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Status of the Positron Range Study in GATE

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Positron Range in Literature

Isotope	Half-life	Branching (%)	E max (MeV)	E mean (MeV)	R max (mm)	R mean (mm)
^{11}C	20.4 min	99.8	0.960	0.386	4.2	1.2
^{13}N	10.0 min	99.8	1.199	0.492	5.5	1.8
^{15}O	2 min	99.9	1.732	0.735	8.4	3.0
^{18}F	**110 min**	**96.9**	**0.634**	**0.250**	**2.4**	**0.6**
^{64}Cu	12.7 h	17.5	0.653	0.278	2.5	0.7
^{89}Zr	78.4 h	22.7	0.902	0.396	3.8	1.3

Table: Properties of pure radioisotopes used in positron emission tomography [1] [2] [3].

- Calculation with Continuous Slowing Down Approxmiation (CSDA) 1 [1]
- Semiempircal calculations 2 [4]

$$R(E) = \int_0^E \frac{dE'}{S(E')} \quad (1)$$

$$R_{mean}(cm) \approx \frac{0.108[E_a^{max}]^{1.14}}{\tilde{n}(gcm^{-3})} \quad (2)$$

GATE Simulation of ^{18}F to obtain Positron Range

Phantom	**Isotope**	**Physicslist and Processes**	**Energy and Production Cuts**	**Actor**	**Activity**
Watersphere $r_{\text{radius}} = 300\,\text{mm}$	^{18}F defined with A=18 Z=9	Multiple Physicslists and DNA Models, Radioactive Decay, Decay	50 eV, 0.05 µm	Annihilation	100 kBq

- Output contains $[x, y, z]$-coordinates of annihilation locations
- r_{mean} calculated as mean of euclidean distances from origin $[0, 0, 0]$

$$r_{mean} = \sqrt{x^2 + y^2 + z^2} \tag{3}$$

- r_{max} defined as the maximum value in the dataset of euclidean distances

F18 Positron Kinetic Energy Spectrum

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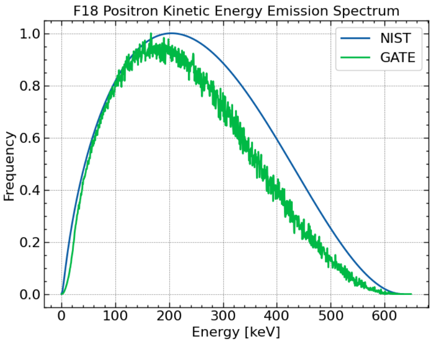
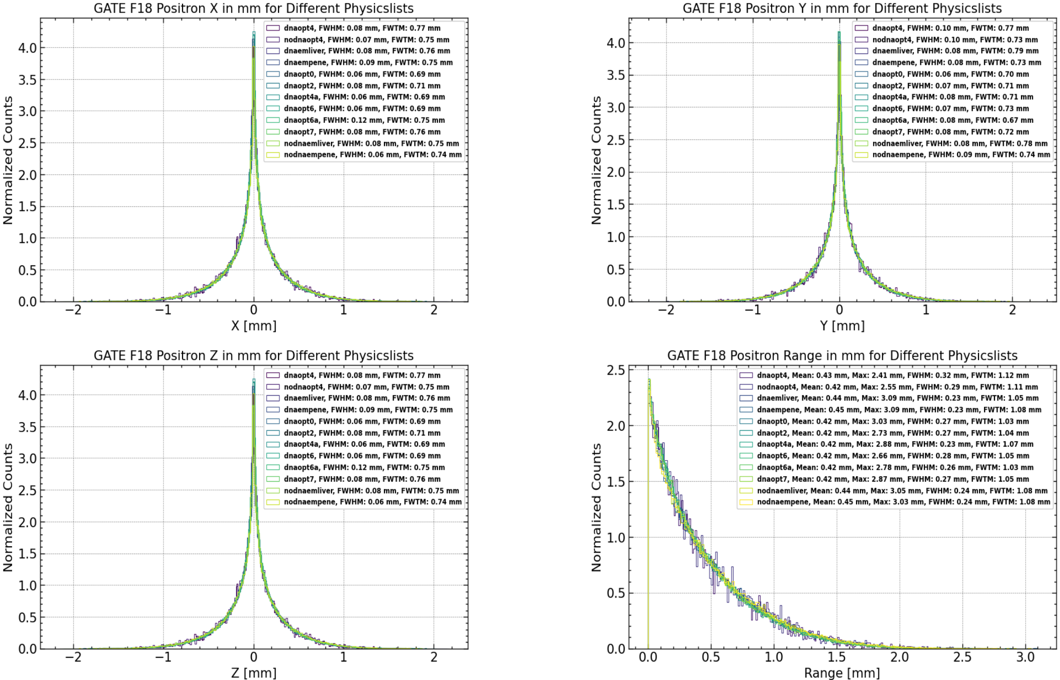


Figure: Simulated kinetic energy spectrum for F18, compared to NIST data.

Isotope	E max (MeV)	E mean (MeV)	R max (mm)	R mean (mm)
^{18}F	0.627	0.235	2.41	0.42

Table: Simulated positron properties of F18.


GATE F18 Positron X in mm for Different Physicslists
dnaopt4, FWHM: 0.08 mm, FWTM: 0.77 mm
nodnaopt4, FWHM: 0.07 mm, FWTM: 0.75 mm
dnaemliver, FWHM: 0.08 mm, FWTM: 0.76 mm
dnaempene, FWHM: 0.09 mm, FWTM: 0.75 mm
dnaopt0, FWHM: 0.06 mm, FWTM: 0.69 mm
dnaopt2, FWHM: 0.08 mm, FWTM: 0.71 mm
dnaopt4a, FWHM: 0.06 mm, FWTM: 0.69 mm
dnaopt6, FWHM: 0.06 mm, FWTM: 0.69 mm
dnaopt6a, FWHM: 0.12 mm, FWTM: 0.75 mm
dnaopt7, FWHM: 0.08 mm, FWTM: 0.76 mm
nodnaemliver, FWHM: 0.08 mm, FWTM: 0.75 mm
nodnaempene, FWHM: 0.06 mm, FWTM: 0.74 mm
Normalized Counts
X [mm]
GATE F18 Positron Y in mm for Different Physicslists
dnaopt4, FWHM: 0.10 mm, FWTM: 0.77 mm
nodnaopt4, FWHM: 0.10 mm, FWTM: 0.73 mm
dnaemliver, FWHM: 0.08 mm, FWTM: 0.79 mm
dnaempene, FWHM: 0.08 mm, FWTM: 0.73 mm
dnaopt0, FWHM: 0.06 mm, FWTM: 0.70 mm
dnaopt2, FWHM: 0.07 mm, FWTM: 0.71 mm
dnaopt4a, FWHM: 0.08 mm, FWTM: 0.71 mm
dnaopt6, FWHM: 0.07 mm, FWTM: 0.73 mm
dnaopt6a, FWHM: 0.08 mm, FWTM: 0.67 mm
dnaopt7, FWHM: 0.08 mm, FWTM: 0.72 mm
nodnaemliver, FWHM: 0.08 mm, FWTM: 0.78 mm
nodnaempene, FWHM: 0.09 mm, FWTM: 0.74 mm
Normalized Counts
Y [mm]
GATE F18 Positron Z in mm for Different Physicslists
dnaopt4, FWHM: 0.08 mm, FWTM: 0.77 mm
nodnaopt4, FWHM: 0.07 mm, FWTM: 0.75 mm
dnaemliver, FWHM: 0.08 mm, FWTM: 0.76 mm
dnaempene, FWHM: 0.09 mm, FWTM: 0.75 mm
dnaopt0, FWHM: 0.06 mm, FWTM: 0.69 mm
dnaopt2, FWHM: 0.08 mm, FWTM: 0.71 mm
dnaopt4a, FWHM: 0.06 mm, FWTM: 0.69 mm
dnaopt6, FWHM: 0.06 mm, FWTM: 0.69 mm
dnaopt6a, FWHM: 0.12 mm, FWTM: 0.75 mm
dnaopt7, FWHM: 0.08 mm, FWTM: 0.76 mm
nodnaemliver, FWHM: 0.08 mm, FWTM: 0.75 mm
nodnaempene, FWHM: 0.06 mm, FWTM: 0.74 mm
Normalized Counts
Z [mm]
GATE F18 Positron Range in mm for Different Physicslists
dnaopt4, Mean: 0.43 mm, Max: 2.41 mm, FWHM: 0.32 mm, FWTM: 1.12 mm
nodnaopt4, Mean: 0.42 mm, Max: 2.55 mm, FWHM: 0.29 mm, FWTM: 1.11 mm
dnaemliver, Mean: 0.44 mm, Max: 3.09 mm, FWHM: 0.23 mm, FWTM: 1.05 mm
dnaempene, Mean: 0.45 mm, Max: 3.09 mm, FWHM: 0.23 mm, FWTM: 1.08 mm
dnaopt0, Mean: 0.42 mm, Max: 3.03 mm, FWHM: 0.27 mm, FWTM: 1.03 mm
dnaopt2, Mean: 0.42 mm, Max: 2.73 mm, FWHM: 0.27 mm, FWTM: 1.04 mm
dnaopt4a, Mean: 0.42 mm, Max: 2.88 mm, FWHM: 0.23 mm, FWTM: 1.07 mm
dnaopt6, Mean: 0.42 mm, Max: 2.66 mm, FWHM: 0.28 mm, FWTM: 1.05 mm
dnaopt6a, Mean: 0.42 mm, Max: 2.78 mm, FWHM: 0.26 mm, FWTM: 1.03 mm
dnaopt7, Mean: 0.42 mm, Max: 2.87 mm, FWHM: 0.27 mm, FWTM: 1.05 mm
nodnaemliver, Mean: 0.44 mm, Max: 3.05 mm, FWHM: 0.24 mm, FWTM: 1.08 mm
nodnaempene, Mean: 0.45 mm, Max: 3.03 mm, FWHM: 0.24 mm, FWTM: 1.08 mm
Normalized Counts
Range [mm]

Physicslist/Model	Positron Range [mm]			
	Mean	Max	FWHM	FWTM
dnaopt4	0.43	2.41	0.32	1.12
nodnaopt4	0.42	2.55	0.29	1.11
dnaemliver	0.44	3.09	0.23	1.05
nodnaemliver	0.44	3.05	0.24	1.08
dnaempene	0.45	3.09	0.23	1.08
nodnaempene	0.45	3.03	0.24	1.08
dnaopt0	0.42	3.03	0.27	1.03
dnaopt2	0.42	2.73	0.27	1.04
dnaopt4a	0.42	2.88	0.23	1.07
dnaopt6	0.42	2.66	0.28	1.05
dnaopt6a	0.42	2.87	0.26	1.03
dnaopt7	0.42	2.87	0.27	1.05

Table: Positron range for different models.

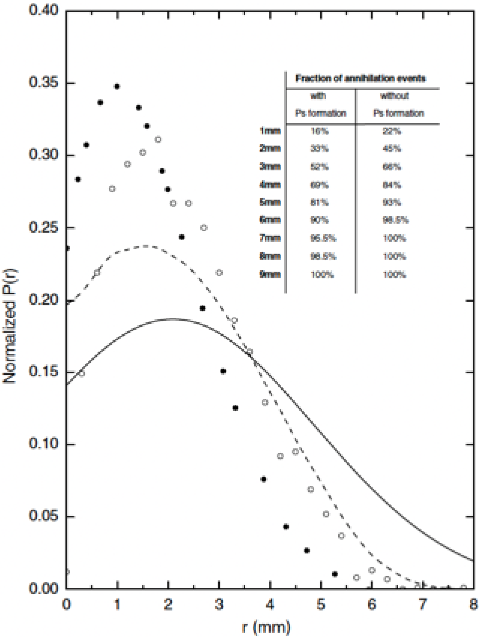
Positronium Formation in Monte Carlo Simulations

- GATE/GEANT4 annihilation cross section computation using Heitler Formula [5]
  $\rightarrow$ does not consider positronium formation [6]

	$R_{mean}(mm)$			
Isotope	**Theory**[7,8]	**Experimental**[9]	**Champion**[6]	**GATE**
^{18}F	**0.6** **0.64**	**0.54**	**0.66**	**0.42**

Table: Comparison of positron range values obtained theoretical, experimentally and in simulation considering positronium formation (Champion Model) [6] [7] [8] [9].

- Champion Model yields $3-10\%$ and $\approx 20\%$ higher values compared to theoretical and experimental data, respectively



- Comparison between annihilation point distribution for ^{15}O, in GEANT3 (solid circles) GEANT4 (open circles) [6]
- Solid and dashed line correspond to taking and not taking positronium formation into account, respectively (Champion Model) [6]

Positronium Parameters

Model	Lifetime (oPs)[10]	Lifetime (pPs)[10]	$\prod_{Ps}$ [6,11,10,12]	$\prod_{e+}$ [6,11,10,12]	$oPs - pPs$ *ratio* [10]
Champion	1848 ± 3 *ps*	131 ± 2 *ps*	83%	17%	3 : 1
Literature	≈ 1800 *ps*	≈ 125 *ps*	$\approx 30\%$	$\approx 70\%$	
			$\approx 36.5\%$	$\approx 63.5\%$	
			$\approx 31.6\%$	$\approx 68.4\%$	

Table: Comparison of positronium formation lifetime and fraction values in water [10] [6] [11] [12].
$\prod_{Ps}$ as *Ps*-formation and $\prod_{e+}$ as annihilation fraction

- Strong overestimation of $\prod_{Ps}$ in Champion Model, more than twice as large as in literature
  $\rightarrow$ overestimated range as result possible

Lifetime

- *oPs* in matter might be trapped in volumes of low electron density [13]
  $\rightarrow$ lifetime may change in range of 142 *ns* to below 1 *ns* [14]

Outlook

Missing in GATE

- Positronium Formation, including cross section and fraction of Ps-formation, $oPs - pPs$ ratio

Problems

- Reference values are theoretical values, not considering positronium formation
- Lack of current experimental data

Possible Solutions

- Proper experiments on positron range and positronium formation in different media
  $\rightarrow$ detector blurring, acollinearity blurring has to be considered
  $\rightarrow$ measuring lifetime, formation ratio, range
- With good experimental data, evaluation of different Ps-formation models may be possible

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