Analytical approaches to gravitational wave modelling

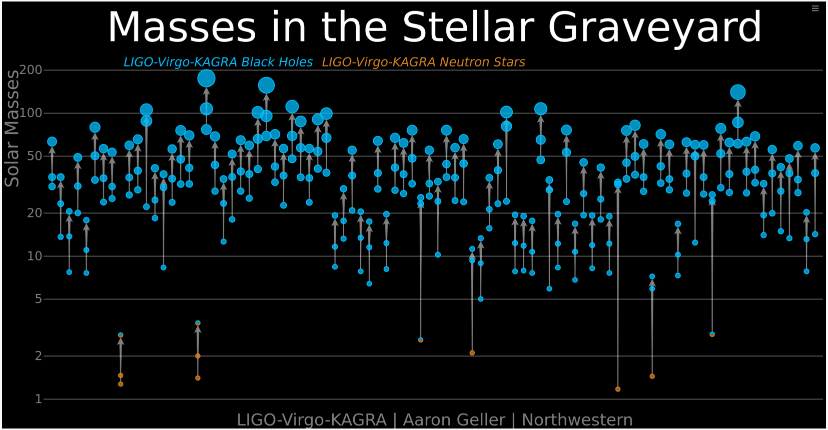
Laura Bernard

"Théorie, Univers et Gravitation" meeting – IV

Annecy-Le-Vieux, Nov. 5-7 2024

LUTH Observatoire de Paris | PSL cnrs

The current gravitational wave universe



The current gravitational wave universe

LIGO/Virgo/KAGRA Public Alerts

- More details about public alerts are provided in the LIGO/Virgo/KAGRA Alerts User Guide.
- Retractions are marked in red. Retraction means that the candidate was manually vetted and is no longer considered a candidate of interest.
- Less-significant events are marked in grey, and are not manually vetted. Consult the LVK Alerts User Guide for more information on significance in O4.
- Less-significant events are not shown by default. Press "**Show All Public Events**" to show significant and less-significant events.

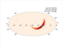
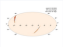
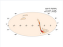
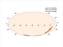
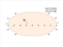
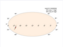
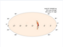
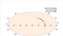
O4 Significant Detection Candidates: **142** (158 Total - 16 Retracted)

O4 Low Significance Detection Candidates: **2484** (Total)

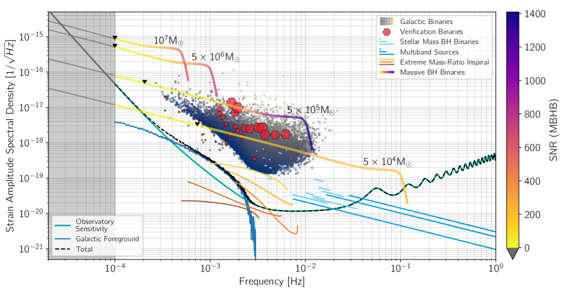
Show All Public Events

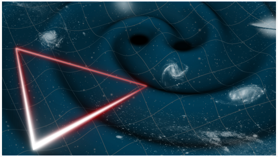
Page 1 of 11. next last »

SORT: EVENT ID (A-Z)

Event ID	Possible Source (Probability)	Significant	UTC	GCN	Location	FAR	Comments
S240930du	BBH (67%), Terrestrial (33%)	Yes	Sept. 30, 2024 23:46:14 UTC	GCN Circular Query Notices | VOE		1 per 2.4747 years	
S240930aa	BBH (>99%)	Yes	Sept. 30, 2024 03:59:59 UTC	GCN Circular Query Notices | VOE		1 per 1.0344e+11 years	
S240925n	BBH (>99%)	Yes	Sept. 25, 2024 00:58:09 UTC	GCN Circular Query Notices | VOE		1 per 7.9146e+11 years	
S240924a	BBH (>99%)	Yes	Sept. 24, 2024 00:03:16 UTC	GCN Circular Query Notices | VOE		1 per 12.869 years	
S240923ct	BBH (>99%)	Yes	Sept. 23, 2024 20:40:06 UTC	GCN Circular Query Notices | VOE		1 per 4.1462e+07 years	
S240922df	BBH (>99%)	Yes	Sept. 22, 2024 14:21:06 UTC	GCN Circular Query Notices | VOE		1 per 2.2729e+16 years	
S240921cw	BBH (>99%)	Yes	Sept. 21, 2024 20:18:35 UTC	GCN Circular Query Notices | VOE		1 per 39.517 years	
S240920dw	BBH (>99%)	Yes	Sept. 20, 2024 12:40:24 UTC	GCN Circular Query Notices | VOE		1 per 3.2668e+43 years	

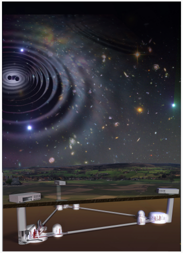
The future gravitational wave universe

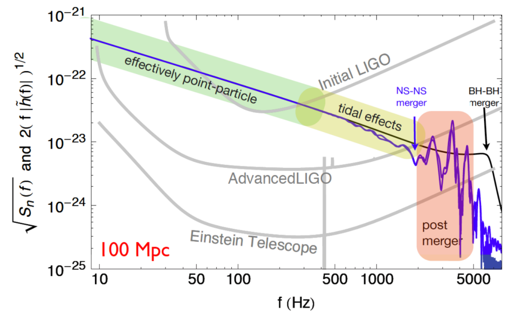




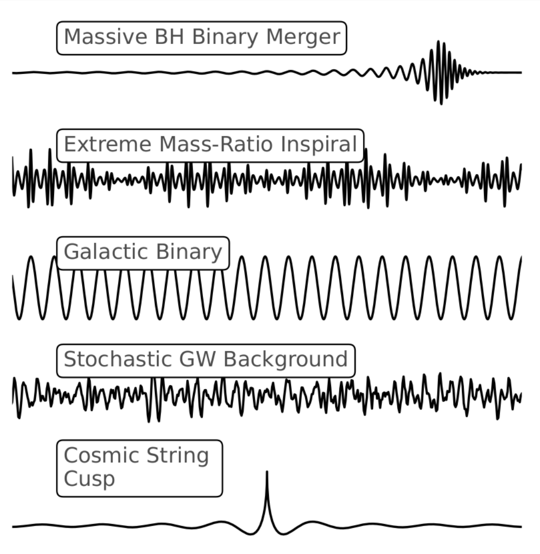
LISA definition study report (2023)

Einstein Telescope science case (2021)



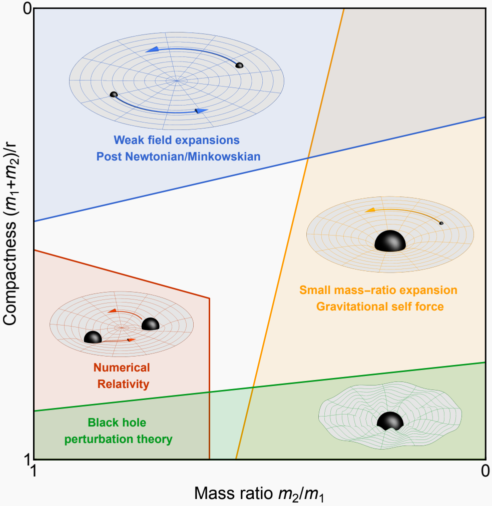


Gravitational waveforms



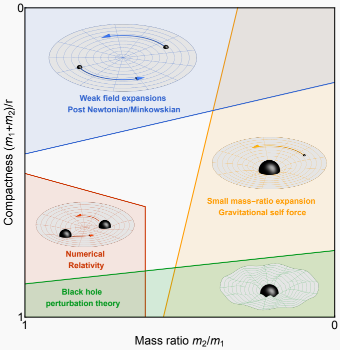
LISA definition study report 2023

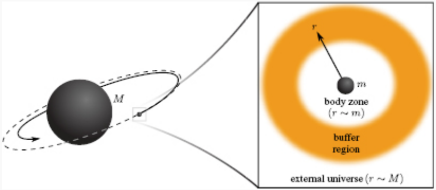
The different methods



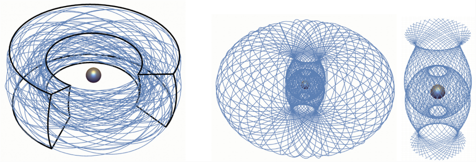
- ▷ Inspiral-Merger-Ringdown (IMR): *effective-one-body, phenomenological & surrogate models*

The different methods: gravitational self-force





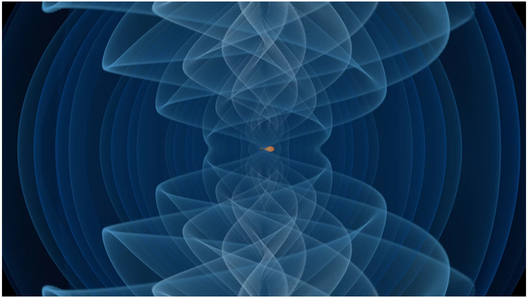
- ▷ extreme mass ratio inspiral
- ▷ expansion in $q = \frac{m_1}{m_2} \ll 1$
- ▷ resonances, par ex. 2:3



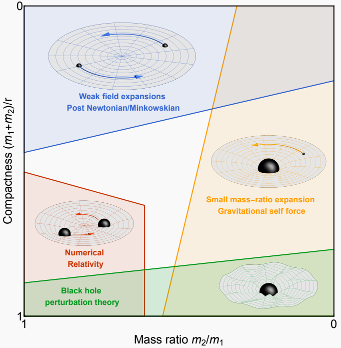
Barack & Pound '18

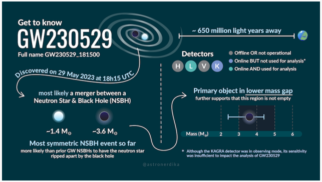
The different methods: numerical relativity

- ▷ solving the full Einstein equations
- ▷ computationally expensive
- ▷ add spins, eccentricity, *etc.*

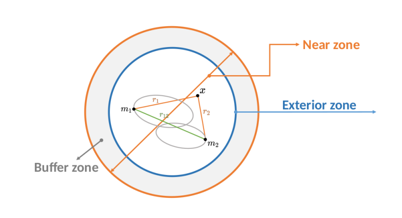


I. Markin, T. Dietrich, H. Pfeiffer, A. Buonanno (Potsdam University and Max Planck Institute for Gravitational Physics)

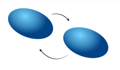




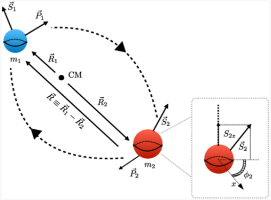
The different methods: post-Newtonian



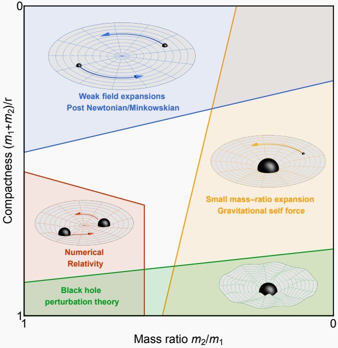
- ▷ expansion in $\epsilon = \frac{v_{12}^2}{c^2} \sim \frac{G(m_1)}{r_{12}c^2} \ll 1$
- ▷ point-particle approximation



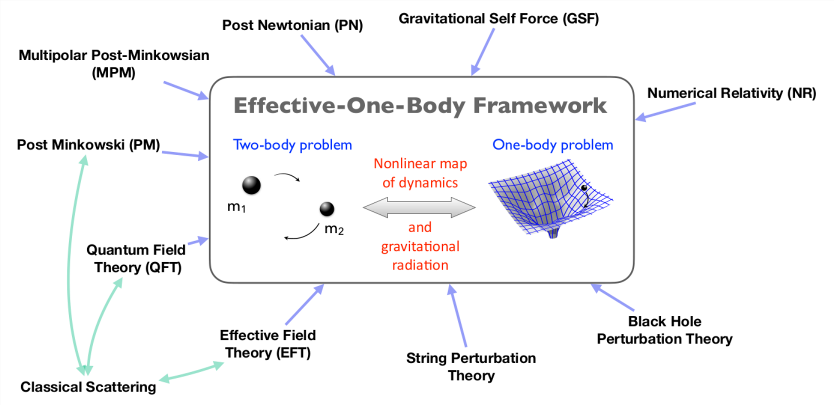
- ▷ add spins, tides, *etc.*



Tanay et al. '23



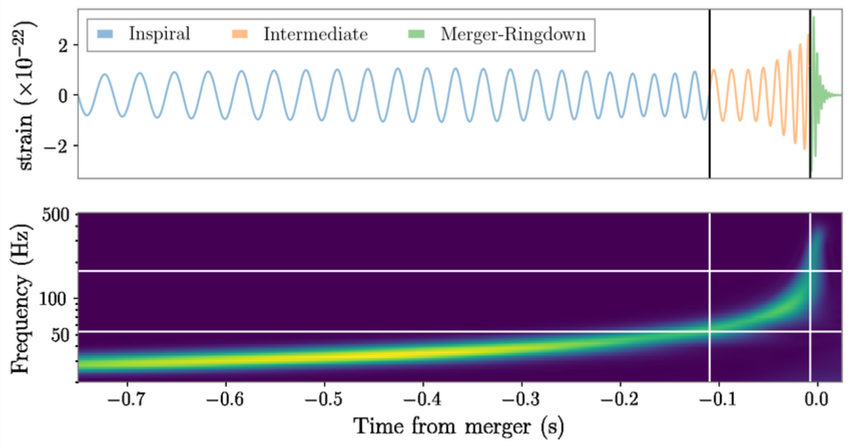
Full IMR waveform: the EOB class



LISA waveform white paper '23

Full IMR waveform: the Phenom class

$$h(f) = \mathcal{A}(f)\, \mathrm{e}^{\psi_n(f)} \qquad \psi_n = \{\varphi_{0,..7},\ \sigma_{0..4},\ \beta_{1..3},\ \alpha_{0..5}\}$$



Kwok et al. '21

The Lagrangian approach

$$S = \int \mathrm{d}^4x \sqrt{-g}\, R\left[g, \partial g, \partial^2 g\right] + S_{\mathsf{m}}\left[y^\rho, v^\rho; g_{\mu\nu}\right]$$

Einstein field equations: $R^{\mu\nu} - \frac{1}{2}\, g^{\mu\nu}\, R = 8\pi\, T^{\mu\nu}$

defining $h^{\mu\nu} \equiv \sqrt{-g}\, g^{\mu\nu} - \eta^{\mu\nu}$, we rewrite it as

$$\underbrace{\Box\, h^{\mu\nu}}_{\text{flat d'Alembertian } \eta^{\rho\sigma}\partial_\rho\partial_\sigma} = \overbrace{\frac{16\pi G}{c^4}|g|T^{\mu\nu}}^{\text{matter fields}} + \underbrace{\Lambda^{\mu\nu}[h, \partial h, \partial^2 h]}_{\text{non-linearities: } \Lambda \sim h\partial^2 h + \partial h \partial h + h\partial h \partial h + \cdots} \equiv \frac{16\pi G}{c^4}\, \tau^{\mu\nu}(\mathbf{x}, t)$$

A short introduction to PN modelling in GR

The Lagrangian approach

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$$\nabla_\nu T^{\mu\nu} = 0 \qquad \Longleftrightarrow \qquad \partial_\nu \tau^{\mu\nu} = 0$$

The hierarchy of scales

Internal zone $r \leq r_s$

- point-particle approximation

$$S_{\mathsf{m}} = -\sum_a m_a \int \mathrm{d}\tau_a$$

- finite-size effects: tides, spins

$$S_{\mathsf{tid.}} = \int \mathrm{d}\tau \left[\mu^{(l)} G_L G^L + \nu^{(l)} H_L H^L\right], \quad G_L = -\nabla_{L-2} C_{\mu_{l-1}\rho\nu_l\sigma} u^\rho u^\sigma$$

$$S_{\mathsf{spin}} = \int \mathrm{d}\tau \left[p_\mu u^\mu + \frac{1}{2} S_{\mu\nu}\Omega^{\mu\nu}\right], \quad S^{\mu\nu} p_\nu = 0 \quad \text{(Spin Sup. Cond.)}$$

- integrating out the internal dofs: one-particle EFT

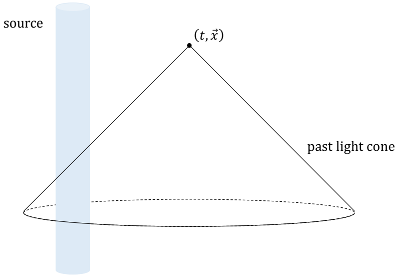
Solving the wave equation: the retarded solution

$$h^{\mu\nu}(\mathbf{x},t) = \frac{16\pi G}{c^4}\left(\Box_{\rm ret}^{-1}\tau^{\mu\nu}\right)(\mathbf{x},t)$$

Flat-space retarded propagator

$$\left(\Box_{\rm ret}^{-1}\tau\right)(\mathbf{x},t) \equiv -\frac{1}{4\pi}\int_{\mathbb{R}^3}\frac{\mathrm{d}^3\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|}\,\tau\left(\mathbf{x}',t-\frac{|\mathbf{x}-\mathbf{x}'|}{c}\right)$$

- This is an integral over the past light cone of the point $(\mathbf{x},t)$



The hierarchy of scales

Internal zone $r \leq r_s$

- point-particle approximation
- finite-size effects: tides, spins
- integrating out the internal dofs: one-particle EFT

Near zone $r_s < r < R$

$$\frac{\tau\left(\mathbf{x}', t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}\right)}{|\mathbf{x}-\mathbf{x}'|} = \frac{\tau\left(\mathbf{x}', t\right)}{|\mathbf{x}-\mathbf{x}'|} - \frac{\dot{\tau}\left(\mathbf{x}', t\right)}{c} + \frac{|\mathbf{x}-\mathbf{x}'|}{2c^2}\ddot{\tau}\left(\mathbf{x}', t\right) + \cdots$$

- generates a PN expansion: $\bar{h}^{\mu\nu} = \sum_{m=2}^{\infty} \frac{1}{c^m} \bar{h}^{\mu\nu}_m$ with

$$\Box \bar{h}^{\mu\nu}_m = 16\pi G \left(\underbrace{T^{\mu\nu}}_{\text{source}} + \underbrace{\Lambda^{\mu\nu}}_{\text{nonlinearities}} \right), \quad \partial_\nu \bar{h}^{\mu\nu}_m = 0$$

  - divergences when $r \gg \lambda_{\mathrm{GW}}$ and $\mathbf{x} \to \mathbf{y}_{1,2}$: **dimensional regularization**
  - conservative orbital dynamics
  - integrating out the potential modes: binding potential

The hierarchy of scales

Internal zone $r \leq r_s$

- ▷ point-particle approximation
- ▷ finite-size effects: tides, spins
- ▷ integrating out the internal dofs: one-particle EFT

Near zone $r_s < r < R$

- ▷ conservative orbital dynamics
- ▷ integrating out the potential modes: binding potential

Wave zone $r \gg \lambda$

$$\frac{\tau\left(\mathbf{x}', t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}\right)}{|\mathbf{x}-\mathbf{x}'|} = \frac{\tau\left(\mathbf{x}', t - \frac{r}{c}\right)}{r} - x'^j \partial_j \left(\frac{\tau\left(\mathbf{x}', t - \frac{r}{c}\right)}{r}\right) + \cdots$$

1. PM expansion: $h^{\alpha\beta}_{\text{ext}} = \sum_{n=1}^{\infty} G^n h^{\alpha\beta}_{(n)}$

$$\Box h^{\alpha\beta}_{(n)} = \Lambda^{\alpha\beta}_n \left[h_{(1)}, ..., h_{(n-1)}\right] \,, \qquad \partial_\beta h^{\alpha\beta}_{(n)} = 0$$

2. most general solution: $h^{\alpha\beta}_{(n)}[\underbrace{I_L, J_L}_{\text{source}}; \underbrace{W_L, X_L, Y_L, Z_L}_{\text{gauge}}]$

- ▷ radiative power loss and gravitational waveform
- ▷ integrating out the radiation modes: EFT of dynamical multipoles

The hierarchy of scales

Internal zone $r \leq r_s$

- point-particle approximation
- finite-size effects: tides, spins
- integrating out the internal dofs: one-particle EFT

Near zone $r_s < r < R$

- conservative orbital dynamics
- integrating out the potential modes: binding potential

Wave zone $r \gg \lambda$

- radiative power loss and gravitational waveform
- integrating out the radiation modes: EFT of dynamical multipoles

Interplay between near and wave zone

- hereditary effects: tails, memory
- radiation-reaction forces

Gravitation waveforms

Gravitational wave field

$$H_{ij}^{TT} = \frac{2G}{c^4 R} P_{ijkl}(\mathbf{N}) \left\{ \ddot{U}_{kl}\left(T - \frac{R}{C}\right) + \mathcal{O}\left(\frac{1}{c}\right) \right\} + \mathcal{O}\left(\frac{1}{R^2}\right)$$

Energy balance equation

$$\langle \frac{\mathrm{d}E}{\mathrm{d}t} \rangle = -\langle \mathcal{F} \rangle \qquad \text{with} \qquad \mathcal{F} \equiv \left(\frac{\mathrm{d}E}{\mathrm{d}t}\right)^{\mathsf{GW}} = \frac{G}{c^5}\left[\dddot{U}_{ij} \dddot{U}_{ij} + \mathcal{O}\left(\frac{1}{c^2}\right) \right]$$

Dynamics period decay $\frac{\mathrm{d}P}{\mathrm{d}t}$, eccentricity $\frac{\mathrm{d}e}{\mathrm{d}t}$

GW modes amplitude $a(t) \propto (t_c - t)^{1/4}$, phase $\phi(t) \propto (t_c - t)^{5/8}$

State-of-the-art in GR - dynamics

	Dynamics				
PN order	non-spinning	spinning			tides
		SO	SS	higher spins	
0	$\surd$	-	-	-	-
1	$\surd$	-	-	-	-
1.5	-	$\surd$	-	-	-
2	$\surd$	-	$\surd$	-	-
2.5	$\surd$	$\surd$	-	-	-
3	$\surd$	-	$\surd$	-	-
3.5	$\surd$	$\surd$	-	$\surd$ (S^3)	-
4	$\surd$	-	$\surd$	$\surd$ (S^4)	-
4.5	$\surd$	$\surd$	-	$\surd$ (S^3)	-
5	$*$	-	$\surd$	$\surd$ (S^4)	$\surd$
5.5	$*$			$\surd$ (S^5)	-
6				$\surd$ (S^6)	$\surd$ (7PN)

State-of-the-art in GR - flux and GW modes

	Dissipative flux				
PN order	non-spinning	spinning			tides
		SO	SS	higher spins	
2.5	$\surd$	-	-	-	-
3	-	-	-	-	-
3.5	$\surd$	-	-	-	-
4	$\surd$	$\surd$	-	-	-
4.5	$\surd$	-	$\surd$	-	-
5	$\surd$	$\surd$	-	-	$\surd$
5.5	$\surd$	$\surd$	$\surd$	-	-
6	$\surd$	$\surd$	$\surd$	$\surd$ (S^3)	$\surd$
6.5	$\surd$	$\surd$	$\surd$	-	$\surd$
7	$\surd$				$\surd$

- Small eccentricity waveforms up to 4PN

Post-Newtonian and post-Minkowskian formalisms

Traditional PN: direct iteration and integration in the direct space

Effective Field Theory: diagramatic and integration in Fourier space

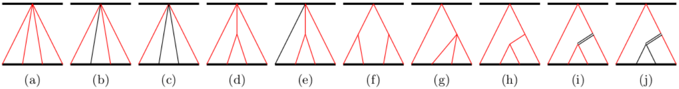


Figure 4: Diagrams contributing at order $G^4 m_1^4 \lambda_2^{(0)}$

Scattering amplitudes: PM expansion, diagrammatic and integration in Fourier space

	0PN		1PN		2PN		3PN		4PN		5PN		6PN		7PN			
1PM	(1	+	v^2	+	v^4	+	v^6	+	v^8	+	v^{10}	+	v^{12}	+	v^{14}	+	...)	G^1
2PM			(1	+	v^2	+	v^4	+	v^6	+	v^8	+	v^{10}	+	v^{12}	+	...)	G^2
3PM					(1	+	v^2	+	v^4	+	v^6	+	v^8	+	v^{10}	+	...)	G^3
4PM							(1	+	v^2	+	v^4	+	v^6	+	v^8	+	...)	G^4
5PM									(1	+	v^2	+	v^4	+	v^6	+	...)	G^5
6PM											(1	+	v^2	+	v^4	+	...)	G^6

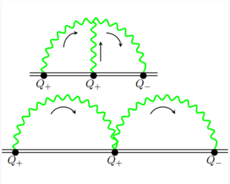
Comparison table of powers used for PN and PM approximations in the case of two non-rotating bodies.

What about 5PN and beyond?

Synergies between different techniques

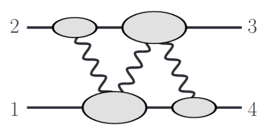
NRGR – EFT techniques

- gravitational potential, 5PN hereditary terms on-going



Scattering amplitudes

- QFT 2-body interaction as an EFT
- extract classical part
- map to bound systems
- 3-loops complete $\longrightarrow$ 4PM



Self force : $\mathcal{O}(\nu)$

- 5PN and 6PN partial

Application to scalar-tensor theories

$$S_{\mathrm{ST}} = \frac{c^3}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\left[R - \frac{1}{2}g^{\alpha\beta}\partial_\alpha\varphi\,\partial_\beta\varphi\right] + S_m\left[\mathfrak{m}, A(\varphi)\,g_{\alpha\beta}\right]$$

Field equations in Einstein frame

$$\begin{cases} \Box h^{\mu\nu} = 16\pi G\,\tau^{\mu\nu} \text{ with } \tau^{\mu\nu} = \underbrace{T^{\mu\nu}}_{\text{source}} + \underbrace{\Lambda^{\mu\nu}}_{\nabla h\cdot\nabla h} + \underbrace{\Lambda^{\mu\nu}_{\mathsf{ST}}}_{\sim\nabla\varphi\cdot\nabla\varphi} \\ \Box\varphi = 16\pi G\,\tau_{\mathsf{s}} \text{ with } \tau_{\mathsf{s}} = \underbrace{T_{\mathsf{S}}}_{\text{source}} + \underbrace{\Sigma_{\mathsf{S}}}_{\sim h\partial^2\varphi+\partial h\partial\varphi+\nabla\varphi\cdot\nabla\varphi} \end{cases}$$

- a good starting point for more complicated theories
  - Einstein-scalar-Gauss-Bonnet, Einstein-Maxwell-dilaton
- PN expansion for $\varphi \sim h^{00}$
- no hair theorem but scalarized neutron stars

Internal zone: point-particles and beyond

Point-like sources

$$S_{\rm pp} = -\int \mathrm{d}\tau_a \, m_a(\phi)$$

- ○ minimal coupling, non-spinning massive objects
- ▷ potentially scalarized objects: $m_a(\phi)$ [Eardley '75]

NSs Tolman–Oppenheimer–Volkoff equations

BHs matching to known (analytical) solutions [Julié 2017, Julié-Berti 2019]

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Spinning particles

- ▷ beyond the mass monopole: coupling to dipole and higher multipoles

$$S_{\rm pp} = -\int \mathrm{d}\tau_a \left[P_\mu \left(\phi, \dots\right) u^\mu - \frac{1}{2} S_{\mu\nu} \left(\phi, \dots\right) \Omega^{\mu\nu} + \cdots \right]$$

  - ▷ spin supplementary condition: $S^{\mu\nu} P_\nu = 0$ [Tulczyjew '59]

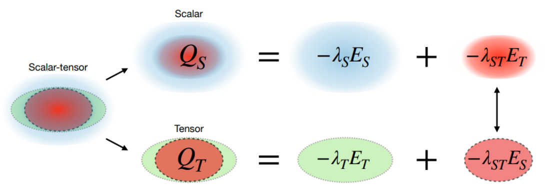
Beyond point particles: scalar tides

Reminder in GR

- electric and magnetic type Love numbers
- effacement principle: start at $5PN \sim \left(\frac{v}{c}\right)^{10}$

In scalar-tensor

- scalar dipole moment $\mathcal{E}_i \propto \partial_i \phi \Rightarrow$ scalar-induced tidal deformability



- enhanced effect wrt GR: $3PN$

Creci et al. '23

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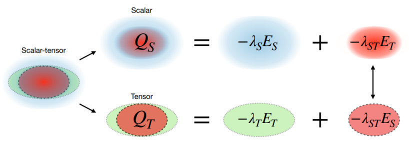

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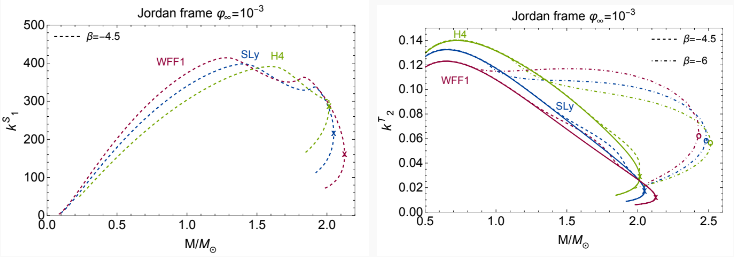


Creci et al. '23

- more important at low frequency (LISA) or highly scalarized objects

Different types of tidal effects

$$S_{\text{tidal}} = -\frac{c}{2}\sum_{a=1,2}\int \mathrm{d}\tau_a \Big\{\lambda_a(\phi)\left(\nabla^{\perp}_{\alpha}\varphi\right)_a\left(\nabla^{\alpha}_{\perp}\varphi\right)_a + \frac{1}{2}\mu_a(\phi)\left(\nabla^{\perp}_{\alpha\beta}\varphi\right)_a\left(\nabla^{\alpha\beta}_{\perp}\varphi\right)_a$$
$$+\nu_a(\phi)\left(\nabla^{\perp}_{\alpha\beta}\varphi\right)_a\left(G^{\alpha\beta}\right)_a - \frac{1}{2c^2}c_a(\phi)\left(G_{\alpha\beta}\right)^a\left(G^{\alpha\beta}\right)_a\Big\}$$

with $(G_{\mu\nu})_a = -c^2\left(C_{\mu\rho\nu\sigma}\right)_a u^{\rho}_a\, u^{\sigma}_a$ an $\nabla^{\perp}_{\mu} \equiv \left(\delta^{\nu}_{\mu} + u_{\mu}u^{\nu}\right)\nabla_{\nu}$

Scalar LO (3PN), NLO (4PN), NNLO (5PN)

- dimensionless scalar tidal deformability: $k_s \equiv \frac{G\,\lambda_s}{c^2\,R^3} \underbrace{\longrightarrow}_{\frac{Gm}{Rc^2}\sim 1}$ 3PN
- leading order in the eoms

$$\Delta\mathbf{a}_{(fs)} \propto \mathbf{a}_{(\mathrm{N})}\cdot\left[\frac{m_2}{m_1}\,\bar{\delta}_2\,k_1^{(s)} + \frac{m_1}{m_2}\,\bar{\delta}_1\,k_2^{(s)}\right]\frac{R^3}{r^3}$$

[LB 2018]

Gravito-scalar LO (5PN)

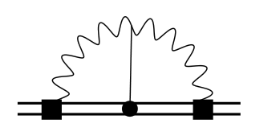
Gravitational (electric and magnetic type) LO (5PN)

[LB, Dones & Mougiakakos, 2023]

The ST equations of motion

$$\frac{\mathrm{d}\mathbf{v_1}}{\mathrm{d}t} = \underbrace{-\frac{G_{\text{eff}}\, m_2}{r_{12}^2}\mathbf{n_{12}} + \frac{\mathbf{A}_{1\text{PN}}}{c^2}}_{\text{conservative terms}} + \underbrace{\frac{\mathbf{A}_{1.5\text{PN}}}{c^3}}_{\text{rad. reac.}} + \underbrace{\frac{\mathbf{A}_{2\text{PN}}}{c^4}}_{\text{cons.}} + \underbrace{\frac{\mathbf{A}_{2.5\text{PN}}}{c^5}}_{\text{rad. reac.}}$$

$$+ \underbrace{\frac{\mathbf{A}_{3\text{PN}}^{\text{inst}}}{c^6}}_{\text{cons, local}} + \underbrace{\frac{\mathbf{A}_{3\text{PN}}^{\text{tail}}}{c^6}}_{\text{cons, nonloc.}} + \underbrace{\frac{\mathbf{A}_{3\text{PN}}^{\text{tidal}}}{c^6}}_{\text{cons, local}}$$



Differences w.r.t. GR

- Dissipative effects start at 1.5PN (*v.s.* 2.5PN in GR)
- Tidal effects start at 3PN (*v.s.* 5PN in GR)
- A conservative scalar tail term at 3PN : $\mathbf{A}_{3\text{PN}}^{\text{tail}} \propto M \int_{-\infty}^{+\infty} \frac{\mathrm{d}t'}{|t-t'|}\, I_{s\,i}^{(4)}(t')$

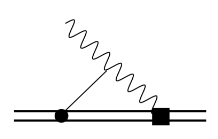
[LB '18, '19, Mougiakakos & LB in prep. ('24)]

The scalar and gravitational fluxes

$$\mathcal{F} = \frac{32c^5\nu^2x^5}{5G_{\text{eff}}}\left[1 + \frac{\mathcal{F}^{\text{grav}}_{1\text{PN}}}{c^2} + \frac{\mathcal{F}^{\text{grav}}_{1.5\text{PN}}}{c^3} + \frac{\mathcal{F}^{\text{grav}}_{2\text{PN}}}{c^4}\right] + \frac{4c^5\nu^2x^4}{3G_{\text{eff}}}\zeta S_-^2 \cdot \frac{\mathcal{F}^{\text{scal, tidal}}_{2\text{PN}}}{c^4}$$

$$+ \frac{4c^5\nu^2x^5}{3G_{\text{eff}}}\zeta S_-^2\left[x^{-1} + \frac{\mathcal{F}^{\text{scal}}_{0\text{PN}}}{c^0} + \frac{\mathcal{F}^{\text{scal}}_{0.5\text{PN}}}{c^1} + \frac{\mathcal{F}^{\text{scal}}_{1\text{PN}}}{c^2} + \frac{\mathcal{F}^{\text{scal}}_{1.5\text{PN}}}{c^3} + \frac{\mathcal{F}^{\text{scal}}_{2\text{PN}}}{c^4}\right]$$

$x \equiv \left(\frac{G_{\text{eff}}\, m\omega}{c^3}\right)^{2/3}, \; \nu \equiv \frac{m_1 m_2}{m^2}$



Differences w.r.t. GR

- Scalar flux starts at -1PN, known at 1.5PN: **2PN in progress**
- Scalar tidal contribution at 2PN
- Scalar memory term at 1.5PN: $\delta U_{ij} \propto \frac{1}{c^3}\int \frac{\mathrm{d}t'}{|t-t'|}\, {I_s}_i^{(2)}(t') {I_s}_j^{(2)}(t')$
- Scalar tail term at 0.5PN: $\delta U_i^s \propto \frac{M}{c^3}\int \mathrm{d}\tau\, \ln\left(\frac{\tau}{\tau_0}\right)\, {I_s}_i^{(3)}(t-\tau)$

[LB, Blanchet & Trestini 2022, Dones, Trestini & LB in prep. ('24)]

Concluding remarks

Small coupling curvature corrections

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left[R - \frac{1}{2} \nabla_a \varphi \nabla^a \varphi + \alpha\, f(\varphi)\, \mathcal{G} \right] + S_m[\psi_{\mathsf{m}};\, g]$$

$$\frac{\mathrm{d}\mathbf{v_1}}{\mathrm{d}t} = \underbrace{\frac{\mathbf{A}_{3\mathrm{PN}}}{c^6}}_{\text{ST}} + \underbrace{\alpha\, f'(\varphi)\, \frac{\mathbf{a}^{(\mathrm{LO})}}{c^2}}_{\text{new th.}}$$

Some examples with hairy BH solutions

- Einstein-scalar-Gauss-Bonnet: $\mathcal{G} = R_{abcd} R^{abcd} - 4 R_{ab} R^{ab} + R^2$
  - $[\alpha] = [\mathrm{length}]^2 \implies$ 3PN effect [Julié & Berti 2018, 2019, van Gemeren et al. 2023, LB, Dones & Mougiakakos 2023]
- Chern-Simmons gravity (parity-violating): $\mathcal{G} = \frac{1}{2}\, \epsilon^{\rho\sigma\alpha\beta}\, R_{\nu\mu\rho\sigma}\, R^{\mu\nu}{}_{\alpha\beta}$
  - $[\alpha] = [\mathrm{length}]^2 \implies$ 4PN effect (spins only) [Loutrel, Tanaka, Yagi & Yunes 2012, 2018]
- Cubic gravity: $\varphi = 1, \quad \mathcal{G} = C_{\mu\nu\rho\sigma}\, C^{\rho\sigma}{}_{\alpha\beta} C^{\mu\nu\alpha\beta}$
  - $[\alpha] = [\mathrm{length}]^4 \implies$ 5PN effect

Future prospects

- Many recent progresses in the simplest theories:
  - scattering angle [Jain 2023, LB, Jain & Mougiakakos in prep.]
  - higher order gravity [LB, Giri & Lehner in prep.]
- Synergies with numerical relativity results: full IMR models
- Important for the future: non-perturbative effects (scalarization), spins, dynamical tides
- Missing theories or very preliminary: Lorentz-violating, DHOST, massive gravity, *etc.*

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Thank you!

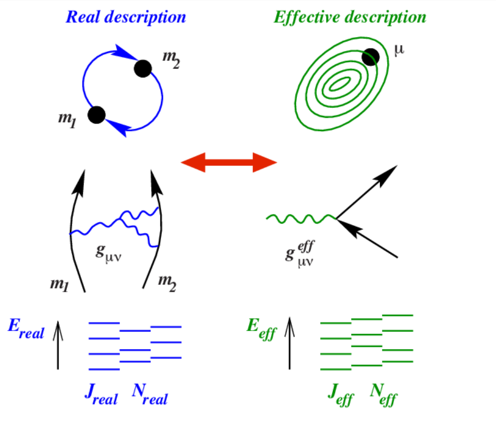
Supplementary

Total number of accumulated cycles

Detector	LIGO/Virgo		ET		LISA	
Masses ($M_\odot$)	1.4×1.4	10×10	1.4×1.4	500×500	$10^5 \times 10^5$	$10^7 \times 10^7$
PN order	cumulative number of cycles					
Newtonian	2 562.599	95.502	744 401.36	37.90	28 095.39	9.534
1PN	143.453	17.879	4 433.85	9.60	618.31	3.386
1.5PN	−94.817	−20.797	−1 005.78	−12.63	−265.70	−5.181
2PN	5.811	2.124	23.94	1.44	11.35	0.677
2.5PN	−8.105	−4.604	−17.01	−3.42	−12.47	−1.821
3PN	1.858	1.731	2.69	1.43	2.59	0.876
3.5PN	−0.627	−0.689	−0.93	−0.59	−0.91	−0.383
4PN	−0.107	−0.064	−0.12	−0.04	−0.12	−0.013
4.5PN	0.098	0.118	0.14	0.10	0.14	0.065

D. Trestini, PhD thesis (2023)

The basics of EOB

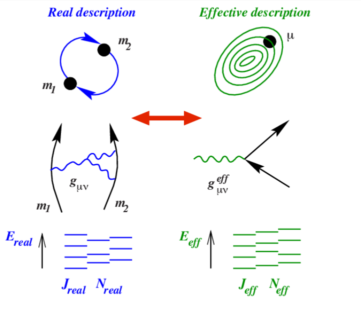


Credits: Buonanno & Sathyaprakash

- Canonical transformation: $(r, \phi, p_r, p_\phi) \longrightarrow (R, \Phi, P_R, P_\Phi)$

$$H_{2\text{body}} \longrightarrow H_{\text{eff}} \longrightarrow H_{EOB} = M\sqrt{1 + 2\nu\,(H_{\text{eff}} - 1)}$$

Current status



EOB Hamiltonian

- 2PN in simple ST theories [Julié 2017]
- 3PN [Julié et al. 2022; Jain et al. 2022]
  - includes non-local in time tail terms
  - extended to include LO contribution in EsGB

Spins

Effective field theory description (as for GR)

$$S_{\mathrm{mat}} = \int \mathrm{d}\tau_A \left[p_\mu u^\mu + \frac{1}{2} S_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{m^2}\mathcal{C}_\varphi{}^* S_\nu^{\ \alpha} u^\nu \nabla_\mu \varphi \underbrace{- \frac{1}{6} J^{\gamma\nu\sigma\alpha} R_{\gamma\nu\sigma\alpha}}_{\text{quadrupole corr.}} \right]_A$$

- obtained by doing $p_\mu \longrightarrow \mathcal{P}_\mu \left(u^\mu, \Omega^{\mu\nu}, g_{\mu\nu}, R_{\mu\nu\rho\sigma}, \varphi, \partial\varphi, \ldots\right)$
  - modified spin supplementary condition $S^{\mu\nu}\mathcal{P}_\nu = 0$
- LO (2PN) dynamics: SS, monopole-quadrupole, scalar-dipole [Loutrel et al. 2018]
- Corrections to the phase at 1.5PN (SO, SS) and 2PN (scalar dipole) [Loutrel et al. 2022]

Scalarizations

- PN results can apply to already (adiabatically) scalarized objects
  - scalar charges computed by matching to known solutions
- dynamical scalarization
  - effective description: [Khalil et al;, 2022]