Composite Supersymmetric Dark Matter

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Introduction : Dark Matter in SUSY formalism

- DM candidate in the MSSM : Lightest SUSY particle (LSP), neutralino or gravitino :

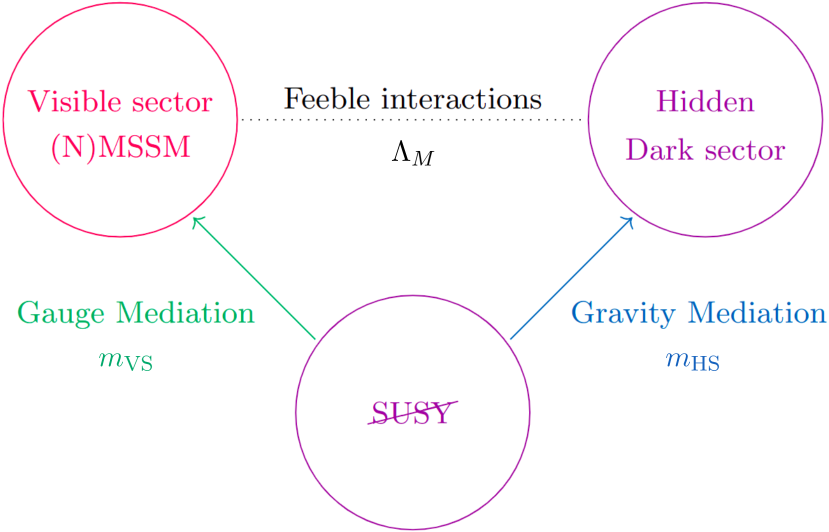
  → No signals detected, strong constraints on MSSM parameter space

- Alternative to LSP : hidden sector, exact SUSY at the DM mass scale

  → Hierarchy between the SUSY breaking scales in the visible and hidden sectors

Content

I. The model


Visible sector
(N)MSSM
Feeble interactions
Λ_M
Hidden
Dark sector
Gauge Mediation
m_{VS}
Gravity Mediation
m_{HS}
~~SUSY~~

I. SUSY breaking mediation : Gravity mediation

- Messenger : chiral superfield X such as $\langle F_X \rangle \neq 0$
- Interaction with the dark sector :

$$\mathcal{L} \supset \int \mathrm{d}^2\theta \left(\frac{c}{M_P} X W^{\alpha}_{\mathrm{HS}} W_{\mathrm{HS}\ \alpha} + \mathrm{h.c.} \right). \quad (1)$$

- SUSY breaking scale in the hidden sector :

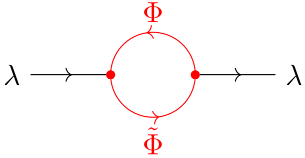
$$m_{\mathrm{HS}} \sim \frac{\langle F_X \rangle}{M_P}. \quad (2)$$

I. SUSY breaking mediation : Gauge mediation

- SUSY breaking parameter : spurion X with $\langle X\rangle = M_G + \theta^2\langle F_X\rangle$
- Messengers : chiral superfields Φ, $\tilde{\Phi}$
- Interaction :

$$\int \mathrm{d}^2\theta X\Phi\tilde{\Phi}. \tag{3}$$

- SUSY Breaking scale in the visible sector :



$$m_{\mathrm{VS}} \sim \frac{g_G^2}{16\pi^2}\frac{\langle F_X\rangle}{M_G}. \tag{4}$$

I. SUSY breaking mediation : order of magnitudes

- Assumptions : $m_{\mathrm{DM}} \geq 100$ keV, $\quad g_G \sim 1$,

$$\longrightarrow \quad \langle F_X \rangle \leq 10^{15} \ \text{GeV}^2, \qquad M_G \leq 10^{10} \ \text{GeV}.$$

- Cosmological Gravitino problem : bound $m_{3/2} \leq 4.7$ eV, DM can decay in gravitinos when

$$\langle F_X \rangle \leq 10^{10} \ \text{GeV}.$$

$\longrightarrow$ But decay channels surpressed by M_P, Λ_M, and $\Delta m \sim m_{3/2} \sim m_{\mathrm{HS}}$.

II. Example of Dark sector : Super Yang Mills (SYM)

- Super Yang Mills : gluons (v^μ) and gluinos (λ) dynamics, SU(N_c) gauge group

- Lagrangian :

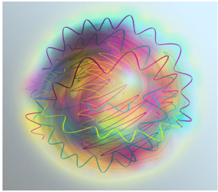
$$\begin{aligned} \mathcal{L}_{\text{SYM}} &= \frac{1}{32\pi}\text{Im}\left(\tau\int \mathrm{d}^2\theta \text{Tr} W^\alpha W_\alpha\right) \\ &= \text{Tr}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - i\lambda\sigma^\mu D_\mu\bar{\lambda} + \frac{1}{2}D^2\right] + \frac{\theta_{\text{SYM}}}{32\pi^2}g^2\text{Tr}F_{\mu\nu}\tilde{F}^{\mu\nu} \end{aligned} \tag{5}$$

- ✳ $\tau = \frac{\theta_{\text{SYM}}}{2\pi} + \frac{4\pi i}{g^2}$
- ✳ $F_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu - ig[v_\mu, v_\nu]$
- ✳ D : Auxiliary field
- ✳ $D_\mu\bar{\lambda} = \partial_\mu\lambda - ig[v_\mu, \lambda]$
- ✳ $\tilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$

II. Example of Dark sector : Super Yang Mills (SYM)

- SYM is expected to confine at a scale Λ : uncoloured bound states made of gluons and gluinos $\rightarrow$ glueballs $(v^{\mu}v_{\mu})$ and gluinoballs $(\lambda\lambda)$
- Suitable dark matter candidates :

✓Electrically neutral ✓Uncoloured
✓Massive ✓Stable



[Image credit : TU Wien]

II. Dark Matter production : defining the feeble interactions

- Simplest mediator fields : heavy chiral superfields $\mathcal{F}_i$ and $\tilde{\mathcal{F}}_i$ in conjugate representation of $\mathrm{SU}(N_c)$

1. If the visible sector is the MSSM, we can add 2 sets of $\mathcal{F}$ fields :

   $i = Q$, $\mathcal{F}_Q$ is a $\mathrm{SU}(2)_{\mathrm{w}}$ doublet, $\quad i = U$, $\mathcal{F}_U$ is a up-type singlet

$$\mathcal{L} \supset \int \mathrm{d}^2\theta \, \left[M_{\mathcal{F}} \left(\mathcal{F}_Q \tilde{\mathcal{F}}_Q + \mathcal{F}_U \tilde{\mathcal{F}}_U\right) + \lambda_M H_u \mathcal{F}_Q \tilde{\mathcal{F}}_U\right] + \text{h.c.} \tag{6}$$

Integrating out the heavy fields, we get a dim 6 operator :

$$\mathcal{L}_{\mathrm{dim6}} \supset \frac{1}{32\pi} \int \mathrm{d}^2\theta \left[\mathrm{Im}\left(\tau \mathrm{Tr} W^\alpha W_\alpha\right) \left(1 + \frac{1}{\Lambda_M^2} H_u H_u^\dagger + \ldots\right)\right] \tag{7}$$

II. Dark Matter production : defining the feeble interactions

2. If the visible sector is the NMSSM :

$$\mathcal{L} \supset \int \mathrm{d}^2\theta \, \left[M_{\mathcal{F}} \mathcal{F}\tilde{\mathcal{F}} + \lambda_M \hat{N} \mathcal{F}\tilde{\mathcal{F}} \right] + \mathrm{h.c} \tag{8}$$

Leading dim 5 operator :

$$\mathcal{L}_{\mathrm{dim5}} \supset \frac{1}{32\pi} \int \mathrm{d}^2\theta \left[\mathrm{Im} \left(\tau \mathrm{Tr} W^\alpha W_\alpha \right) \left(1 + \frac{1}{\Lambda_M} (\hat{N} + \hat{N}^\dagger) + ... \right) \right] \tag{9}$$

- Both cases : Λ_M determines the production rate in the dark sector

$$\frac{1}{\Lambda_M} \sim \frac{\lambda_M}{4\pi M_{\mathcal{F}}} \tag{10}$$

II. Dark Matter production

- Thermally decoupled sectors and non-renormalizable operator $\rightarrow$ UV freeze-in mechanism
- Dark Matter production before confinement in the hidden sector
- Hypotheses for a UV freeze-in mechanism : [Elahi, Kolda, Unwin 2015]

  - Gluon n_v and gluinos n_λ number densities are initially negligible
  - Dark and visible sectors are never at thermal equilibrium
  - Confinement energy of gluons/gluinos in glueballs/gluinoballs is negligible

II. Dark Matter production

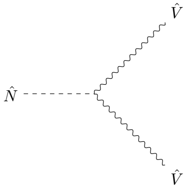
$\ast$ K_1 : Bessel function of 2nd kind

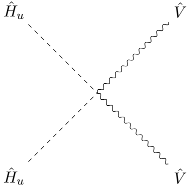
- Boltzmann equations (simplified) :

$$\frac{\mathrm{d}n_{\mathrm{HS}}}{\mathrm{d}t} + 3Hn_{\mathrm{HS}} \simeq \frac{T}{512\pi^5} \int_0^\infty \mathrm{d}s |\mathcal{M}|^2 \sqrt{s} K_1(\sqrt{s}/T). \tag{11}$$

- Amplitudes for both cases :

$$|\mathcal{M}_{\mathrm{dim5}}|^2 \sim \frac{N_c}{\Lambda_M^2} s \qquad \text{and} \qquad |\mathcal{M}_{\mathrm{dim6}}|^2 \sim \frac{N_c}{\Lambda_M^4} s^2. \tag{12}$$





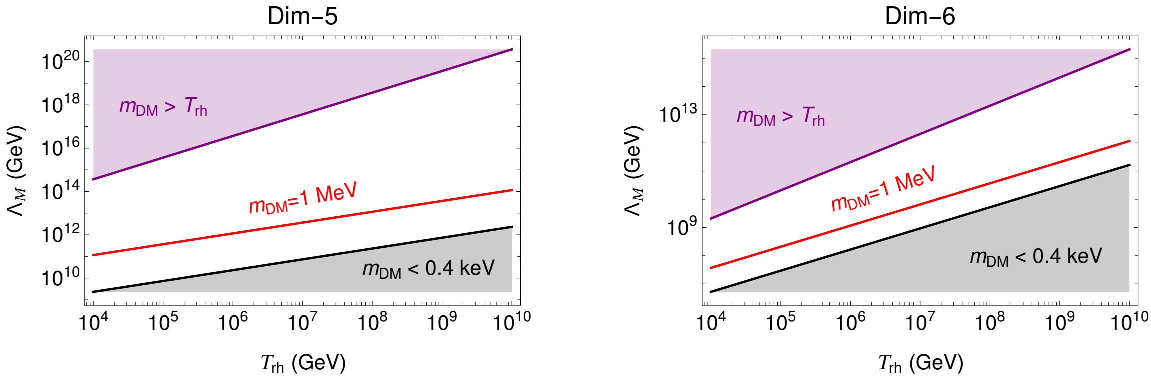
II. Dark Matter production

- DM relic density $\Omega_{\rm DM} = \frac{m_{\rm DM} Y_{\rm DM} s_0}{\rho_c}$:

$$\Omega_{\rm dim5} h^2 \simeq 0.134 \times 10^{21} N_c \frac{T_{\rm rh} m_{\rm DM}}{\Lambda_M^2}$$

and

$$\Omega_{\rm dim6} h^2 \simeq 0.185 \times 10^{21} N_c \frac{T_{\rm rh}^3 m_{\rm DM}}{\Lambda_M^4}$$



III. Low energy Dynamics : Veneziano-Yankeliowicz effective theory

- Difficulty to describe the confined theory with SYM [Veneziano, Yankeliowicz 1982]

- Veneziano and Yankeliowicz idea : introduction of a chiral superfield S such as

$$S = \phi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y), \qquad y_\mu = x_\mu + i\theta\sigma_\mu\bar{\theta}. \tag{13}$$

$$\phi(y) \equiv \frac{\beta(g)}{2g}\lambda^\alpha\lambda_\alpha, \qquad \sqrt{2}\psi_\alpha(y) \equiv -\frac{\beta(g)}{2g}\left(-i\lambda_\alpha D + (\sigma^{\mu\nu}\lambda)_\alpha F_{\mu\nu}\right), \tag{14}$$

$$F(y) \equiv -\frac{\beta(g)}{g}\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{i}{2}\bar{\lambda}\bar{\sigma}\bar{\nabla}\lambda + \frac{1}{2}D^2 - \frac{i}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{i}{2}\partial_\mu J^{\mu 5}\right). \tag{15}$$

III. Low energy Dynamics : Veneziano-Yankeliowicz effective theory

- Veneziano-Yankeliowicz Lagrangian :

$$\mathcal{L}_{\mathrm{VY}}^{N_c} = \frac{9N_c^2}{\alpha}(S^\dagger S)^{\frac{1}{3}}\Big|_D + \left[\frac{2N_c}{3}\, S\left(\log\left(\frac{S}{\Lambda^3}\right)^{N_c} - N_c\right)\Big|_F + \text{h.c.}\right] \tag{16}$$

✳ Λ : dynamical energy scale	✳ α : order 1 parameter

Issue : glueballs appear in the auxiliary field

III. Low energy Dynamics : Veneziano-Yankeliowicz effective theory

- Idea : add a glueball chiral superfield χ :

$$\chi = \phi_\chi + \sqrt{2}\theta\psi_\chi + \theta^2 F_\chi \qquad [\chi] = 0 \tag{17}$$

[Merlatti, Sannino 2004]

- Generalization of $\mathcal{L}_{\mathrm{VY}}^{N_c}$:

$$\begin{aligned} \mathcal{L}_{\mathrm{gVY}}^{N_c} &= \frac{9N_c^2}{\alpha}(S^\dagger S)^{\frac{1}{3}}\left(1 + \gamma\chi\chi^\dagger\right)\Big|_D \\ &+ \frac{2N_c}{3}S\left(\log\left(\frac{S}{\Lambda^3}\right)^{N_c} - N_c - N_c \ln\left(-e\frac{\chi}{N_c}\ln\chi^N\right)\right)\Big|_F + \mathrm{h.c.} \end{aligned} \tag{18}$$

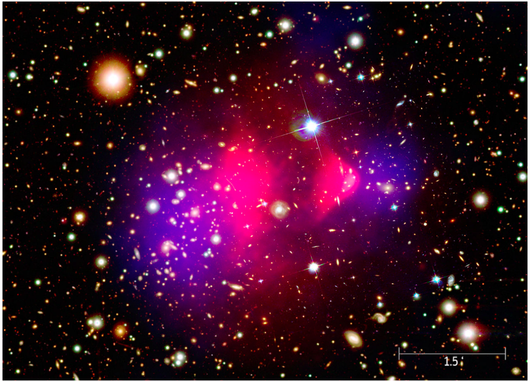
$\ast$ γ : fixed parameter

$\longrightarrow$ Develop to get the scalar potential $V(\phi, \bar{\phi}, \phi_\chi, \bar{\phi}_\chi)$

III. Bound on Dark Matter mass

- Use bounds on self-scattering interactions from Bullet clusters :

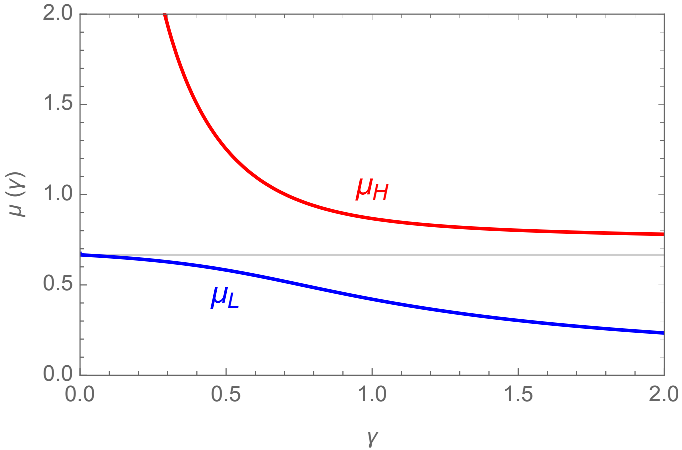
$$\frac{\sigma_{\text{DM}}}{m_{\text{DM}}} \leq 2 \text{ cm.g}^{-1} \qquad \text{[Robertson, Massey, Eke 2016]} \qquad (19)$$



[Image credit : Chandra 2004]

III. Bound on Dark Matter mass

- Diagonalize scalar potential to get mass eigenstates ϕ_L (light) and ϕ_H (heavy) :



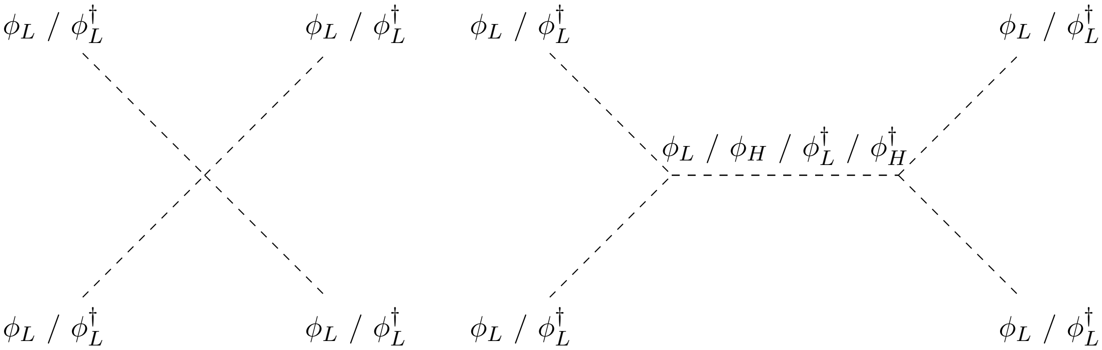
$$m_{L,H} = \alpha\Lambda\ \mu_{L,H}(\gamma). \tag{20}$$

$$m_S = \frac{2}{3}\alpha\Lambda. \tag{21}$$

III. Bound on Dark Matter mass

- Terms in the Lagrangian allowing scattering of the light eigenstate :

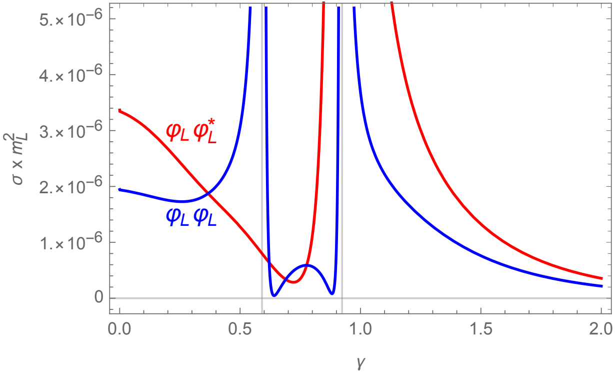
$$\mathcal{L} \supset C_{31}\ (\varphi_L^3 \varphi_L^\dagger + \text{h.c.}) + C_{22}\ \varphi_L^2 (\varphi_L^\dagger)^2 + m_L \left(c_{21}\ \varphi_L^2 \varphi_L^\dagger + \text{h.c.} \right) \\ + m_H \left(c_{20}^H\ \varphi_L^2 \varphi_H^\dagger + c_{11}^H\ \varphi_L \varphi_L^\dagger \varphi_H^\dagger + \text{h.c.} \right) . \tag{22}$$



III. Bound on Dark Matter mass

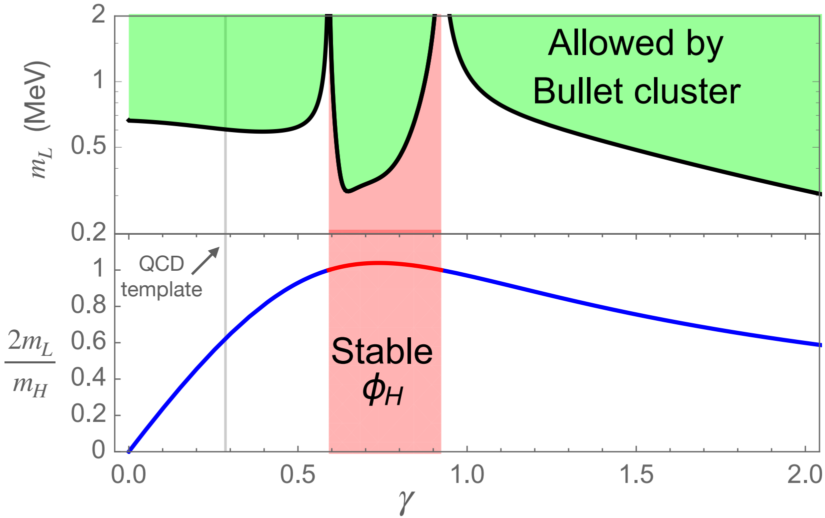
- DM cross section :

$$\sigma(\varphi_L\varphi_L^\dagger) = \sum_f \frac{|\mathcal{A}(\varphi_L\varphi_L^\dagger \to f)|^2}{128\pi\ m_L^2}\,, \quad \sigma(\varphi_L\varphi_L) = \sum_f \frac{|\mathcal{A}(\varphi_L\varphi_L \to f)|^2}{128\pi\ m_L^2}\,, \tag{23}$$



$$\sigma_{\mathrm{DM}} = \frac{\sigma(\varphi_L\varphi_L^\dagger) + \sigma(\varphi_L\varphi_L)}{2} \sim \frac{\alpha^6}{N_c^4}\frac{|\tilde{A}(\gamma)|^2}{128\pi m_L^2} \tag{24}$$

III. Bound on Dark Matter mass



$\alpha = 1,\ N_C = 3$

Conclusion

- SUSY hidden sectors offer new possibilities for Dark Matter
- Example with a SYM hidden sectors where predictions can be made :
  - Dark matter are gluons and gluinos bound states called glueballs and gluinoballs
  - DM production through UV Freeze-in
  - Constraints on the DM mass using Bullet Cluster data
- Outlooks :
  - Constraints from Domain Walls in SYM theory
  - Possibility to construct the same kind of model using different SUSY theories

Why Supersymmetry (SUSY) ?

- SUSY is the natural extension of Poincaré algebra, attractive formalism
- Superfields in superspace $(y_\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}})$ with $y_\mu = x_\mu + i\theta\sigma_\mu\bar{\theta}$:
  - Chiral : $\Phi(y,\theta) = \phi(y) + \sqrt{2}\psi(y)\theta + F(y)\theta^2, \qquad \bar{D}_{\dot{\alpha}}\Phi = 0.$
  - Vector : $V(y,\theta,\bar{\theta}) = \theta\sigma^\mu\bar{\theta}v_\mu(y) + i\theta^2\bar{\theta}\bar{\lambda}(y) - i\theta\bar{\theta}^2\lambda(y) + \frac{1}{2}\theta^2\bar{\theta}^2(D(y) - i\partial_\mu v^\mu(y)).$

* ϕ : scalar field
* ψ_α, λ_α : spinors
* v^μ : vector field
* F, D : auxiliary fields

Dark Matter in SUSY formalism

- Minimal Supersymmetric Standard Model (MSSM) :

	Superfield	$SU(3)$	$SU(2_L)$	$U(1)_Y$	Particles
Quarks/Squarks	$\hat{Q}$	3	2	1/6	(u_L, d_L), $(\tilde{u}_L, \tilde{d}_L)$
	$\hat{U}^c$	$\bar{3}$	1	$-2/3$	$\bar{u}_R$, $\tilde{u}_R^*$
	$\hat{D}^c$	$\bar{3}$	1	1/3	$\bar{d}_R$, $\tilde{d}_R^*$
Leptons/Sleptons	$\hat{L}$	1	2	$-1/2$	(ν_L, e_L), $(\tilde{\nu}_L, \tilde{e}_L)$
	$\hat{E}^c$	1	1	1	$\bar{e}_R$, $\tilde{e}_R^*$
Higgs/Higgsinos	$\hat{H}_u$	1	2	1/2	$(H_u, \tilde{h}_u)$
	$\hat{H}_d$	1	2	$-1/2$	$(H_d, \tilde{h}_d)$
Gauge/Gauginos	$\hat{G}^a$	8	1	0	G^μ, $\tilde{g}$
	$\hat{W}^i$	1	3	0	W_i^μ, $\tilde{w}_i$
	$\hat{B}$	1	1	0	B^μ, $\tilde{b}$

- Next to Minimal Supersymmetric Standard Model (NMSSM) : MSSM + $\hat{N}$ superfield

Dark Matter production

- Comoving number density (yield) $Y_{\text{HS}} = \frac{n_{\text{HS}}}{s_e}$:

$$Y_{\text{dim5}} \simeq \frac{45 M_P N_c}{128\pi^7 1.66 g_*^s \sqrt{g_*^\rho}} \frac{T_{\text{rh}}}{\Lambda_M^2} \quad \text{and} \quad Y_{\text{dim6}} \simeq \frac{1485 M_P N_c}{1024\pi^7 1.66 g_*^s \sqrt{g_*^\rho}} \frac{T_{\text{rh}}^3}{\Lambda_M^4}. \tag{25}$$

g_*^s / g_*^ρ : number of effective degrees of freedom

- DM relic density $\Omega_{\text{DM}} = \frac{m_{\text{DM}} Y_{\text{DM}} s_0}{\rho_c}$:

$$\Omega_{\text{dim5}} h^2 \simeq 0.134 \times 10^{21} N_c \frac{T_{\text{rh}} m_{\text{DM}}}{\Lambda_M^2} \quad \text{and} \quad \Omega_{\text{dim6}} h^2 \simeq 0.185 \times 10^{21} N_c \frac{T_{\text{rh}}^3 m_{\text{DM}}}{\Lambda_M^4}. \tag{26}$$

Scalar potential

- Developing the gVY Lagrangian gives the interactions between the scalar parts of the glueballs (ϕ_χ) and the gluinoballs (ϕ) :

$$V(\phi, \bar{\phi}, \phi_\chi, \bar{\phi}_\chi) = (\phi\bar{\phi})^{\frac{2}{3}} \frac{4N^2\alpha}{9} \Bigg[\left| \log \left(\frac{\phi}{-e\Lambda^3 \phi_\chi \log \phi_\chi} \right) \right|^2 + \frac{1 + \gamma \phi_\chi \bar{\phi}_\chi}{9\gamma} \left| \frac{\log \phi_\chi + 1}{\phi_\chi \log \phi_\chi} \right|^2$$
$$+ \frac{\log \phi_\chi + 1}{3 \log \phi_\chi} \log \left(\frac{\bar{\phi}}{-e\Lambda^3 \bar{\phi}_\chi \log \bar{\phi}_\chi} \right) + \frac{\log \bar{\phi}_\chi + 1}{3 \log \bar{\phi}_\chi} \log \left(\frac{\phi}{-e\Lambda^3 \phi_\chi \log \phi_\chi} \right) \Bigg] .$$

Bound on Dark Matter mass

- We note :

$$C_x = \frac{\alpha^3}{N_c^2}\, F_x(\gamma)\,, \quad c_x = \sqrt{\frac{\alpha^3}{N_c^2}}\, f_x(\gamma)\,, \quad c_x^H = \sqrt{\frac{\alpha^3}{N_c^2}}\, f_x^H(\gamma) \tag{27}$$

- Amplitudes for the different scattering processes at 0 velocity, $\zeta = m_L^2/m_H^2$:

$$i\mathcal{A}(\varphi_L\varphi_L^\dagger \to \varphi_L\varphi_L^\dagger) = \frac{\alpha^3}{N_c^2}\left[4F_{22} + \frac{20}{3}f_{21}^2 + 4(f_{20}^H)^2 + 4(f_{11}^H)^2\left(1 - \frac{1}{4\zeta - 1}\right)\right], \tag{28}$$

$$i\mathcal{A}(\varphi_L\varphi_L^\dagger \to \varphi_L\varphi_L) = \frac{\alpha^3}{N_c^2}\left[6F_{31} + \frac{20}{3}f_{21}^2 + 2f_{20}^H f_{11}^H\left(2 - \frac{1}{4\zeta - 1}\right)\right], \tag{29}$$

$$i\mathcal{A}(\varphi_L\varphi_L^\dagger \to \varphi_L^\dagger\varphi_L^\dagger) = i\mathcal{A}(\varphi_L\varphi_L^\dagger \to \varphi_L\varphi_L)\,, \tag{30}$$

$$i\mathcal{A}(\varphi_L\varphi_L \to \varphi_L\varphi_L) = \frac{\alpha^3}{N_c^2}\left[4F_{22} + \frac{20}{3}f_{21}^2 - 4(f_{20}^H)^2\left(\frac{1}{4\zeta - 1}\right) + 8(f_{11}^H)^2\right], \tag{31}$$

$$i\mathcal{A}(\varphi_L\varphi_L \to \varphi_L\varphi_L^\dagger) = i\mathcal{A}(\varphi_L\varphi_L^\dagger \to \varphi_L\varphi_L)\,. \tag{32}$$