

# Weak lensing Higher Order Statistics theoretical prediction

Joint ARGOS-TITAN-TOSCA workshop,  
6th June 2024

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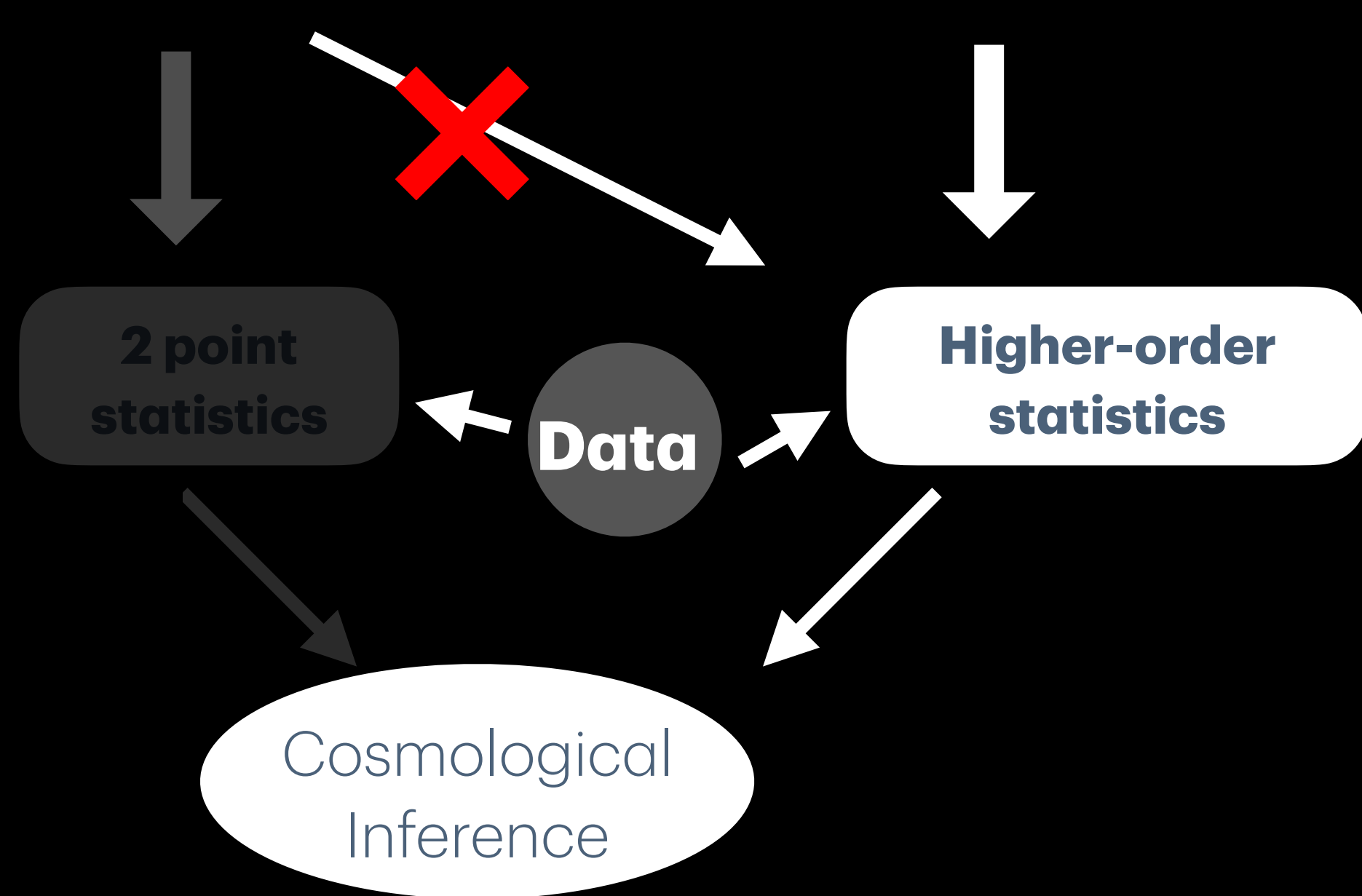




**Theory**

**Simulations**

**Multiple realisations,  
Highly resource  
intensive!**



**My work:**

**Have a theory for the  
wavelet  $l_1$ -norm**

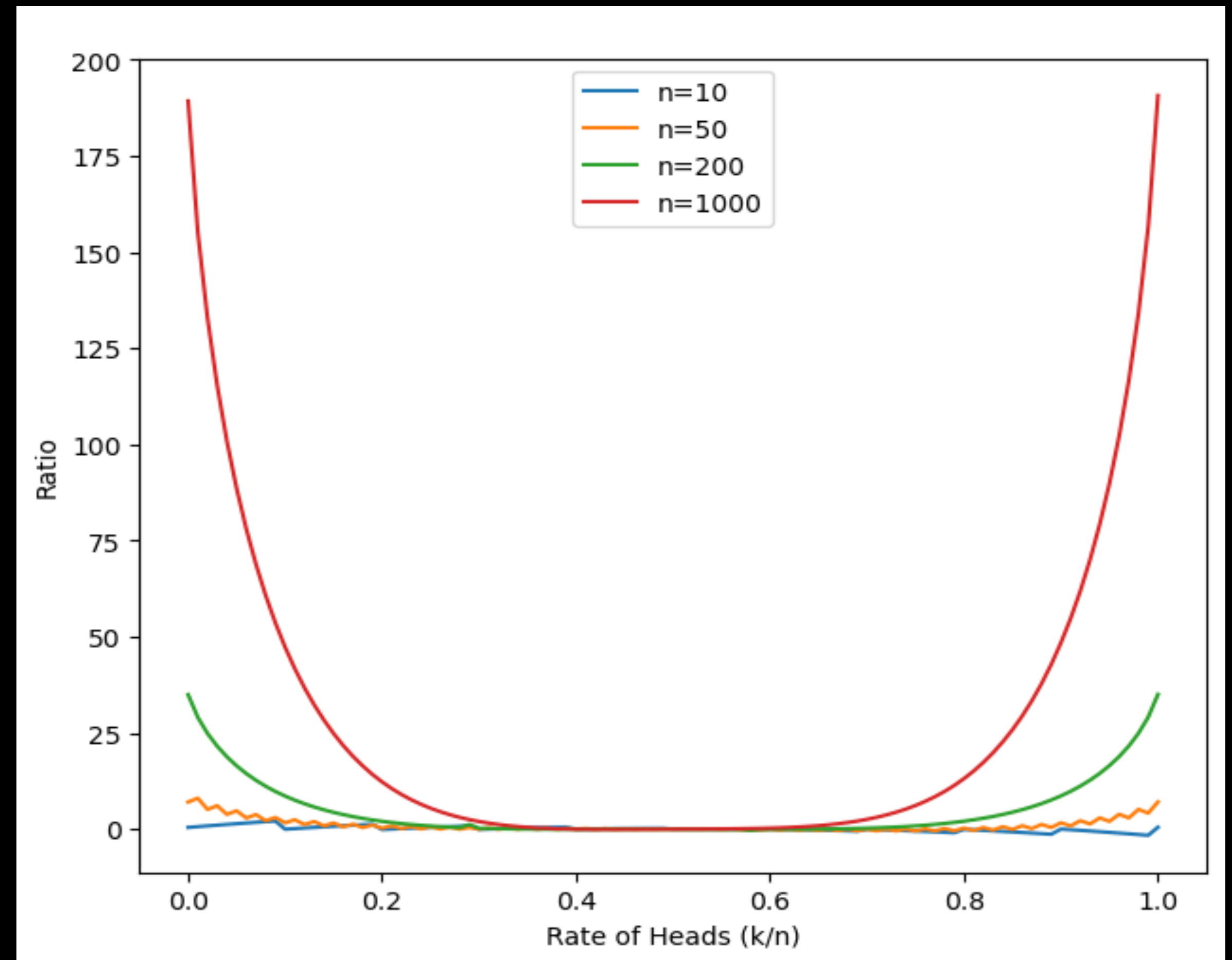


# One-point PDF from Large Deviation Theory

**Large Deviation Theory** —> A framework to predict one-PDF in **mildly non-linear regime** from the 1st principles of Cosmology

The approximation by the central limit theorem may not be accurate if  $x$  is far from  $E[X]$  and  $N$  is not sufficiently large.

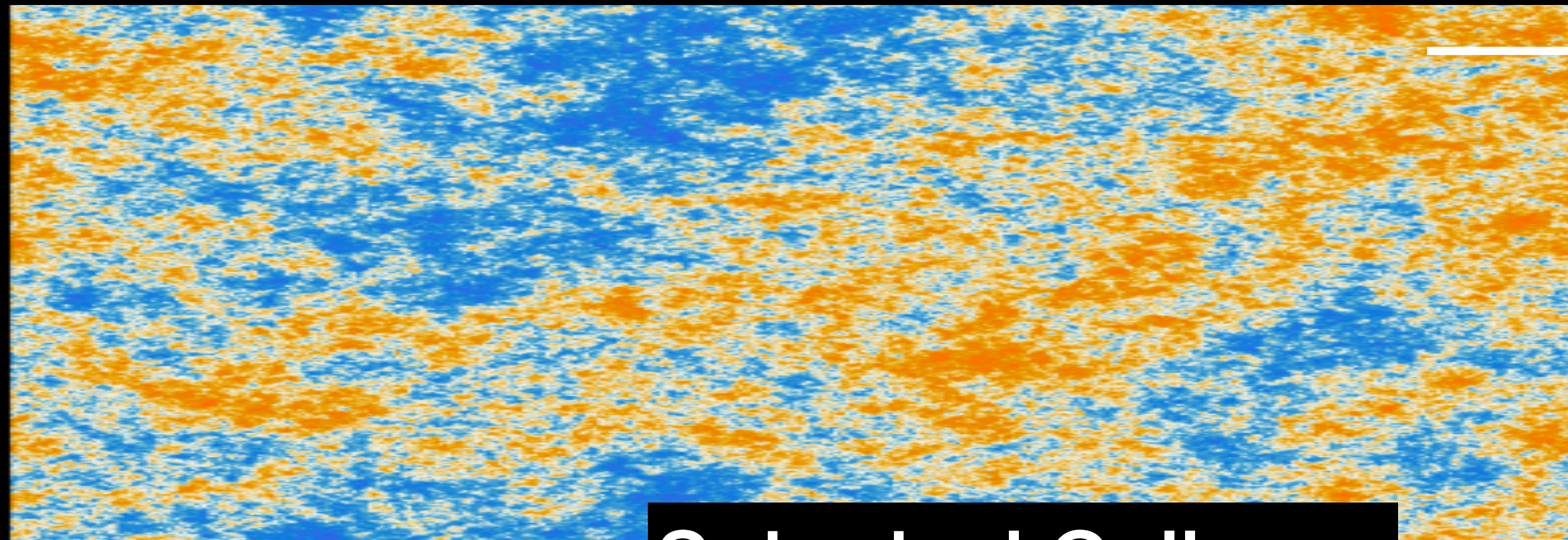
LDP: The probability of rare events happening decreases exponentially with the size of the sample, and the rate function quantifies how unlikely these events are.





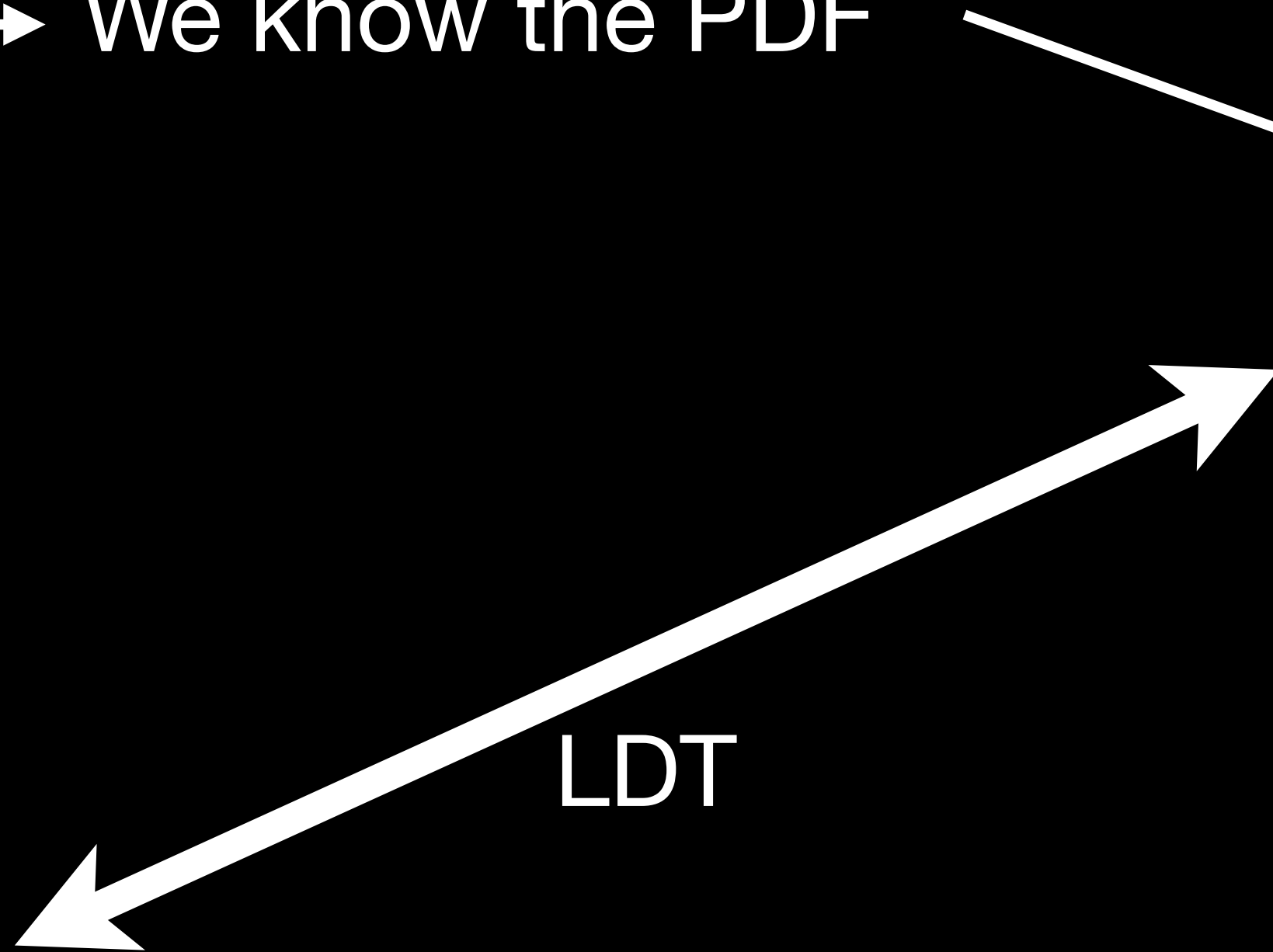
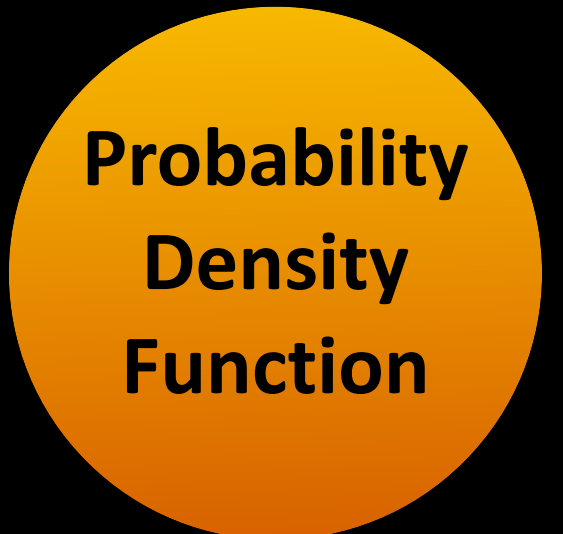
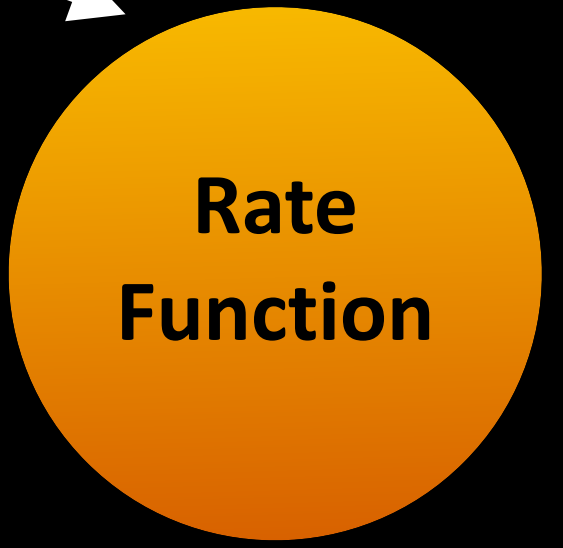
# One-point PDF from Large Deviation Theory

**Large Deviation Theory** —> A framework to predict one-PDF in **mildly non-linear regime** from the 1st principles of Cosmology



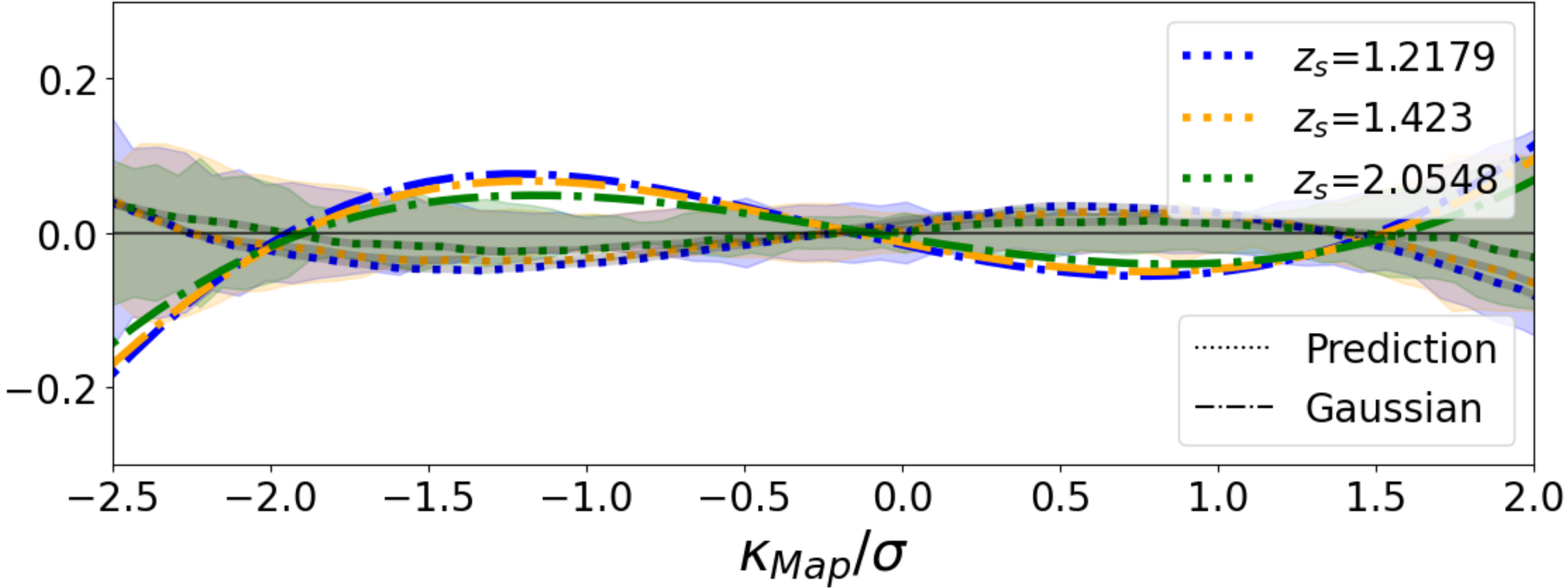
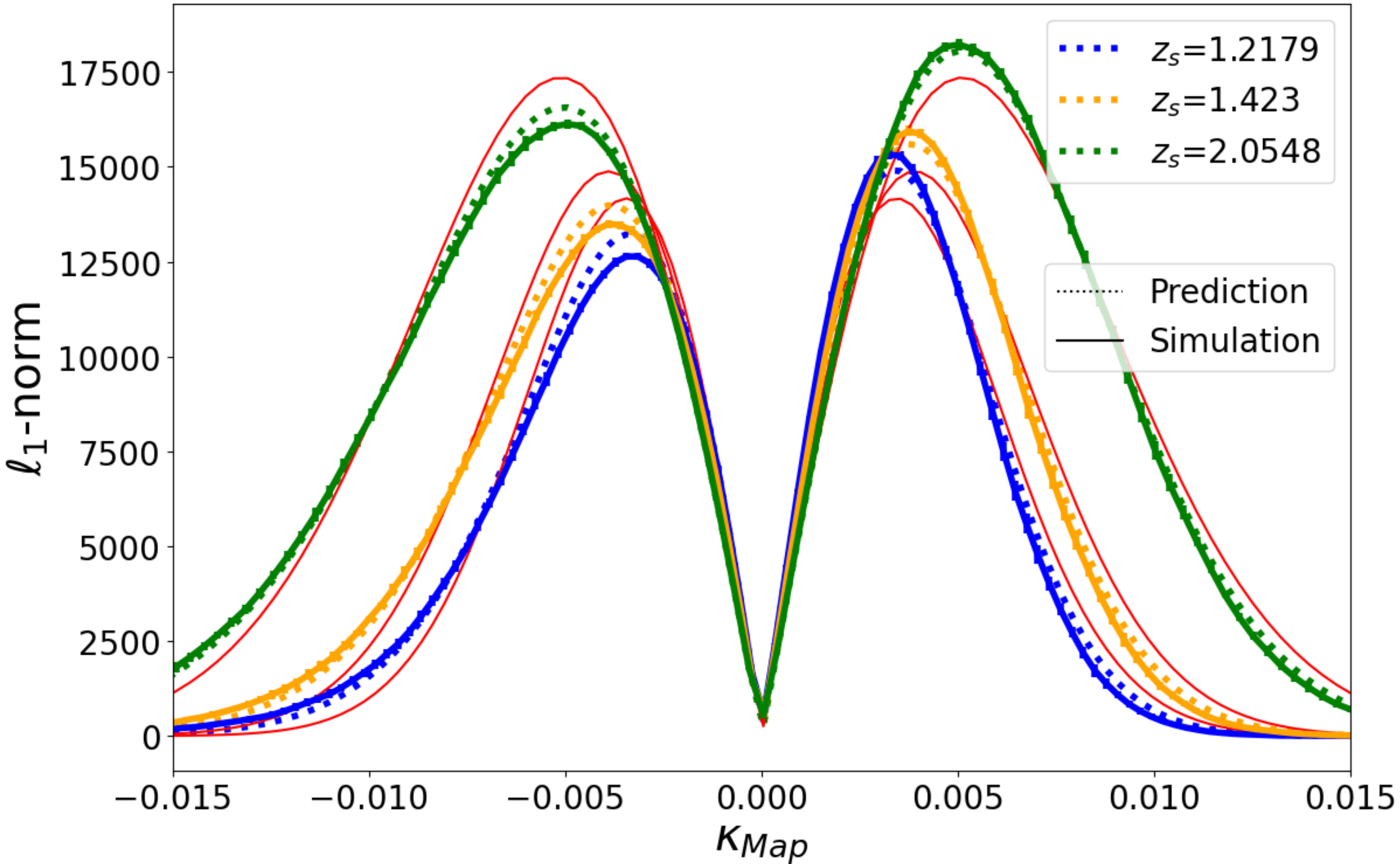
We know the PDF

**Spherical Collapse**





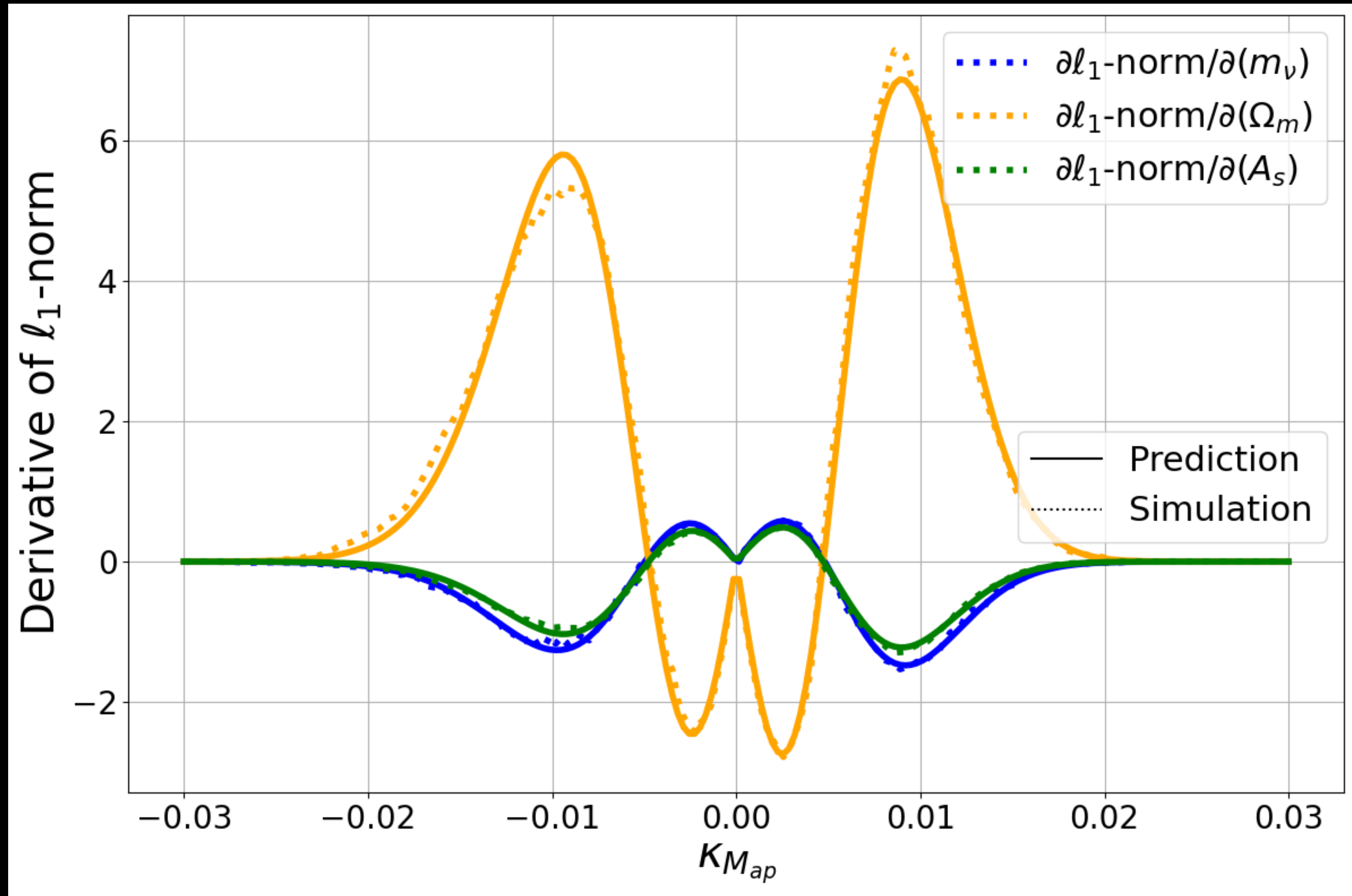
Simulation used: Takahashi's



[Vilasini+ Submitted]

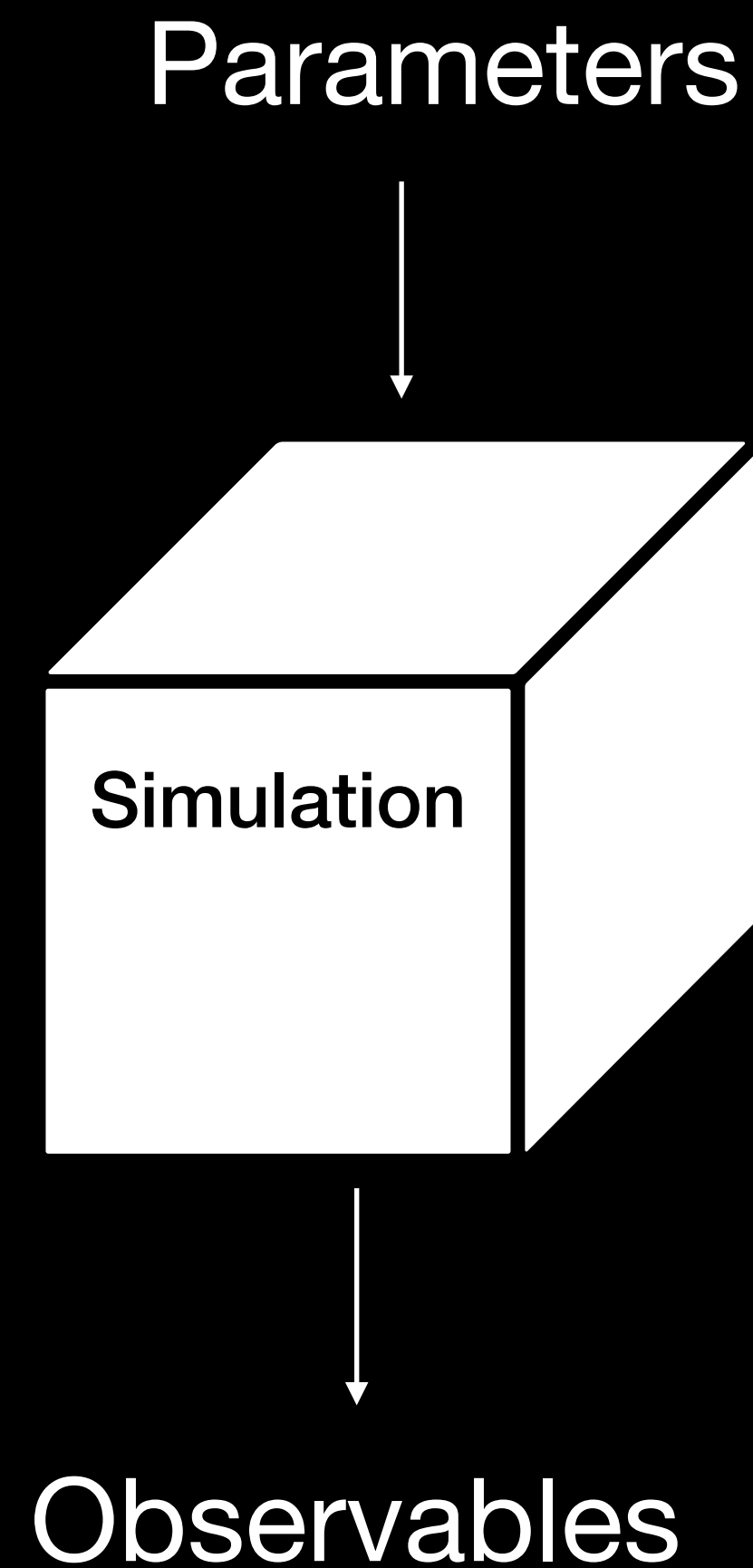


Simulation used:  
MassiveNus



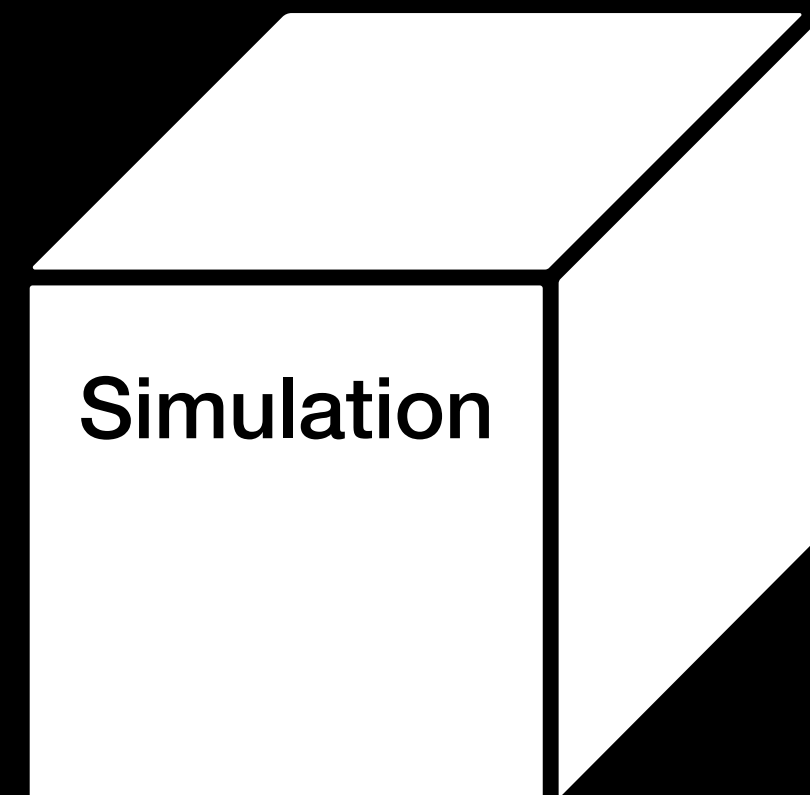


# Forward modelling



- Directly extracts information the convergence map
- Systematic effects can be directly incorporated
- Combination of multiple probes



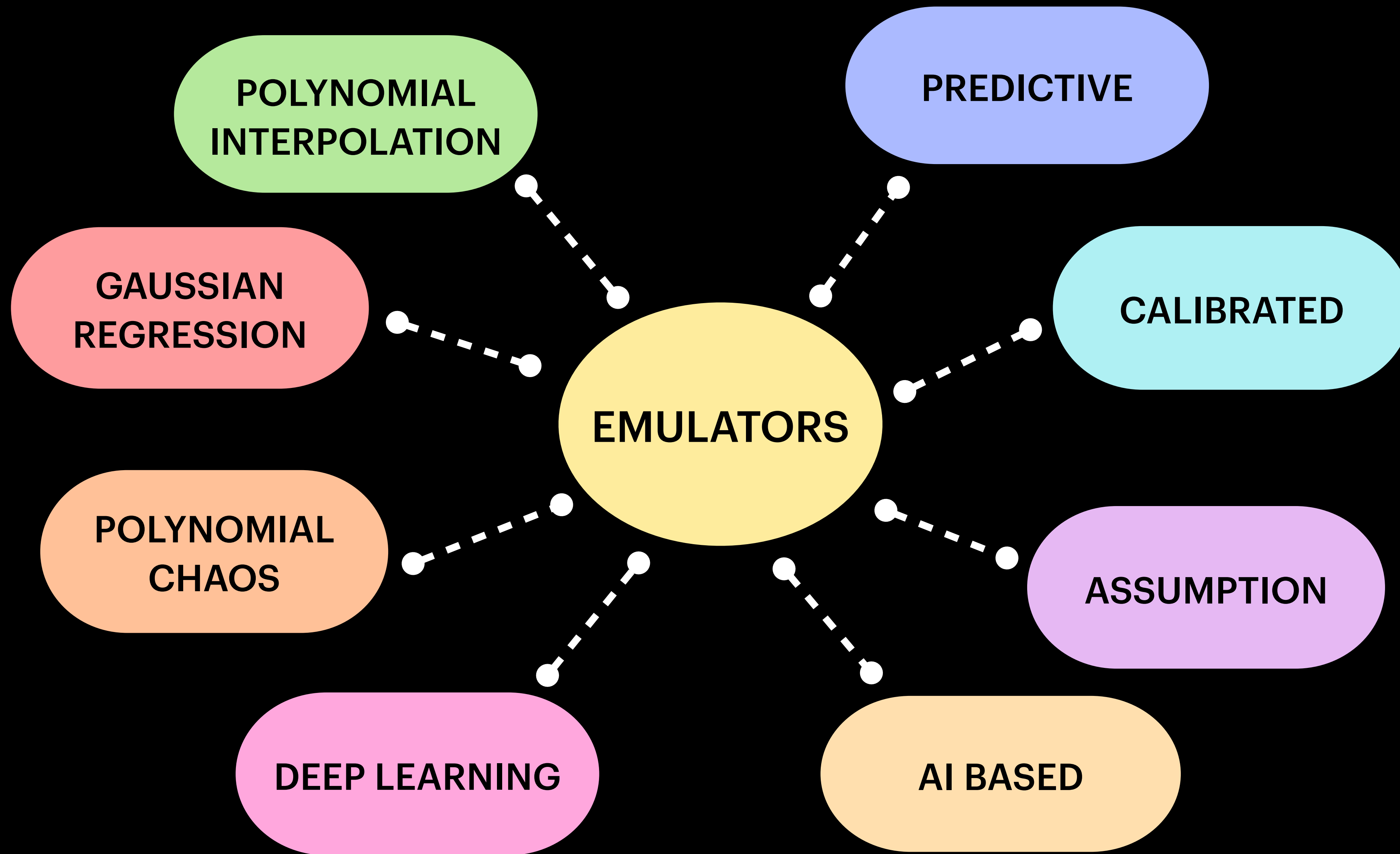


Full N-body simulations — —> resource intensive!

Emulators

- Can reproduce the **summary statistics** and maps produced from an N-body simulation, but faster







- Emulators can reproduce the **summary statistics**



- Emulators can reproduce the correct **2 point summary statistics**
  - ***Might not be the case for HOS.***

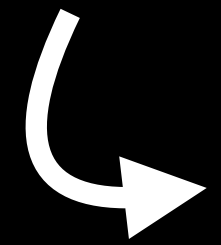


**How to have a convergence map  
with the correct HOS incorporated in  
that as well?**



# Emulator based on theory prediction

- ***Need something that will have the correct higher order statistics as well!***



**Main ingredient:** inverse CDF transformation

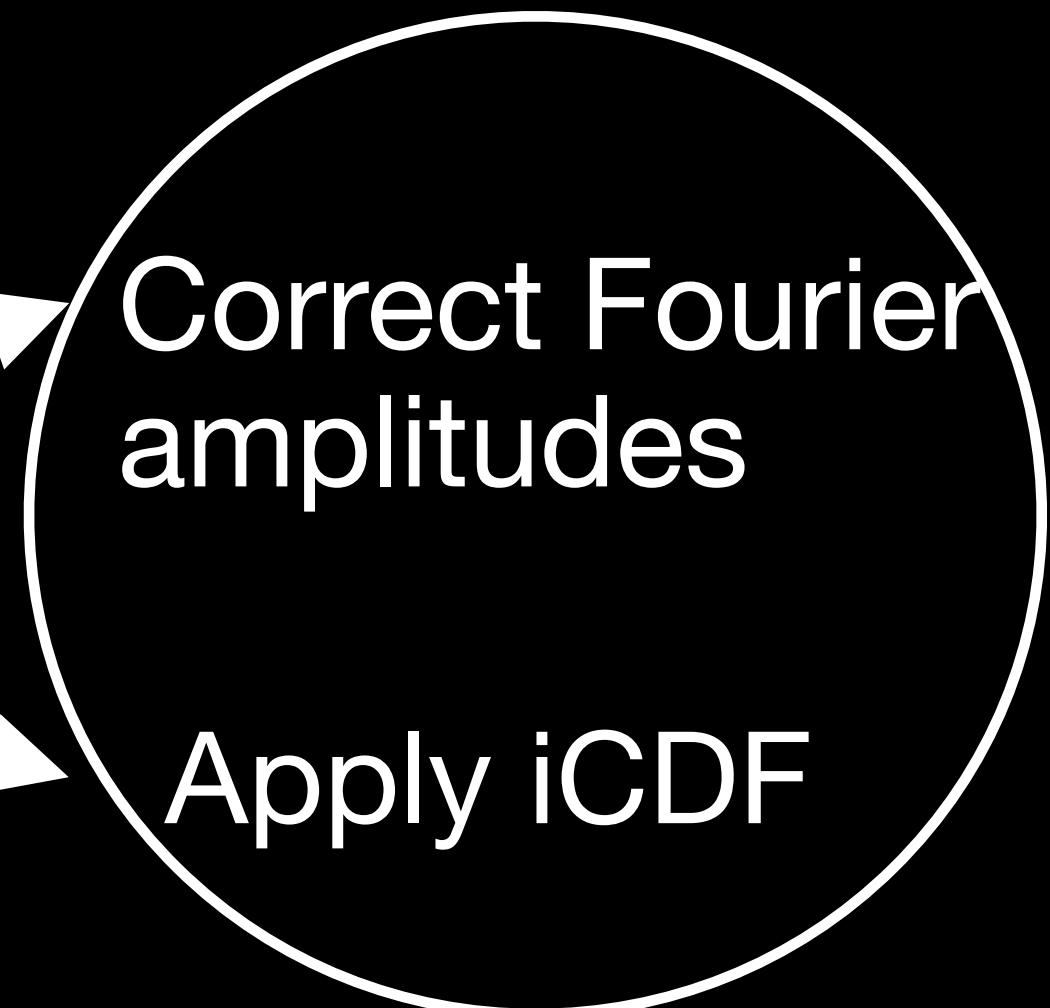
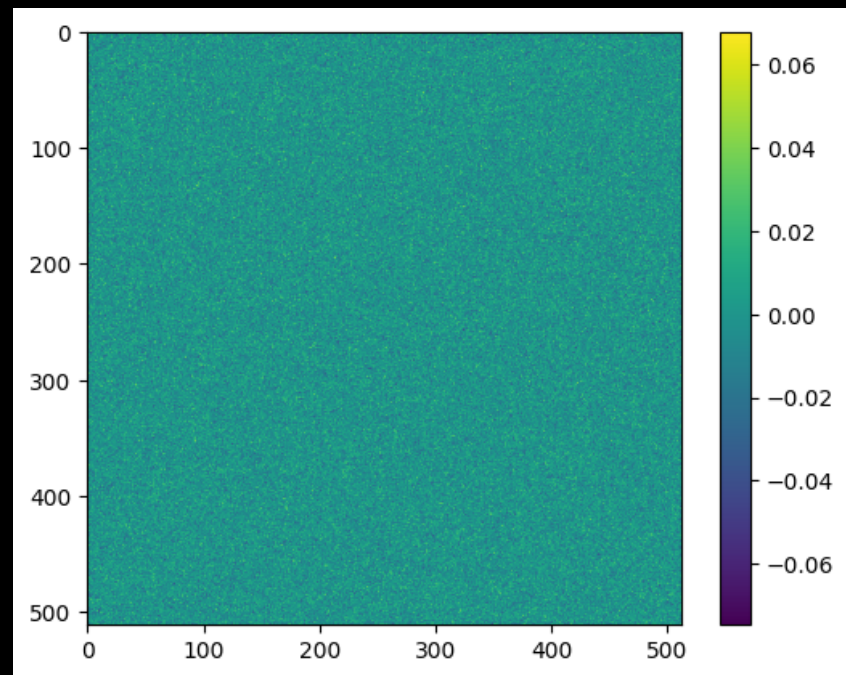
The goal is to transform the histogram of the input image to match the target histogram.

$$I_{\text{transformed}}(i,j) = F_{\text{target}}^{-1}(F_I(I(i,j)))$$

$F_I(I(i,j))$  is the CDF value of the input image intensity at (i,j)

$F_{\text{target}}^{-1}$  is the inverse of the target CDF

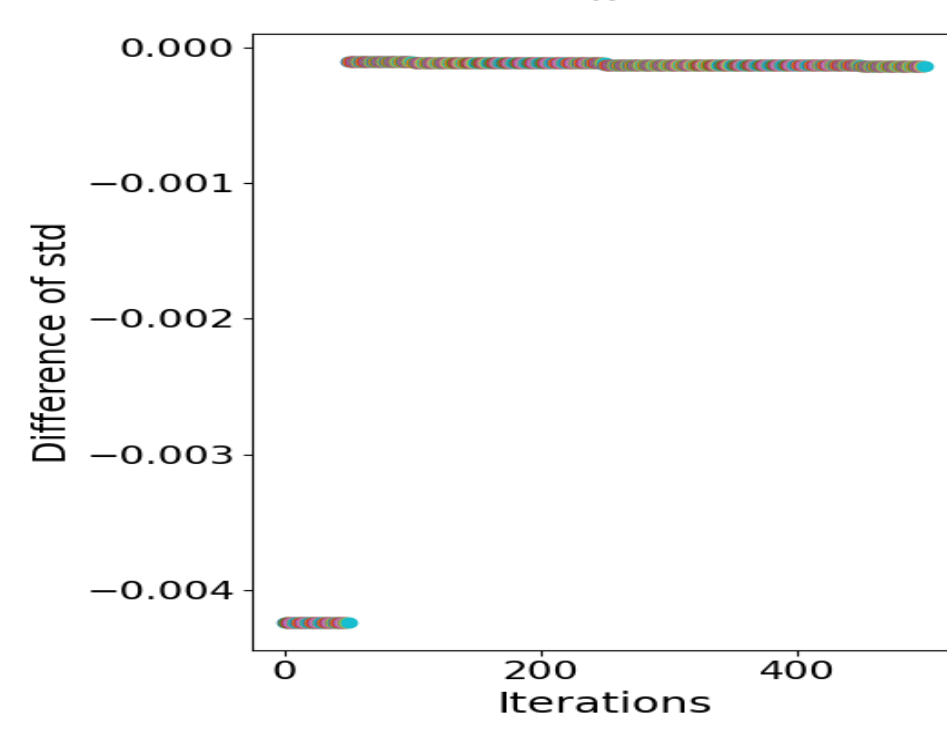
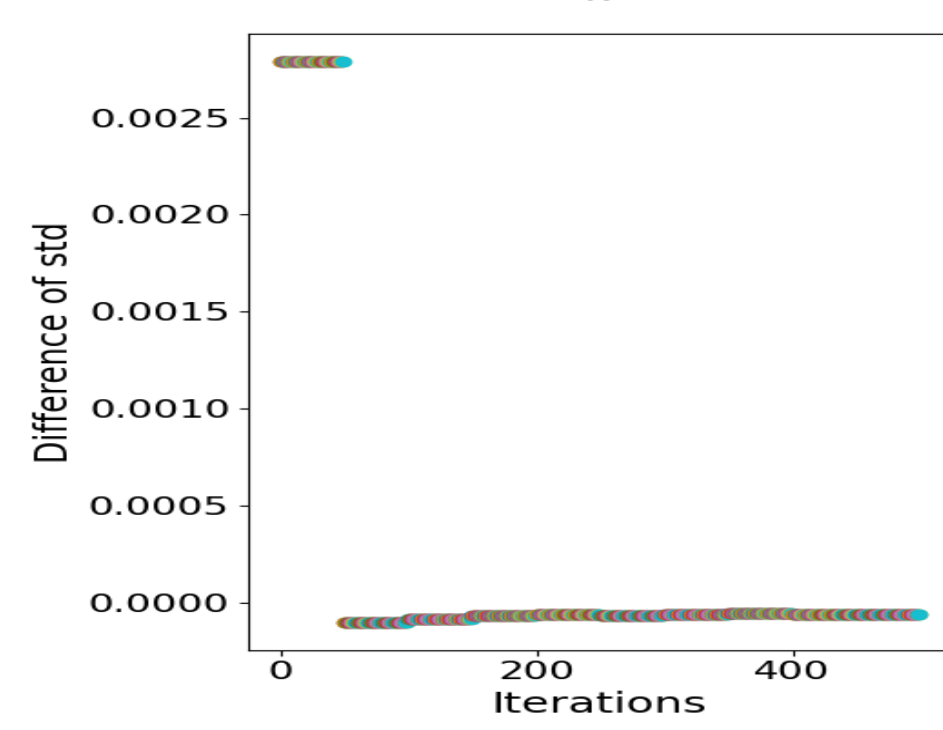
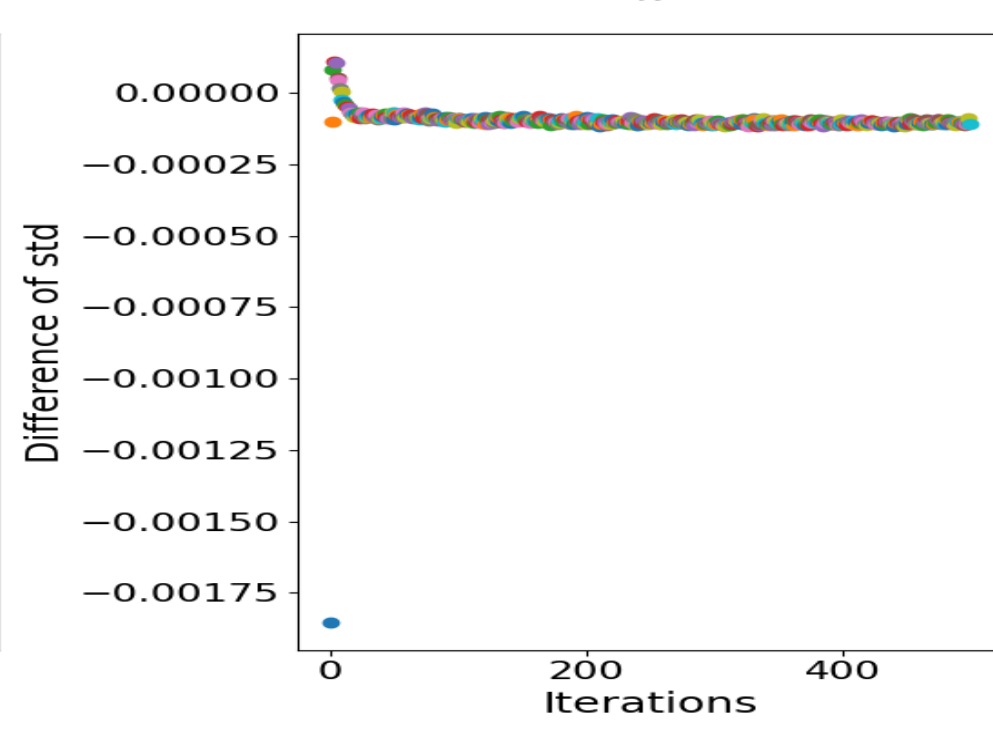
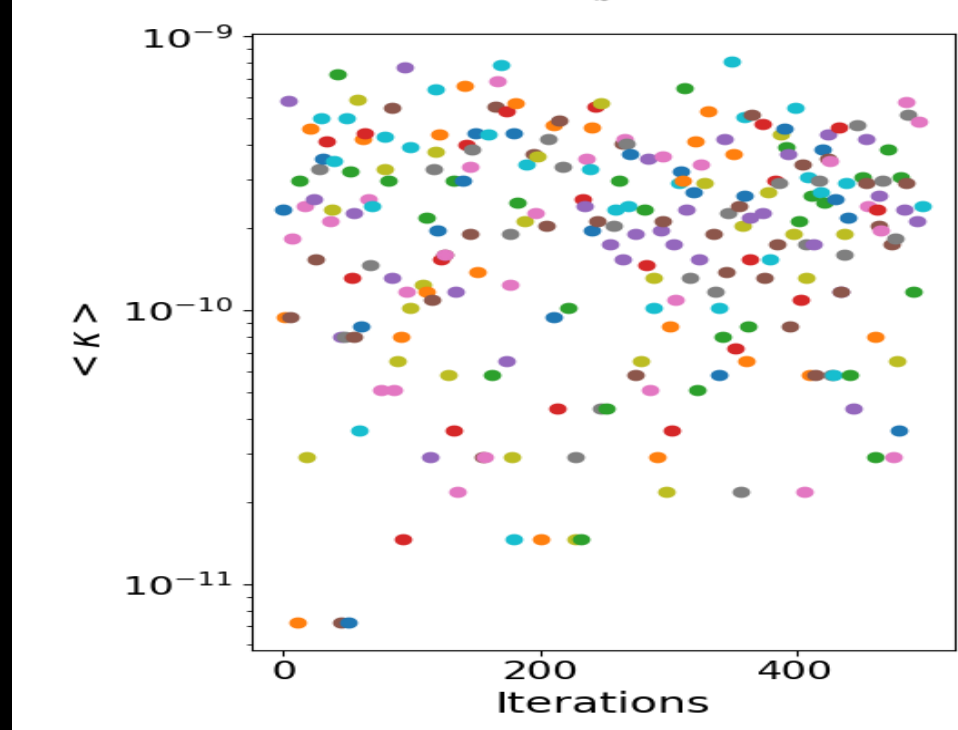
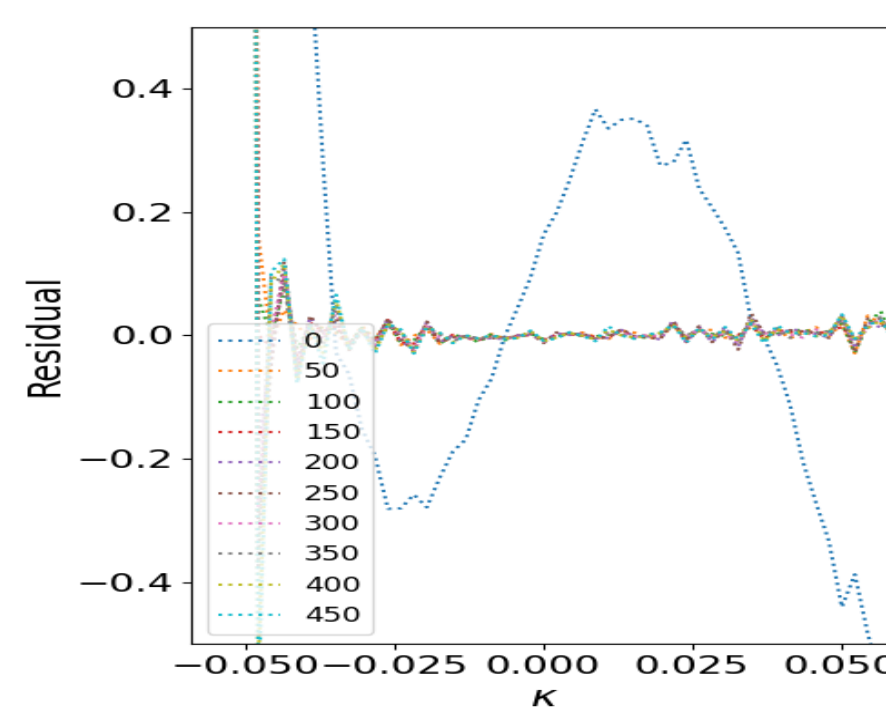
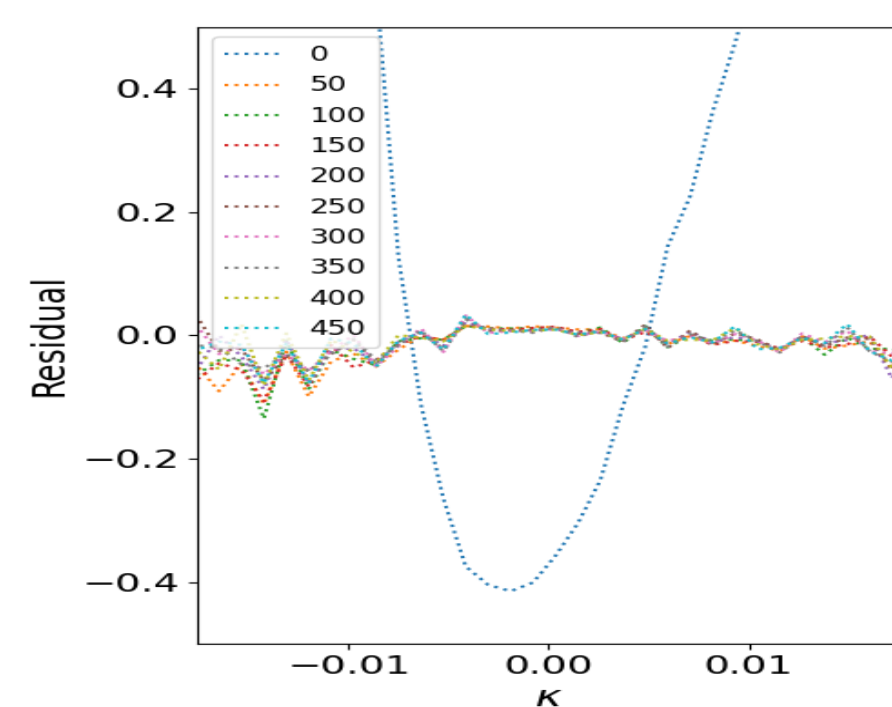
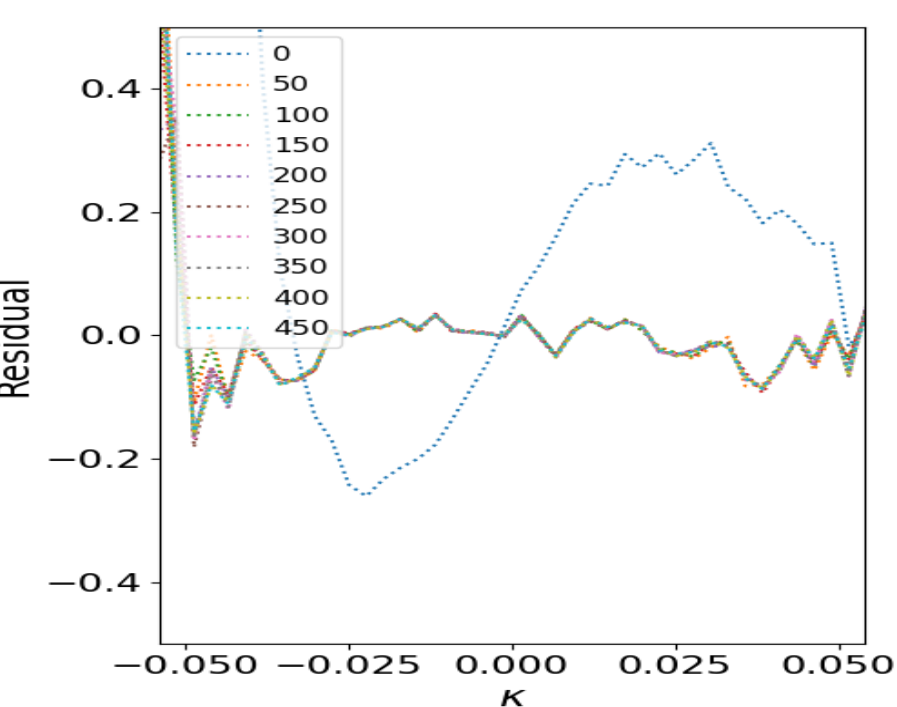
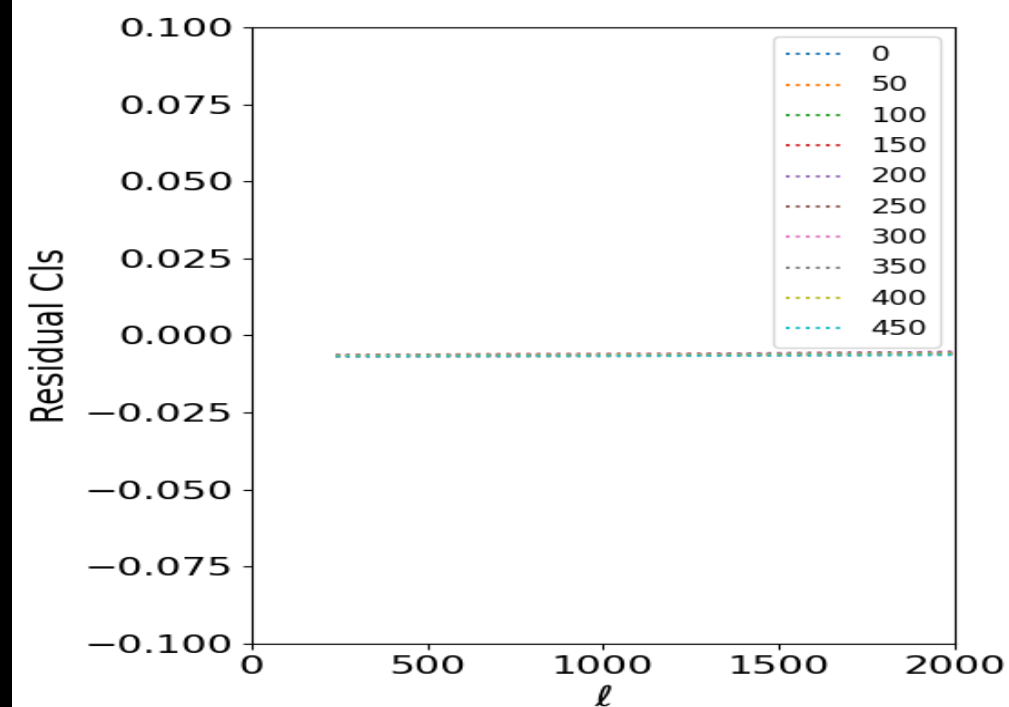
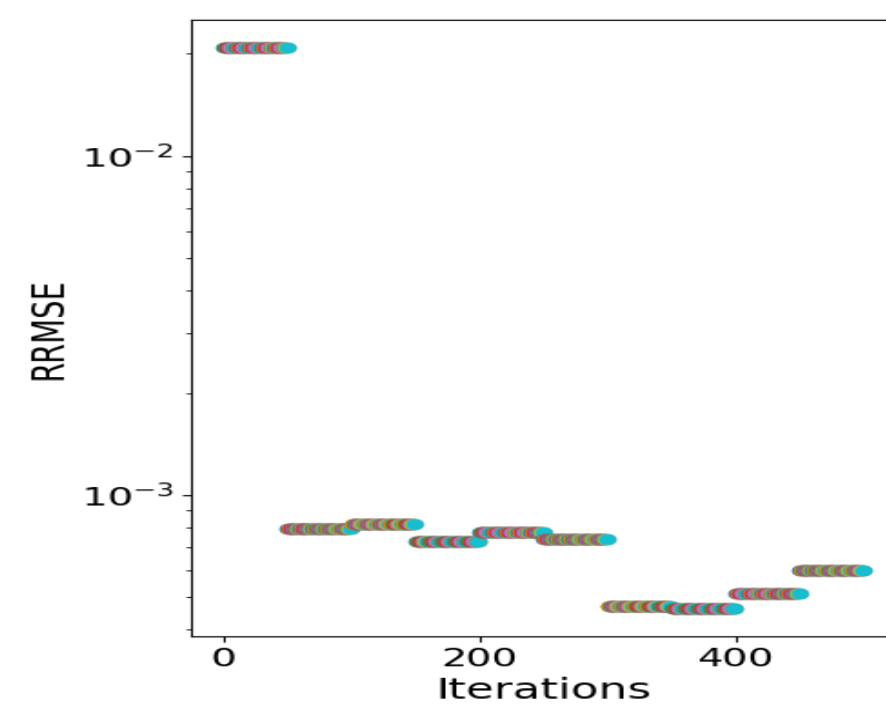
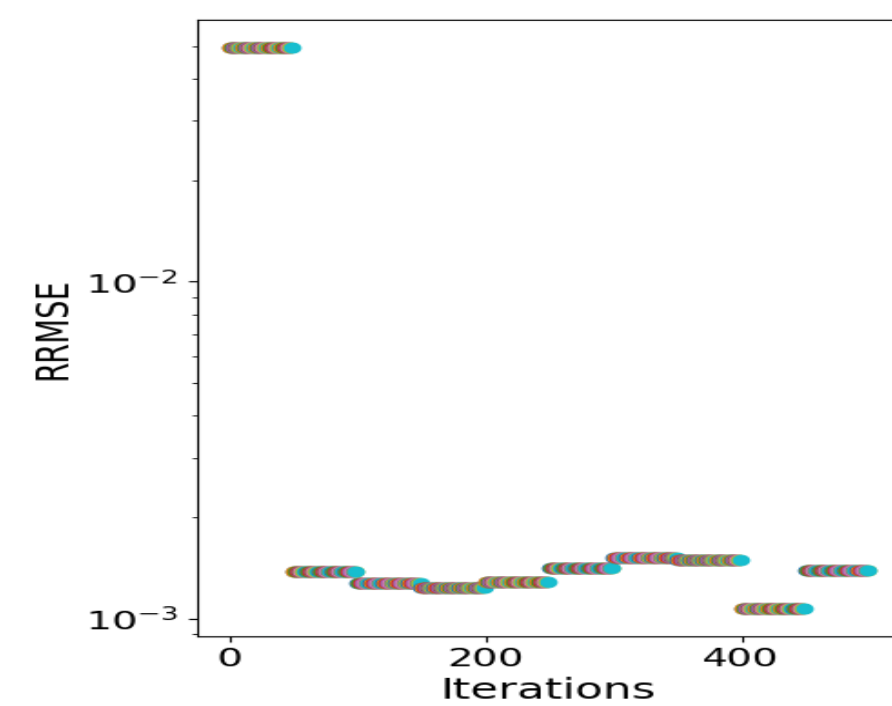
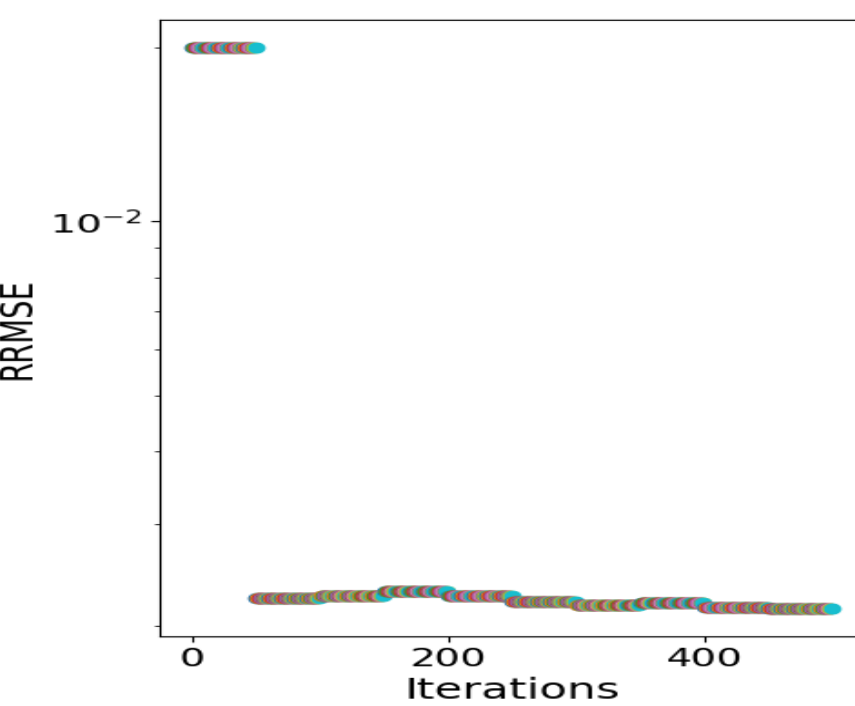
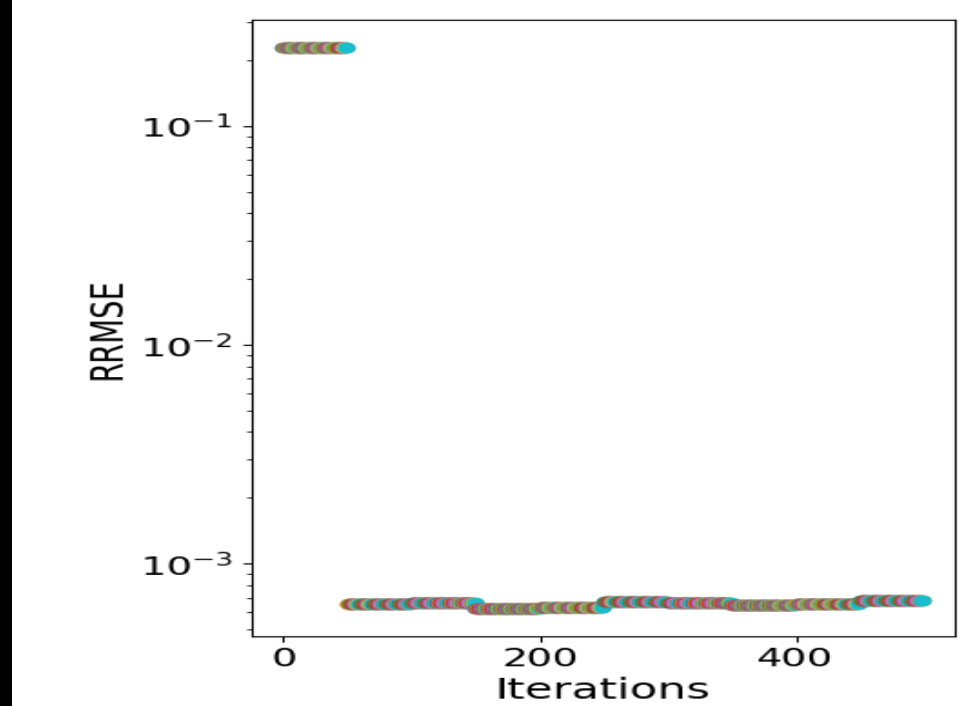
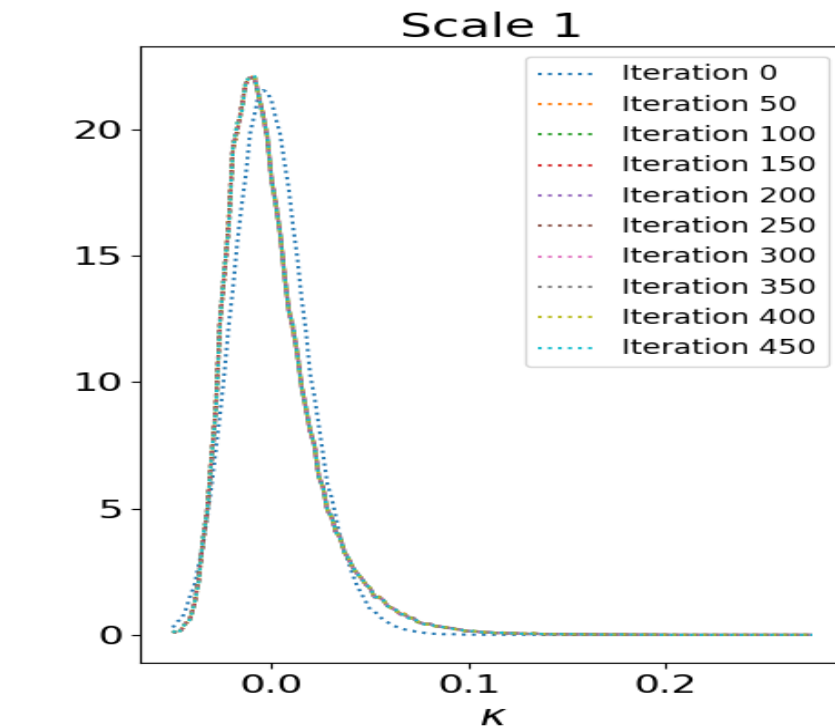
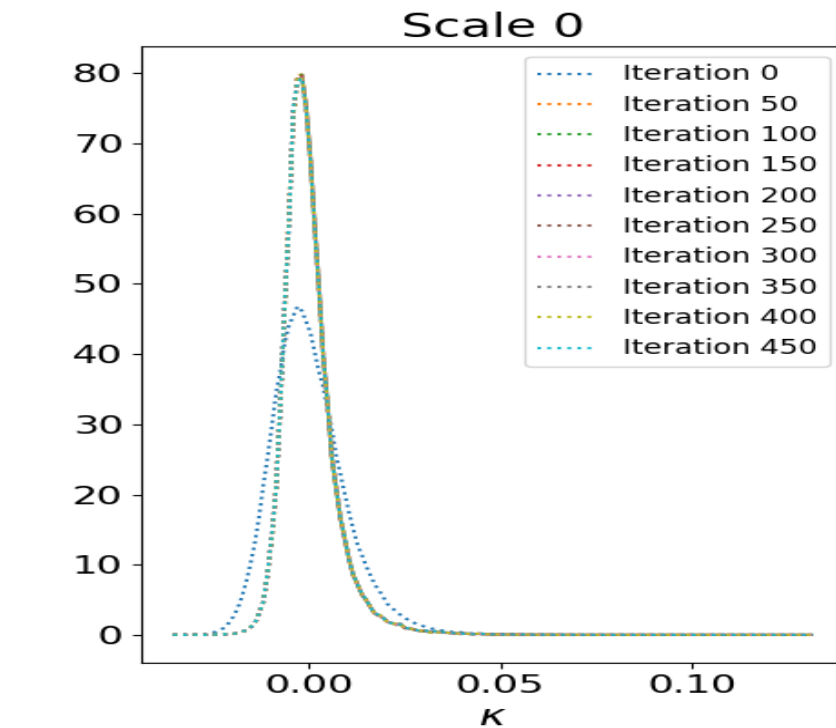
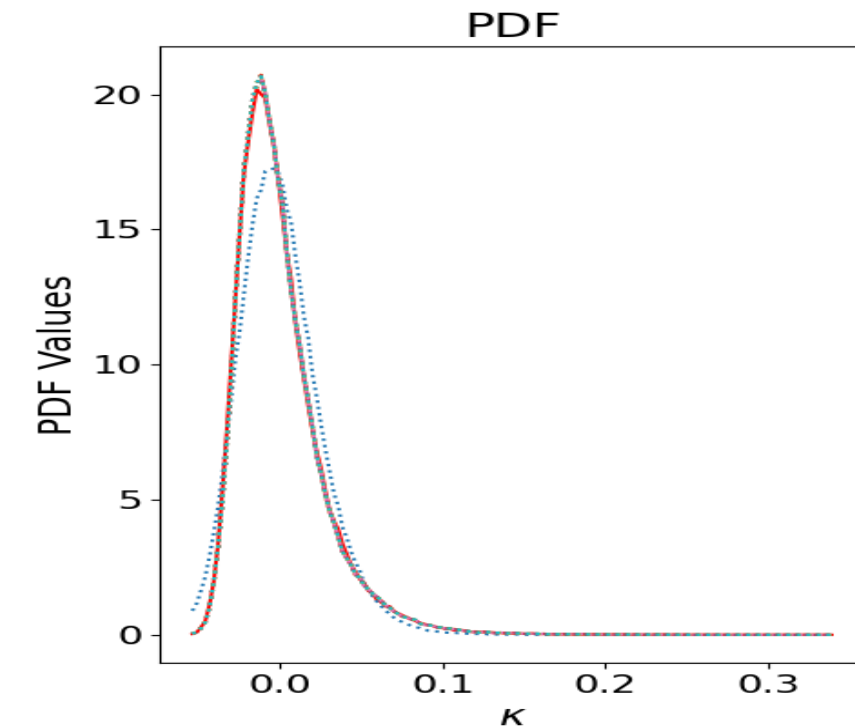
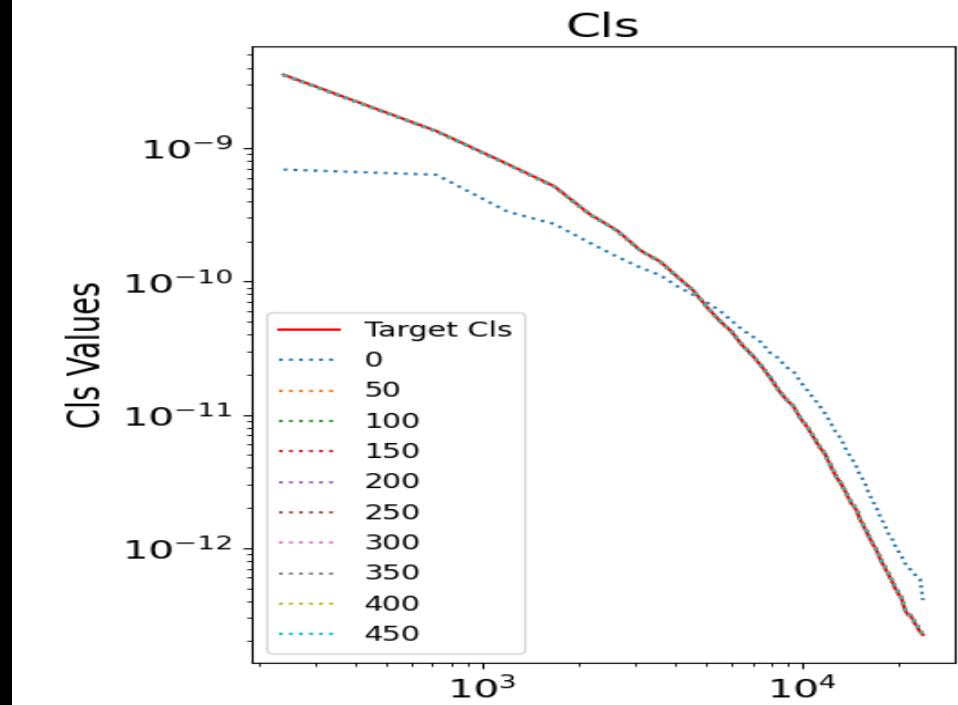




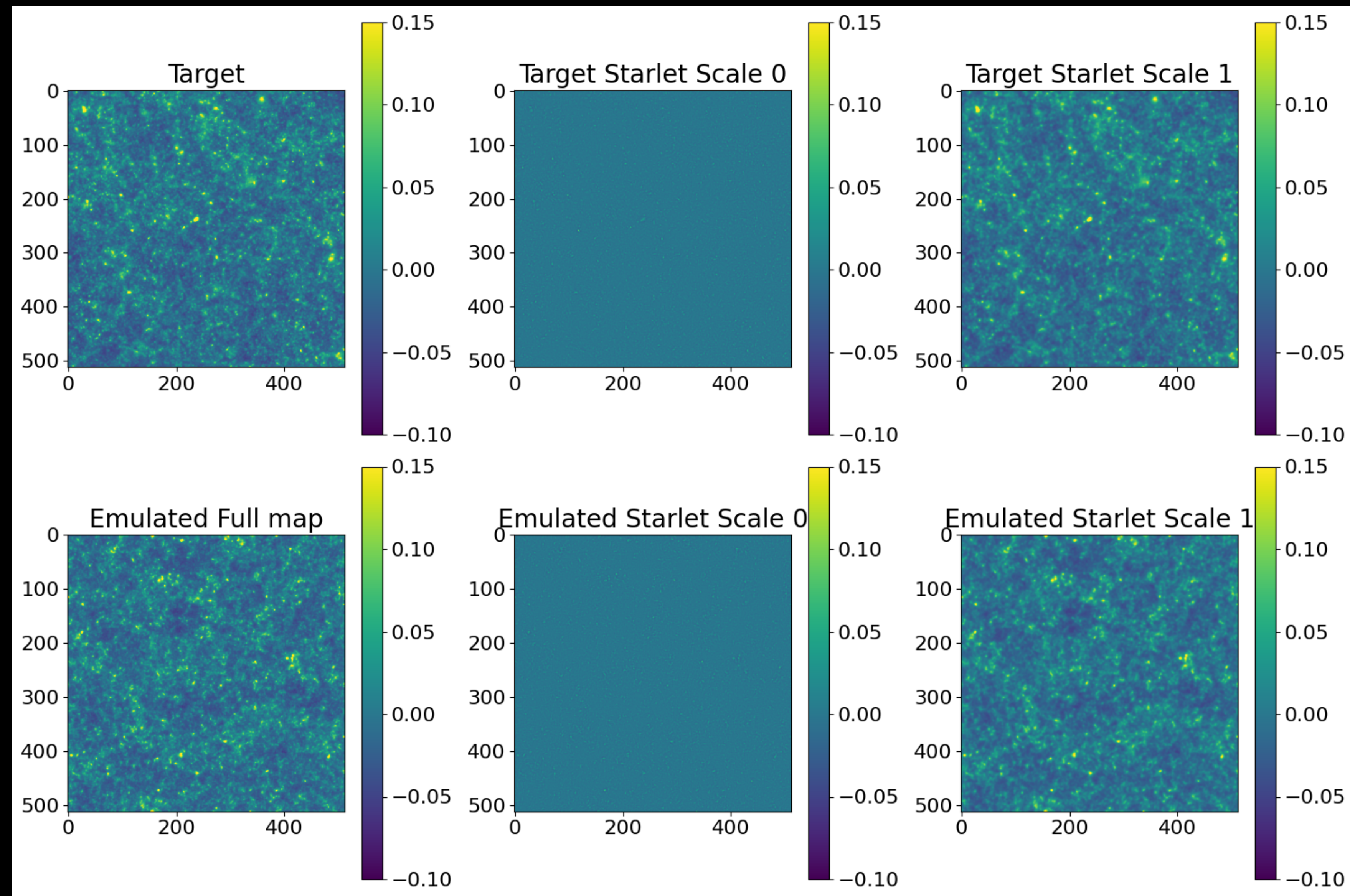
$$\frac{1}{2}(I_{transformed, Cls} + I_{transformed, CDF}) = I_{transformed}$$

Iterate till the error stabilises





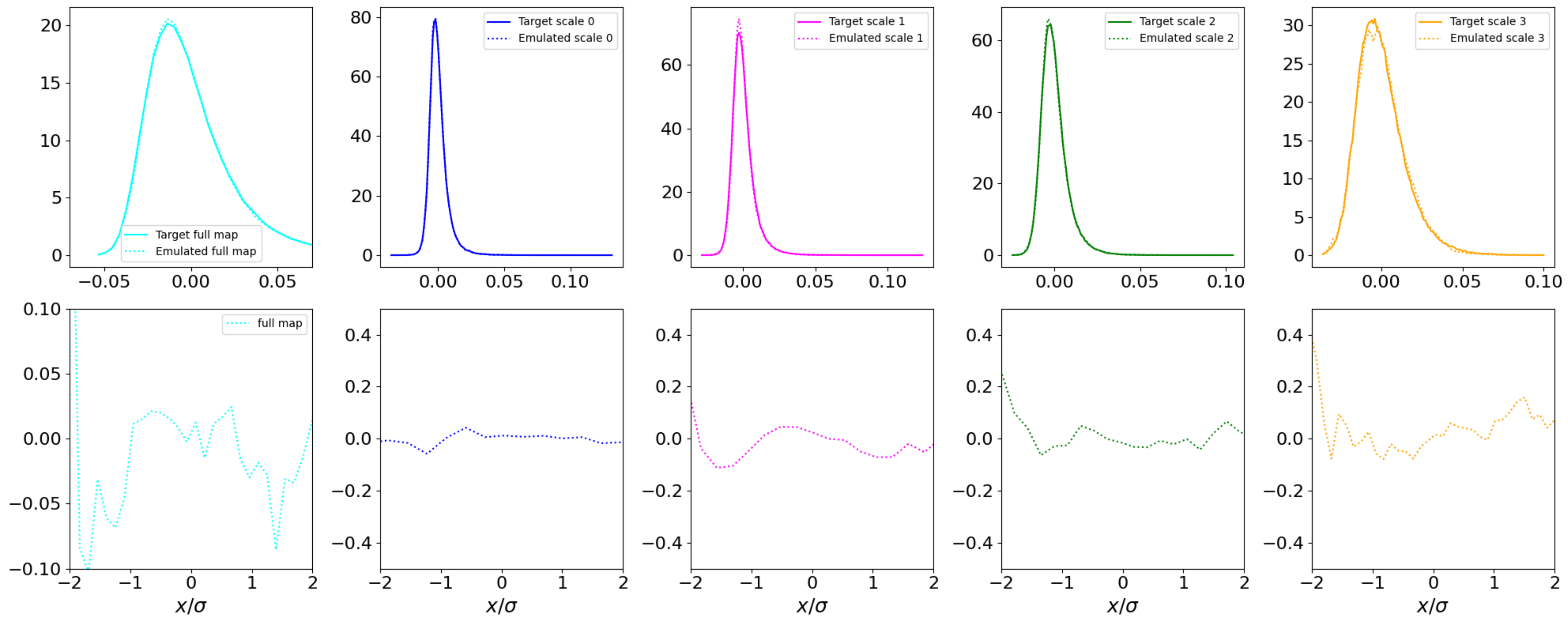




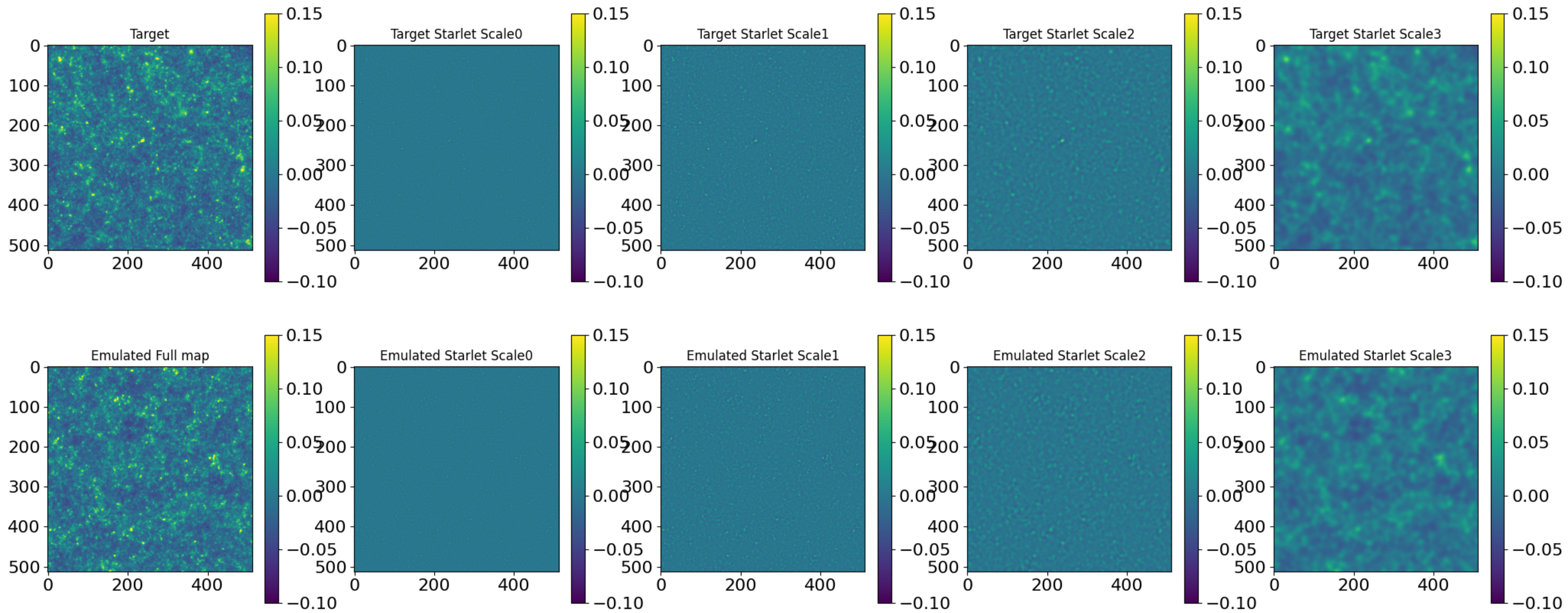
## Standard Deviation

	Target	Emulated
Full map	0.02691	0.2680
Scale 0	0.00703	0.00697
Scale 1	0.0236	0.0235













**Thank you!**



# EXTRA SLIDES

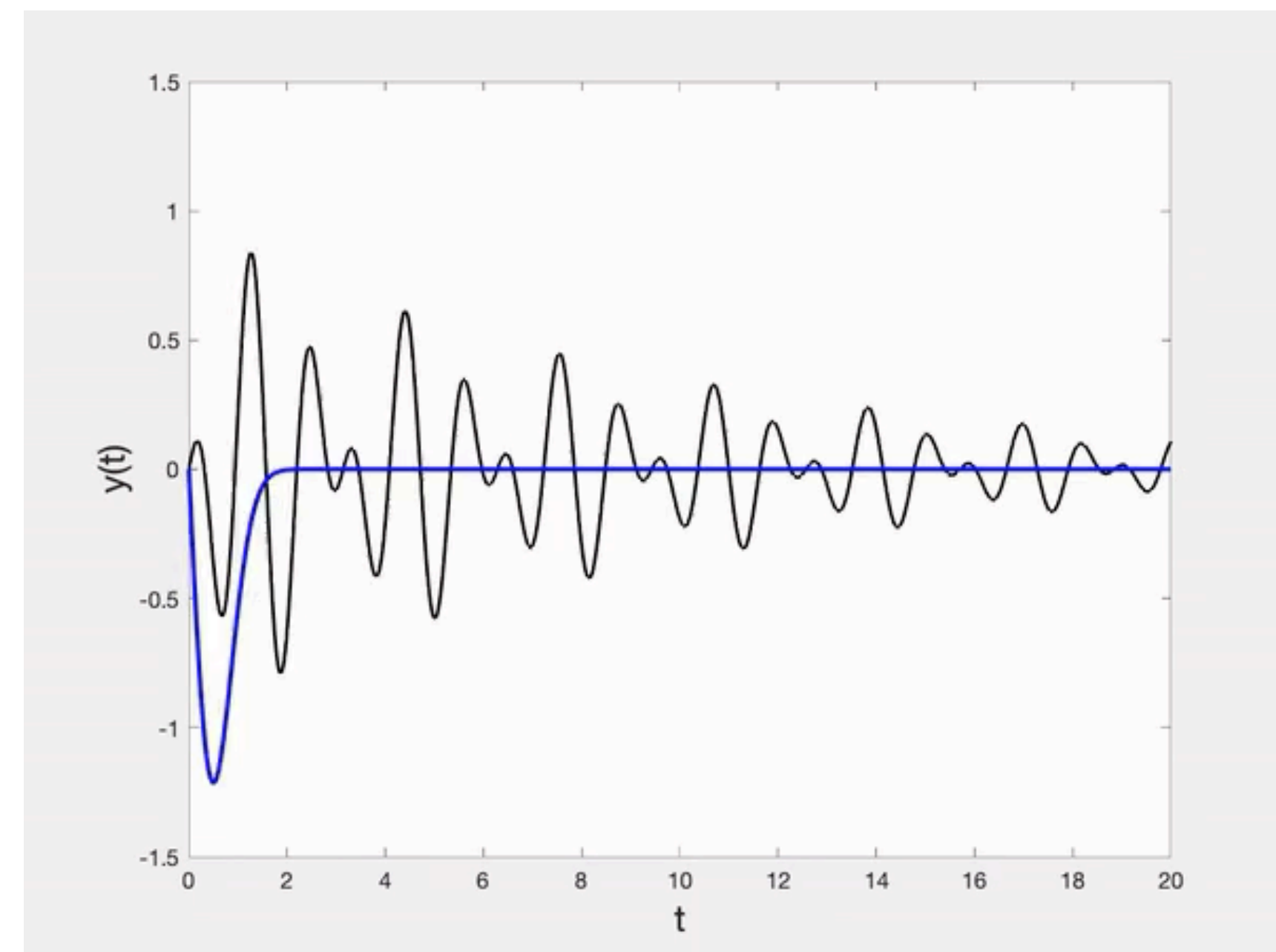


# Wavelets:

A set of mathematical function that is defined by the following properties:

- Highly localized in space/time
- Has a vanishing mean

- A useful tool in analyzing signals where there are sharp spikes and discontinuities



## The Continuous Wavelet Transform

$$W(a, b) = K \int_{-\infty}^{+\infty} \psi^* \left( \frac{x - b}{a} \right) f(x) dx$$

where:

- $W(a, b)$  is the wavelet coefficient of the function  $f(x)$
- $\psi(x)$  is the analyzing wavelet
- $a (> 0)$  is the scale parameter
- $b$  is the position parameter

In Fourier space, we have:  $\hat{W}(a, \nu) = \sqrt{a} \hat{f}(\nu) \hat{\psi}^*(a\nu)$

When the scale  $a$  varies, the filter  $\hat{\psi}^*(a\nu)$  is only reduced or dilated while keeping the same pattern.



Jean Morlet



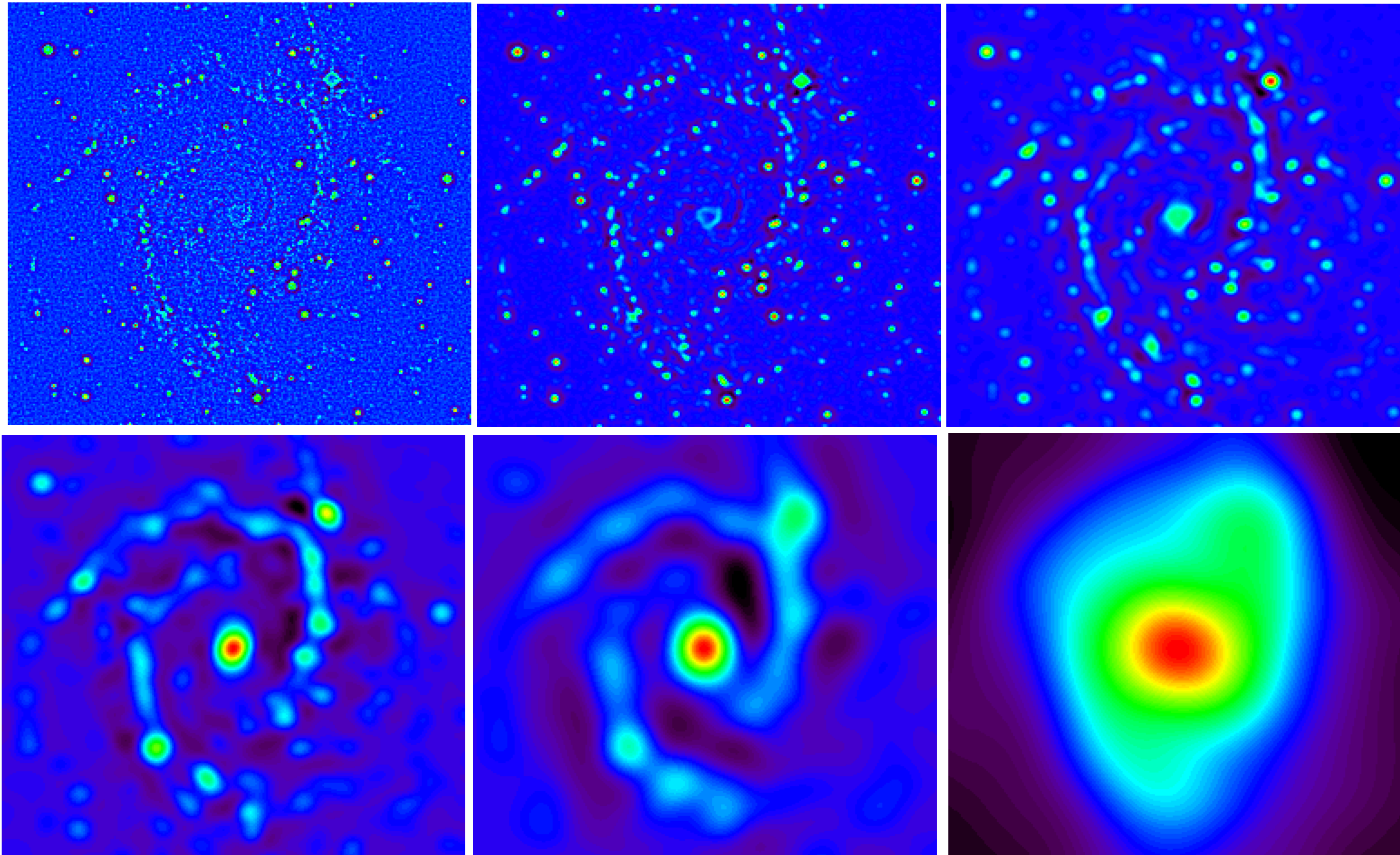
# The STARLET Transform

Isotropic Undecimated Wavelet Transform (a trous algorithm)

$$\varphi = B_3 \text{ - spline, } \frac{1}{2}\psi\left(\frac{x}{2}\right) = \frac{1}{2}\varphi\left(\frac{x}{2}\right) - \varphi(x)$$

$$h = [1,4,6,4,1]/16, \quad g = \delta - h, \quad \tilde{h} = \tilde{g} = \delta$$

$$I(k,l) = c_{J,k,l} + \sum_{j=1}^J w_{j,k,l}$$







According to various theories the word "Biryani" is either derived from farsi word "brishtah" (fried onions) or biryan (frying).

<https://en.wikipedia.org/wiki/Biryani>

**Pulao** - The vegetable/meat is cooked with pre-ground masala paste. No whole spices, cashews, etc.

**Biryani** - Vegetable/meat is either (1) Cooked along with rice or (2) Layered with 80% cooked rice. Whole spices are added. Toppings include fried cashews, raisins, chopped mint, chopped coriander, etc. Saffron milk and ghee is added generously.

[link to difference.](#)

## Pulao



**Biryani**

Notice the layering →



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## Fried rice



Shubra Kitchen

