Carroll Physics and Flat Holography
Facts and Expectations

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Symmetry: a fundamental tool

Poincaré group: isometry of Minkowski

- designs the *relativistic dynamics*
- organizes the particle spectrum: *mass* and *spin*
- shapes the *scattering amplitudes*
- constrains the theory of *fundamental interactions*

Limits and extensions of Poincaré

Classical non-relativistic limit: $v/c \to 0$

- Poincaré group $\to$ Galilean group
- Minkowski spacetime $\to$ $t \in \mathbb{R}$ & $\mathbf{x} \in \mathbb{E}_3$

Exotic *ultra-local* limit $v/c \to \infty$ [Lévy-Leblond '65; Sen Gupta '66]

- Poincaré group $\to$ Carroll group
- Minkowski spacetime $\to$ Carrollian manifold

Unexpected extension [Bondi, van der Burg, Metzner '62; Sachs '62]

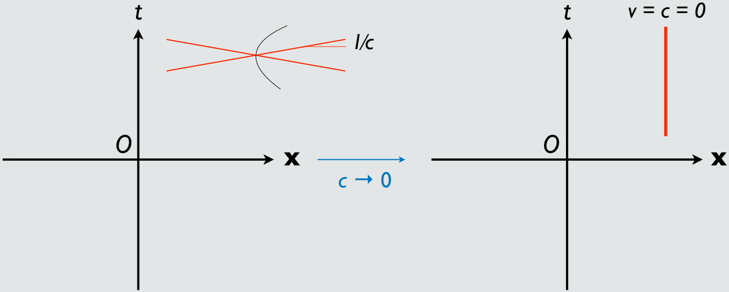
the asymptotic symmetry group of 4-dim asymptotically flat spacetimes is *infinite-dimensional:* BMS_4

Questions, challenges and aim of the talk

- Physical/geometric occurrences of Carroll
- Relationship Carroll-BMS
- Application to flat-space holography

Ultra-local limit

$c \to 0$



By law: *motion is forbidden*

…unless you allow for *tachyons* or kindred excitations…

THROUGH THE LOOKING GLASS [LEWIS CARROLL 1871]

"Well, in our country," said Alice, still panting a little, "you'd generally get to somewhere else if you run very fast for a long time, as we've been doing."
"A slow sort of country!" said the Queen. "Now, here, you see, it takes all the running you can do, to keep in the same place. If you want to get somewhere else, you must run at least twice as fast as that!"

Geometrically: spacetime with degenerate metric

- $\mathrm{d}s^2 = 0 \times \mathrm{d}t^2 + \mathrm{d}\mathbf{x}^2 \quad \eta_{\mu\nu} \to \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
- time-like kernel $\quad n^\mu \to \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ zero-norm vector

Invariance of the Carrollian geometry: isometry group

- 4 translations $t \to t + t_0 \quad x^i \to x^i + x_0^i$
- 6 point transformations
  - 3 rotations $x^i \to R^i{}_j x^j$
  - 3 boosts with $v^i = c^2 B^i + \mathrm{O}\left(c^4\right) \to 0$

$|B^i| < {}^1\!/_c \to +\infty$ is *"all the running you can do"*

$$\begin{cases} t' = \lim_{c\to 0} \gamma\left(t - \frac{\mathbf{v}\cdot\mathbf{x}}{c^2}\right) = t - \mathbf{B}\cdot\mathbf{x} \\ \mathbf{x}'_{\parallel} = \lim_{c\to 0} \gamma\left(\mathbf{x}_{\parallel} - \mathbf{v}_{\parallel} t\right) = \mathbf{x}_{\parallel} \\ \mathbf{x}'_{\perp} = \mathbf{x}_{\perp} \end{cases}$$

$\to$ *relative* time & absolute Euclidean space

Carrollian is "dual" to Galilean

Formally Inönü–Wigner contraction

Carroll group $\equiv$ translations $\ltimes$ rotations & Carroll boosts

Important remarks

No motion but Carrollian dynamics exists

- Fluids: $\partial_\mu T^{\mu\nu} = 0 \underset{c\to 0}{\longrightarrow}$ time & space Carrollian equations
- Fields: $\Box\Phi = 0 \underset{c\to 0}{\longrightarrow}$ Carrollian scalar field e.g. $\partial_t^2\Phi = 0$

All this can be extended for curved Carrollian manifolds

- Carrollian geometry: degenerate metric plus kernel

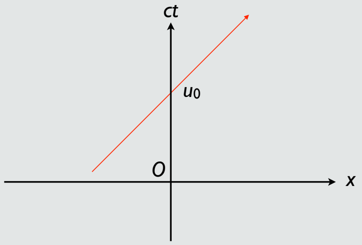
$$\mathrm{d}s^2 = q_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu \qquad q_{\mu\nu}n^\nu = 0$$

- Extra equipment: connection $\to$ torsion & curvature
  Note: Levi–Civita *is not* unique $\to$ physical implications
- Carroll group: acts on the tangent space

Example: the lightcone

Back to Minkowski: light rays along x

- $ct - x = u_0$ constant: light-like (null) hypersurface (3-dim)



- the induced metric on the 3-dim null hypersurface is *degenerate:* $\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 = \mathrm{d}y^2 + \mathrm{d}z^2$

In general

Null hypersurfaces yield Carrollian geometries

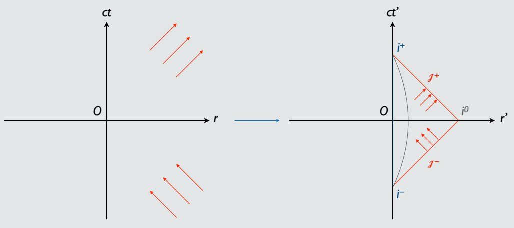
- Black-hole horizons — ordinary Carrollian (*not Galilean!*)
- Asymptotic null infinity in gravitational fields produced by localized mass distributions — conformal Carrollian

Null infinity $\mathscr{I}^{\pm}$ — the realm of radiation

Minkowski again $ct \pm r = \tan(ct' \pm r') \to \{t', r'\}$ compact

$$
\begin{aligned}
ds^2 &= -c^2 dt^2 + dr^2 + r^2 \left(d\vartheta^2 + \sin^2\vartheta d\varphi^2\right) \\
&= \Omega(t', r') \left(-c^2 dt'^2 + dr'^2 + f(r') \text{ sphere}\right)
\end{aligned}
$$

- radial light rays: constant $ct \pm r$ i.e. constant $ct' \pm r'$



- incoming radiation originates from $\mathscr{I}^{-}$
- outgoing radiation heads towards $\mathscr{I}^{+}$
- $\mathscr{I}^{-} \cup \mathscr{I}^{+}$ is the *null conformal boundary*

The same holds for gravitational radiation

Solving Einstein's equations with localized matter $\rightarrow \{\mathscr{M}, \mathrm{d}s^2\}$

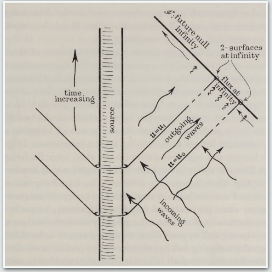


Figure: Newman–Penrose 1968

Key message

Einstein dynamics in $\{\mathscr{M}, \mathrm{d}s^2\} \leftrightarrow$ Carrollian dynamics on $\mathscr{I}^\pm$

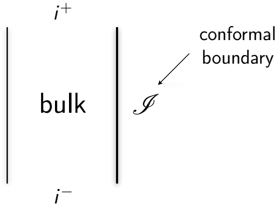
AdS/CFT I — building Einstein spacetimes

Solving Einstein's equations $E_{MN} + \Lambda G_{MN} = 0 \quad \Lambda < 0$

- choose a gauge with a radial coordinate r, expand $\mathrm{d}s^2_{\text{bulk}}$
  impose bry. conditions, solve iteratively for all $f(u, \vartheta, \varphi)$s
- finite solution space without generic gravitational radiation
  conformal bry. 1st and 2nd fundamental forms: $g_{\mu\nu}$ & $T_{\mu\nu}$

$$\nabla_\mu T^{\mu\nu} = 0$$

- asymptotic symmetries (Dirichlet bry. conditions) in 4 dim
  conformal group of the 3-dim bry. $\equiv SO(3,2) \longrightarrow 10$



AdS/CFT II — holographic principle

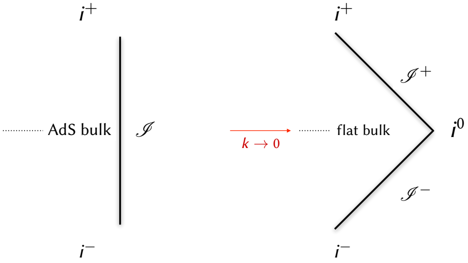
AdS/CFT correspondence in a nutshell

- incarnation of old 't Hooft & Susskind ideas on gravity dofs
- *fundamental theories:* type IIB string and $N = 4$ SYM
- *holographic:* dual field theory on a codim-1 time-like hypersurface — the conformal boundary $\mathscr{I}$
  - bulk field = leading + subleading
  - $\langle \text{subleading} \rangle_{\text{bry. CFT}} = \frac{\delta I_{\text{bulk}}}{\delta \text{leading}}$ vevs & sources
- *pivotal ingredients:* conjugate to each other
  - conserved bry. en.–mom. tensor (sublead. — vev) $T_{\mu\nu} \sim {}^{1}\!/_{r}$
  - the bry. metric (leading — source) $g_{\mu\nu} \sim r^2$

AdS versus flat I — asymptotics

From AdS$_n$ to flat$_n$ dim $n = d + 2$

$\Lambda = -\frac{(n-1)(n-2)}{2} k^2 \to 0$ $c_{\text{bry.}} = k \times R_{\text{celestial sphere}} \times c_{\text{bulk}}$



$\mathscr{I}^{\pm}$ is a null hypersurface $\to$ Carrollian geometry

asymptotic isometry group BMS$_n$ $\equiv$ Lorentz $\ltimes$ Supertransls.
$\equiv$ conformal Carrollian group $\mathfrak{CCarr}(n-1)$

Flat versus AdS II — Carrollian boundary fields

Solving Einstein for asymptotically flat spacetimes dim 4

- Ricci-flat spacetimes have ∞-dim solution space
  1. Carrollian boundary geometry (degenerate metric & kernel)
  2. radiation (shear tensor)
     ↳ *free — part of the boundary Carrollian connection*
  3. Carrollian momenta (stress, matter flux, energy flux)
     ↳ *generalized Carrollian conservation laws*
  4. Chthonian fields (deeper in $^1/_r$)
     ↳ *evolution Carrollian equations*
- all information is reached as $\Lambda = -3k^2 \to 0$ carefully
  Laurent expansion of AdS en.–mom. tensor $\to$ 3 & 4
  ↳ $T^{\mu\nu} = \sum_{n\in\mathbb{Z}} k^{2n} T^{\mu\nu}_{(n)}$

Form of the metric — expansion in $r^{\#}$ — no logs

$$\mathrm{d}s^2_{\text{bulk}} = 2\frac{\mathrm{u}}{k^2}(\mathrm{d}r + r\mathrm{A}) + r^2\mathrm{d}s^2 + r\mathscr{C}_{AB}\theta^A\theta^B + \frac{1}{k^4}\mathscr{F}_{AB}\theta^A\theta^B + \sum_{s=1}^{\infty}\frac{1}{r^s}\left(f_{(s)}\frac{\mathrm{u}^2}{k^4} + 2\frac{\mathrm{u}}{k^2}f_{(s)A}\theta^A + f_{(s)AB}\theta^A\theta^B\right)$$

using a boundary Cartan frame adapted to a congruence u

- $\mathrm{d}s^2 = \eta_{AB}\theta^A\theta^B = -\left(\theta^{\hat{0}}\right)^2 + \delta_{ab}\theta^a\theta^b$
- $\mathrm{u} = -k\theta^{\hat{0}} = -k^2\left(\Omega\mathrm{d}u - b_i\mathrm{d}x^i\right) = k^2\mu$

with

- $f_{(s)A}$ & $f_{(s)AB}$ transverse wrt u
- all coefficients of definite Weyl weight

FOLLOWING THE AdS_4/CFT_3 PARADIGM

- dual CFT_3 on the Carrollian conformal boundary $\mathscr{I}^{\pm}$
- must be invariant under $BMS_4 \equiv SL(2,\mathbb{C}) \ltimes sT \equiv \mathfrak{CCarr}_3$

COMPARAISON N'EST PAS RAISON

- the vev–source relationship is blured
- clearly multi-sector (scattering, bound states & deep dofs)

possibly *non-local, non-unitary* or *non-holographic*

Great deal of novelties to handle — slow progress

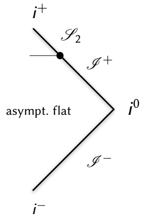
FLAT/CFT = $\lim_{k\to 0}$ ADS/CFT

Consider appropriate bulk diagrams — harsh task requires care

Preliminary results beyond the classical solution reconstruction for the radiative sector

THE CELESTIAL SPHERE

- $\mathscr{S}_2$ is a cut of $\mathscr{I}^\pm$
- Lorentz group $SL(2, \mathbb{C})$ acts on $\mathscr{S}_2$ as Möbius transfs.



Möbius are conformal transformations … *il n'y a qu'un pas …*

Utterly different path: $\text{Flat}_4/\text{CFT}_2$ *celestial* approach

Framework
- $\mathscr{S}_2 \equiv$ "spatial section" of the Carrollian bry.
- 2-dim en.–mom. tensor $\sim \int_{\mathbb{C}} \int_{\mathbb{R}} \mathscr{N}_{ab}$ (news)

Feature mostly designed for recasting *radiation S*-matrix on Minkowski & soft theorems — not fund. theor.
$\langle \text{scat. ampl.} \rangle_{\text{Mink.}} \sim \langle \text{correl.} \rangle_{\mathscr{S}_2}$

not bound to gravity — could have been elaborated in the 80ies

$\mathscr{S}_2$ conformal weights are the bulk-boost eigenvalues $\in 1 + \mathrm{i}\,\mathbb{R}$

Exotic CFT_2 with intriguing results and recent efforts to recast them in the appropriate Carrollian language — more is needed

Carrollian physics and flat holography

Facts

- Carrollian structures are well understood
- bulk Einstein's equations are mapped onto Carrollian boundary dynamics
- an infinite number of data emerge in the solution space
- a genuine boundary energy–momentum tensor is available
- satisfactory understanding of
  - gravitational losses reflecting Carroll boost breaking
  - charges, magnetic charges and duality properties

Expectations

- work out genuine Carrollian CFTs in 3 dim
- elaborate on Carrollian–Chthonian holography
  - shear/news scattering sector at order r
  - energy–momentum & Chthonian sector at $1/r^m \forall m \geq 1$
- make contact with the celestial approach

Starring: A. Campoleoni, L. Ciambelli, A. Delfante, A. Fiorucci, R. Leigh, C. Marteau, N. Mittal, S. Pekar, A. Petkou, M. Petropoulos, D. Rivera, R. Ruzziconi, K. Siampos, M. Vilatte