16 April 2024

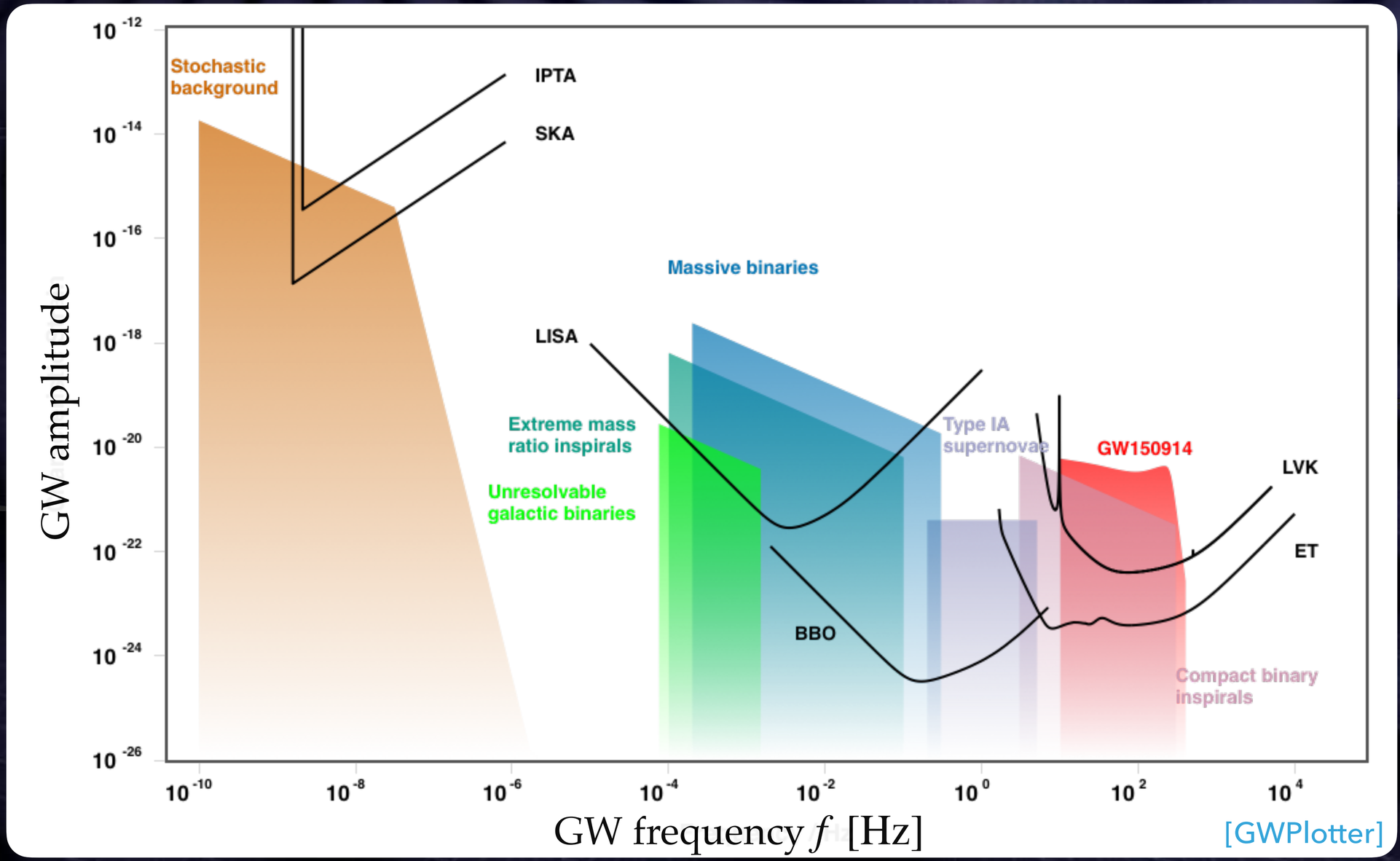
Laboratoire de Physique Théorique Annecy

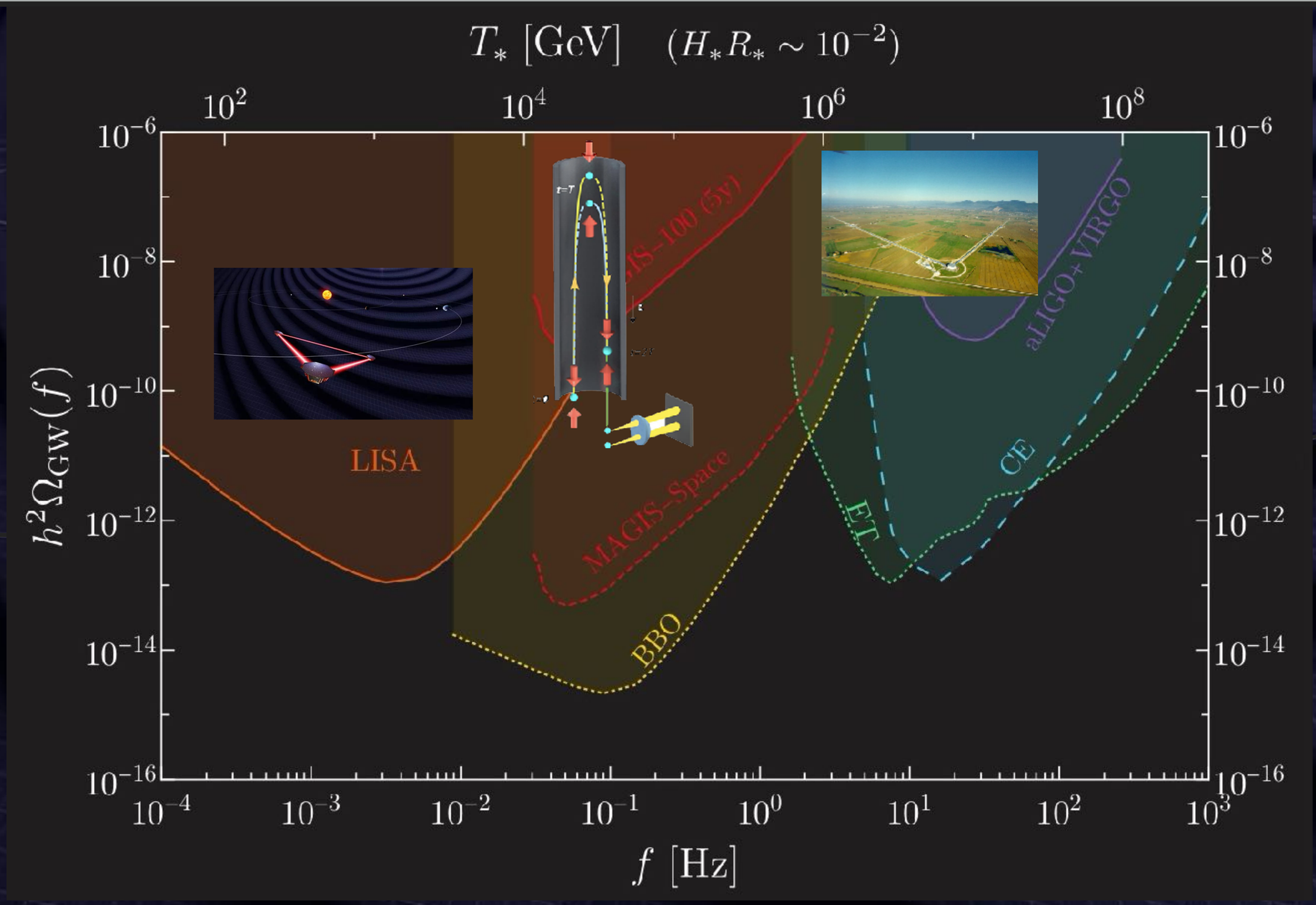
Utilising the Causal Spectrum of GW Backgrounds: Implications for Pulsar Timing Arrays

Davide Racco

ETH zürich

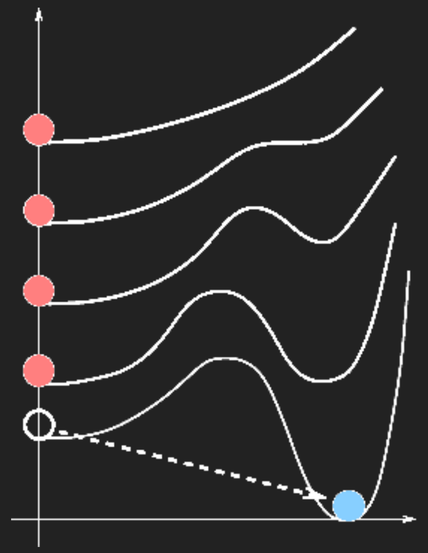
Universität Zürich[UZH]

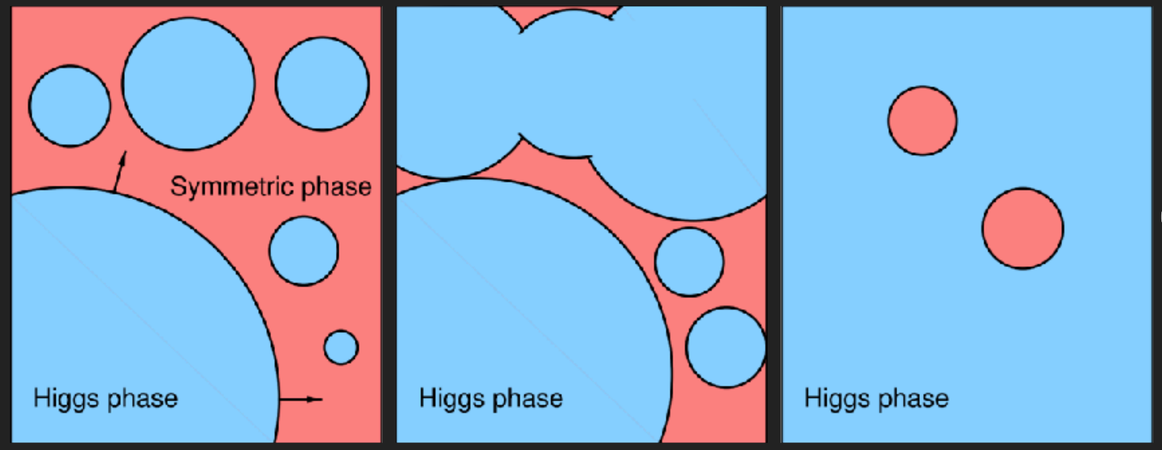

Stochastic background
IPTA
SKA
Massive binaries
LISA
Extreme mass ratio inspirals
Unresolvable galactic binaries
Type IA supernovae
GW150914
LVK
ET
BBO
Compact binary inspirals
GW amplitude
GW frequency f [Hz]
[GWPlotter]


T_* [GeV] $(H_* R_* \sim 10^{-2})$
10^2
10^4
10^6
10^8
$h^2\Omega_{\rm GW}(f)$
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
f [Hz]
LISA
MAGIS-100 (5y)
MAGIS-Space
BBO
ET
CE
aLIGO+VIRGO

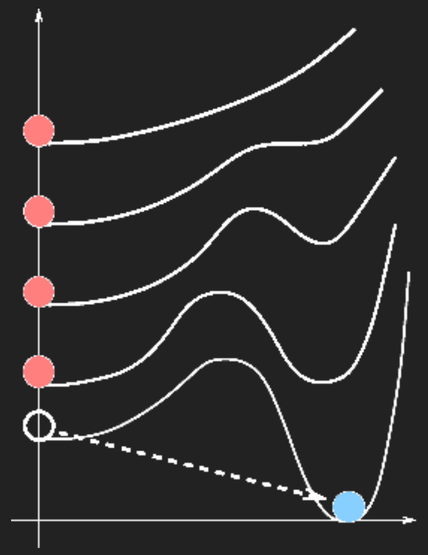
- GWs from primordial source **local** in *space* and *time*
- Phase transitions, preheating, peak in $\mathscr{P}_\zeta$ …

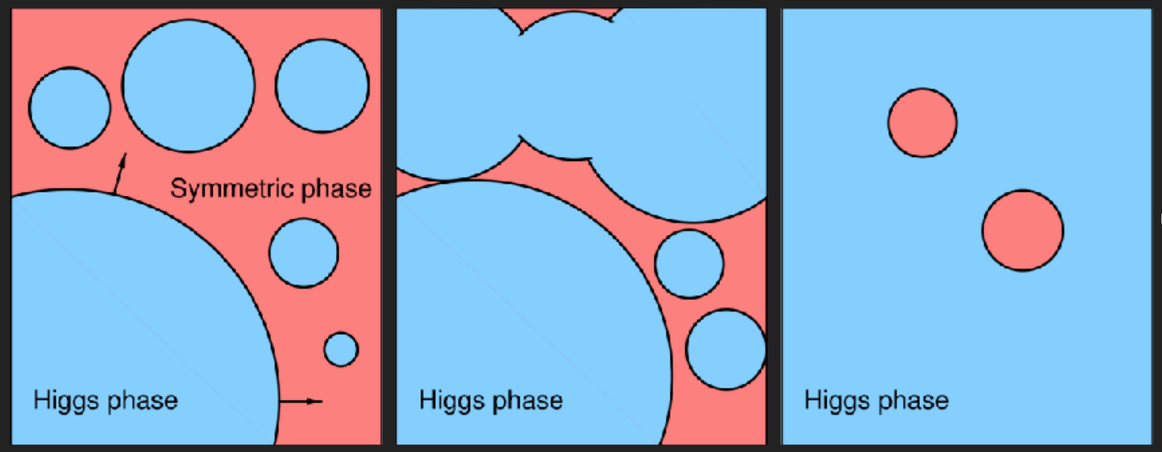
- GWs from primordial source **local** in *space* and *time*
- Phase transitions, preheating, peak in $\mathscr{P}_\zeta$ …



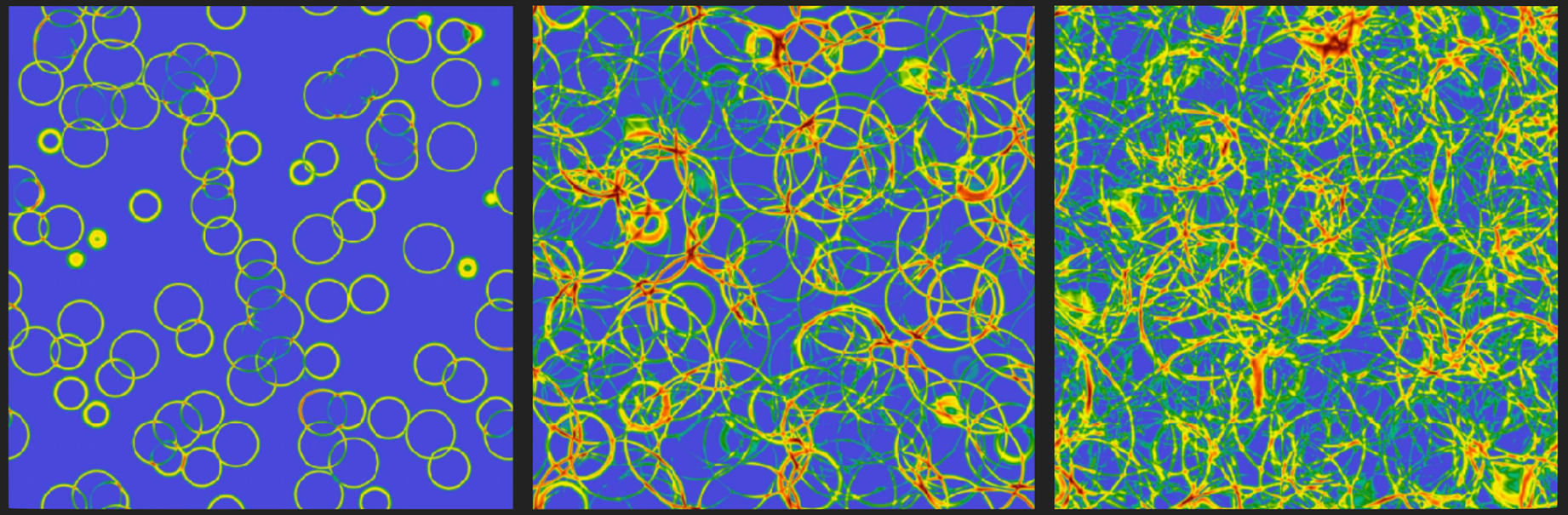


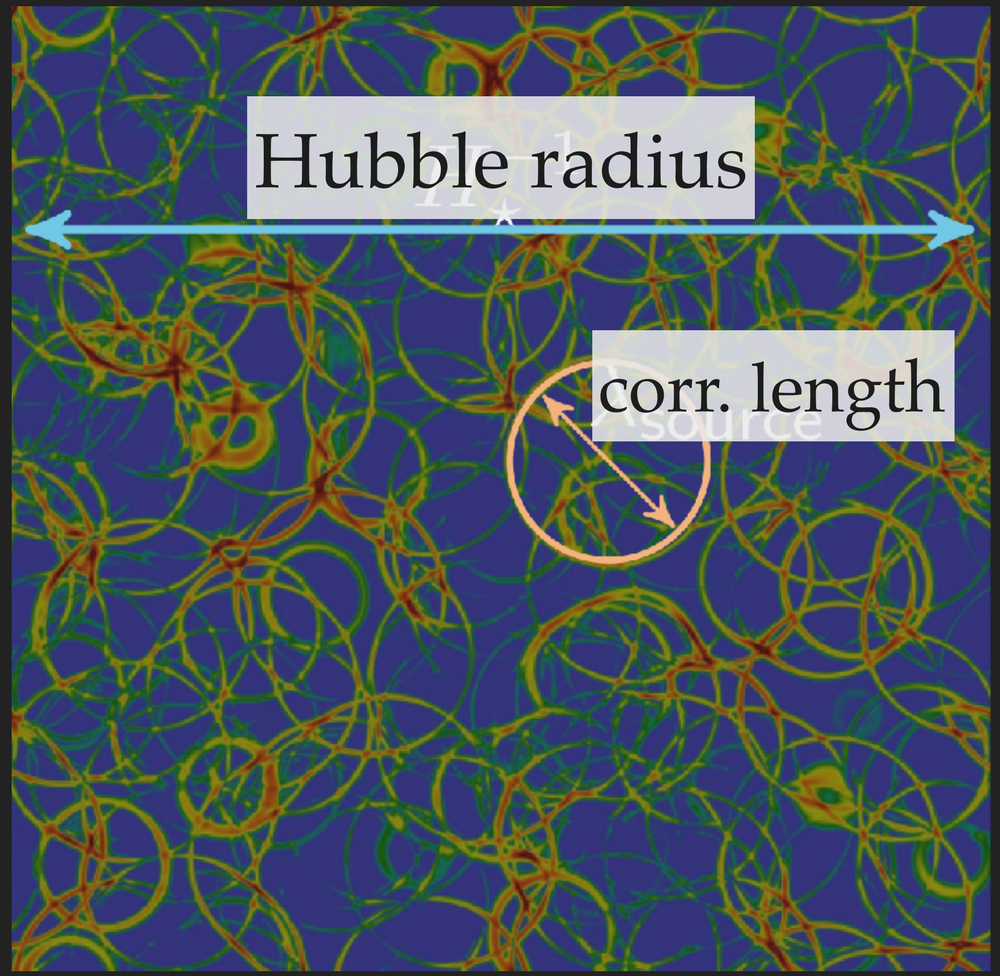
- GWs from primordial source **local** in *space* and *time*
- Phase transitions, preheating, peak in $\mathscr{P}_\zeta$ …

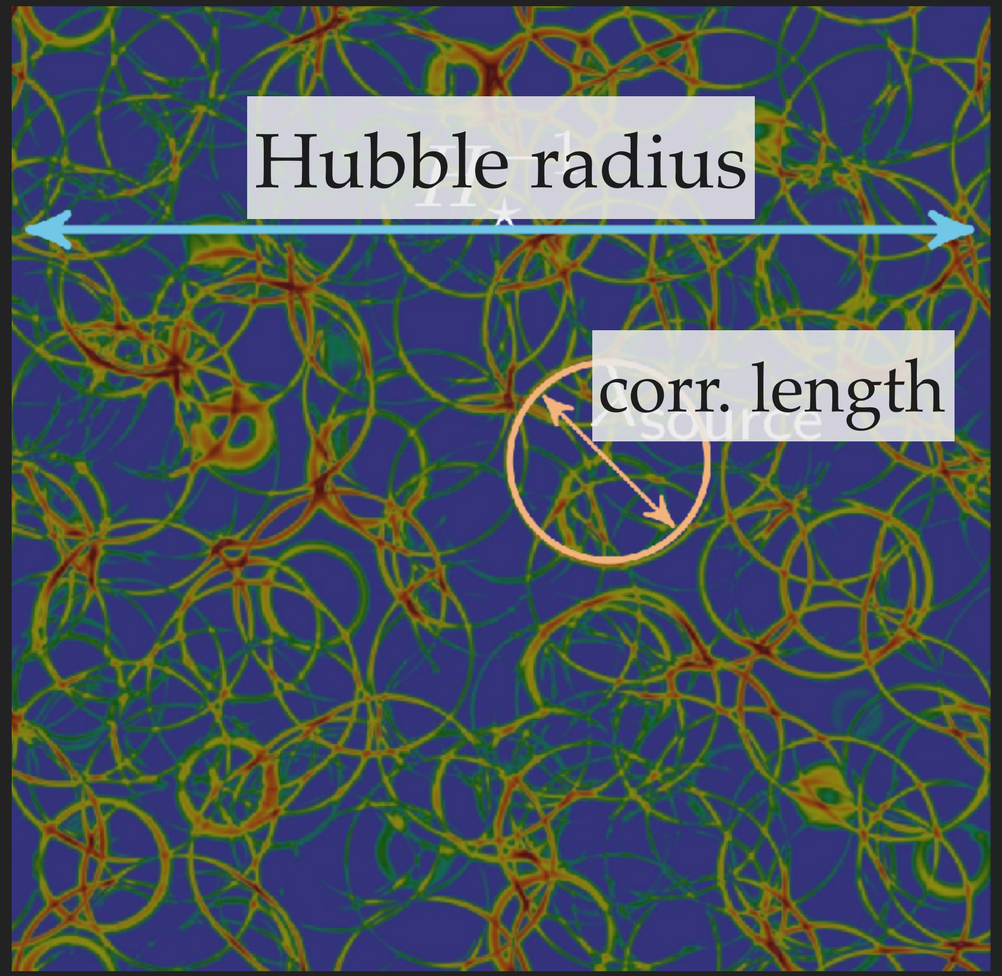




['15 Hindmarsh+]



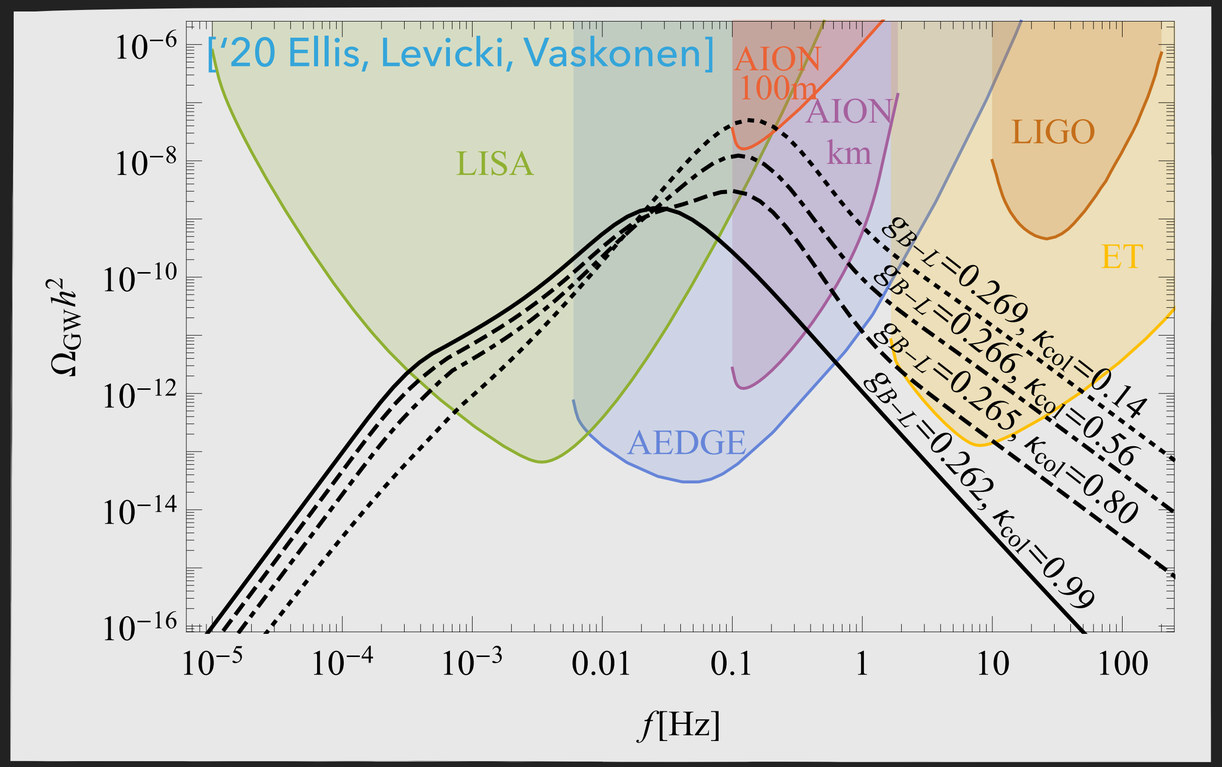

Hubble radius
corr. length



$$\langle \Pi(0)\, \Pi(d \gg \lambda_{\text{source}}) \rangle = 0 \quad \Longrightarrow$$

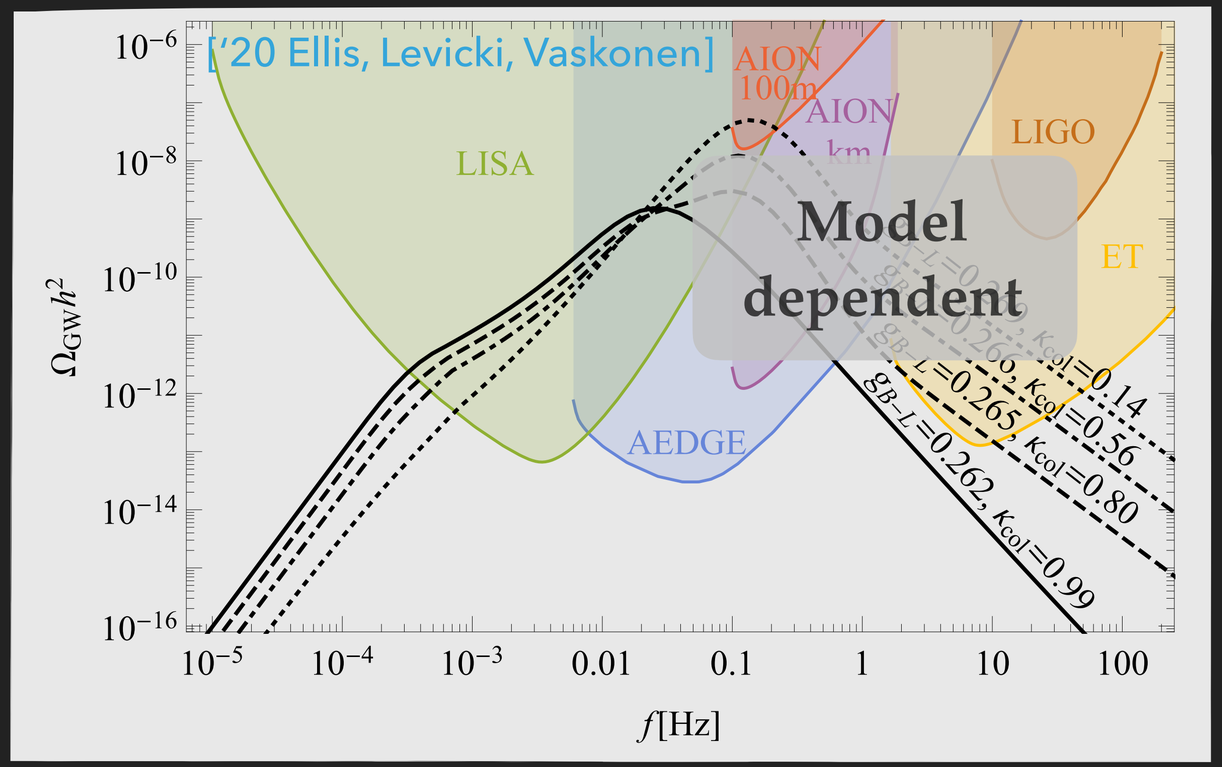
$$\langle \widetilde{\Pi}(k)\, \widetilde{\Pi}(-k) \rangle \overset{k \ll \lambda_{\text{source}}^{-1}}{\longrightarrow} \text{constant}$$

UNIVERSALITY OF THE CAUSALITY TAIL



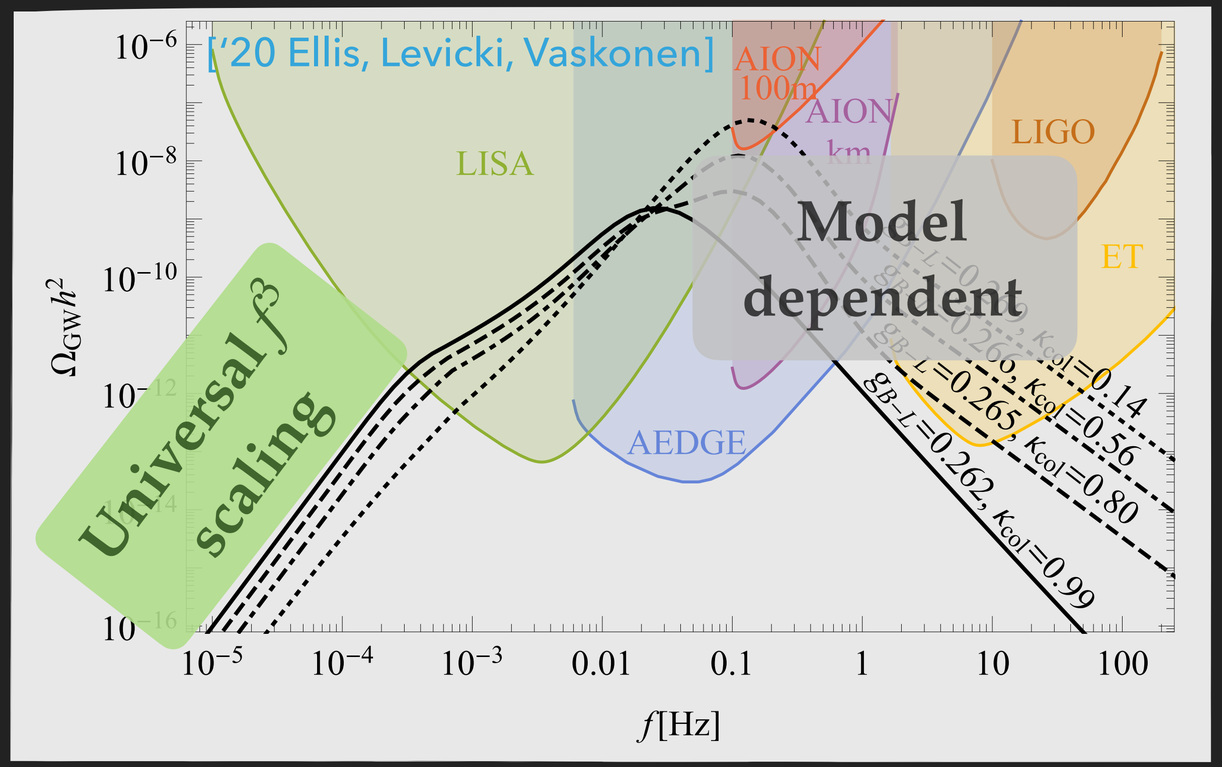
['03 Seto, Yokoyama;
'05 Boyle, Steinhardt;
'06 Watanabe, Komatsu;
'09 Caprini, Durrer, Konstandin, Servant;
'18 Caprini, Figueroa;
'18 Saikawa, Shirai;
'18 Cui, Lewicki, Morrissey, Wells;
'19 D'Eramo, Schmitz;
...]

UNIVERSALITY OF THE CAUSALITY TAIL

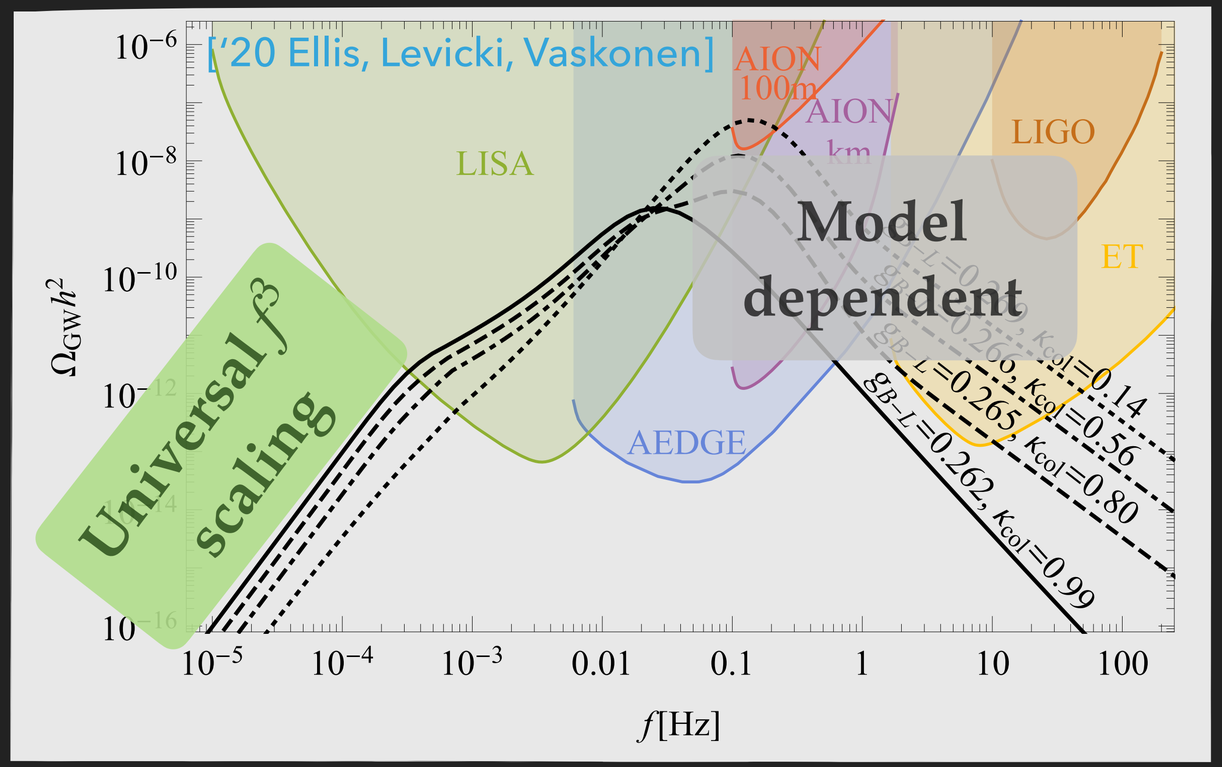


['03 Seto, Yokoyama;
'05 Boyle, Steinhardt;
'06 Watanabe, Komatsu;
'09 Caprini, Durrer, Konstandin, Servant;
'18 Caprini, Figueroa;
'18 Saikawa, Shirai;
'18 Cui, Lewicki, Morrissey, Wells;
'19 D'Eramo, Schmitz;
...]

UNIVERSALITY OF THE CAUSALITY TAIL



['03 Seto, Yokoyama;
'05 Boyle, Steinhardt;
'06 Watanabe, Komatsu;
'09 Caprini, Durrer, Konstandin, Servant;
'18 Caprini, Figueroa;
'18 Saikawa, Shirai;
'18 Cui, Lewicki, Morrissey, Wells;
'19 D'Eramo, Schmitz;
...]



['03 Seto, Yokoyama;
'05 Boyle, Steinhardt;
'06 Watanabe, Komatsu;
'09 Caprini, Durrer, Konstandin, Servant;
'18 Caprini, Figueroa;
'18 Saikawa, Shirai;
'18 Cui, Lewicki, Morrissey, Wells;
'19 D'Eramo, Schmitz;
...]

Causality tail

- Low-f spectrum GWs ⇔ universal (model-independent) probe of the Universe

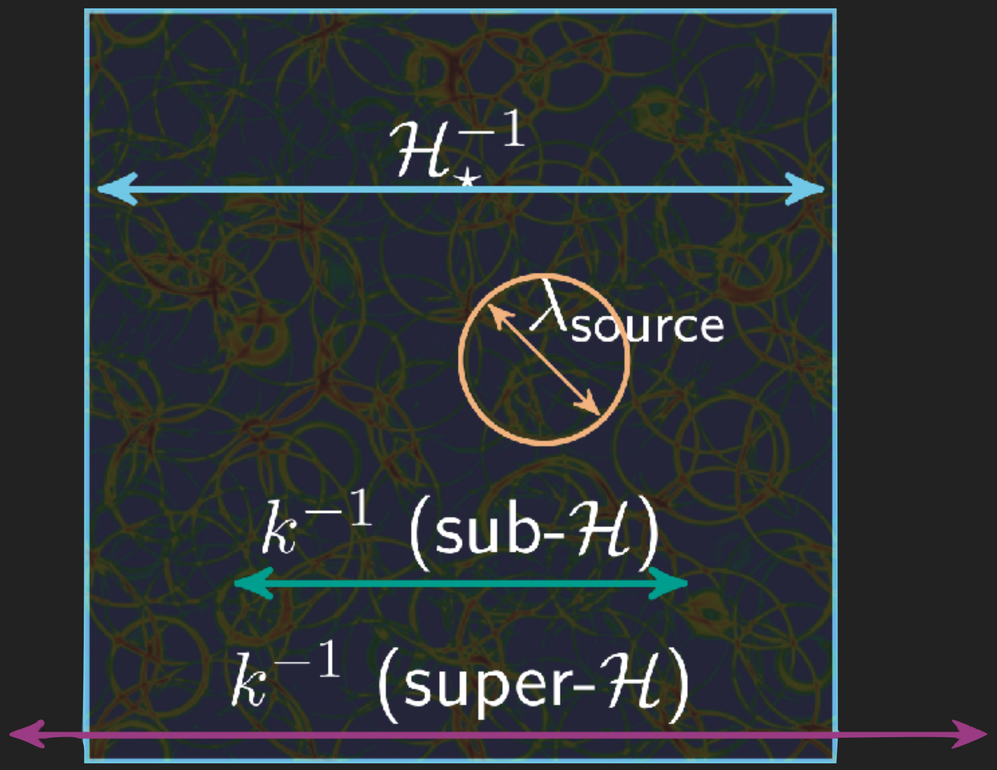
wavelength $k^{-1} \gg$ corr. length λ_{source}

period $f^{-1} \gg$ duration of phase transition β^{-1}

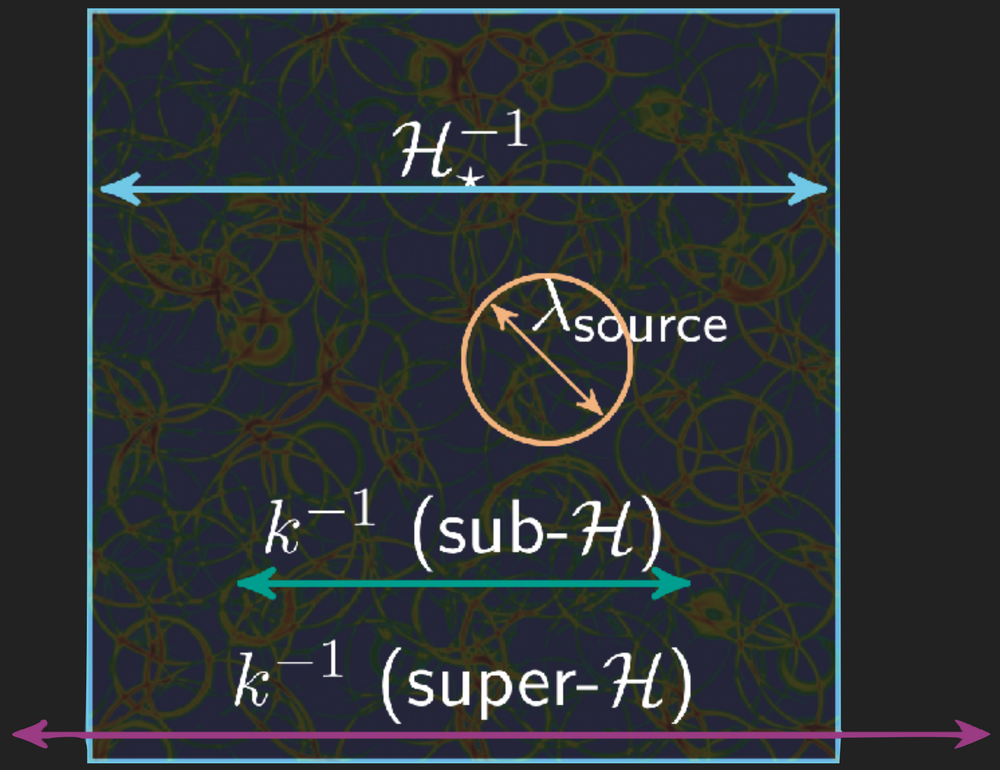
$$\Box h = J_\star \, \delta(\tau - \tau_\star) \quad \Longrightarrow \quad \begin{cases} h = 0 \\ h' = J_\star \end{cases}$$

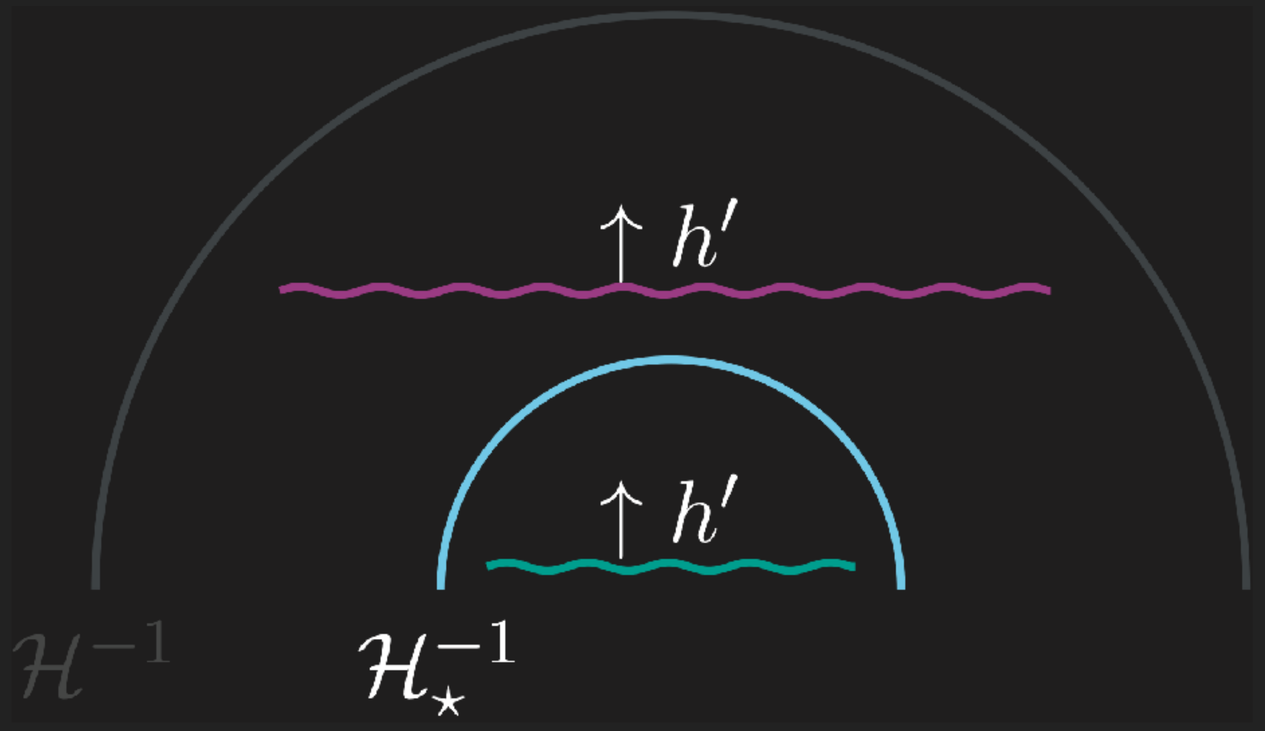
$$|h(\tau)| \sim \underbrace{\frac{a_\star}{a}}_{\text{redshift}} \cdot \underbrace{\frac{1}{k}}_{\text{for RD}} \cdot J_\star$$

$$\frac{\mathrm{d}\,\Omega_{\text{GW}}}{\mathrm{d}\ln k} \sim \underbrace{k^3}_{\text{phase space}} \cdot \underbrace{k^2}_{\rho_{\text{GW}} \sim h'^2} \cdot \underbrace{\frac{1}{k^2}}_{\text{for RD}} \cdot \underbrace{P_\Pi(k)}_{k \text{ ind. from causality}} \sim k^3 \,.$$

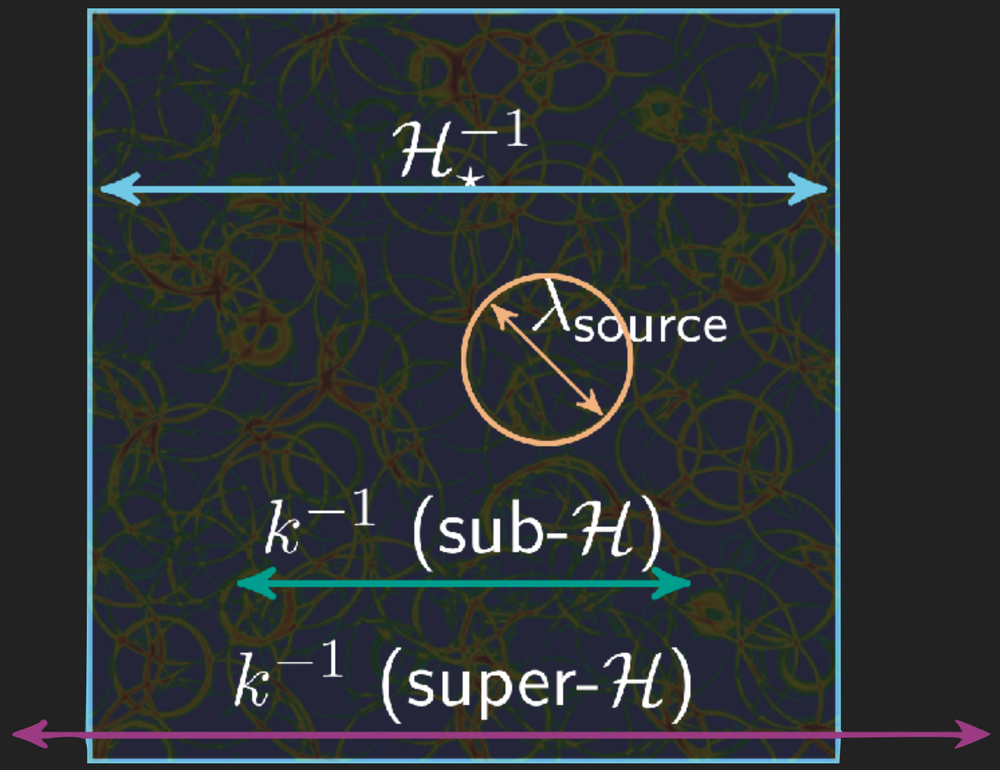

$\mathcal{H}_\star^{-1}$
λ_{source}
k^{-1} (sub-$\mathcal{H}$)
k^{-1} (super-$\mathcal{H}$)

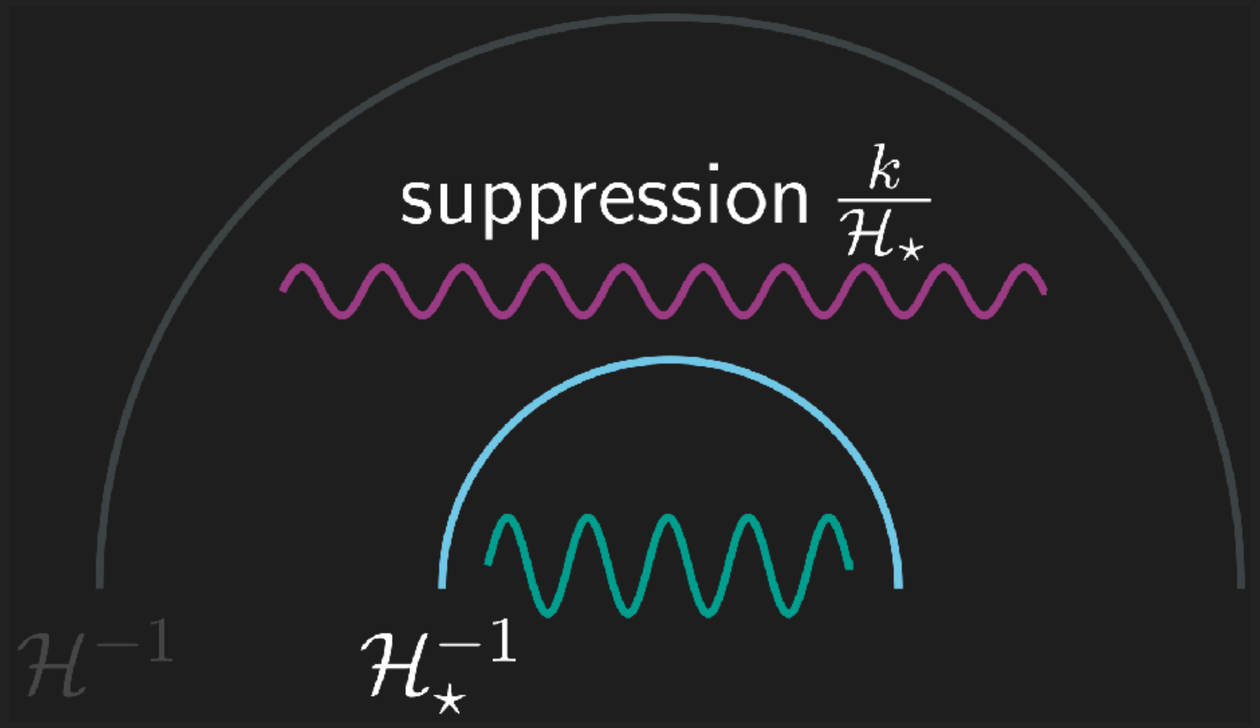
$$h'' + 2\mathcal{H}\,h' + k^2 h = 0$$



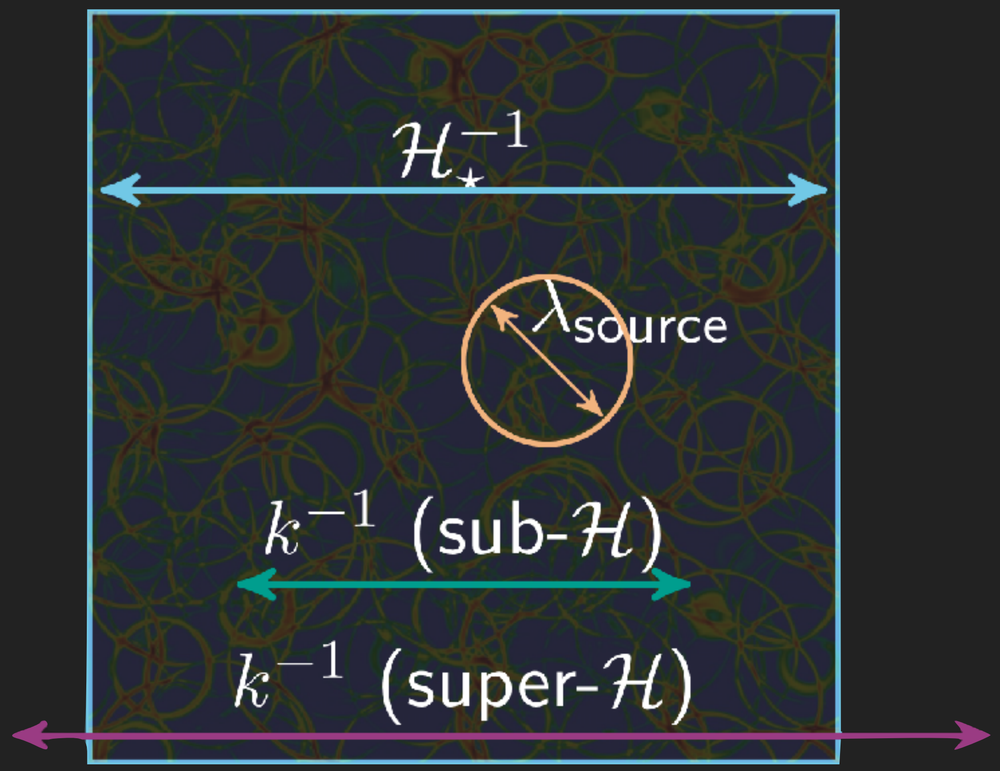


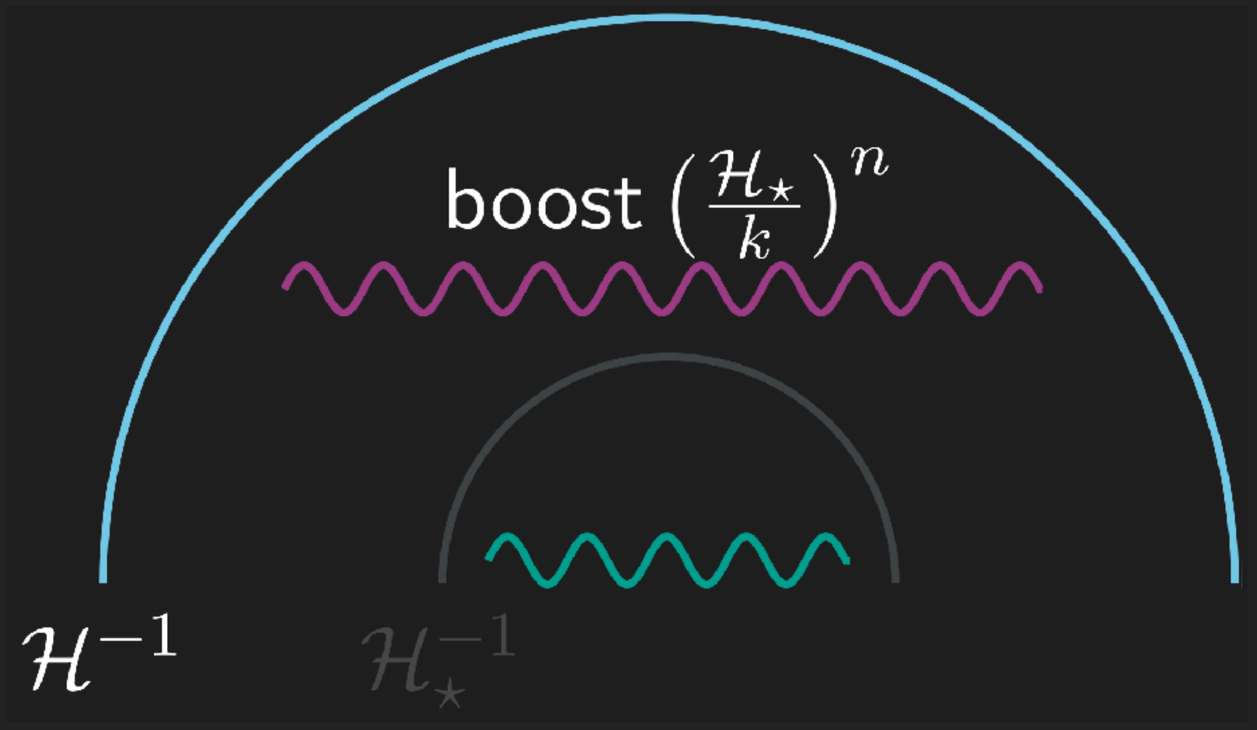
$$h'' + 2\mathcal{H}\,h' + k^2 h = 0$$





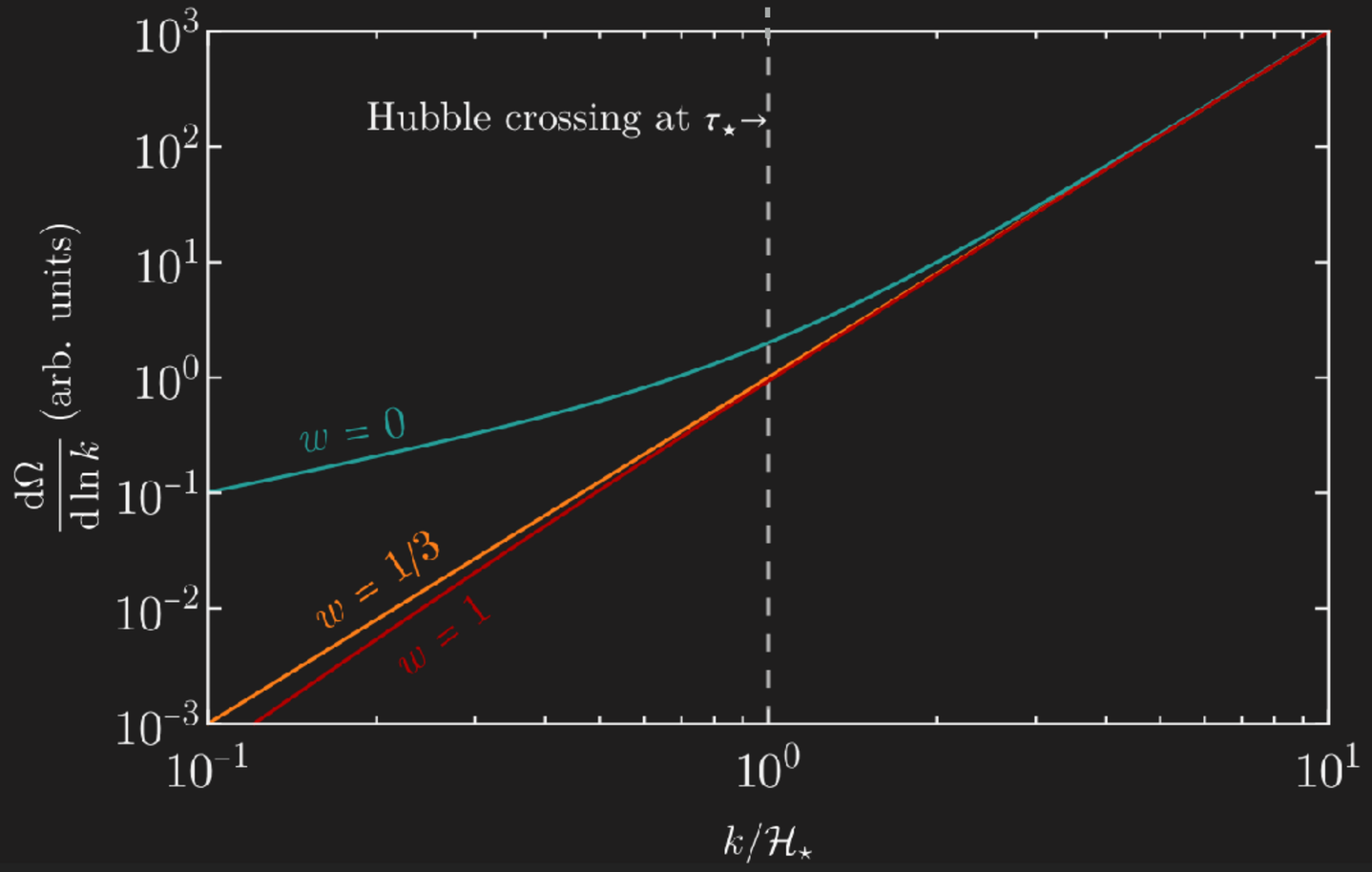
$$h'' + 2\mathcal{H}\,h' + k^2 h = 0$$





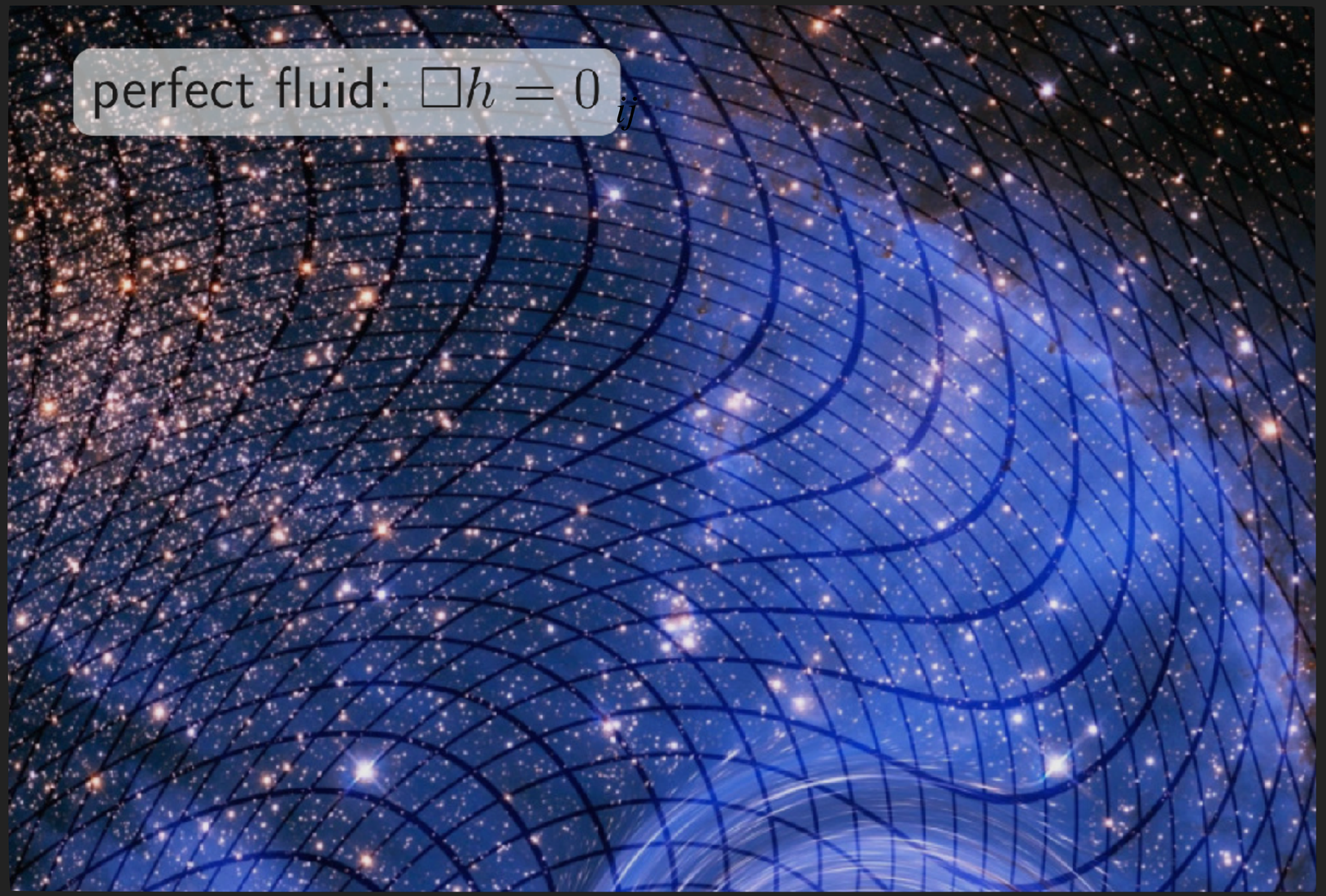
$$\Omega_{\rm GW} \sim \begin{cases} k^3 & \text{RD} \\ k & \text{MD} \end{cases} \text{(super-horizon)}$$

$\Omega_{\rm GW} \sim k^3$ (sub-horizon)



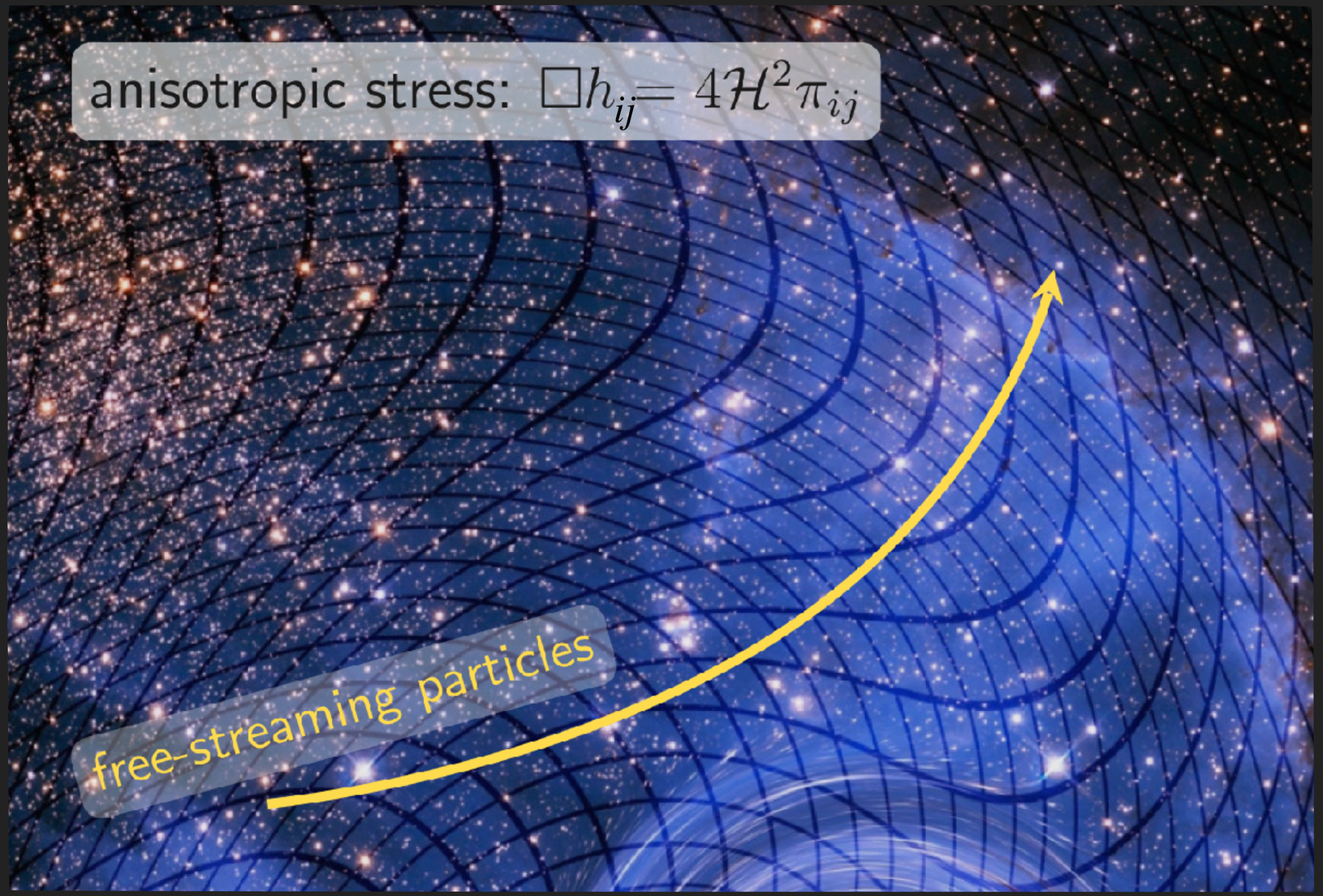
- Causality tail ⇔ probe equation-of-state w

EFFECTS ON GW PROPAGATION

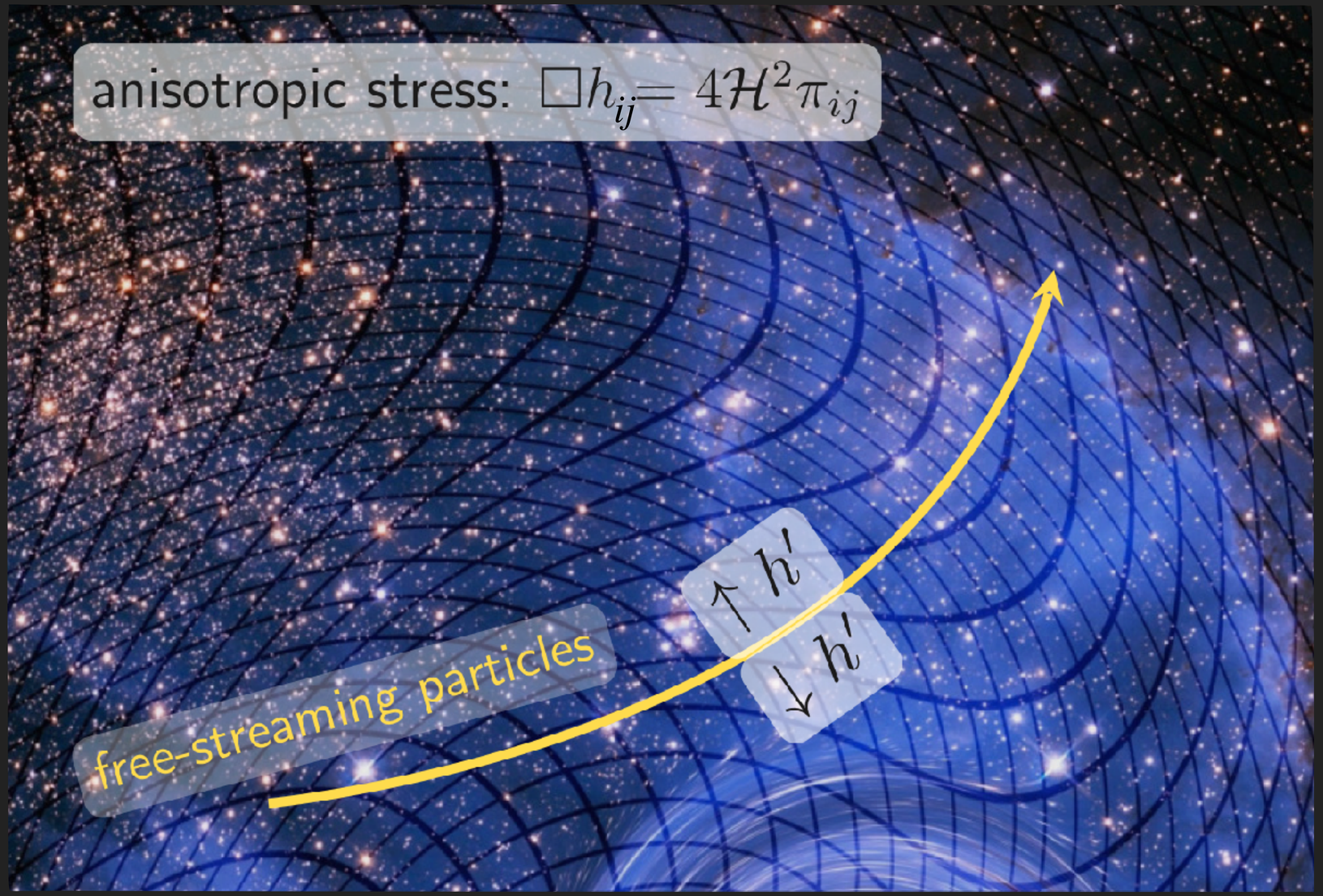


['86 Bond]
['04 Weinberg]
['06 Watanabe, Komatsu]

EFFECTS ON GW PROPAGATION

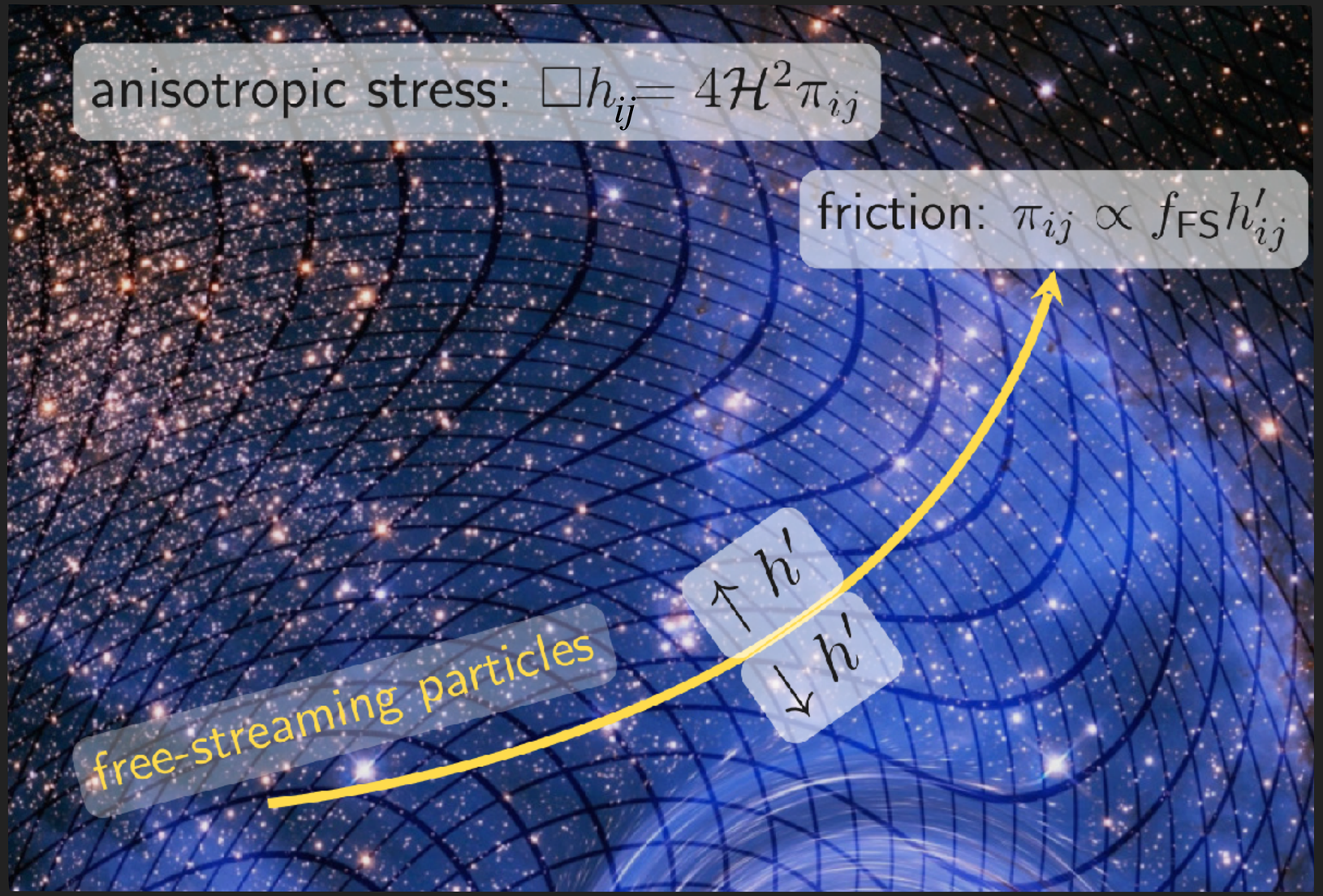


['86 Bond]
['04 Weinberg]
['06 Watanabe, Komatsu]



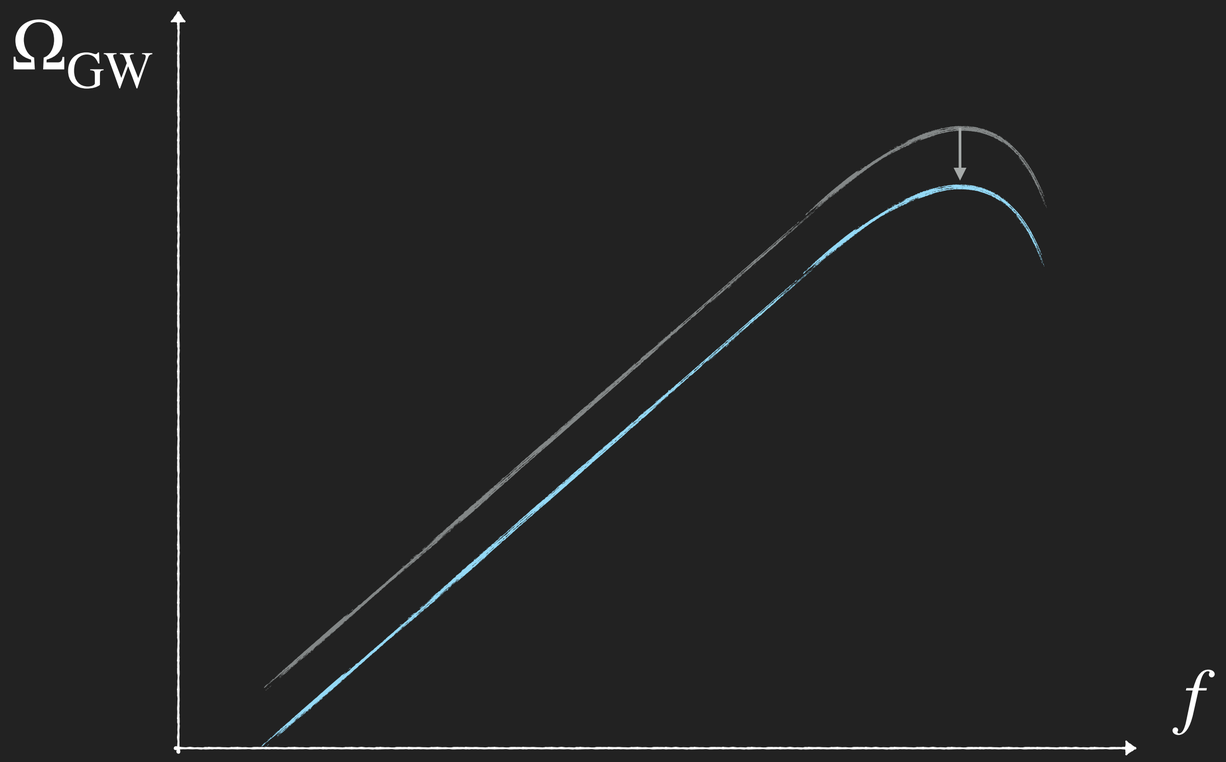
['86 Bond]
['04 Weinberg]
['06 Watanabe, Komatsu]

EFFECTS ON GW PROPAGATION



['86 Bond]
['04 Weinberg]
['06 Watanabe, Komatsu]

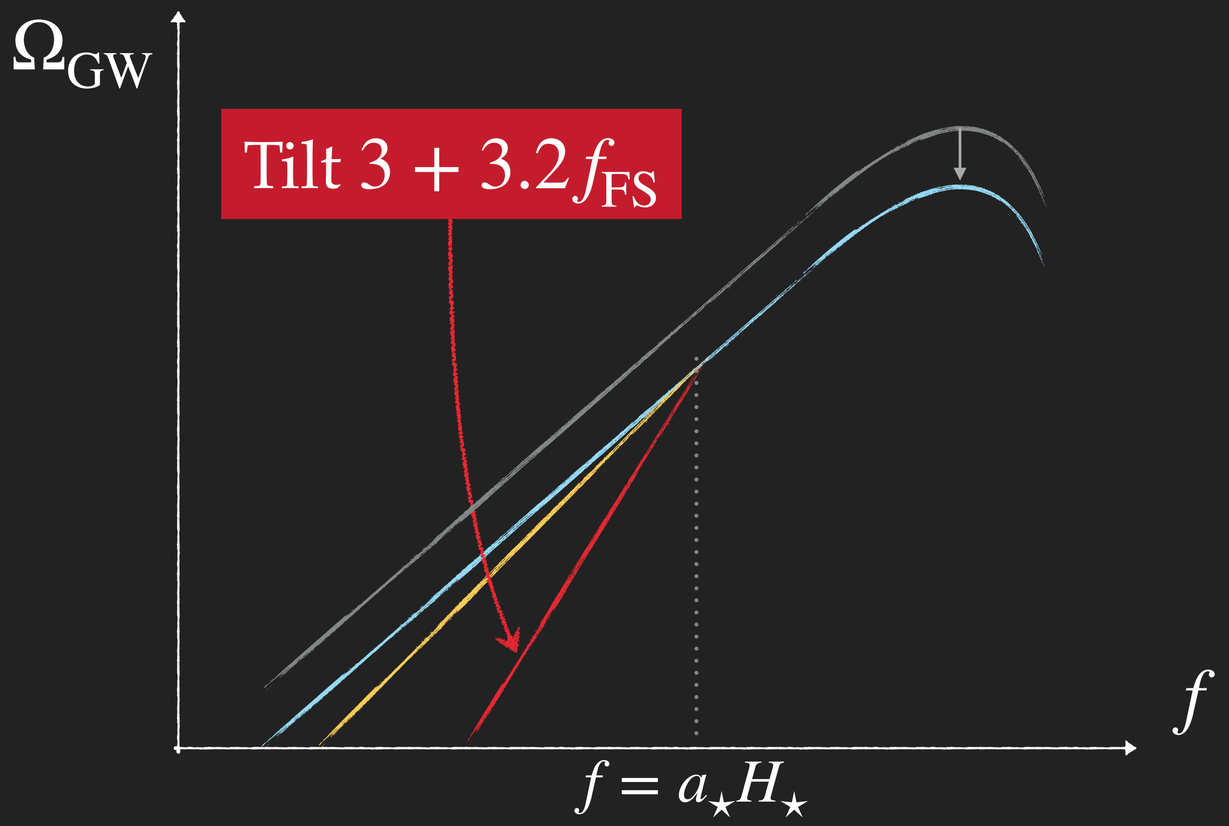
- Weinberg damping: $h' \neq 0$ at horizon crossing
- Frequency-independent ['04 Weinberg]



- Weinberg damping: $h' \neq 0$ at horizon crossing
- Frequency-independent [‘04 Weinberg]

- Causality-limited sources: **also** affected at GW *production*
- Frequency-dependent

[‘20 Hook, Marques-Tavares, **DR**]

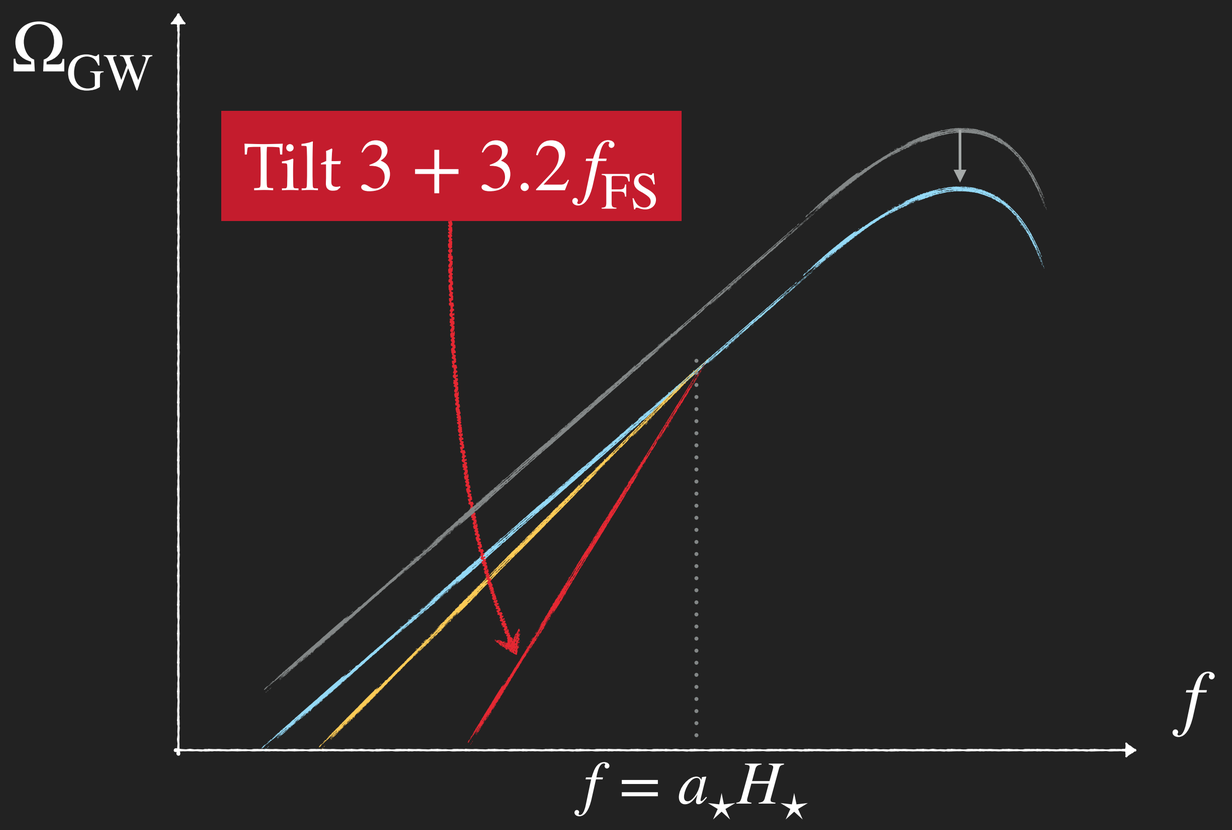


- Weinberg damping: $h' \neq 0$ at horizon crossing
- Frequency-independent [‘04 Weinberg]

- Causality-limited sources: **also** affected at GW *production*
- Frequency-dependent

[‘20 Hook, Marques-Tavares, **DR**]

- Causality tail ⇔ probe new free-streaming particles before BBN

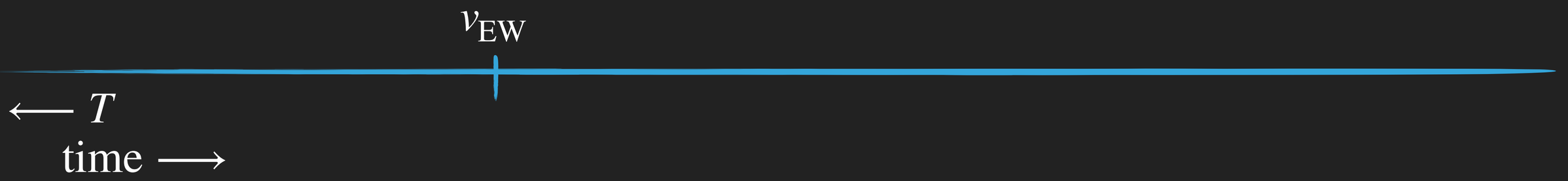


- SM particles are massless $\Rightarrow w = \frac{1}{3}$

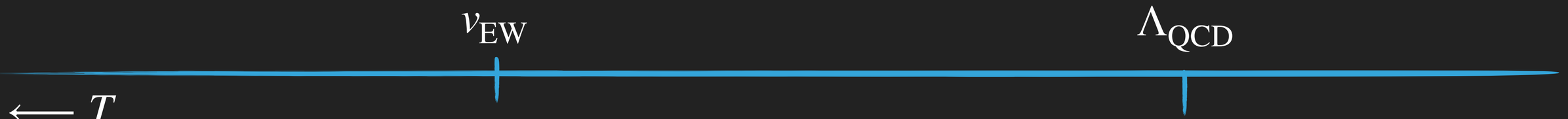
$\longleftarrow T$

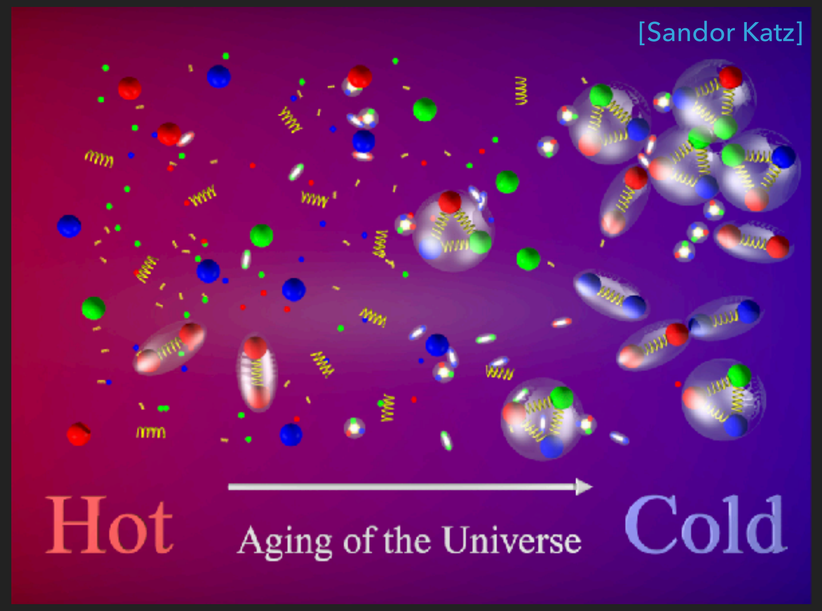
time $\longrightarrow$

- SM particles are massless $\Rightarrow w = \frac{1}{3}$
- Heavy particle decays $\Rightarrow \Omega_{\rm GW} \sim g_\star(T)^{1/3}$

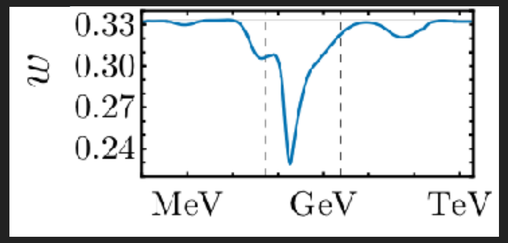


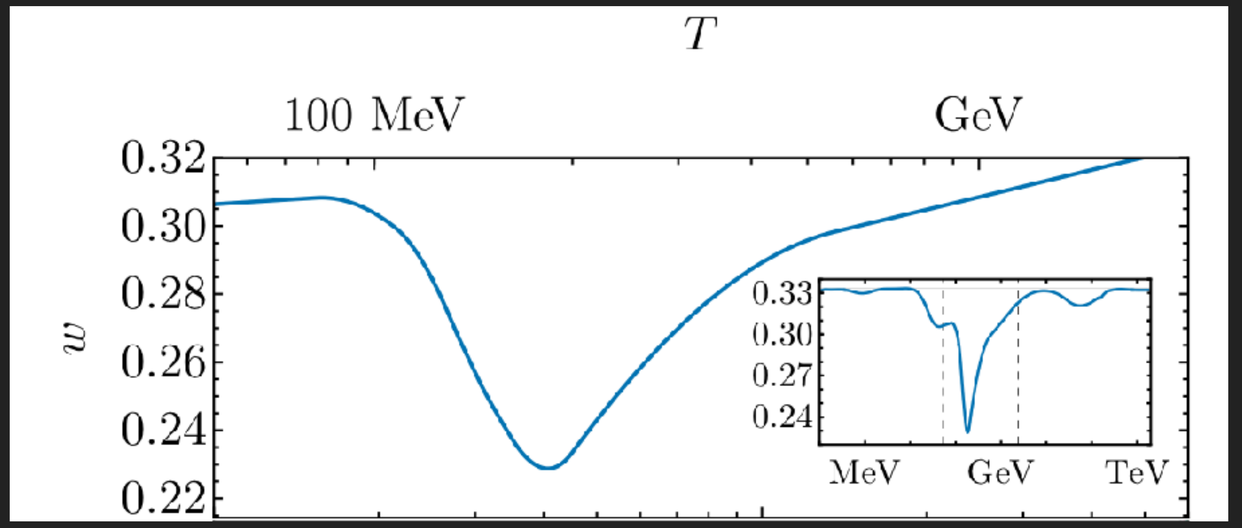
- SM particles are massless $\Rightarrow w = \frac{1}{3}$
- Heavy particle decays $\Rightarrow \Omega_{\rm GW} \sim g_\star(T)^{1/3}$
- QCD confines
- w deviates from $\frac{1}{3}$

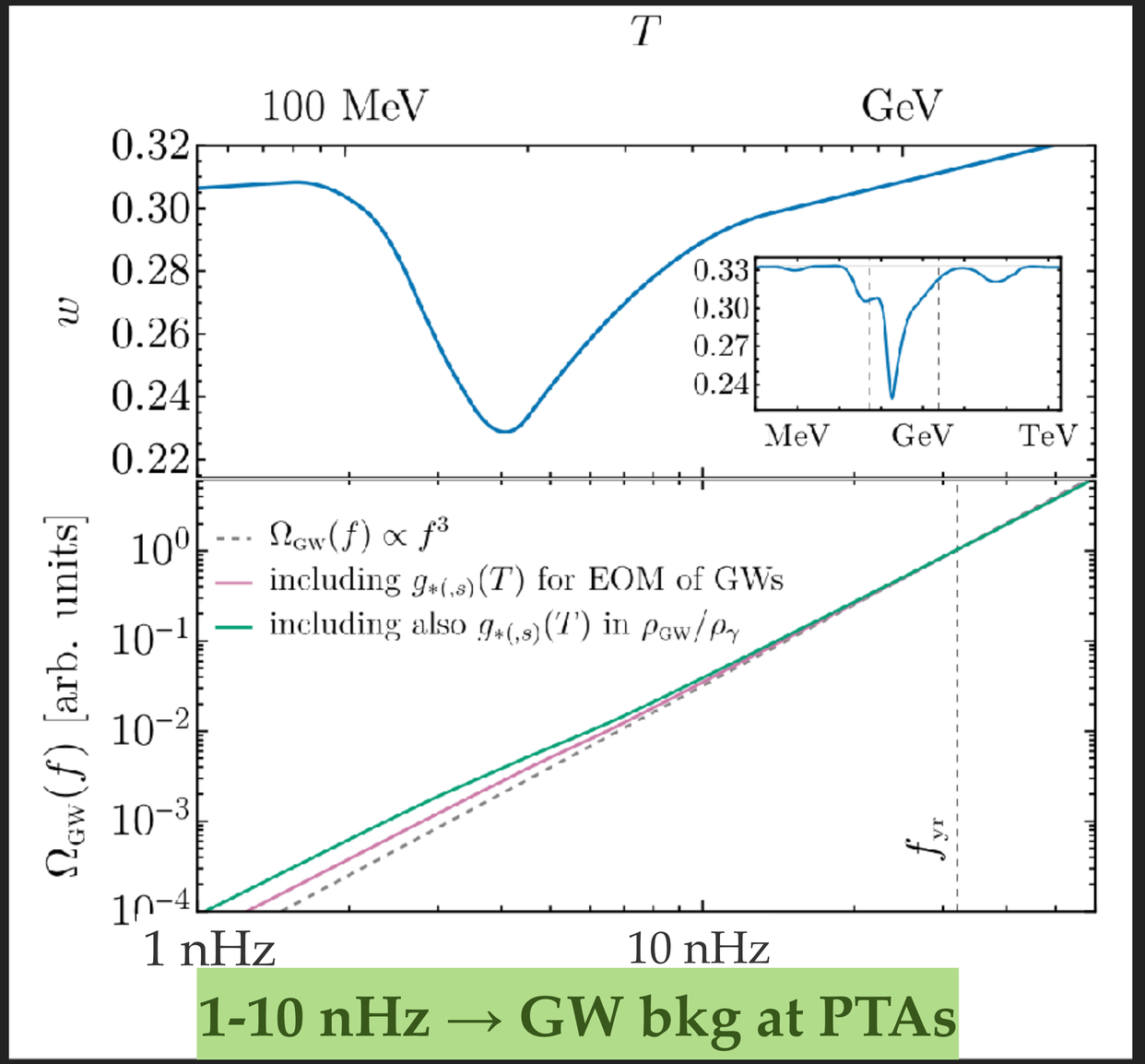




STANDARD MODEL EFFECT ON THE CAUSALITY TAIL



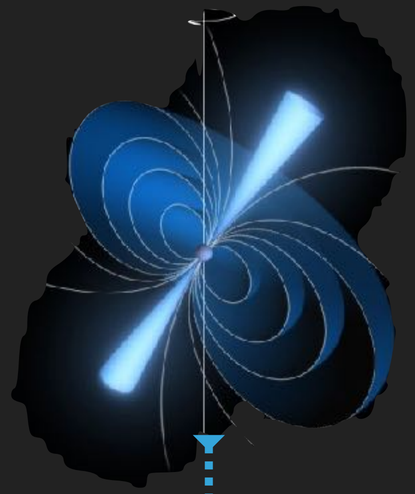

T
100 MeV
GeV
0.32
0.30
0.28
0.26
0.24
0.22
w
0.33
0.30
0.27
0.24
MeV
GeV
TeV



1-10 nHz → GW bkg at PTAs

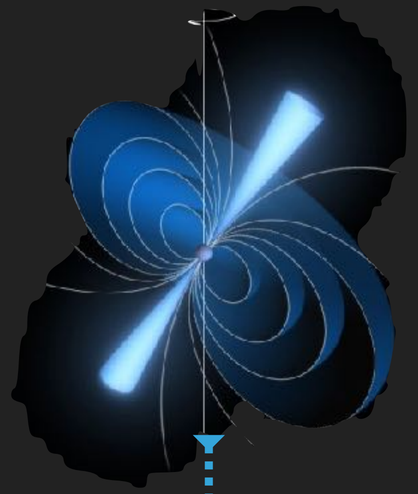
PULSAR TIMING ARRAY MEASUREMENTS

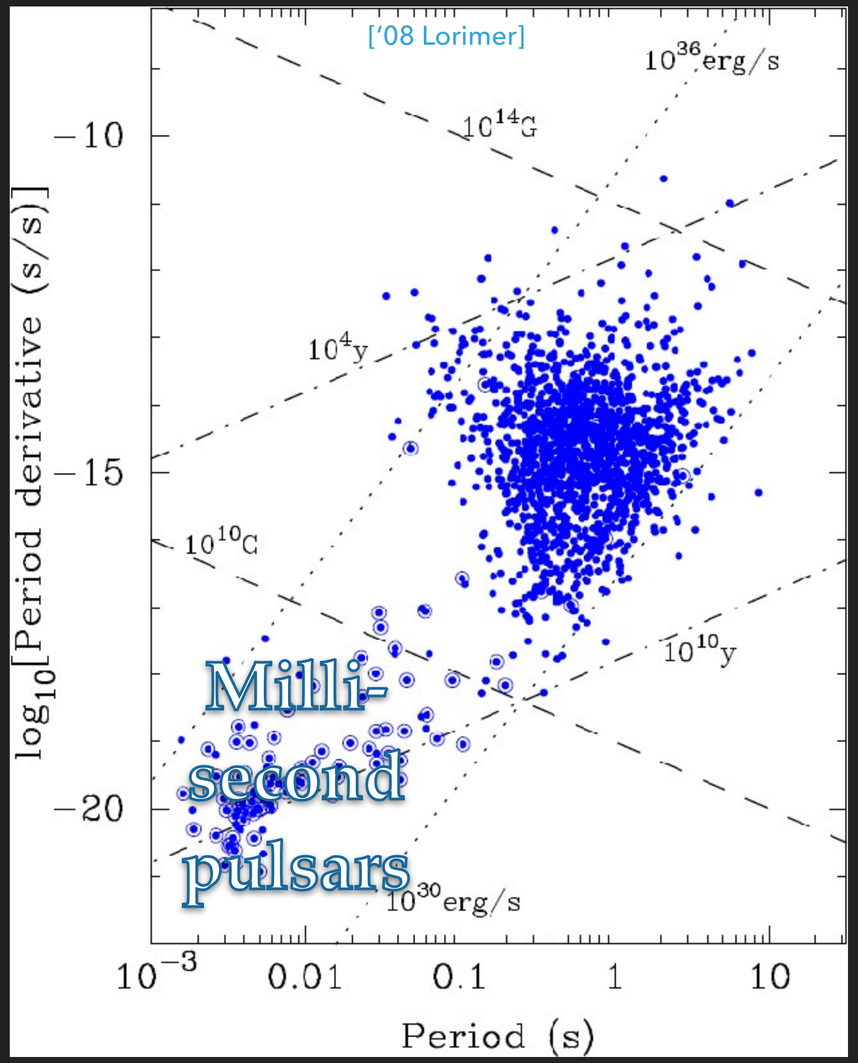
['78 Sazhin; '79 Detweiler;
'83 Hellings, Downs]



PULSAR TIMING ARRAY MEASUREMENTS

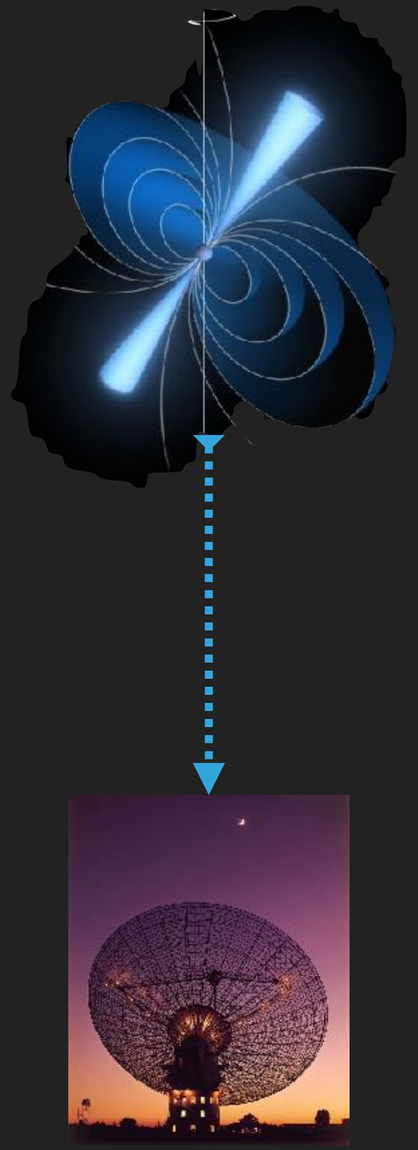
['78 Sazhin; '79 Detweiler;
'83 Hellings, Downs]



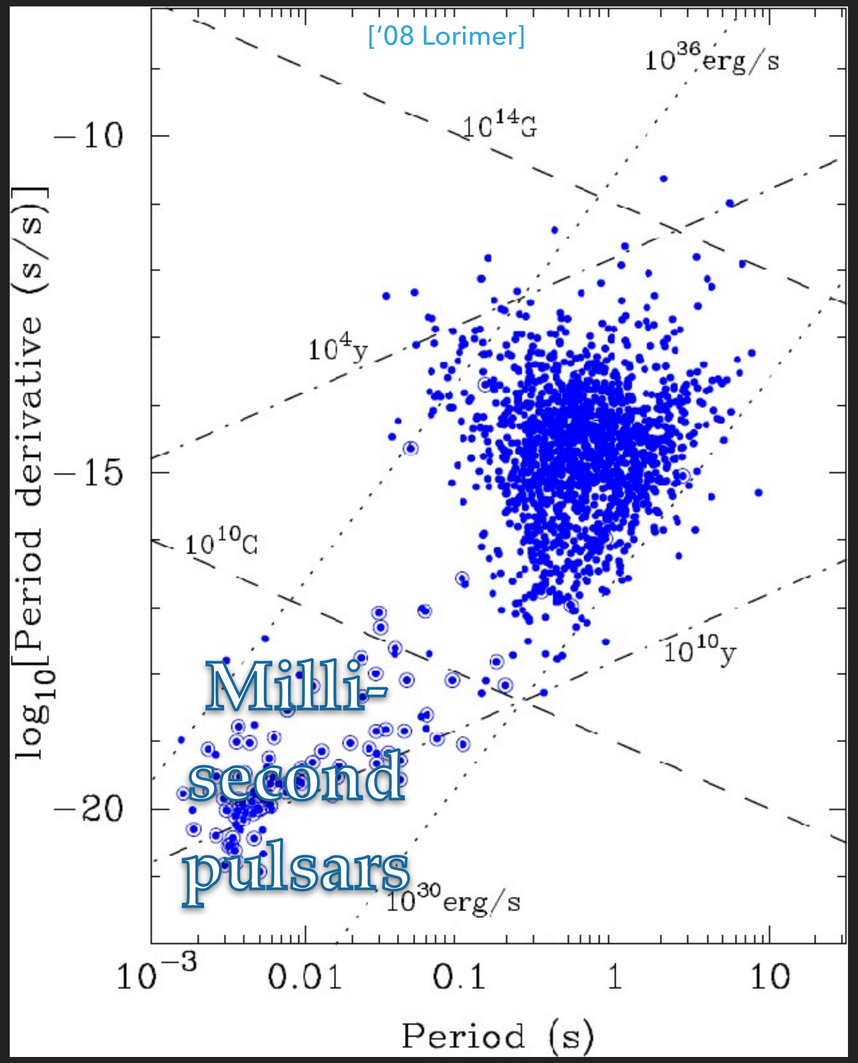


PULSAR TIMING ARRAY MEASUREMENTS

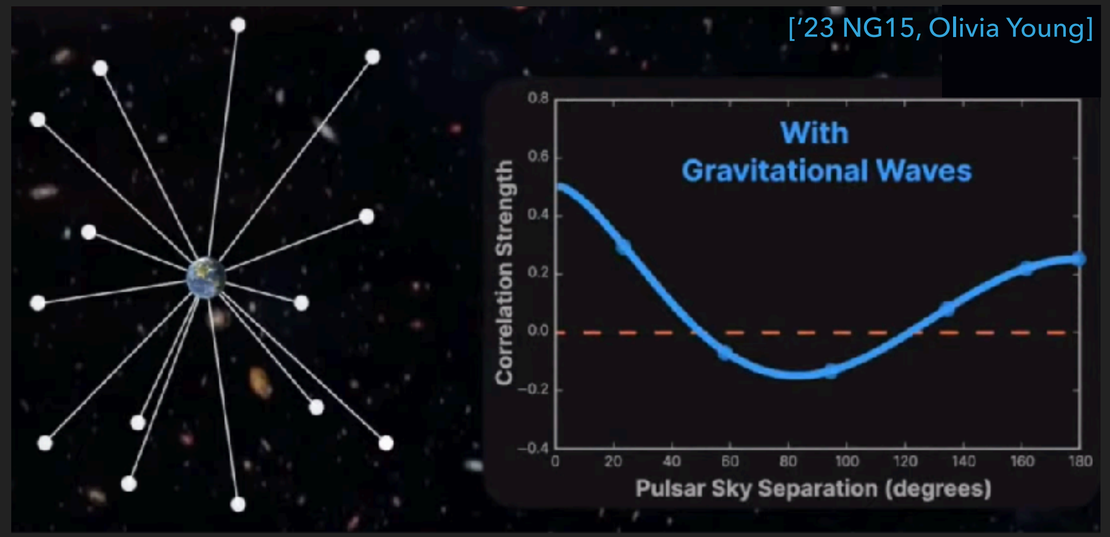
['78 Sazhin; '79 Detweiler; '83 Hellings, Downs]



['08 Lorimer]

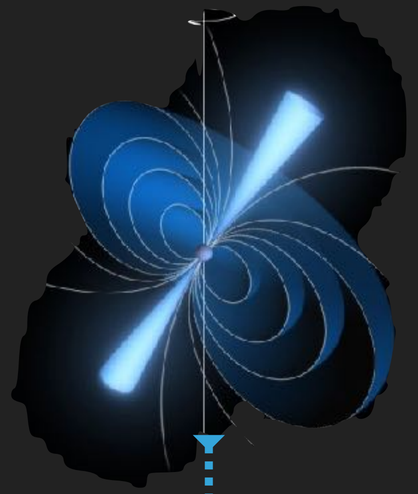


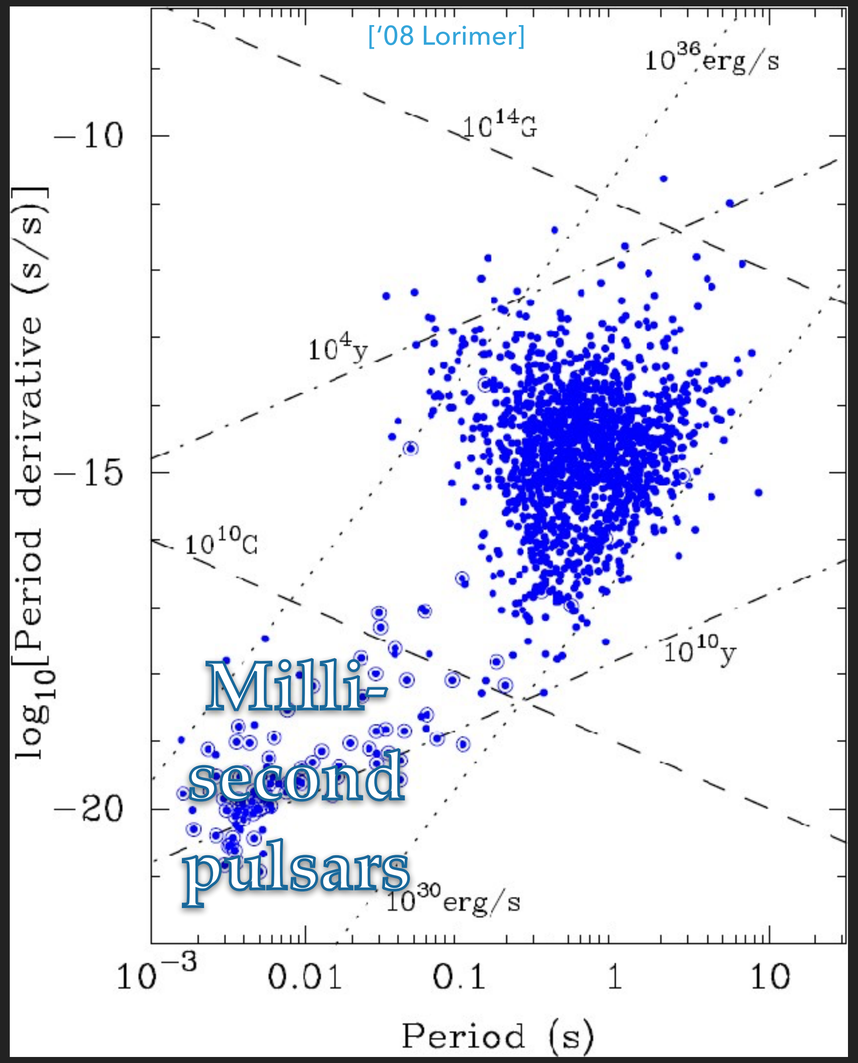
['23 NG15, Olivia Young]

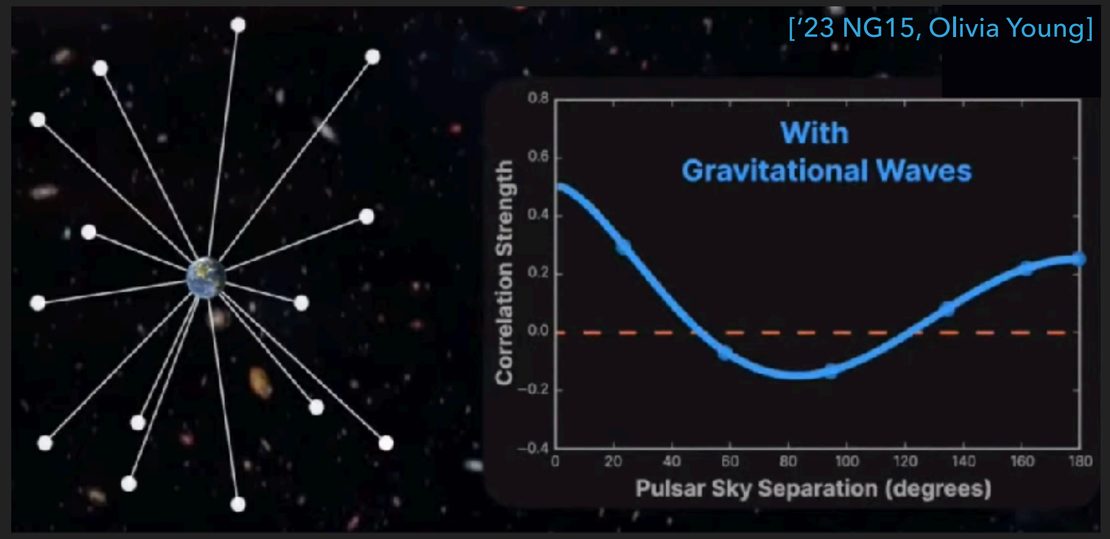


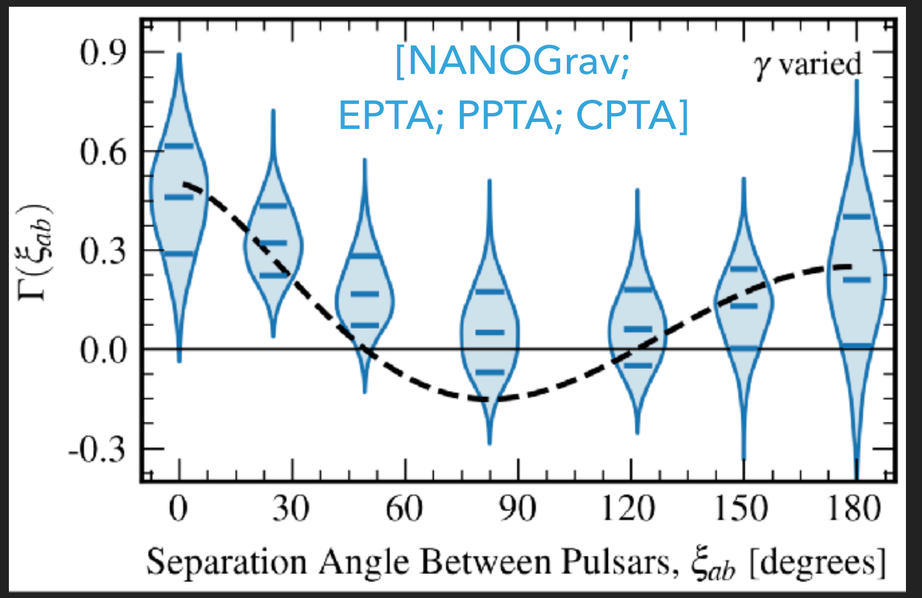
PULSAR TIMING ARRAY MEASUREMENTS

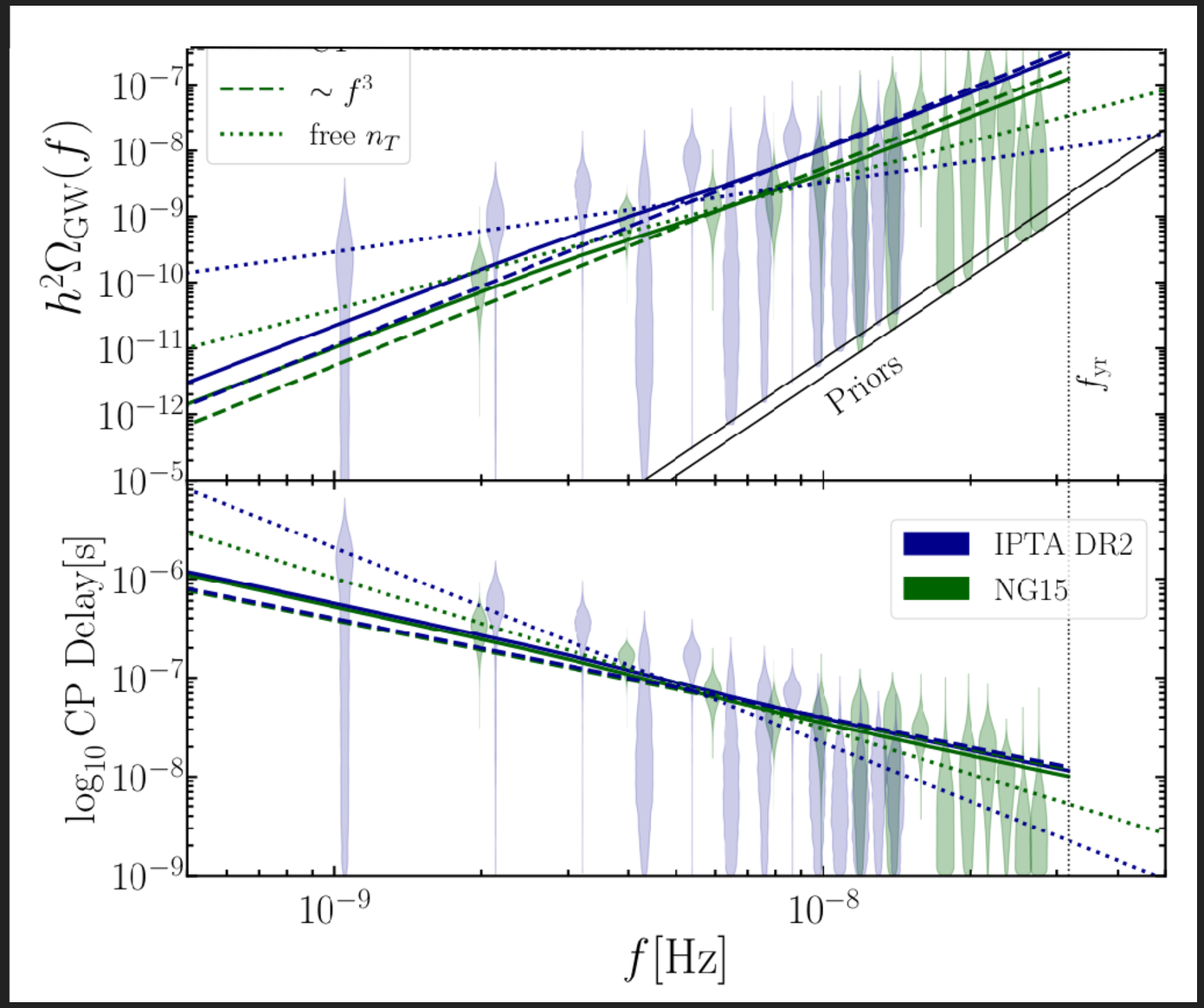
['78 Sazhin; '79 Detweiler; '83 Hellings, Downs]

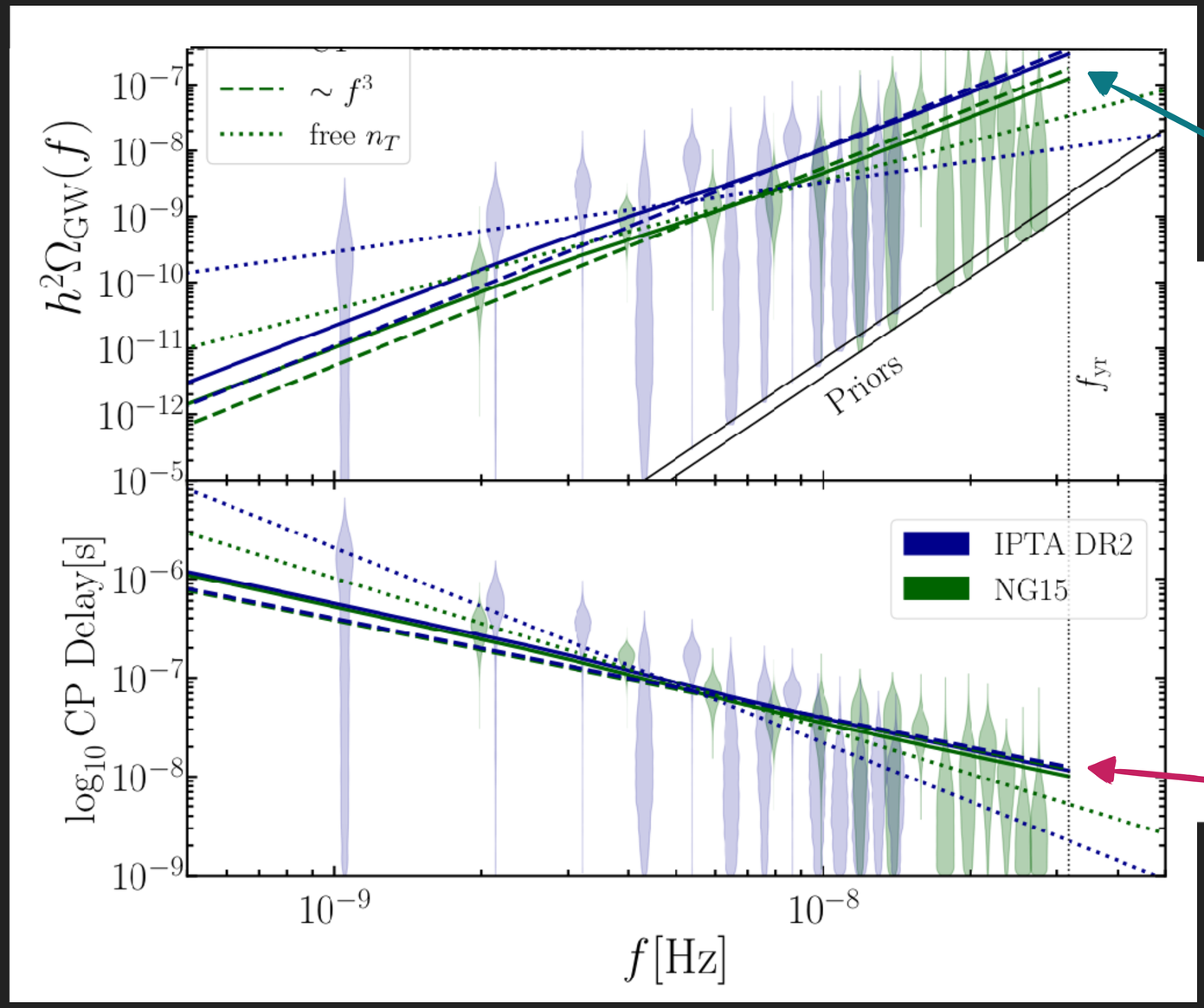




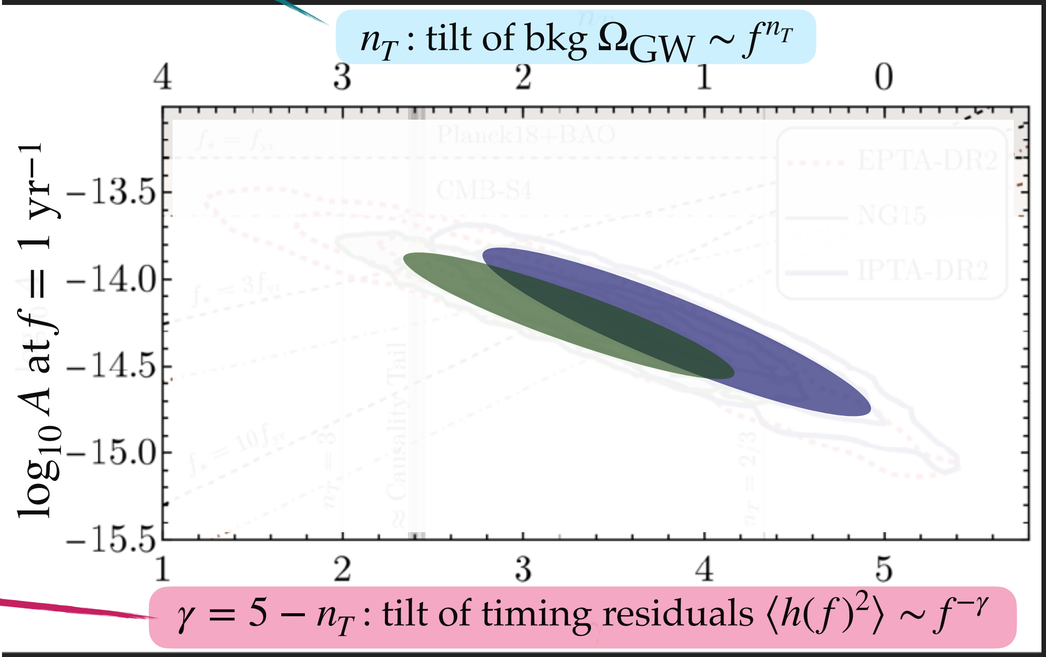





$\sim f^3$
free n_T
$h^2\Omega_{\rm GW}(f)$
Priors
$f_{\rm yr}$
IPTA DR2
NG15
$\log_{10}$ CP Delay[s]
f [Hz]



n_T : tilt of bkg $\Omega_{\rm GW} \sim f^{n_T}$



$\gamma = 5 - n_T$: tilt of timing residuals $\langle h(f)^2 \rangle \sim f^{-\gamma}$

- Earth motion

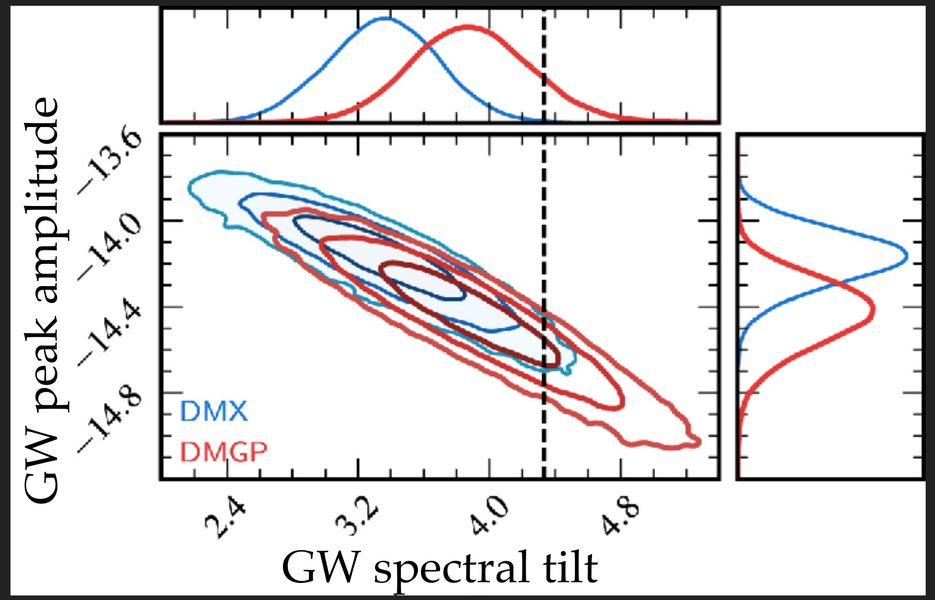
- Pulsar evolution: timing model

- Earth motion

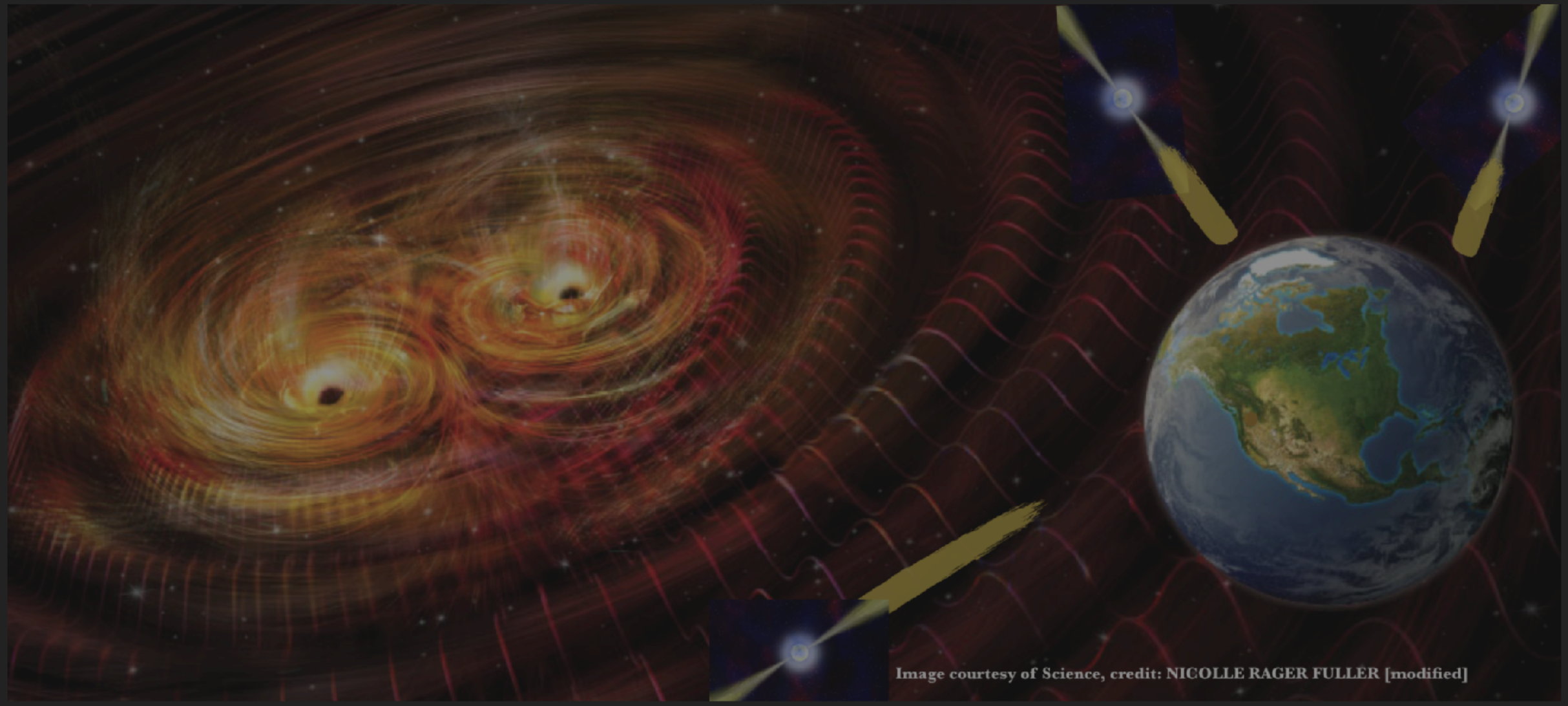
- Pulsar evolution: timing model

- Dispersion measure: ionised gas clouds

['23 NANOGrav-15]



ORIGIN OF THE SIGNAL: ASTROPHYSICAL INTERPRETATION



ORIGIN OF THE SIGNAL: ASTROPHYSICAL INTERPRETATION

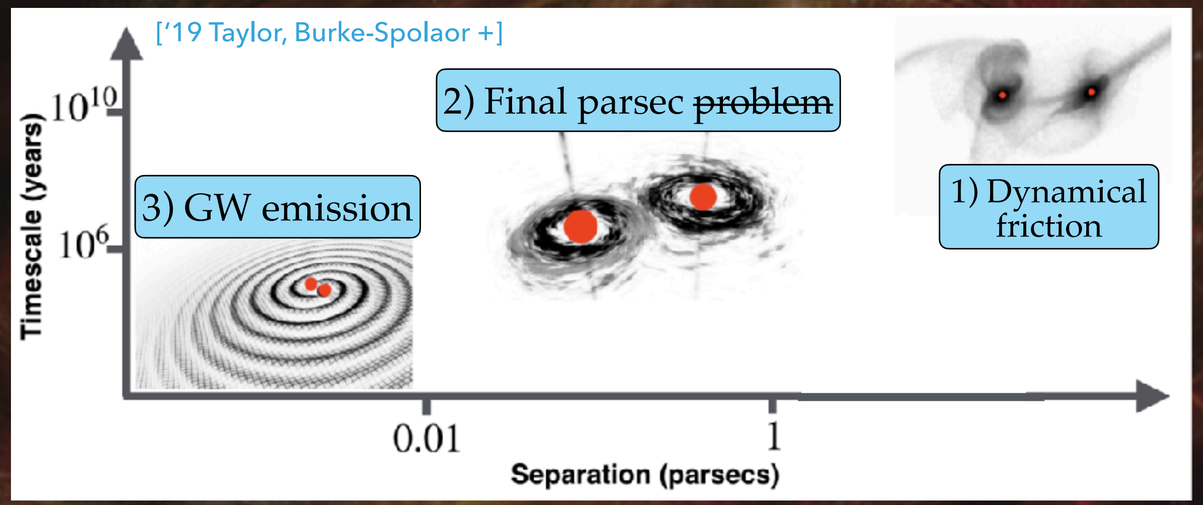
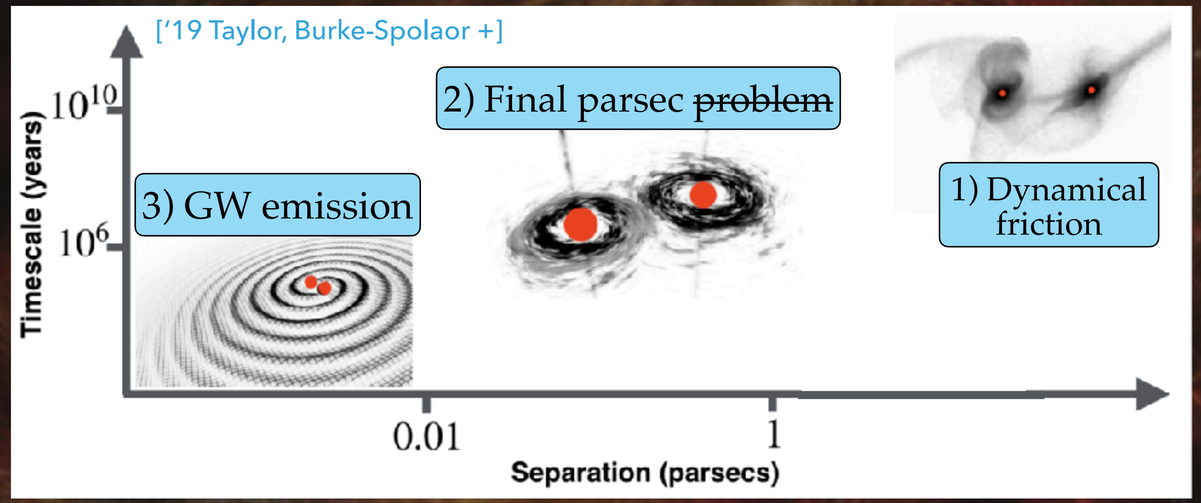
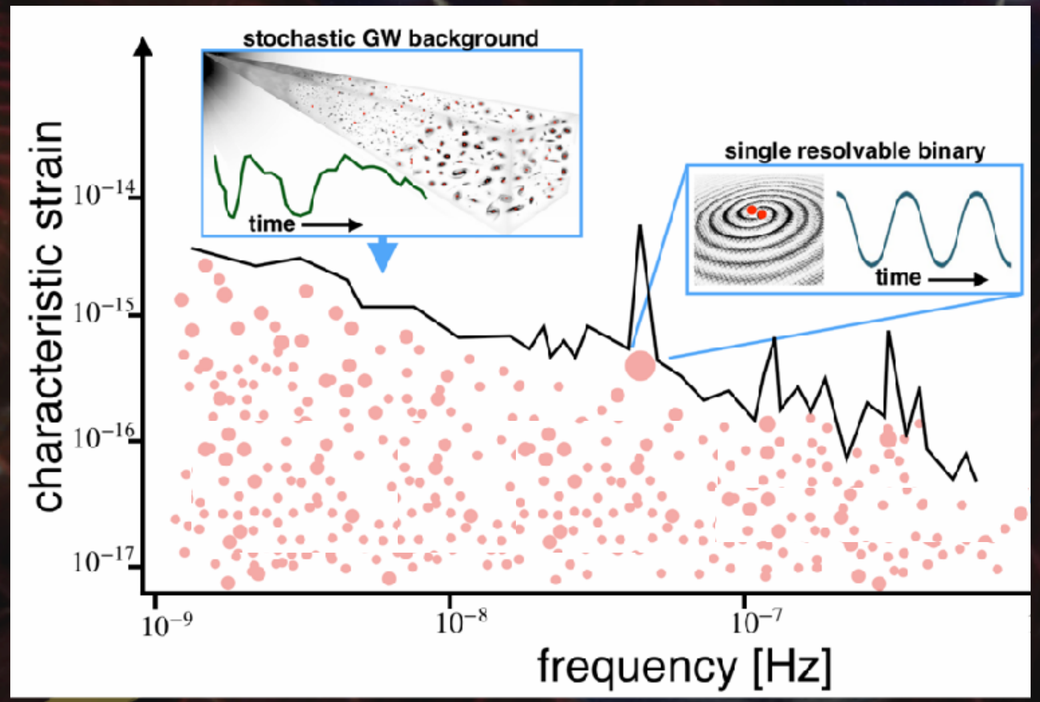


Image courtesy of Science, credit: NICOLLE RAGER FULLER [modified]

ORIGIN OF THE SIGNAL: ASTROPHYSICAL INTERPRETATION





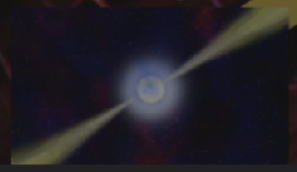
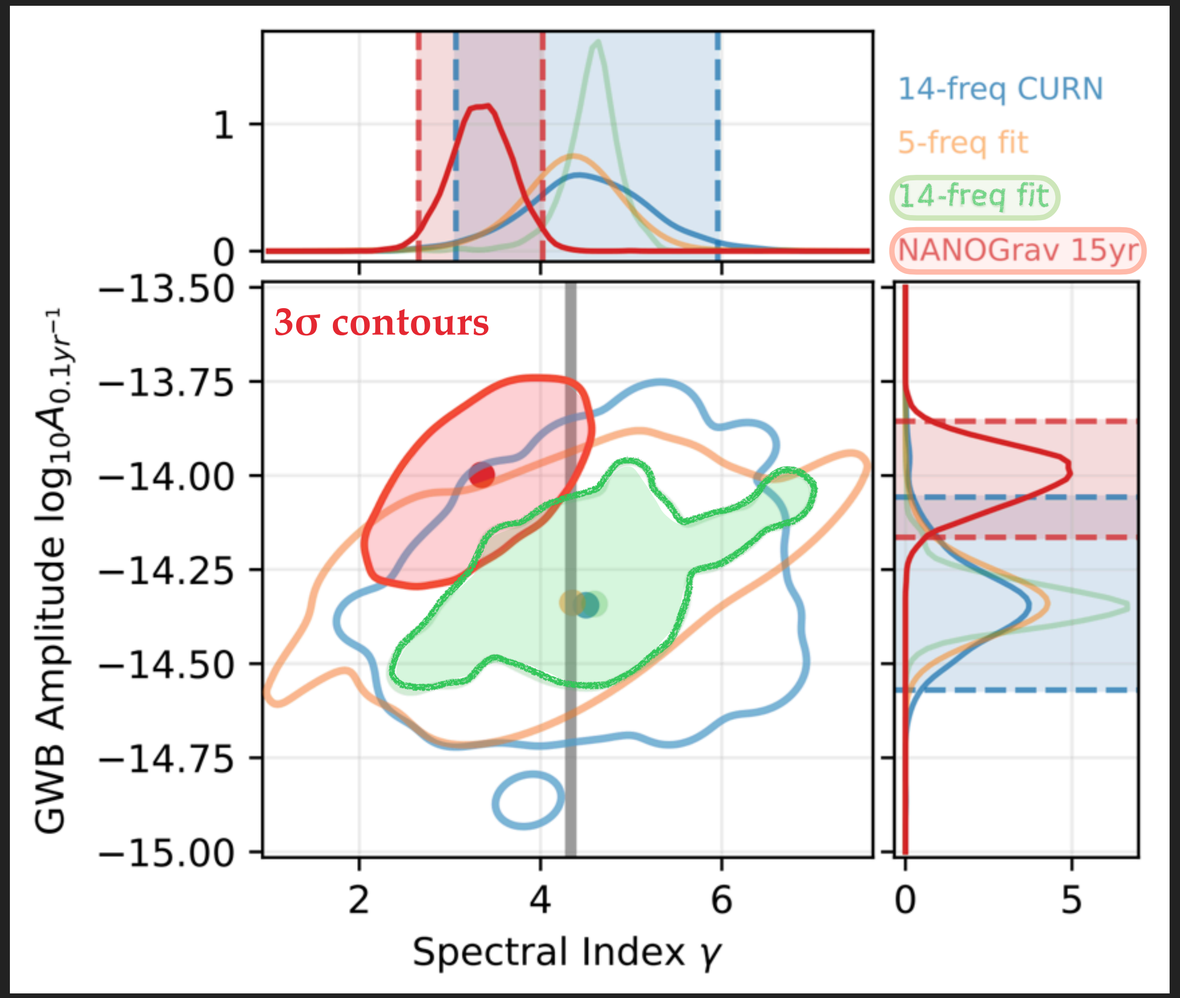


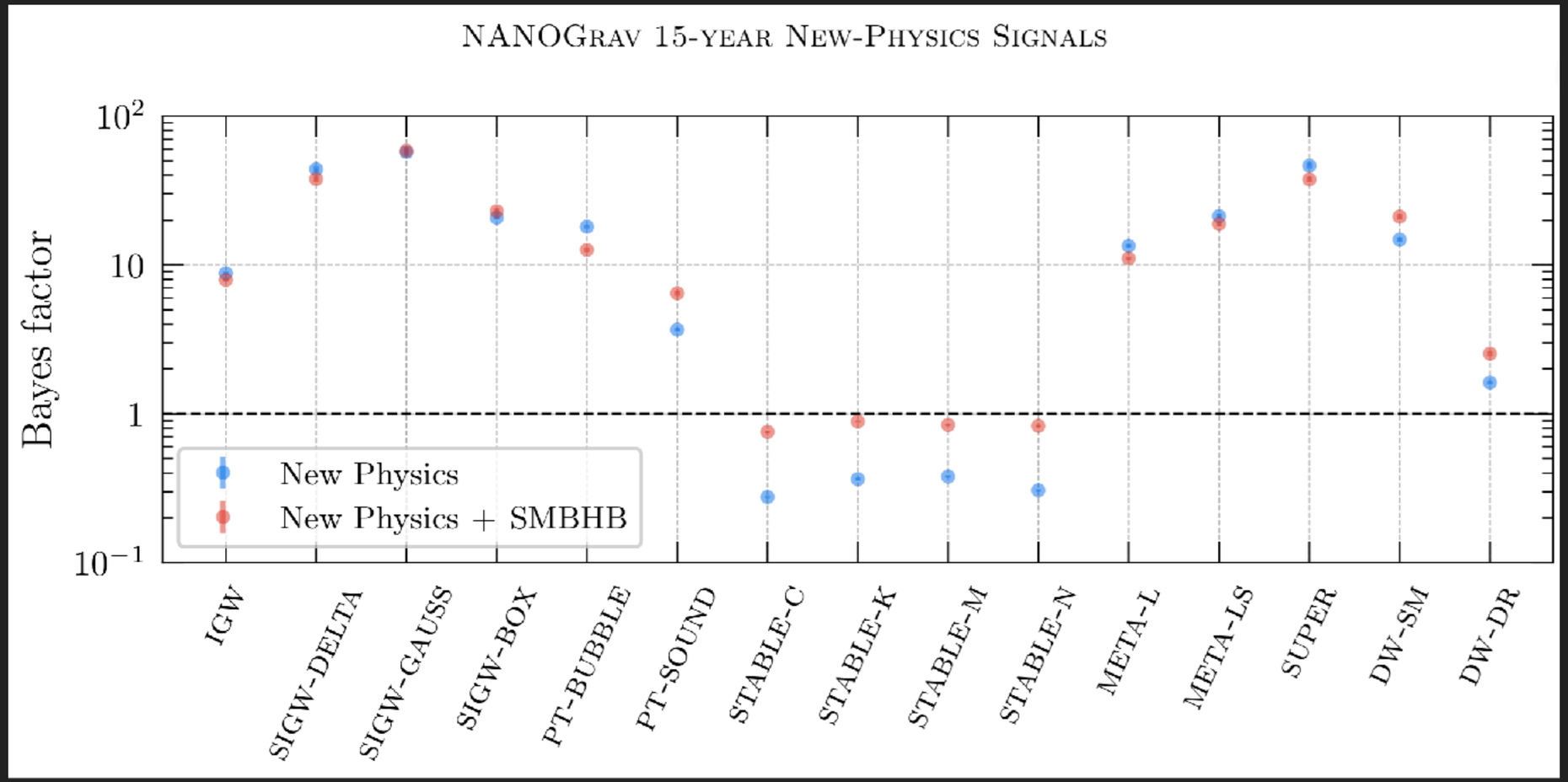
Image courtesy of Science, credit: NICOLLE RAGER FULLER [modified]

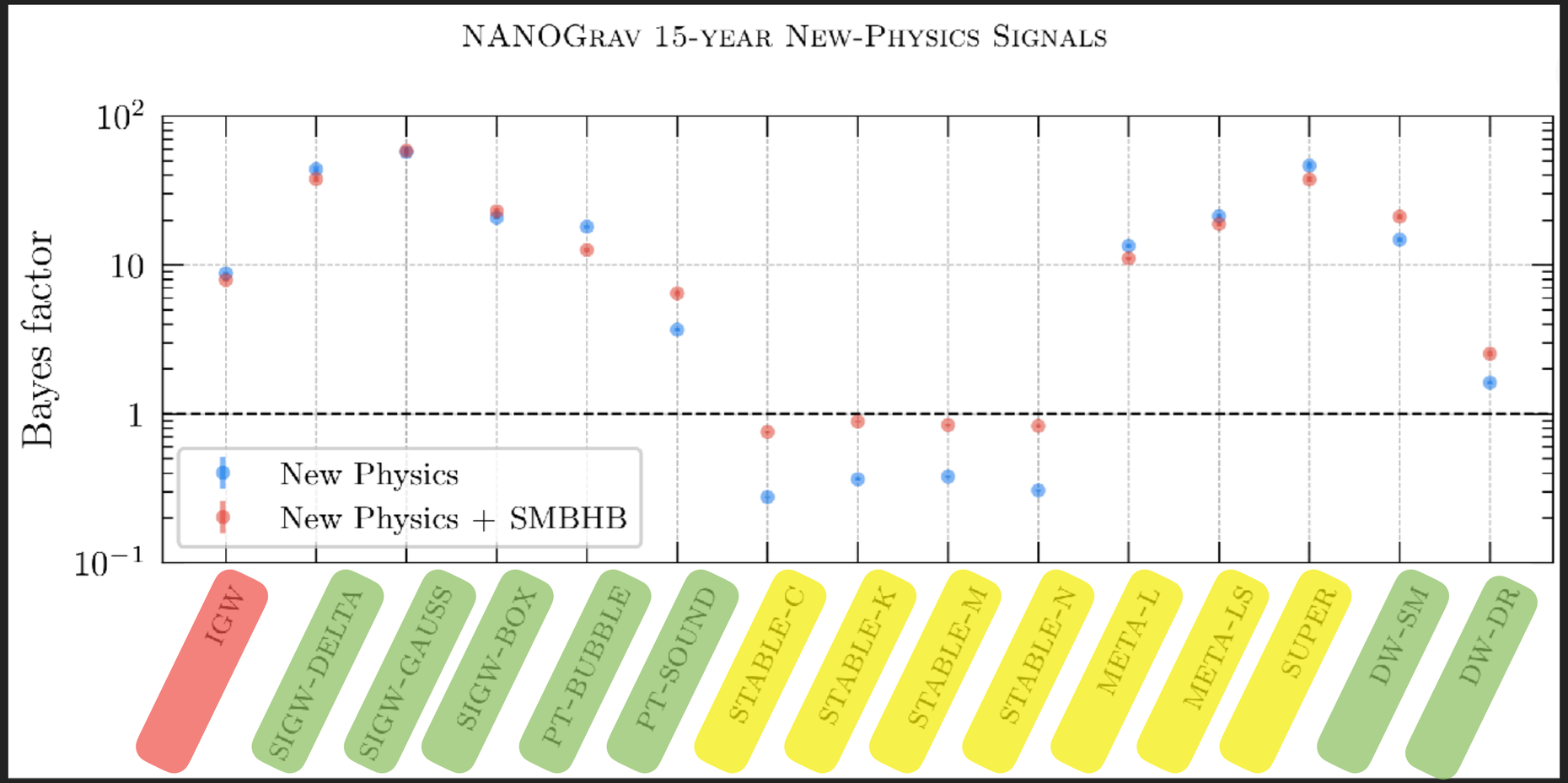
['23 Becsy+ (NG15)]



- Handle to discriminate different origins: identify **robust** spectral features

ORIGIN OF THE SIGNAL: PRIMORDIAL INTERPRETATION



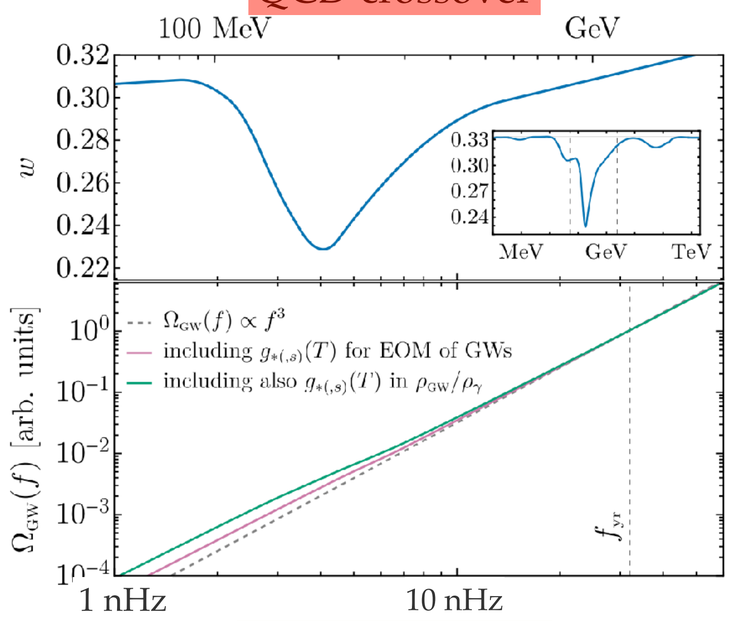


inflationary GWB: no CT

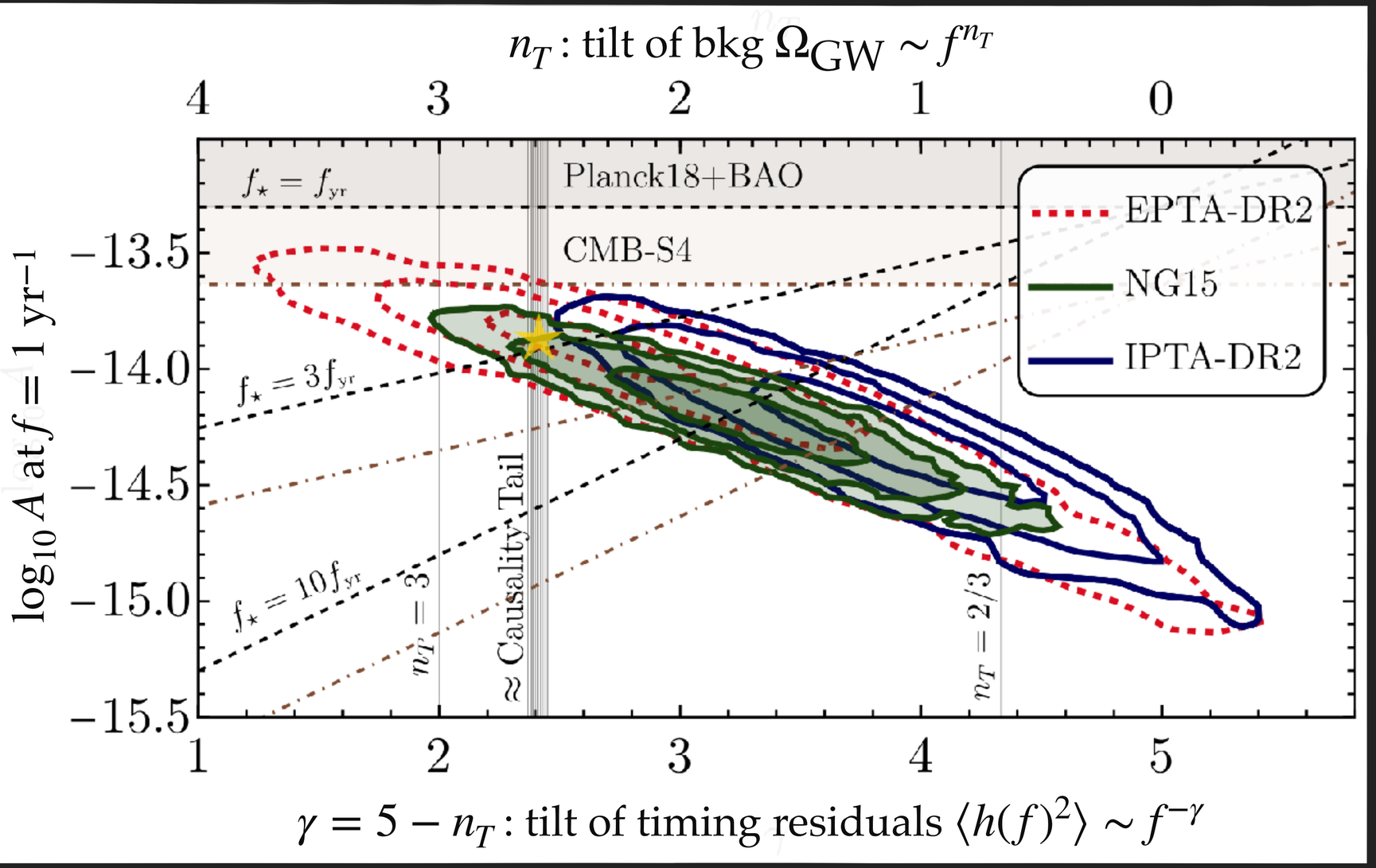
must go to CT at low f

can go to CT at low f

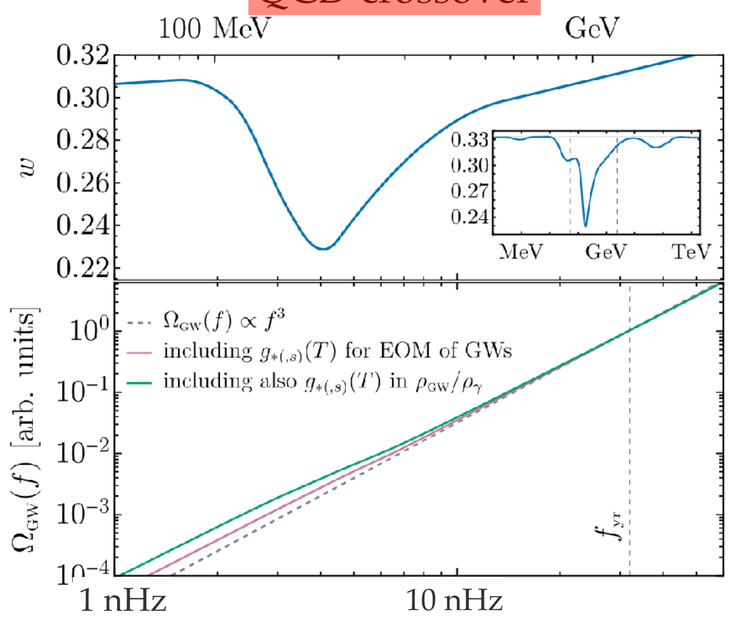
QCD crossover



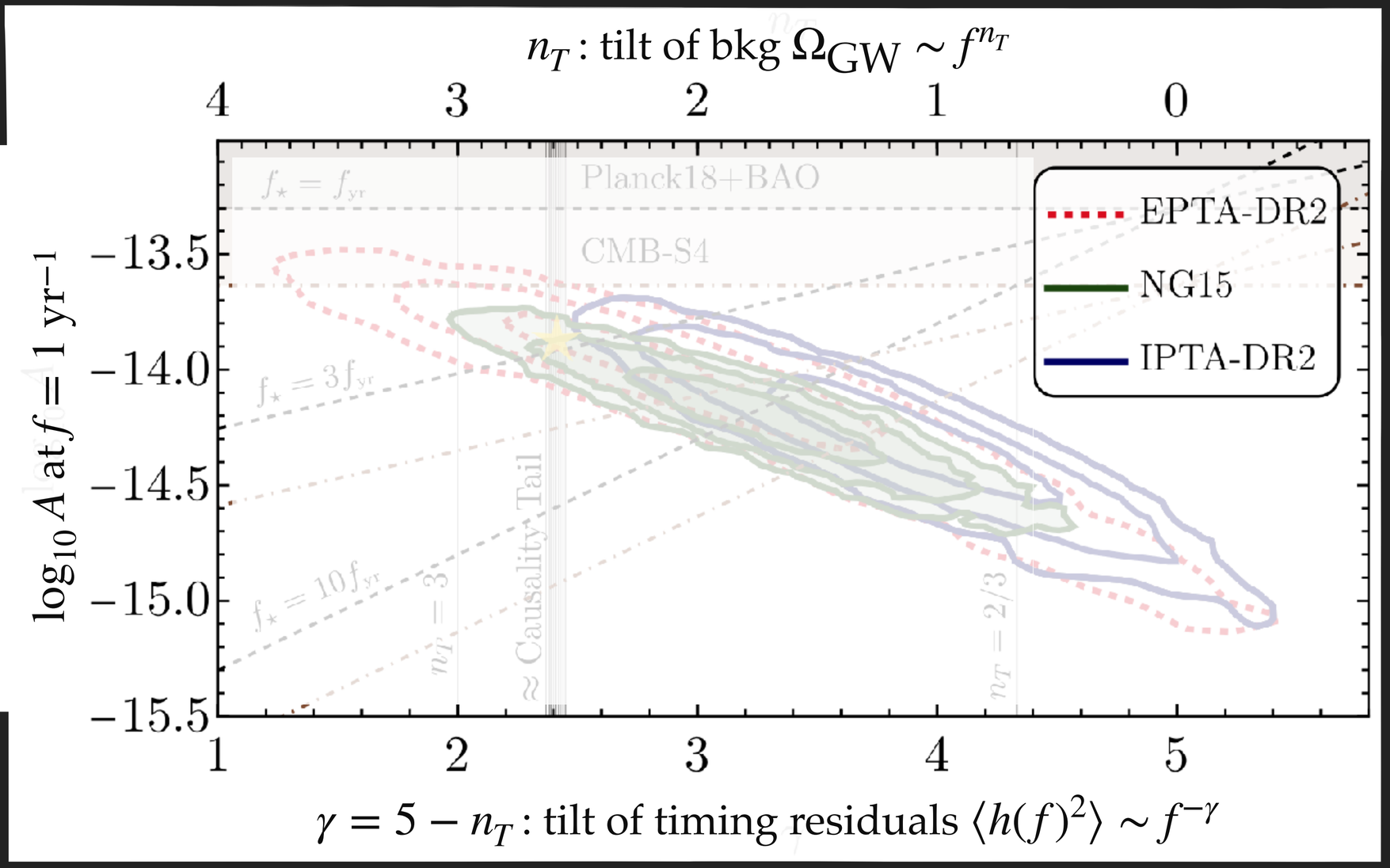
GW bkg at PTAs

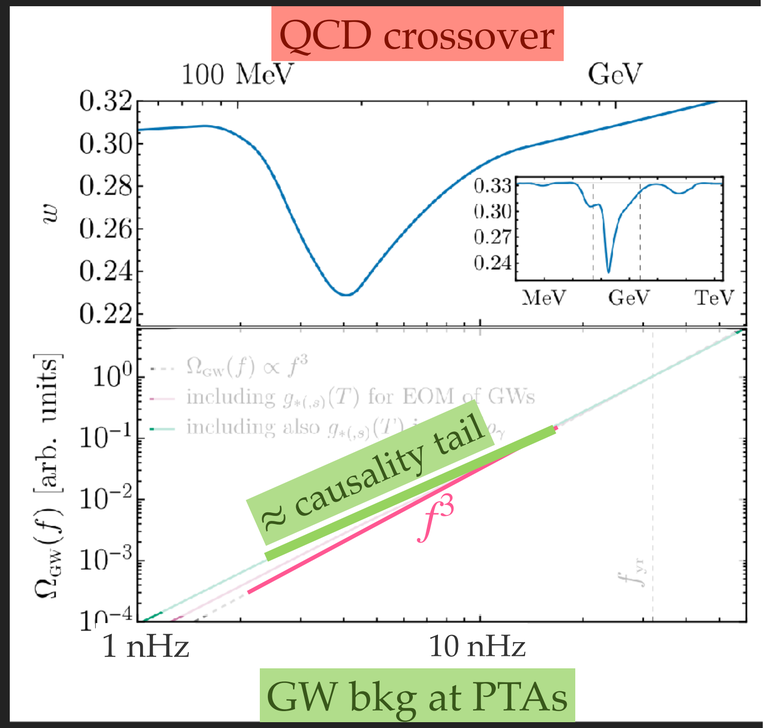


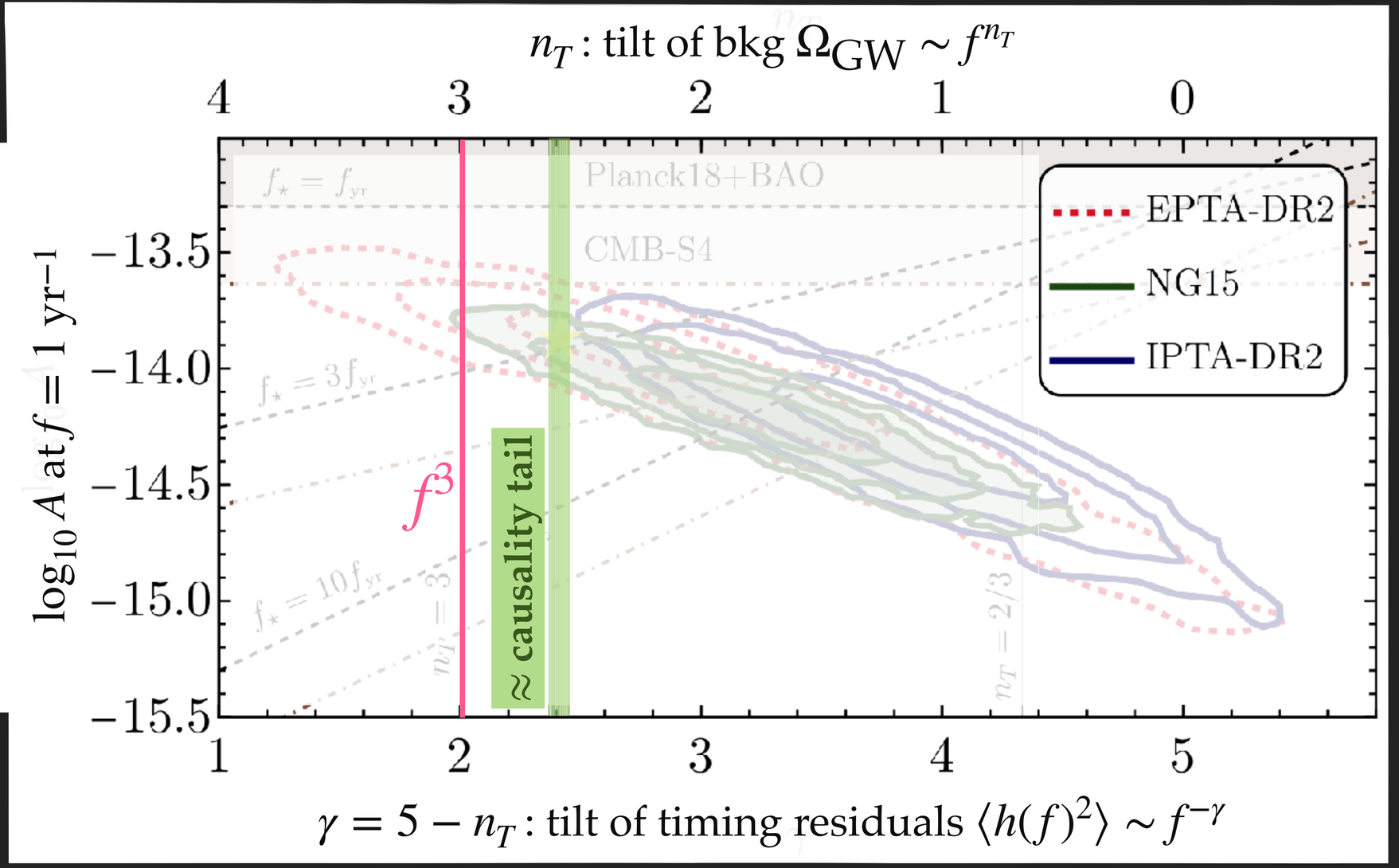
QCD crossover

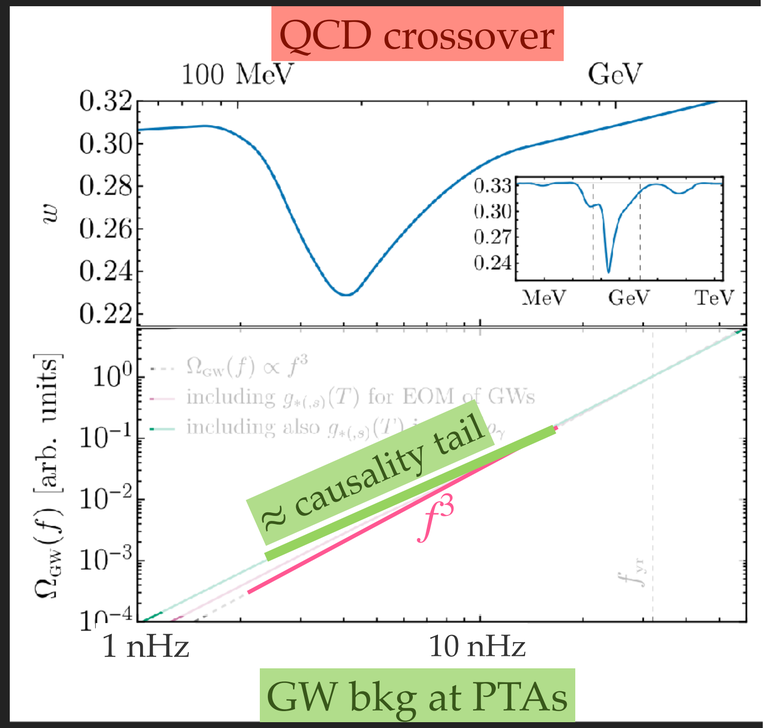


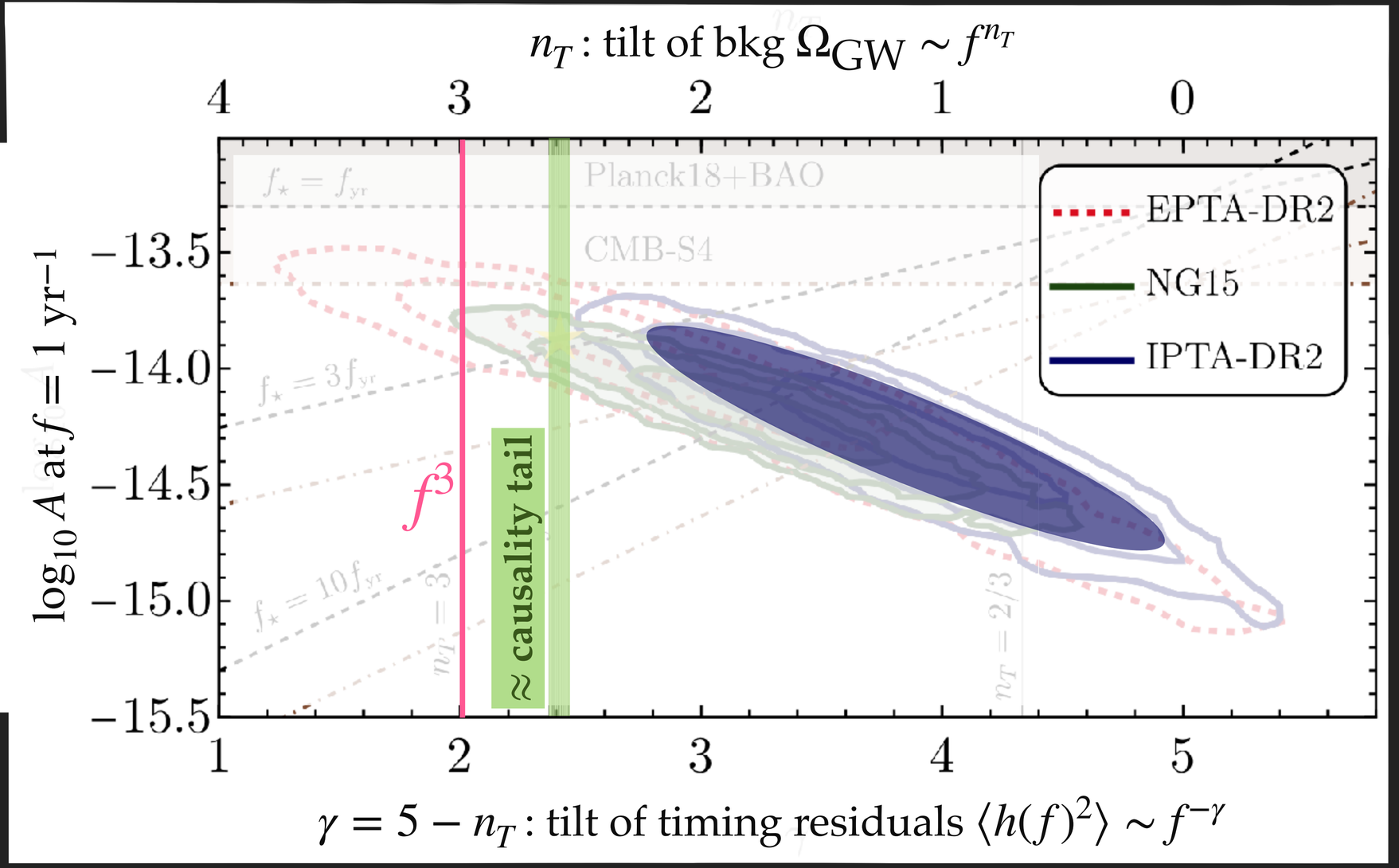
GW bkg at PTAs

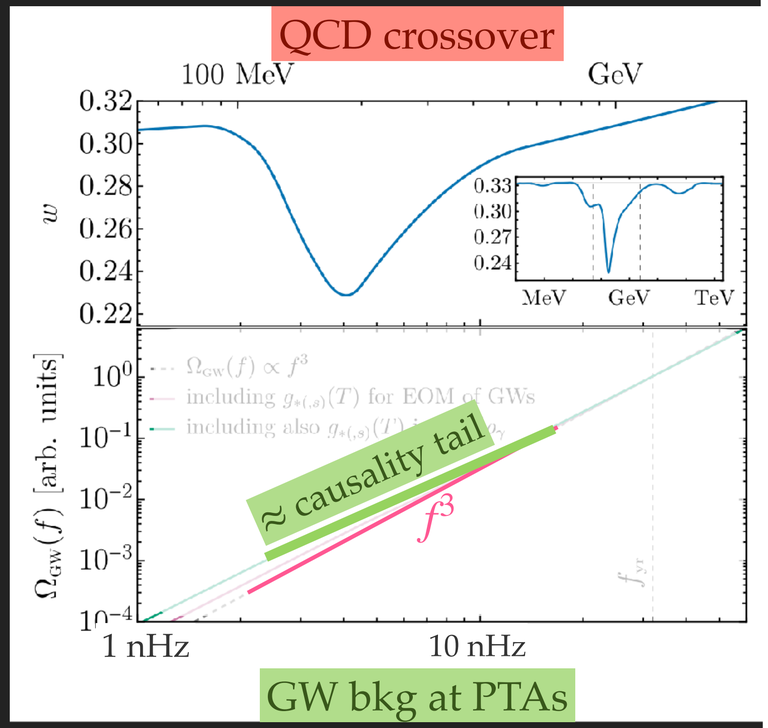


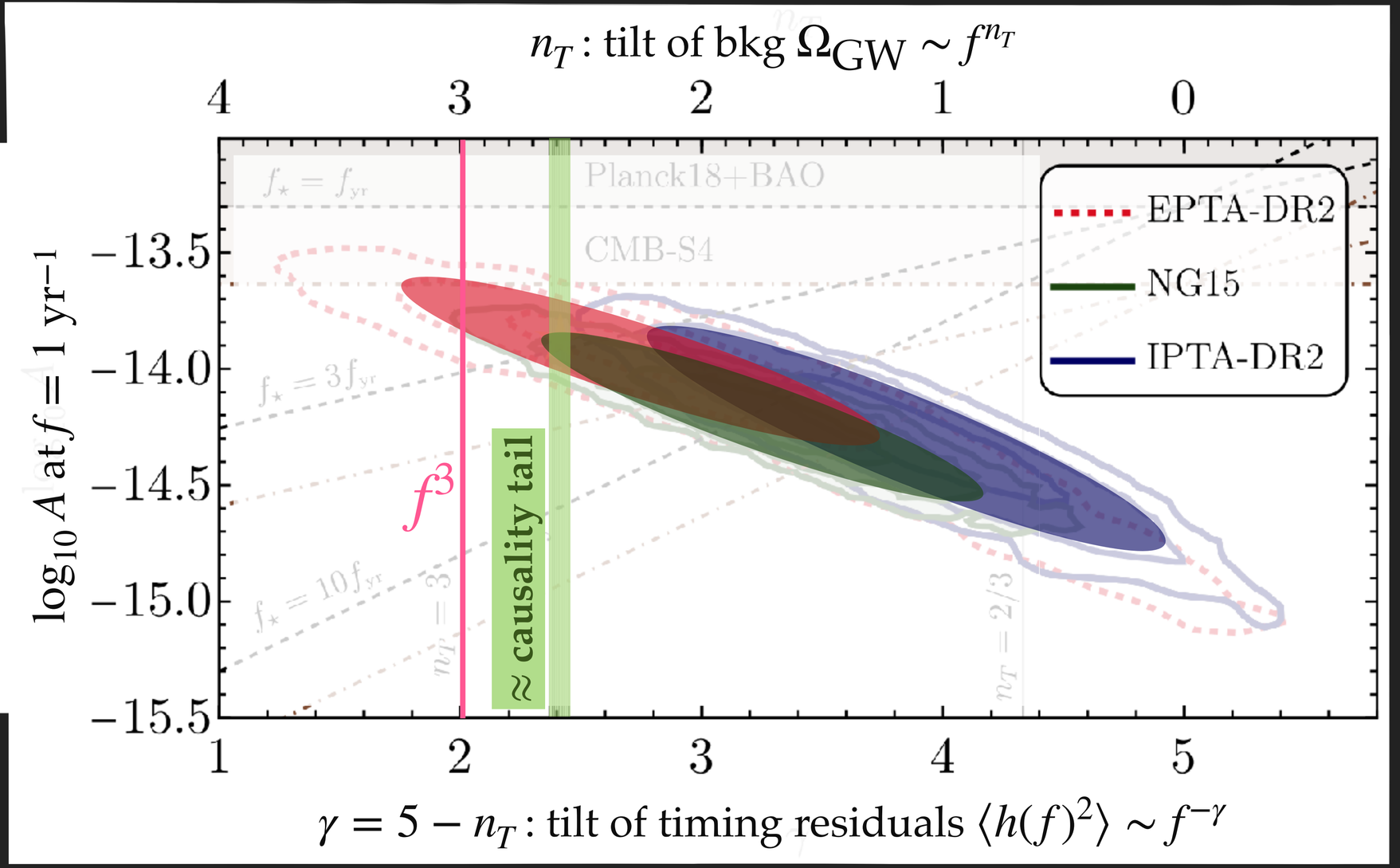

QCD crossover
100 MeV
GeV
MeV
GeV
TeV
w
$\Omega_{\rm GW}(f)$ [arb. units]
$\Omega_{\rm GW}(f) \propto f^3$
including $g_{*(,s)}(T)$ for EOM of GWs
including also $g_{*(,s)}(T)$ in ρ_γ
≈ causality tail
f^3
$f_{\rm yr}$
1 nHz
10 nHz
GW bkg at PTAs


n_T : tilt of bkg $\Omega_{\rm GW} \sim f^{n_T}$
$\log_{10} A$ at $f = 1\ \mathrm{yr}^{-1}$
$\gamma = 5 - n_T$: tilt of timing residuals $\langle h(f)^2 \rangle \sim f^{-\gamma}$
Planck18+BAO
CMB-S4
EPTA-DR2
NG15
IPTA-DR2
$f_\star = f_{\rm yr}$
$f_\star = 3 f_{\rm yr}$
$f_\star = 10 f_{\rm yr}$
$n_T = 3$
$n_T = 2/3$
f^3
≈ causality tail


QCD crossover
100 MeV
GeV
MeV
GeV
TeV
w
$\Omega_{\rm GW}(f)$ [arb. units]
$\Omega_{\rm GW}(f) \propto f^3$
including $g_{*(,s)}(T)$ for EOM of GWs
including also $g_{*(,s)}(T)$ in ρ_γ
$\approx$ causality tail
f^3
$f_{\rm yr}$
1 nHz
10 nHz
GW bkg at PTAs


n_T : tilt of bkg $\Omega_{\rm GW} \sim f^{n_T}$
$\log_{10} A$ at $f = 1$ yr^{-1}
$\gamma = 5 - n_T$: tilt of timing residuals $\langle h(f)^2 \rangle \sim f^{-\gamma}$
Planck18+BAO
CMB-S4
EPTA-DR2
NG15
IPTA-DR2
$f_\star = f_{\rm yr}$
$f_\star = 3 f_{\rm yr}$
$f_\star = 10 f_{\rm yr}$
$n_T = 3$
$n_T = 2/3$
f^3
$\approx$ causality tail


QCD crossover
100 MeV
GeV
0.32
0.30
0.28
0.26
0.24
0.22
w
0.33
0.30
0.27
0.24
MeV
GeV
TeV
$\Omega_{\rm GW}(f)$ [arb. units]
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$\Omega_{\rm GW}(f) \propto f^3$
including $g_{*(,s)}(T)$ for EOM of GWs
including also $g_{*(,s)}(T)$ in ρ_γ
≈ causality tail
f^3
$f_{\rm yr}$
1 nHz
10 nHz
GW bkg at PTAs

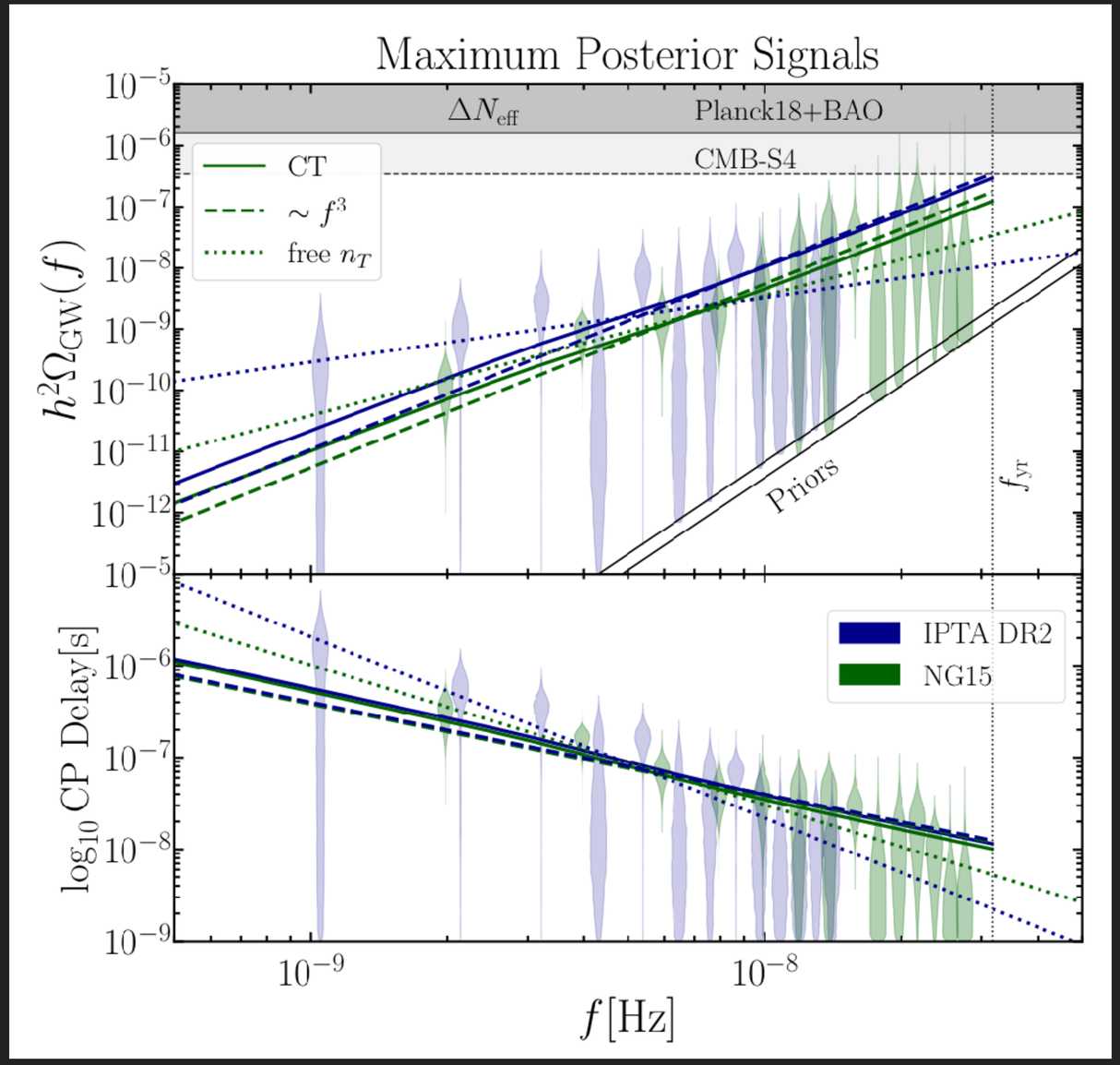

n_T : tilt of bkg $\Omega_{\rm GW} \sim f^{n_T}$
4
3
2
1
0
$f_\star = f_{\rm yr}$
Planck18+BAO
CMB-S4
EPTA-DR2
NG15
IPTA-DR2
$f_\star = 3 f_{\rm yr}$
$f_\star = 10 f_{\rm yr}$
$n_T = 3$
f^3
≈ causality tail
$n_T = 2/3$
$\log_{10} A$ at $f = 1$ yr^{-1}
−13.5
−14.0
−14.5
−15.0
−15.5
1
2
3
4
5
$\gamma = 5 - n_T$: tilt of timing residuals $\langle h(f)^2 \rangle \sim f^{-\gamma}$

- Bayesian model comparison: causality tail vs. f^3

$$\log_{10}\mathcal{B} = \begin{cases} 1.6 & \text{(NANOGrav-15)} \\ 1.3 & \text{(IPTA-DR2)} \end{cases}$$

(*strong* evidence)

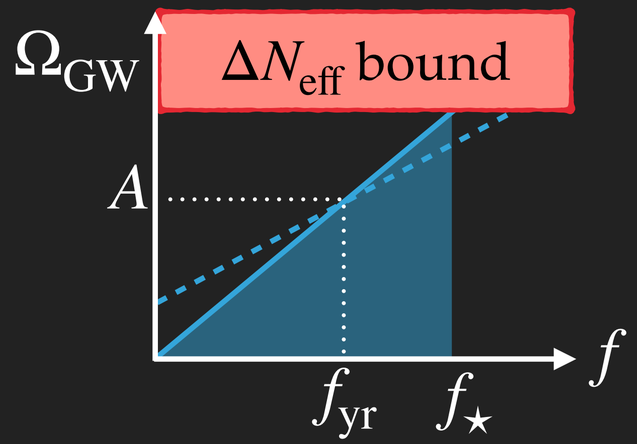
- Must account for causality tail in model comparison for PTAs



BOUND FROM $\Delta N_{\rm eff}$ FOR PRIMORDIAL SIGNAL

- Primordial GWs are relativistic energy not coupled to SM bath
- $\Rightarrow \Delta N_{\rm eff}$ constrained by BBN, CMB

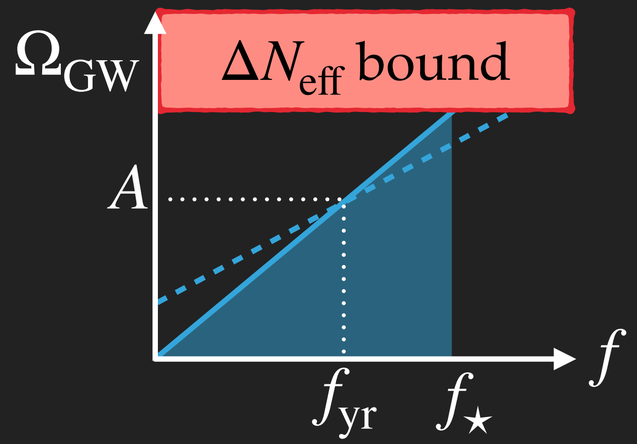
$$\Omega_{\rm CGW} h^2 = 1.6 \cdot 10^{-6} \cdot \frac{\Delta N_{\rm eff}^{\rm CGW}}{0.28}$$

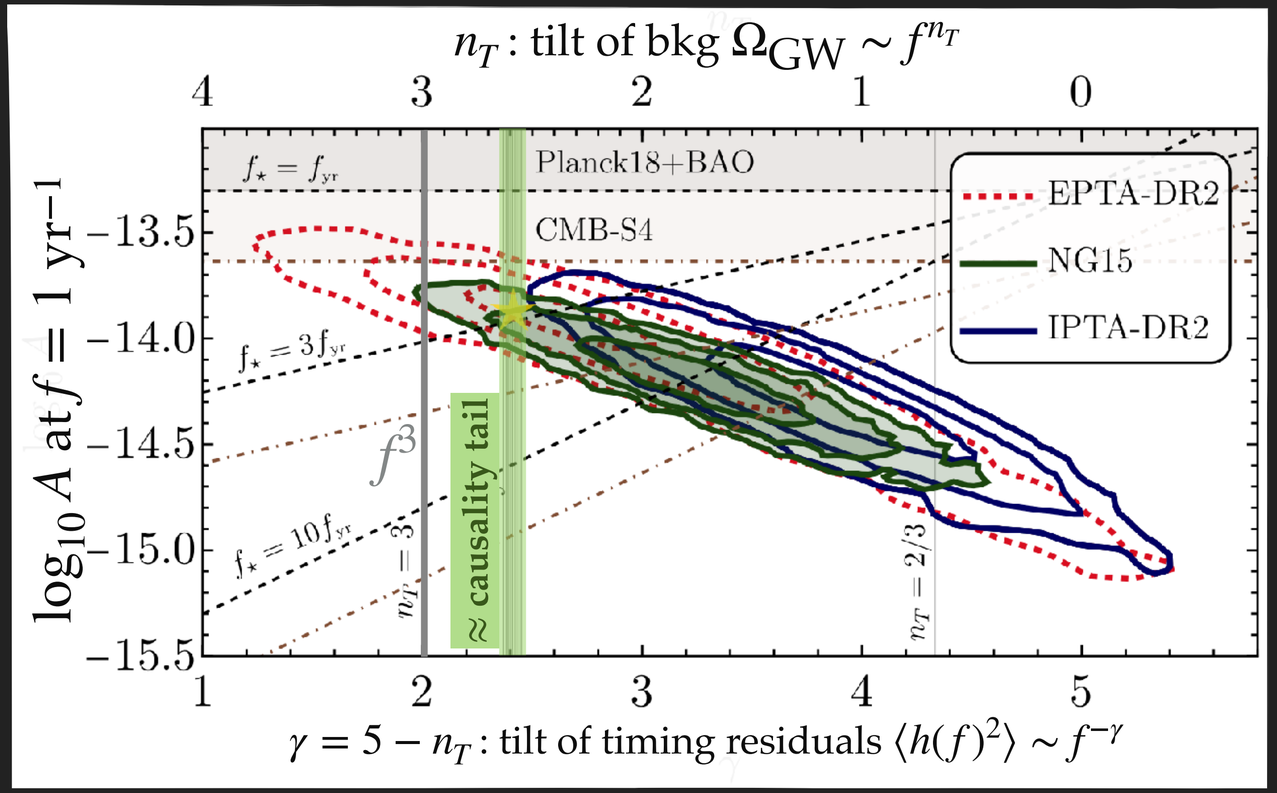


BOUND FROM $\Delta N_{\rm eff}$ FOR PRIMORDIAL SIGNAL

- Primordial GWs are relativistic energy not coupled to SM bath
- $\Rightarrow \Delta N_{\rm eff}$ constrained by BBN, CMB

$$\Omega_{\rm CGW} h^2 = 1.6 \cdot 10^{-6} \cdot \frac{\Delta N_{\rm eff}^{\rm CGW}}{0.28}$$



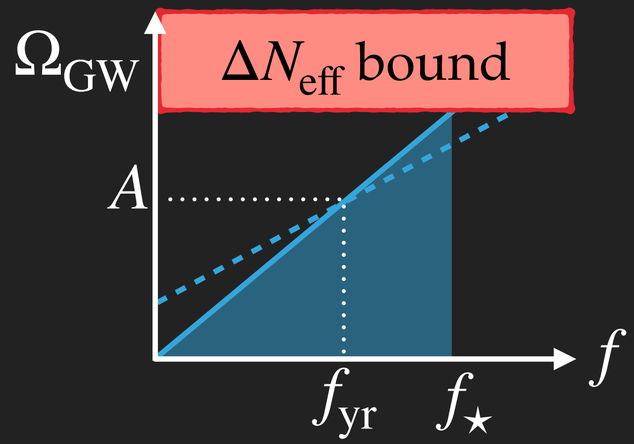


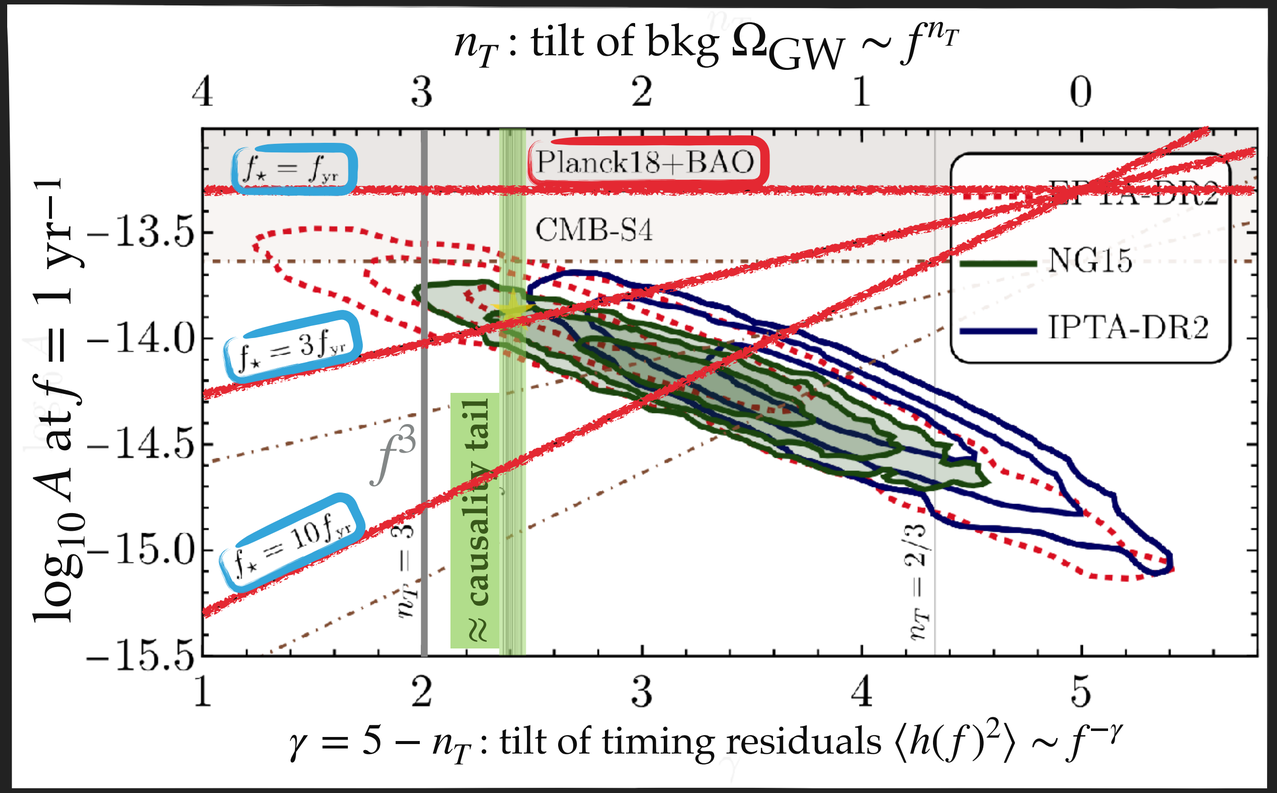
['23 Franciolini, **DR**, Rompineve]

BOUND FROM $\Delta N_{\rm eff}$ FOR PRIMORDIAL SIGNAL

- Primordial GWs are relativistic energy not coupled to SM bath
- $\Rightarrow \Delta N_{\rm eff}$ constrained by BBN, CMB

$$\Omega_{\rm CGW}h^2 = 1.6\cdot 10^{-6}\cdot\frac{\Delta N_{\rm eff}^{\rm CGW}}{0.28}$$



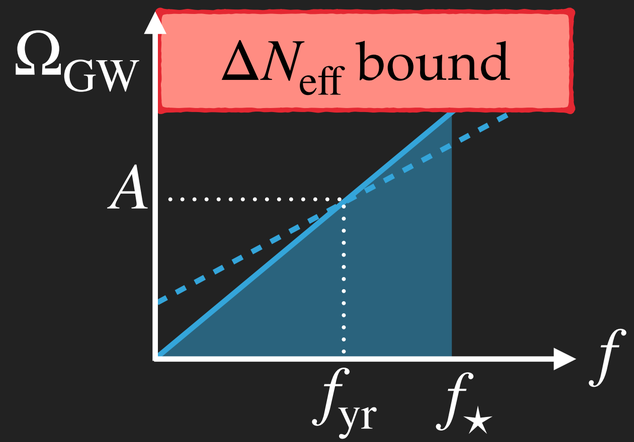


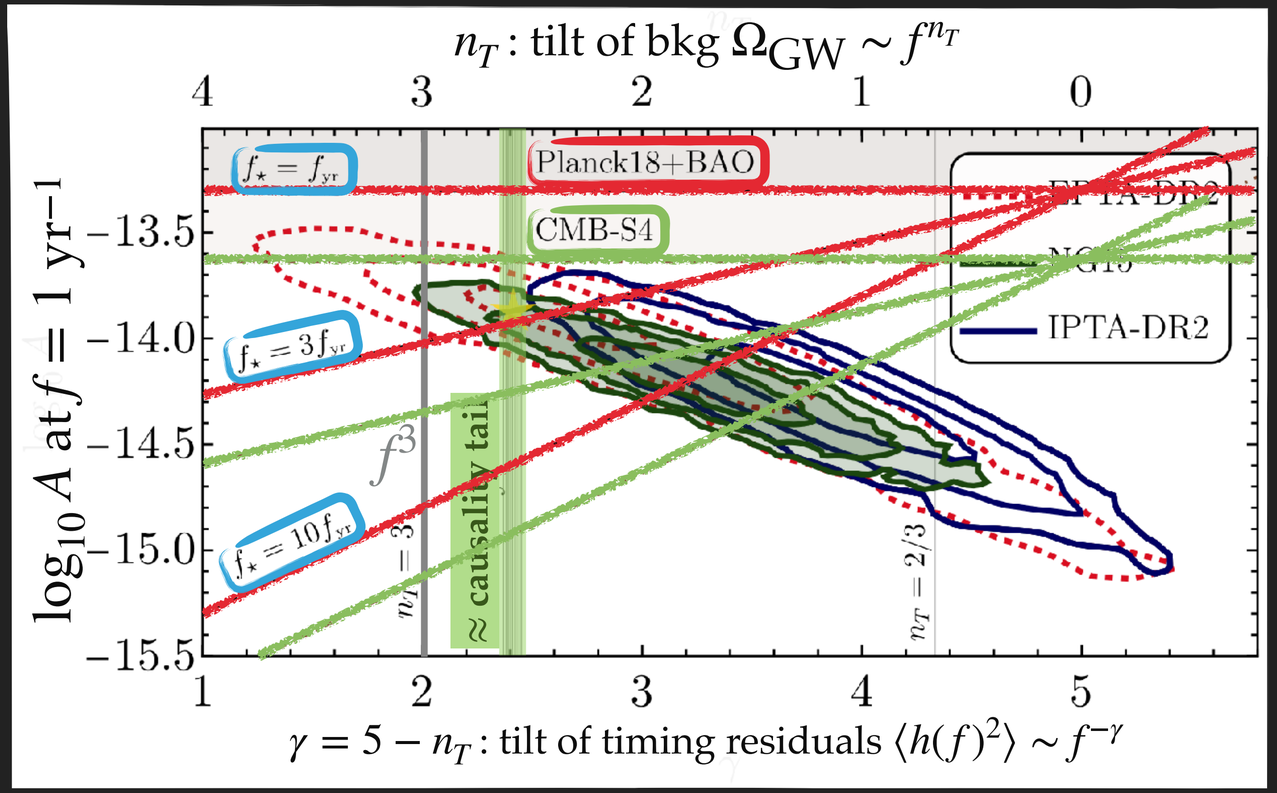
['23 Franciolini, **DR**, Rompineve]

BOUND FROM $\Delta N_{\rm eff}$ FOR PRIMORDIAL SIGNAL

- Primordial GWs are relativistic energy not coupled to SM bath
- $\Rightarrow \Delta N_{\rm eff}$ constrained by BBN, CMB

$$\Omega_{\rm CGW} h^2 = 1.6 \cdot 10^{-6} \cdot \frac{\Delta N_{\rm eff}^{\rm CGW}}{0.28}$$

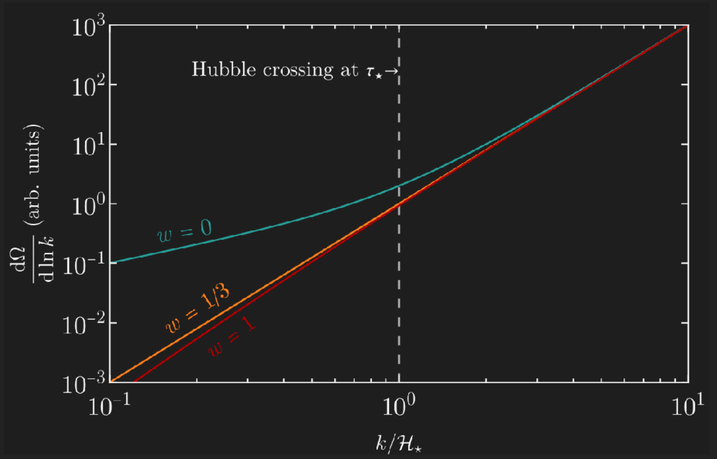


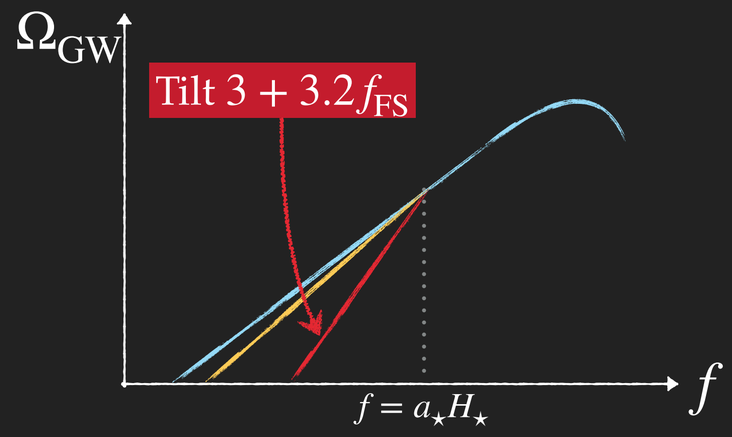


['23 Franciolini, **DR**, Rompineve]

- Primordial GWB can't grow until too large $f_\star$
- Signature in CMB if GWB is primordial!

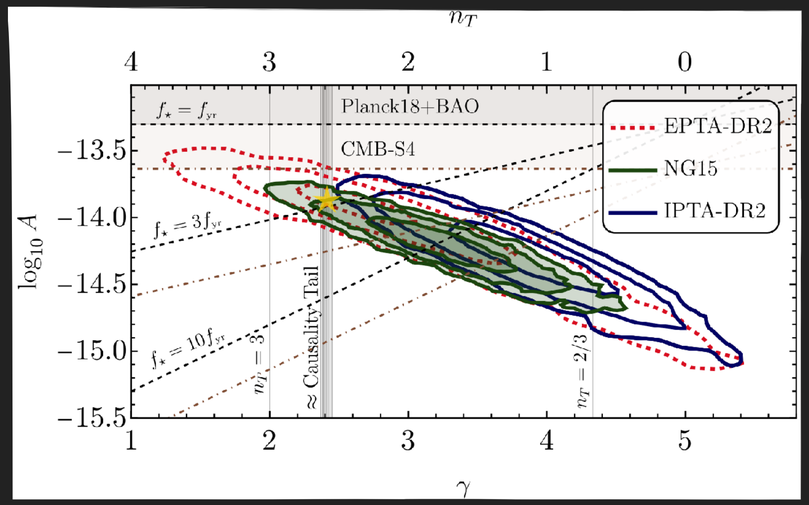
- Causality tail ⇔ probe w and $f_{\rm FS}$ independently of the primordial GW source





Hook, Marques-Tavares, **DR**
JHEP (2021)

- SM effects on the causality tail must be included at PTAs



Franciolini, **DR**, Rompineve
Phys. Rev. Lett. (2024)

Buchalter Cosmology Prize 2023

9 — 20 September 2024

POLLICA PHYSICS CENTRE

Fundamental Physics and Gravitational Wave Detectors

Gravitational waves (GWs) from compact binaries have been detected in the kHz regime, and Pulsar Timing Arrays are revealing a GW background in the nHz band. Experimental ideas spanning from nHz to MHz are being explored to cover the gravitational spectrum. This is an exciting time to explore what GWs can uncover about fundamental physics and cosmology by detecting astrophysical and primordial GW sources, as well as exploring effects that could be induced by various Dark Matter candidates.

This workshop will unite astrophysicists, cosmologists, and particle physicists to discuss open questions and define theoretical targets, guiding the field in experimental strategies across the gravitational spectrum to learn about our Universe. All of this is to be set against the backdrop of the medieval town of Pollica in southwestern Italy (Cilento region).

ORGANIZING COMMITTEE

Sebastian Baum (RWTH Aachen)

Djuna Croon (Durham University)

Paolo Pani (Sapienza University & INFN Roma)

Davide Racco (ETH & University of Zurich)

Géraldine Servant (DESY & Universität Hamburg)

Participants will include:

Bruce Allen (AEI Potsdam)

Chiara Caprini (CERN & U. Geneva)

Reed Essick (CITA Toronto)

José María Ezquiaga (NBI Copenhagen)

Daniel Figueroa (IFIC Valencia)

Maya Fishbach (CITA Toronto)

Daniel Holz (U. of Chicago)

Andrea Mitridate (DESY)

Samaya Nissanke (UvA)

Chris Overstreet (Johns Hopkins)

Marco Peloso (Padova U.)

Antoine Petiteau (CEA)

Nataliya Porayko (MPIFR Bonn)

Nicholas Rodd (LBNL Berkeley)

Jorinde Van De Vis (Leiden U.)

Sarah Vigeland (UW Milwaukee)

Matias Zaldarriaga (IAS Princeton)

Sponsored by

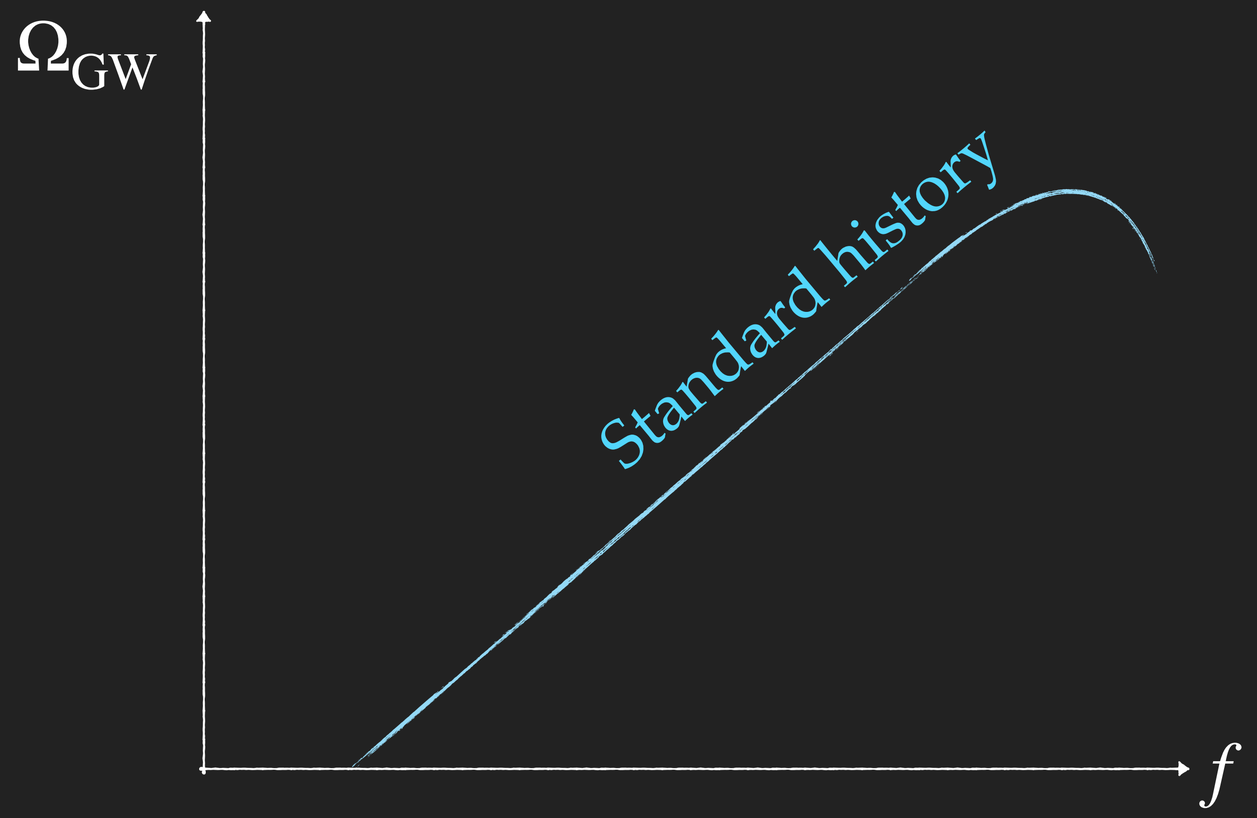
COMUNE DI POLLICA · FUTURE FOOD INSTITUTE · INFN · SIMONS FOUNDATION

APPLICATIONS ARE OPEN!

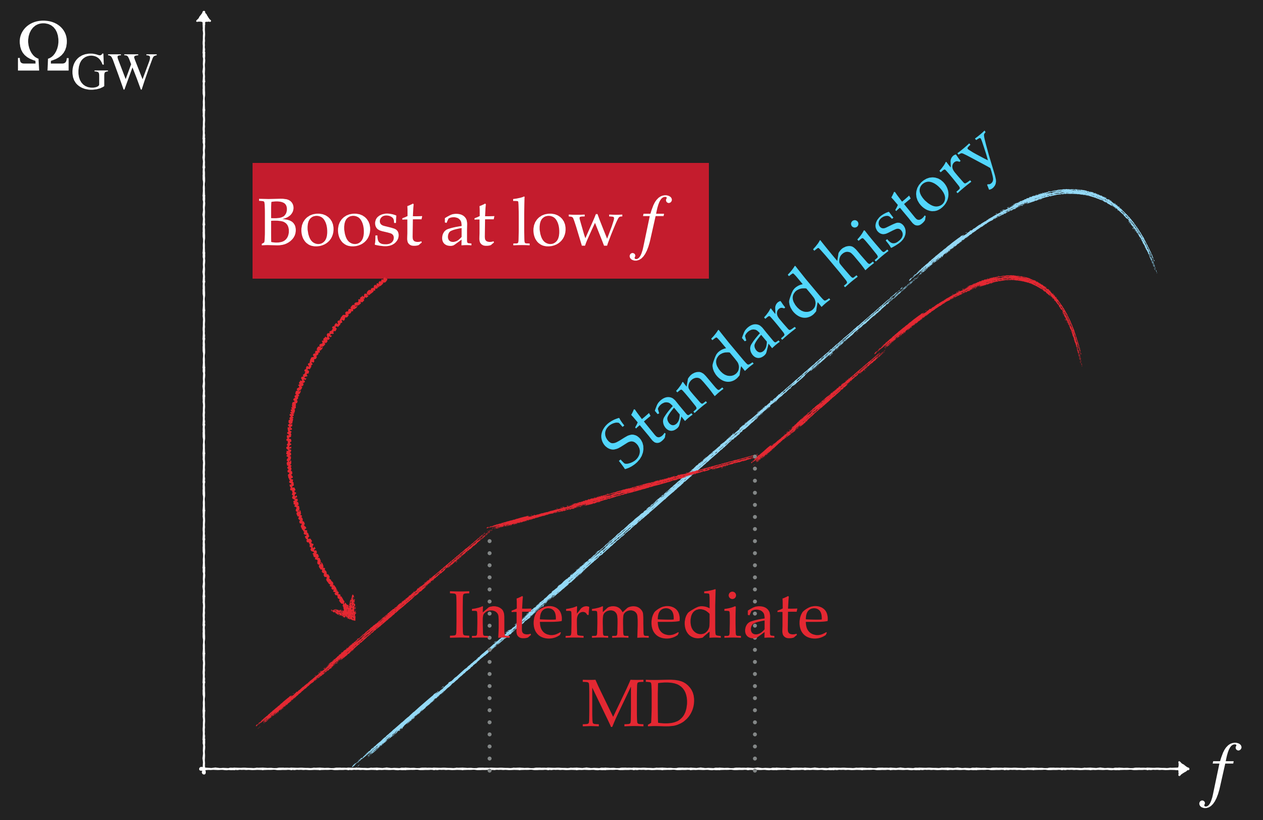
Thank you for your attention!

BACKUP SLIDES

- Intermediate MD phase → amplify low-f spectrum

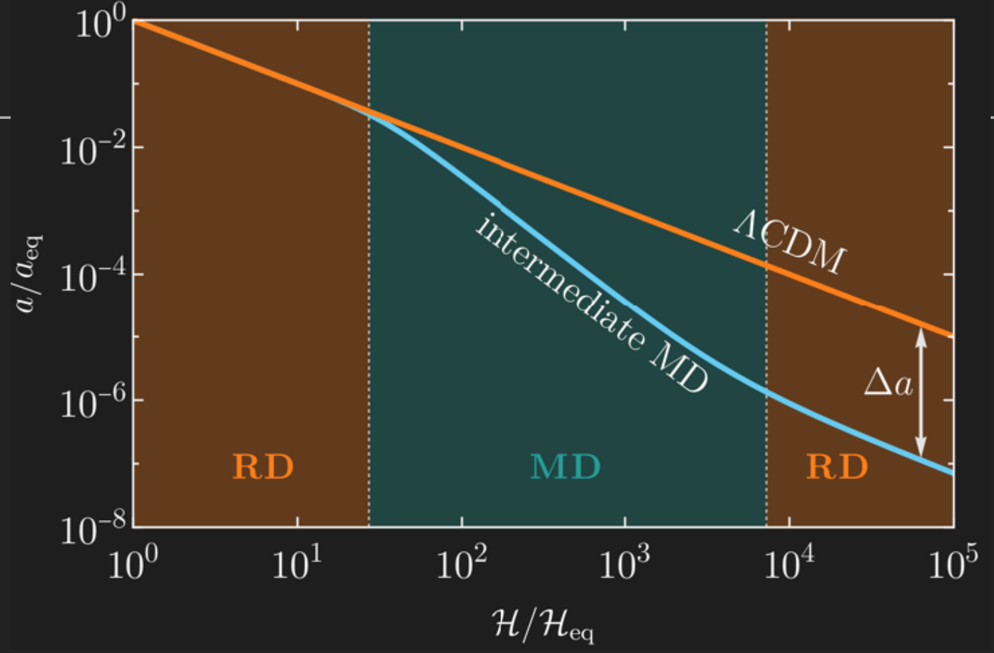


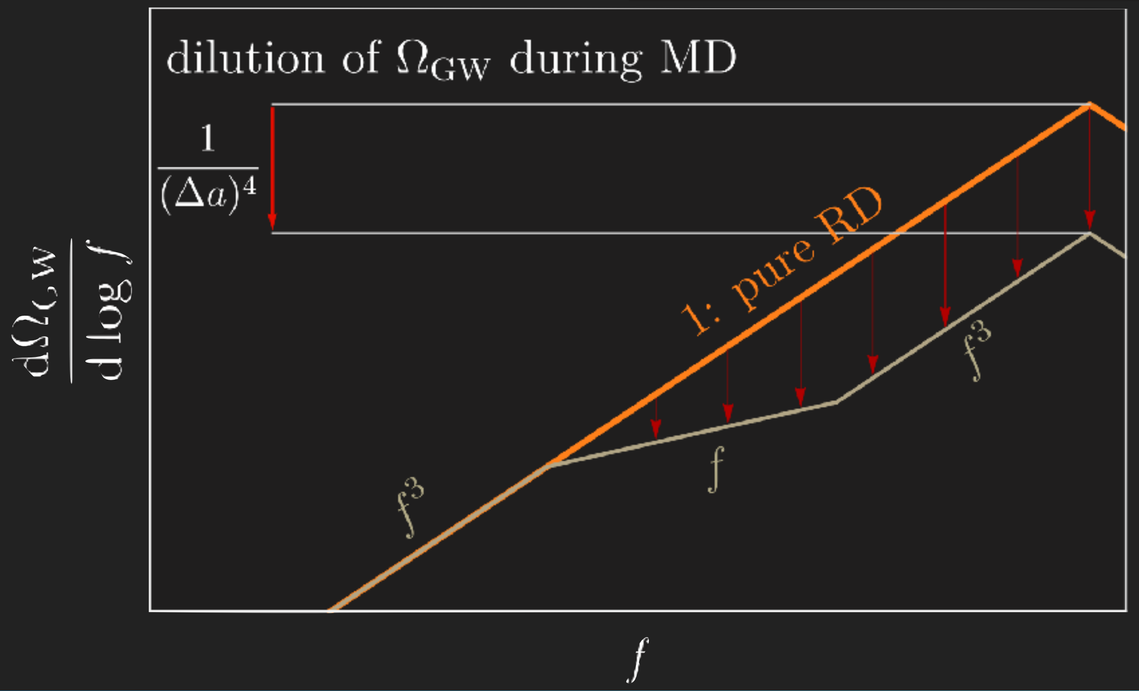
- Intermediate MD phase → amplify low-f spectrum

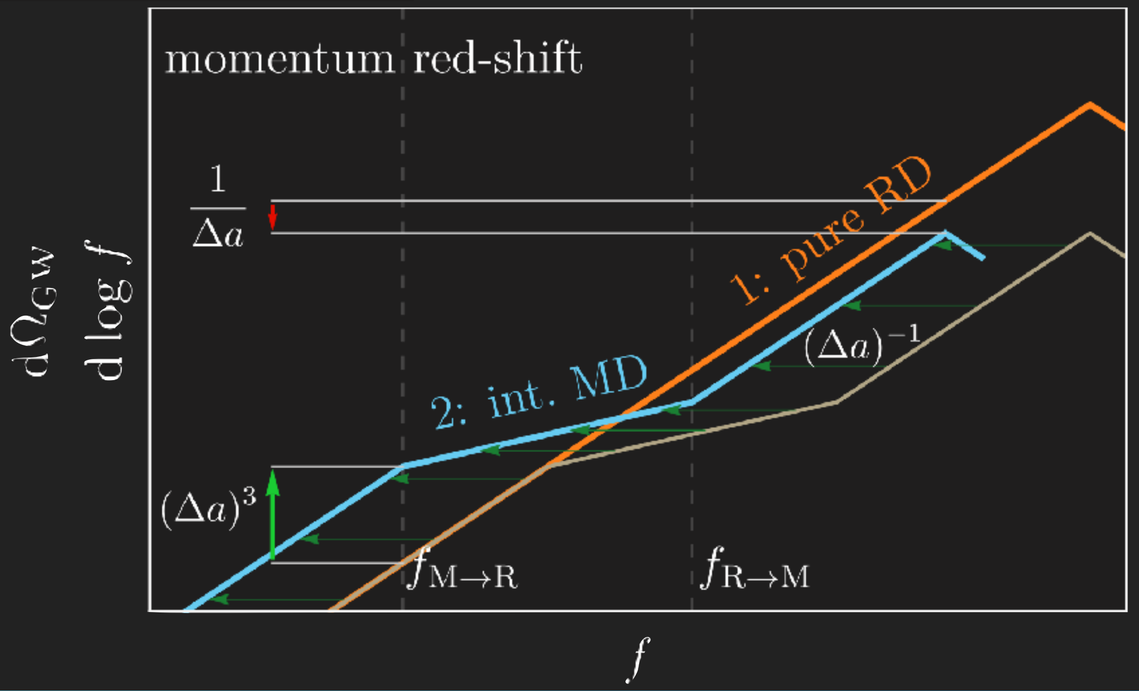


INTERMEDIATE MD

['20 Hook, Marques-Tavares, **DR**]

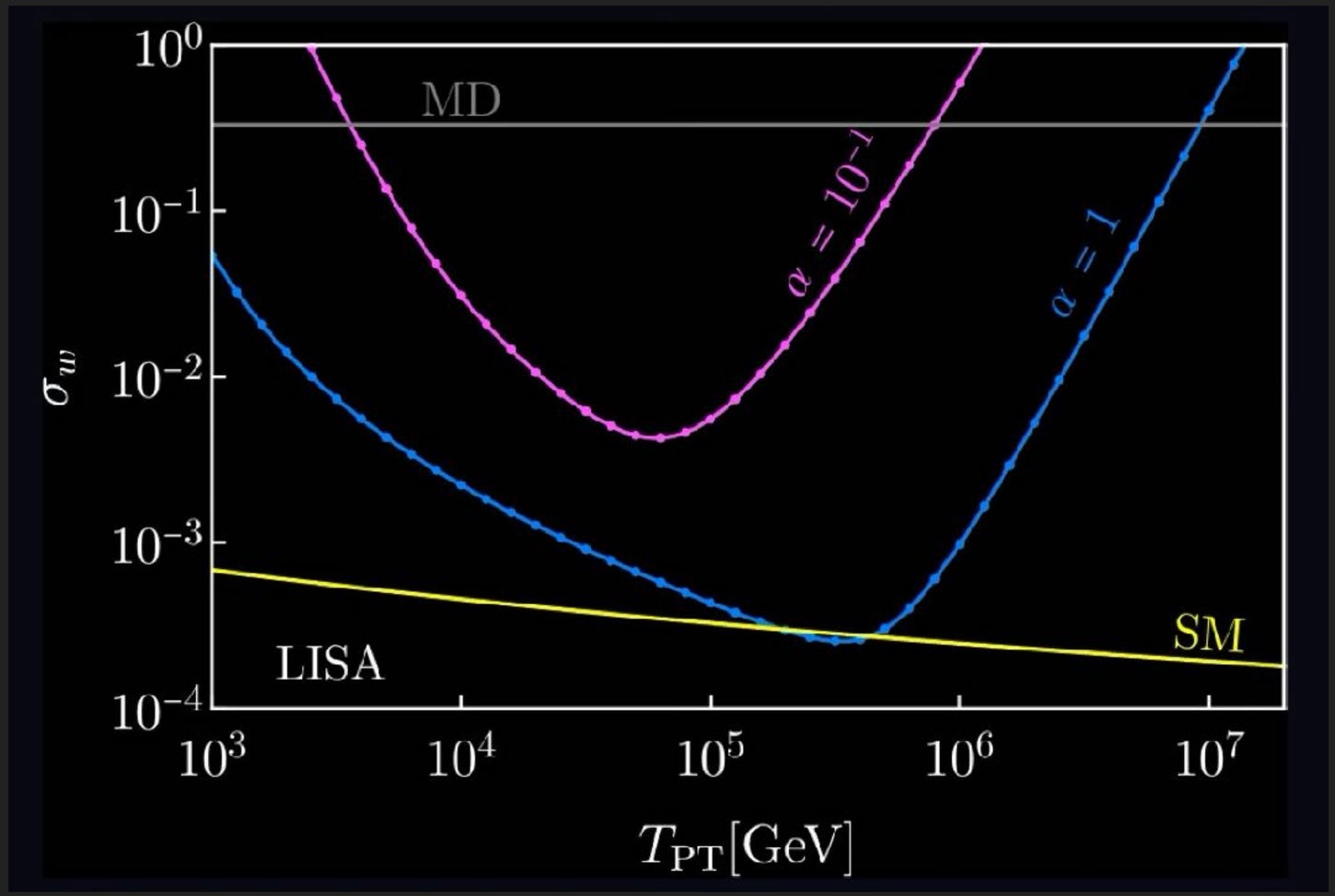


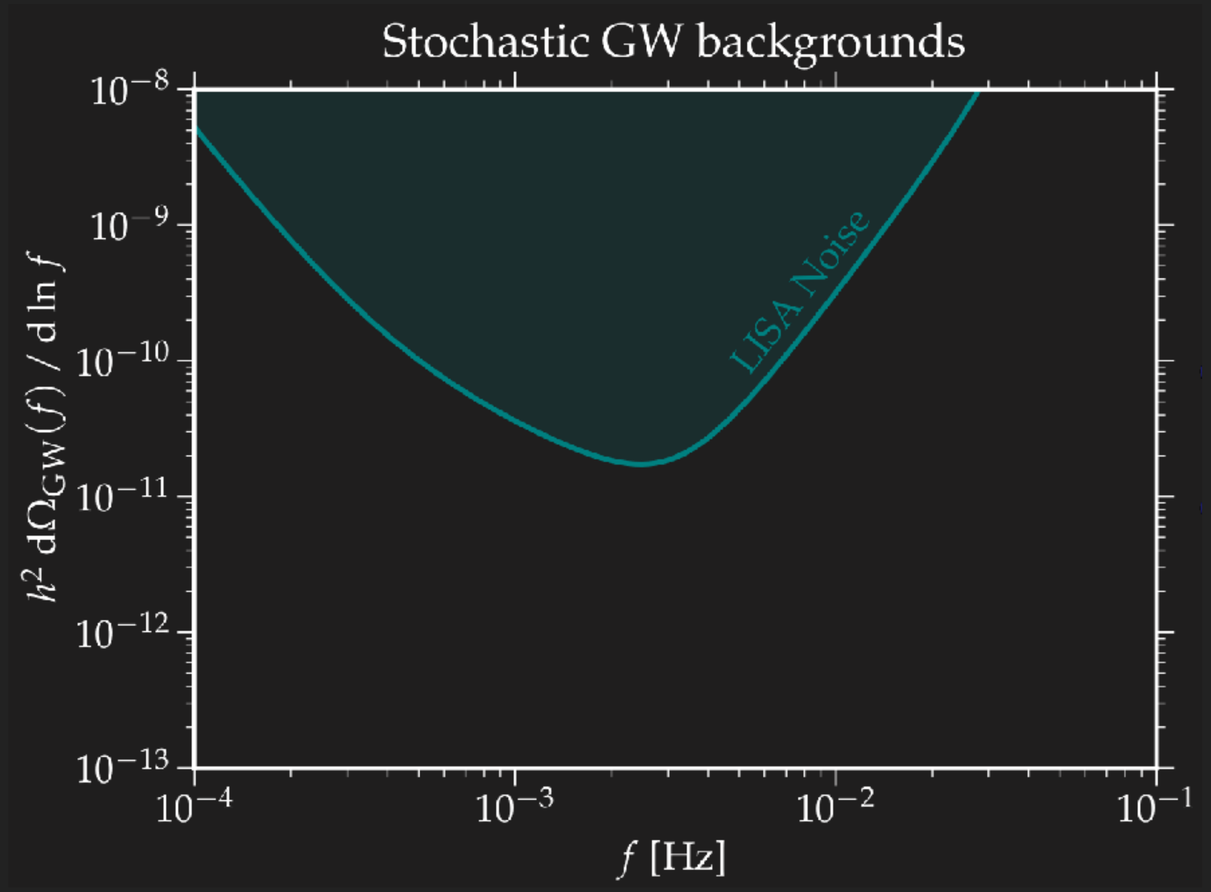


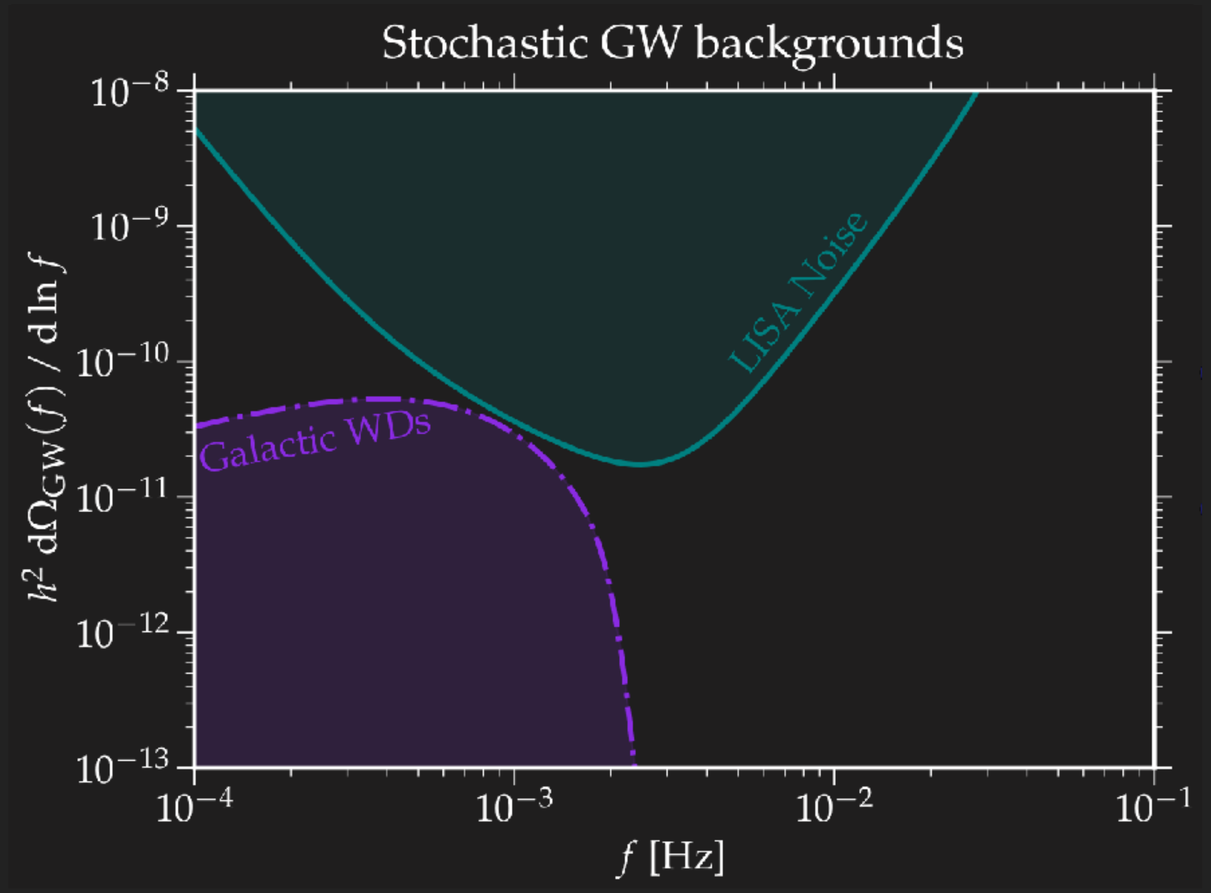


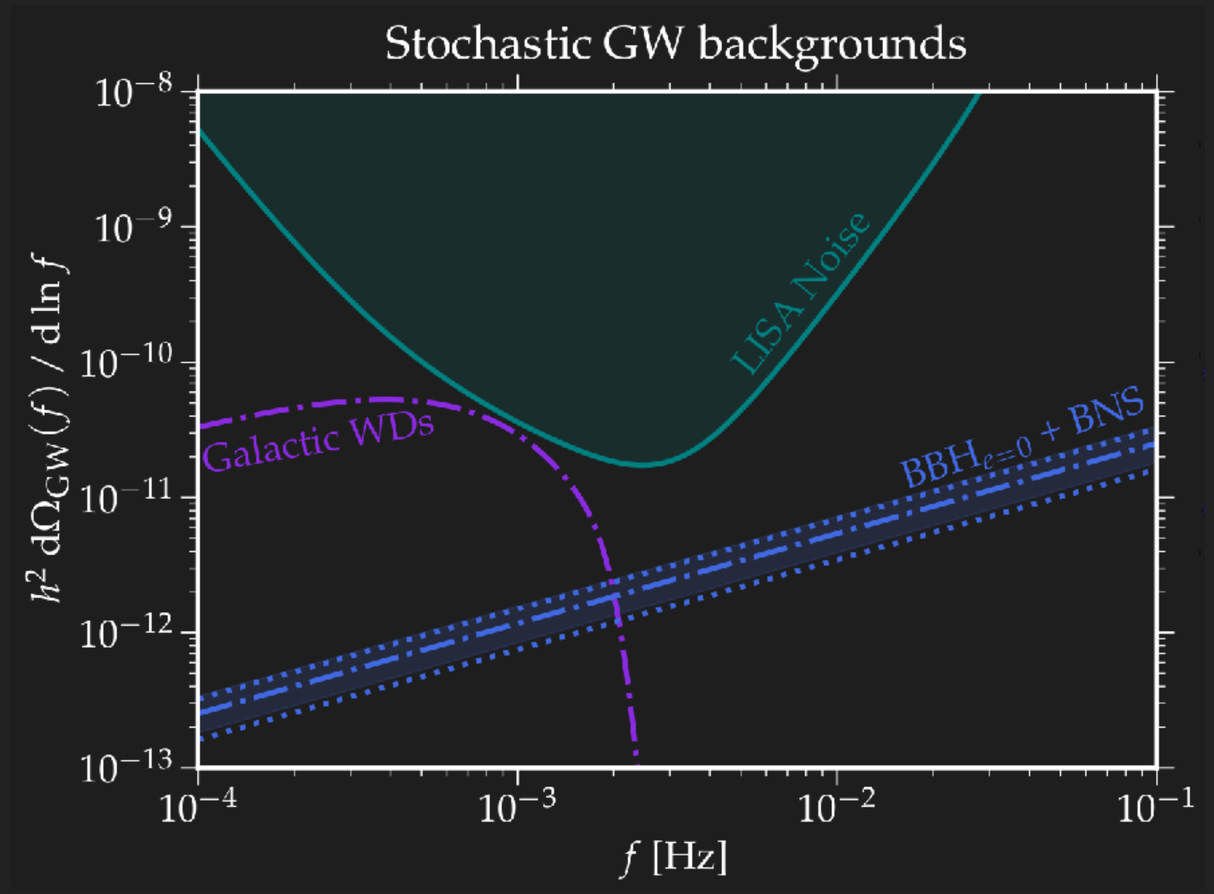
HOW WELL CAN WE DETECT FEATURES IN THE CAUSALITY TAIL?

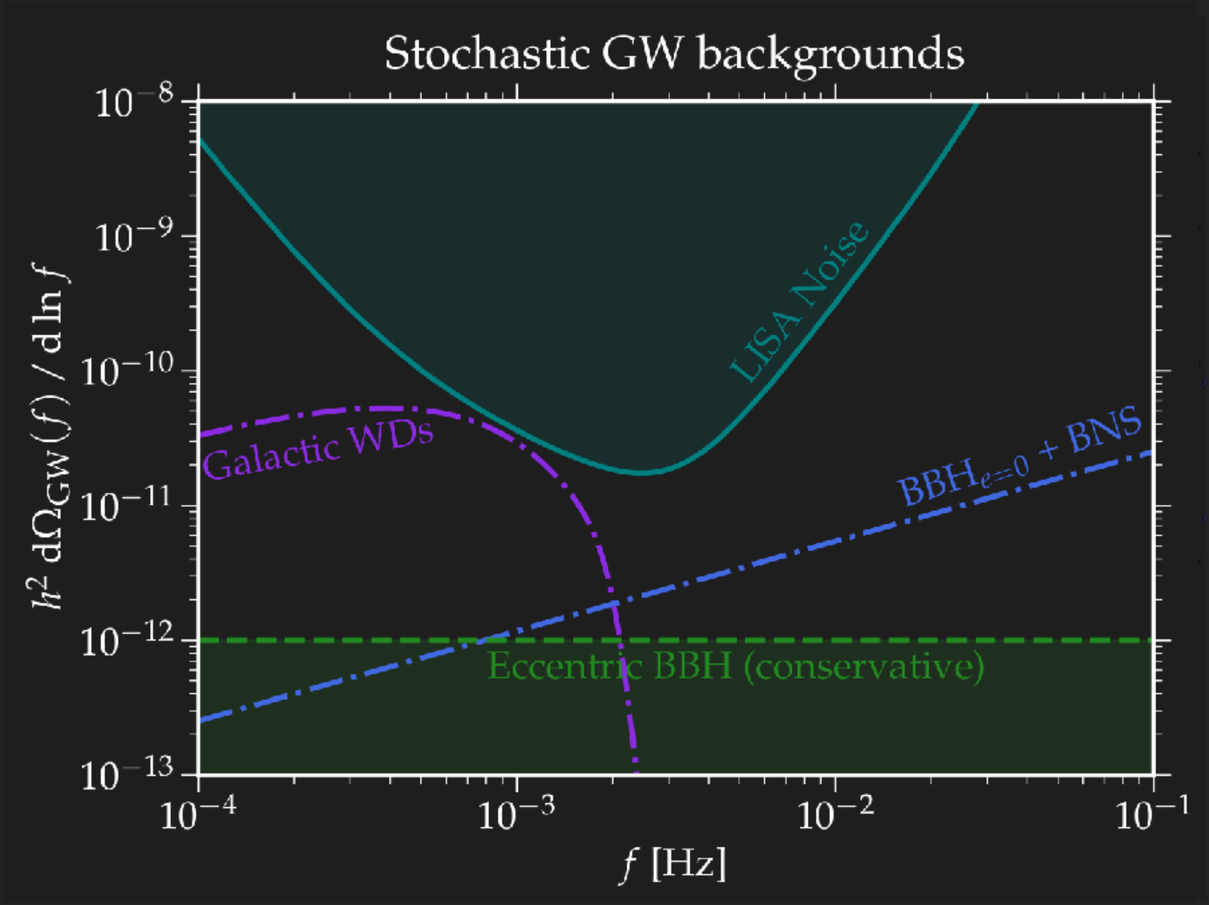
['22 Brzeminski, Hook, Marques-Tavares]

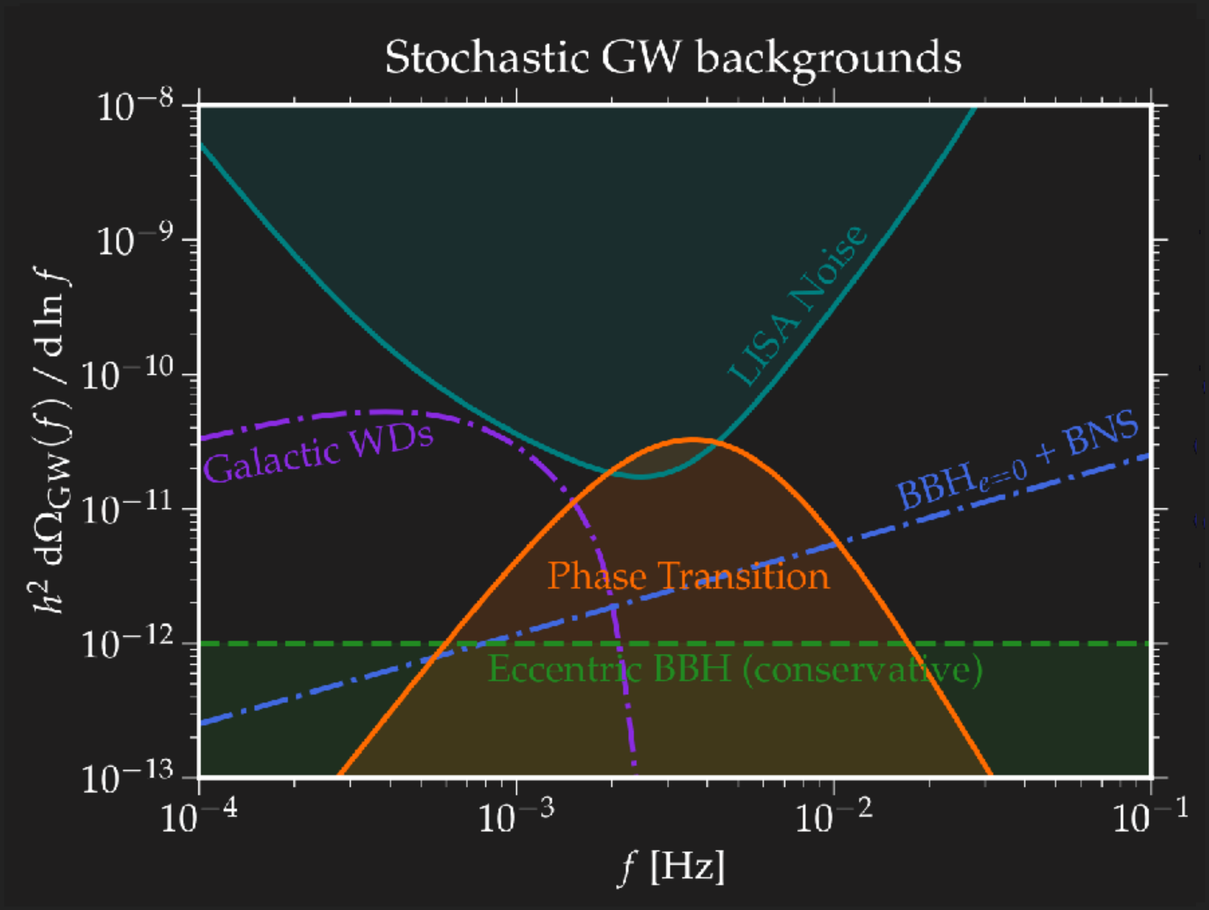


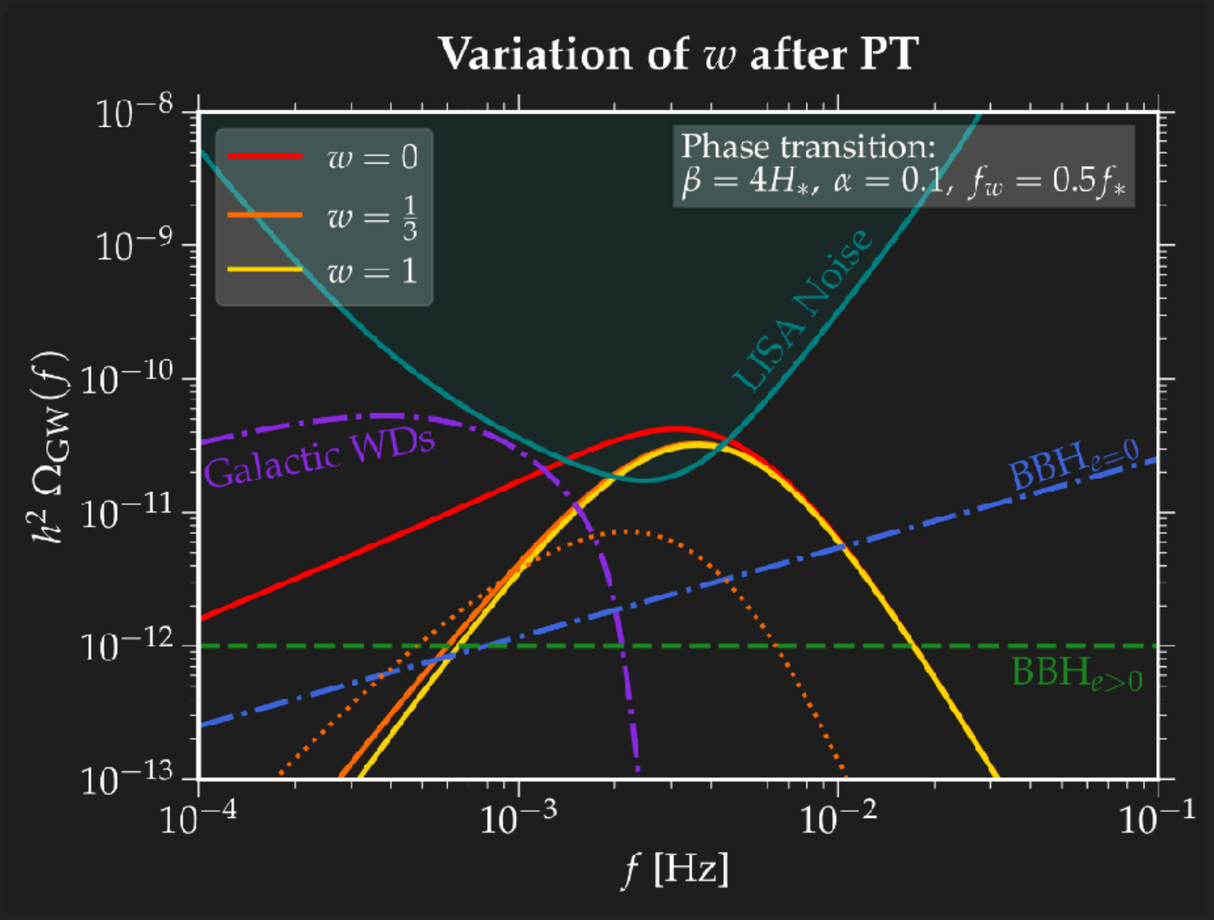

Stochastic GW backgrounds
LISA Noise
$h^2\,\mathrm{d}\Omega_{\mathrm{GW}}(f)\,/\,\mathrm{d}\ln f$
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
f [Hz]

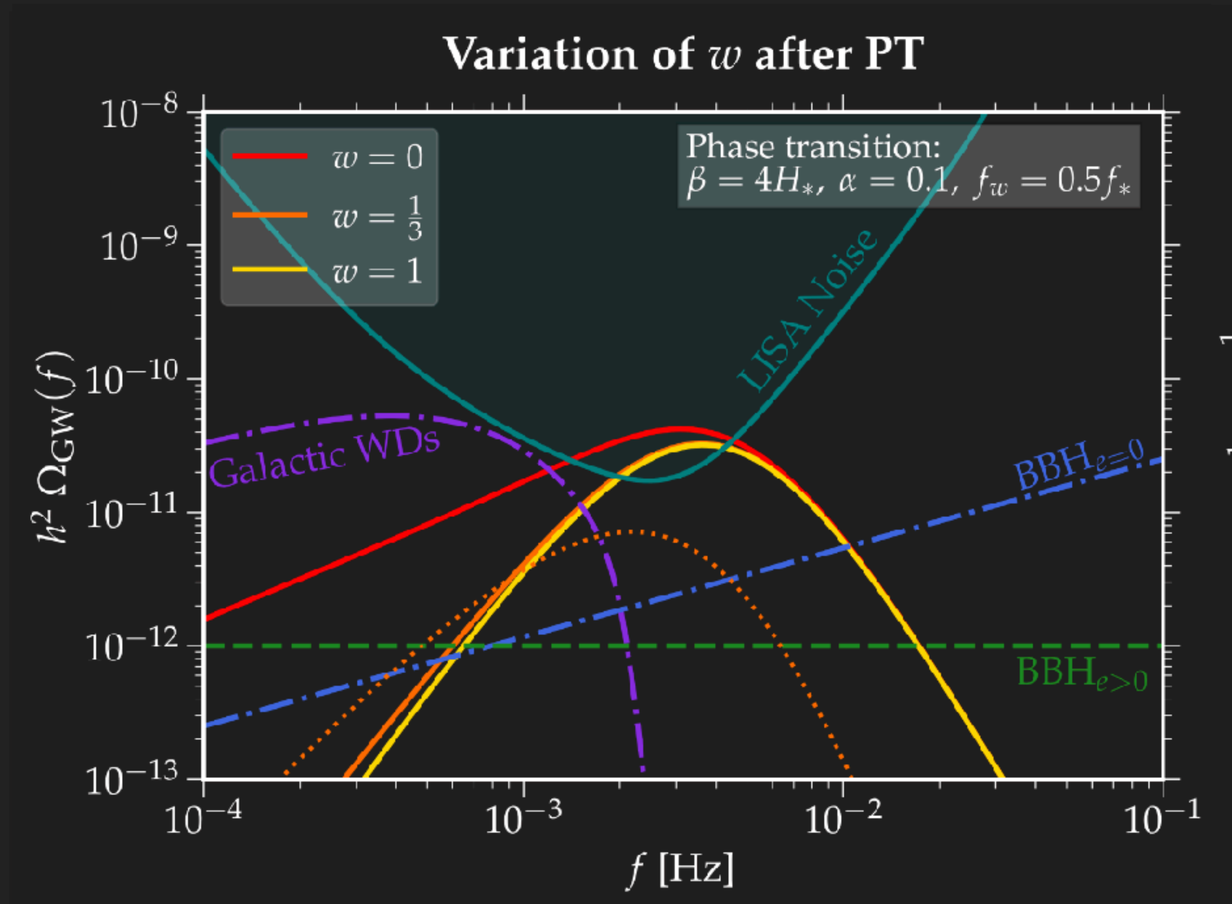

Stochastic GW backgrounds
LISA Noise
Galactic WDs
$h^2\,\mathrm{d}\Omega_{\mathrm{GW}}(f)\,/\,\mathrm{d}\ln f$
f [Hz]
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-4}
10^{-3}
10^{-2}
10^{-1}

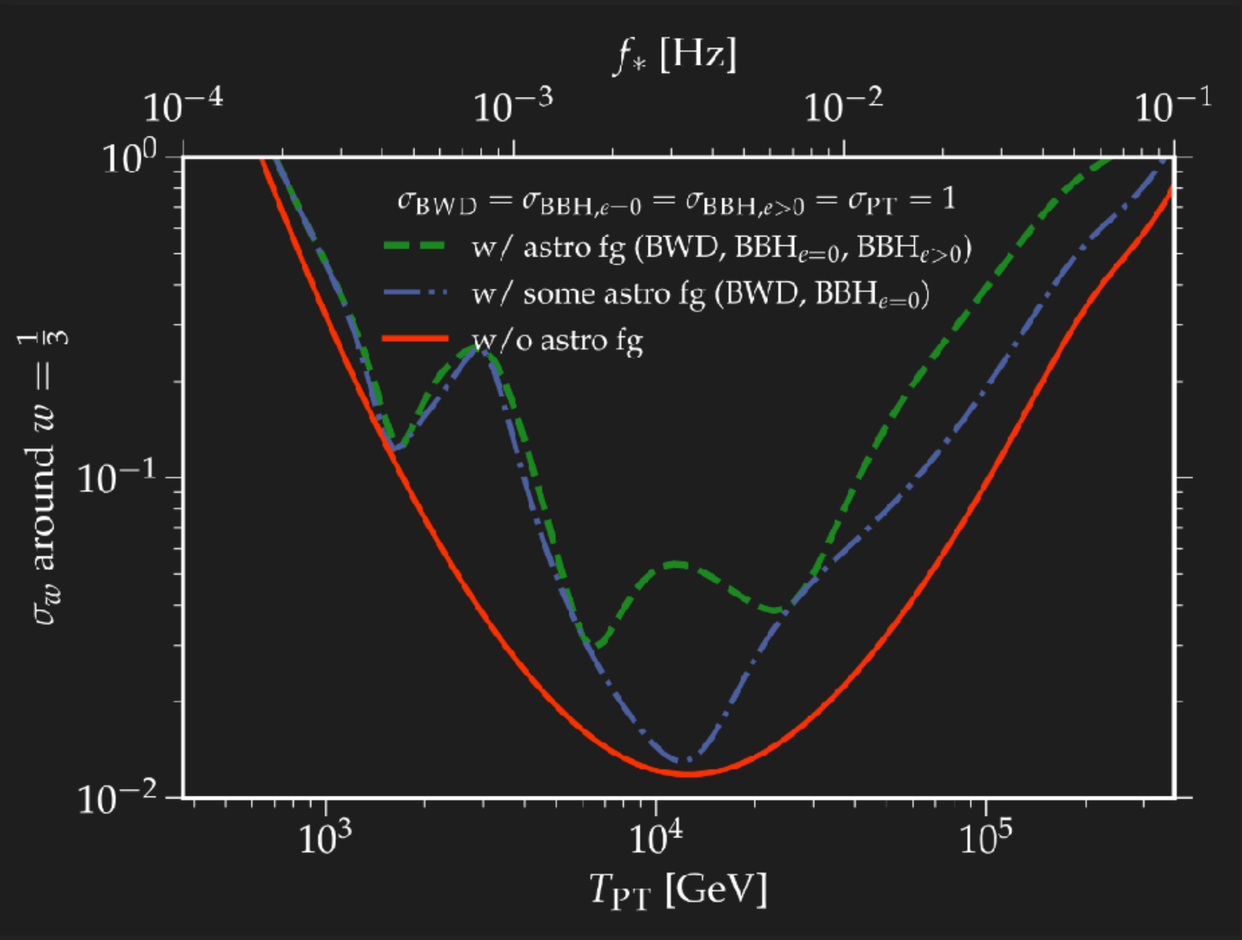

Stochastic GW backgrounds
LISA Noise
Galactic WDs
BBH$_{e=0}$ + BNS
$h^2\,\mathrm{d}\Omega_{\mathrm{GW}}(f)\,/\,\mathrm{d}\ln f$
f [Hz]
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-4}
10^{-3}
10^{-2}
10^{-1}

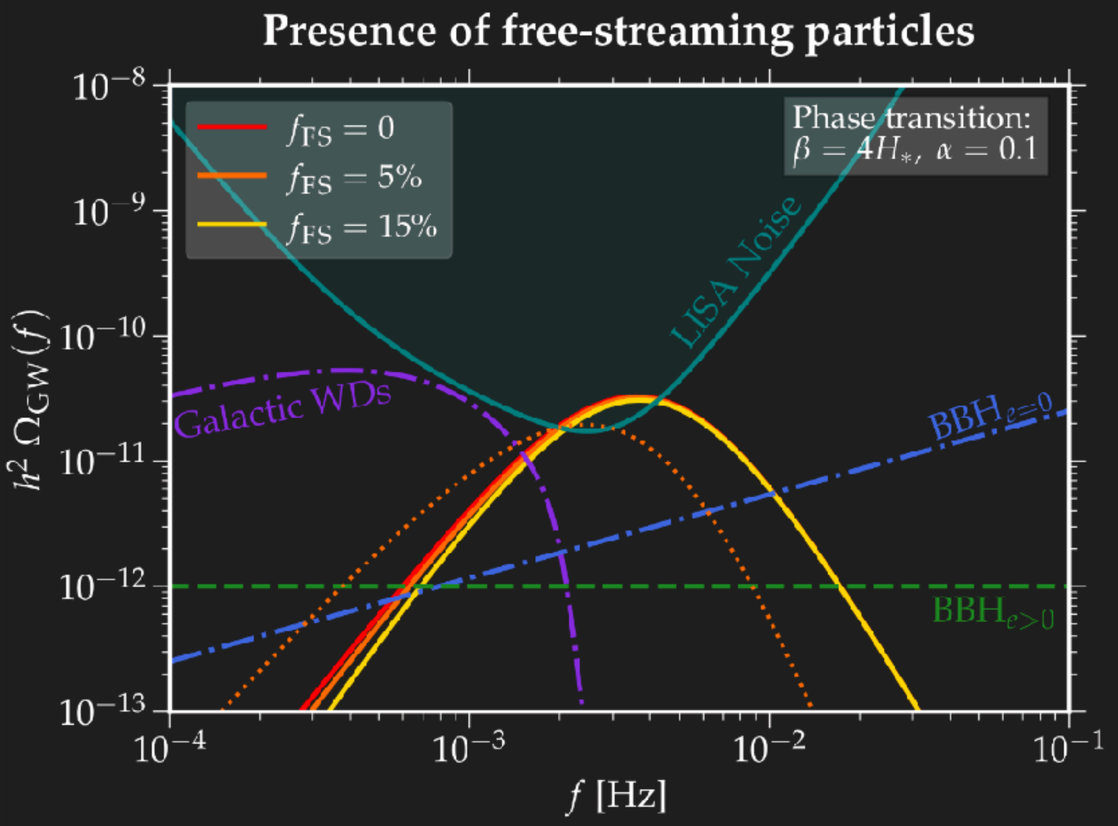

Stochastic GW backgrounds
LISA Noise
Galactic WDs
BBH$_{e=0}$ + BNS
Eccentric BBH (conservative)
$h^2\,\mathrm{d}\Omega_{\mathrm{GW}}(f)\,/\,\mathrm{d}\ln f$
f [Hz]
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-4}
10^{-3}
10^{-2}
10^{-1}

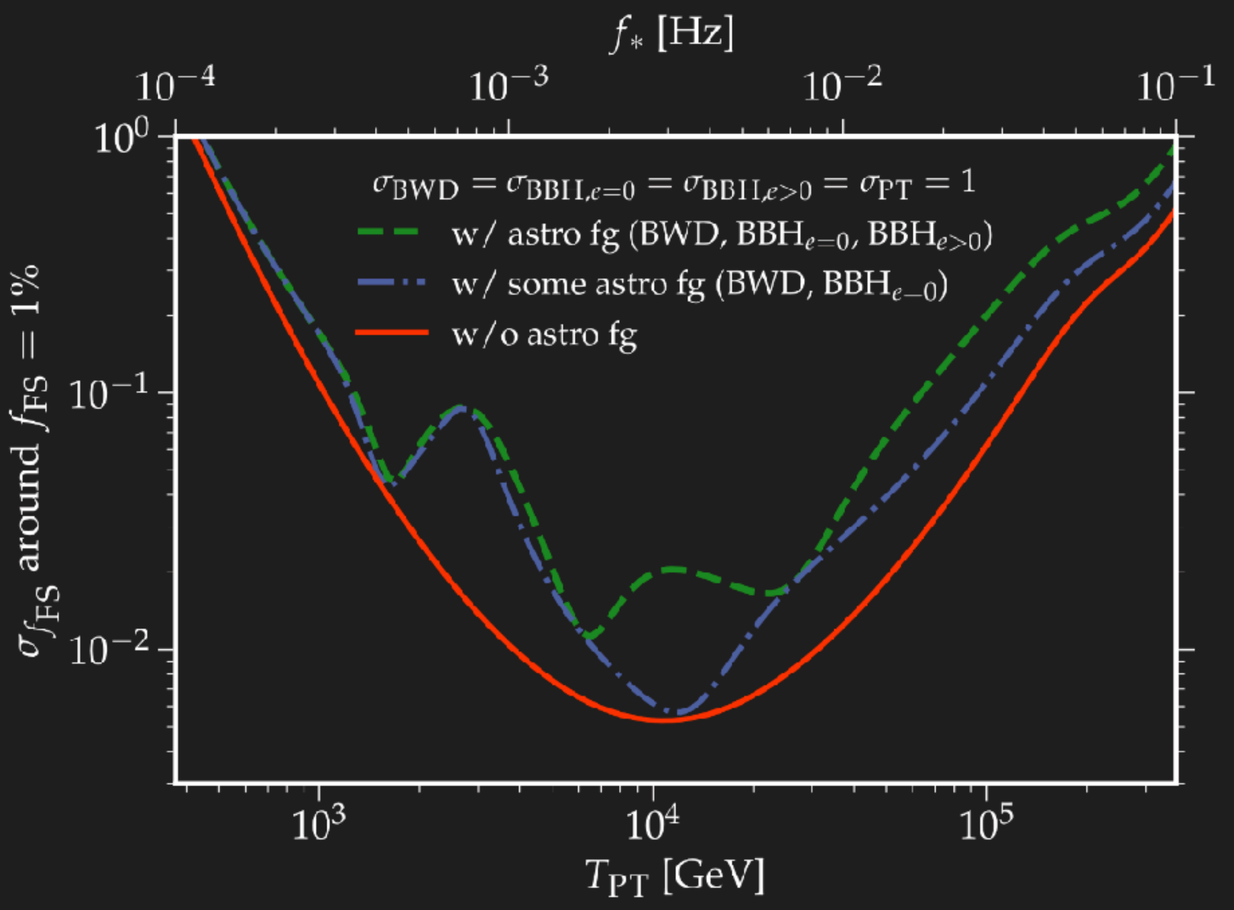

Stochastic GW backgrounds
LISA Noise
Galactic WDs
BBH$_{e=0}$ + BNS
Phase Transition
Eccentric BBH (conservative)
$h^2\,\mathrm{d}\Omega_{\mathrm{GW}}(f)\,/\,\mathrm{d}\ln f$
f [Hz]
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-4}
10^{-3}
10^{-2}
10^{-1}


Variation of w after PT
Phase transition:
$\beta = 4H_*$, $\alpha = 0.1$, $f_w = 0.5 f_*$
$w = 0$
$w = \frac{1}{3}$
$w = 1$
LISA Noise
Galactic WDs
BBH$_{e=0}$
BBH$_{e>0}$
$h^2\,\Omega_{\rm GW}(f)$
f [Hz]
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-4}
10^{-3}
10^{-2}
10^{-1}


Variation of w after PT
$w = 0$
$w = \frac{1}{3}$
$w = 1$
Phase transition:
$\beta = 4H_*$, $\alpha = 0.1$, $f_w = 0.5 f_*$
LISA Noise
Galactic WDs
$\mathrm{BBH}_{e=0}$
$\mathrm{BBH}_{e>0}$
$h^2\,\Omega_{\mathrm{GW}}(f)$
f [Hz]


f_* [Hz]
$\sigma_{\mathrm{BWD}} = \sigma_{\mathrm{BBH},e=0} = \sigma_{\mathrm{BBH},e>0} = \sigma_{\mathrm{PT}} = 1$
w/ astro fg (BWD, $\mathrm{BBH}_{e=0}$, $\mathrm{BBH}_{e>0}$)
w/ some astro fg (BWD, $\mathrm{BBH}_{e=0}$)
w/o astro fg
σ_w around $w = \frac{1}{3}$
T_{PT} [GeV]


Presence of free-streaming particles
$f_{\rm FS} = 0$
$f_{\rm FS} = 5\%$
$f_{\rm FS} = 15\%$
Phase transition:
$\beta = 4H_*,\ \alpha = 0.1$
LISA Noise
Galactic WDs
BBH$_{e=0}$
BBH$_{e>0}$
$h^2\,\Omega_{\rm GW}(f)$
f [Hz]


f_* [Hz]
$\sigma_{\rm BWD} = \sigma_{\rm BBH,e=0} = \sigma_{\rm BBH,e>0} = \sigma_{\rm PT} = 1$
w/ astro fg (BWD, BBH$_{e=0}$, BBH$_{e>0}$)
w/ some astro fg (BWD, BBH$_{e=0}$)
w/o astro fg
$\sigma_{f_{\rm FS}}$ around $f_{\rm FS} = 1\%$
$T_{\rm PT}$ [GeV]

- **Standard Model**: free-streaming *neutrinos* ($T \lesssim 2$ MeV).
  Primordial GWs: $h' = 0$ until horizon-entry $\Rightarrow$ frequency-ind. suppression.

$$h'' + 2\mathcal{H}h' + k^2 h = -24 f_\nu \mathcal{H}^2 \int_{\tau_0}^{\tau} \mathrm{d}\tau' \frac{j_2[k(\tau-\tau')]}{k^2(\tau-\tau')^2} h'(\tau')$$

- **Phase transitions**: with extra FS species, also suppression at *generation*:
  $h'(\tau_\star) \neq 0$. ['20 Hook, Marques-Tavares, **DR**]

sub-$\mathcal{H}$ no effect

super-$\mathcal{H}$ $h'' + \underbrace{2\mathcal{H}h'}_{\text{friction}} + \underbrace{\left(k^2 + \frac{8 f_{\rm FS}}{5}\mathcal{H}^2\right)}_{\text{mass term}} h = 0$

Schematic derivation of Weinberg damping

[Weinberg '04; Watanabe, Komatsu '06]

- $$h''_{ij} + 2\mathcal{H}h'_{ij} + k^2 h_{ij} = 4\mathcal{H}^2 \pi_{ij}$$

$$T_{ij} = p\, g_{ij} + a^2 \pi_{ij}\,, \qquad T^{(\nu)}_{ij} = \frac{1}{\sqrt{-g}} \int \frac{\mathrm{d}^3 q}{q^0} q_i q_j F^{(\nu)}(q)$$

- Boltzmann eq.:

$$0 = \frac{\mathrm{d}F}{\mathrm{d}t} = \frac{\partial F}{\partial \tau} + \frac{\mathrm{d}x^i}{\mathrm{d}t}\frac{\partial F}{\partial x^i} + \frac{\mathrm{d}p^0}{\mathrm{d}t}\frac{\partial F}{\partial p^0}$$

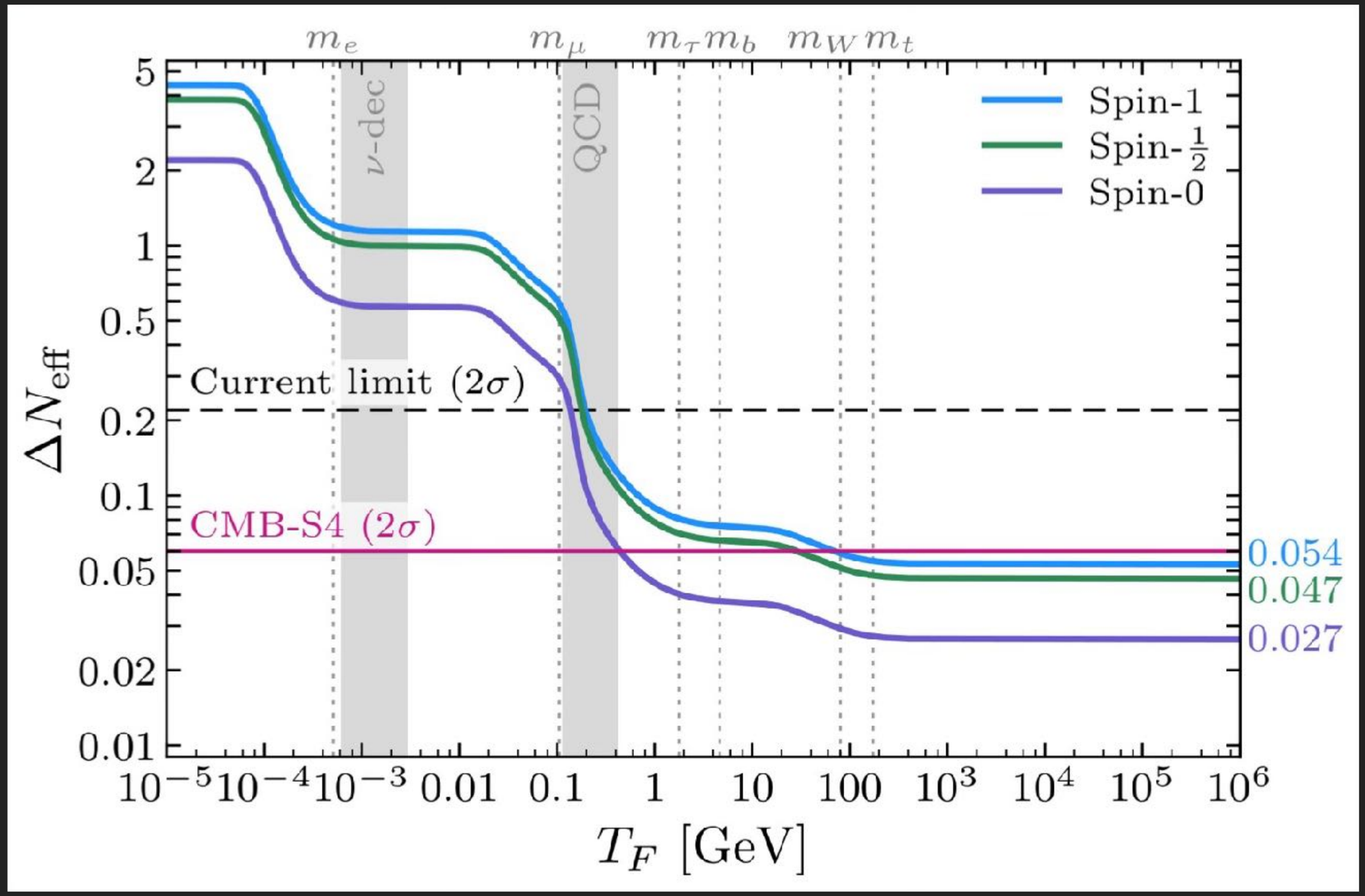
- Geodesic eq.:

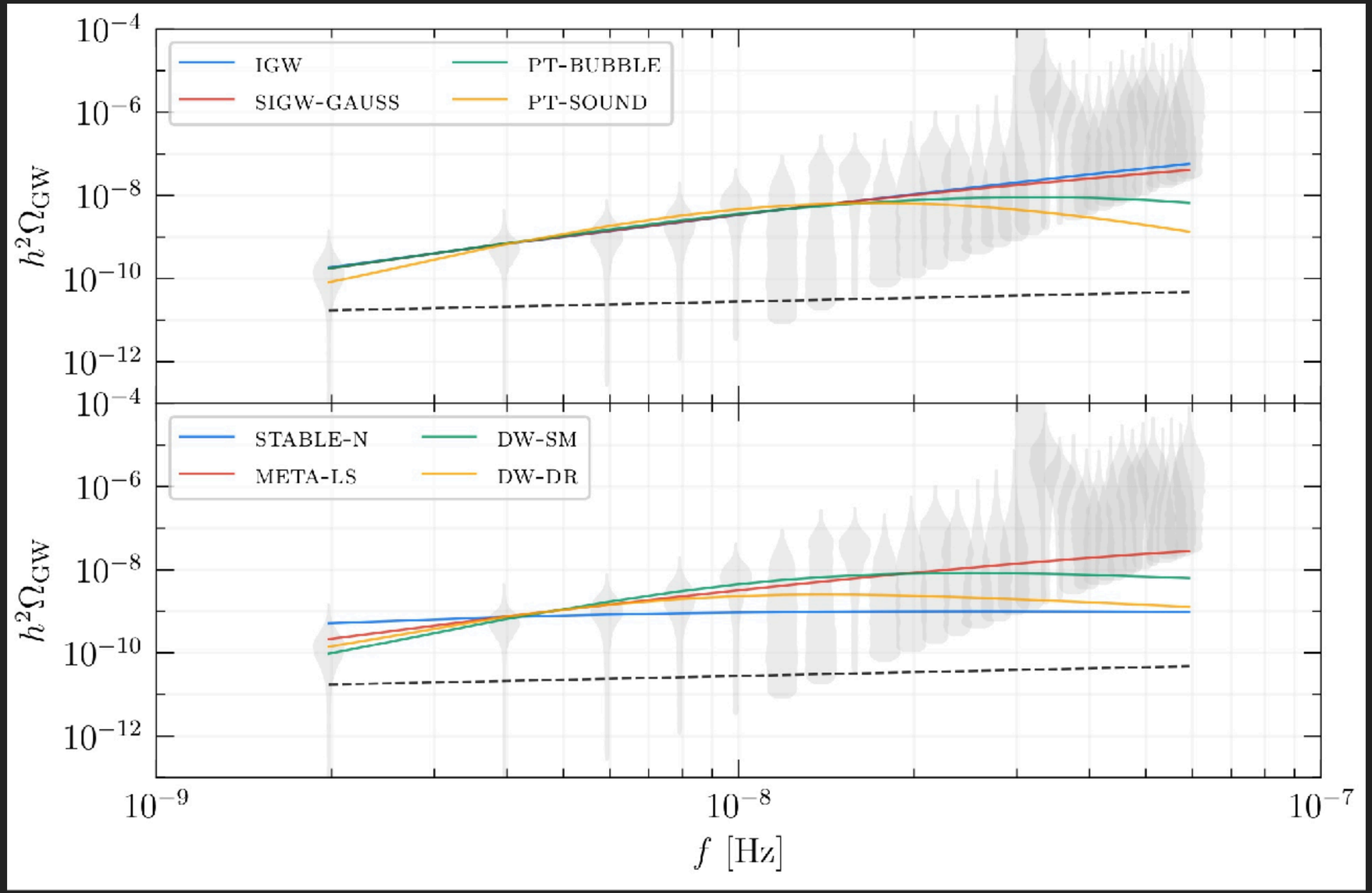
$$\frac{\mathrm{d}p^\mu}{\mathrm{d}\lambda} = -\Gamma^\mu_{\alpha\beta} p^\alpha p^\beta \implies \frac{1}{p^0}\frac{\mathrm{d}p^0}{\mathrm{d}t} = -H - \frac{1}{2}\frac{\partial h_{ij}}{\partial t}\frac{p^i p^j}{(p^0)^2}$$

- ν's lose (or gain) energy depending on the sign of h'.

$$h'' + 2\mathcal{H}h' + k^2 h = -24 f_\nu \mathcal{H}^2 \int_{\tau_0}^{\tau} \mathrm{d}\tau' \frac{j_2[k(\tau-\tau')]}{k^2(\tau-\tau')^2} h'(\tau')$$

REACH OF CMB MEASUREMENTS FOR $\Delta N_{\rm eff}$




IGW
SIGW-GAUSS
PT-BUBBLE
PT-SOUND
STABLE-N
META-LS
DW-SM
DW-DR
$h^2\Omega_{\rm GW}$
f [Hz]
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-9}
10^{-8}
10^{-7}

TESTING PRIMORDIAL BLACK HOLES AS DARK MATTER

['18 Espinosa, **DR**, Riotto]

['18 Bartolo, De Luca, Franciolini, Peloso, **DR**, Riotto]