Spinning Waveforms from Scattering Amplitudes in Modified Gravity

Panagiotis Marinellis

In collaboration with Adam Falkowski:
[2407.16457], [24XX.XXXXX]

IJCLab Irène Joliot-Curie Laboratoire de Physique des 2 Infinis

DMLab

université PARIS-SACLAY

Plan for this talk:

1. Motivation

2. Scattering Amplitudes and Observables

3. Scalar-tensor theories: Examples, compact objects, scalar hair and scattering waveforms

4. Outlook

1. Motivation

Image Credit: EGO*

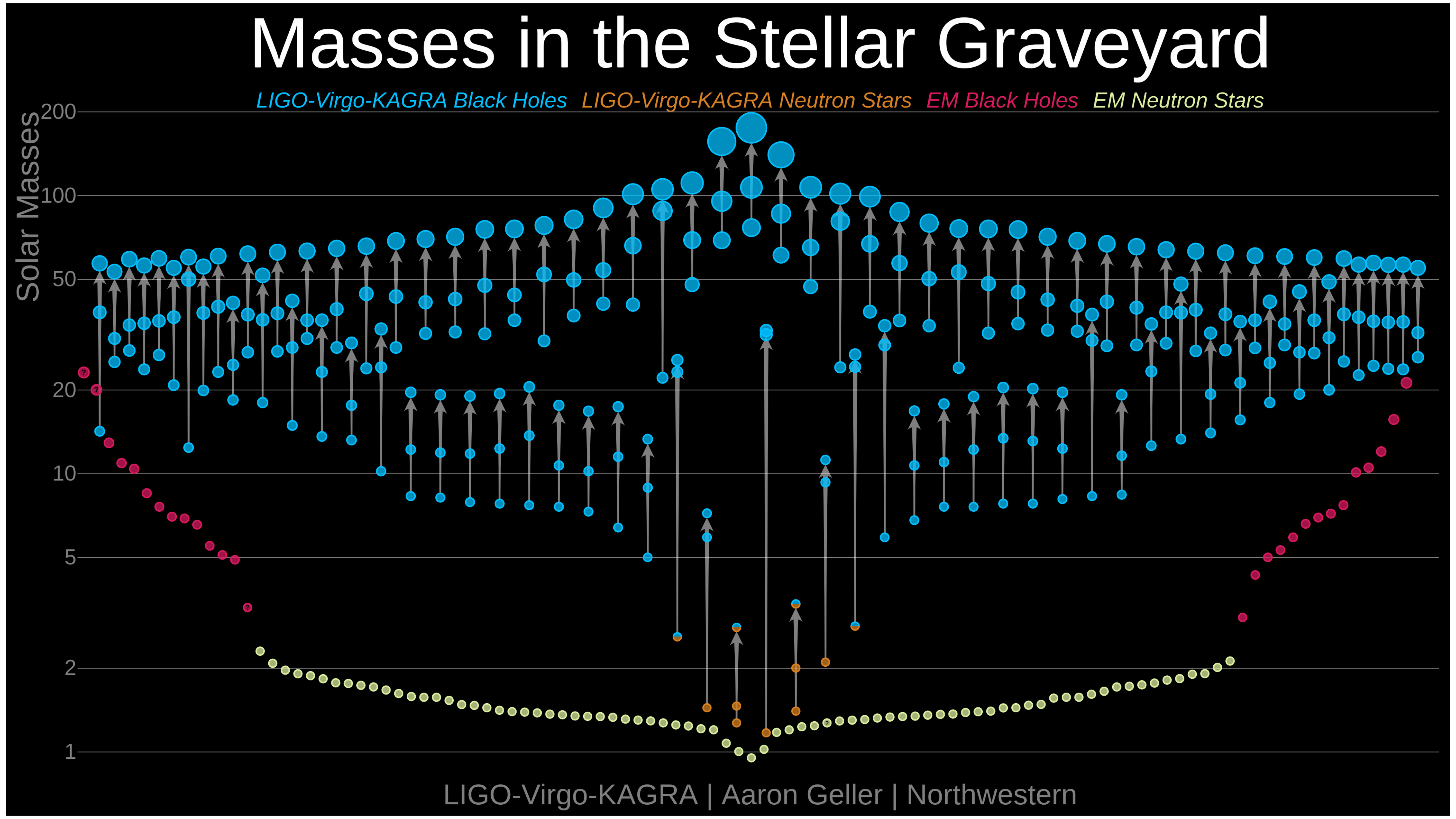
→ LIGO/VIRGO collaboration: First detection of **Gravitational Waves** (GWs) in **2015**

1. Motivation

Image Credit: EGO*

LIGO/VIRGO collaboration: First detection of **Gravitational Waves** (GWs) in **2015**

Has since then inspired an unprecedented interest in GW detection, especially with the upcoming **new generation of GW interferometers** (ET, Cosmic Explorer, LISA)



Credit:LIGO-Virgo/Aaron Geller/Northwestern University*

1. Motivation

Image Credit: EGO*

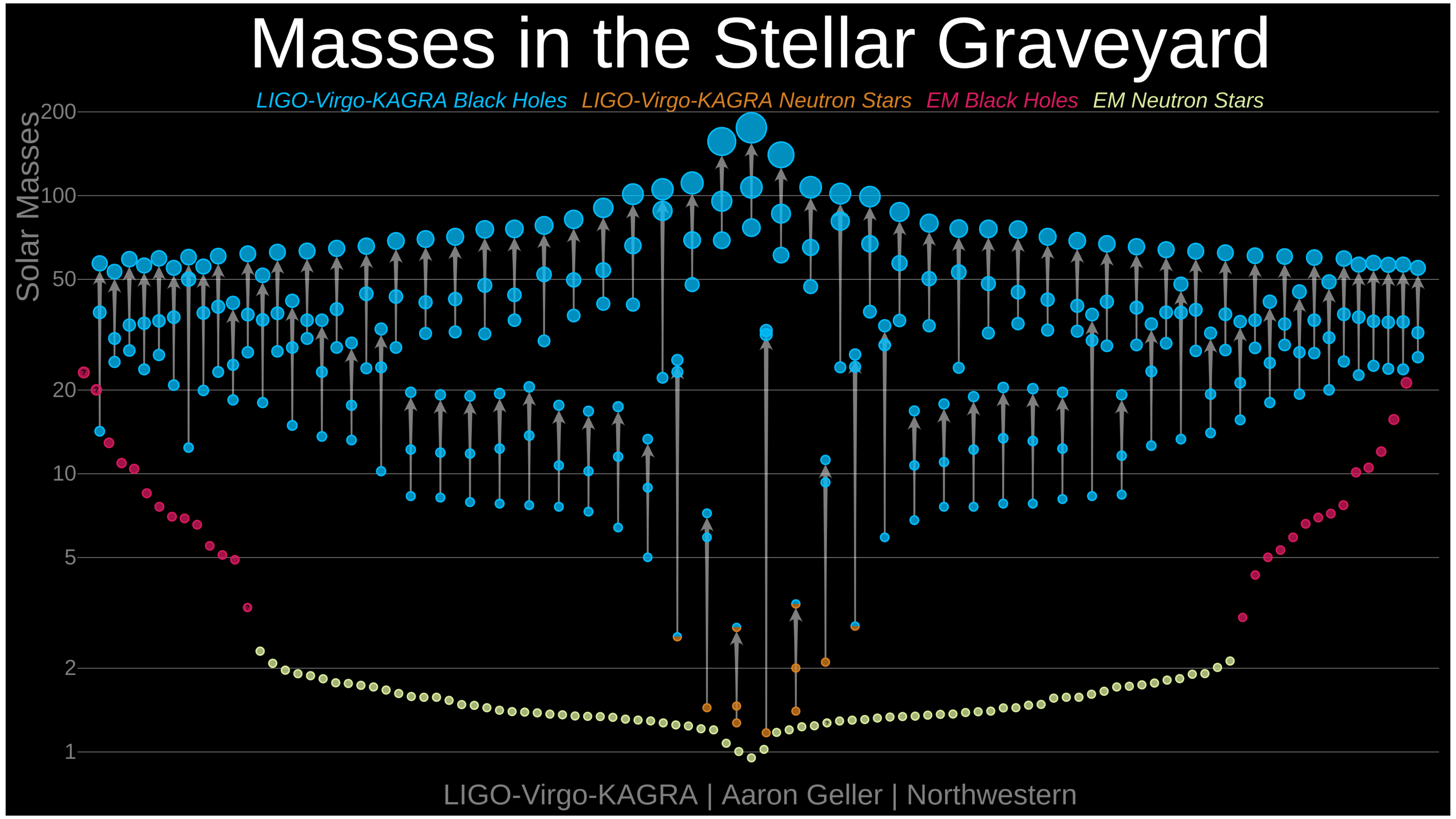
LIGO/VIRGO collaboration: First detection of **Gravitational Waves** (GWs) in **2015**

Has since then inspired an unprecedented interest in GW detection, especially with the upcoming **new generation of GW interferometers** (ET, Cosmic Explorer, LISA)

New era of **high precision** measurements of GWs:

Highly accurate GW templates

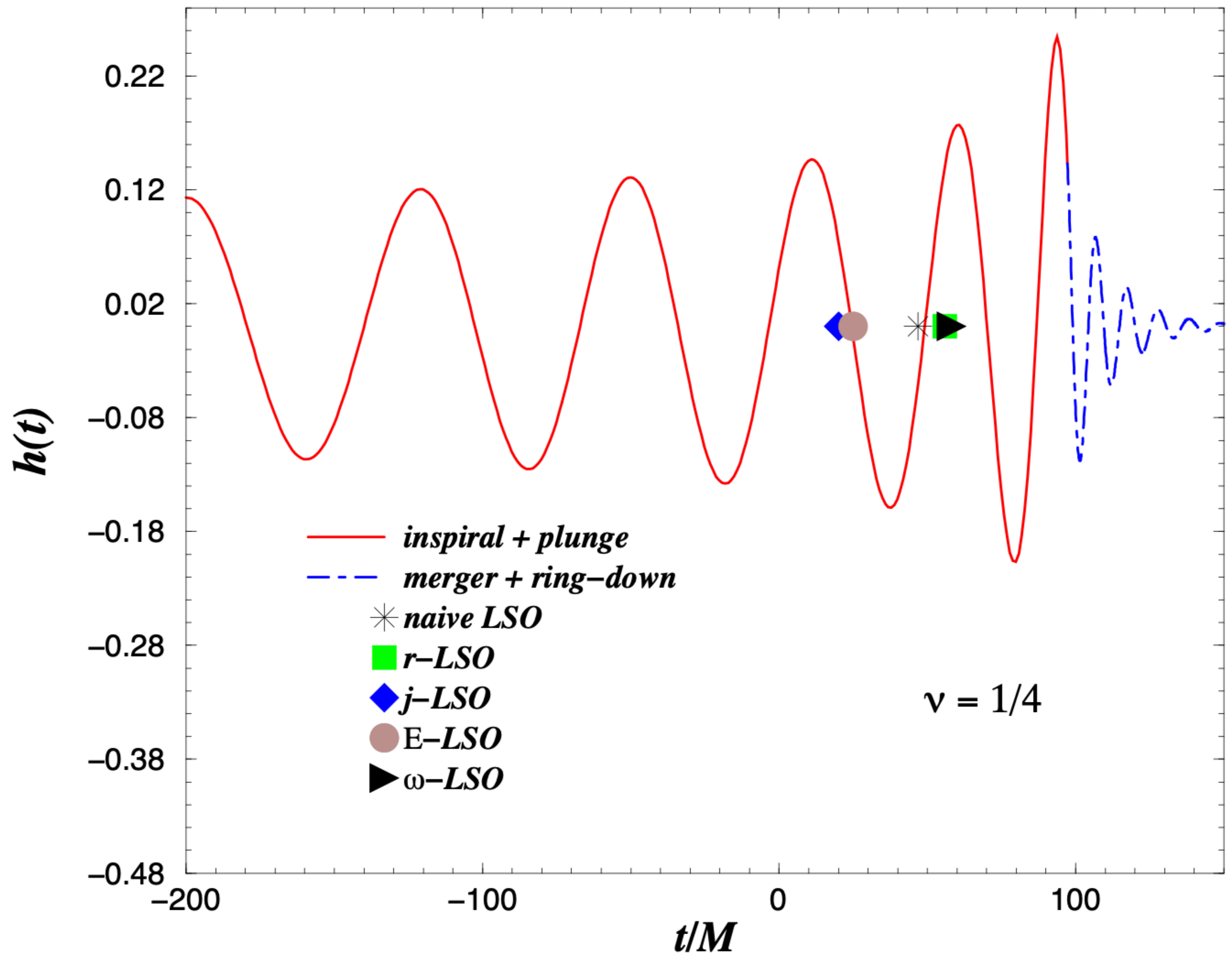
New window to test General Relativity (GR)



Credit:LIGO-Virgo/Aaron Geller/Northwestern University*

The phases of the binary problem:

Phys.Rev.D 62 (2000) 064015 [Buonanno, Damour]*



The phases of the binary problem:

Phys.Rev.D 62 (2000) 064015 [Buonanno, Damour]*

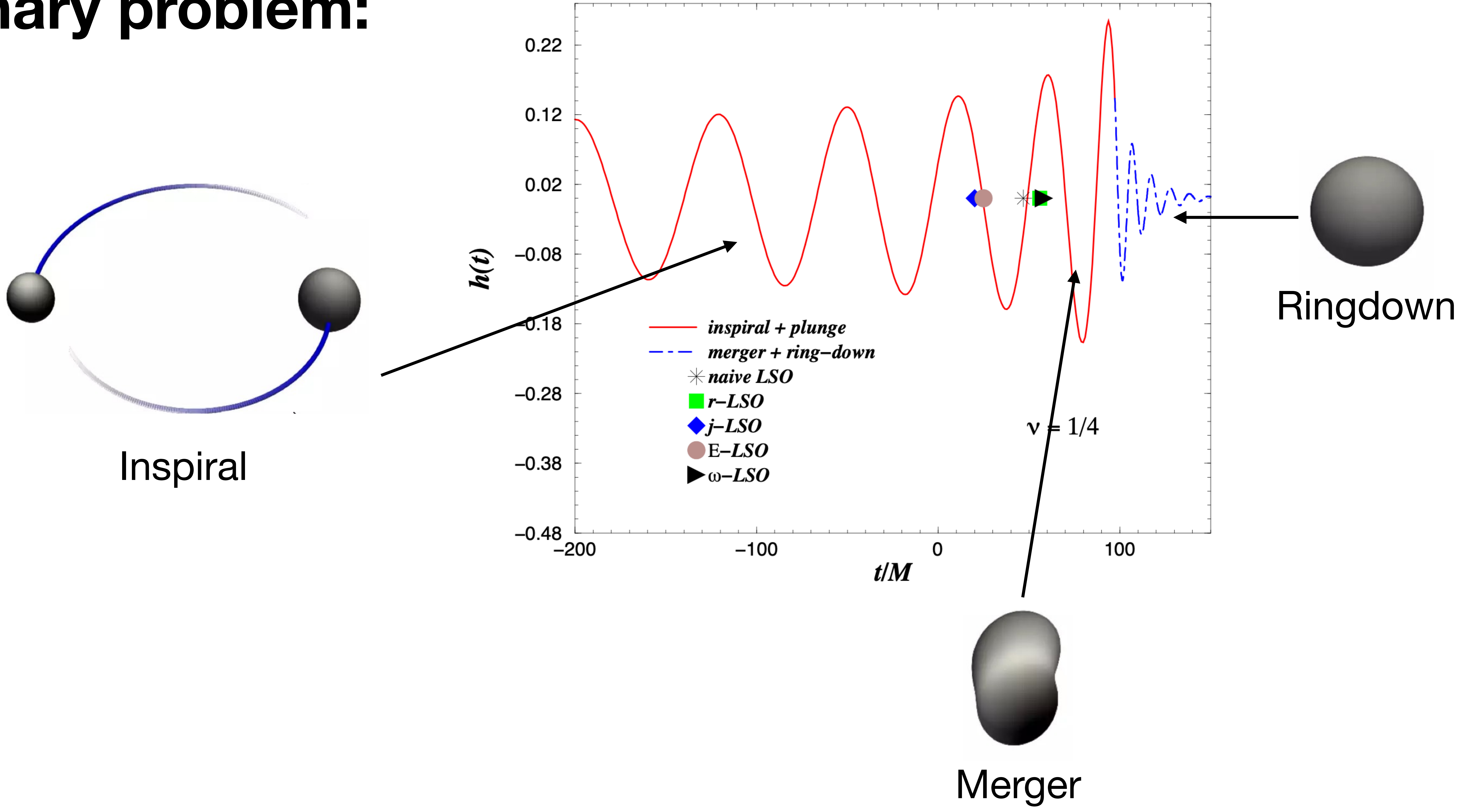
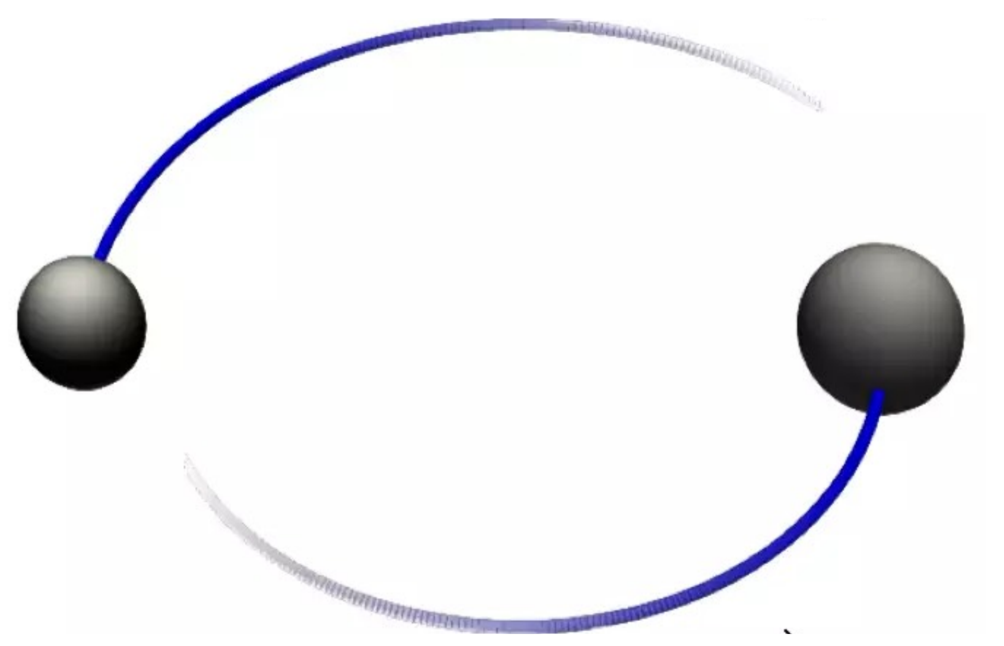


Illustration Credit: Max-Planck-Gesellschaft*

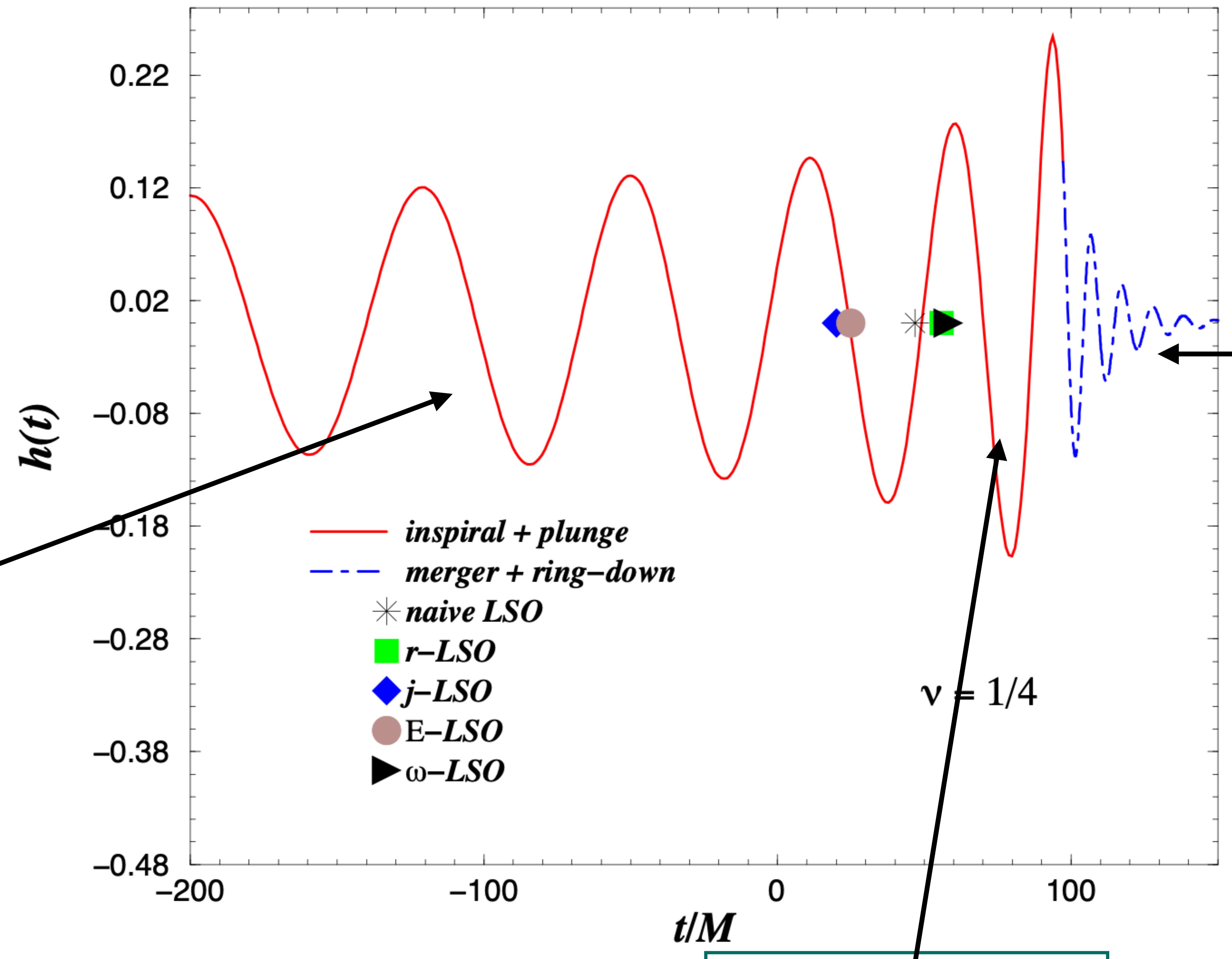
The phases of the binary problem:

Analytical approaches



Inspiral

Phys.Rev.D 62 (2000) 064015 [Buonanno, Damour]*



Ringdown

Black Hole Perturbation Theory (BHPT)

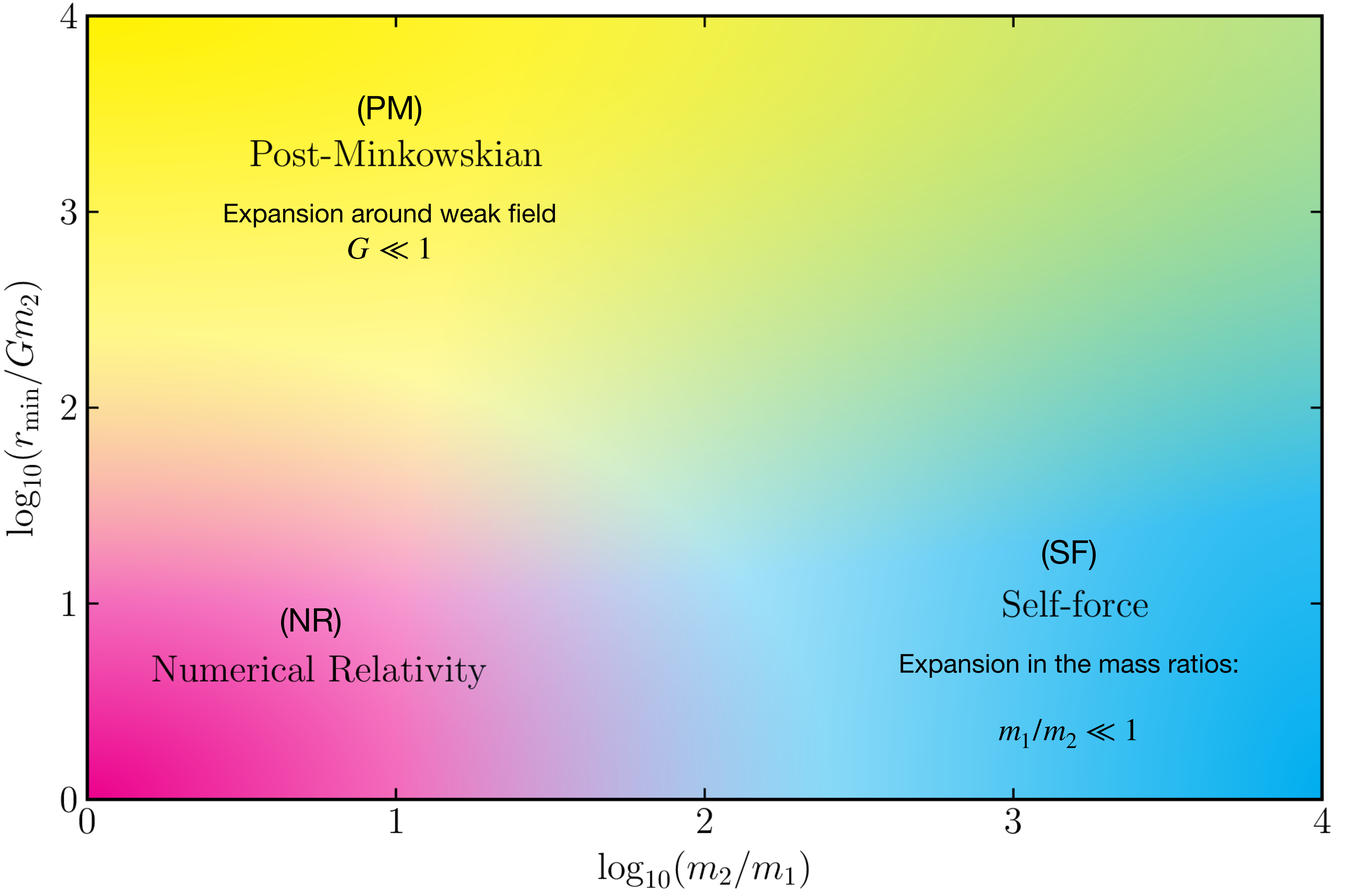
Merger

Numerical Relativity

Illustration Credit: Max-Planck-Gesellschaft*

Analytical approaches:

Phys.Rev.D 108 (2023) 2, 024025 [Barack, Bern, Herrmann, Long, Parra-Martinez, Roiban, Ruf, Shen, Solon, Teng, Zeng]



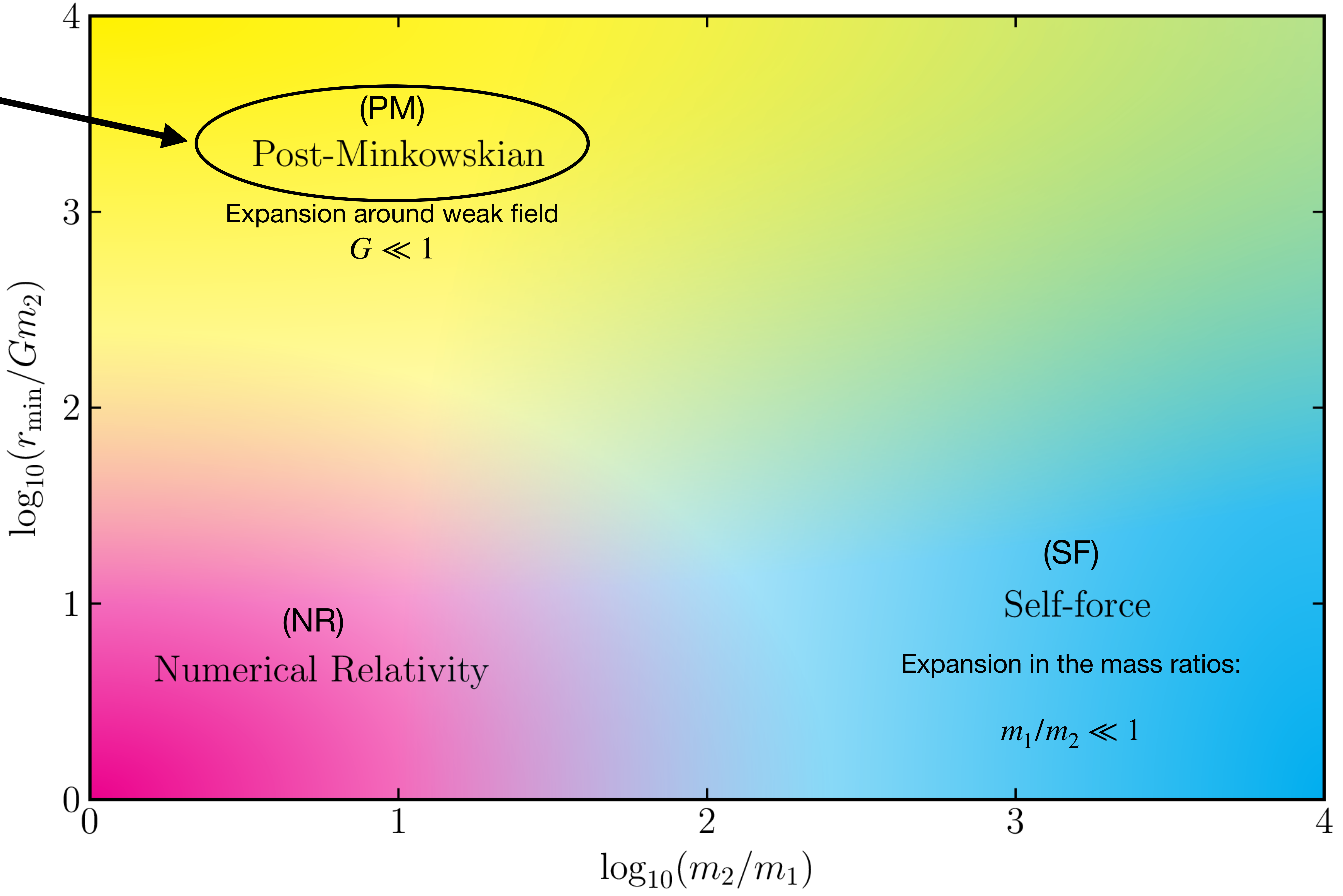
Analytical approaches:

- Enormous program for accurately determining the binary dynamics and computing waveforms **at all orders in** $\frac{v}{c}$, in contrast with Post-Newtonian (PN) approaches.

- Inspiration from **techniques** used in **Scattering Amplitudes and Effective Field Theory**

[Alaverdian, Aoude, Bautista, Ben-Shahar, Bern, Bini, Brandhuber, Brown, Buonanno, Cachazo, Cangemi, Chiodaroli, Chen, Cordero, Cristofoli, de la Cruz, Damour, Damgaard, De Angelis, Driesse, Elkhidir, Gatica, Georgoudis, Goldberger, Gowdy, Gonzo, Guevara, Haddad, Heissenberg, Helset, Herrmann, Holstein, Huang, Huang, Jakobsen, Johansson, Kim, Kraus, Kosmopoulos, Kosower, Lee, Levi, Lin, Liu, Luna, Matasan, Maybee, Menezes, Mogull, Mougiakakos, Moynihan, Novichkov, O'Connell, Ochirov, Parra-Martinez, Pichini, Plefka, Porto, Riva, Roiban, Ross, Rothstein, Ruf, Russo, Saketh, Sauer, Scheopner, Sergola, Shen, Siemonsen, Smirnov, Smirnov, Steinhoff, Teng, Travaglini, Vanhove, Vazquez-Holm, di Vecchia, Veneziano, Vernizzi, Vines, Wong, Xu, Yang, Yin, Zeng, et al…]

Phys.Rev.D 108 (2023) 2, 024025 [Barack, Bern, Herrmann, Long, Parra-Martinez, Roiban, Ruf, Shen, Solon, Teng, Zeng]



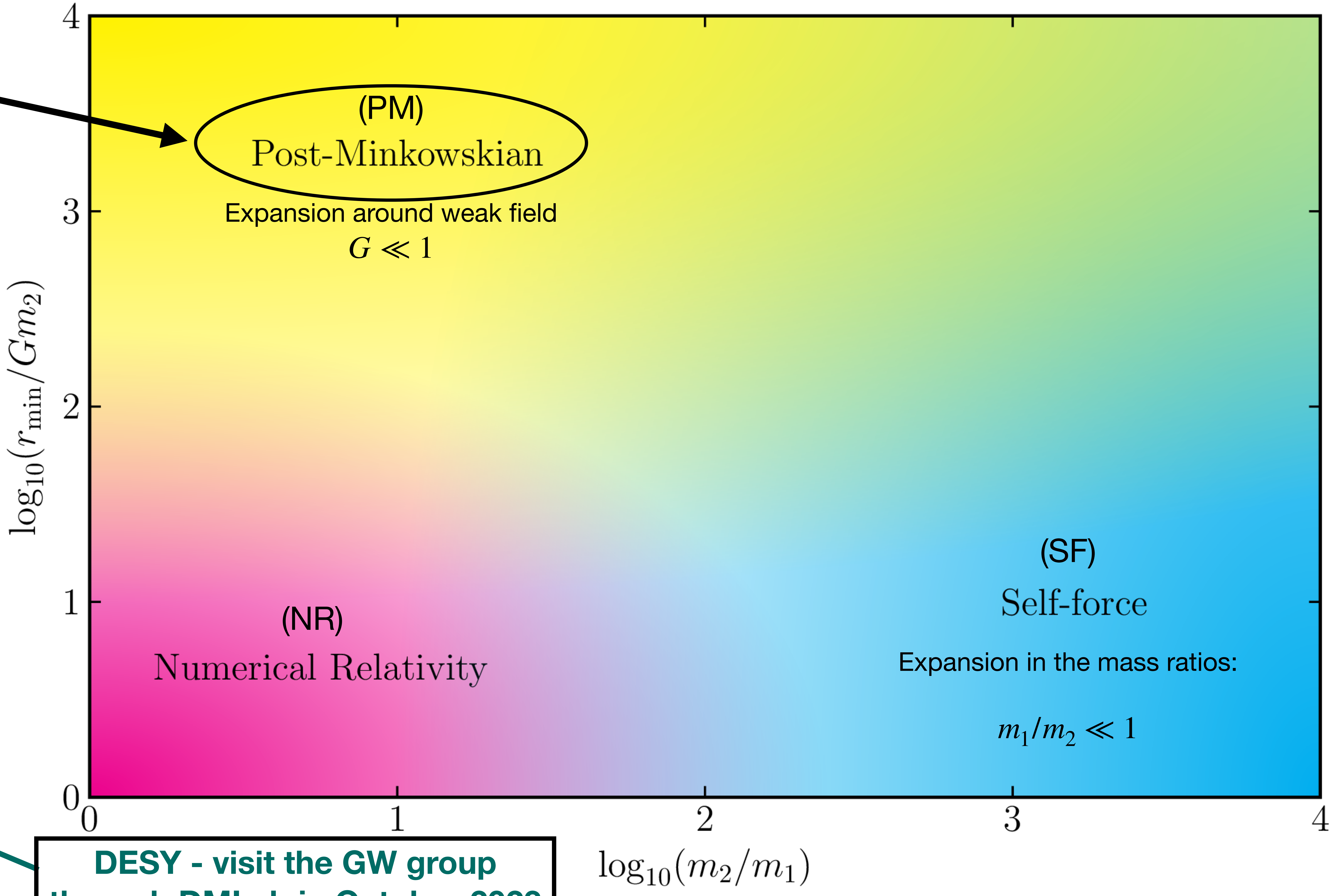
Analytical approaches:

- Enormous program for accurately determining the binary dynamics and computing waveforms **at all orders in** $\frac{v}{c}$, in contrast with Post-Newtonian (PN) approaches.

- Inspiration from **techniques** used in **Scattering Amplitudes and Effective Field Theory**

[Alaverdian, Aoude, Bautista, Ben-Shahar, Bern, Bini, Brandhuber, Brown, Buonanno, Cachazo, Cangemi, Chiodaroli, Chen, Cordero, Cristofoli, de la Cruz, Damour, Damgaard, De Angelis, Driesse, Elkhidir, Gatica, Georgoudis, Goldberger, Gowdy, Gonzo, Guevara, Haddad, Heissenberg, Helset, Herrmann, Holstein, Huang, Huang, Jakobsen, Johansson, Kim, Kraus, Kosmopoulos, Kosower, Lee, Levi, Lin, Liu, Luna, Matasan, Maybee, Menezes, Mogull, Mougiakakos, Moynihan, Novichkov, O'Connell, Ochirov, Parra-Martinez, Pichini, Plefka, Porto, Riva, Roiban, Ross, Rothstein, Ruf, Russo, Saketh, Sauer, Scheopner, Sergola, Shen, Siemonsen, Smirnov, Smirnov, Steinhoff, Teng, Travaglini, Vanhove, Vazquez-Holm, di Vecchia, Veneziano, Vernizzi, Vines, Wong, Xu, Yang, Yin, Zeng, et al…]

Phys.Rev.D 108 (2023) 2, 024025 [Barack, Bern, Herrmann, Long, Parra-Martinez, Roiban, Ruf, Shen, Solon, Teng, Zeng]

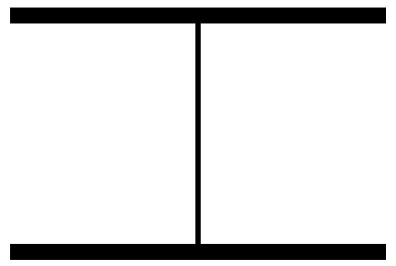


DESY - visit the GW group through DMLab in October 2023

Results obtained in standard GR:

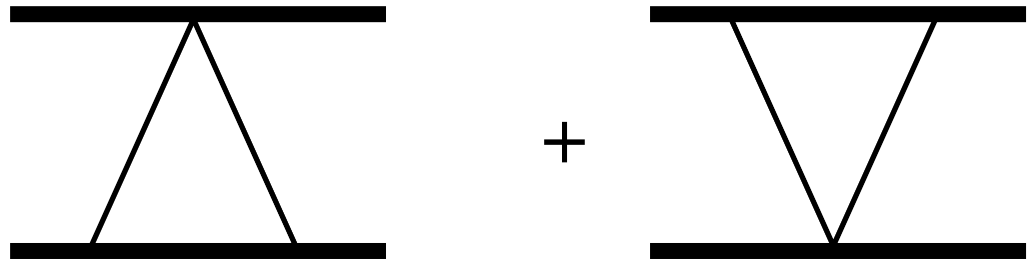
JHEP 10 (2021) 148 [Herrmann, Parra-Martinez, Ruf, Zeng]

tree:



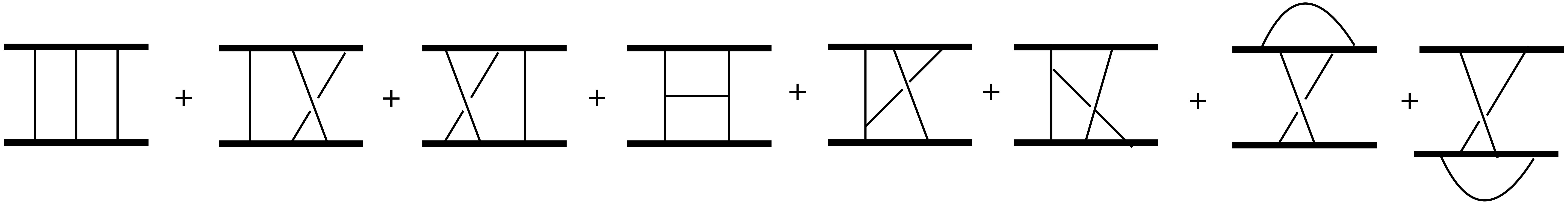
$$\Delta p_{1,\mathrm{GR}}^{\mu,(0)} = \frac{GM^2\nu}{|b|}\frac{2(2\sigma^2-1)}{\sqrt{\sigma^2-1}}\frac{b^\mu}{|b|}\,.$$

1-loop:



$$\Delta p_{1,\perp}^{\mu,(1)} = \frac{G^2M^3\nu}{|b|^2}\frac{3\pi}{4}\frac{(5\sigma^2-1)}{\sqrt{\sigma^2-1}}\frac{b^\mu}{|b|}$$

2-loop:



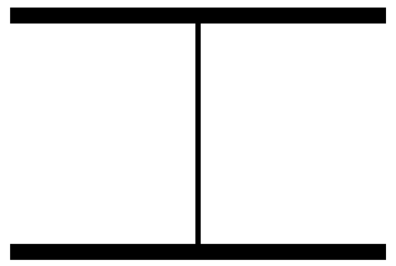
$$\Delta p_{1,\perp,\mathrm{cons}}^{\mu,(2)} = \frac{G^3M^4\nu}{|b|^3}\frac{2}{\sqrt{\sigma^2-1}}\frac{b^\mu}{|b|}\Bigg[h^2(\sigma,\nu)\left(16\sigma^2-\frac{1}{(\sigma^2-1)^2}\right) - \frac{4}{3}\nu\,\sigma\left(14\sigma^2+25\right) - 8\nu\left(4\sigma^4-12\sigma^2-3\right)\frac{\mathrm{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}}\Bigg]$$

$$\Delta p_{1,\mathrm{rad}}^{\mu,(2)} = \frac{G^3M^4\nu^2}{|b|^3}\Bigg\{\frac{4}{\sqrt{\sigma^2-1}}\frac{b^\mu}{|b|}\Bigg[f_1^{\mathrm{LS}}(\sigma)+f_3^{\mathrm{LS}}(\sigma)\frac{\sigma\,\mathrm{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}}\Bigg] + \pi\,\check{u}_2^\mu\Bigg[f_1(\sigma)+f_2(\sigma)\log\left(\frac{\sigma+1}{2}\right)+f_3(\sigma)\frac{\sigma\,\mathrm{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}}\Bigg]\Bigg\}$$

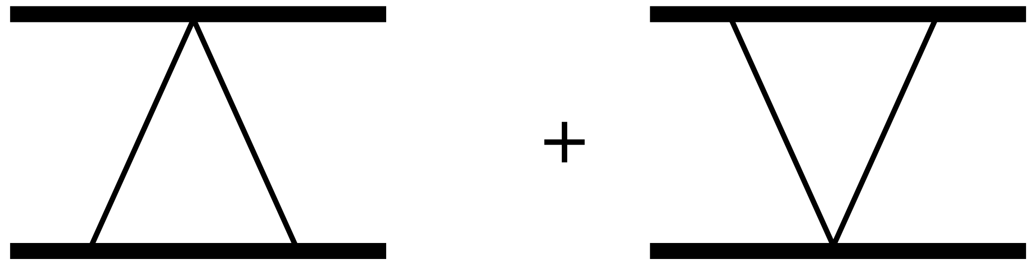
…

Results obtained in standard GR:

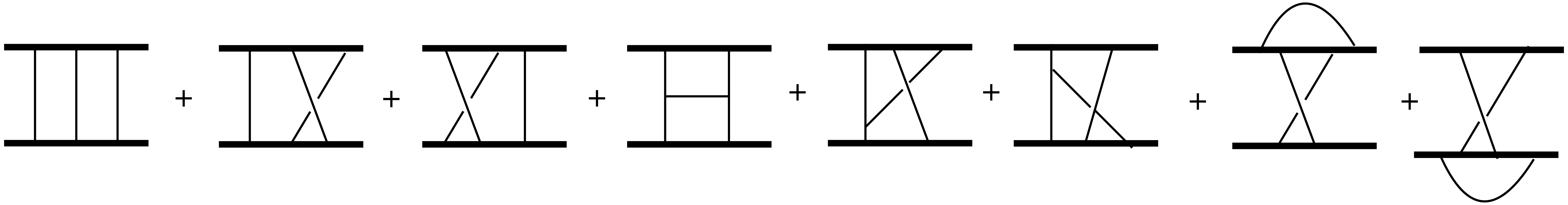
JHEP 10 (2021) 148 [Herrmann, Parra-Martinez, Ruf, Zeng]

tree: 

$$\Delta p^{\mu,(0)}_{1,\mathrm{GR}} = \frac{GM^2\nu}{|b|}\,\frac{2(2\sigma^2-1)}{\sqrt{\sigma^2-1}}\,\frac{b^\mu}{|b|}\,.$$

1-loop: 

$$\Delta p^{\mu,(1)}_{1,\perp} = \frac{G^2M^3\nu}{|b|^2}\,\frac{3\pi}{4}\,\frac{(5\sigma^2-1)}{\sqrt{\sigma^2-1}}\,\frac{b^\mu}{|b|}$$

2-loop: 

$$\Delta p^{\mu,(2)}_{1,\perp,\mathrm{cons}} = \frac{G^3M^4\nu}{|b|^3}\,\frac{2}{\sqrt{\sigma^2-1}}\,\frac{b^\mu}{|b|}\left[h^2(\sigma,\nu)\left(16\sigma^2-\frac{1}{(\sigma^2-1)^2}\right)\right.$$
$$\left. -\frac{4}{3}\nu\,\sigma\,(14\sigma^2+25) - 8\nu\,(4\sigma^4-12\sigma^2-3)\,\frac{\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}}\right]$$

$$\Delta p^{\mu,(2)}_{1,\mathrm{rad}} = \frac{G^3M^4\nu^2}{|b|^3}\left\{\frac{4}{\sqrt{\sigma^2-1}}\,\frac{b^\mu}{|b|}\left[f_1^{\mathrm{LS}}(\sigma)+f_3^{\mathrm{LS}}(\sigma)\,\frac{\sigma\,\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}}\right]\right.$$
$$\left. +\,\pi\,\check{u}_2^\mu\left[f_1(\sigma)+f_2(\sigma)\log\left(\frac{\sigma+1}{2}\right)+f_3(\sigma)\,\frac{\sigma\,\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}}\right]\right\}$$

… → **State of the art is pushing towards 4-loops (5PM) binary dynamics**

Phys.Rev.Lett. 132 (2024) 24, 241402 [Driesse, Jakobsen, Mogull, Plefka, Sauer, Usovitsch]
arxiv: 2406.01554 [Bern, Herrmann, Roiban, Ruf, Smirnov, Smirnov, Zeng]

Results obtained in standard GR:

LO order spinning waveform obtained from different approaches (consensus up to $\mathcal{O}(a^4)$):

Phys.Rev.D 110 (2024) 4, L041502 [De Angelis, Novichkov, Gonzo]
Phys.Rev.D 109 (2024) 3, 036007 [Aoude, Haddad, Heissenberg, Helset]
JHEP 02 (2024) 026 [Brandhuber, Brown, Chen, Gowdy, Travaglini]

e.g.: LO spinless waveform:

$$h_f(x)|_{\mathfrak{a}_i=0} = \sum_{i=1}^{2} \frac{\tilde{r}^{-,\mu\nu}_{(i),0} + \tilde{r}^{+,\mu\nu}_{(i),0}}{(p_i\cdot\rho)^2}\mathcal{I}_{(i),\mu\nu}(b_0)$$

$$\mathcal{I}^{\mu\nu}_{(1)}(b) = \frac{K^{\mu\nu}_{(1)}(v_1\cdot K_{(1)}\cdot\rho) - 2(v_1\cdot K_{(1)})^{(\mu}(\rho\cdot K_{(1)})^{\nu)}}{4\pi(\gamma^2-1)(\rho\cdot v_2)^2|b|^2|\boldsymbol{b}|_{(1)}|b|^2_{2\mathrm{d}}}$$

$$r^{+,\mu\nu}_{(1),0} = \langle k|p_1p_2\gamma^\mu p_1|k\rangle\langle k|p_1p_2\gamma^\nu p_1|k\rangle$$
$$r^{+,\mu\nu}_{(1),1} = \langle k|p_1p_2\gamma^\mu p_1|k\rangle\langle k|p_1p_2\gamma^\nu \mathfrak{a}_1|k\rangle$$
$$r^{+,\mu\nu}_{(1),2} = \langle k|p_1p_2\gamma^\mu \mathfrak{a}_1|k\rangle\langle k|p_1p_2\gamma^\nu \mathfrak{a}_1|k\rangle$$
$$r^{+,\mu\nu}_{(1),3} = \langle k|p_1p_2\gamma^\mu \mathfrak{a}_1|k\rangle\langle k|\mathfrak{a}_1p_2\gamma^\nu \mathfrak{a}_1|k\rangle$$
$$r^{+,\mu\nu}_{(1),4} = \langle k|\mathfrak{a}_1p_2\gamma^\mu \mathfrak{a}_1|k\rangle\langle k|\mathfrak{a}_1p_2\gamma^\nu \mathfrak{a}_1|k\rangle$$

$$r^{-,\mu\nu}_{(1),0} = m_1^4\langle k|p_2\gamma^\mu|k\rangle\langle k|p_2\gamma^\nu|k\rangle,$$
$$r^{-,\mu\nu}_{(1),1} = m_1^2\langle k|p_2\gamma^\mu|k\rangle\langle k|\mathfrak{a}_1p_1p_2\gamma^\nu|k\rangle$$
$$r^{-,\mu\nu}_{(1),2} = \langle k|\mathfrak{a}_1p_1p_2\gamma^\mu|k\rangle\langle k|\mathfrak{a}_1p_1p_2\gamma^\nu|k\rangle,$$
$$r^{-,\mu\nu}_{(1),3} = \frac{1}{m_1^2}\langle k|\mathfrak{a}_1p_1p_2\gamma^\mu|k\rangle\langle k|\mathfrak{a}_1p_1p_2\gamma^\nu p_1\mathfrak{a}_1|k\rangle$$
$$r^{-,\mu\nu}_{(1),4} = \frac{1}{m_1^4}\langle k|\mathfrak{a}_1p_1p_2\gamma^\mu p_1\mathfrak{a}_1|k\rangle\langle k|\mathfrak{a}_1p_1p_2\gamma^\nu p_1\mathfrak{a}_1|k\rangle$$

Results obtained in standard GR:

LO order spinning waveform obtained from different approaches (consensus up to $\mathcal{O}(a^4)$):

Phys.Rev.D 110 (2024) 4, L041502 [De Angelis, Novichkov, Gonzo]
Phys.Rev.D 109 (2024) 3, 036007 [Aoude, Haddad, Heissenberg, Helset]
JHEP 02 (2024) 026 [Brandhuber, Brown, Chen, Gowdy, Travaglini]

e.g.: LO spinless waveform:

$$h_f(x)|_{\mathfrak{a}_i=0} = \sum_{i=1}^{2} \frac{\tilde{r}^{-,\mu\nu}_{(i),0} + \tilde{r}^{+,\mu\nu}_{(i),0}}{(p_i\cdot\rho)^2}\mathcal{I}_{(i),\mu\nu}(b_0)$$

$$\mathcal{I}^{\mu\nu}_{(1)}(b) = \frac{K^{\mu\nu}_{(1)}(v_1\cdot K_{(1)}\cdot\rho) - 2(v_1\cdot K_{(1)})^{(\mu}(\rho\cdot K_{(1)})^{\nu)}}{4\pi(\gamma^2-1)(\rho\cdot v_2)^2|b|^2|\boldsymbol{b}|_{(1)}|b|^2_{2\mathrm{d}}}$$

$$r^{+,\mu\nu}_{(1),0} = \langle k|p_1p_2\gamma^\mu p_1|k\rangle\langle k|p_1p_2\gamma^\nu p_1|k\rangle$$
$$r^{+,\mu\nu}_{(1),1} = \langle k|p_1p_2\gamma^\mu p_1|k\rangle\langle k|p_1p_2\gamma^\nu \mathfrak{a}_1|k\rangle$$
$$r^{+,\mu\nu}_{(1),2} = \langle k|p_1p_2\gamma^\mu \mathfrak{a}_1|k\rangle\langle k|p_1p_2\gamma^\nu \mathfrak{a}_1|k\rangle$$
$$r^{+,\mu\nu}_{(1),3} = \langle k|p_1p_2\gamma^\mu \mathfrak{a}_1|k\rangle\langle k|\mathfrak{a}_1p_2\gamma^\nu \mathfrak{a}_1|k\rangle$$
$$r^{+,\mu\nu}_{(1),4} = \langle k|\mathfrak{a}_1p_2\gamma^\mu \mathfrak{a}_1|k\rangle\langle k|\mathfrak{a}_1p_2\gamma^\nu \mathfrak{a}_1|k\rangle$$

$$r^{-,\mu\nu}_{(1),0} = m_1^4\langle k|p_2\gamma^\mu|k\rangle\langle k|p_2\gamma^\nu|k\rangle,$$
$$r^{-,\mu\nu}_{(1),1} = m_1^2\langle k|p_2\gamma^\mu|k\rangle\langle k|\mathfrak{a}_1p_1p_2\gamma^\nu|k\rangle$$
$$r^{-,\mu\nu}_{(1),2} = \langle k|\mathfrak{a}_1p_1p_2\gamma^\mu|k\rangle\langle k|\mathfrak{a}_1p_1p_2\gamma^\nu|k\rangle,$$
$$r^{-,\mu\nu}_{(1),3} = \frac{1}{m_1^2}\langle k|\mathfrak{a}_1p_1p_2\gamma^\mu|k\rangle\langle k|\mathfrak{a}_1p_1p_2\gamma^\nu p_1\mathfrak{a}_1|k\rangle$$
$$r^{-,\mu\nu}_{(1),4} = \frac{1}{m_1^4}\langle k|\mathfrak{a}_1p_1p_2\gamma^\mu p_1\mathfrak{a}_1|k\rangle\langle k|\mathfrak{a}_1p_1p_2\gamma^\nu p_1\mathfrak{a}_1|k\rangle$$

NLO order waveform also looks like a closed case:

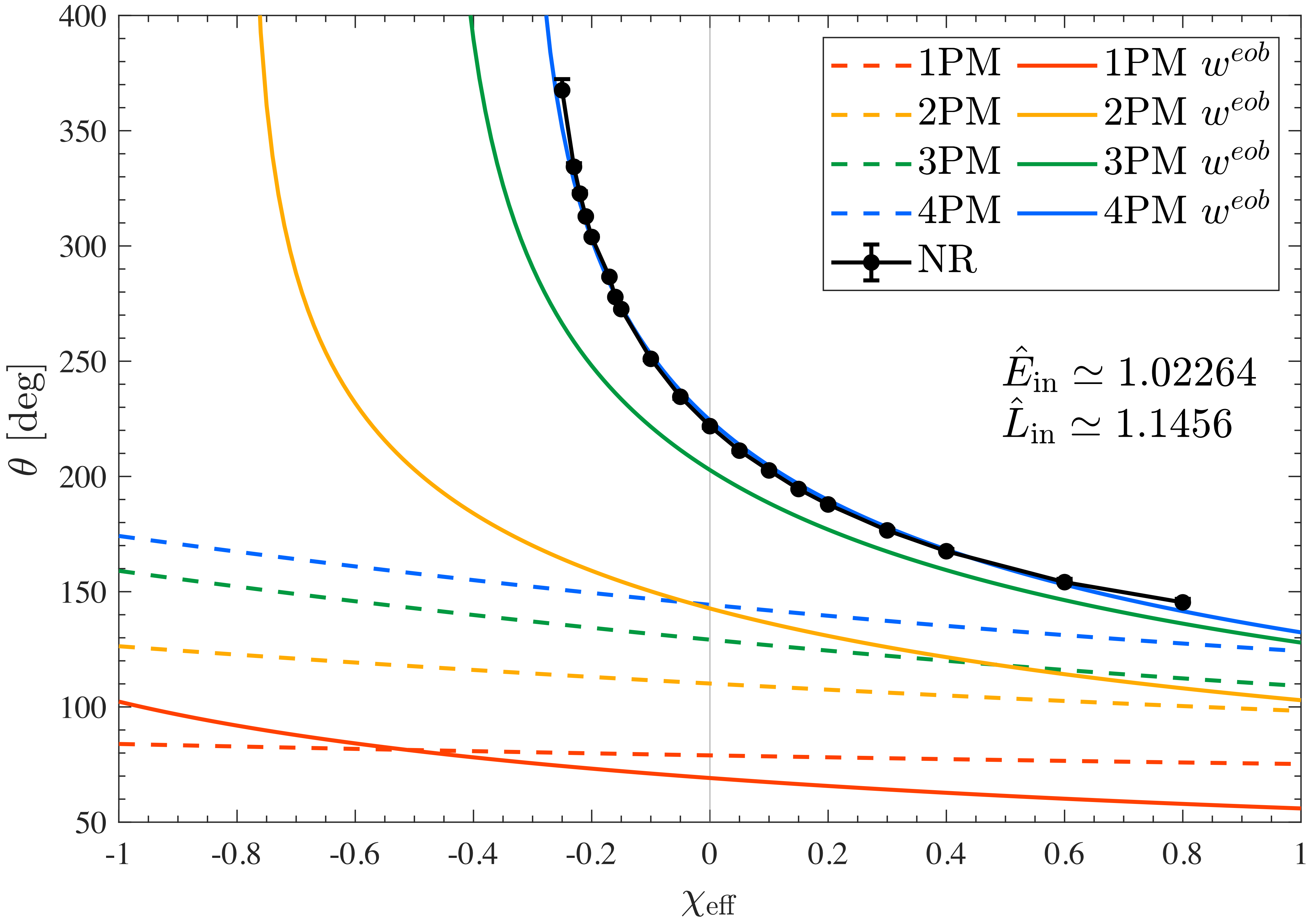
JHEP 06 (2023) 048 [Brandhuber, Brown, Chen, De Angelis, Gowdy, Travaglini]
JHEP 07 (2024) 272 [Elkhidir, O'Connell, Sergola, Vazquez-Holm]
JHEP 06 (2023) 004 [Herderschee, Roiban, Teng]
JHEP 2023 (2023) 06, 126 [Georgoudis, Heissenberg, Vazquez-Holm]
JHEP 01 (2024) 139 [Caron-Huot, Giroux, Hannesdottir, Mizera]

Recent result for NLO linear-in-spin effects:

arxiv: 2312.14859 [Bohnenblust, Ita, Kraus, Schlenk]

PM expansion to the test and bound orbits:

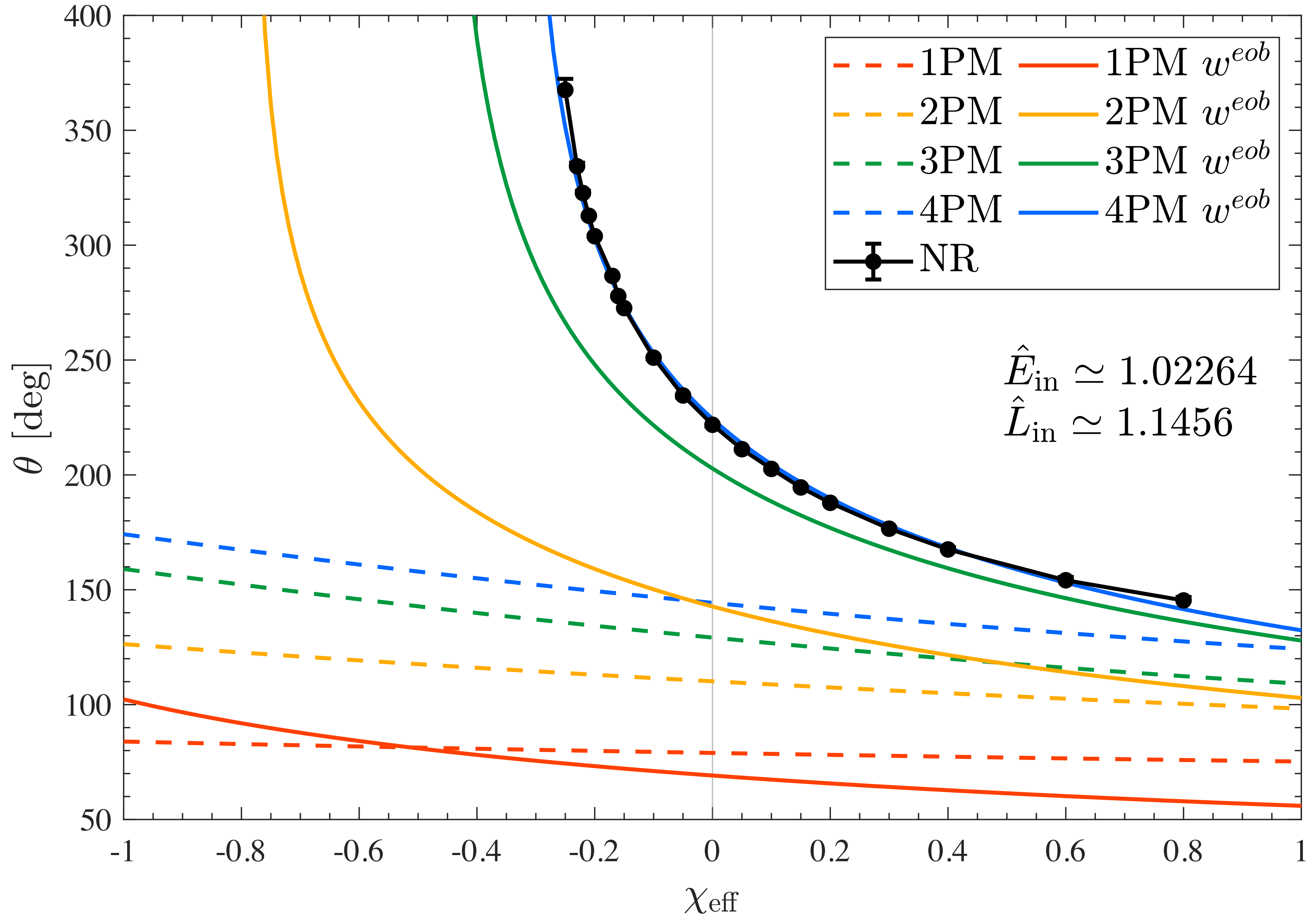
Phys.Rev.D 108 (2023) 12, 124016 [Rettegno, Pratten, Thomas, Schmidt, Damour]



Already PM is doing very well for black hole (BH) scattering!

PM expansion to the test and bound orbits:

Phys.Rev.D 108 (2023) 12, 124016 [Rettegno, Pratten, Thomas, Schmidt, Damour]



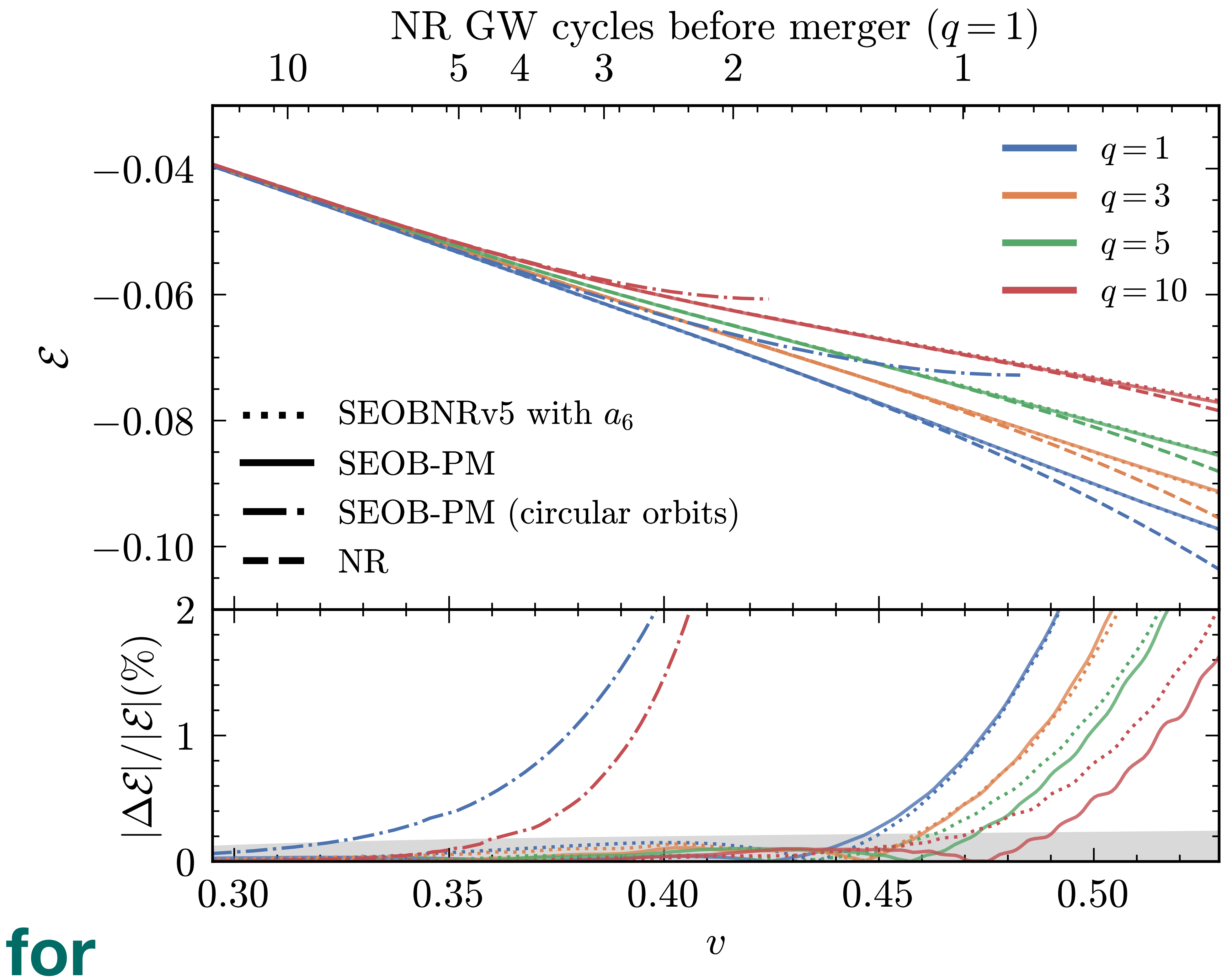
Already PM is doing very well for black hole (BH) scattering!

Bound to Boundary map for binary dynamics:

JHEP 01 (2020) 072 [Kälin, Porto]
JHEP 02 (2020) 120 [Kälin, Porto]
JHEP 04 (2022) 154, JHEP 07 (2022) 002 (erratum) [Cho, Kälin, Porto]

Recent work on a waveform map:

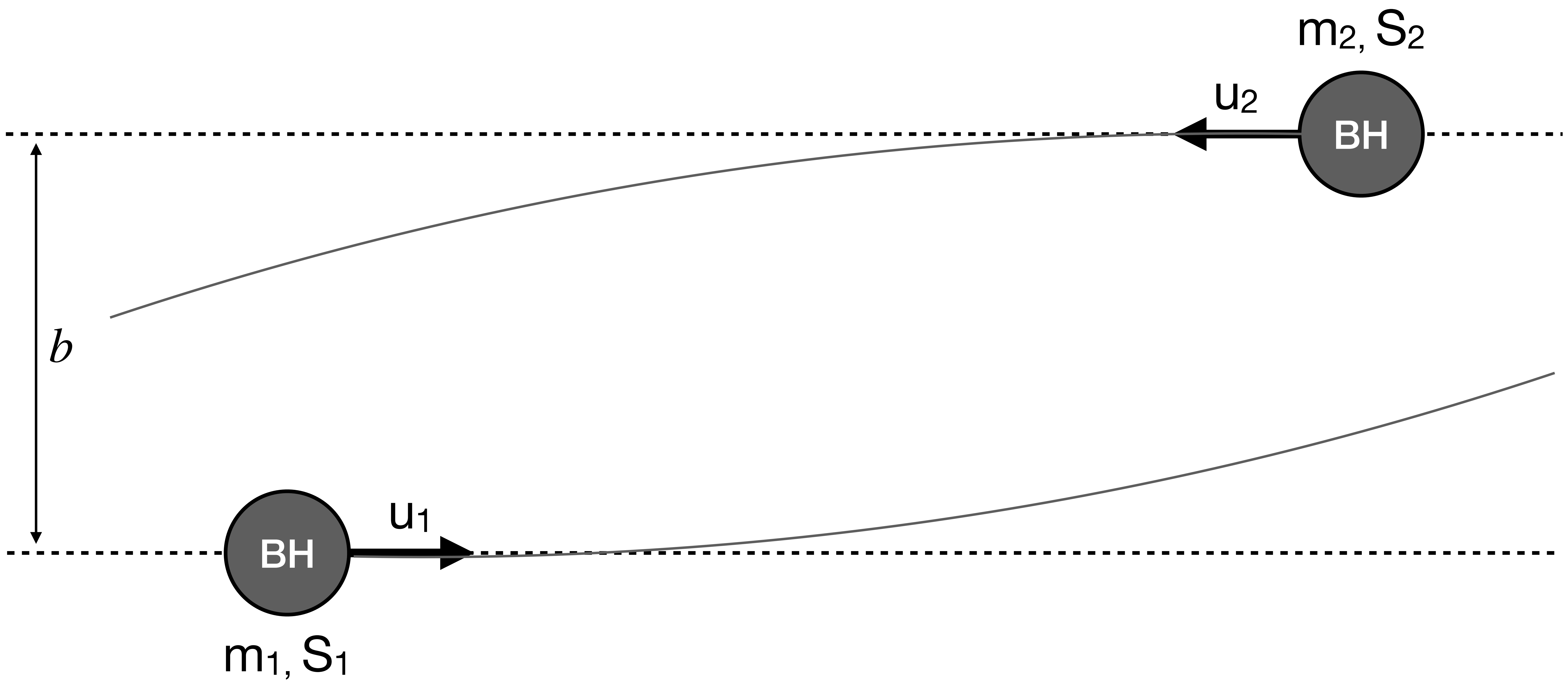
JHEP 05 (2024) 034 [Adamo, Gonzo, Ilderton]



Good agreement for the bound case as well!

arxiv: 2405.19181 [Buonanno, Mogull, Patil, Pompili]

2. Scattering Amplitudes and Observables



Weak field expansion:

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{2}{M_{Pl}} h_{\mu\nu}$$

PM expansion:

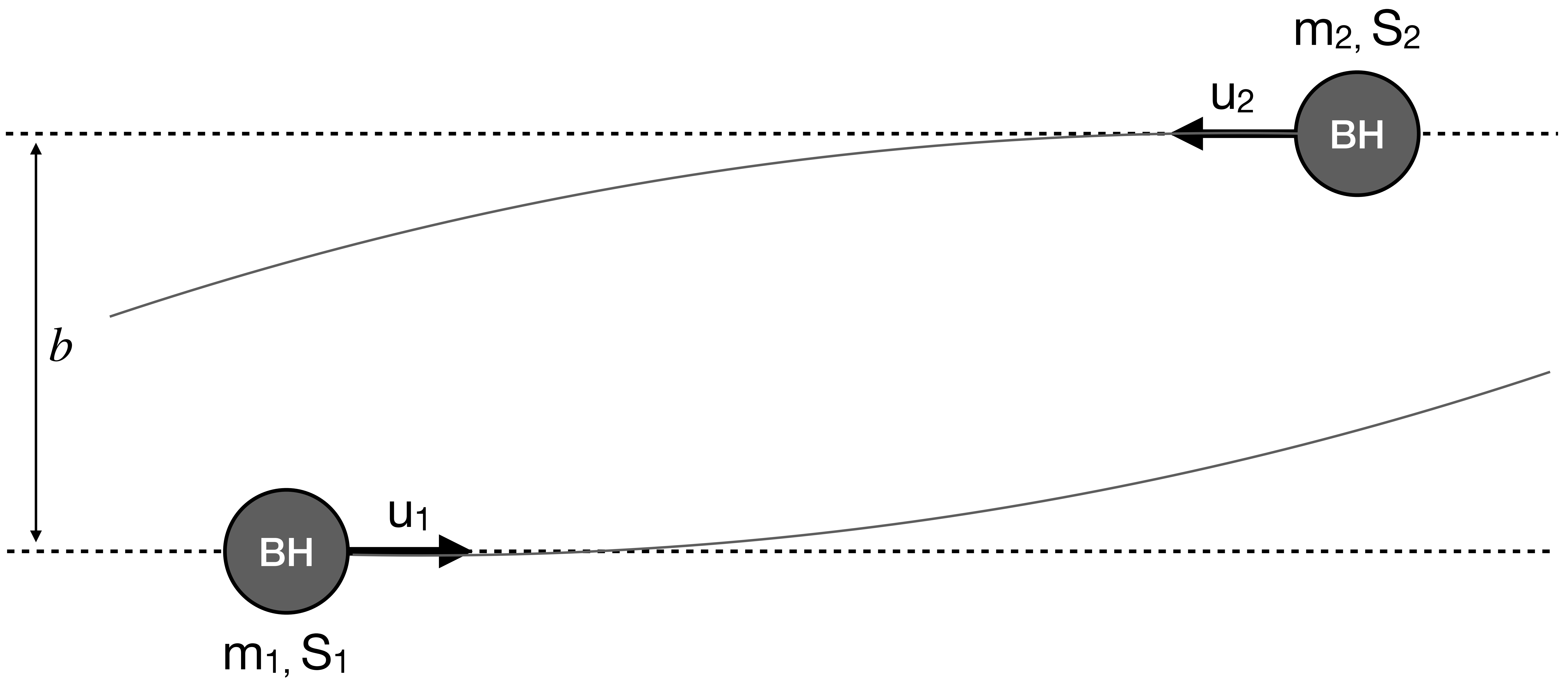
$$R_s \ll b$$

Spin expansion:

$$\frac{S}{m} \ll b$$

- **Focus**: **Classical scattering problem in GR**

2. Scattering Amplitudes and Observables



Weak field expansion:

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{2}{M_{Pl}} h_{\mu\nu}$$

PM expansion:

$$R_s \ll b$$

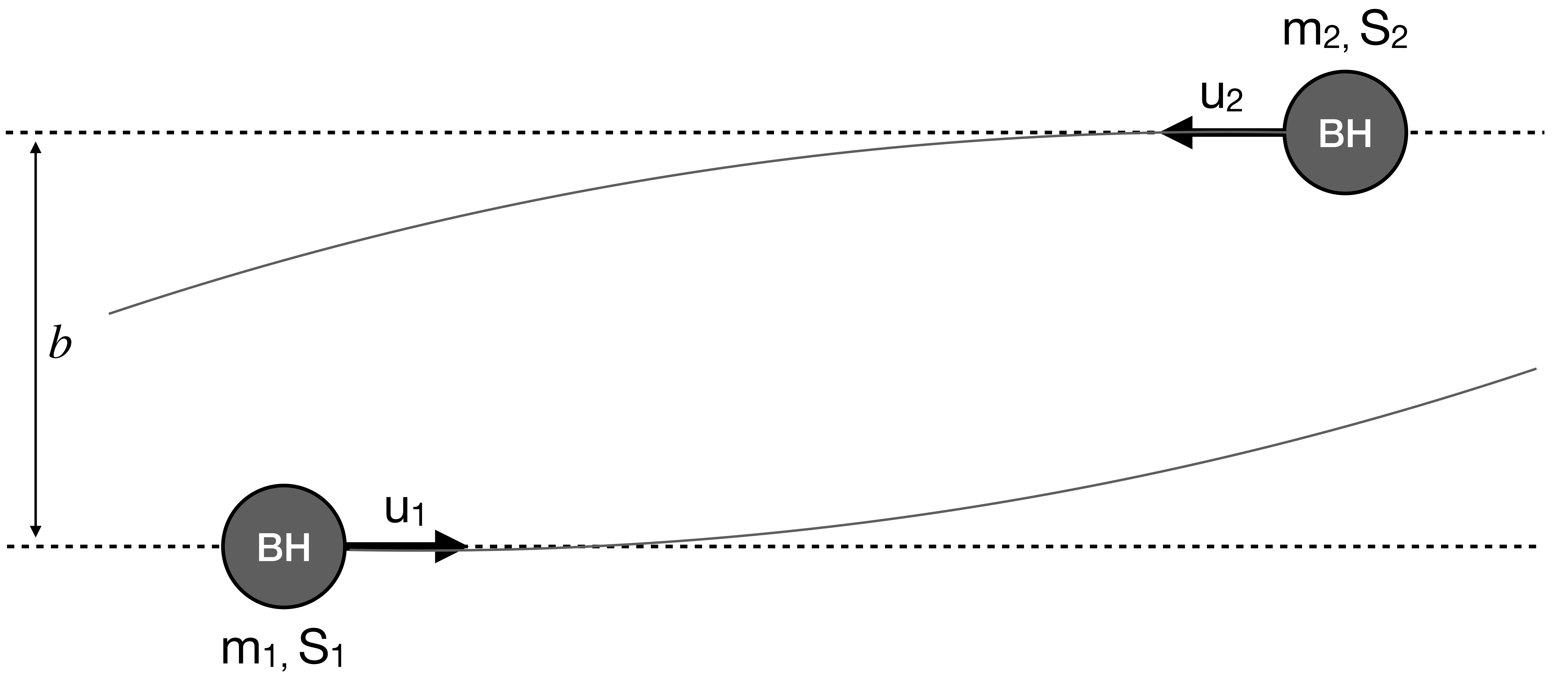
Spin expansion:

$$\frac{S}{m} \ll b$$

- **Focus**: **Classical scattering problem in GR**

⟶ Can we describe this problem using the Scattering Amplitudes used in QFT?

2. Scattering Amplitudes and Observables



Weak field expansion:

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{2}{M_{Pl}} h_{\mu\nu}$$

PM expansion:

$$R_s \ll b$$

Spin expansion:

$$\frac{S}{m} \ll b$$

- **Focus**: **Classical scattering problem in GR**

⟶ Can we describe this problem using the Scattering Amplitudes used in QFT?

Yes! ⟶ Use of the **KMOC formalism**

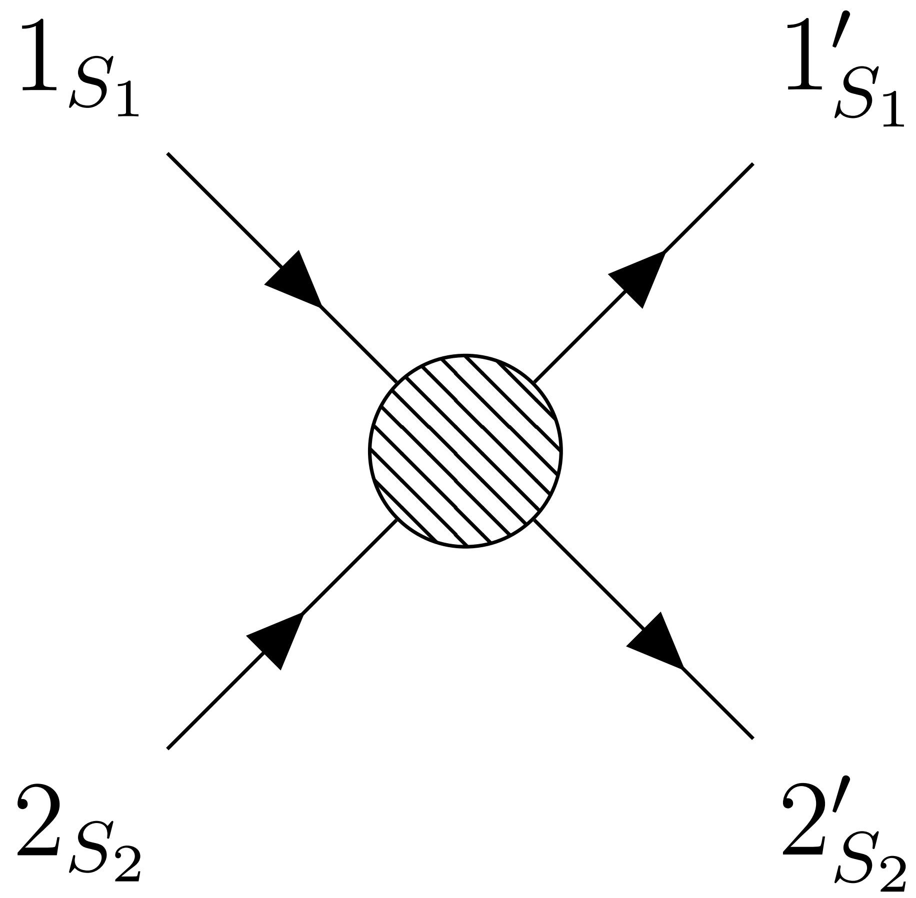
KMOC (Kosower Maybee O'Connell) formalism

JHEP 02 (2019) 137 [Kosower, Maybee, O'Connell]
Phys.Rev.D 106 (2022) 5, 0567007 [Cristofoli, Gonzo, Kosower, O'Connell]

<u>Idea</u>: Relate **Scattering Amplitudes** directly to **classical observables**

⟶ Extract the classical piece of the amplitude through an "$\hbar$" counting prescription

e.g.:



⟶

Classical Impulse:

$$\Delta p_1^\mu = p_{1,fin.}^\mu - p_{1,in.}^\mu$$

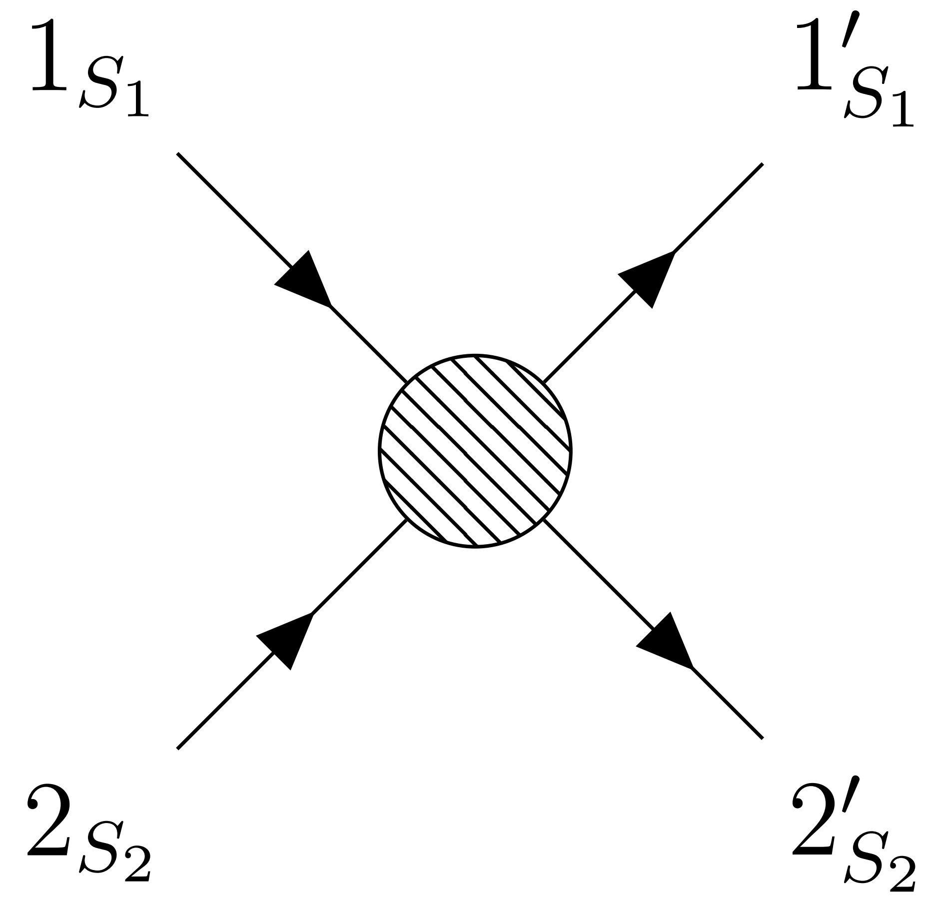
KMOC (Kosower Maybee O'Connell) formalism

JHEP 02 (2019) 137 [Kosower, Maybee, O'Connell]
Phys.Rev.D 106 (2022) 5, 0567007 [Cristofoli, Gonzo, Kosower, O'Connell]

Idea: Relate **Scattering Amplitudes** directly to **classical observables**

⟶ Extract the classical piece of the amplitude through an "$\hbar$" counting prescription

e.g.:



⟶

Classical Impulse:

$$\Delta p_1^\mu = p^\mu_{1,fin.} - p^\mu_{1,in.}$$

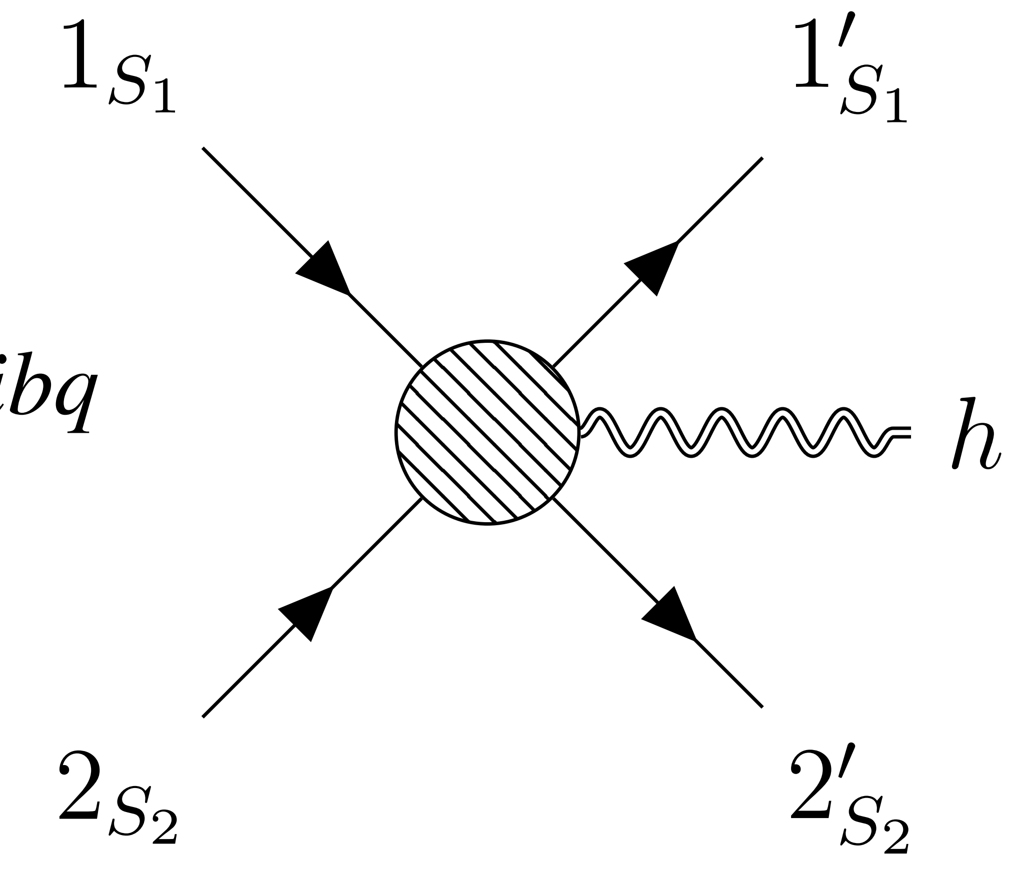
At leading order: $\Delta p_1^{\mu,LO} = \frac{i}{4}\left\langle\left\langle \hbar^2 \int \hat{d}^4 q \hat{\delta}(q\cdot p_1)\hat{\delta}(q\cdot p_2) e^{-ib\cdot q} q^\mu \mathcal{M}^{LO}(p_1, p_2 \to p_1 + \hbar q, p_2 - \hbar q)\right\rangle\right\rangle$

Waveforms at leading order:

strain

on-shell measure

$$h_{GR}(t) \equiv h_+ + ih_\times \sim \int d\omega e^{-i\omega t} \int d\Phi(q) e^{-ibq}$$



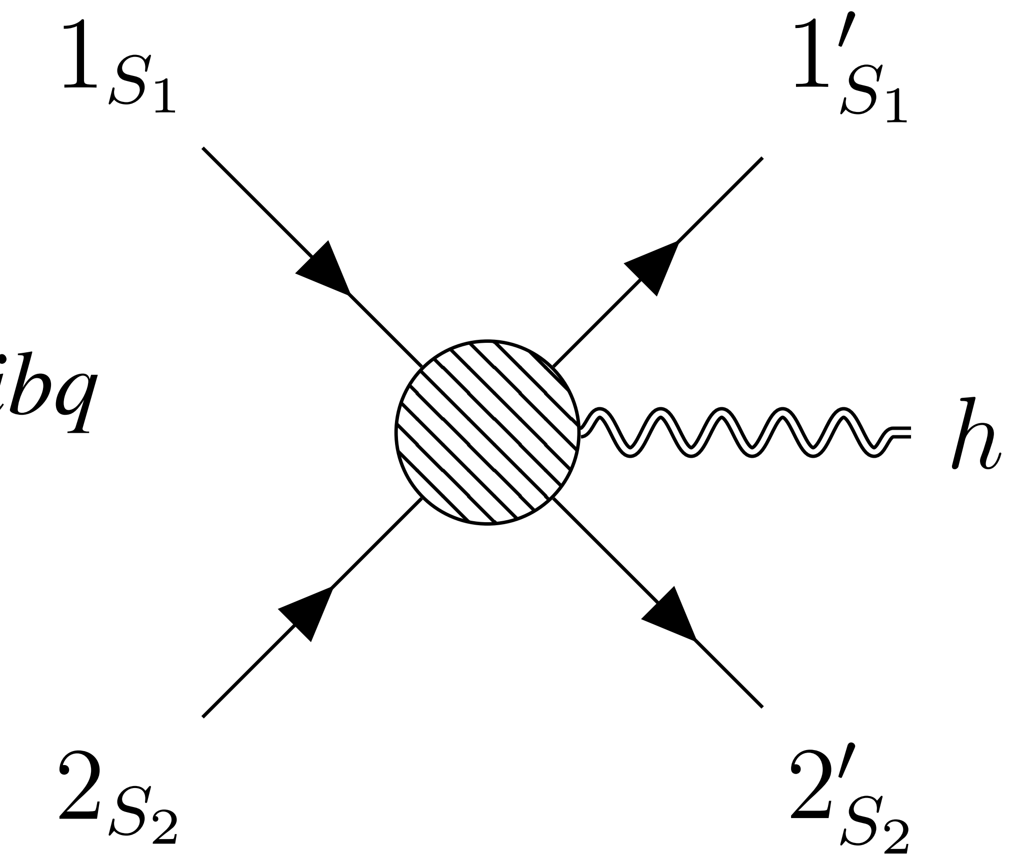
$$\mathcal{M}_5^{\mathrm{cl}}(q,k)\,|_{k^\mu=\omega n^\mu}$$

Waveforms at leading order:

strain

on-shell measure

$$h_{GR}(t) \equiv h_+ + ih_\times \sim \int d\omega e^{-i\omega t} \int d\Phi(q) e^{-ibq}$$



$$\mathcal{M}_5^{\text{cl}}(q,k)\,|_{k^\mu=\omega n^\mu}$$

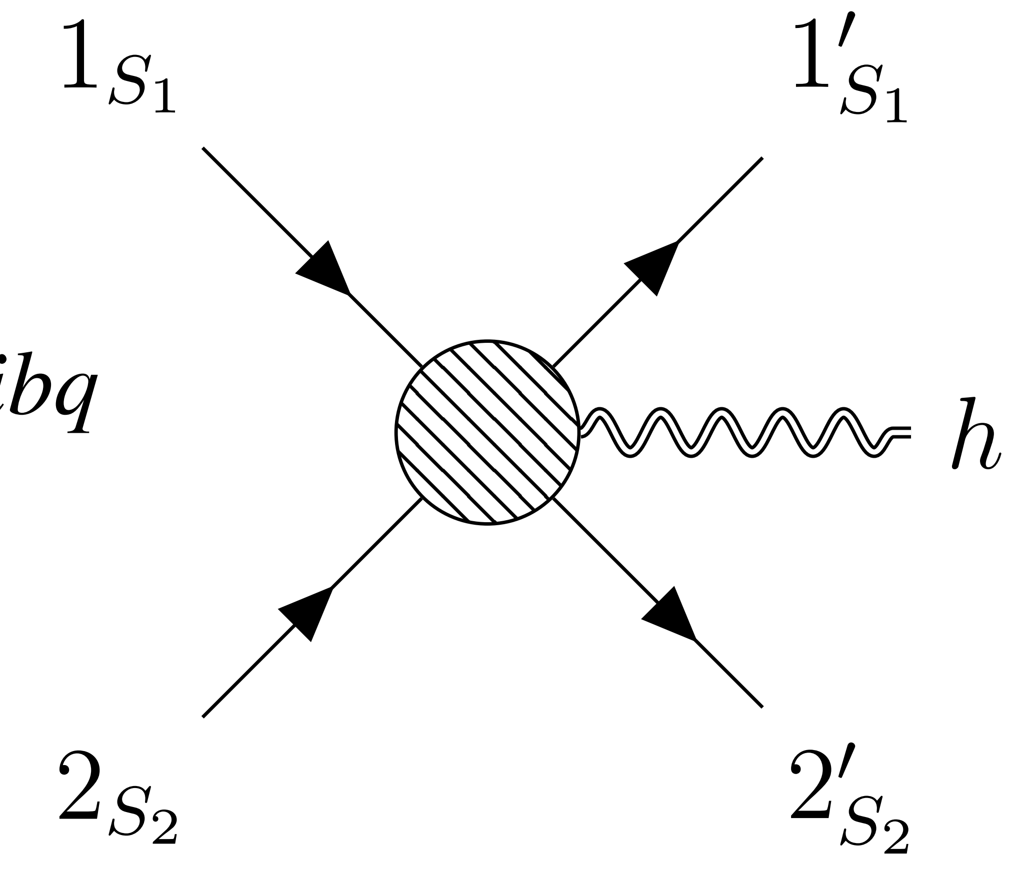
But, **why?**

Waveforms at leading order:

strain

on-shell measure

$$h_{GR}(t) \equiv h_+ + ih_\times \sim \int d\omega e^{-i\omega t} \int d\Phi(q) e^{-ibq}$$



$$\mathscr{M}_5^{\mathrm{cl}}(q,k)\,|_{k^\mu=\omega n^\mu}$$

But, **why?**

- Computations organized in a **perturbative expansion** using a **simple algorithm**.
- **Analytic results**, in places where either PN approximations or NR was used before.
- Can exploit many **modern techniques used in particle physics** to simplify the problem.
- Can straightforwardly **extend to beyond GR predictions**

3. Scalar-tensor theories: Examples, compact objects, scalar hair and scattering waveforms

⟶ **Scalar-tensor theories** have long stood as a promising avenue to study **extensions of GR**

⟶ They consist of gravity theories with the introduction of an additional **massless scalar** degree of freedom

$$S_{GR}[g_{\mu\nu}] \rightarrow S_{ST}[g_{\mu\nu}, \phi]$$

3. Scalar-tensor theories: Examples, compact objects, scalar hair and scattering waveforms

→ **Scalar-tensor theories** have long stood as a promising avenue to study **extensions of GR**

→ They consist of gravity theories with the introduction of an additional **massless scalar** degree of freedom

$$S_{GR}[g_{\mu\nu}] \rightarrow S_{ST}[g_{\mu\nu}, \phi]$$

Example: Scalar Gauss-Bonnet and Dynamical Chern Simons gravity

$$S = S_{EH}[R] + S_{SGB,DCS}[\phi, g_{\mu\nu}] + S_m[\Psi_m, \mathscr{A}(\phi)g_{\mu\nu}] \quad ,$$

- $S_{EH} = \int d^4x \frac{M_{Pl}^2}{2}\sqrt{-g}R$
- $S_{SGB,DCS} = \int d^4x\sqrt{-g}\left[M_{Pl}\left(\frac{\alpha}{\Lambda^2}f(\phi)\mathscr{G} + \frac{\tilde{\alpha}}{\Lambda^2}\phi R\tilde{R}\right) + \frac{1}{2}(\partial^\mu\phi\partial_\mu\phi)\right]$

3. Scalar-tensor theories: Examples, compact objects, scalar hair and scattering waveforms

→ **Scalar-tensor theories** have long stood as a promising avenue to study **extensions of GR**

→ They consist of gravity theories with the introduction of an additional **massless scalar** degree of freedom

$$S_{GR}[g_{\mu\nu}] \to S_{ST}[g_{\mu\nu}, \phi]$$

Example: Scalar Gauss-Bonnet and Dynamical Chern Simons gravity

$$S = S_{EH}[R] + S_{SGB,DCS}[\phi, g_{\mu\nu}] + S_m[\Psi_m, \mathscr{A}(\phi) g_{\mu\nu}] \quad ,$$

$$R^{\mu\nu\rho\sigma}\tilde{R}_{\mu\nu\rho\sigma} \quad , \quad \tilde{R}^{\mu}{}_{\nu\rho\sigma} = \frac{1}{2}\varepsilon_{\rho\sigma}{}^{\alpha\beta}R^{\mu}{}_{\nu\alpha\beta}$$

- $S_{EH} = \int d^4x \frac{M_{Pl}^2}{2}\sqrt{-g}R$

$$R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 4R^{\mu\nu}R_{\mu\nu} + R^2$$

- $S_{SGB,DCS} = \int d^4x \sqrt{-g}\left[M_{Pl}\left(\frac{\alpha}{\Lambda^2}f(\phi)\mathscr{G} + \frac{\tilde{\alpha}}{\Lambda^2}\phi R\tilde{R}\right) + \frac{1}{2}\left(\partial^\mu\phi\partial_\mu\phi\right)\right]$

3. Scalar-tensor theories: Examples, compact objects, scalar hair and scattering waveforms

→ **Scalar-tensor theories** have long stood as a promising avenue to study **extensions of GR**

→ They consist of gravity theories with the introduction of an additional **massless scalar** degree of freedom

$$S_{GR}[g_{\mu\nu}] \rightarrow S_{ST}[g_{\mu\nu}, \phi]$$

Phys.Rev.D 107 (2023) 4, 044030 [Silva, Ghosh, Buonanno]
arXiv:2406.13654 [Julié, Pompili, Buonanno]
Phys.Rev.Lett. 126 (2021) 18, 181101 [Silva, Holgado, Cárdenas-Avendaño, Yunes]

Example: Scalar Gauss-Bonnet and Dynamical Chern Simons gravity

$$S = S_{EH}[R] + S_{SGB,DCS}[\phi, g_{\mu\nu}] + S_m[\Psi_m, \mathcal{A}(\phi) g_{\mu\nu}] \quad ,$$

- $S_{EH} = \int d^4x \frac{M_{Pl}^2}{2} \sqrt{-g} R$
- $S_{SGB,DCS} = \int d^4x \sqrt{-g} \left[M_{Pl} \left(\frac{\alpha}{\Lambda^2} f(\phi) \mathcal{G} + \frac{\tilde{\alpha}}{\Lambda^2} \phi R\tilde{R} \right) + \frac{1}{2} (\partial^\mu \phi \partial_\mu \phi) \right]$

$\mathcal{G}$: $R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - 4R^{\mu\nu} R_{\mu\nu} + R^2$

$R\tilde{R}$: $R^{\mu\nu\rho\sigma} \tilde{R}_{\mu\nu\rho\sigma} \quad , \quad \tilde{R}^\mu{}_{\nu\rho\sigma} = \frac{1}{2} \varepsilon_{\rho\sigma}{}^{\alpha\beta} R^\mu{}_{\nu\alpha\beta}$

Already GW observations are used to constrain them!!!

Experimental window:

$$\frac{\sqrt{\alpha}}{\Lambda} \lesssim 0.22km \quad , \quad \frac{\sqrt{\tilde{\alpha}}}{\Lambda} \lesssim 9.5km$$

Scalar hair in scalar-tensor theories:

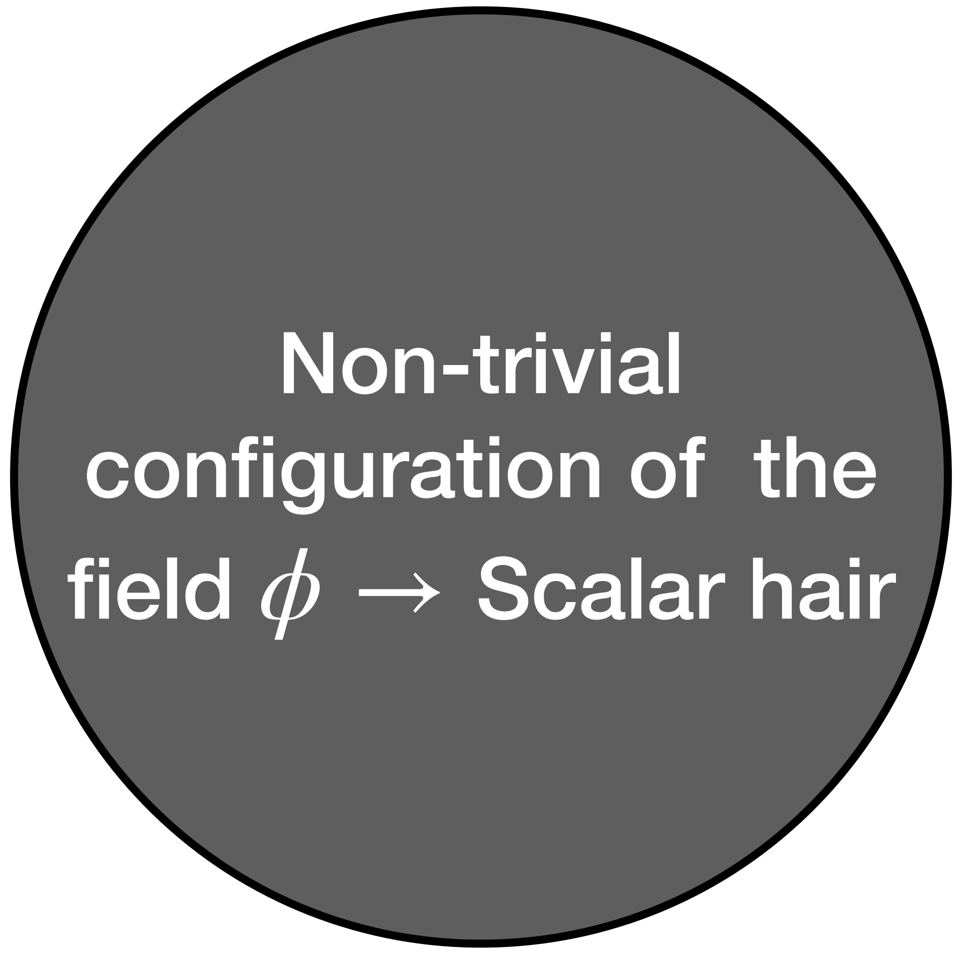
Compact objects can acquire scalar hair in ST theories ⟶ Exactly the case for SGB and DCS!

$$\phi = \frac{c_1}{r} + \frac{c_2}{r^2} + \dots$$

BH solution in ST theory

Scalar hair in scalar-tensor theories:

Compact objects can acquire scalar hair in ST theories → Exactly the case for SGB and DCS!



BH solution in ST theory

Far zone

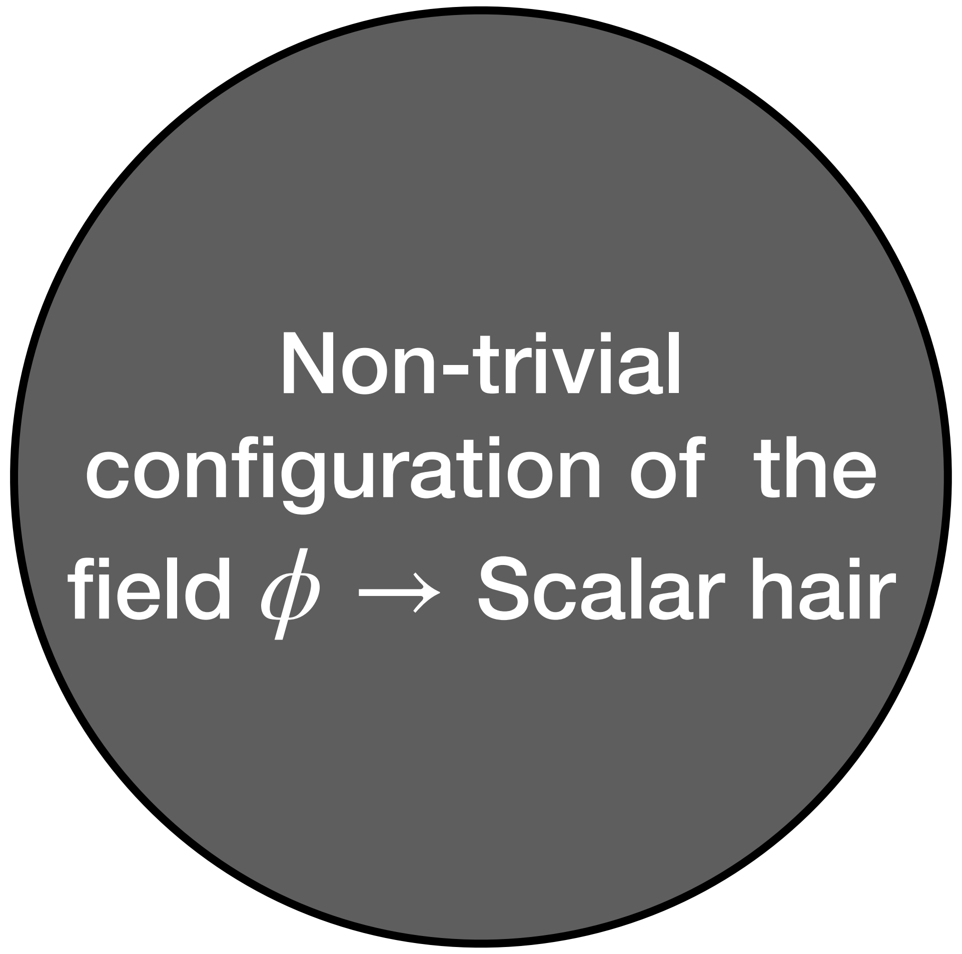
$x \to \infty$

"Monopole hair" "Dipole hair"

$$\phi = \frac{c_1}{r} + \frac{c_2}{r^2} + \dots$$

Scalar hair in scalar-tensor theories:

Compact objects can acquire scalar hair in ST theories ⟶ Exactly the case for SGB and DCS!



BH solution in ST theory

Far zone

$x \rightarrow \infty$

"Monopole hair" "Dipole hair"

$$\phi = \frac{c_1}{r} + \frac{c_2}{r^2} + \ldots$$

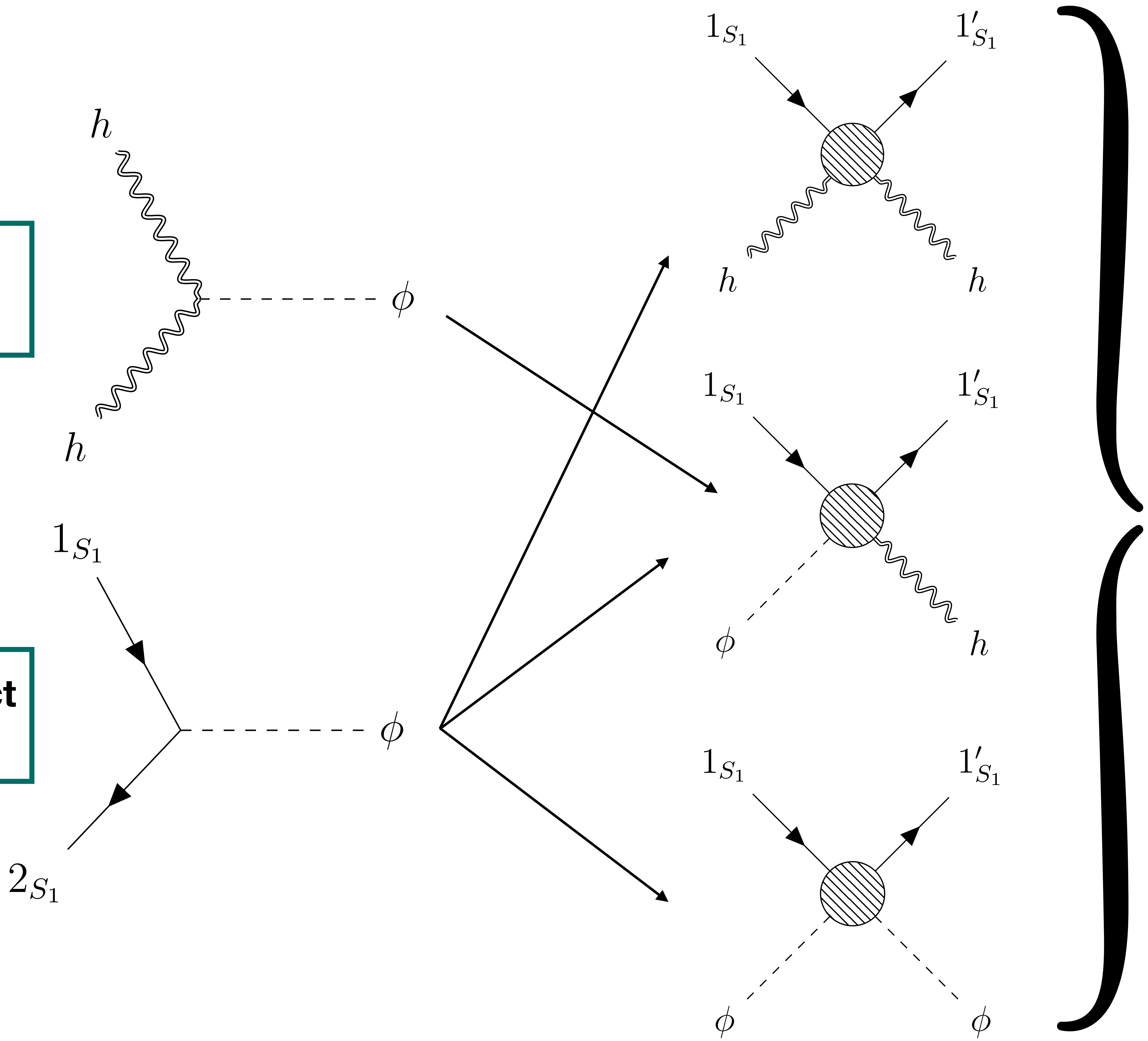
Have to also account for that effect as well!

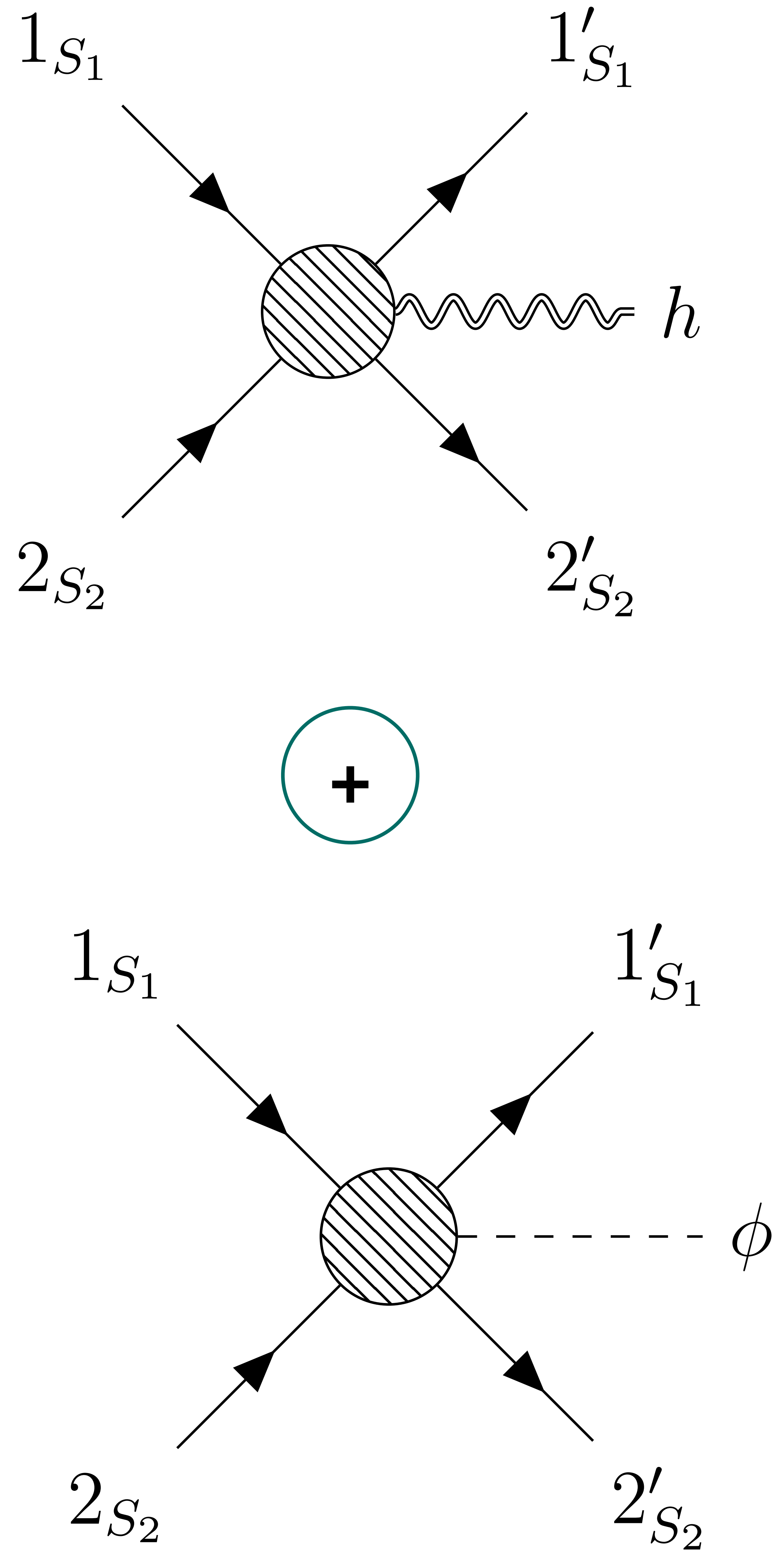
Corrections to Waveforms

→ **New interactions modify the waveforms:**

All compact objects:

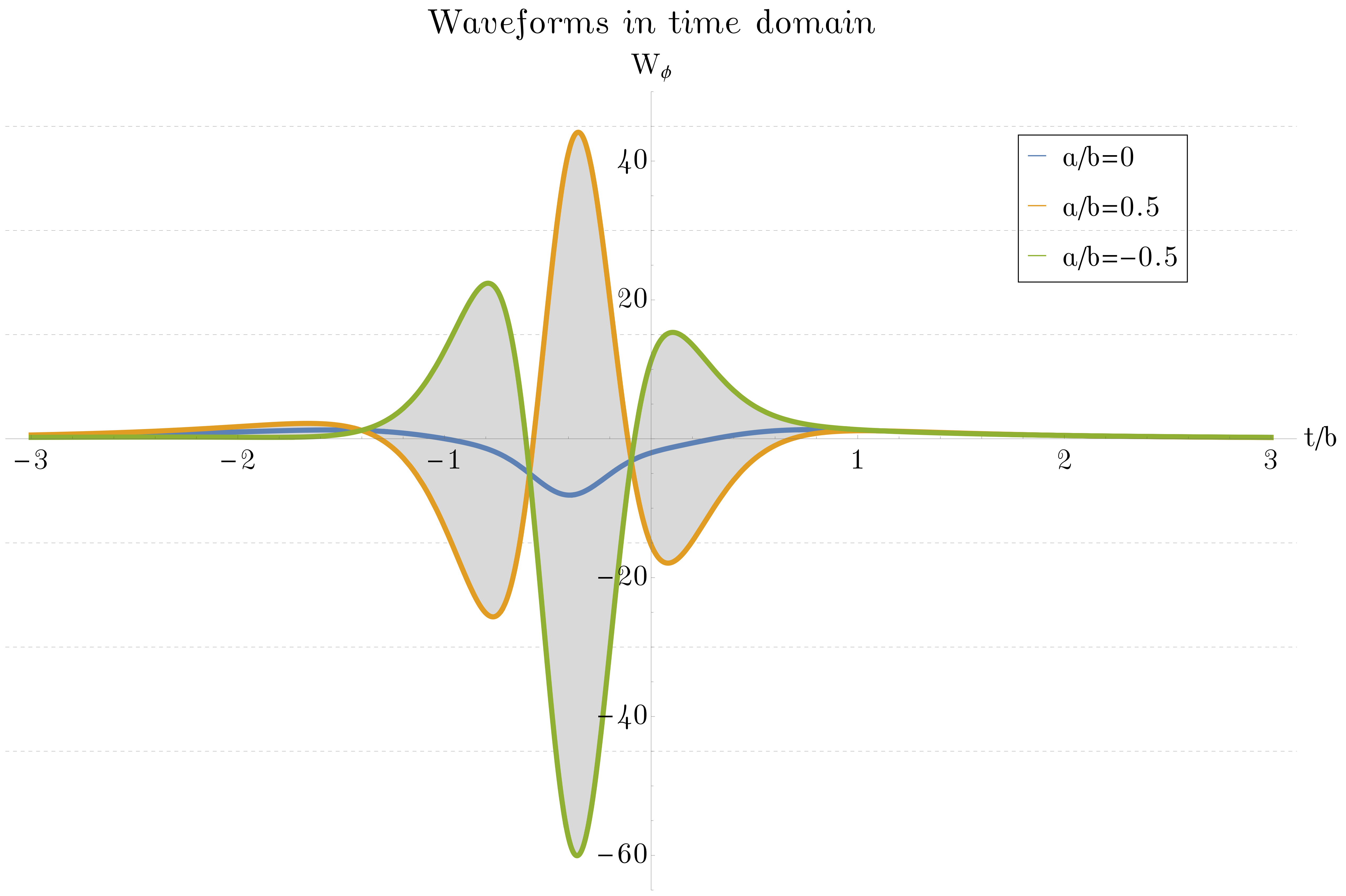
Hairy compact objects:



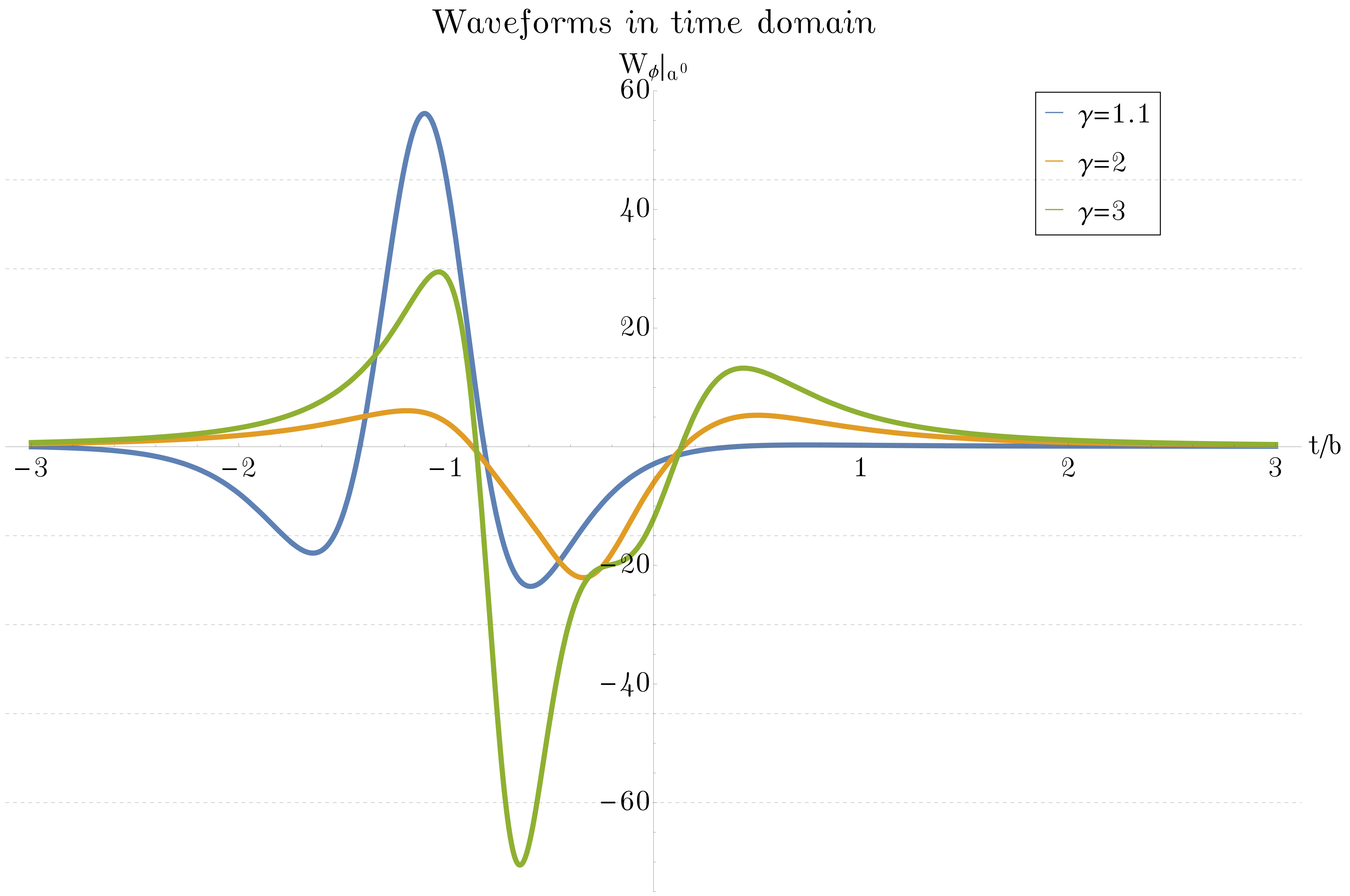


Results:

The scalar waveform for no-hair compact objects in scalar Gauss-Bonnet:



1. Scalar waveform for the SGB case up to linear order in spin for different values of the spin magnitude $a \in [-b/2, b/2]$.



2. Scalar piece of the SGB waveform for different values of γ.

Results:

The scalar waveform for no-hair compact objects in scalar Gauss-Bonnet:

$$W_\phi = \frac{m_1 m_2}{8\pi^2 b^3 M_{Pl}^3 \Lambda^2 (\hat{u}_1 n)^2 \sqrt{\gamma^2-1}} \Bigg(\alpha \Bigg\{ -\frac{d^2}{dz^2}\left[\frac{1}{\sqrt{z^2+1}} \frac{2(\gamma^2-1)^2(\hat{u}_2 n)^2\big[\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b}n)z\big]}{\big[\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b}n)z\big]^2 + (\tilde{v}n)^2(z^2+1)}\right] + \big[(\hat{u}_1 n) + \gamma\big(2\gamma^2-3\big)(\hat{u}_2 n)\big]\frac{2z^2-1}{(z^2+1)^{5/2}} + \big(2\gamma^2-1\big)(\tilde{b}n)\frac{3z}{(z^2+1)^{5/2}} \Bigg\}$$

$$+\frac{2\alpha}{b}\frac{d^3}{dz^3}\mathrm{Re}\Bigg\{\frac{1}{\sqrt{z^2+1}}\frac{(\hat{u}_2 n) - \gamma(\hat{u}_1 n) + \gamma(\tilde{b}n)z + i\gamma(\tilde{v}n)\sqrt{z^2+1}}{\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b}n)z + i(\tilde{v}n)\sqrt{z^2+1}}\left[z(\tilde{v}n) - i\sqrt{z^2+1}(\tilde{b}n)\right] \times \left[\frac{(a_1 n)}{(\hat{u}_1 n)} - \gamma(a_1\hat{u}_2) - (a_2\hat{u}_1) - (a_1^A - a_2^A)(z\tilde{b}^A + i\sqrt{z^2+1}\tilde{v}^A)\right]\Bigg\}$$

$$-\sqrt{\gamma^2-1}\frac{C_1}{b}\frac{d^3}{dz^3}\Bigg\{\frac{1}{\sqrt{z^2+1}}\mathrm{Re}\Bigg[\Bigg(z\big[(\tilde{v}n)a_1^A + (a_1\tilde{v})n^A\big] - i\sqrt{z^2+1}\big[(\tilde{b}n)a_1^A + (a_1\tilde{b})n^A\big]\Bigg) \times \Bigg(\hat{u}_2^A - \gamma\hat{u}_1^A + \gamma\big[z\tilde{b}^A + i\sqrt{z^2+1}\tilde{v}^A\big]\Bigg)\Bigg]\Bigg\}\Bigg)\Bigg|_{z=T_1} + (1\leftrightarrow 2) + \mathcal{O}(a^2)\,.$$

Results:

The scalar waveform for no-hair compact objects in scalar Gauss-Bonnet:

$$
\begin{aligned}
W_\phi = \frac{m_1 m_2}{8\pi^2 b^3 M_{Pl}^3 \Lambda^2 (\hat{u}_1 n)^2 \sqrt{\gamma^2-1}} \Bigg(\alpha \Bigg\{ -\frac{d^2}{dz^2}\Bigg[\frac{1}{\sqrt{z^2+1}} \frac{2(\gamma^2-1)^2(\hat{u}_2 n)^2\big[\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b} n) z\big]}{\big[\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b} n) z\big]^2 + (\tilde{v} n)^2 (z^2+1)}\Bigg] + \big[(\hat{u}_1 n) + \gamma\big(2\gamma^2 - 3\big)(\hat{u}_2 n)\big]\frac{2z^2-1}{(z^2+1)^{5/2}} + \big(2\gamma^2-1\big)(\tilde{b} n)\frac{3z}{(z^2+1)^{5/2}} \Bigg\} \\
+ \frac{2\alpha}{b}\frac{d^3}{dz^3}\text{Re}\Bigg\{ \frac{1}{\sqrt{z^2+1}} \frac{(\hat{u}_2 n) - \gamma(\hat{u}_1 n) + \gamma(\tilde{b} n) z + i\gamma(\tilde{v} n)\sqrt{z^2+1}}{\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b} n) z + i(\tilde{v} n)\sqrt{z^2+1}} \Big[z(\tilde{v} n) - i\sqrt{z^2+1}(\tilde{b} n)\Big] \times \Bigg[\frac{(a_1 n)}{(\hat{u}_1 n)} - \gamma(a_1 \hat{u}_2) - (a_2 \hat{u}_1) - (a_1^A - a_2^A)(z\tilde{b}^A + i\sqrt{z^2+1}\,\tilde{v}^A)\Bigg]\Bigg\} \\
-\sqrt{\gamma^2-1}\frac{C_1}{b}\frac{d^3}{dz^3}\Bigg\{ \frac{1}{\sqrt{z^2+1}} \text{Re}\Bigg[\Bigg(z\big[(\tilde{v} n) a_1^A + (a_1 \tilde{v}) n^A\big] - i\sqrt{z^2+1}\big[(\tilde{b} n) a_1^A + (a_1 \tilde{b}) n^A\big]\Bigg) \times \Bigg(\hat{u}_2^A - \gamma \hat{u}_1^A + \gamma\big[z\tilde{b}^A + i\sqrt{z^2+1}\,\tilde{v}^A\big]\Bigg)\Bigg]\Bigg\}\Bigg)\Bigg|_{z=T_1} + (1 \leftrightarrow 2) + \mathcal{O}(a^2)\,.
\end{aligned}
$$

Connect to observables: Power emitted in scalar radiation

$$\frac{dP_\phi}{d\Omega} = (\partial_t W_\phi)^2$$

$$\frac{dP_\phi}{d\Omega}\Bigg|_{\mathcal{O}(a^0)} \sim \frac{\beta^6}{b^8} \quad \xrightarrow{\text{For closed orbits}} \quad \frac{dP_\phi}{d\Omega}\Bigg|_{\mathcal{O}(a^0)} \sim \beta^{22}$$

$$\frac{dP_\phi}{d\Omega}\Bigg|_{\mathcal{O}(a^1)} \sim \frac{\beta^4}{b^{10}} \quad \xrightarrow{\text{For closed orbits}} \quad \frac{dP_\phi}{d\Omega}\Bigg|_{\mathcal{O}(a^1)} \sim \beta^{24}$$

Results:

The scalar waveform for no-hair compact objects in scalar Gauss-Bonnet:

$$
\begin{aligned}
W_\phi = \frac{m_1 m_2}{8\pi^2 b^3 M_{Pl}^3 \Lambda^2 (\hat{u}_1 n)^2 \sqrt{\gamma^2-1}} \Bigg(\alpha \Bigg\{ -\frac{d^2}{dz^2}\Bigg[\frac{1}{\sqrt{z^2+1}} \frac{2(\gamma^2-1)^2(\hat{u}_2 n)^2\big[\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b} n) z\big]}{\big[\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b} n) z\big]^2 + (\tilde{v} n)^2 (z^2+1)} \Bigg] + \big[(\hat{u}_1 n) + \gamma\big(2\gamma^2-3\big)(\hat{u}_2 n)\big] \frac{2z^2-1}{(z^2+1)^{5/2}} + \big(2\gamma^2-1\big)(\tilde{b} n) \frac{3z}{(z^2+1)^{5/2}} \Bigg\} \\
+ \frac{2\alpha}{b}\frac{d^3}{dz^3} \mathrm{Re} \Bigg\{ \frac{1}{\sqrt{z^2+1}} \frac{(\hat{u}_2 n) - \gamma(\hat{u}_1 n) + \gamma(\tilde{b} n) z + i\gamma(\tilde{v} n)\sqrt{z^2+1}}{\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b} n) z + i(\tilde{v} n)\sqrt{z^2+1}} \Big[z(\tilde{v} n) - i\sqrt{z^2+1}(\tilde{b} n) \Big] \times \Bigg[\frac{(a_1 n)}{(\hat{u}_1 n)} - \gamma(a_1 \hat{u}_2) - (a_2 \hat{u}_1) - (a_1^A - a_2^A)(z\tilde{b}^A + i\sqrt{z^2+1}\,\tilde{v}^A) \Bigg] \Bigg\} \\
- \sqrt{\gamma^2-1}\,\frac{C_1}{b}\frac{d^3}{dz^3} \Bigg\{ \frac{1}{\sqrt{z^2+1}} \mathrm{Re} \Bigg[\Bigg(z\big[(\tilde{v} n) a_1^A + (a_1 \tilde{v}) n^A\big] - i\sqrt{z^2+1}\,\big[(\tilde{b} n) a_1^A + (a_1 \tilde{b}) n^A\big] \Bigg) \times \Bigg(\hat{u}_2^A - \gamma \hat{u}_1^A + \gamma\big[z\tilde{b}^A + i\sqrt{z^2+1}\,\tilde{v}^A\big] \Bigg) \Bigg] \Bigg\} \Bigg) \Bigg|_{z=T_1} + (1 \leftrightarrow 2) + \mathcal{O}(a^2)\,.
\end{aligned}
$$

Connect to observables: Power emitted in scalar radiation

$$\frac{dP_\phi}{d\Omega} = (\partial_t W_\phi)^2$$

$$\frac{dP_\phi}{d\Omega}\bigg|_{\mathcal{O}(a^0)} \sim \frac{\beta^6}{b^8} \quad \xrightarrow{\text{For closed orbits}} \quad \frac{dP_\phi}{d\Omega}\bigg|_{\mathcal{O}(a^0)} \sim \beta^{22}$$

Bigger suppression compared to β^8 previously computed with GR methods

Phys.Rev.D 85 (2012) 064022, Phys.Rev.D 93 (2016) 2, 029902 (erratum) [Yagi, Stein, Yunes, Tanaka]

$$\frac{dP_\phi}{d\Omega}\bigg|_{\mathcal{O}(a^1)} \sim \frac{\beta^4}{b^{10}} \quad \xrightarrow{\text{For closed orbits}} \quad \frac{dP_\phi}{d\Omega}\bigg|_{\mathcal{O}(a^1)} \sim \beta^{24}$$

Results:

The scalar waveform for hairy compact objects in SGB/DCS-Spinless part:

$$W_\phi^{(0)} = -\frac{m_1 m_2}{32\pi^2 M_{Pl}^3 \sqrt{\gamma^2-1}(\hat{u}_1 n)^2 b}\frac{1}{\sqrt{1+T_1^2}}\left\{\frac{(\gamma^2-1)[c_1(\hat{u}_2 n)^2 + c_2(\hat{u}_1 n)^2][\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b}n)T_1]}{[-(\hat{u}_1 n) + y(\hat{u}_2 n) + (\tilde{b}n)T_1]^2 + (\tilde{v}n)^2(1+T_1^2)}\right.$$

$$\left. -\frac{c_1}{2}\frac{(\hat{u}_1 n) + (2\gamma^2-3)y(\hat{u}_2 n) - (2\gamma^2-1)(\tilde{b}n)T_1}{y^2-1} + \frac{C_1^{(0)}}{2}c_2(\hat{u}_1 n)\right\} + (1 \leftrightarrow 2)\,.$$

$$\left.\frac{dP_\phi}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim \frac{1}{b^4}$$

For closed orbits →

$$\left.\frac{dP_\phi}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim \beta^8$$

→ **Agreement with existing PN results for SGB!**

Phys.Rev.D 85 (2012) 064022, Phys.Rev.D 93 (2016) 2, 029902 (erratum) [Yagi, Stein, Yunes, Tanaka]

Results:

The scalar waveform for hairy compact objects in SGB/DCS-Spinless part:

e.g.: Phys.Rev.D 100 (2019) 10, 104061 [Julié, Berti]

scalar "monopole" charges $c_i = c_i(m_i, a_i, \frac{\alpha}{\Lambda})$: Can be matched to existing results!

$$W_\phi^{(0)} = -\frac{m_1 m_2}{32\pi^2 M_{Pl}^3 \sqrt{\gamma^2-1}(\hat{u}_1 n)^2 b} \frac{1}{\sqrt{1+T_1^2}} \left\{ \frac{(\gamma^2-1)[c_1(\hat{u}_2 n)^2 + c_2(\hat{u}_1 n)^2][\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b}n)T_1]}{[-(\hat{u}_1 n) + y(\hat{u}_2 n) + (\tilde{b}n)T_1]^2 + (\tilde{v}n)^2(1+T_1^2)} \right.$$

$$\left. -\frac{c_1}{2}\frac{(\hat{u}_1 n) + (2\gamma^2-3)y(\hat{u}_2 n) - (2\gamma^2-1)(\tilde{b}n)T_1}{y^2-1} + \frac{C_1^{(0)}}{2} c_2(\hat{u}_1 n) \right\} + (1 \leftrightarrow 2)\,.$$

$$\left.\frac{dP_\phi}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim \frac{1}{b^4}$$

For closed orbits →

$$\left.\frac{dP_\phi}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim \beta^8$$

→ **Agreement with existing PN results for SGB!**

Phys.Rev.D 85 (2012) 064022, Phys.Rev.D 93 (2016) 2, 029902 (erratum) [Yagi, Stein, Yunes, Tanaka]

Results:

The gravitational waveform for hairy compact objects in SGB/DCS-Spinless part:

$$W_h^{(0)} = -\frac{c_1 c_2 m_1 m_2}{512\pi^2 M_{Pl}^3 \sqrt{\gamma^2-1}(\hat{u}_1 n)^2 b}\frac{1}{\sqrt{1+T_1^2}}\mathrm{Re}\left\{\frac{(\lambda_n[\gamma\hat{u}_2\sigma + T_1\tilde{b}\sigma + i\sqrt{T_1^2+1}\tilde{v}\sigma]\hat{u}_1\bar{\sigma}\lambda_n)^2}{y(\hat{u}_2 n) - (\hat{u}_1 n) + T_1(\tilde{b}n) + i\sqrt{T_1^2+1}(\tilde{u}n)}\right\} + (1 \leftrightarrow 2)\,.$$

$$\left.\frac{dP_h}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim \frac{\beta^2}{b^4} \quad \xrightarrow{\text{For closed orbits}} \quad \left.\frac{dP_h}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim \beta^{10} \longrightarrow$$

Expected suppression compared to scalar radiation!

Phys.Rev.D 85 (2012) 064022, Phys.Rev.D 93 (2016) 2, 029902 (erratum) [Yagi, Stein, Yunes, Tanaka]

→ Can show that there are no linear-in-spin corrections to $W_h^{(1)}$ at this order!

Results:

The gravitational waveform for hairy compact objects in SGB/DCS-Spinless part:

$$W_h^{(0)} = -\frac{c_1 c_2 m_1 m_2}{512\pi^2 M_{Pl}^3 \sqrt{\gamma^2-1}(\hat{u}_1 n)^2 b} \frac{1}{\sqrt{1+T_1^2}} \mathrm{Re}\left\{ \frac{(\lambda_n[\gamma \hat{u}_2 \sigma + T_1 \tilde{b}\sigma + i\sqrt{T_1^2+1}\tilde{v}\sigma]\hat{u}_1\bar{\sigma}\lambda_n)^2}{y(\hat{u}_2 n) - (\hat{u}_1 n) + T_1(\tilde{b}n) + i\sqrt{T_1^2+1}(\tilde{u}n)} \right\} + (1 \leftrightarrow 2)\,.$$

$$\left.\frac{dP_h}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim \frac{\beta^2}{b^4} \xrightarrow{\text{For closed orbits}} \left.\frac{dP_h}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim \beta^{10} \longrightarrow$$

Expected suppression compared to scalar radiation!

Phys.Rev.D 85 (2012) 064022, Phys.Rev.D 93 (2016) 2, 029902 (erratum) [Yagi, Stein, Yunes, Tanaka]

⟶ Can show that there are no linear-in-spin corrections to $W_h^{(1)}$ at this order!

Comments:

- Notice the **simplicity** of the waveform when expressed in the **spinor** language!
- In fact, we also derived that the NR **power** emitted is **exactly the same as in GR** up to setting $c_1 c_2 \to 1$
- We describe how to model dipole, quadruple, … hair and compute the **dipole corrections**

4. Outlook

- Scatttering Amplitudes techniques can be proven to be extremely useful in the quest for precision measurements in the GW era, proving results to all orders in velocity.

- Computations for spinning binaries remarkably simplify in the on-shell language.

- Recasting already known problems in the amplitudes' language makes the search for beyond GR effects easier to handle and essentially the usual QFT methods can be used.

4. Outlook

- Scatttering Amplitudes techniques can be proven to be extremely useful in the quest for precision measurements in the GW era, proving results to all orders in velocity.

- Computations for spinning binaries remarkably simplify in the on-shell language.

- Recasting already known problems in the amplitudes' language makes the search for beyond GR effects easier to handle and essentially the usual QFT methods can be used.

What's next?

- Employing similar methods to study GR (and beyond) effects, where current PN results are poor.

- A QFT framework to study the self-force expansion as well? Already promising work towards that direction.

- Dive more deeply into the synergy of QFT and GR and consider other applications as well, e.g. emergence of couplings in ST theories from integrating out arbitrary spinning heavy particles.

The universe seems to be extremely loud!

Artwork by Penelope Cowley*

The universe seems to be extremely loud!

Thank you for your attention!:)

Artwork by Penelope Cowley*

Backup slides

The scalar waveform for dynamical Chern-Simons:

$$W_{\tilde{\phi}} = \frac{m_1 m_2}{8\pi^2 M_{Pl}^3 \Lambda^2 (\hat{u}_1 n)^2 b^3}\Bigg(2\tilde{\alpha}(\tilde{v}n)\frac{d^2}{dz^2}\Bigg\{\frac{1}{\sqrt{z^2+1}}\Bigg[\gamma z - (\gamma^2-1)(\hat{u}_2 n)\frac{z\big[\gamma(\hat{u}_2 n) - (\hat{u}_1 n)\big] - (\tilde{b}n)}{\big[\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b}n)z\big]^2 + (\tilde{v}n)^2(z^2+1)}\Bigg]\Bigg\}$$

$$+\frac{\tilde{\alpha}}{b\sqrt{\gamma^2-1}}\mathrm{Re}\frac{d^3}{dz^3}\Bigg\{\frac{1}{\sqrt{z^2+1}}\Bigg(\frac{(a_1 n)}{(\hat{u}_1 n)} - \gamma(a_1\hat{u}_2) - (a_2\hat{u}_1) + (a_2^A - a_1^A)\big[z\tilde{b}^A + i\sqrt{z^2+1}\,\tilde{v}^A\big]\Bigg)\Bigg(\frac{2(\gamma^2-1)^2(\hat{u}_2 n)^2}{\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b}n)z + i(\tilde{v}n)\sqrt{z^2+1}} - (\hat{u}_1 n) - \gamma\big(2\gamma^2-3\big)(\hat{u}_2 n) + \big(2\gamma^2-1\big)\big[z(\tilde{b}n) + i\sqrt{z^2+1}(\tilde{v}n)\big]\Bigg)\Bigg\}$$

$$-\frac{1}{\sqrt{\gamma^2-1}}\frac{\tilde{C}_1 a_1^A}{b}\frac{d^3}{dz^3}\Bigg\{\frac{1}{\sqrt{z^2+1}}\Bigg[(2\gamma^2-1)\big[z^2(\tilde{b}n)\tilde{b}^A - (z^2+1)(\tilde{v}n)\tilde{v}^A\big] - (\gamma^2-1)n^A + \gamma(\gamma^2-2)(\hat{u}_1 n)\hat{u}_2^A - (\gamma^2-2)(\hat{u}_2 n)\hat{u}_2^A + z\gamma(\tilde{b}n)\hat{u}_2^A - z\gamma^2(\hat{u}_1 n)\tilde{b}^A + z\gamma(\hat{u}_2 n)\tilde{b}^A\Bigg]\Bigg\}\Bigg)\Bigg|_{z=T_1} + (1\leftrightarrow 2) + \mathcal{O}(a^2)\,.$$

LO Scalar Waveforms for CC coupling-Spinning part:

$$W_{\phi}^{(1)} = \frac{m_1 m_2}{32\pi^2 M_{Pl}^3 (\hat{u}_1 n)^2 b^2}\frac{\partial}{\partial z}\Bigg[\frac{1}{\sqrt{z^2+1}}\mathrm{Re}\Bigg\{c_1\big[(\tilde{v}n)z - is_1(\tilde{b}n)\sqrt{z^2+1}\big]\big[-(\hat{u}_1 a_2) + z(\tilde{b}a_2) + is_1\sqrt{z^2+1}(\tilde{v}a_2)\big]\Bigg(\frac{\gamma}{\gamma^2-1} - \frac{(\hat{u}_2 n)}{-\hat{u}_1 n + \gamma(\hat{u}_2 n) + z(\tilde{b}n) + is_1\sqrt{z^2+1}(\tilde{v}n)}\Bigg)$$

$$-\frac{c_2(\hat{u}_1 n)}{-\hat{u}_1 n + \gamma(\hat{u}_2 n) + z(\tilde{b}n) + is_1\sqrt{z^2+1}(\tilde{v}n)}\Bigg\{[is_1(\tilde{b}n)\sqrt{z^2+1} - (\tilde{v}n)z](\hat{u}_2 a_1) + \big[\gamma(\tilde{v}n) + is_1[\gamma(\hat{u}_1 n) - (\hat{u}_2 n)]\sqrt{z^2+1}\big](\tilde{b}a_1) + \big[[(\hat{u}_2 n) - \gamma(\hat{u}_1 n)]z - \gamma(\tilde{b}n)\big](\tilde{v}a_1)\Bigg\}\Bigg\}\Bigg] + (1\leftrightarrow 2)\,.$$

Backup slides

LO Scalar Waveforms from dipole charges:

$$W^{(1)}_{dip,\phi} = \frac{m_1 m_2}{32\pi^2 M_{Pl}^3 (\hat{u}_1 n)^2 b^2} \frac{\partial}{\partial z} \Bigg[\frac{1}{\sqrt{z^2+1}} \mathrm{Re} \Bigg\{ c_1 \Big[(\tilde{v}n) z - i s_1 (\tilde{b}n) \sqrt{z^2+1} \Big] \Big[-(\hat{u}_1 a_2) + z(\tilde{b}a_2) + i s_1 \sqrt{z^2+1} (\tilde{v}a_2) \Big] \Big(\frac{\gamma}{\gamma^2-1} - \frac{(\hat{u}_2 n)}{-\hat{u}_1 n + \gamma(\hat{u}_2 n) + z(\tilde{b}n) + i s_1 \sqrt{z^2+1} (\tilde{v}n)} \Big)$$

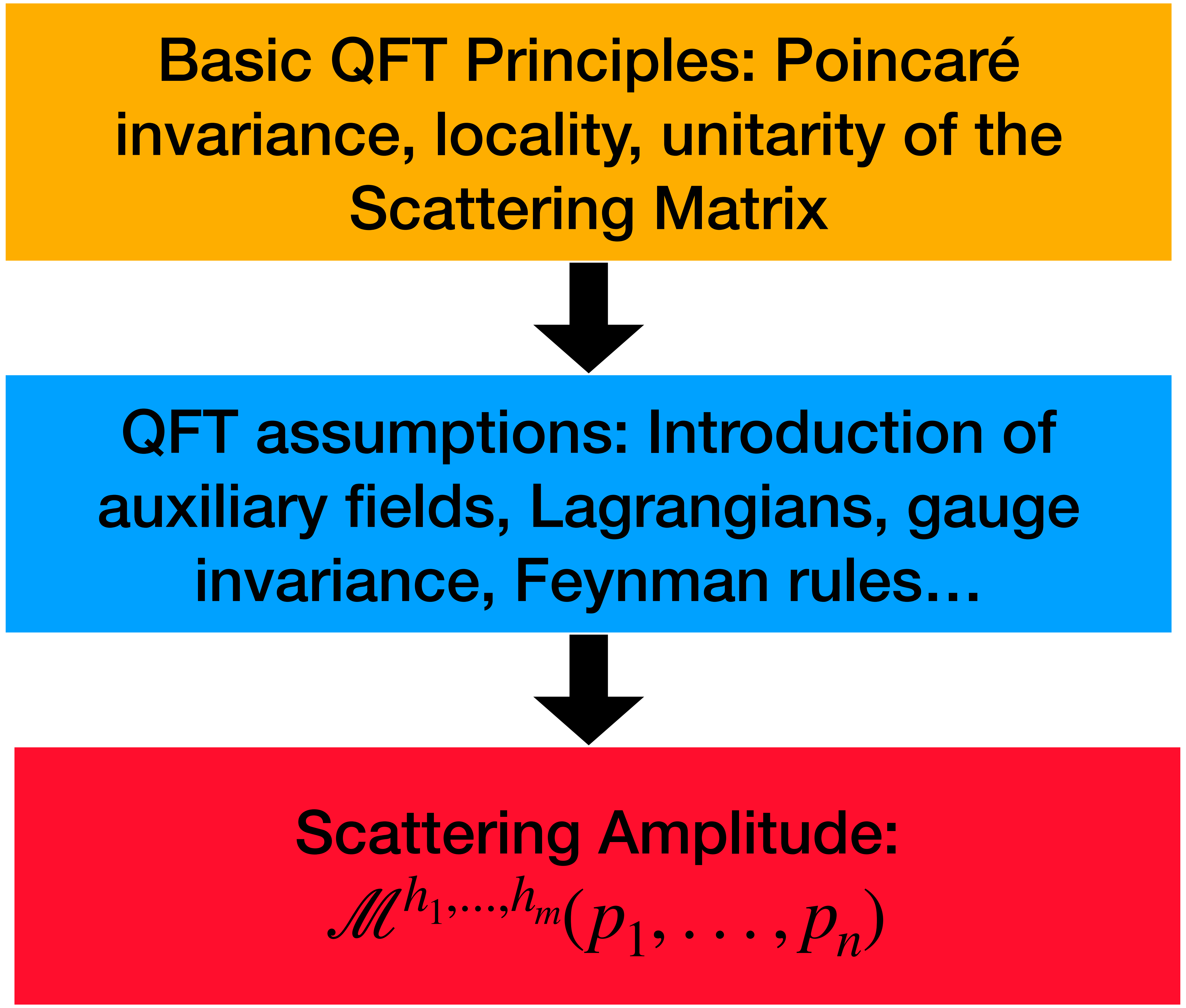
$$- \frac{c_2 (\hat{u}_1 n)}{-\hat{u}_1 n + \gamma(\hat{u}_2 n) + z(\tilde{b}n) + i s_1 \sqrt{z^2+1} (\tilde{v}n)} \Big\{ [i s_1 (\tilde{b}n) \sqrt{z^2+1} - (\tilde{v}n) z] (\hat{u}_2 a_1) + \Big[\gamma(\tilde{v}n) + i s_1 [\gamma(\hat{u}_1 n) - (\hat{u}_2 n)] \sqrt{z^2+1} \Big] (\tilde{b}a_1) + \Big[[(\hat{u}_2 n) - \gamma(\hat{u}_1 n)] z - \gamma(\tilde{b}n) \Big] (\tilde{v}a_1) \Big\} \Bigg\} \Bigg] + (1 \leftrightarrow 2)\,.$$

LO Gravitational Waveforms from dipole charges:

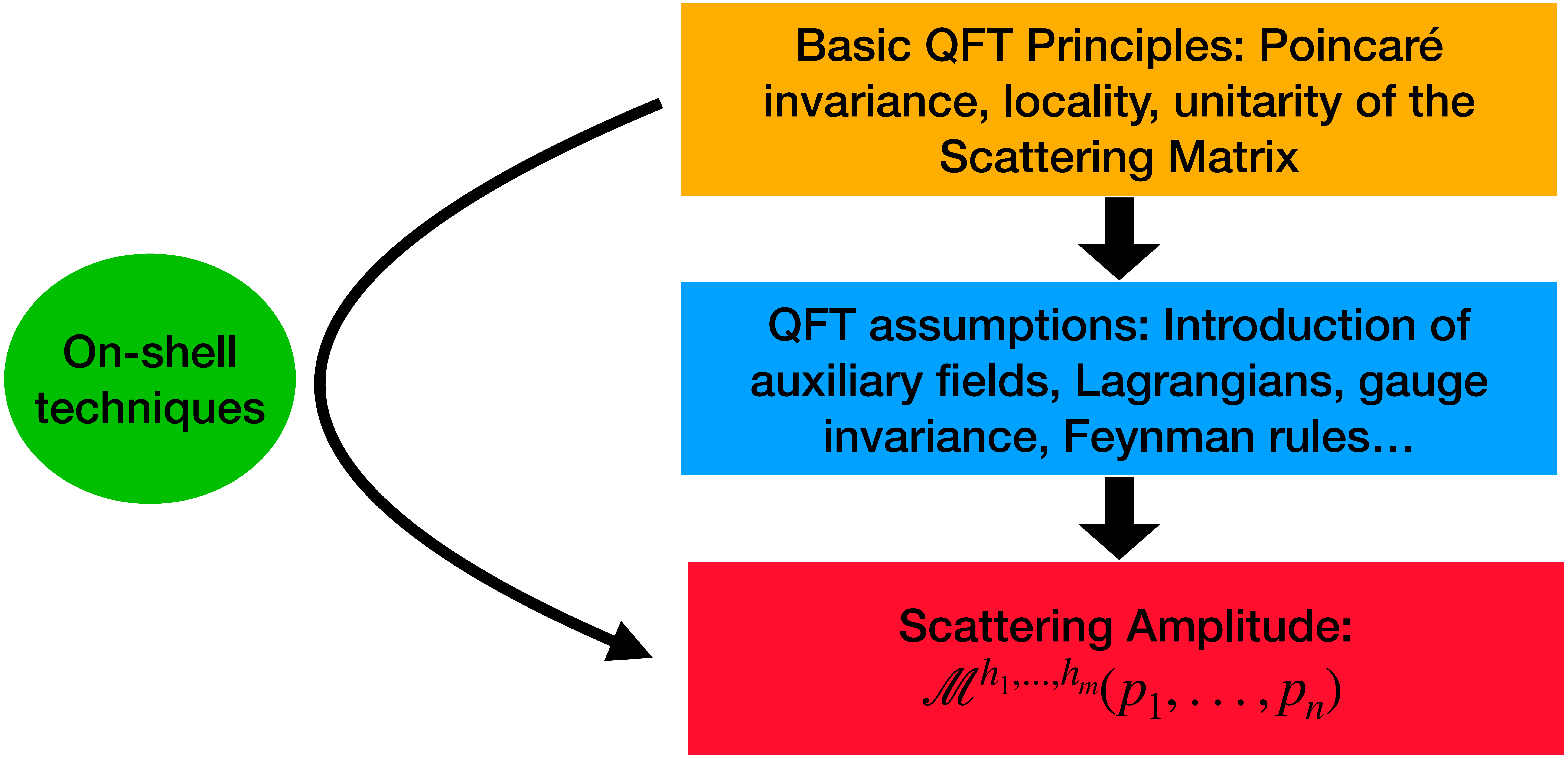
$$W^{(1)}_{dip,h} = - \frac{C_d c_2 m_1 m_2 \varepsilon^{\mu\nu\rho\alpha}}{1024\pi^2 M_{Pl}^3 \sqrt{\gamma^2-1}} \frac{\partial}{\partial z} \Bigg\{ \frac{1}{\sqrt{z^2+1}} \text{"Re"} \Bigg\{ \frac{(\lambda_n [\gamma \hat{u}_{2\mu} + z\tilde{b}_\mu + i\sqrt{z^2+1}\,\tilde{v}_\mu] \hat{u}_{1\nu} \sigma_\alpha \bar{\sigma}_\beta \lambda_n)}{-(\hat{u}_1 n) + \gamma(\hat{u}_2 n) + z(\tilde{b}n) + i\sqrt{z^2+1}(\tilde{v}n)}$$

$$\times (\lambda_n [\gamma \hat{u}_2 \sigma + z\tilde{b}\sigma + i\sqrt{z^2+1}\,\tilde{v}\sigma] (\hat{u}_1 \bar{\sigma}) \lambda_n) \Big[\frac{(n a_1)}{(\hat{u}_1 n)} - \gamma(\hat{u}_2 a_1) - z(\tilde{b}a_1) - i\sqrt{z^2+1}(\tilde{v}a_1) \Big] \Bigg\} \Bigg|_{z=T_1} + + (1 \leftrightarrow 2)\,.$$

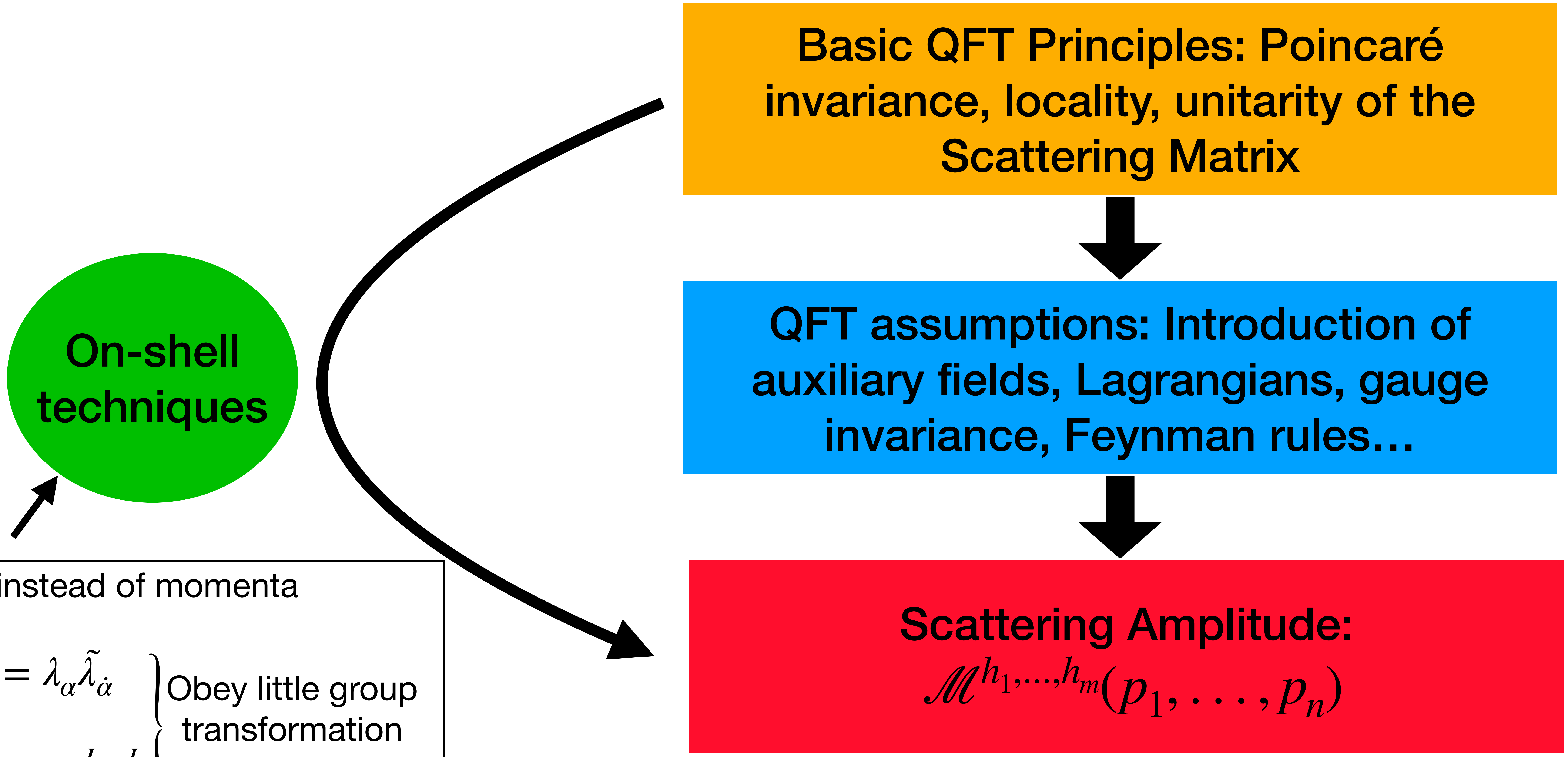
On-shell techniques:



On-shell techniques:



On-shell techniques:



Use spinors as variables instead of momenta

Massless spinors: $p_\mu \sigma^\mu_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}}$

Massive spinors: $p_\mu \sigma^\mu_{\alpha\dot{\alpha}} = \varepsilon_{IJ} \chi^I_\alpha \tilde{\chi}^J_{\dot{\alpha}}$

} Obey little group transformation rules

$\langle ij \rangle = \varepsilon_{\alpha\beta} \lambda_i^\alpha \lambda_j^\beta \quad , \quad [ij] = \varepsilon_{\dot{\alpha}\dot{\beta}} \tilde{\lambda}_i^{\dot{\alpha}} \tilde{\lambda}_j^{\dot{\beta}}$

$\langle \mathbf{ij} \rangle = \langle i^I j^J \rangle \quad , \quad [\mathbf{ij}] = [i^I j^J]$

(2S+1) symmetrized structures

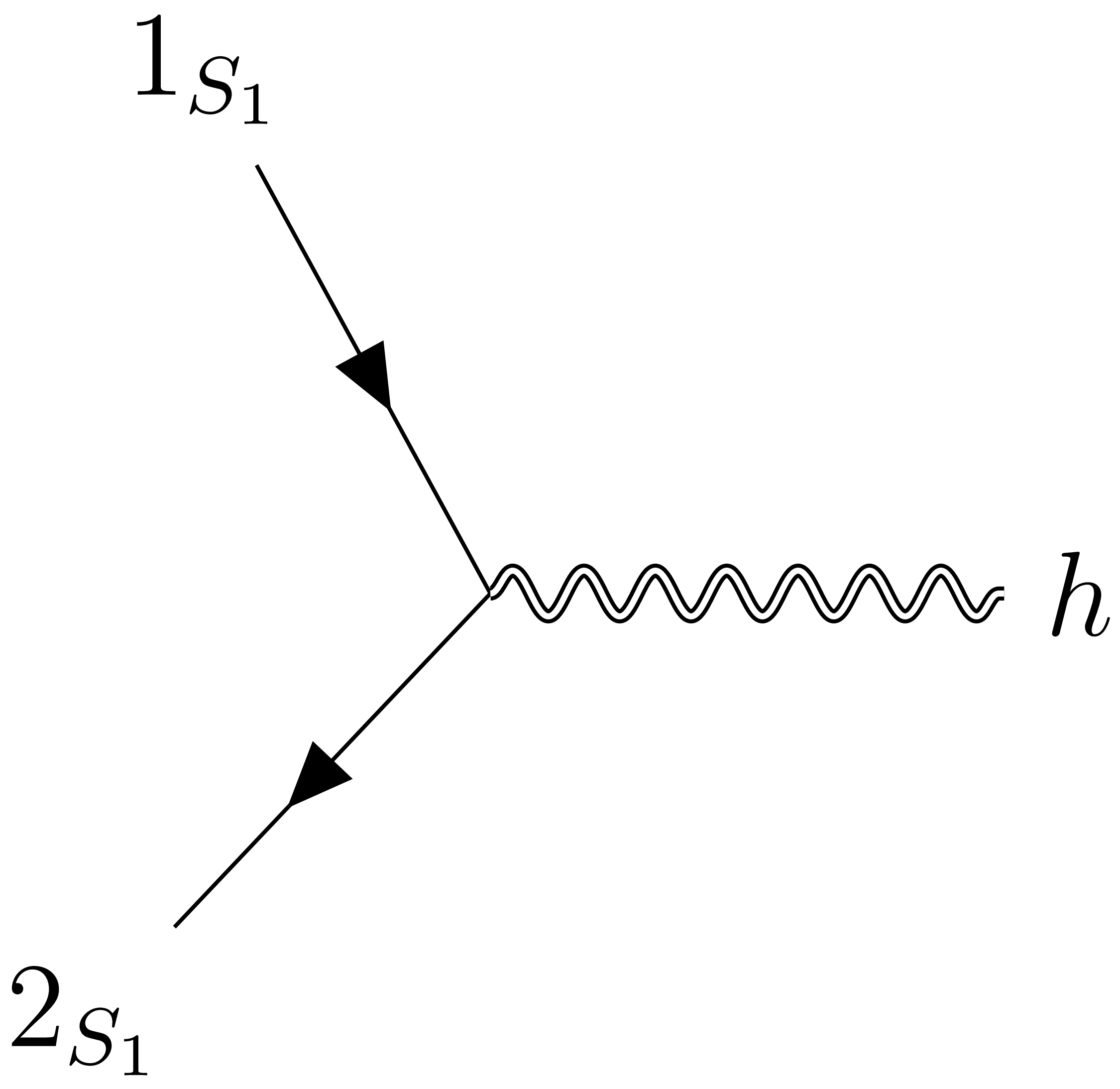
On-shell techniques:

<u>Idea</u>: Build the **on-shell 3-point amplitudes** of the theory

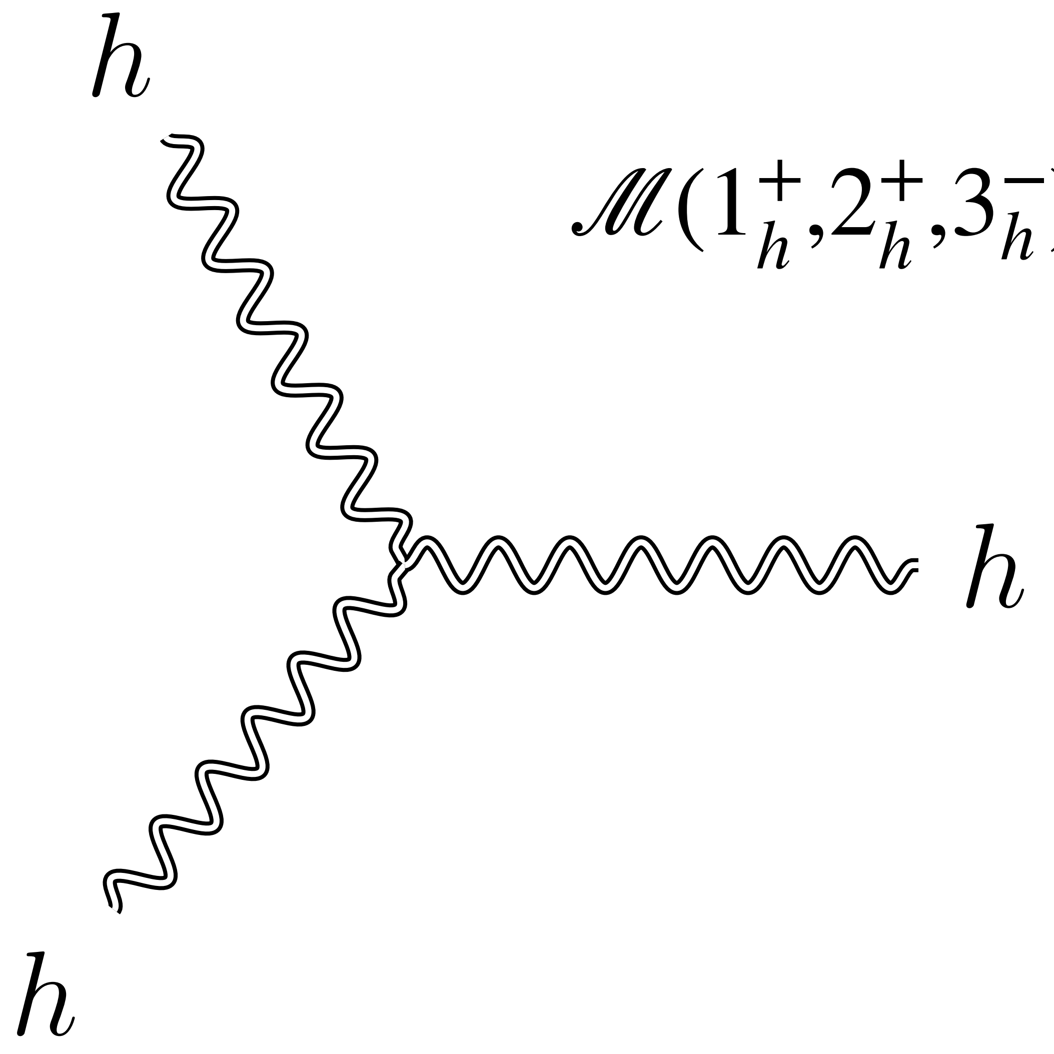
e.g.: **Spinning matter in GR**

$$\mathcal{M}\left[1_\Phi 2_{\bar\Phi} 3_h^-\right] = -\frac{\langle 3|p_1|\tilde\zeta]^2}{M_{Pl}[3\tilde\zeta]^2}\frac{[\mathbf{21}]^{2S}}{m^{2S}},$$

$$\mathcal{M}\left[1_\Phi 2_{\bar\Phi} 3_h^+\right] = -\frac{\langle \zeta|p_1|3]^2}{M_{Pl}\langle 3\zeta\rangle^2}\frac{\langle\mathbf{21}\rangle^{2S}}{m^{2S}},$$



,



$$\mathcal{M}(1_h^-,2_h^-,3_h^+) = -\frac{1}{M_{Pl}}\frac{\langle 12\rangle^6}{\langle 13\rangle^2\langle 23\rangle^2}$$

$$\mathcal{M}(1_h^+,2_h^+,3_h^-) = -\frac{1}{M_{Pl}}\frac{[12]^6}{[13]^2[23]^2}.$$

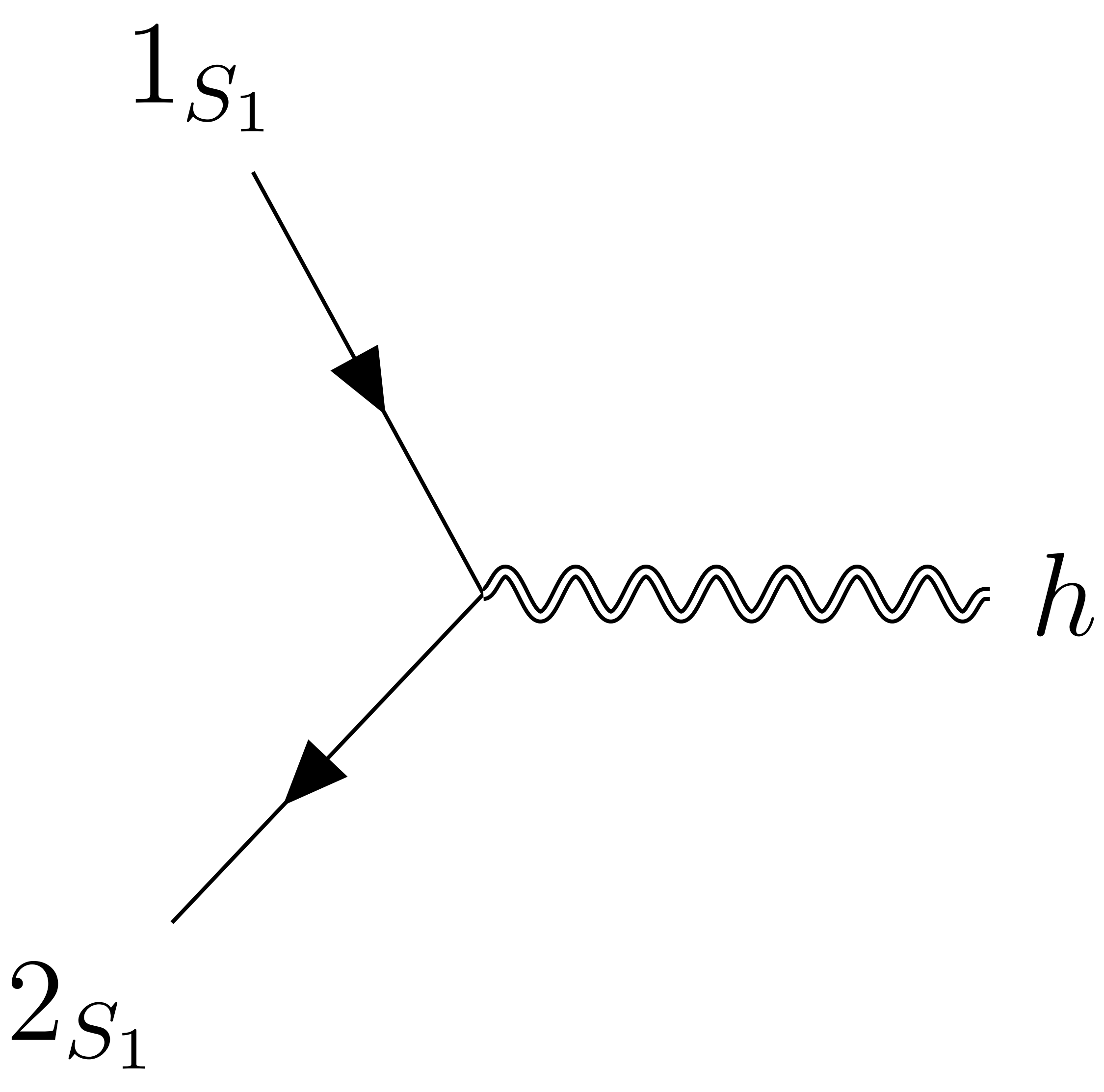
On-shell techniques:

<u>Idea</u>: Build the **on-shell 3-point amplitudes** of the theory

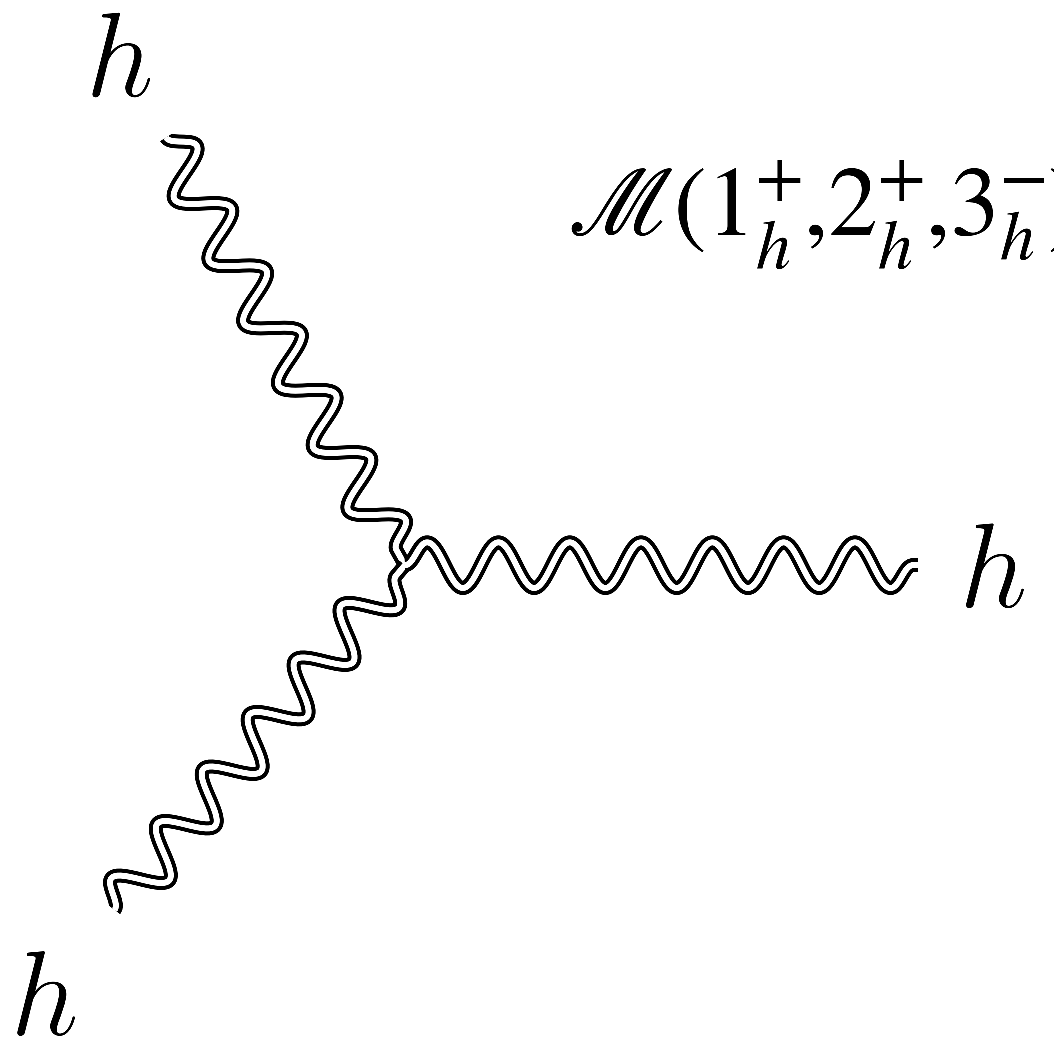
e.g.: **Spinning matter in GR**

$$\mathcal{M}\left[1_\Phi 2_{\bar\Phi} 3_h^-\right] = -\frac{\langle 3|p_1|\tilde\zeta]^2}{M_{Pl}[3\tilde\zeta]^2}\frac{[\mathbf{21}]^{2S}}{m^{2S}},$$

$$\mathcal{M}\left[1_\Phi 2_{\bar\Phi} 3_h^+\right] = -\frac{\langle\zeta|p_1|3]^2}{M_{Pl}\langle 3\zeta\rangle^2}\frac{\langle\mathbf{21}\rangle^{2S}}{m^{2S}},$$



,



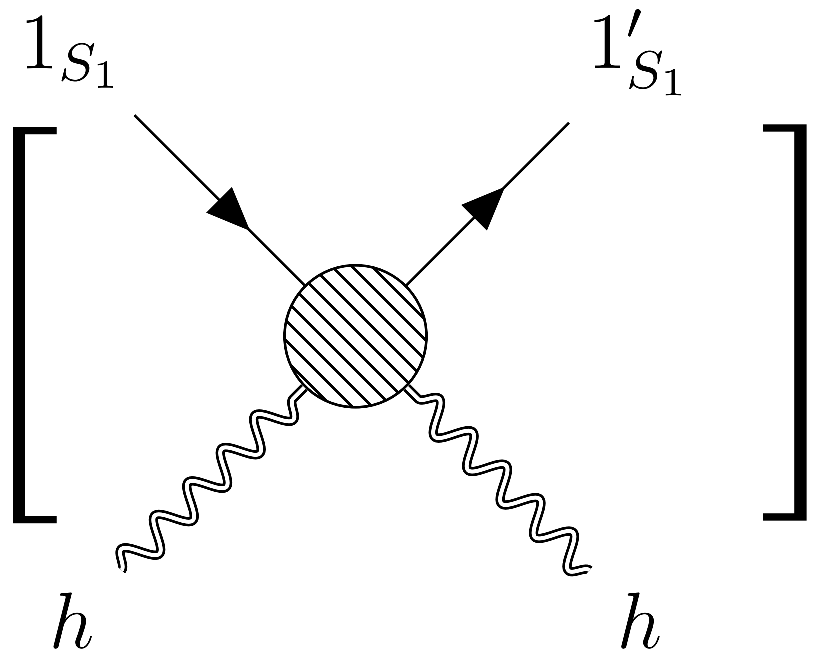
$$\mathcal{M}(1_h^-,2_h^-,3_h^+) = -\frac{1}{M_{Pl}}\frac{\langle 12\rangle^6}{\langle 13\rangle^2\langle 23\rangle^2}$$

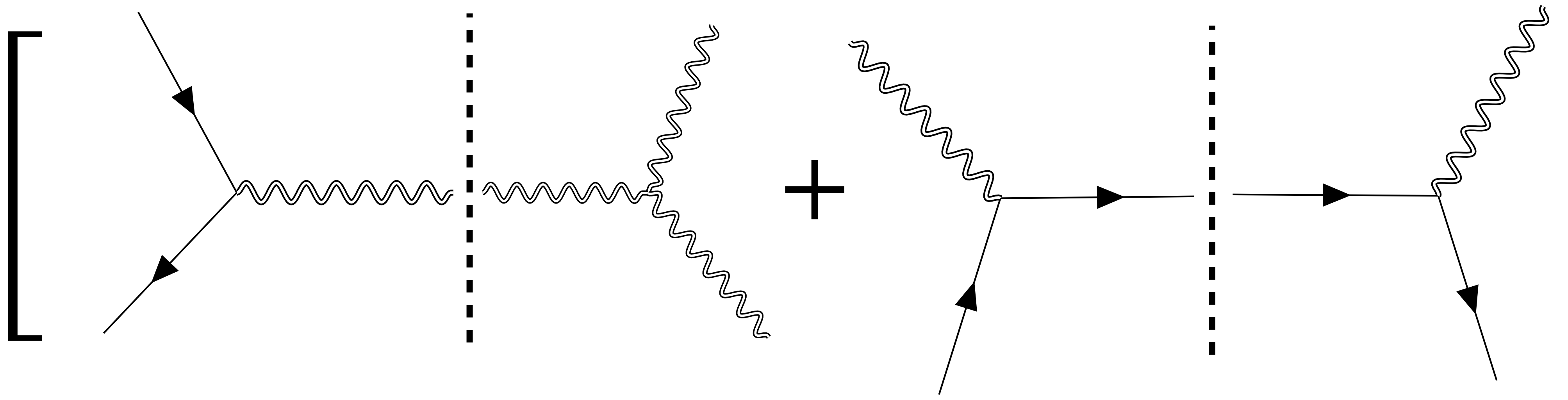
$$\mathcal{M}(1_h^+,2_h^+,3_h^-) = -\frac{1}{M_{Pl}}\frac{[12]^6}{[13]^2[23]^2}.$$

⟶ Build higher-point amplitudes from their **residues** at kinematic poles in the **complex** plane

e.g.:

$$\sum_{i=2,3,4} \text{Res}_{(p_1+p_i)^2\to 0}\left[\ \cdots\ \right]\Bigg|_{tree} = -\left[\ \cdots\ + \ \cdots\ + (t\leftrightarrow u)\right]$$

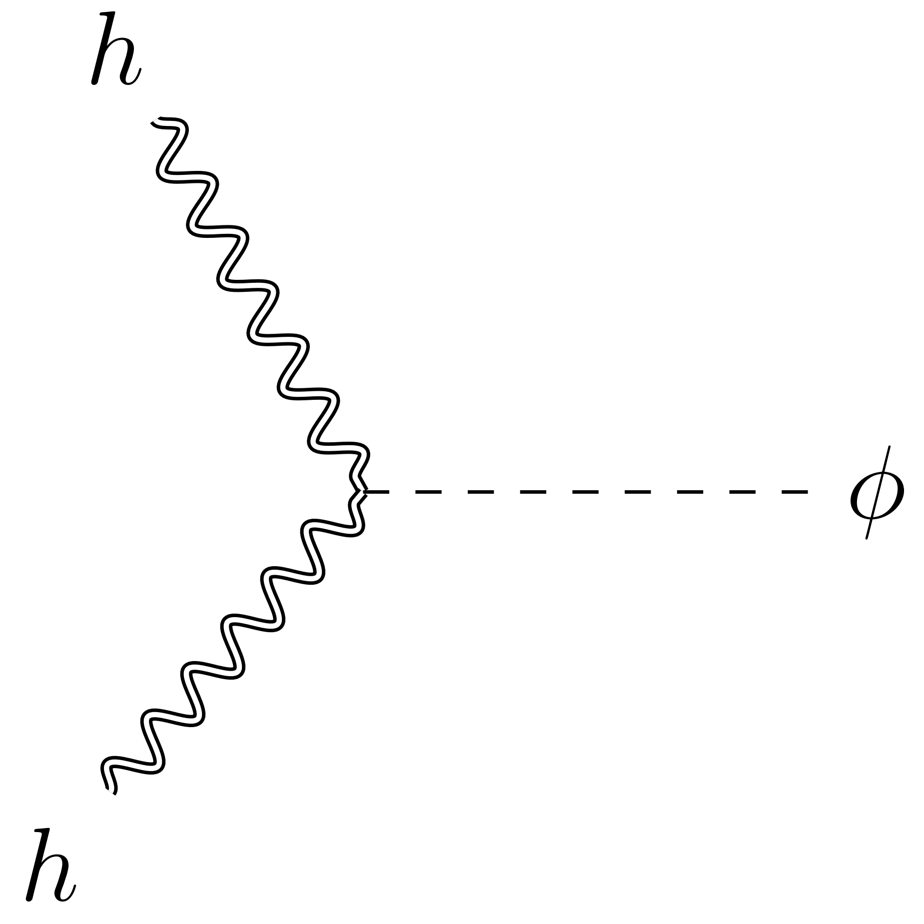




On-shell amplitudes:

Let's work by expanding $f(\phi) \approx c + \phi + \mathcal{O}(\phi^2)$.

Naively, this action produces an extra 3-point on-shell amplitude which we should consider:

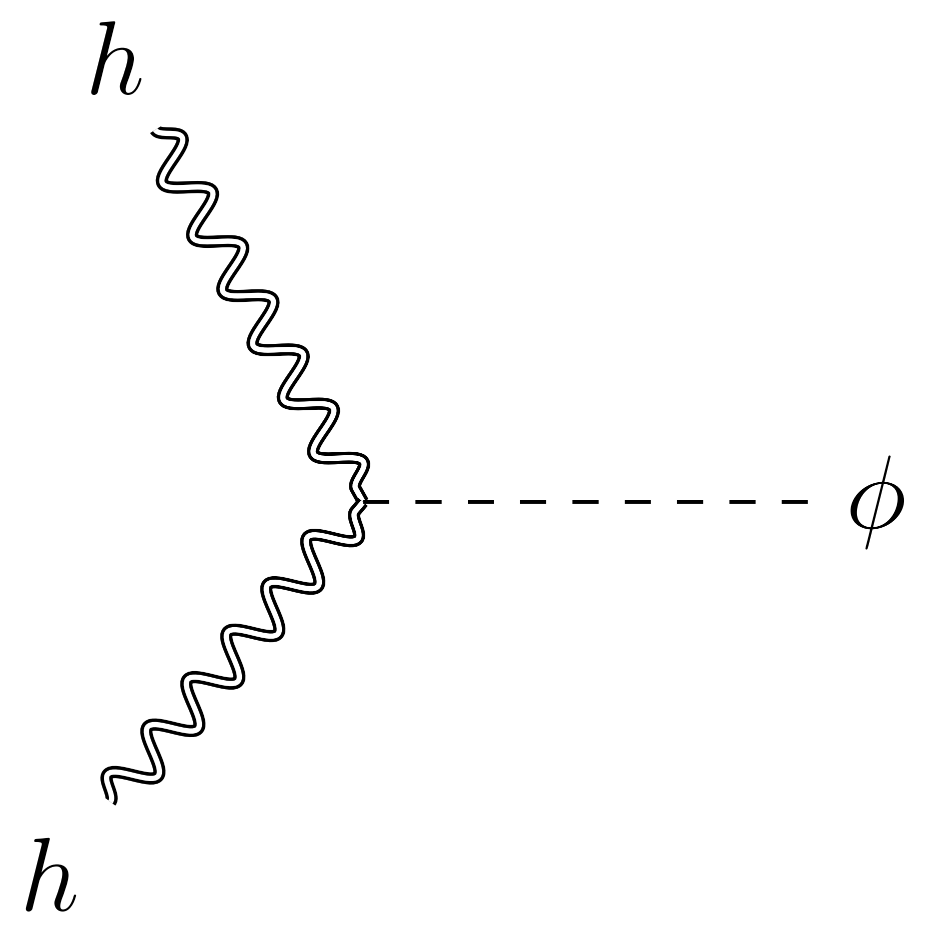


$$\sim \frac{\alpha}{\Lambda^2} \Big/ \frac{\tilde{\alpha}}{\Lambda^2}$$

On-shell amplitudes:

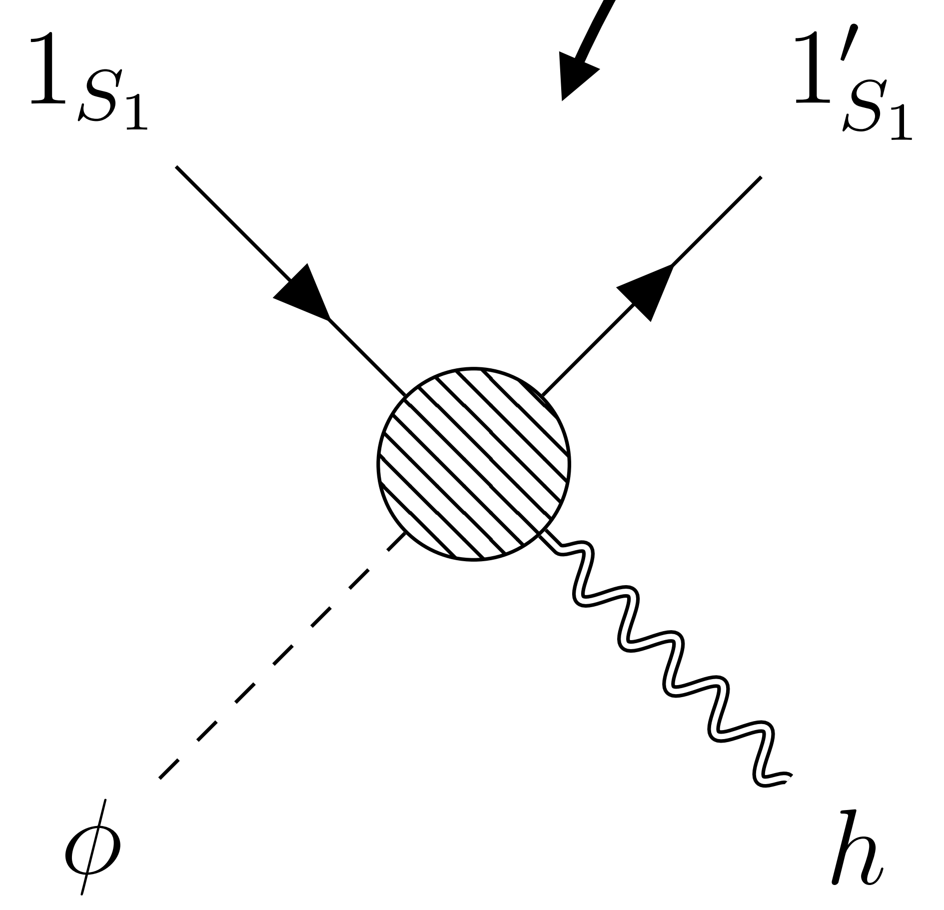
Let’s work by expanding $f(\phi) \approx c + \phi + \mathcal{O}(\phi^2)$.

Naively, this action produces an extra 3-point on-shell amplitude which we should consider:



$$\sim \frac{\alpha}{\Lambda^2} \Big/ \frac{\tilde{\alpha}}{\Lambda^2}$$

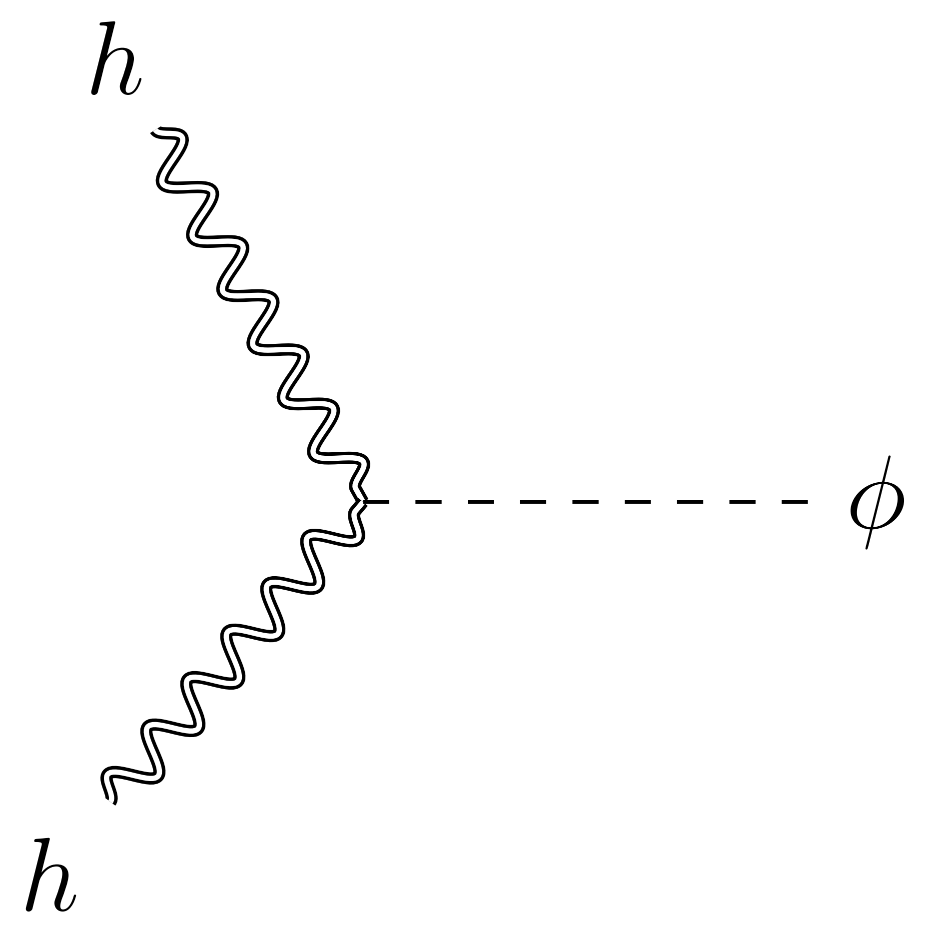
Gives an extra contribution to the 4-point amplitudes



On-shell amplitudes:

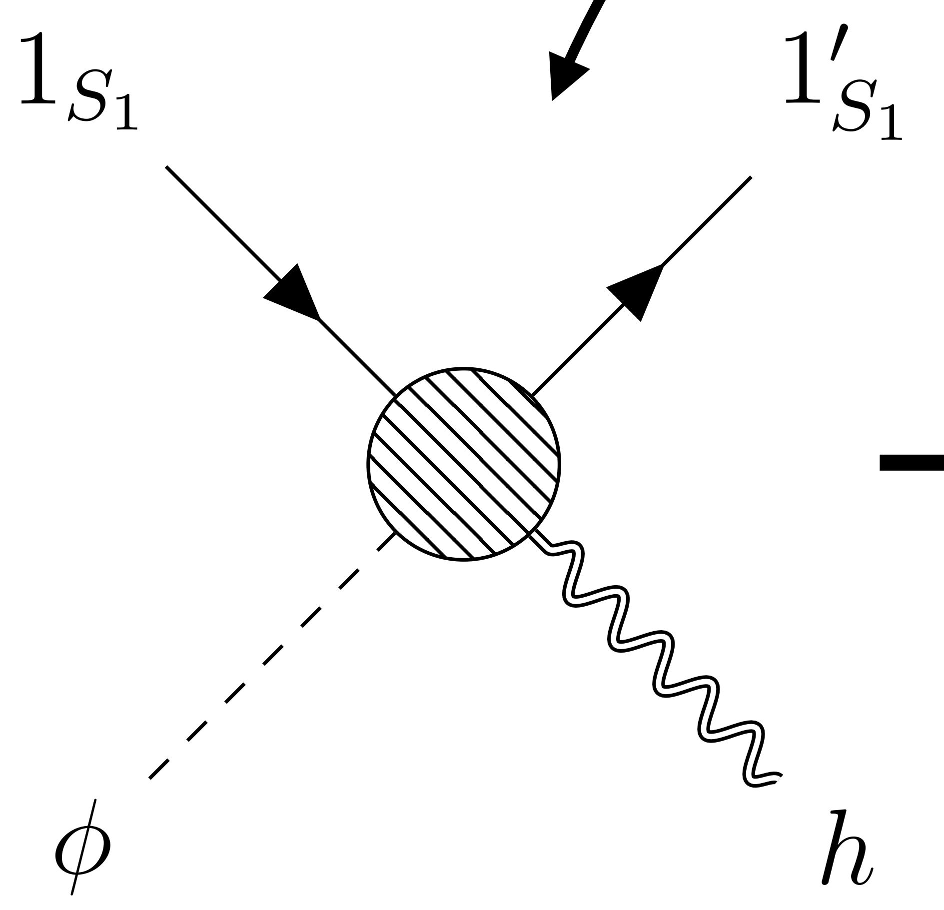
Let's work by expanding $f(\phi) \approx c + \phi + \mathcal{O}(\phi^2)$.

Naively, this action produces an extra 3-point on-shell amplitude which we should consider:

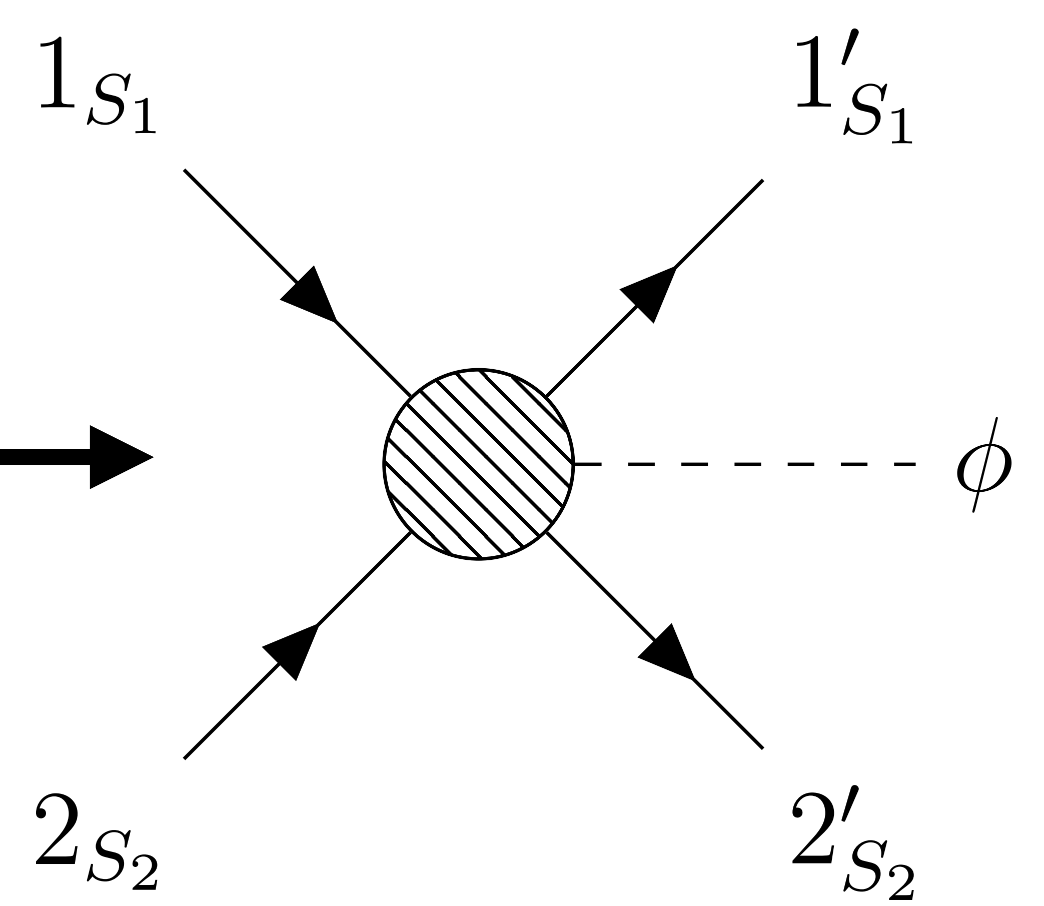


$$\sim \frac{\alpha}{\Lambda^2} \Big/ \frac{\tilde{\alpha}}{\Lambda^2}$$

Gives an extra contribution to the 4-point amplitudes



Contributes to the leading order 5-point amplitude



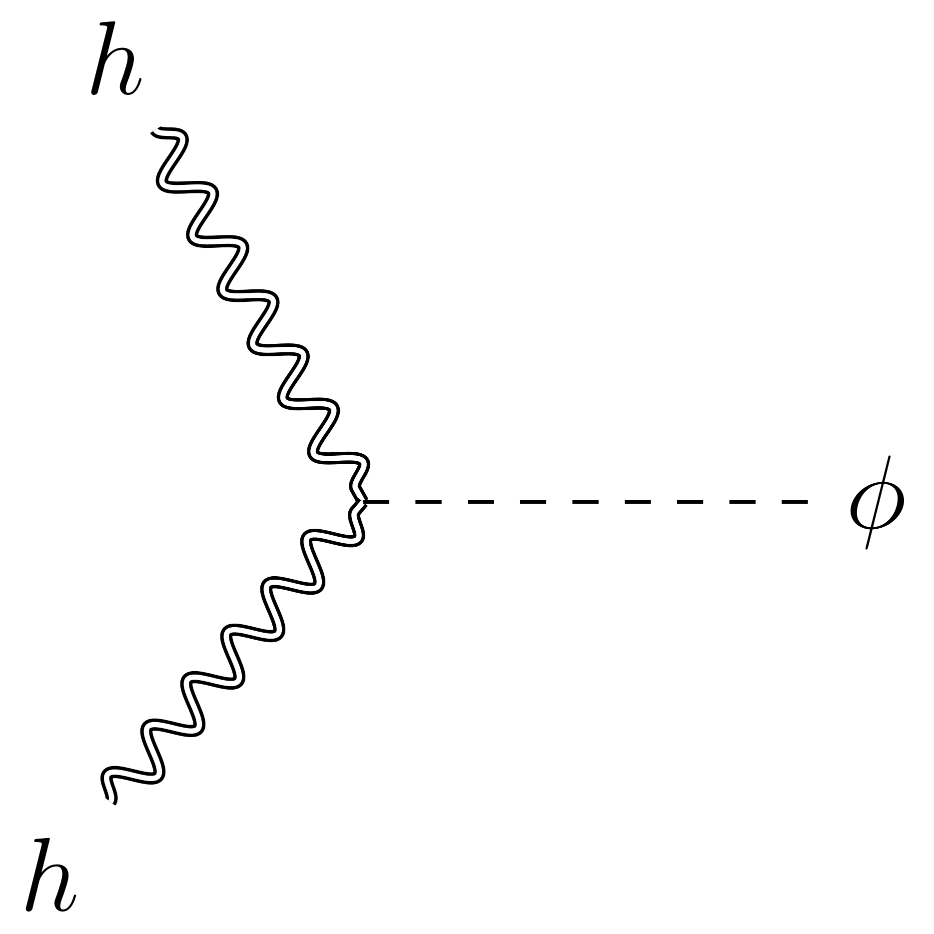
Can compute the Scalar Waveform in the KMOC formalism!

On-shell amplitudes:

*** Have to include <u>contact terms'</u> deformations which can contribute classically: Can be done order by order in the spin expansion, but need a matching procedure to fix their coefficients**

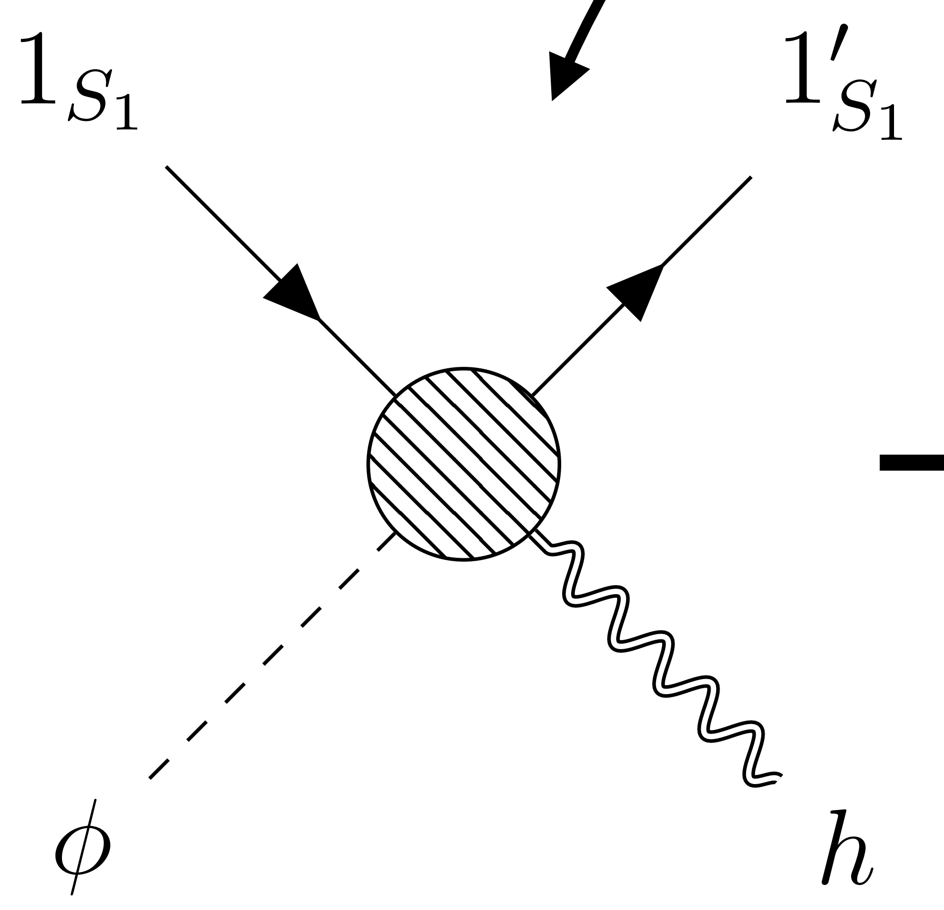
Let's work by expanding $f(\phi) \approx c + \phi + \mathcal{O}(\phi^2)$.

Naively, this action produces an extra 3-point on-shell amplitude which we should consider:

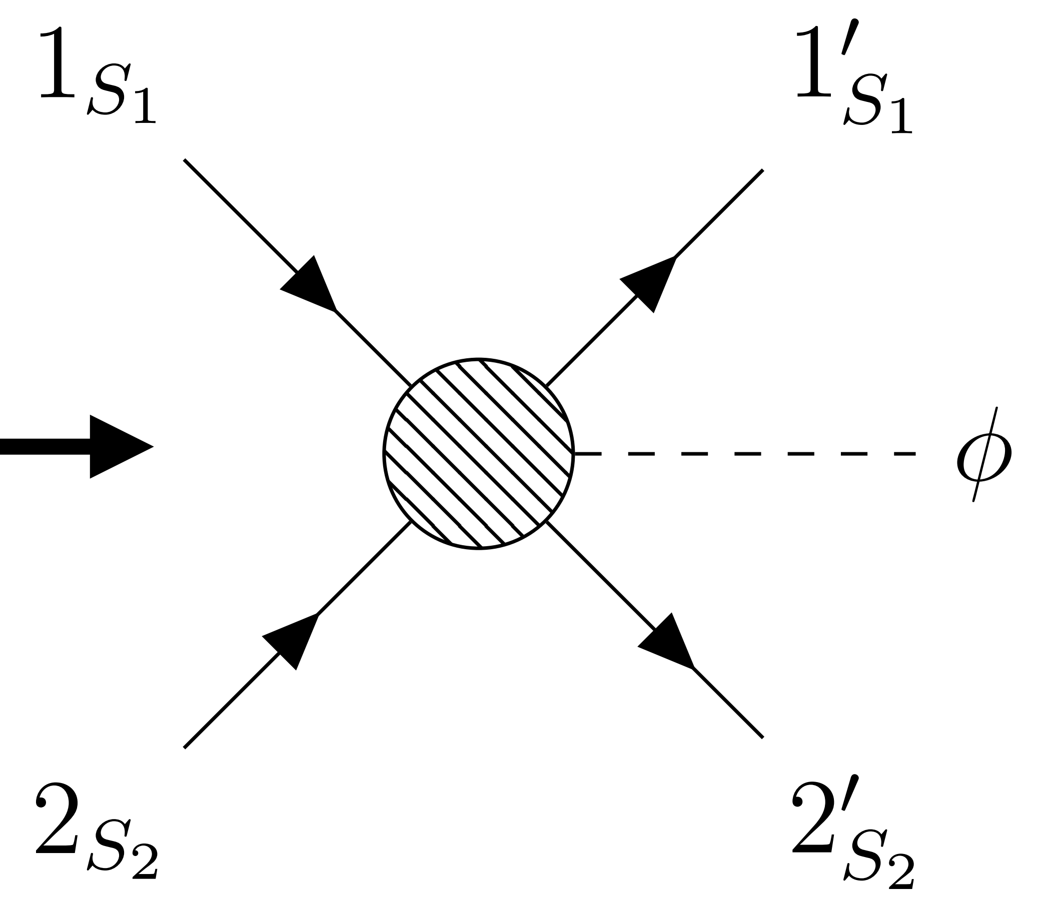


$$\sim \frac{\alpha}{\Lambda^2} \Big/ \frac{\tilde{\alpha}}{\Lambda^2}$$

Gives an extra contribution to the 4-point amplitudes *



Contributes to the leading order 5-point amplitude



Can compute the Scalar Waveform in the KMOC formalism!

Results:

The scalar waveform for no-hair compact objects in scalar Gauss-Bonnet:

$$
\begin{aligned}
W_\phi = \frac{m_1 m_2}{8\pi^2 b^3 M_{Pl}^3 \Lambda^2 (\hat{u}_1 n)^2 \sqrt{\gamma^2-1}} \Bigg(\alpha \Bigg\{ -\frac{d^2}{dz^2} \Bigg[\frac{1}{\sqrt{z^2+1}} \frac{2(\gamma^2-1)^2 (\hat{u}_2 n)^2 \big[\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b} n) z\big]}{\big[\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b} n) z\big]^2 + (\tilde{v} n)^2 (z^2+1)} \Bigg] + \big[(\hat{u}_1 n) + \gamma\big(2\gamma^2-3\big)(\hat{u}_2 n)\big] \frac{2z^2-1}{(z^2+1)^{5/2}} + \big(2\gamma^2-1\big)(\tilde{b} n) \frac{3z}{(z^2+1)^{5/2}} \Bigg\} \\
+ \frac{2\alpha}{b} \frac{d^3}{dz^3} \mathrm{Re} \Bigg\{ \frac{1}{\sqrt{z^2+1}} \frac{(\hat{u}_2 n) - \gamma(\hat{u}_1 n) + \gamma(\tilde{b} n) z + i\gamma(\tilde{v} n)\sqrt{z^2+1}}{\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b} n) z + i(\tilde{v} n)\sqrt{z^2+1}} \Big[z(\tilde{v} n) - i\sqrt{z^2+1}(\tilde{b} n) \Big] \times \Bigg[\frac{(a_1 n)}{(\hat{u}_1 n)} - \gamma(a_1 \hat{u}_2) - (a_2 \hat{u}_1) - (a_1^A - a_2^A)(z\tilde{b}^A + i\sqrt{z^2+1}\,\tilde{v}^A) \Bigg] \Bigg\} \\
- \sqrt{\gamma^2-1} \frac{C_1}{b} \frac{d^3}{dz^3} \Bigg\{ \frac{1}{\sqrt{z^2+1}} \mathrm{Re} \Bigg[\Bigg(z\big[(\tilde{v} n) a_1^A + (a_1 \tilde{v}) n^A\big] - i\sqrt{z^2+1}\big[(\tilde{b} n) a_1^A + (a_1 \tilde{b}) n^A\big] \Bigg) \times \Bigg(\hat{u}_2^A - \gamma \hat{u}_1^A + \gamma\big[z\tilde{b}^A + i\sqrt{z^2+1}\,\tilde{v}^A\big] \Bigg) \Bigg] \Bigg\} \Bigg) \Bigg|_{z=T_1} + (1 \leftrightarrow 2) + \mathcal{O}(a^2) \, .
\end{aligned}
$$

Connect to observables: Power emitted in scalar radiation

$$\frac{dP_\phi}{d\Omega} = (\partial_t W_\phi)^2$$

$$\frac{dP_\phi}{d\Omega}\Bigg|_{\mathcal{O}(a^0)} \sim \frac{\beta^6}{b^8} \quad \xrightarrow{\text{For closed orbits}} \quad \frac{dP_\phi}{d\Omega}\Bigg|_{\mathcal{O}(a^0)} \sim \beta^{22}$$

→ **Bigger suppression** compared to β^8 previously computed with GR methods

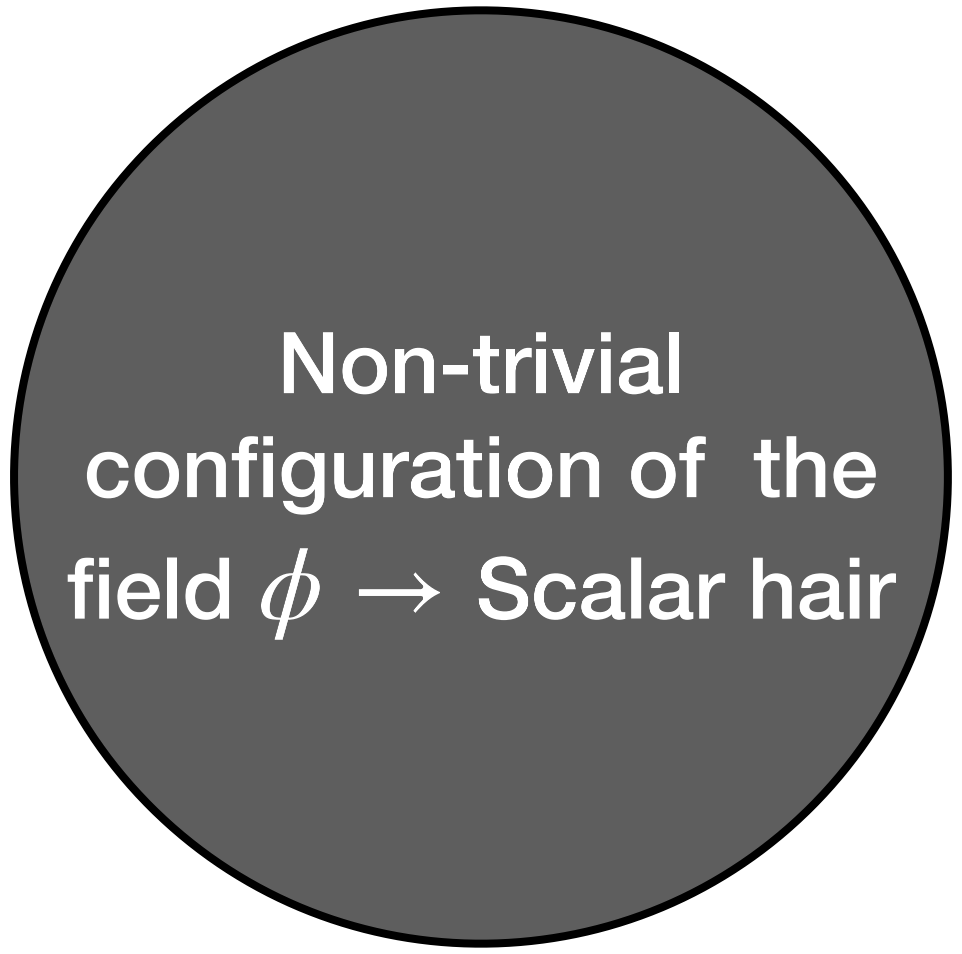
Phys.Rev.D 85 (2012) 064022, Phys.Rev.D 93 (2016) 2, 029902 (erratum) [Yagi, Stein, Yunes, Tanaka]

$$\frac{dP_\phi}{d\Omega}\Bigg|_{\mathcal{O}(a^1)} \sim \frac{\beta^4}{b^{10}} \quad \xrightarrow{\text{For closed orbits}} \quad \frac{dP_\phi}{d\Omega}\Bigg|_{\mathcal{O}(a^1)} \sim \beta^{24}$$

So what's the difference?

Scalar hair in scalar-tensor theories:

Compact objects can acquire scalar hair in ST theories ⟶ Exactly the case for SGB and DCS!



BH solution in ST theory

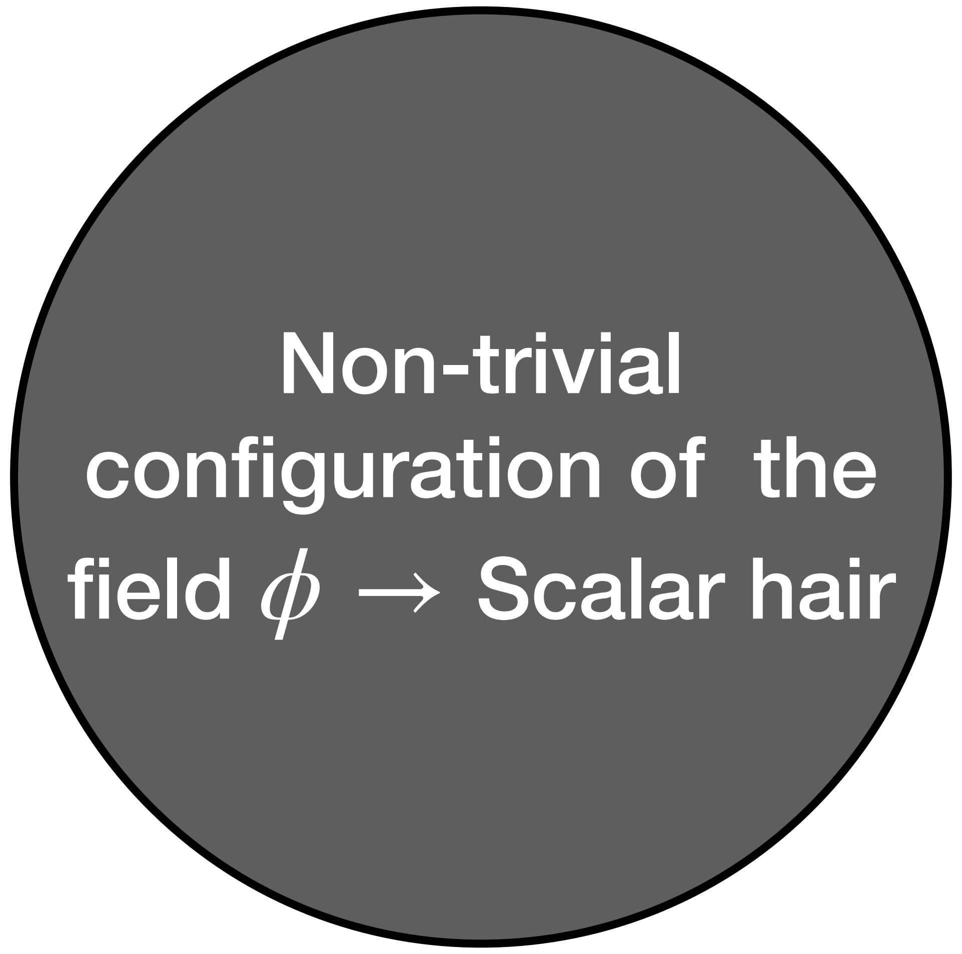
Far zone

$x \rightarrow \infty$

$$\phi = \frac{c_1}{r} + \frac{c_2}{r^2} + \dots$$

Scalar hair in scalar-tensor theories:

Compact objects can acquire scalar hair in ST theories ⟶ Exactly the case for SGB and DCS!



BH solution in ST theory

Far zone

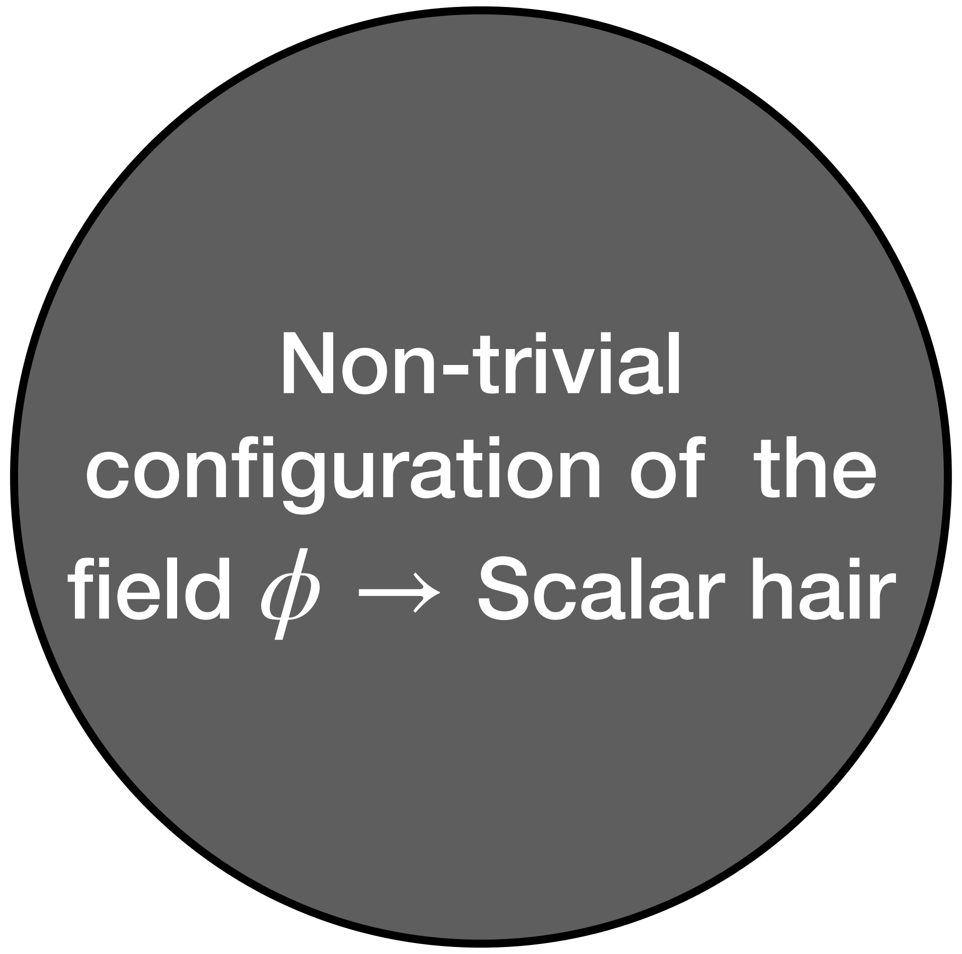
$x \rightarrow \infty$

“Monopole hair” “Dipole hair”

$$\phi = \frac{c_1}{r} + \frac{c_2}{r^2} + \dots$$

Scalar hair in scalar-tensor theories:

Compact objects can acquire scalar hair in ST theories ⟶ Exactly the case for SGB and DCS!



BH solution in ST theory

Far zone

$x \to \infty$

"Monopole hair" (c_1)   "Dipole hair" (c_2)

$$\phi = \frac{c_1}{r} + \frac{c_2}{r^2} + \dots$$

So how can we model this behaviour with amplitudes?

The on-shell way again:

We model the BH as a point-particle interacting with the scalar field in a ST fashion

Most general effective metric that respects causality is:

$$\tilde{g}_{\mu\nu} = \underbrace{\exp\left[C\left(\frac{\phi}{M_{Pl}}\right)\right]g_{\mu\nu}}_{\text{Conformal coupling}} + \underbrace{D\left(\frac{\phi}{M_{Pl}}\right)\frac{D_\mu\phi D_\nu\phi}{M_{Pl}^2\Lambda^2}}_{\text{Disformal coupling}},$$

The on-shell way again:

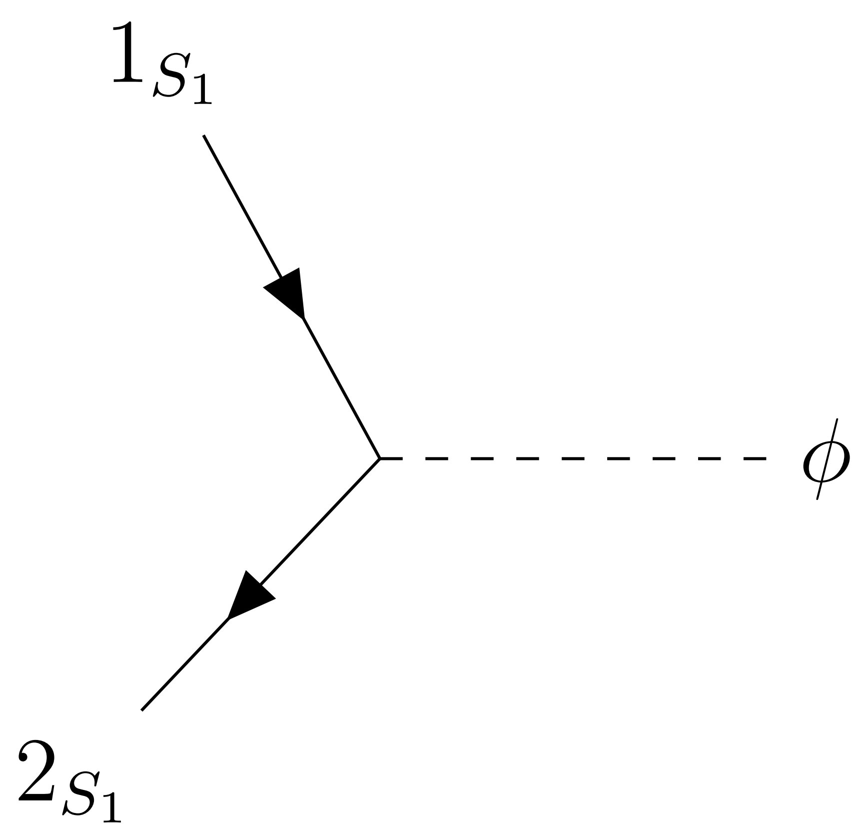
We model the BH as a point-particle interacting with the scalar field in a ST fashion

Neglect it, heavily suppressed

Most general effective metric that respects causality is: $\tilde{g}_{\mu\nu} = \underbrace{\exp\Big[C\big(\frac{\phi}{M_{Pl}}\big)\Big] g_{\mu\nu}}_{\text{Conformal coupling}} + \underbrace{D\big(\frac{\phi}{M_{Pl}}\big)\frac{D_\mu \phi D_\nu \phi}{M_{Pl}^2 \Lambda^2}}_{\text{Disformal coupling}},$

Generate 3-point amplitudes for arbitrary spinning BH:

$$\exp\Big[C\big(\frac{\phi}{M_{Pl}}\big)\Big] \approx 1 + c\frac{\phi}{M_{Pl}}$$



The on-shell way again:

We model the BH as a point-particle interacting with the scalar field in a ST fashion

Neglect it, heavily suppressed

Most general effective metric that respects causality is: $\tilde{g}_{\mu\nu} = \underbrace{\exp\left[C\left(\frac{\phi}{M_{Pl}}\right)\right] g_{\mu\nu}}_{\text{Conformal coupling}} + \underbrace{D\left(\frac{\phi}{M_{Pl}}\right)\frac{D_\mu \phi D_\nu \phi}{M_{Pl}^2 \Lambda^2}}_{\text{Disformal coupling}},$

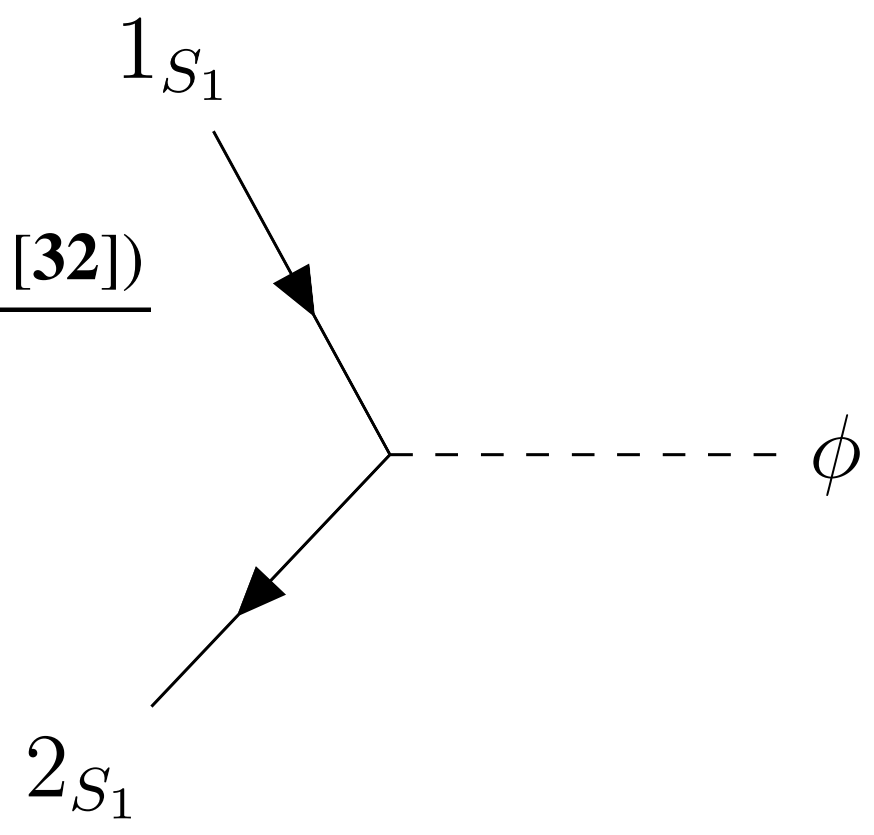
Conformal coupling   Disformal coupling

Generate 3-point amplitudes for arbitrary spinning BH:

$$\exp\left[C\left(\frac{\phi}{M_{Pl}}\right)\right] \approx 1 + c\frac{\phi}{M_{Pl}}$$

$$\mathcal{M}_{3,bos.}(1_\phi, 2_A, 3_A) = \frac{c_1}{M_{Pl}} \frac{\langle \mathbf{32}\rangle^S [\mathbf{32}]^S}{m^{2S-2}}$$

$$\mathcal{M}_{3,ferm.}(1_\phi, 2_\Psi, 3_{\bar{\Psi}}) = \frac{c_1}{M_{Pl}} \frac{\langle \mathbf{32}\rangle^{S-1/2} [\mathbf{32}]^{S-1/2} (\langle \mathbf{32}\rangle + [\mathbf{32}])}{m^{2S-2}}$$



Simple mass redefinition at all spin orders:

$$m \rightarrow e^{C/2} m$$

Resemblance to skeletonized action used in GR literature:

Astrophysical Journal, vol. 196, Mar. 1, 1975, pt. 2, p. L59-L62. [Eardley]

1992 Class. Quantum Grav. 9 2093 [Damour, Esposito-Farese]

The on-shell way again:

We model the BH as a point-particle interacting with the scalar field in a ST fashion

Neglect it, heavily suppressed

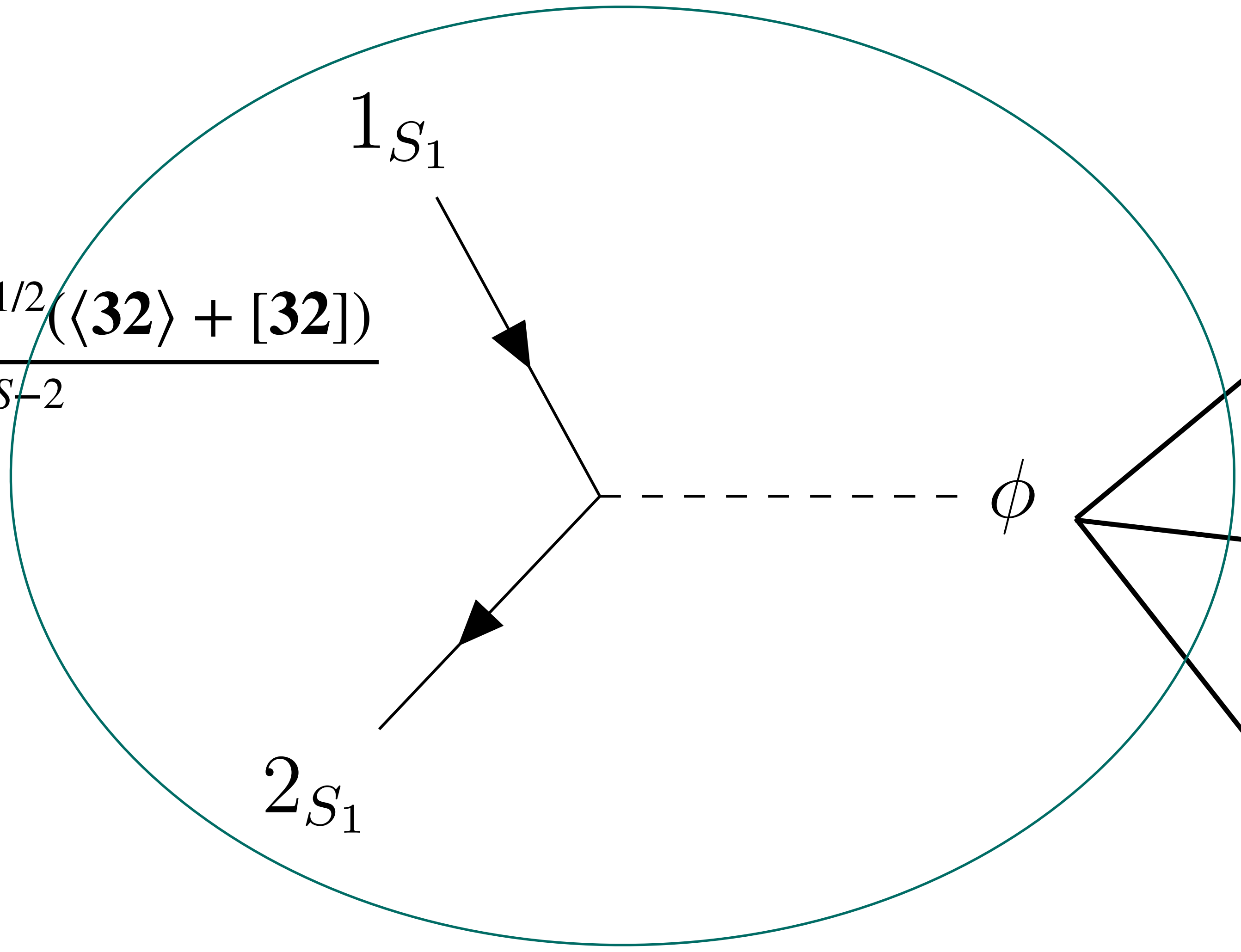
Most general effective metric that respects causality is:

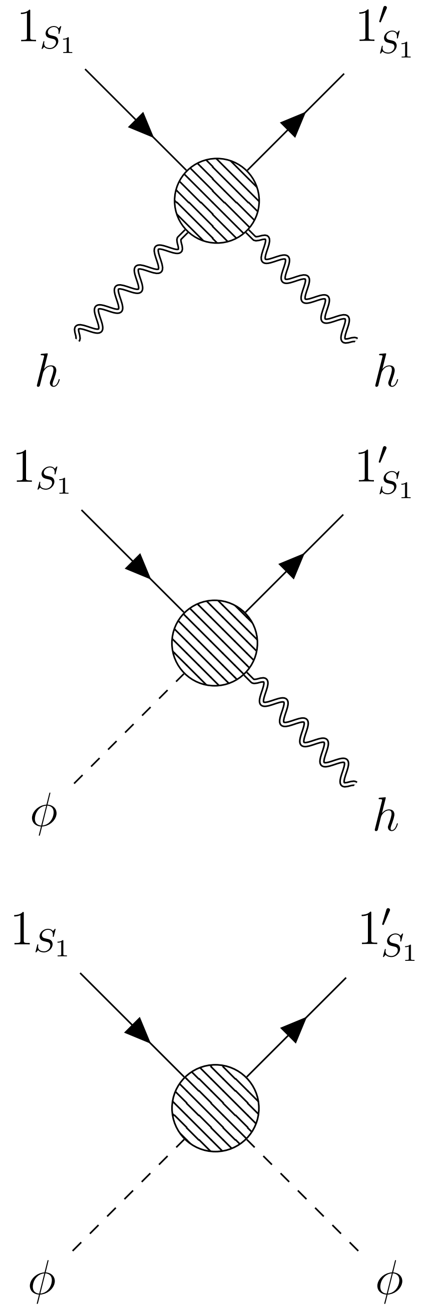
$$\tilde{g}_{\mu\nu} = \underbrace{\exp\left[C\left(\frac{\phi}{M_{Pl}}\right)\right] g_{\mu\nu}}_{\text{Conformal coupling}} + \underbrace{D\left(\frac{\phi}{M_{Pl}}\right)\frac{D_\mu\phi D_\nu\phi}{M_{Pl}^2\Lambda^2}}_{\text{Disformal coupling}},$$

Generate 3-point amplitudes for arbitrary spinning BH:

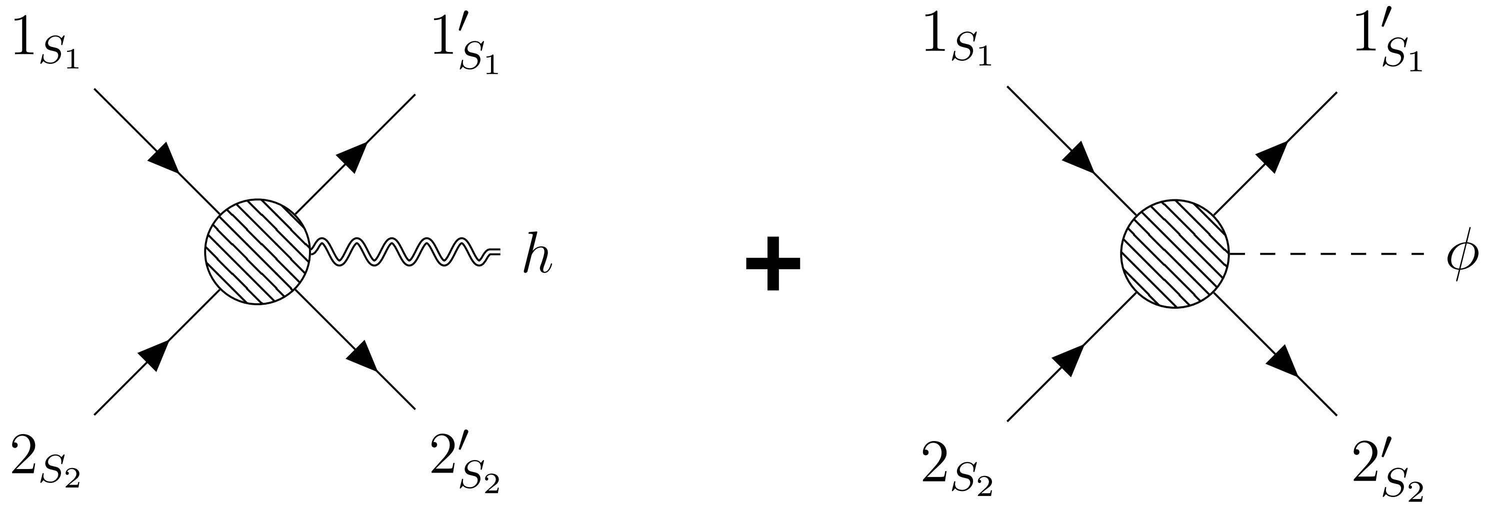
$$\mathcal{M}_{3,bos.}(1_\phi, 2_A, 3_A) = \frac{c_1}{M_{Pl}}\frac{\langle \mathbf{32}\rangle^S [\mathbf{32}]^S}{m^{2S-2}}$$

$$\mathcal{M}_{3,ferm.}(1_\phi, 2_\Psi, 3_{\bar{\Psi}}) = \frac{c_1}{M_{Pl}}\frac{\langle \mathbf{32}\rangle^{S-1/2} [\mathbf{32}]^{S-1/2}(\langle \mathbf{32}\rangle + [\mathbf{32}])}{m^{2S-2}}$$





$$\exp\left[C\left(\frac{\phi}{M_{Pl}}\right)\right] \approx 1 + c\frac{\phi}{M_{Pl}}$$



Simple mass redefinition at all spin orders:

$$m \rightarrow e^{C/2} m$$

Resemblance to skeletonized action used in GR literature:

Astrophysical Journal, vol. 196, Mar. 1, 1975, pt. 2, p. L59-L62. [Eardley]

1992 Class. Quantum Grav. 9 2093 [Damour, Esposito-Farese]