Black Holes beyond General Relativity

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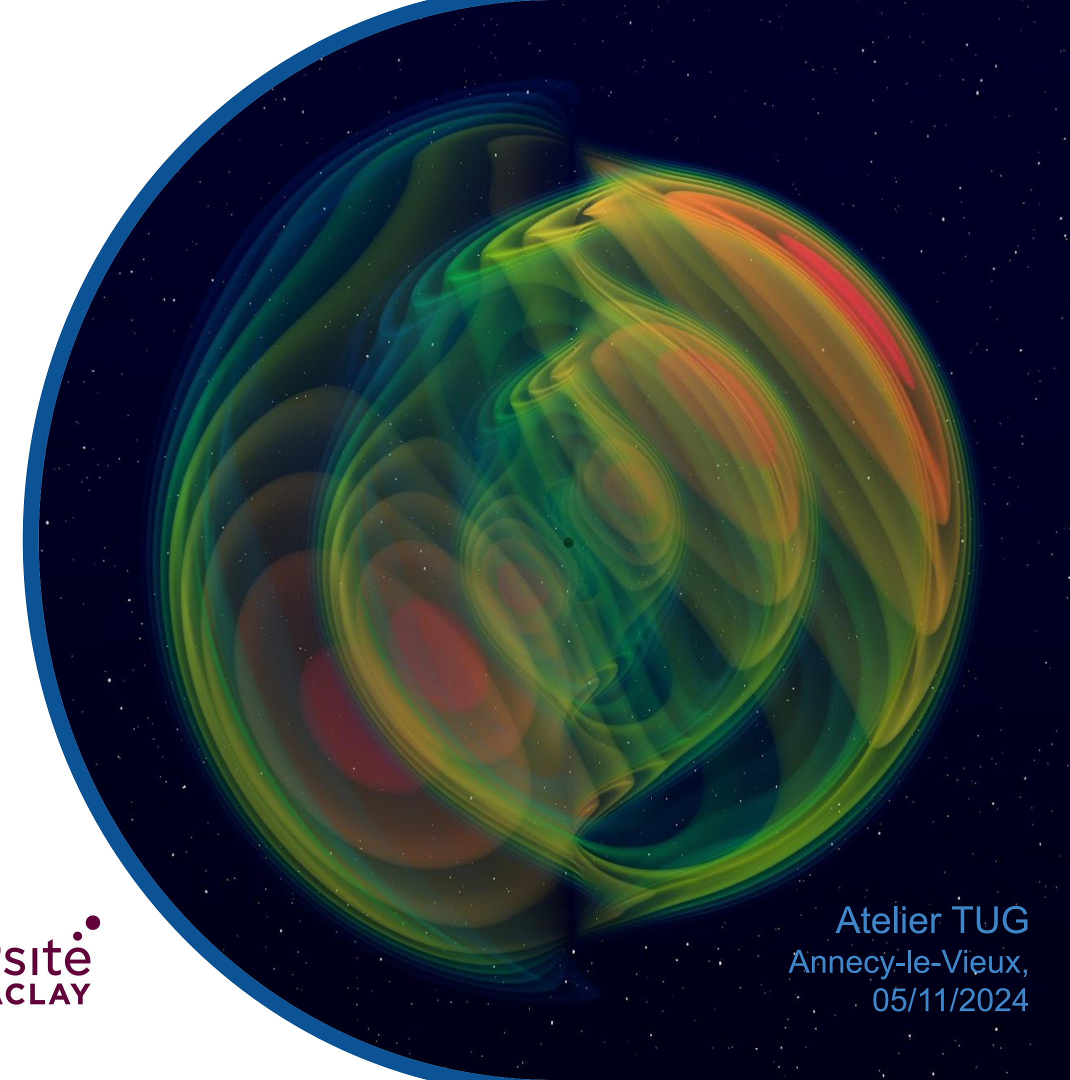
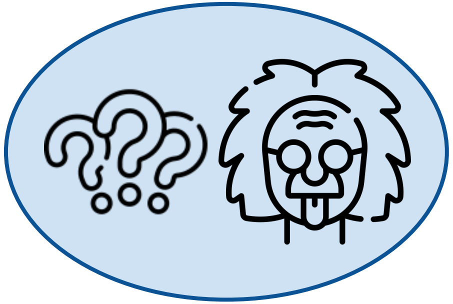
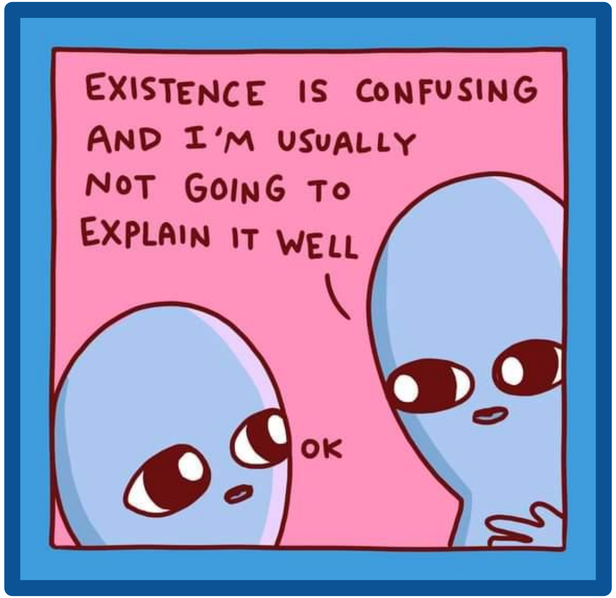


Atelier TUG
Annecy-le-Vieux,
05/11/2024

How different can black holes be
in theories beyond
general relativity?





‘Need’ to go beyond GR

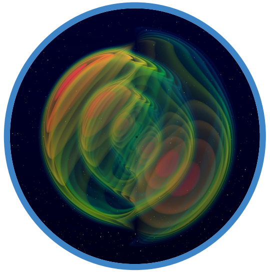
dark sector, quantisation, singularities…

- add **degrees of freedom** and/or add derivatives
- less/more symmetry
- extra dimensions
- …

Expectation:

deviations from GR more evident in strong gravity

black holes



BH solutions
in alternative theories
generically different

sometimes similar, sometimes very different
(sometimes identical)

e.g.
stationary & axisymmetric
vacuum solutions
generically not Kerr

- exactly Kerr
  stealth solutions
  (typically, differences at perturbation level)
- parametrically close
- quantitatively very different

interested in
qualitative, conceptual differences

[not quantitative, not observational]

Example: Einstein–æther

Focus on one example:

Einstein-æther Theory
(æ-theory)

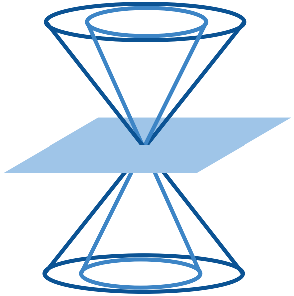
vector-tensor alternative to GR

[Jacobson, Mattingly PRD **64**, 024028 [gr-qc/0007031]]

written in terms of

$g_{\alpha\beta}$ metric

u_μ æther



specific, but

↳ motivated

↳ representative

Æ-theory: Lagrangian

$$\mathcal{L} = R - \left[\frac{1}{3}c_\vartheta \vartheta^2 + c_\sigma \sigma^2 + c_\omega \omega^2 - c_\mathfrak{a} \mathfrak{a}^2\right]$$

$$p_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$$

$$\begin{cases} \mathfrak{a}_\mu &= u^\alpha \nabla_\alpha u_\mu \quad \text{(acceleration)} \\ \vartheta &= \nabla_\mu u^\mu \quad \text{(expansion)} \end{cases} \qquad \begin{cases} \sigma_{\mu\nu} &= \nabla_{(\mu} u_{\nu)} + u_{(\mu} \mathfrak{a}_{\nu)} - \frac{\vartheta}{3} p_{\mu\nu} \quad \text{(shear)} \\ \omega_{\mu\nu} &= \nabla_{[\mu} u_{\nu]} + u_{[\mu} \mathfrak{a}_{\nu]} \quad \text{(twist)} \end{cases}$$

\+ constraint

$$\mathcal{L} + \zeta(u^\mu u_\mu + 1)$$

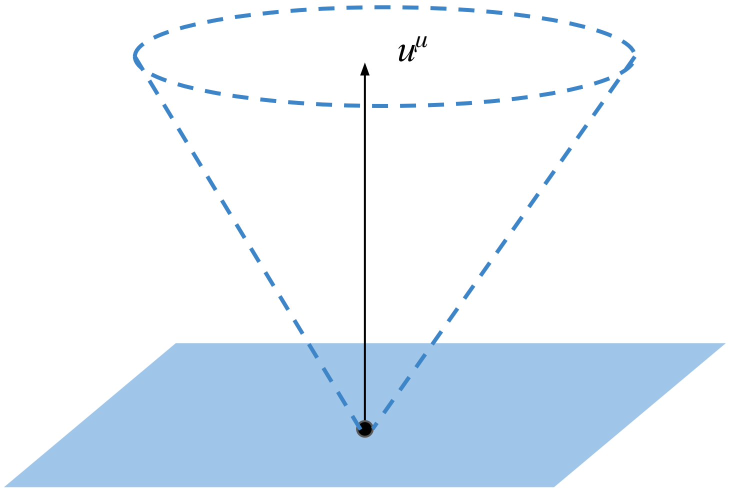
unit-norm
constraint

Lorentz Invariance Violations

$u_\mu(x^\alpha)$ everywhere timelike,
gives preferred time direction

has to be dynamical to ensure
general covariance

Breaks boost invariance:
Lorentz Invariance Violations
(LIV)



[new notion of causality]

Why LIV? Why æ-theory?

LIV sounds bad… but

several quantum gravity scenarios point to UV violations/deformations of LI	LIV can help build QFTs of quantum gravity

æ-theory is
EFT for LIV
in gravity

THE FIRST FEW TIMES EINSTEIN IMAGINED FLYING ALONGSIDE A BEAM OF LIGHT, HE DIDN'T HAVE ANY PARTICULAR INSIGHTS.

[xkcd]

Why LIV? Why æ-theory?

LIV sounds bad… but

several quantum gravity scenarios point to UV violations/deformations of LI | LIV can help build QFTs of quantum gravity

moreover, if

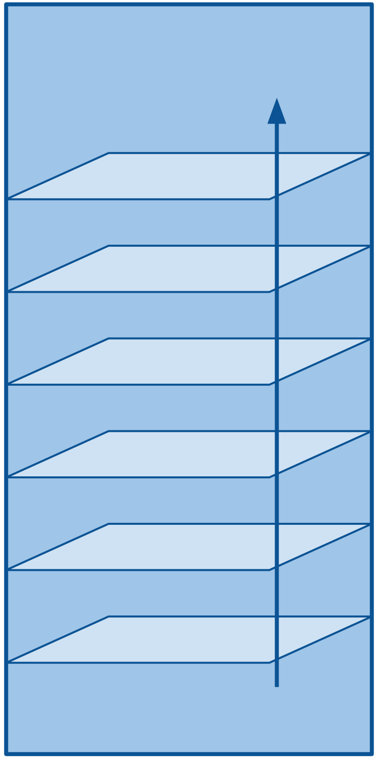
$$u_\mu = \frac{\nabla_\mu T}{\sqrt{-\nabla_\alpha T \nabla^\alpha T}}$$

æ-theory → scalar-tensor *khronometric theory* (∈ UDHOST)

low energy limit of non-projectable Hořava gravity

renormalisable QFT of gravity

can be completed to full Hořava



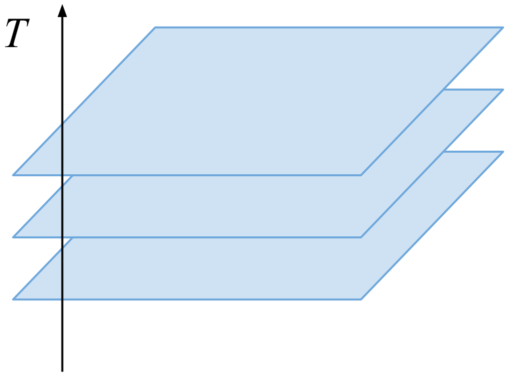
Why LIV? Why æ-theory?

Adding derivatives while avoiding ghosts:

$$\nabla_\mu = -u_\mu \left(\underline{u^\alpha \nabla_\alpha} \right) + \underline{p_\mu^{\ \alpha} \nabla_\alpha}$$

'temporal – spatial' splitting

✗ $u_\mu \left(u^\alpha \nabla_\alpha \right)$, $p_\mu^{\ \alpha} \nabla_\alpha$ ✓



khronometric theory

$$u_\mu = \frac{\nabla_\mu T}{\sqrt{-\nabla_\alpha T \nabla^\alpha T}}$$

$\Updownarrow$

$$\omega_{\mu\nu} = 0$$

→ twist vanishes (Frobenius)

Hypersurface **O**rthogonality → foliation by hypersurfaces of simultaneity

Consequences of LIV

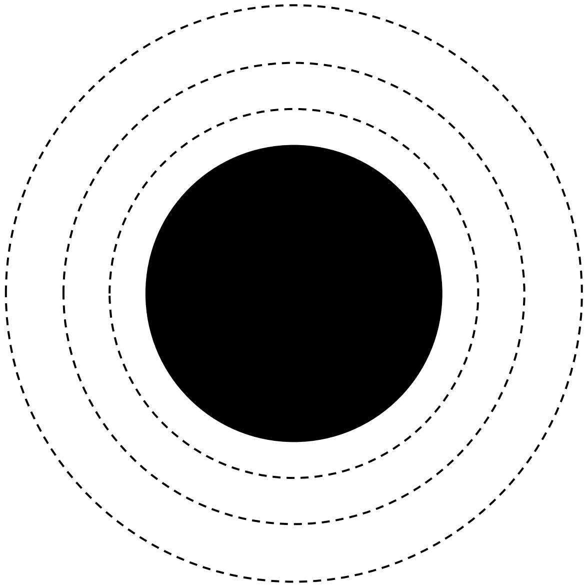
Massless modes not bound to
(metric) light cone

↓

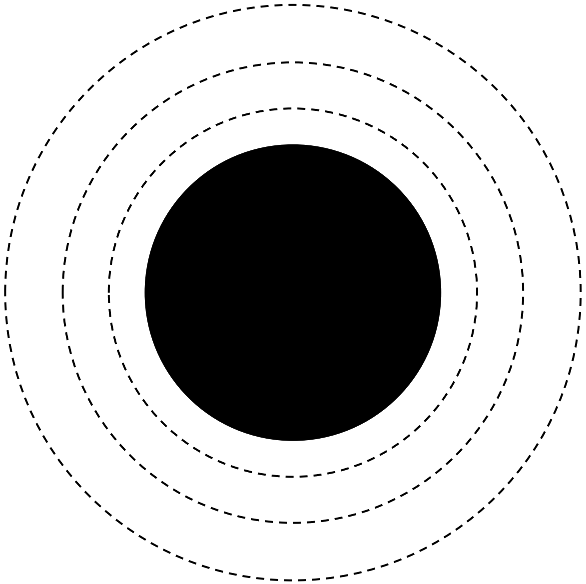
different
propagation speeds

↓

multiple
nested horizons



Consequences of LIV



Not a quirk!

Similar behaviour e.g. in
- scalar-tensor
- multi-metric
- …

LI broken ‘spontaneously’

Note: equivalence principle(s) also violated

[SEP = GWEP + LLI + LPI]

Consequences of LIV—cont.

Adding higher derivatives

↓

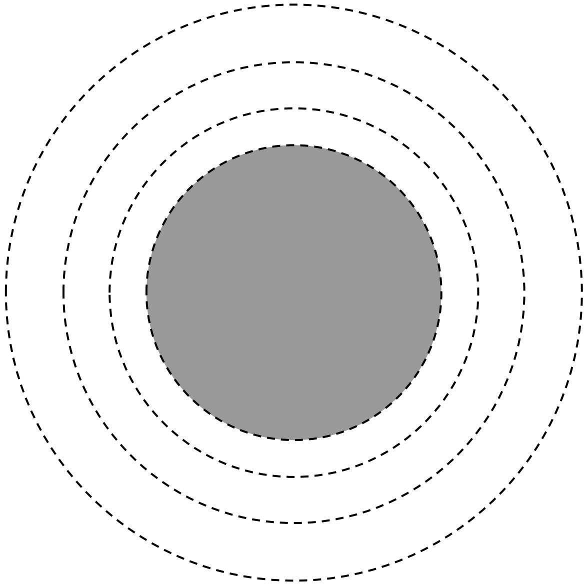
non-linear dispersion relations:

$$\omega^2(\vec{k}) = |\vec{k}|^2 + \frac{|\vec{k}|^4}{\Lambda^2} + \dots$$

[unbounded propagation speeds]

↓

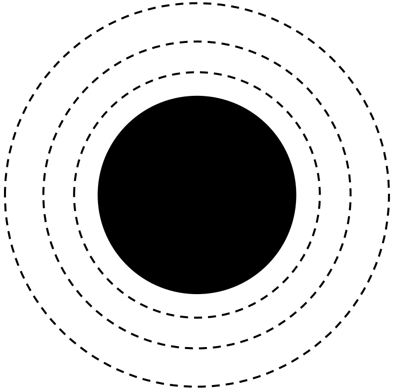
metric horizons
are permeable

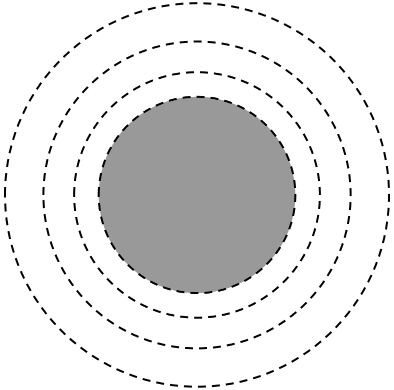


Black holes (?)

It appears BHs are
low-energy phenomenon…

… except

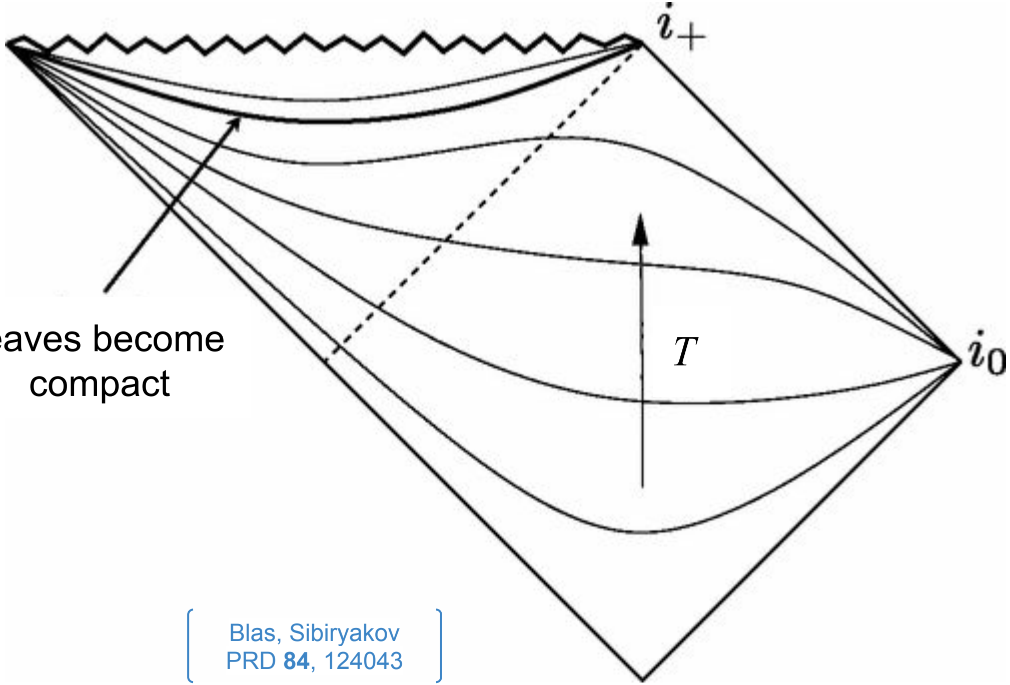




Black Holes (!)

A spherically symmetric surprise!

[sph. sym. ⇒ h.o.]



leaves become compact

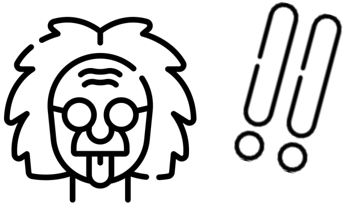
Blas, Sibiryakov
PRD **84**, 124043

Plot leaves of constant

(preferred) time

on top of Penrose diagram:

they change topology!



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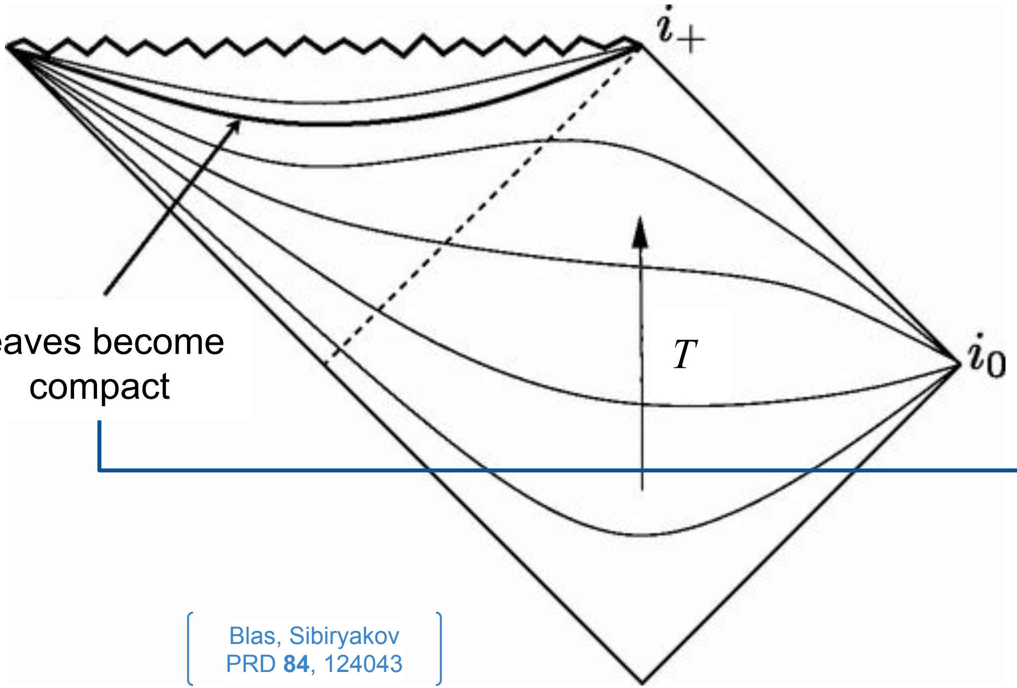


leaves become compact

Blas, Sibiryakov
PRD **84**, 124043

universal horizon
(UH)

traps all future-directed causal curves

similar to GR horizons:

- quasi-local characterisation

$$u_\mu \, \chi^\mu|_{\mathrm{UH}} = 0$$

- mechanics
- Hawking radiation

How general is this? ?

Introducing rotation

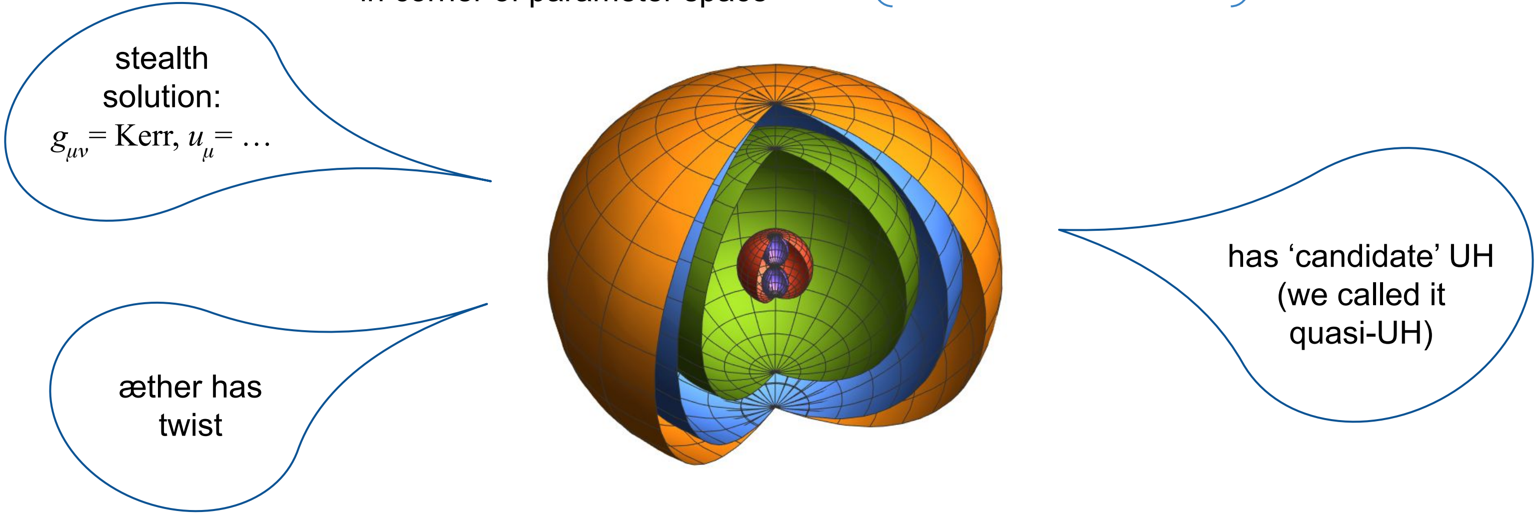
In h.o. case, UHs exist
(though no example apart from spherical symmetry)

Bhattacharyya, Colombo, Sotiriou, Class. Quantum Grav. 33, 235003 (2016) [1509.01558]

What about non-h.o. case?

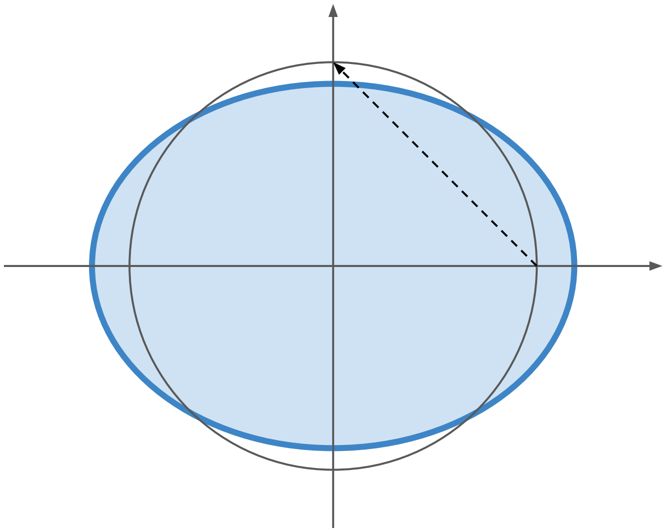
We found:
analytical solution
in corner of parameter space

Franzin, Liberati, **JM**
PRD **109**, 084028
[2312.06891]

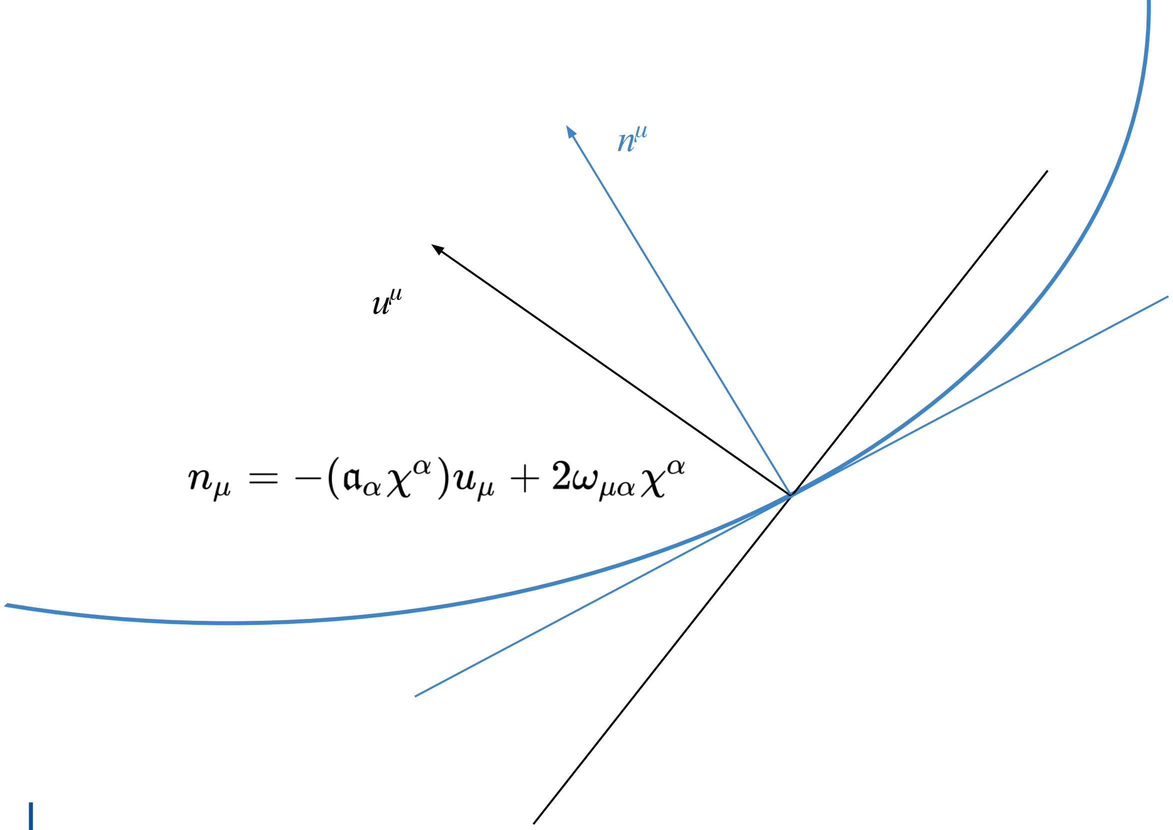


Quasi UH

QUH not a causal boundary



$$n_\mu = -(\mathfrak{a}_\alpha \chi^\alpha) u_\mu + 2\omega_{\mu\alpha}\chi^\alpha$$



QUH not orthogonal to u_μ

it can be escaped following a causal curve

Quasi UH

Still…

1 Better understanding of UHs in non-h.o. setting:

they can exist, but 'fragile'

[Nathan W. Pyle]

2 QUHs might still be interesting phenomenologically

escaping is 'difficult':
need high (group) velocity in a particular direction

momentum always directed 'inwards', but not tangent to trajectories

great example of differences between phase/group/front velocity

work in progress… Del Porro, Liberati, **JM** ???

Upshot

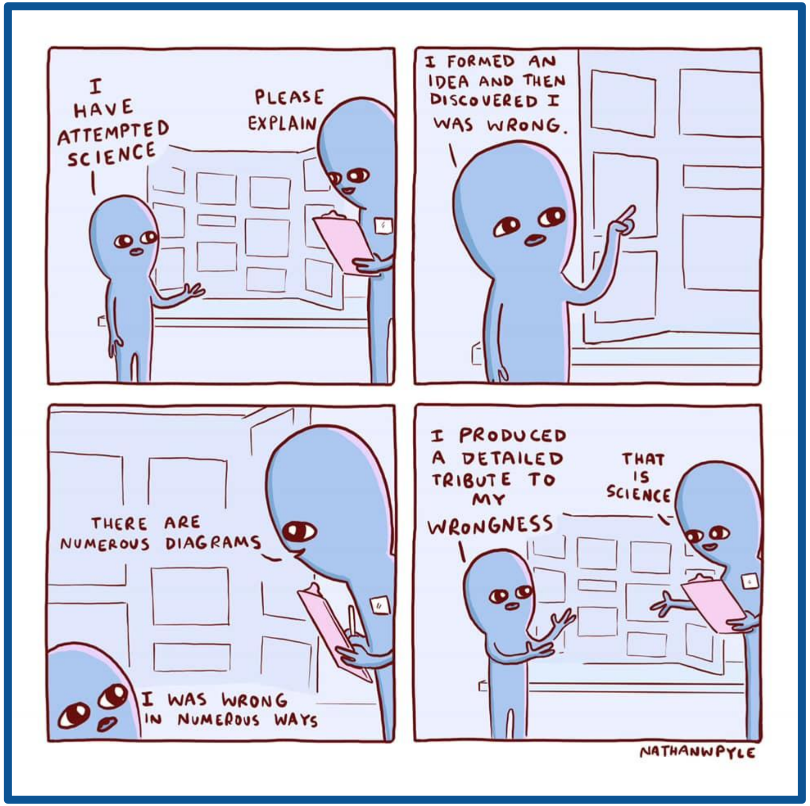
BHs beyond GR
can be deceiving…

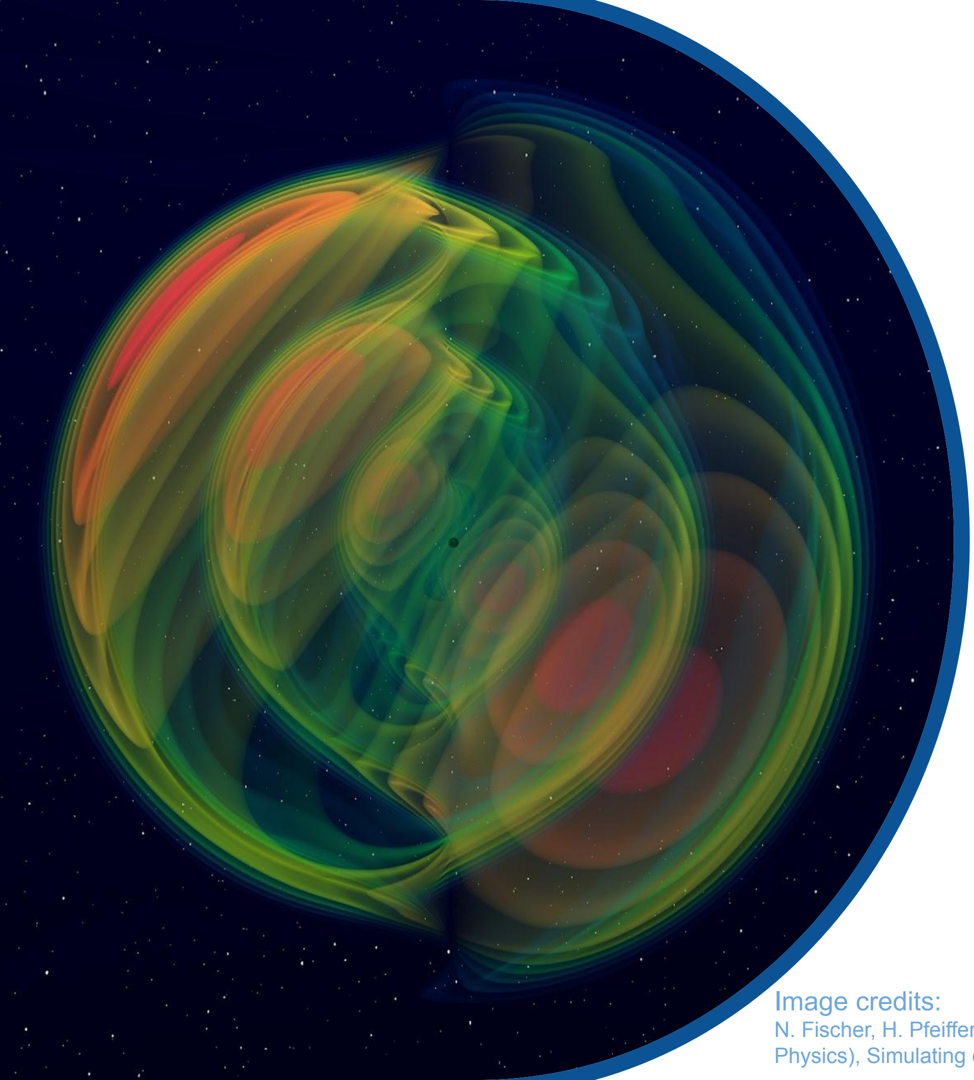
sometimes, different
notion of causality

- BHs dark, but not ‘black’
- new causal boundaries (UHs)
- …

Stability?
Phenomenology?

Be careful, don’t take
anything for granted!





Thanks!

Get in touch

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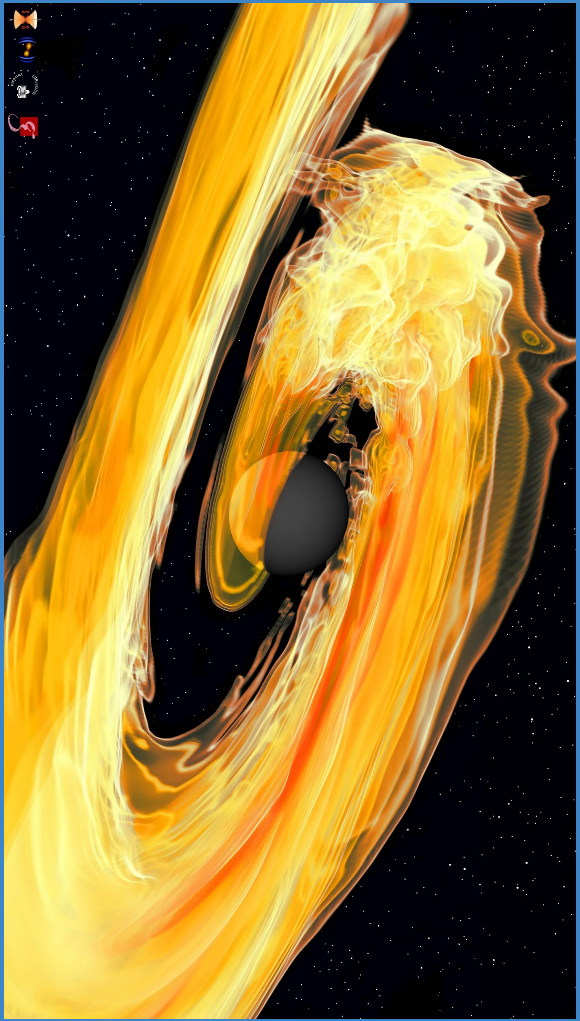
Image credits:
N. Fischer, H. Pfeiffer, A. Buonanno (Max Planck Institute for Gravitational Physics), Simulating eXtreme Spacetimes (SXS) Collaboration

Backup Slides



Stealth solution

$$c_\sigma = c_\omega = c_{\mathfrak{a}} = 0$$

'minimal æ-theory'

$$\sqrt{-g}\mathcal{L} = \sqrt{-g}\left\{R - \left[\frac{1}{3}c_\vartheta \vartheta^2 + c_\sigma \sigma^2 + c_\omega \omega^2 - c_{\mathfrak{a}} \mathfrak{a}^2\right]\right\}$$

choose Kerr
(Ricci flat)

solve

$$\begin{cases} \nabla_\mu u^\mu = 0 \\ u_\mu u^\mu = -1 \end{cases}$$

underdetermined problem

solution contains some arbitrariness

N.B.

Other 'minimal' choices possible,
this is the simplest one that is non trivial

Metric

Kerr in Boyer–Lindquist coordinates

$$ds^2 = -\left(1 - \frac{2Mr}{\Sigma}\right) \mathrm{d}t^2 + \frac{\Sigma}{\Delta}\mathrm{d}r^2 + \Sigma \mathrm{d}\theta^2 - \frac{4Mra\sin^2\theta}{\Sigma}\mathrm{d}t\mathrm{d}\varphi + \frac{A\sin^2\theta}{\Sigma}\mathrm{d}\varphi^2$$

$$\Sigma = r^2 + a^2\cos^2\theta$$
$$\Delta = r^2 - 2Mr + a^2$$
$$A = (r^2 + a^2)^2 - \Delta a^2 \sin^2\theta$$

Æther

Lie-dragged along Killing vectors

$$u_\mu(x^\alpha) = u_\mu(r, \theta)$$

One simple solution is

$$u_\mu = \left\{ \mp\sqrt{\frac{\Sigma\Delta + M^4\Theta^2}{A}}, -\frac{M^2\Theta}{\Delta}, 0, 0 \right\}_\mu$$

$$u^\mu = \left\{ \pm\frac{A}{\Delta\Sigma}\sqrt{\frac{\Sigma\Delta + M^4\Theta^2}{A}}, -\frac{M^2\Theta}{\Sigma}, 0, \pm\frac{2Mar}{\Delta\Sigma}\sqrt{\frac{\Sigma\Delta + M^4\Theta^2}{A}} \right\}^\mu$$

$u_\theta = 0$

$u_\varphi = 0$

'ZAMO' solution
more or less required

$$\omega_{\mu\nu} \neq 0,\ \sigma_{\mu\nu} \neq 0,\ \mathfrak{a}_\mu \neq 0$$

$\Theta(\theta)$ free function of angle

Fixing Θ

Many possibilities…

One interesting option:

define

$$r_{\rm QUH}(\theta) : \quad \partial_r(\Delta\Sigma)\big|_{r=r_{\rm QUH}} = 0$$

then

$$M^4\Theta^2 = -\Delta\Sigma\big|_{r=r_{\rm QUH}}$$

so that

$$(u_\mu \chi^\mu)\big|_{r=r_{\rm QUH}} = 0$$

(also, pick the ± so that it changes sign)

$r_{\rm QUH}$ is a perfect candidate for a UH

(‘Q’ stands for ‘quasi’)

Probing the QUH

Use toy model of matter with non-linear DRs

test scalar, with modified KG eq.

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\phi + \frac{1}{\Lambda^2}\left(p^{\mu\nu}\nabla_\mu\nabla_\nu\right)^2\phi = 0$$

WKB approximation

$$\phi\left(x^\alpha\right) = A\left(x^\alpha\right)e^{iS(x^\alpha)}$$

$$\nabla_\mu A, \nabla_\mu\nabla_\nu S, \dots \ll \partial_\mu S =: k_\mu$$

Three (?) equations

dispersion relation (DR)

$$k_\mu k^\mu + \frac{1}{\Lambda^2}\left(p^{\mu\nu}k_\mu k_\nu\right)^2 = 0$$

conservation of Killing energy

$$k_\mu \chi^\mu = -\Omega$$

conservation of Killing angular momentum

$$k_\mu \psi^\mu = m$$

Disformal transformations

The field redefinition

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - D\, u_\mu u_\nu$$

$$\tilde{u}_\mu = \frac{u_\mu}{\sqrt{1+D}}$$

is an internal map of æ-theory
(and T-theory)

$$\tilde{c}_\vartheta = (1+D)c_\vartheta - 2D$$

$$\tilde{c}_\sigma = 1 + (1+D)(c_\sigma - 1)$$

$$\tilde{c}_\omega = 1 + \frac{c_\omega - 1}{1+D}$$

$$\tilde{c}_\mathfrak{a} = c_\mathfrak{a}$$

Disforming
stealth Kerr,

we get new solutions
(of ‘different’ æ-theories)

resulting metric

- depends on D (3-param family)
- is non-stealth
- is non-circular
- its (metric) horizons not Killing

…

Abstract

Black holes beyond general relativity can be different from their general-relativistic counterparts—but in what ways and to what extent? I will explore this question discussing the particular example of Einstein–aether theory, an alternative to general relativity that displays a rich phenomenology and admits, among other things, faster-than-light signals. I will explain how the existence of such signals puts the very notion of black hole into question, but also describe how the serendipitous discovery of the so-called ‘universal horizons’ could, perhaps, salvage black holes from certain demise.