Numerical Relativity in effective field theories of gravity

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École Normale Supérieure

November 05 2024: Théorie, Univers et Gravitation – TUG, LAPTh – Annecy, November 05 – November 07 2024

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POUR LE DEVELOPPEMENT CULTUREL ET ARTISTIQUE

Part I: Gravitational Effective Field Theories

Part II: Well-posedness with Pau Figueras and Áron Kovács

Part III: Nonlinear evolution (Quadratic Gravity) with Hyun Lim

Bonus: What about ghost instabilities? with Cédric Deffayet, Shinji Mukhoyama, and Alexander Vikman

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Part I: Gravitational Effective Field Theories

The Effective Field Theory Framework ...

Assuming a given

(i)	IR **field content**	**metric ***
(ii)	IR **symmetries**	**Lorentz invariance**
(iii)	**expansion** scale/scheme	**derivative / curvature**

we expand the effective action in all possible operators.

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... introduces rules to modified gravity.

Joint derivative/curvature expansion ...

$$\mathcal{L}^{(1)}_{\mathsf{EFT}} = \mathsf{M}^2_{\mathsf{Pl}}\,\mathsf{R}$$

$$\mathcal{L}^{(2)}_{\mathsf{EFT}} = \left[\alpha_0 \mathsf{R}_{ab}\mathsf{R}^{ab} - \beta_0\,\mathsf{R}^2\right]$$

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After reduction via
- (i) index symmetries
- (ii) geometric identities
- (iii) 4D-specific identities (e.g. Gauss-Bonnet)

see Fulling CQG 9 (1992); Martin-Garcia, Yllanes, Portugal, CPC 179 (2008)

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... before field redefinitions.

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order-by-order field redefinitions of the form

$$g_{ab} \to g_{ab} + c_1\, g_{ab}\, X + c_2\, X_{ab}$$

can remove any term containing Ricci variables

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Part II: Well-posedness

Noakes, JMP 24, 1846 (1983);
Figueras, Held, Kovacs, 2407.08775

- **General Relativity (Leray weights)**
- **Quadratic Gravity**
- **Cubic Gravity, Quartic Gravity, and beyond …**

Noakes, JMP 24, 1846 (1983);
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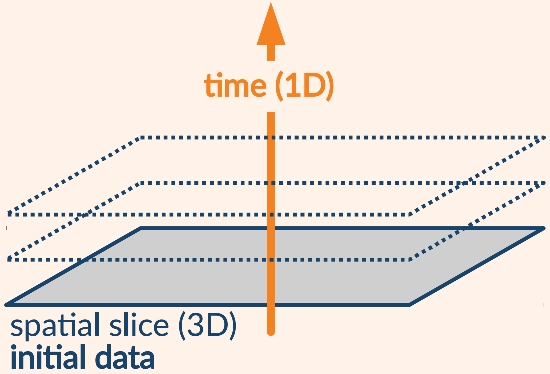
A well-posed initial value problem (IVP) ...



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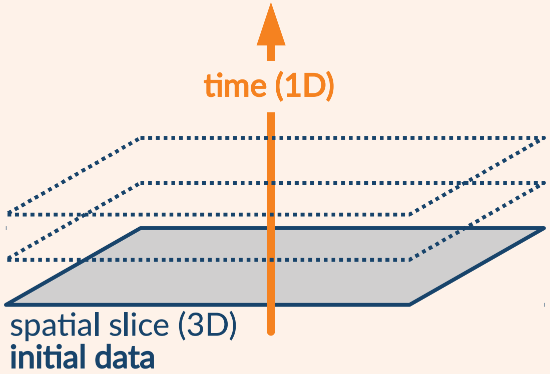

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Formal proof of existence and uniqueness
Yvonne Choquet-Bruhat ‘52

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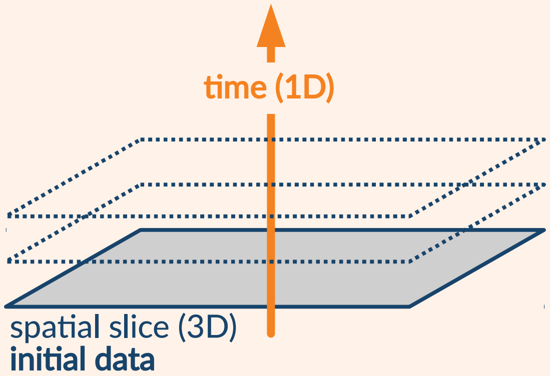

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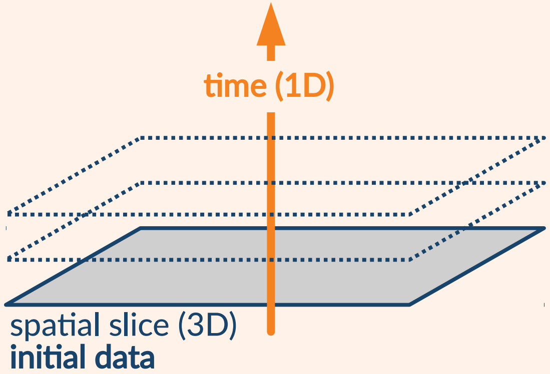

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... and for the EFT (at fixed order)

Figueras, Held, Kovacs, 2407.08775

Leray weights ...

Choquet-Bruhat, DeWitt-Morette, 1982

Let i be an index labeling a system of second order equations E_i for the variables v_i .

The principal part can be diagonalised if there exist positive integers s_i and t_i such that

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... give a prescription to diagonalise the principal part.

Leray weights: A simple example ...

- System of PDEs:

$$\Box u = v$$

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... that can be diagonalised.

Leray weights: A simple example ...

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General Relativity (in harmonic gauge) ...

- Gauge potential: $F^a \equiv -g^{cd}\Gamma^a_{cd}$
- Ricci curvature: $R_{ab} = \Box g_{ab} + g_{c(a}\nabla_{b)}F^c + \mathcal{O}(g, \partial g)$

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- In **harmonic gauge**, i.e., $F^a = 0$
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  are of **wave-like form**.

For constraint propagation see
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... is already in wave-like form.

Quadratic Gravity ...

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Stelle, PRD 16 (1977) 953-969
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2nd-order variables

$$\Box g_{ab} \sim R_{ab} \equiv S_{ab} + \frac{1}{4} g_{ab} R$$

massless spin-2 (graviton)

$$\Box R = m_0^2 R$$

massive spin-0 (scalar)

$$\Box S_{ab} = -\frac{1}{3} \left(\frac{m_2^2}{m_0^2} - 1 \right) (\nabla_a \nabla_b R) - 2 S^{cd} C_{acbd} + \mathcal{O}_{\text{lower order}}$$

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- For equal masses, the 2nd-order field equations of Quadratic Gravity are of wave-like form.

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… admits Leray weights for its 2nd order field equations.

Cubic Gravity (after suitable field redefinitions) ...

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order-reduced 2nd-order field equations

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Higher order EFT (after suitable field redefinitions) …

Figueras, Held, Kovacs, 2407.08775

- Inductively, this extends to $\mathcal{L}^{(n)}_{\mathrm{reg}} = \sum_{k=0}^{n} \left[\alpha_k \, R^{ab} \, \Box^k \, R_{ab} - \beta_k \, R \, \Box^k \, R \right]$ with $\alpha_n = 2\beta_n$

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Well-posed initial value formulation ...

$$\mathcal{L}^{(1)}_{\mathrm{EFT}} = M_{Pl}^2 \, R \quad ✓$$

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Goroff, Sagnotti, Nucl.Phys.B 266 (1986)
Bueno, Cano, PRD 94 (2016) 10
de Rham, Francfort, Zhang, PRD 102 (2020) 2

$$\mathcal{L}^{(4)}_{\mathrm{EFT}} = \frac{1}{M_{Pl}^4}\Big[\alpha_2 R^{ab} \Box^2 R_{ab} - \beta_2 R \Box^2 R + \gamma_{4,1} (C_{abcd} C^{abcd})^2 + \gamma_{4,2} (C_{abcd} \, {}^*C^{abcd})^2 + \dots\Big]$$

Endlich, Gorbenko, Huang, Senatore, JHEP 09, 122 (2017)

order-by-order field redefinitions of the form

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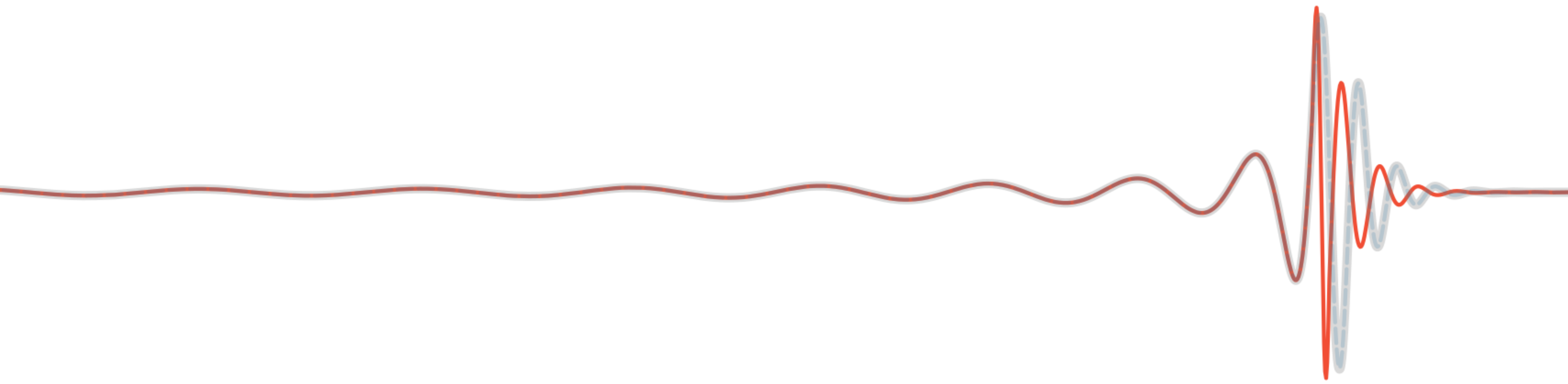
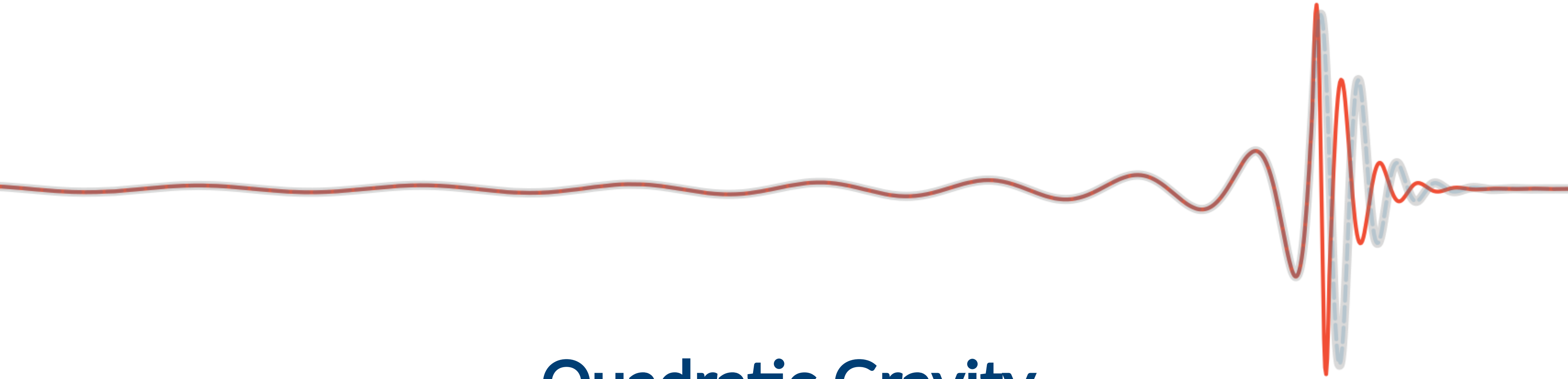
... of general effective field theories of gravity.



Part III: Nonlinear evolution (Quadratic Gravity)

Held, Lim, PRD 104 (2021) 8
Held, Lim, PRD 108 (2023) 10
& to appear



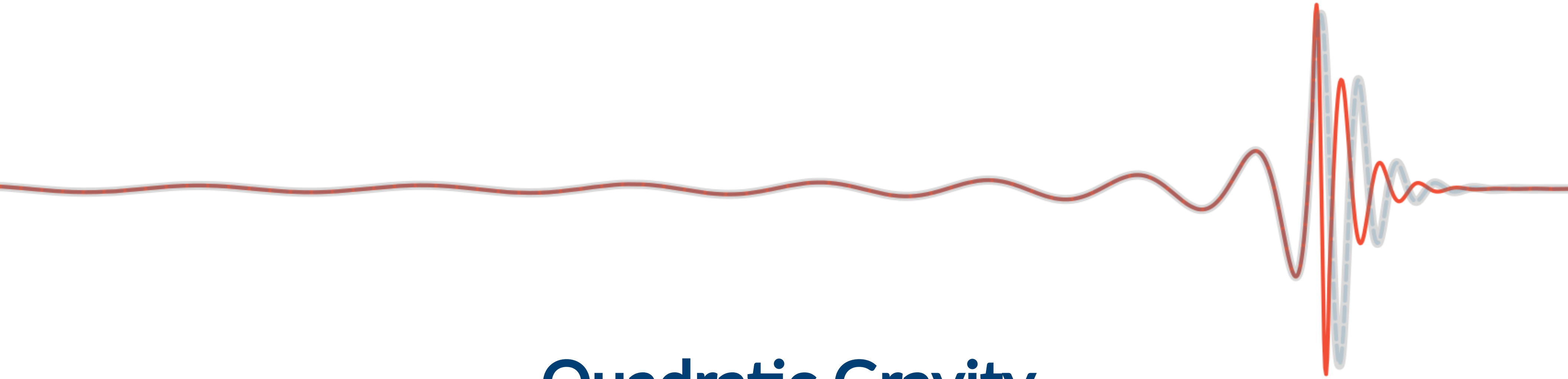
Quadratic Gravity

$$S = \int d^4x \sqrt{|g|}\, M_{\mathrm{Planck}}^2 \left[R + \frac{1}{12 m_0^2} R^2 + \frac{1}{4 m_2^2} C_{abcd} C^{abcd} \right]$$

massless spin-2

massive spin-0

massive spin-2



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massless spin-2 massive spin-0 massive spin-2

as a benchmark model to proceed to cubic/quartic terms

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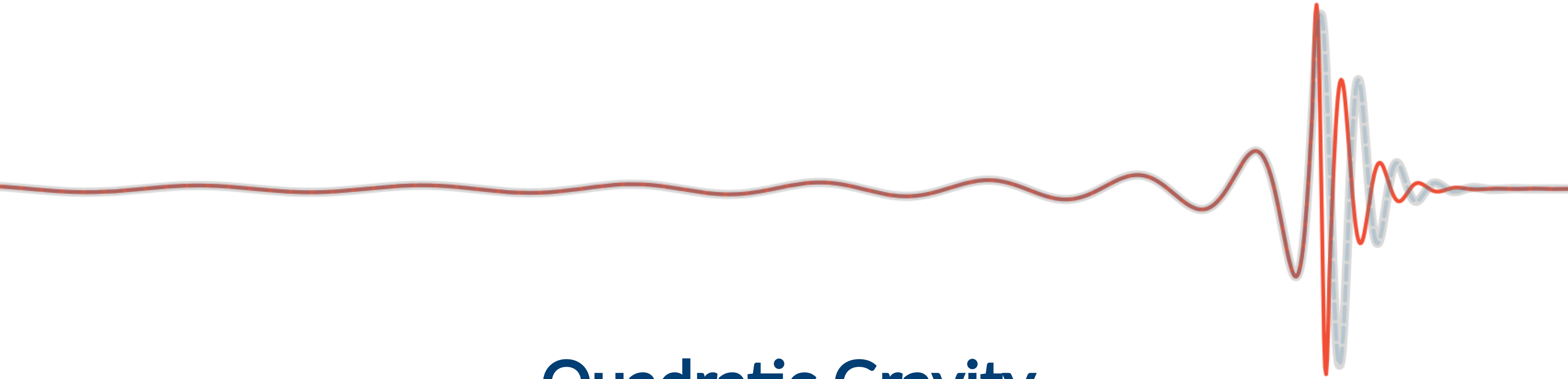
massless spin-2 | massive spin-0 | massive spin-2

as the leading-order terms before field redefinitions / in non-vacuum situations



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massless spin-2 — massive spin-0 — massive spin-2

as a fundamental theory of gravity

Stelle, PRD 16 (1977) 953-969
Avramidi, Barvinsky, PLB 159 (1985) 269-274
Donoghue, Menezes, PRD 104 (2021) 4

- numerically stable evolution
- isolated black holes: stability & transitions
- black-hole binaries & waveforms

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Held, Lim, PRD 104 (2021) 8
Held, Lim, PRD 108 (2023) 10
& to appear

Numerical Evolution of Quadratic Gravity …

Held, Lim, PRD 108 (2023) 10

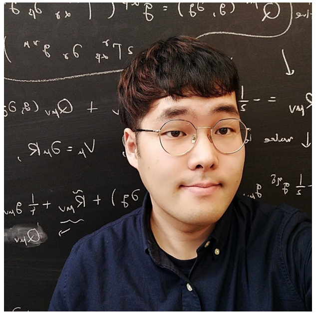
Dendro-GR [adapted]

- parallelized adaptive mesh refinement
- wavelet adaptive multiresolution
- 4th order finite differencing
- 4th order Runge-Kutta

Fernando et.Al. 2018

Numerical Evolution of Quadratic Gravity ...

Held, Lim, PRD 108 (2023) 10



Hyun Lim
Los Alamos

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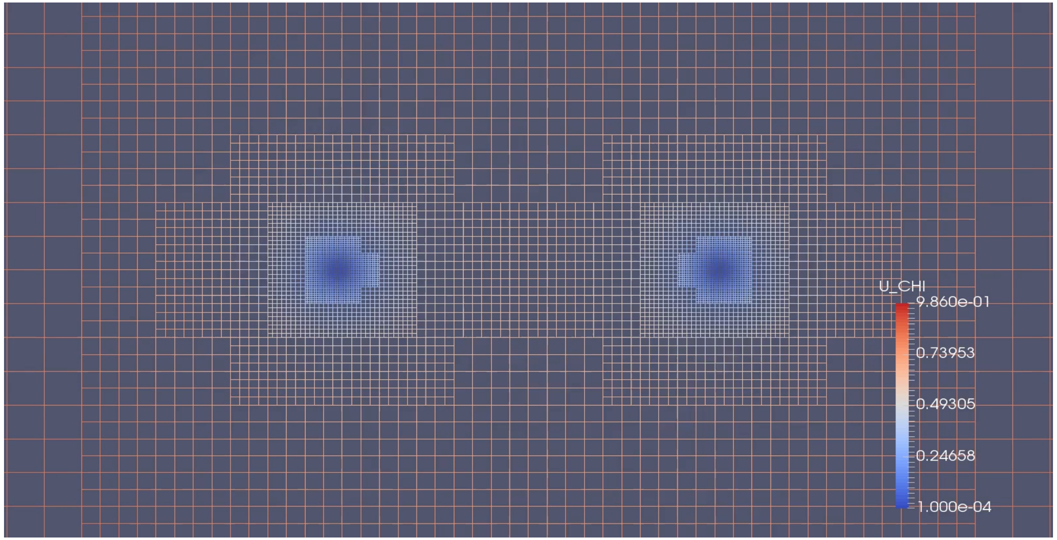
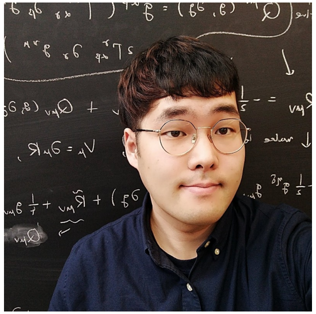


Dendro-GR (Fernando et.Al. 2018),
https://github.com/paralab/Dendro-GR

Numerical Evolution of Quadratic Gravity …

Held, Lim, PRD 108 (2023) 10



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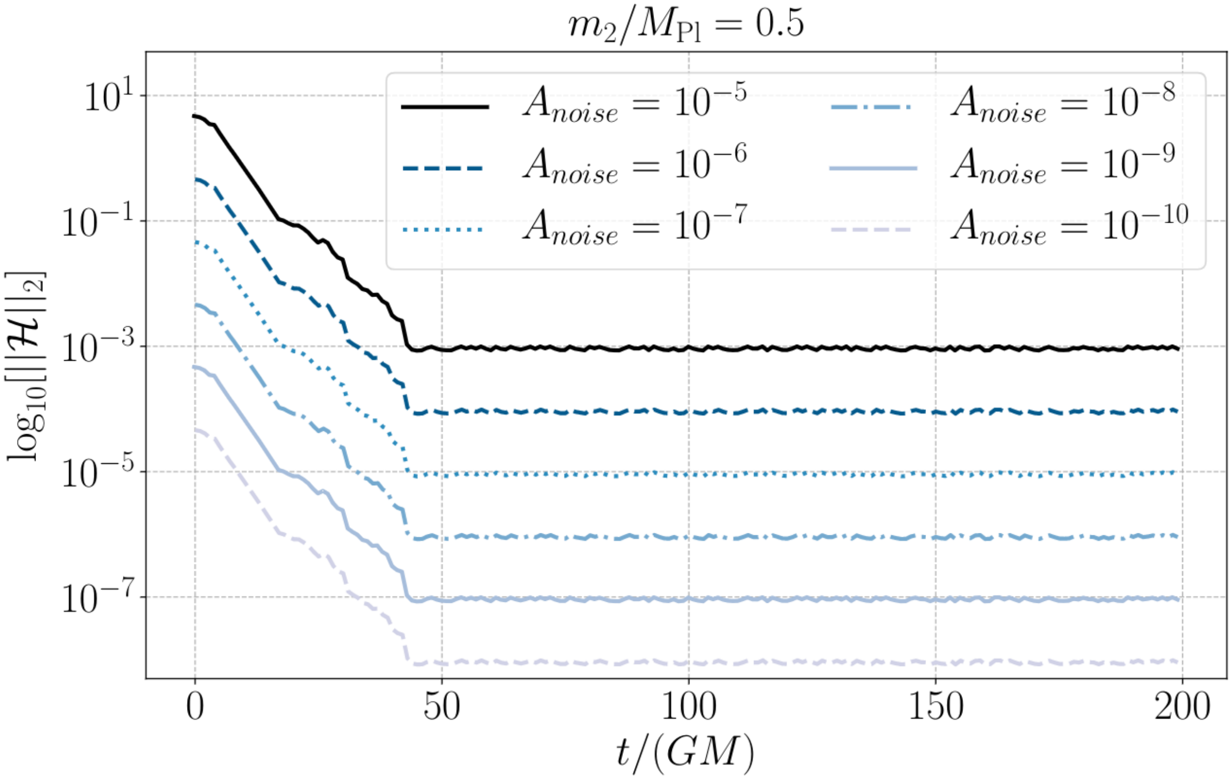
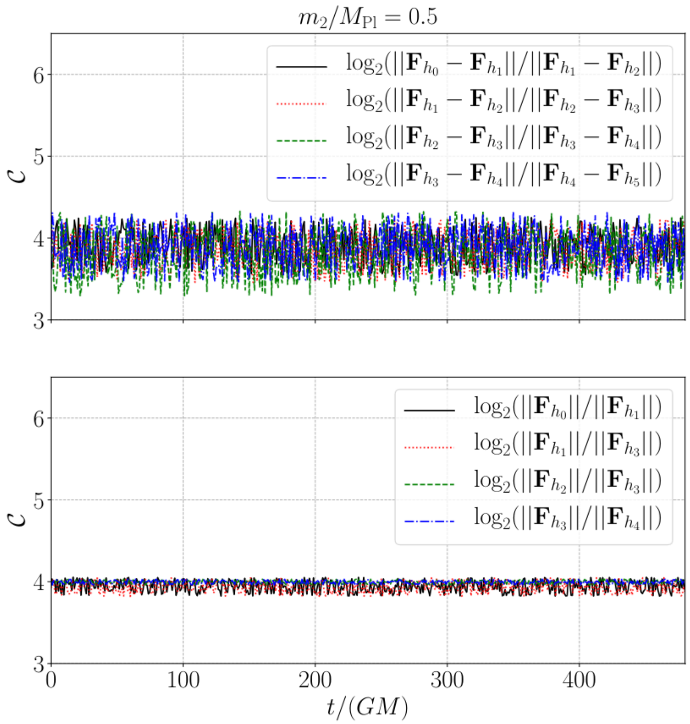
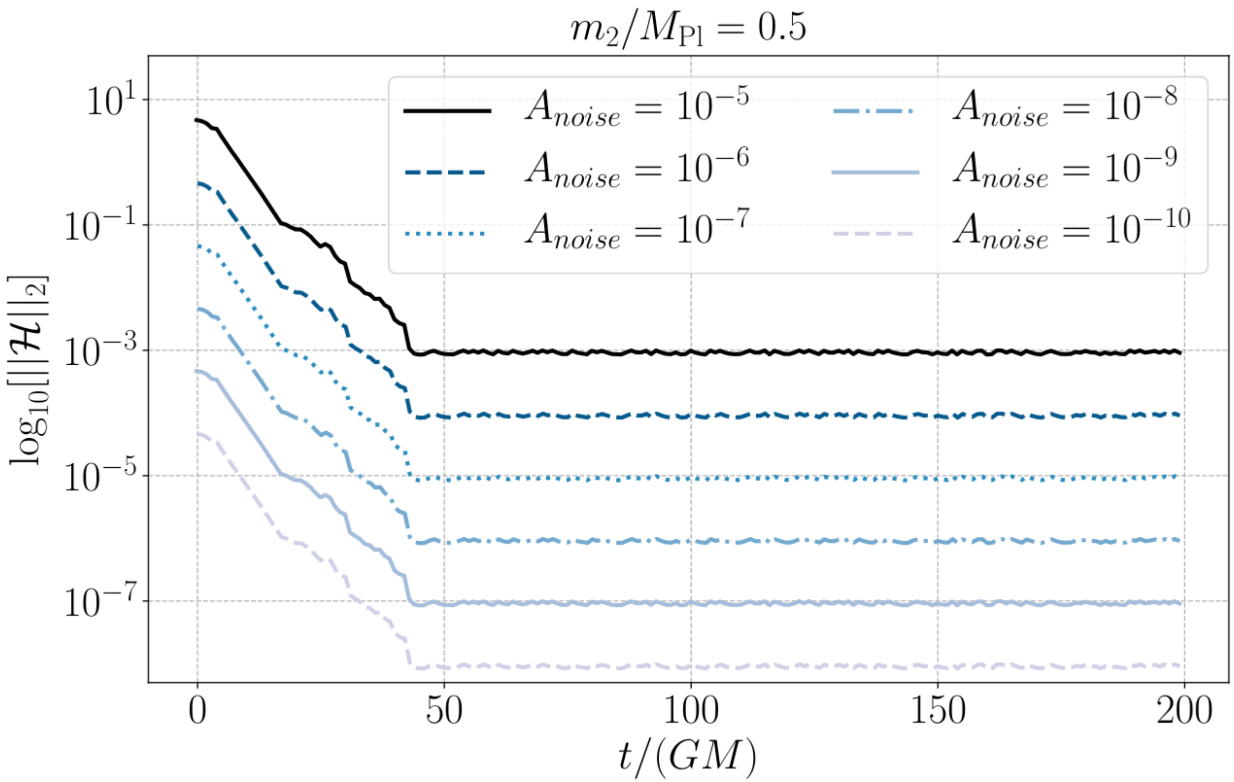


Numerical Evolution of Quadratic Gravity …

Held, Lim, PRD 108 (2023) 10





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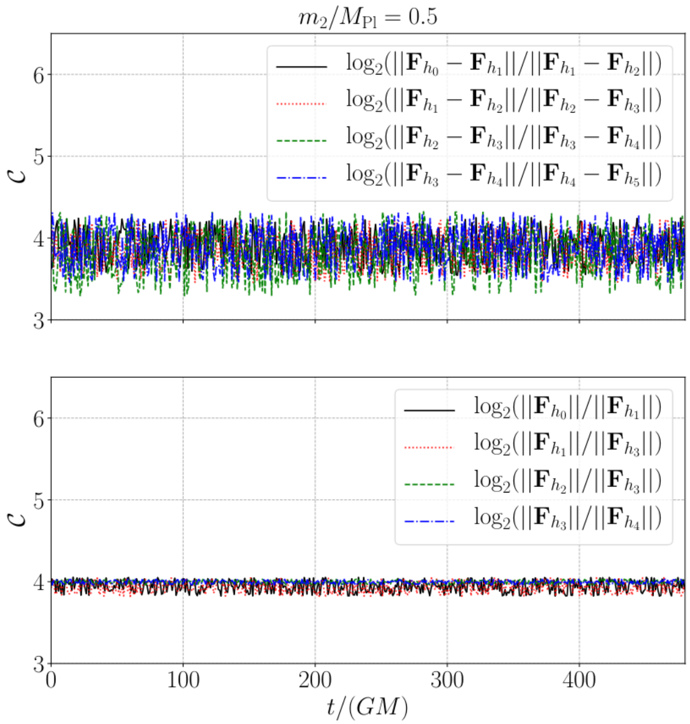

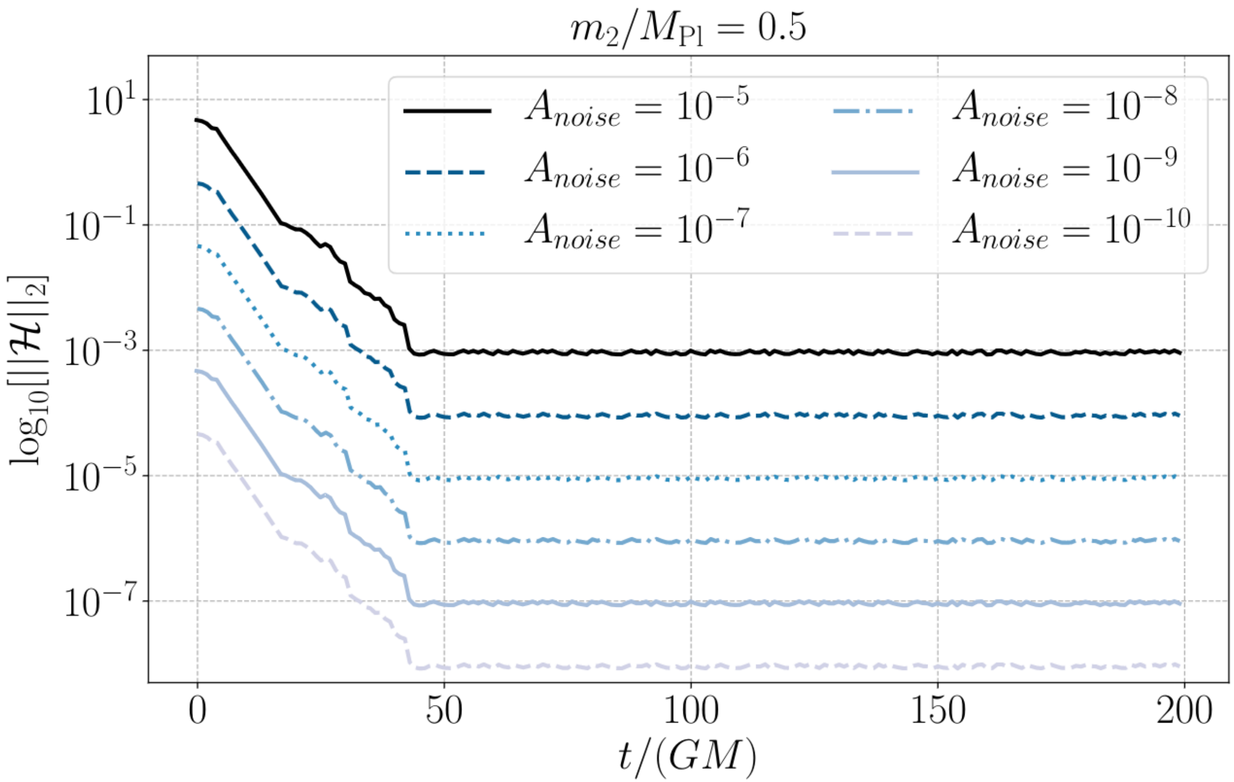

… is numerically stable.

- numerically stable evolution
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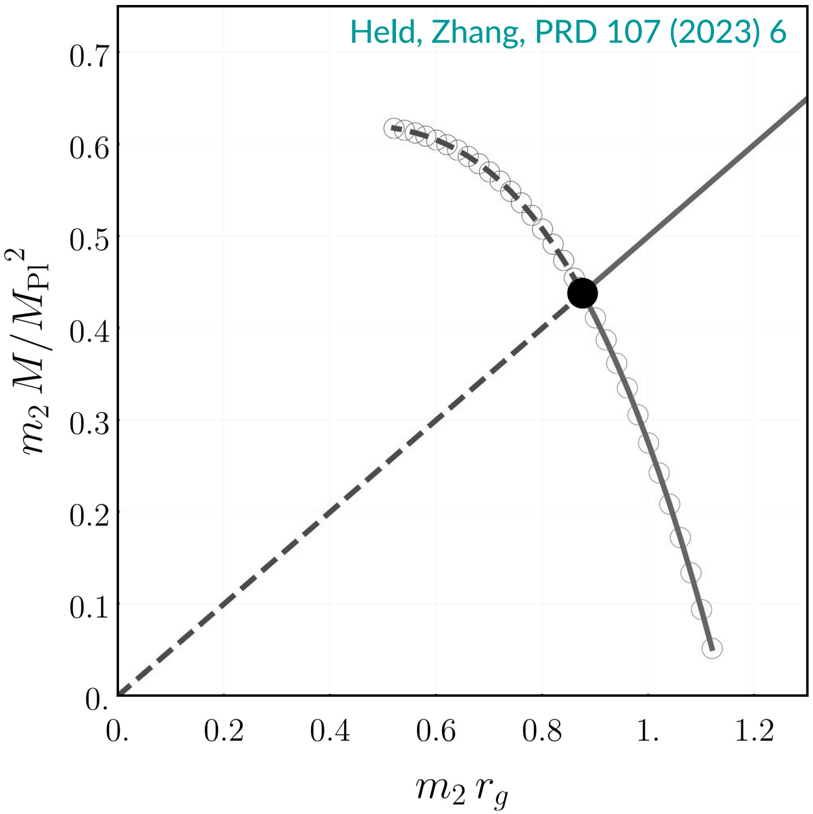
Part III: Nonlinear evolution (Quadratic Gravity)

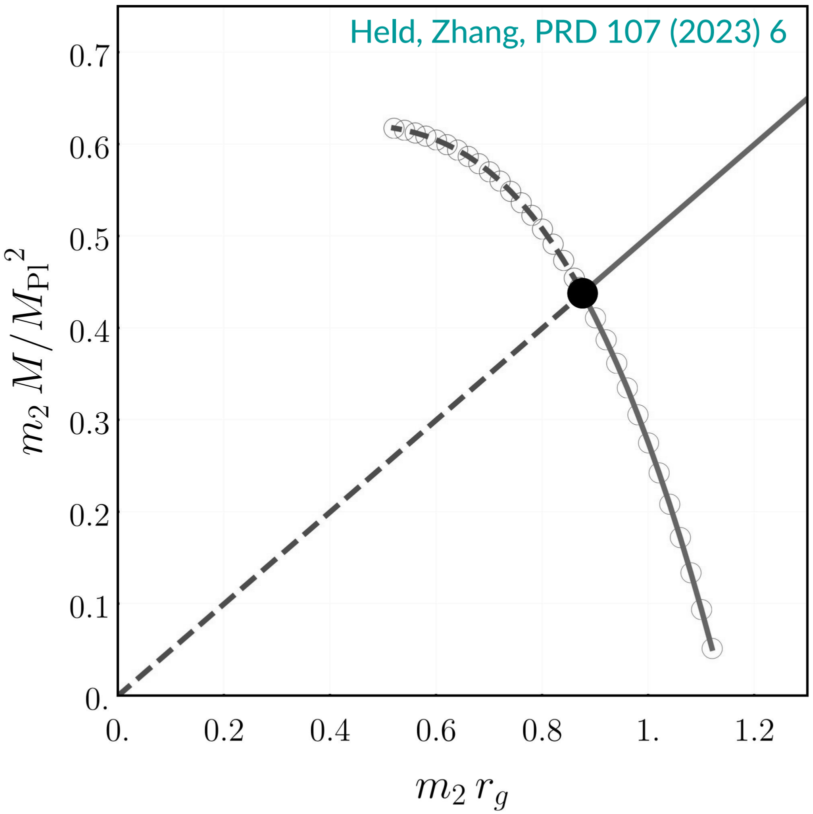
Held, Lim, PRD 104 (2021) 8
Held, Lim, PRD 108 (2023) 10
& upcoming

Isolated black holes ...



see also
Lu, Perkins, , Stelle, PRL 114 (2015) 17 & PRD 92 (2015) 12
Svarc, Pravdova, Miskovsky, PRD 107 (2023) 2

Isolated black holes ...



see also
Lu, Perkins, , Stelle, PRL 114 (2015) 17 & PRD 92 (2015) 12
Svarc, Pravdova, Miskovsky, PRD 107 (2023) 2

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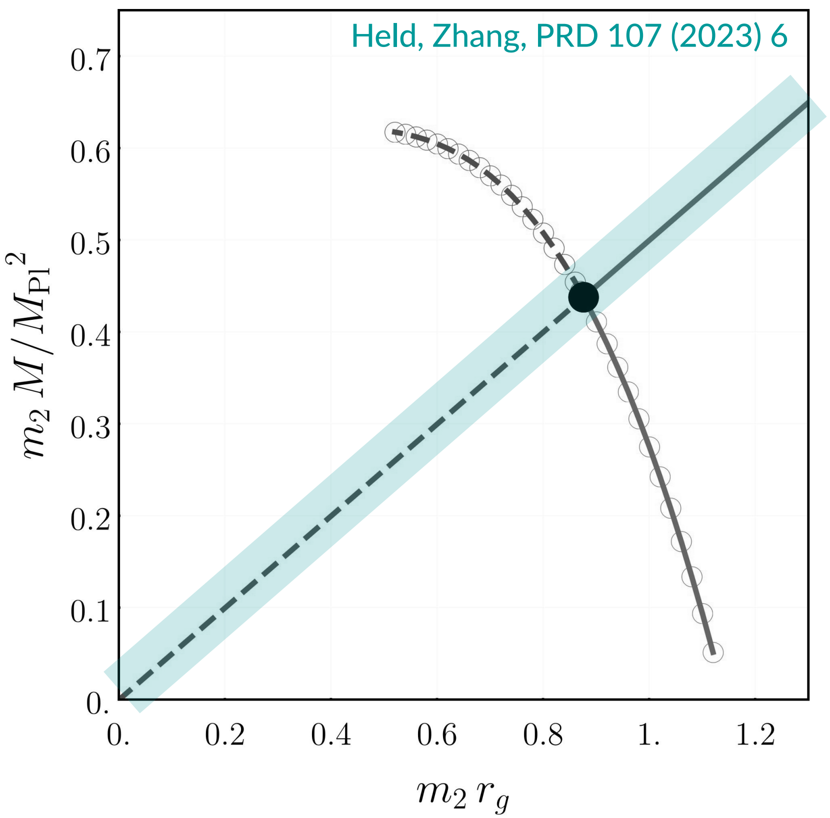
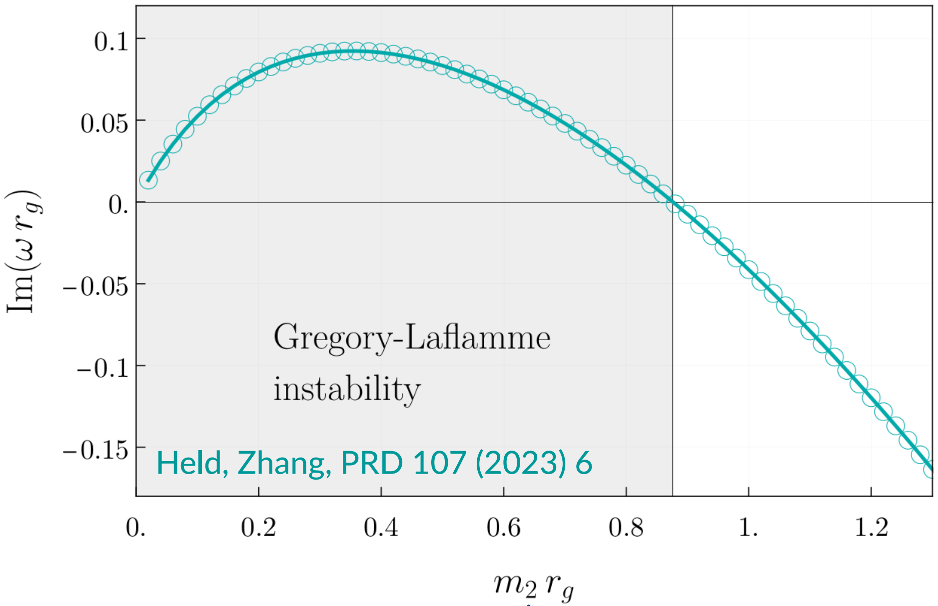
Isolated black holes ...





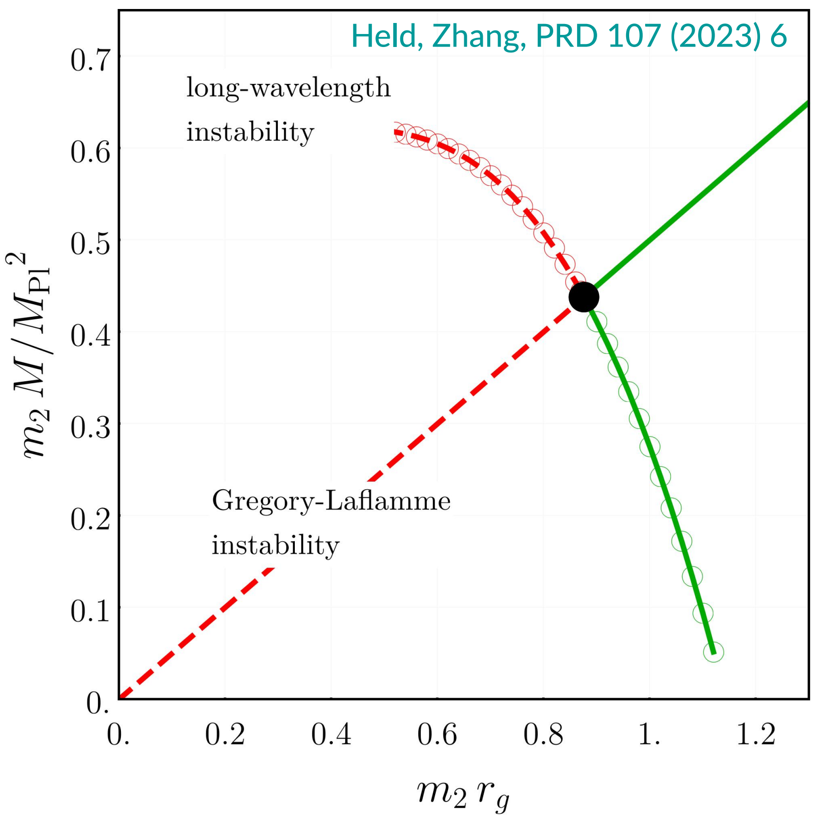
$$\mathrm{GM}\, m_2 < 0.43$$

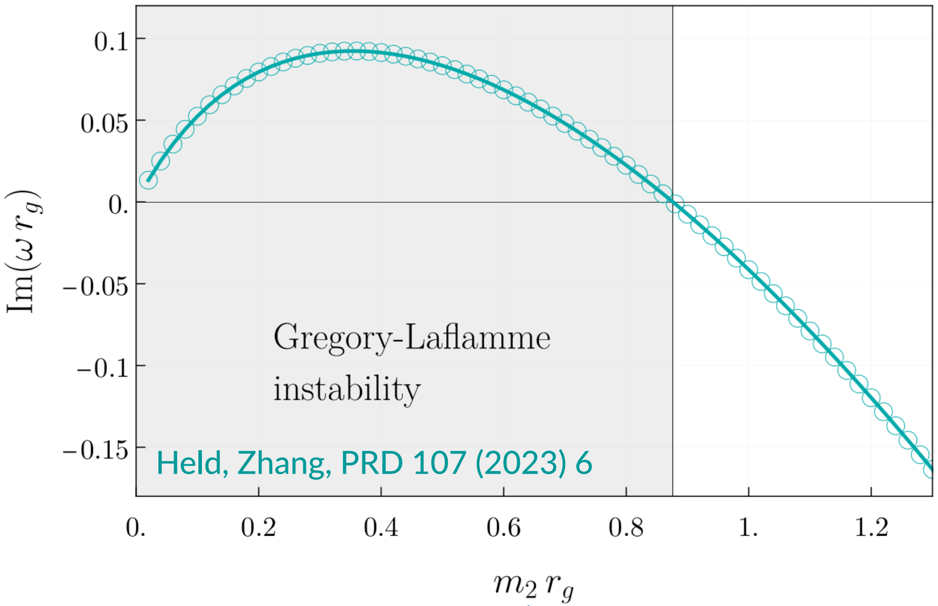
instability condition

$$\frac{t_{GL}}{\mathrm{GM}} \sim \frac{1}{\mathrm{GM}\,\mathrm{Im}(\omega)}$$

growth rate

Isolated black holes ...





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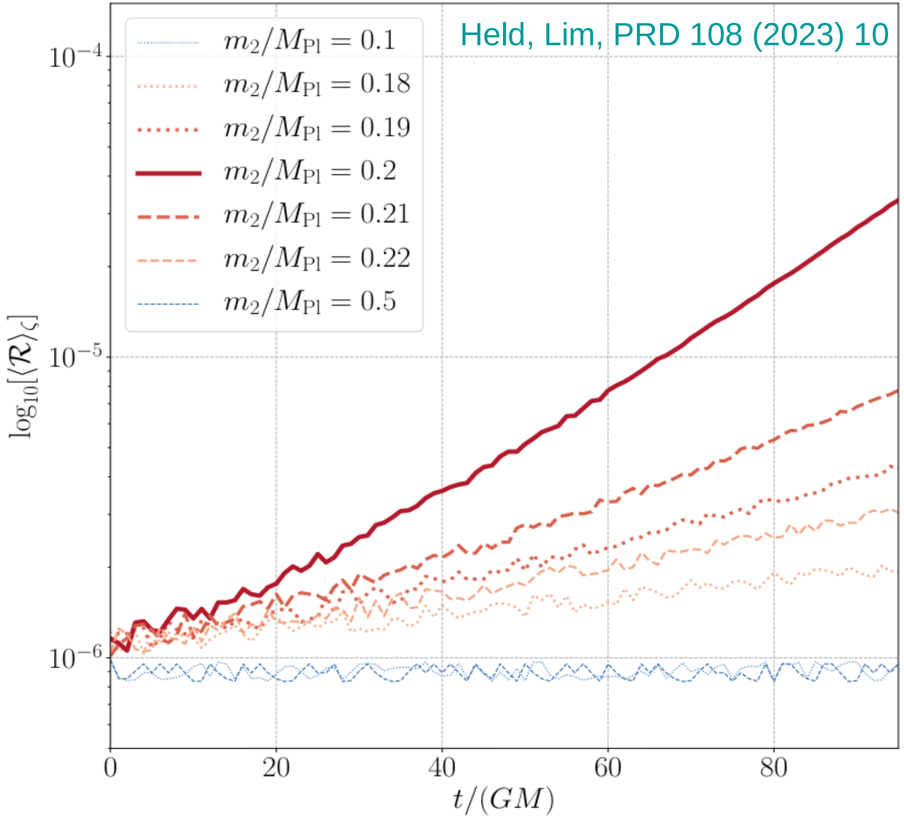
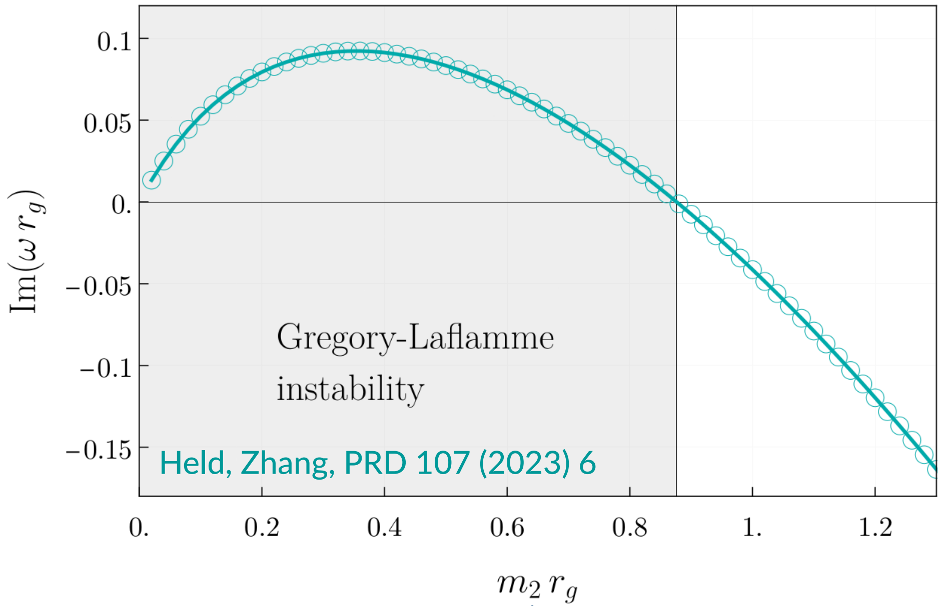
... are stable/unstable if sufficiently large/small.

Isolated black holes ...





$$GM\, m_2 < 0.43$$

instability condition

$$\frac{t_{GL}}{GM} \sim \frac{1}{GM\, \mathrm{Im}(\omega)}$$

growth rate

... are stable/unstable if sufficiently large/small.

Nonlinear evolution of the linear instability ...

Nonlinear evolution of the linear instability …

- **Nontrivial crosscheck available**: two completely independent setups and evolution codes
  Held, Lim, PRD 108 (2023) 10
  East, Siemonsen PRD 108 (2023) 12

Nonlinear evolution of the linear instability …

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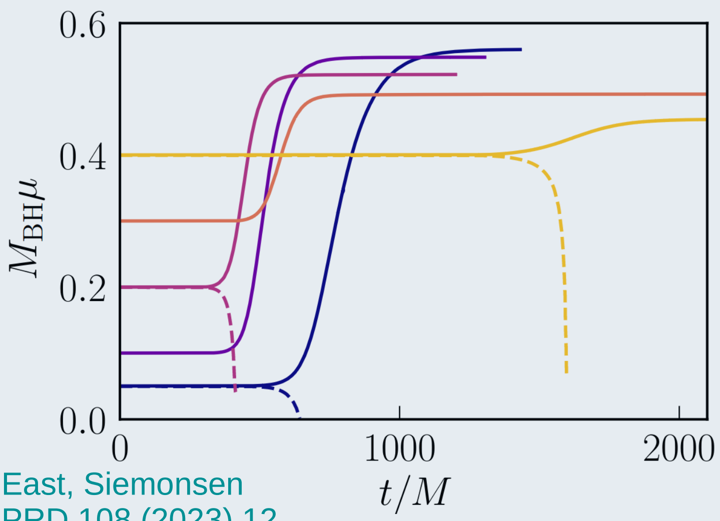
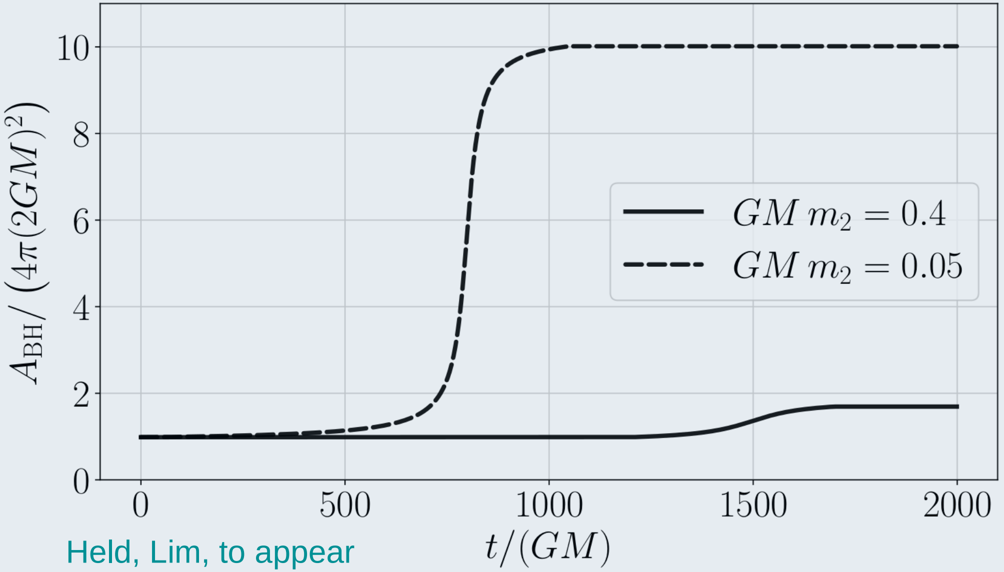


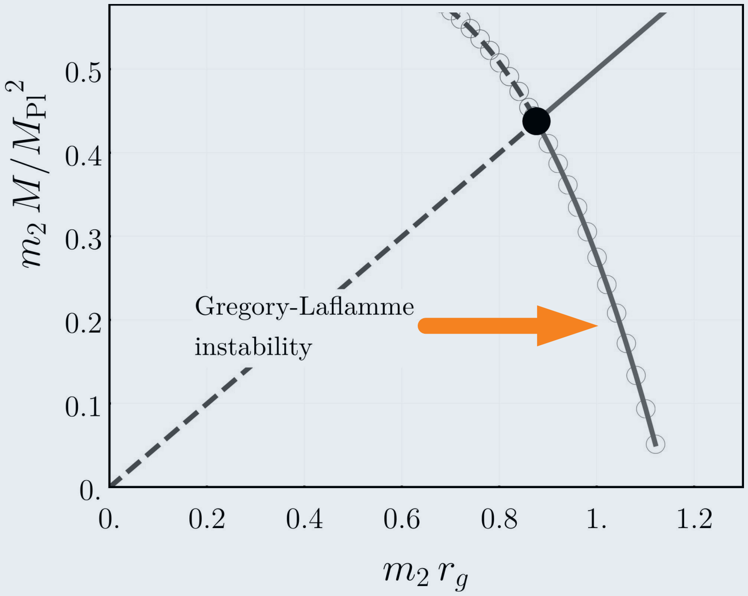
East, Siemonsen
PRD 108 (2023) 12

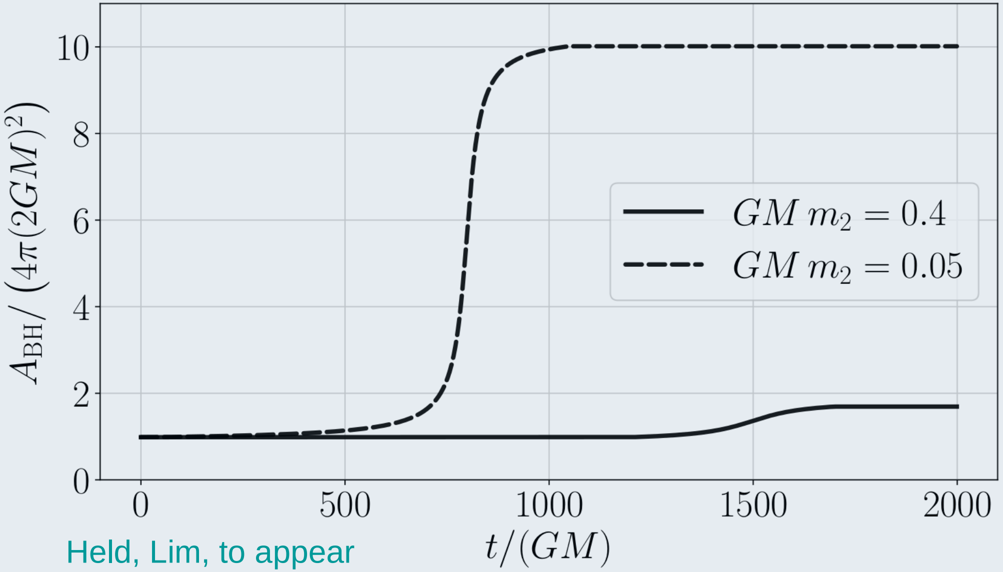


Held, Lim, to appear

Nonlinear evolution of the linear instability …

- **Nontrivial crosscheck available:** two completely independent setups and evolution codes
  Held, Lim, PRD 108 (2023) 10
  East, Siemonsen PRD 108 (2023) 12





Held, Lim, to appear

… can lead to the non-GR branch.

- numerically stable evolution
- isolated black holes: stability & transitions
- black-hole binaries & waveforms

Part III: Nonlinear evolution (Quadratic Gravity)

Held, Lim, PRD 104 (2021) 8
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& to appear

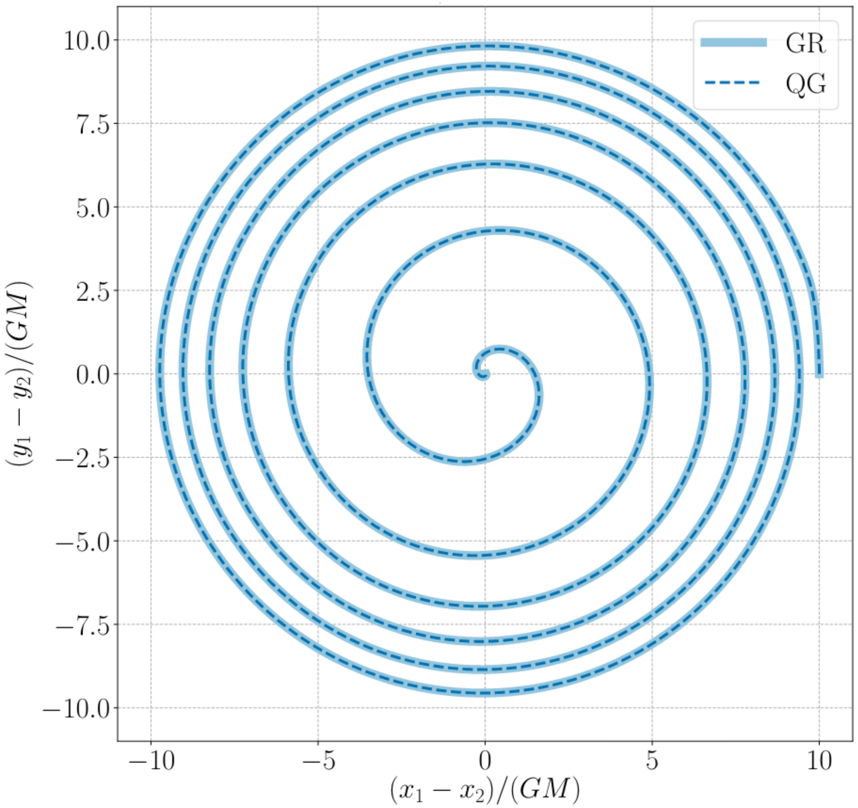
Waveforms ...

$GM\, m_2 \gg 1$	**no deviations**
$GM\, m_2 \sim 1$	**quantitative deviations**
$GM\, m_2 \lesssim 1$	**qualitative deviations**

Waveforms ...

		EFT regime of validity
$GM\,m_2 \gg 1$	no deviations	
$GM\,m_2 \sim 1$	quantitative deviations	
$GM\,m_2 \lesssim 1$	qualitative deviations	

Waveforms for $GM\, m_2 \gg 1$ ...



GW150914 initial data
[EinsteinToolkit]

Held, Lim, PRD 108 (2023) 10

... perfectly mimic those of GR.

Waveforms for $G M\, m_2 \sim 0.43$ ...

QG masses		Binary parameters			
$G\, m_0\, M_2$	$G\, m_2\, M_2$	$\sqrt{G}\, M_1$	$q = \frac{M_1}{M_2}$	$a_{z,1}$	$a_{z,2}$
1	0.4	1	5	-0.696	0

Waveforms for $GM\, m_2 \sim 0.43$ ...

QG masses		Binary parameters			
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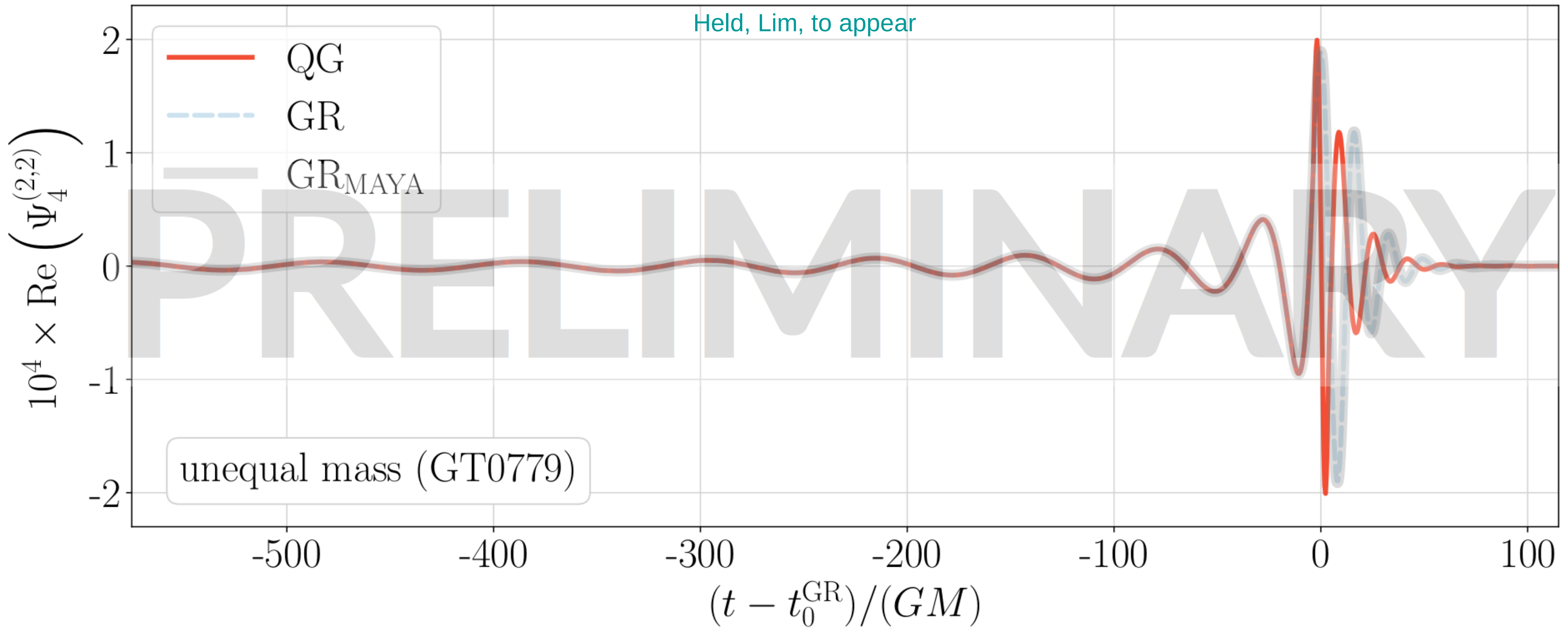

Waveforms for $GM\,m_2 \sim 0.43$ ...

QG masses		Binary parameters			
$G\,m_0\,M_2$	$G\,m_2\,M_2$	$\sqrt{G}\,M_1$	$q = \frac{M_1}{M_2}$	$a_{z,1}$	$a_{z,2}$
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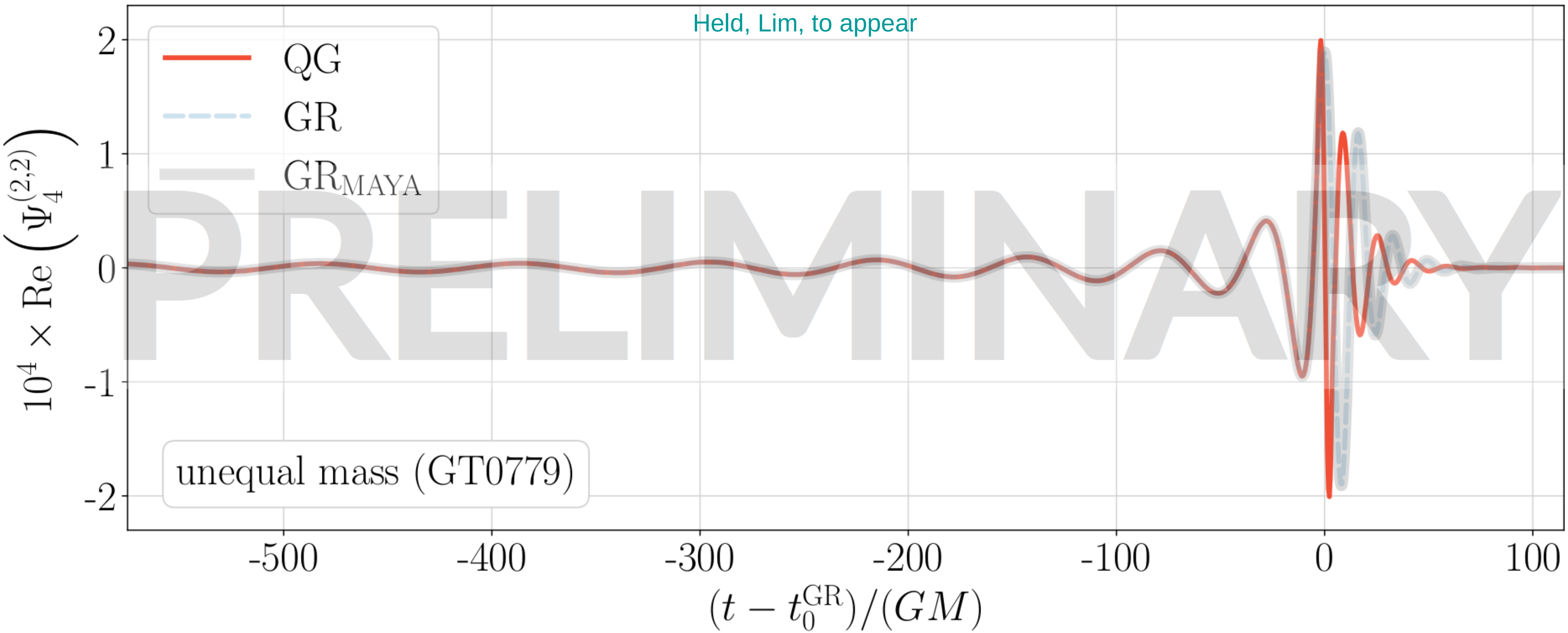
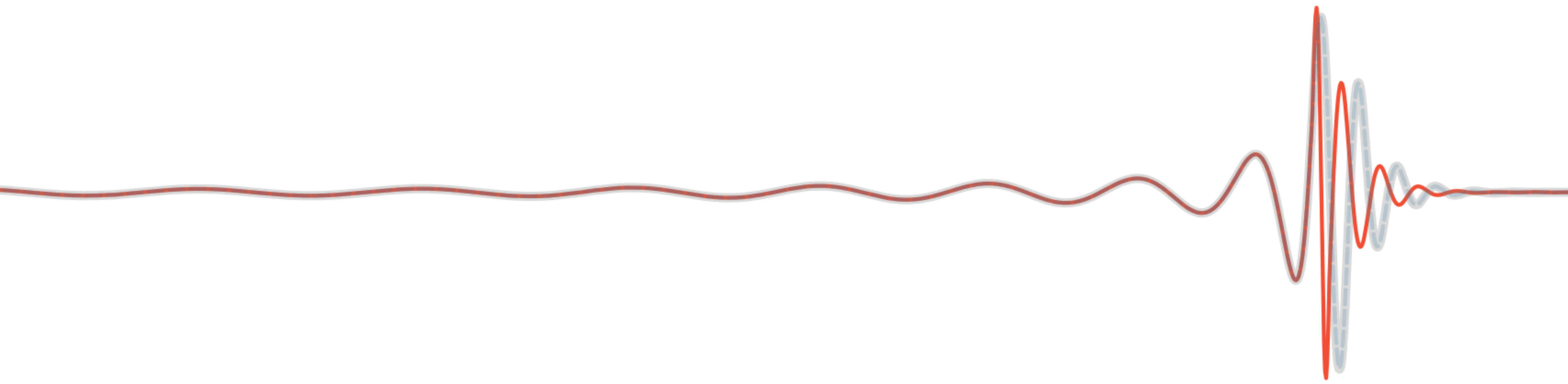


... deviate quantitatively.



There is a feasible pathway to obtain strong-field predictions and waveforms in the effective field theory of gravity.

Noakes, JMP 24, 1846 (1983);
Figueras, Held, Kovacs, 2407.08775

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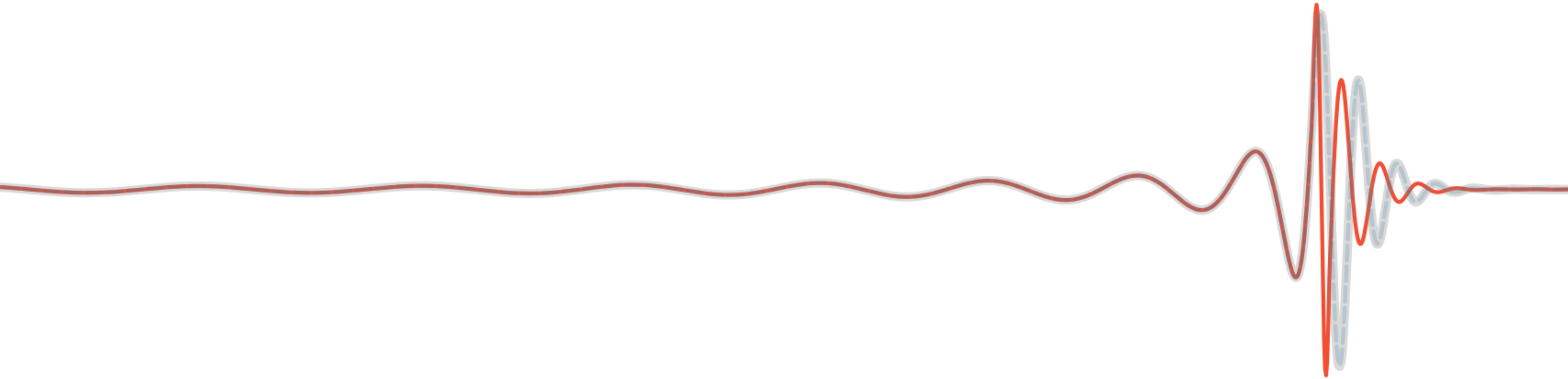
Held, Lim, PRD 108 (2023) 10
& to appear



– thank you –