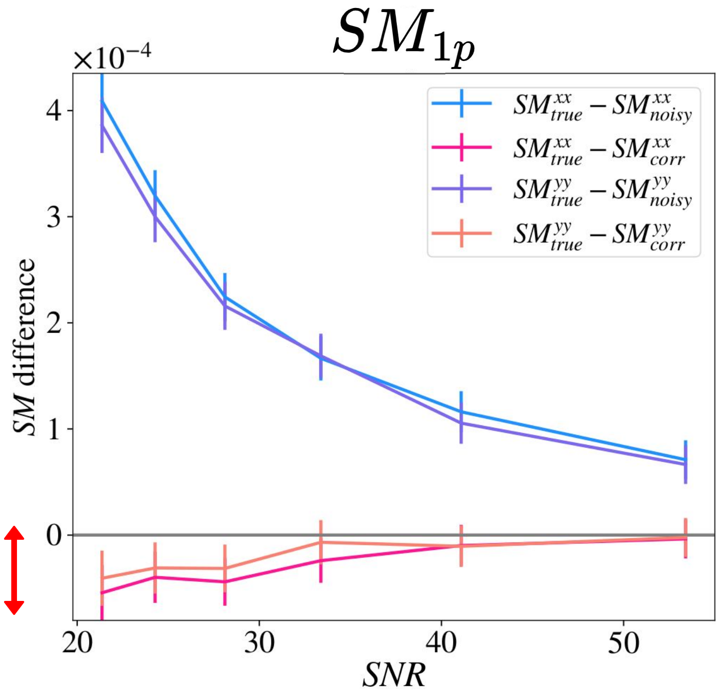


Second moments and flux correlated !

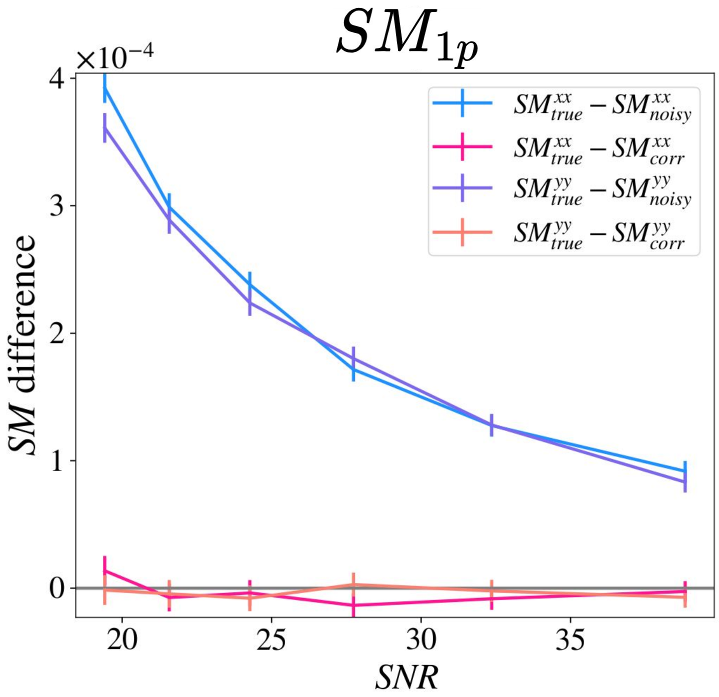
$$\xcancel{M_c = \frac{M_{mes} - B_M}{F_{mes} - B_F}} \longrightarrow M_c = \left(\frac{M}{F}\right)_{mes} - B_{M/F}$$

Results : R correction

$$M_c = \frac{M_{mes} - B_M}{F_{mes} - B_F}$$



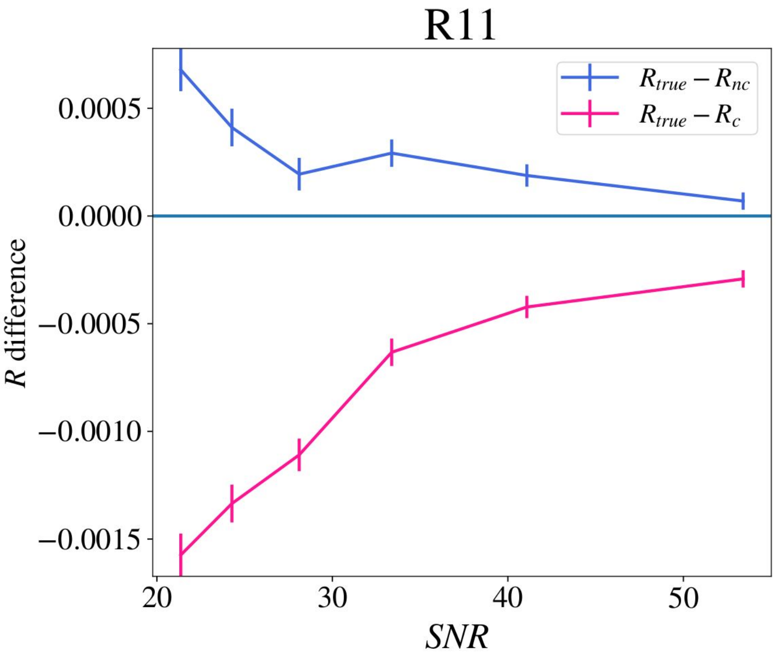
$$M_c = \left(\frac{M}{F}\right)_{mes} - B_{M/F}$$

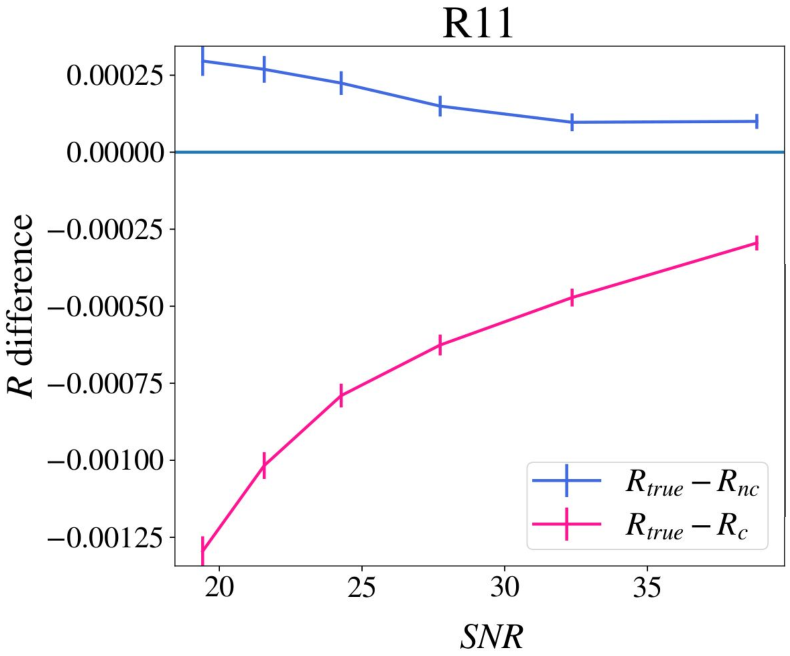


Results : R correction

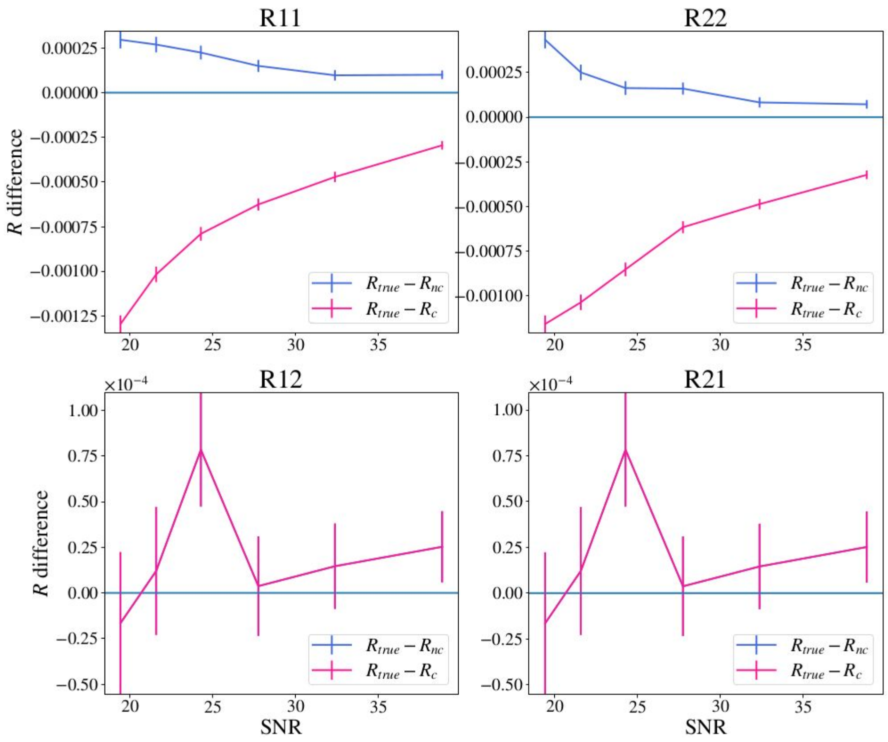
$$M_c = \frac{M_{mes} - B_M}{F_{mes} - B_F}$$

$$M_c = \left(\frac{M}{F}\right)_{mes} - B_{M/F}$$





Results : R correction



Perspectives :

- Combining numerical and analytical derivatives
- M derivatives wrt shear in Fourier space (disappearance of SSB ?)
- Calculate the true noise bias prediction (JAX, Schoenholz and Cubuk, 2019)

Conclusion

- LSST : future-generation survey → **precision cosmology**
- Bias in shear measurement : limit to be set on **m** ($< 10^{-3}$)
- Development of a **self-calibrated shear estimator**

  → Independent of galaxy profile

  → No distortion applied to the original image

  → Satisfactory results on basic (noise-free) tests

- Noisy simulations : **analytical formulas to correct noise bias**

  → Good correction of ellipticity bias

  → Auto-calibration factor : promising initial results, work in progress

Thank you

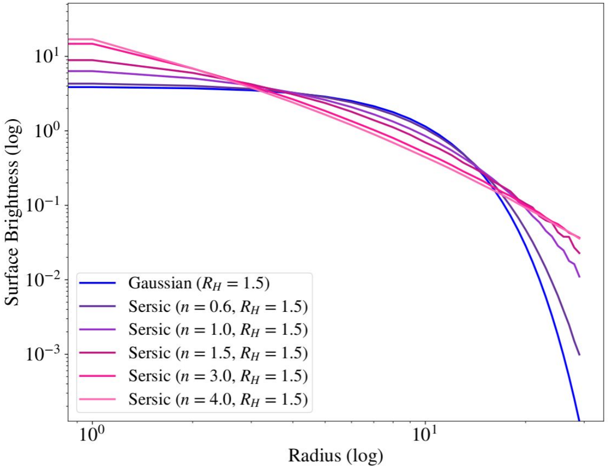
Backup

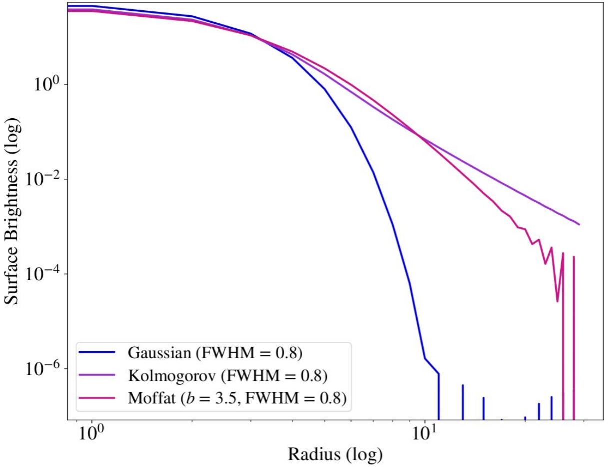
Simulations

- Images simulated with the ***GalSim*** package (Rowe et al. 2015)
- Used profiles :

  - **Galaxies** : Gaussian, Sersic, Bulge + Disk

  - **PSF** : Gaussian, Kolmogorov, Moffat





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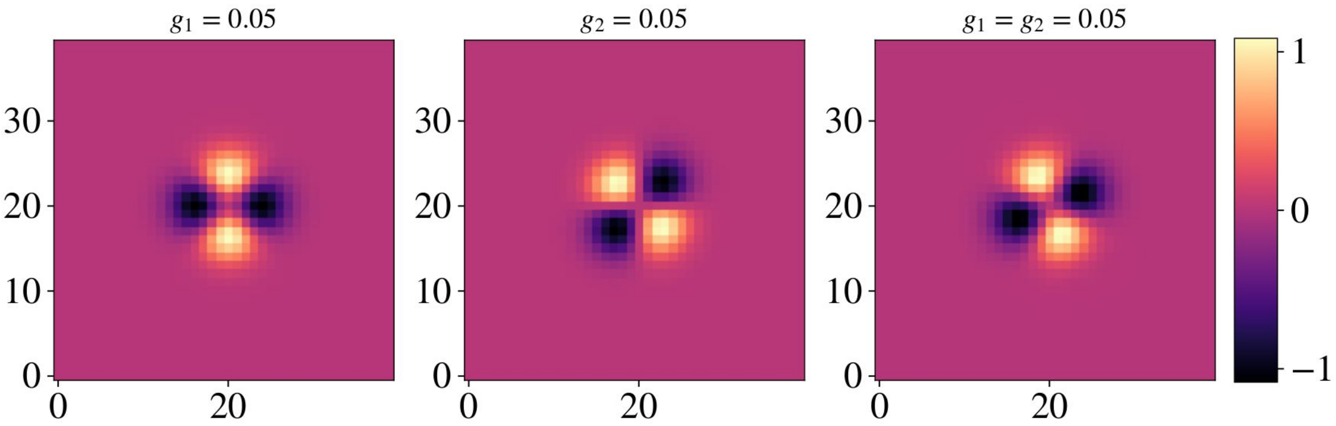


Simulations

- Images simulated with the ***GalSim*** package (Rowe et al. 2015)
- Used profiles :

  - **Galaxies** : Gaussian, Sersic, Bulge + Disk

  - **PSF** : Gaussian, Kolmogorov, Moffat

- Sizes choices : **Trace Ratio (TR)** :

  - Fixed PSF size (FWHM = 0.8 arcsec)

  - Fixed W size (Gaussienne FWHM = 1 arcsec)

  - Simulations with **TR ~ 1.5**

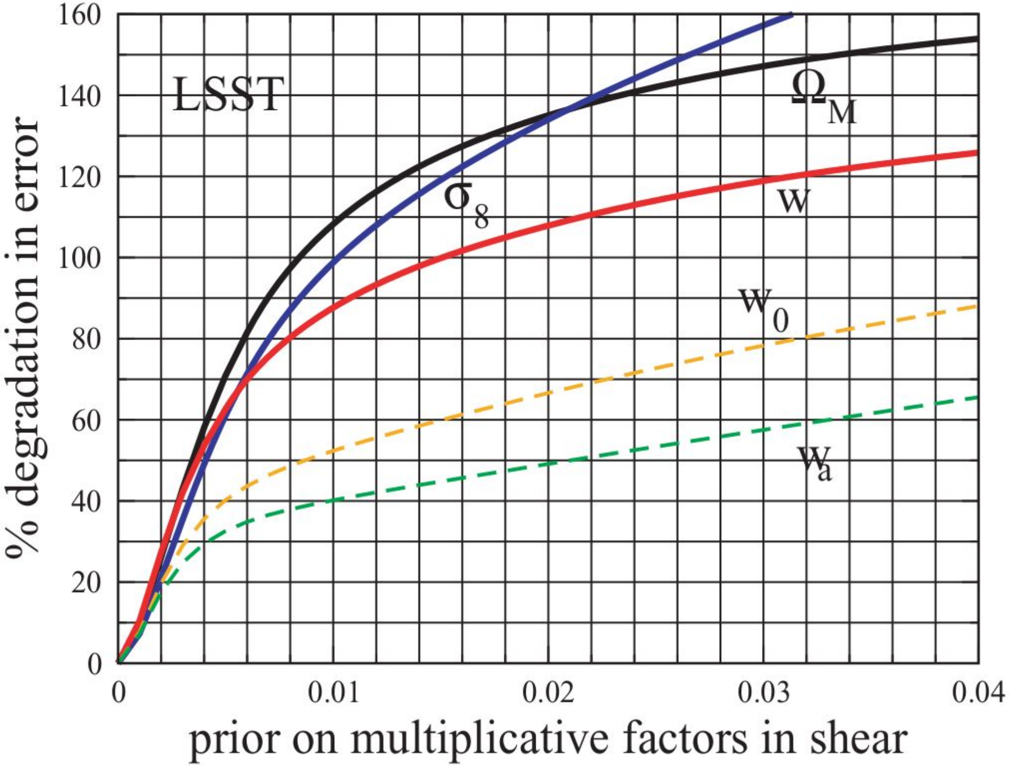
→ Mandelbaum et al. 2018, Zuntz et al. 2018

$$M_{image} = \sum_{i,j}^{N} X_i X_j^T W_i [I_0 \circledast \psi]_i$$

$$TR = \frac{Tr(\boxed{M_{image}})}{Tr(\boxed{M_{PSF}})}$$

$$M_{PSF} = \sum_{i,j}^{N} X_i X_j^T W_i \psi_i$$

Impact of multiplicative bias on cosmological parameters



Huterer 2006

Shear correlation functions

2 shear components for each galaxy → one tangential and one crossed :

$$\gamma_t = -\Re\left(\gamma e^{-2i\Phi}\right)$$

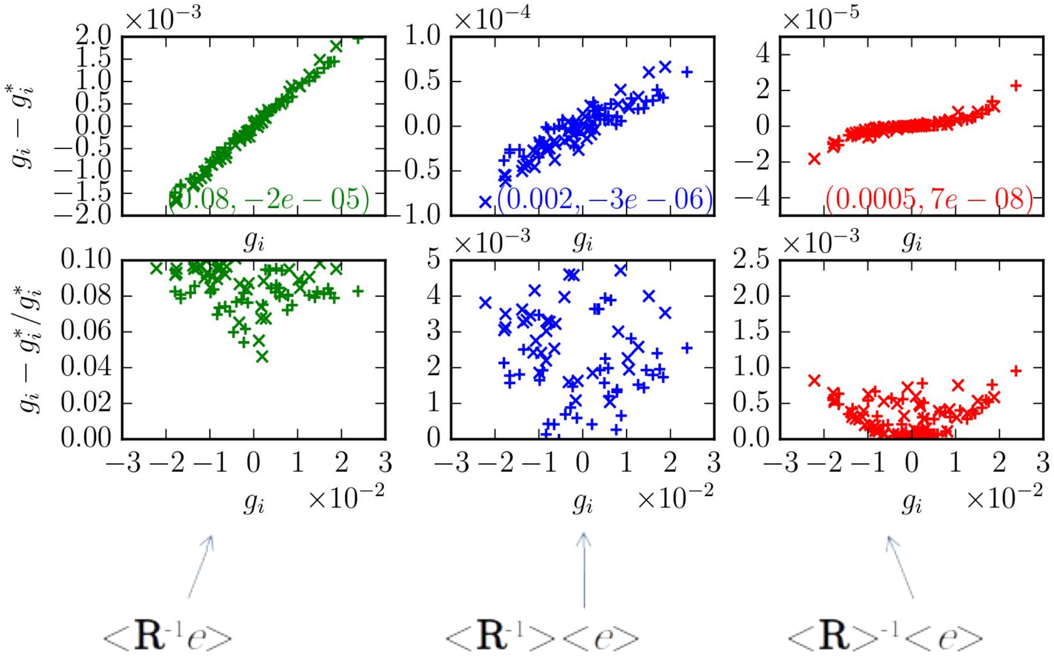
$$\gamma_\times = -\Im\left(\gamma e^{-2i\Phi}\right)$$

Then the 2 non-zero components of the 2pt correlation function :

$$\xi_+ = \langle \gamma\gamma* \rangle(\theta) = \langle \gamma_t\gamma_t \rangle + \langle \gamma_\times\gamma_\times \rangle$$

$$\xi_- = \Re\left[\langle \gamma\gamma \rangle(\theta) e^{-4i\Phi}\right] = \langle \gamma_t\gamma_t \rangle - \langle \gamma_\times\gamma_\times \rangle$$

Shear estimation formalism



Upper panel : absolute difference between input and output shear

Lower panel : relative difference

$$\boxed{\langle g \rangle = \langle R \rangle^{-1} \langle e \rangle}$$

Credit : Maude Le Jeune

Other shear estimation methods : Moments

KSB (1995) :

- **Weighted quadrupole moments** (with iteratively adjusted W)
- Strong PSF assumptions
- Setup adjustment with data

DEIMOS (2011) :

- Same idea as KSB
- **Mathematically exact PSF deconvolution** (no assumption on galaxy or PSF profiles)

Other shear estimation methods : Model fitting

lensfit (2007) :

- **7-parameter galaxy model fit** : galaxy position, flux, scale-length, bulge-to-disc ratio, galaxy ellipticity
- **Bayesian method** : each exposure fitted independently

HSM (2003) :

- **Adaptive moments :** Gaussian weight matched to the image
- **Minimise** →

$$E = \frac{1}{2} \int_{R^2} \left| I(\boldsymbol{x}) - \underbrace{A \exp\left[-\frac{1}{2} (\boldsymbol{x} - \boldsymbol{x}_0)^{\mathrm{T}} \mathbf{M}^{-1} (\boldsymbol{x} - \boldsymbol{x}_0) \right]}_{W} \right|^2 \mathrm{d}^2 \boldsymbol{x}$$

- In practice : Gaussian model fitting

Metacalibration (Sheldon et al. 2017)

Image after shear application : $I(s) = P \otimes [\mathbf{s}(P^{-1} \otimes I)]$

with s the shear operator and P the atmospheric seeing + PSF + pixel response function.

To remove noise amplified by deconvolution, creation of a **dilated PSF Γ :** $\Gamma(x) = P((1 + 2\boxed{|\gamma|})x)$ ← **shear distorsion**

New sheared image : $I(s) = \Gamma \otimes [\mathbf{s} * (P^{-1} \otimes I)]$

→ This procedure introduces correlated anisotropic noise, which can lead to a systematic multiplicative bias.

Estimation method :

$$\boxed{\langle \gamma \rangle \approx \langle \boldsymbol{R}_\gamma \rangle^{-1} \langle \boldsymbol{e} \rangle \approx \langle \boldsymbol{R}_\gamma \rangle^{-1} \langle \boldsymbol{R}_\gamma \gamma \rangle \, .}$$

$$\left\{ \begin{aligned} \langle \boldsymbol{e} \rangle &\approx \int d\boldsymbol{e} \frac{\partial P(\boldsymbol{e})\boldsymbol{e}}{\partial \gamma}\bigg|_{\gamma=0} \gamma \, d\boldsymbol{e} = \langle \boldsymbol{R}_\gamma \gamma \rangle \\ \langle \boldsymbol{R}_\gamma \rangle &= \int \frac{\partial P(\boldsymbol{e})\boldsymbol{e}}{\partial \gamma}\bigg|_{\gamma=0} d\boldsymbol{e} \approx \int d\boldsymbol{e} \left(\frac{P^{+}\boldsymbol{e}_i^{+} - P^{-}\boldsymbol{e}_i^{-}}{\Delta \gamma_j} \right) d\boldsymbol{e} \\ &= \frac{\langle e_i^{+} \rangle - \langle e_i^{-} \rangle}{\Delta \gamma_j}, \qquad (10) \end{aligned} \right.$$

Shear application to seconds moments

Theoretical second moments (after shear application) :

$$\boxed{M(S) = SMS^T = A^2 \begin{pmatrix} a & b \\ c & d \end{pmatrix}}$$

$$S = \overset{= A}{\frac{1}{\sqrt{1-g^2}}} \begin{pmatrix} 1 + g_1 & g_2 \\ g_2 & 1 - g_1 \end{pmatrix}$$

If g1 ≠ 0 and g2 = 0 :

$$a = (1 + g_1)^2 M_{xx}$$
$$b = (1 - g_1^2) * M_{xy}$$
$$c = (1 - g_1^2) * M_{yx}$$
$$d = (1 - g_1)^2 * M_{yy}$$

If g2 ≠ 0 and g1 = 0 :

$$a = M_{xx} + g_2 * (M_{xy} + Myx + g_2 * M_{yy})$$
$$b = M_{xy} + g_2 * (M_{xx} + M_{yy} + g_2 * M_{yx})$$
$$c = M_{yx} + g_2 * (M_{xx} + M_{yy} + g_2 * M_{xy})$$
$$d = M_{yy} + g_2 * (M_{xy} + Myx + g_2 * M_{xx})$$

Pixel second moments calculation

$$M_{pix} = \int_{pixel} (\vec{X} - \vec{X}_c)(\vec{X} - \vec{X}_c)^T d^2\vec{X} / \int_{pixel} d^2\vec{X}$$

Xc : pixel center
M : Jacobian

$$(\vec{X} - \vec{X}_c) \quad = M(\vec{i} - \vec{i}_c)$$
$$d^2\vec{X} = \ |det(M)|\ d^2\vec{i}$$

change of variable

$$M_{pix} = \int_{pixel} (\vec{X} - \vec{X}_c)(\vec{X} - \vec{X}_c)^T d^2\vec{X} \ / \ \int_{pixel} d^2\vec{X}$$
$$= |det(M)| \int_{pixel} M(\vec{i} - \vec{i}_c)(\vec{i} - \vec{i}_c)^T M^T \ d^2\vec{i} \ / \ |det(M)| \int_{pixel} d^2\vec{i}$$
$$= M\left[\int_{pixel} (\vec{i} - \vec{i}_c)(\vec{i} - \vec{i}_c)^T \ d^2\vec{i}\right] M^T$$
$$= MM^T/12$$

$$\int_0^1 (x - 1/2)^2 dx = 1/12$$

Simulation parameters

- **General parameters :**

- 40 random shear values (between -0.03 et 0.03)
- Averaging each over 20 pairs of random and opposite ellipticities (between -0.3 et 0.3)
- Pixel scale : 0.2 arcsec/pixel

- **Noisy simulations :**

- Noise image simulated with a normal law (mean = 0)
- SNR values between ~17 and ~35
- Mean of each point over 10^4 realisations

Position estimation on images

Estimation of x0 and y0 by minimizing the implicit function :

$$f(x_0, I) = \frac{\sum_i (x_i - x_0) W(x_i - x_0) I_i}{\sum_i W(x_i - x_0) I_i}$$

→ Iterative calculation until reaching 10^{-4}

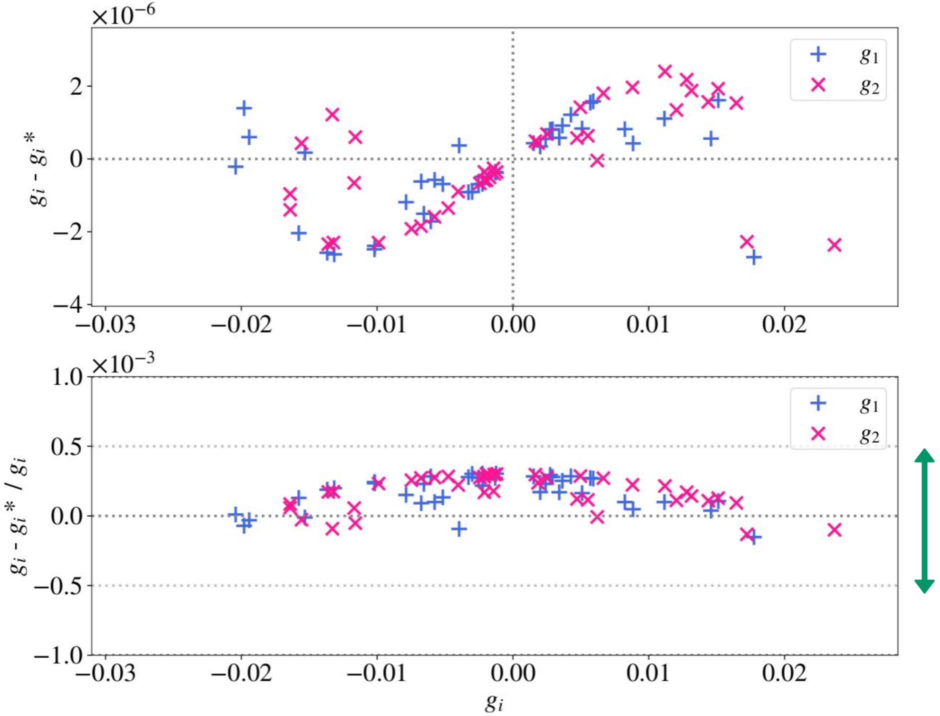
Gaussians galaxies

$$M_{image} = \sum_{i,j}^{N} X_i X_j^T W_i [I_0 \circledast \psi]_i$$

$$TR = \frac{Tr(\boxed{M_{image}})}{Tr(\boxed{M_{PSF}})}$$

$$M_{PSF} = \sum_{i,j}^{N} X_i X_j^T W_i \psi_i$$

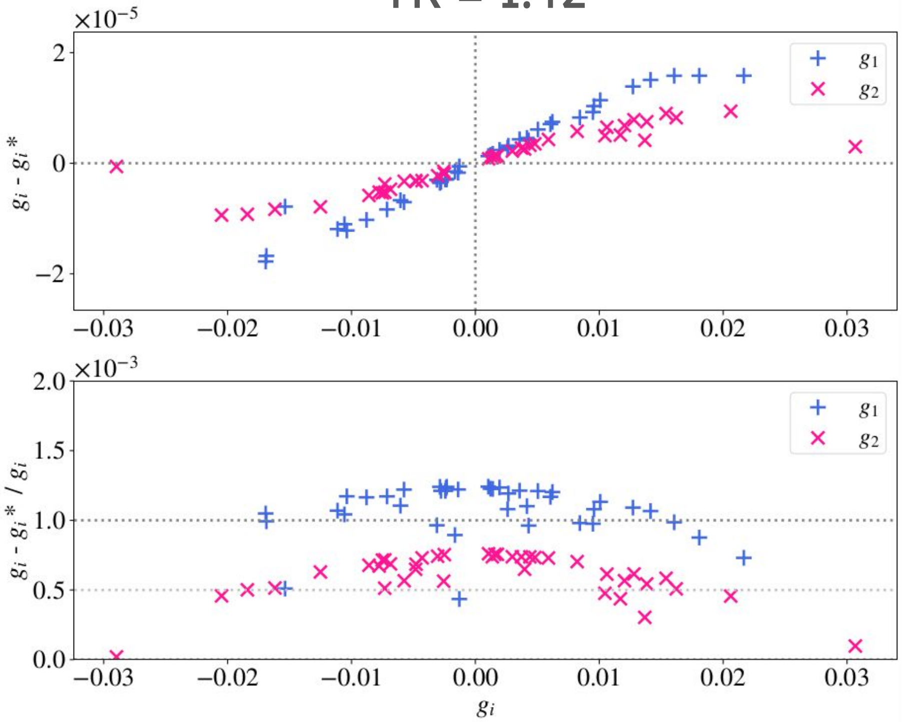
- Gaussian PSF
- TR = 1.5



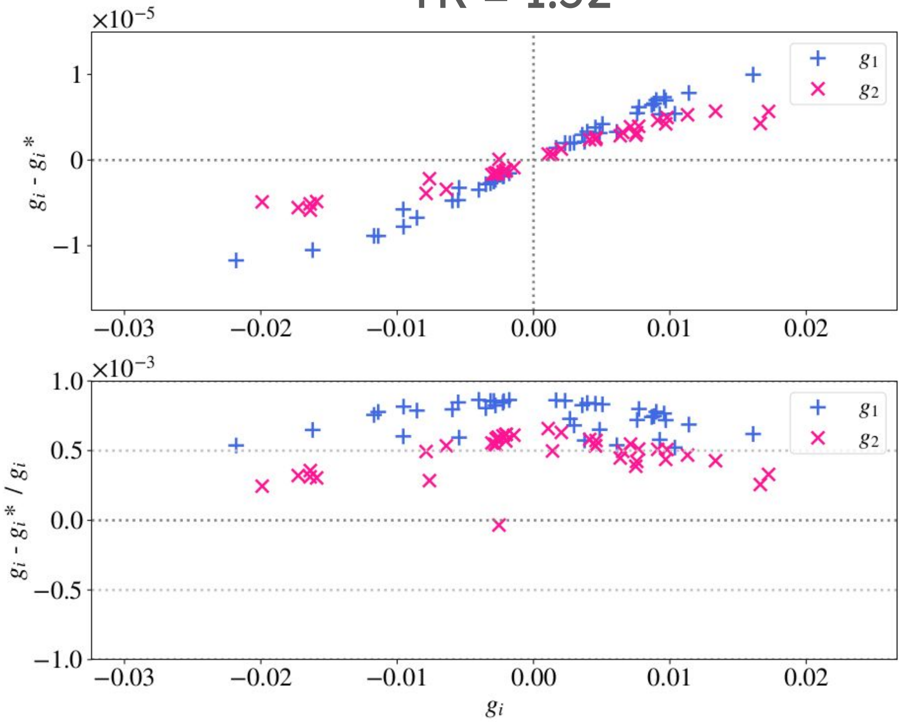
bias < 1‰

Estimations : TR limits (Sersic n=1.0)

TR = 1.42

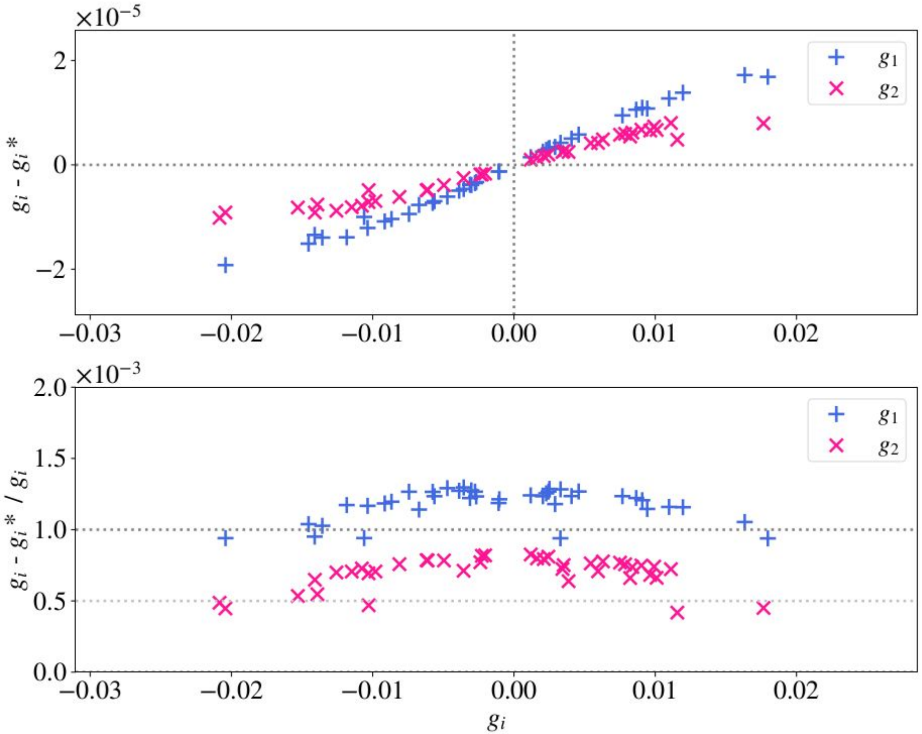


TR = 1.52

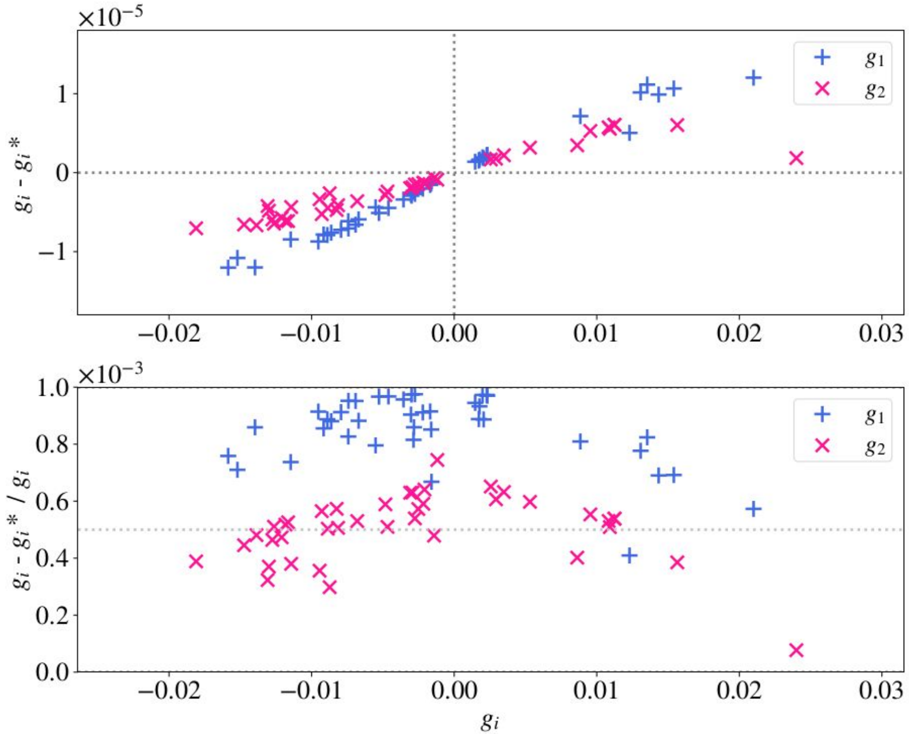


Estimations : TR limits (Sersic n=4.0)

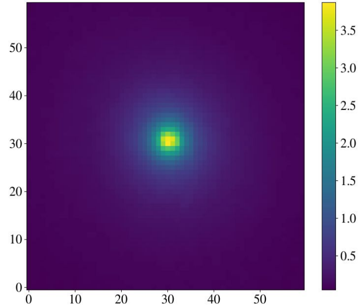
TR = 1.54



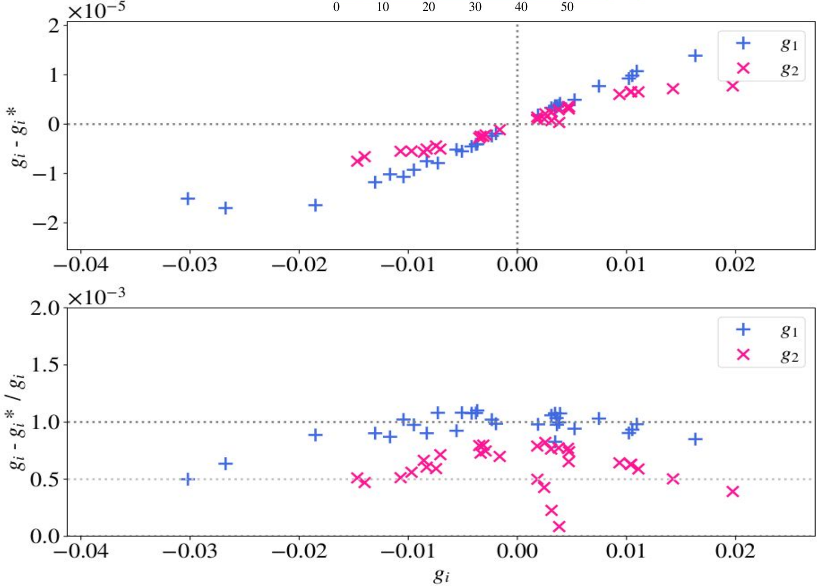
TR = 1.63

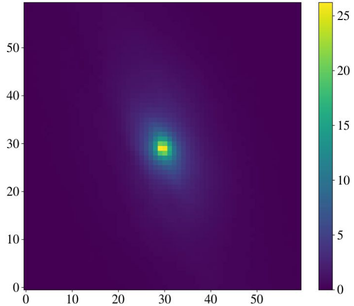


Estimations : TR limits (*COSMOS*)

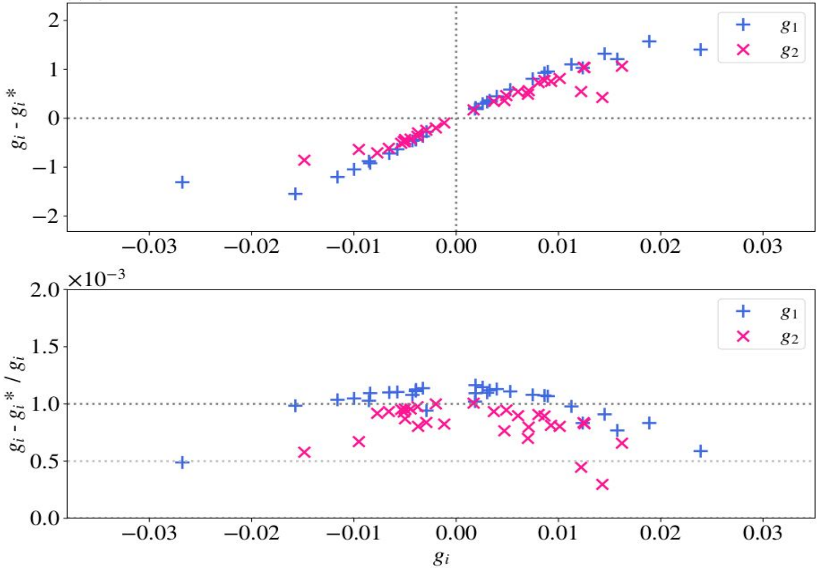


TR = 1.57



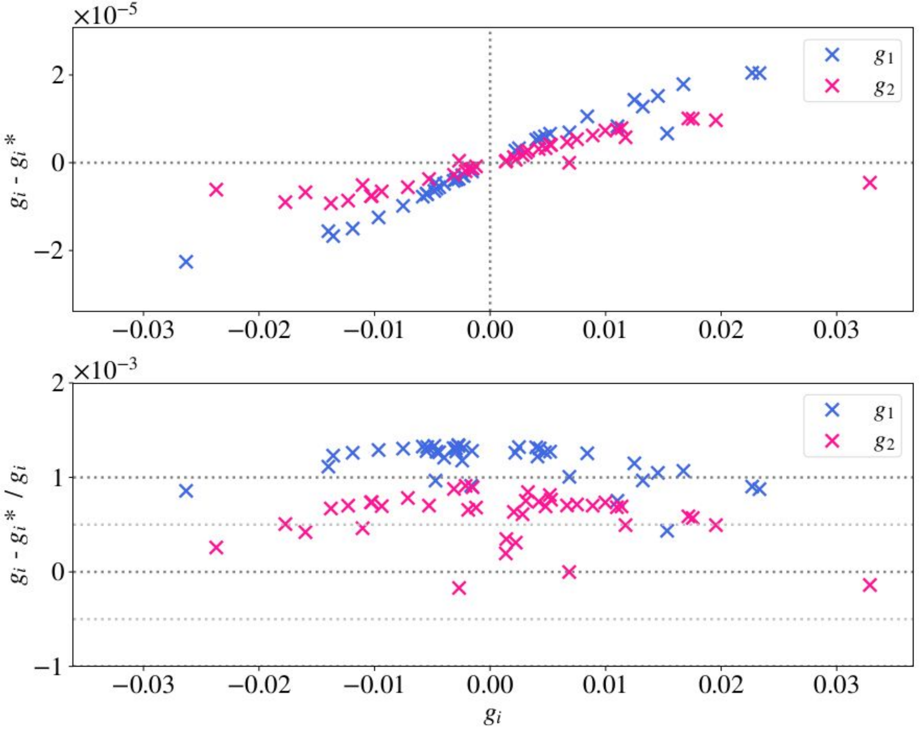


TR = 1.82

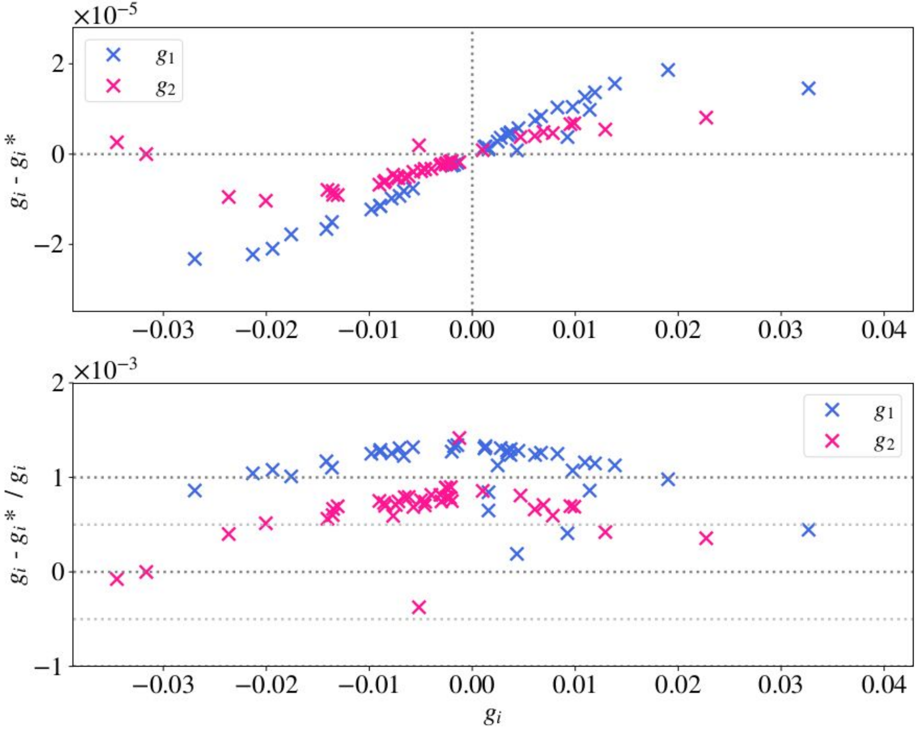


Sersic estimation : different stamp sizes (TR = 1.5)

N = 100



N = 60



Position variance calculation

$$f(x_0, I) = \sum (x_i - x_0)W(x_i - x_0)I_i$$

$$\frac{\partial x_0}{\partial I_i} = -\frac{\frac{\partial f}{\partial I_i}}{\frac{\partial f}{\partial x_0}} \left\{ \begin{aligned} \frac{\partial f}{\partial I_i} &= (x_i - x_0)W_i \\ \frac{\partial f}{\partial x_0} &= -\sum WI + M_W^{-1} \sum (x_i - x_0)(x_i - x_0)^T WI \\ &= -F\mathbb{1} + M_W^{-1} M_P^* F \end{aligned} \right.$$

$$\Rightarrow \frac{\partial x_0}{\partial I_i} = \frac{1}{F}[\mathbb{1} - M_W^{-1} M_P]^{-1}(x_i - x_0)W_i$$

When noise is added (ε) :

$$\begin{aligned} \delta x_0 &= \frac{\partial x_0}{\partial I_i}\epsilon_i \\ &= K(x_i - x_0)W_i\epsilon_i \end{aligned}$$

$$\boxed{\begin{aligned} \sigma(x_0)^2 &= \sum (\frac{\partial x_0}{\partial \epsilon_i})^2 \sigma_{(}\epsilon_i)^2 \\ &= \sum (\frac{\partial x_0}{\partial I_i})^2 \sigma_{noise}^2 \\ &= K^2 \sigma_{noise}^2 \sum (x_i - x_0)(x_i - x_0)^T W_i^2 \\ &= K^2 \sigma_{noise}^2 \frac{M_W}{2} \sum W_i^2 \end{aligned}}$$

Analytical prediction of flux noise bias

$$F = \sum W(x - x_0) I$$

$$t_1 = \frac{1}{2} \frac{d^2 F}{d x_0 d x_0} V = -F tr(M_W^{-1} V_x) + tr(M_W^{-1} m_I M_W^{-1} V_x)$$

$$t_2 = \frac{d^2 F}{d x_0 d I_k} C \; = tr(M_W^{-1} K M_{W^2})$$

Noise bias on unnormalized 2nd moment analytical calculation

$$m_I(I+n) = m_I(I) + \underbrace{\sum_k \left(\frac{dm_I}{dx_0} + \frac{dm_I}{dI_k}\right) n_k}_{\mathbf{0}} + \frac{1}{2}\sum_k \left(\underset{\substack{\downarrow \\ \textbf{4 terms}}}{\frac{d^2m_I}{dx_0^2}} + 2\underset{\substack{\downarrow \\ \textbf{2 terms}}}{\frac{d^2m_I}{dx_0 dI_k}} + \underbrace{\frac{d^2m_I}{dI_k^2}}_{\mathbf{0}}\right) n_k^2$$

$$t1 = 2FV_x$$

$$t2 = -2(V_x M_W^{-1} m_i)$$

$$t3 = t2$$

$$t4 = -m_i Tr(V_x M_W^{-1}) + M_{4i}(M_W^{-1} V_x M_W^{-1})$$

$$t5 = -4K M_{W^2}$$

$$t6 = M_{4W^2}(K M_W^{-1})$$

Noise bias on normalized 2nd moment analytical calculation

$$m_I F^{-1}(I+n) = m_I F^{-1}(I) + \underbrace{\sum_k \frac{d(m_I F^{-1})}{dI_k} n_k}_{\mathbf{0}} + \underset{\substack{\big\downarrow \\ \textbf{5 terms}}}{\frac{1}{2}\sum_{kl} \frac{d^2(m_I F^{-1})}{dI_k dI_l} n_k n_l}$$

$$\left.\begin{aligned} t1 &= B[m_I]/F \\ t2 &= -B[F]m_I/F^2 \\ t3 = t4 &= -2M(W^2)/F^2 \\ t5 &= 2(m_I/F^3)\sum W_k^2 \end{aligned}\right\}$$

B[mI] = noise bias on unnormalized second moments

B[F] = noise bias on flux

Auto calibration factor

$$\mathbf{R} = \frac{1}{2\epsilon}\begin{pmatrix} \boxed{R_{11}} & \boxed{R_{12}} \\ \boxed{R_{21}} & \boxed{R_{22}} \end{pmatrix}$$

$$\boxed{R_{11}} = (M_{xx}^{1+} - M_{xx}^{1-}) - (M_{yy}^{1+} - M_{yy}^{1-})$$

$$\boxed{R_{12}} = 2(M_{xy}^{1+} - M_{xy}^{1-})$$

$$\boxed{R_{21}} = (M_{xx}^{2+} - M_{xx}^{2-}) - (M_{yy}^{2+} - M_{yy}^{2-})$$

$$\boxed{R_{22}} = 2(M_{xy}^{2+} - M_{xy}^{2-})$$