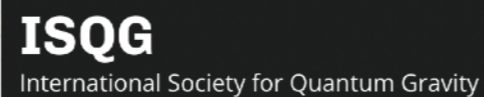


TUG Workshop - Annecy 2024

the Spinfoam Framework for Quantum Gravity

Etera Livine

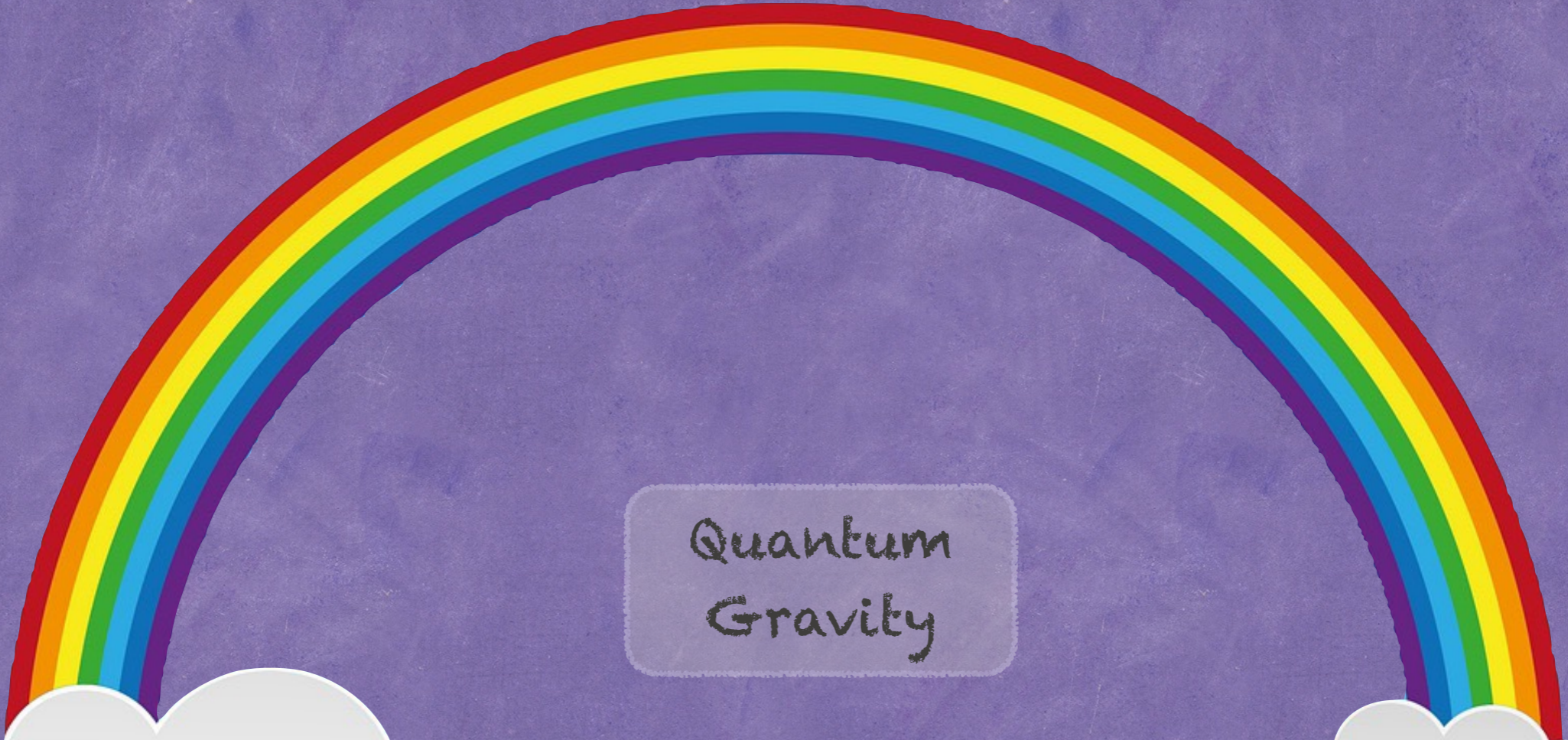
Laboratoire de Physique LP ENSL & CNRS



Quantum Gravity

What is the fundamental structure of space-time ?

What is the UV completion of GR and QFTs ?



Quantum
Gravity

Quantizing
Gravity

Gravitizing
the Quantum

Re-inventing the structure of space-time
and what is meant by "geometry"



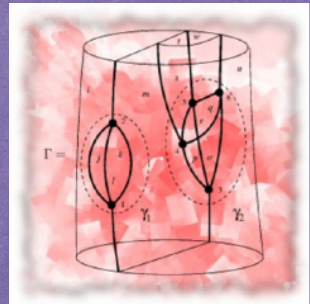
Quantum
Gravity

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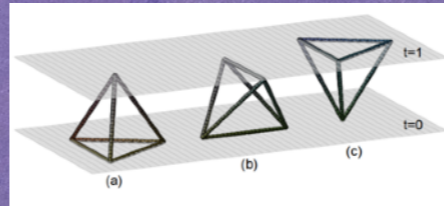
Quantum
Gravity

Gravity from
Thermodynamics



Loop Quantum
Gravity

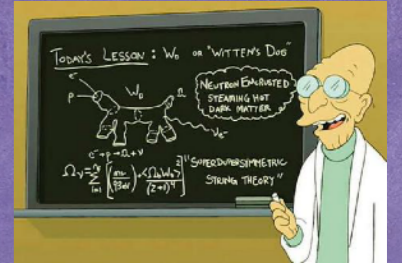
Non-Commutative
Geometry



Matrix Models

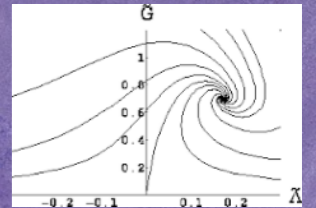
Quantum
Gravity

Supergravity



String
Theory

Non-Perturbative
Renormalization
Group



Effective Field
Theory

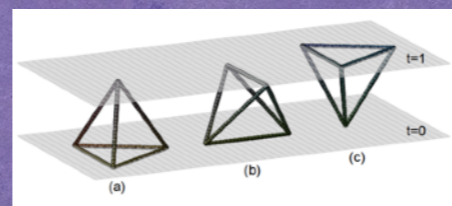
Gravity from Thermodynamics

Dynamical Triangulations

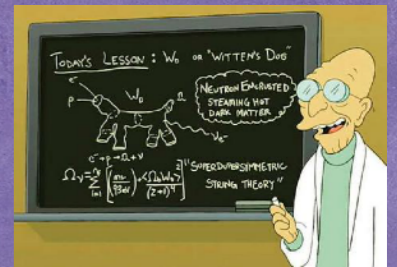
Holography, Celestial Holography

Graphity

Causal Sets



Supergravity

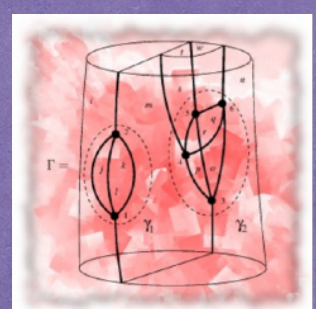


String Theory

Matrix Models

Quantum Gravity

Asymptotic Safety



Loop Quantum Gravity

Non-Perturbative Renormalization Group

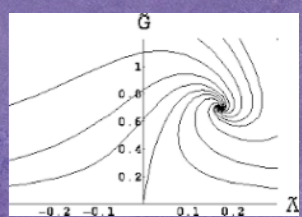
Geometro-dynamics

Topological QFT

Group Field Theory

Non-Commutative Geometry

Effective Field Theory

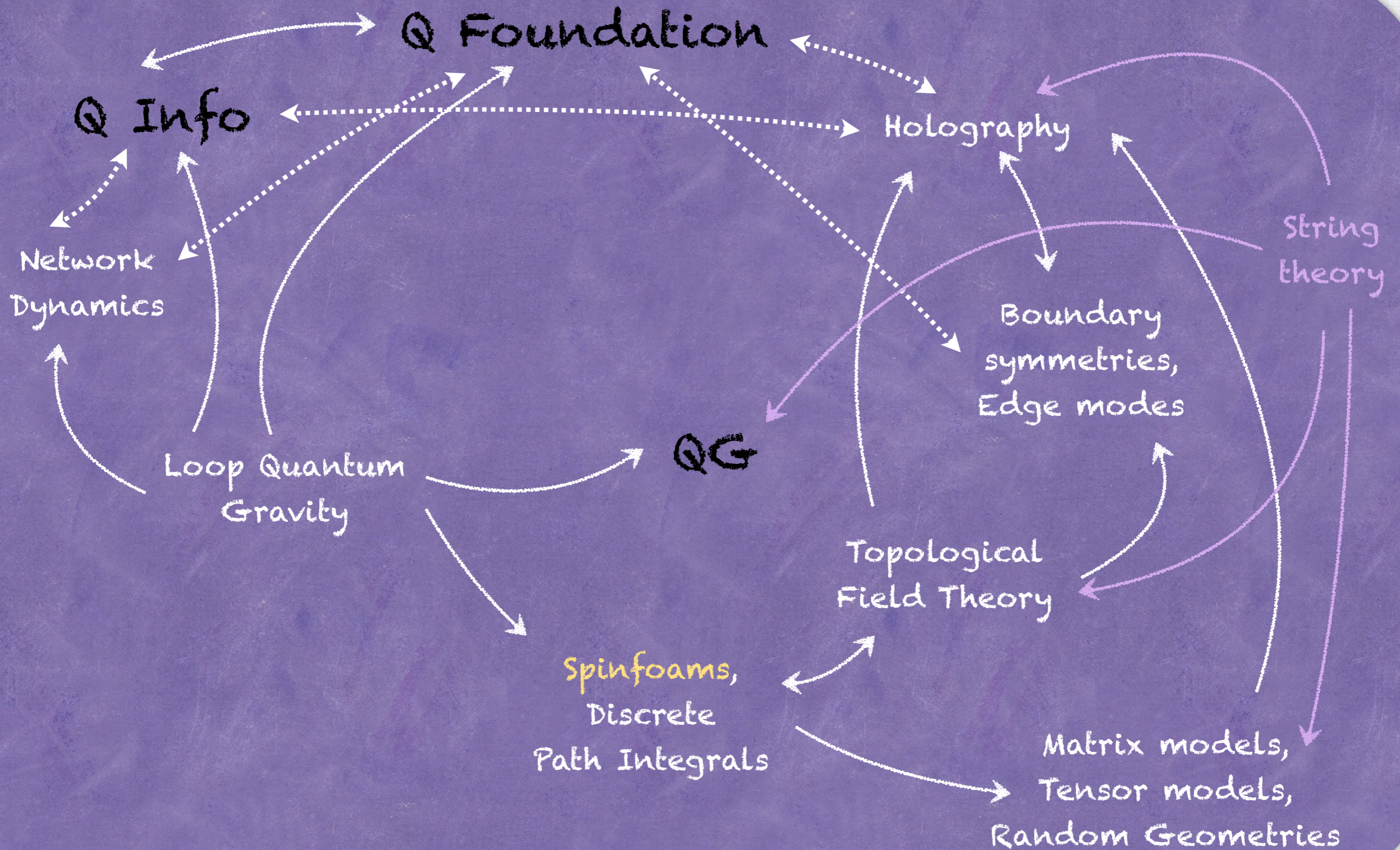


Generalised Geometry

Relative Locality

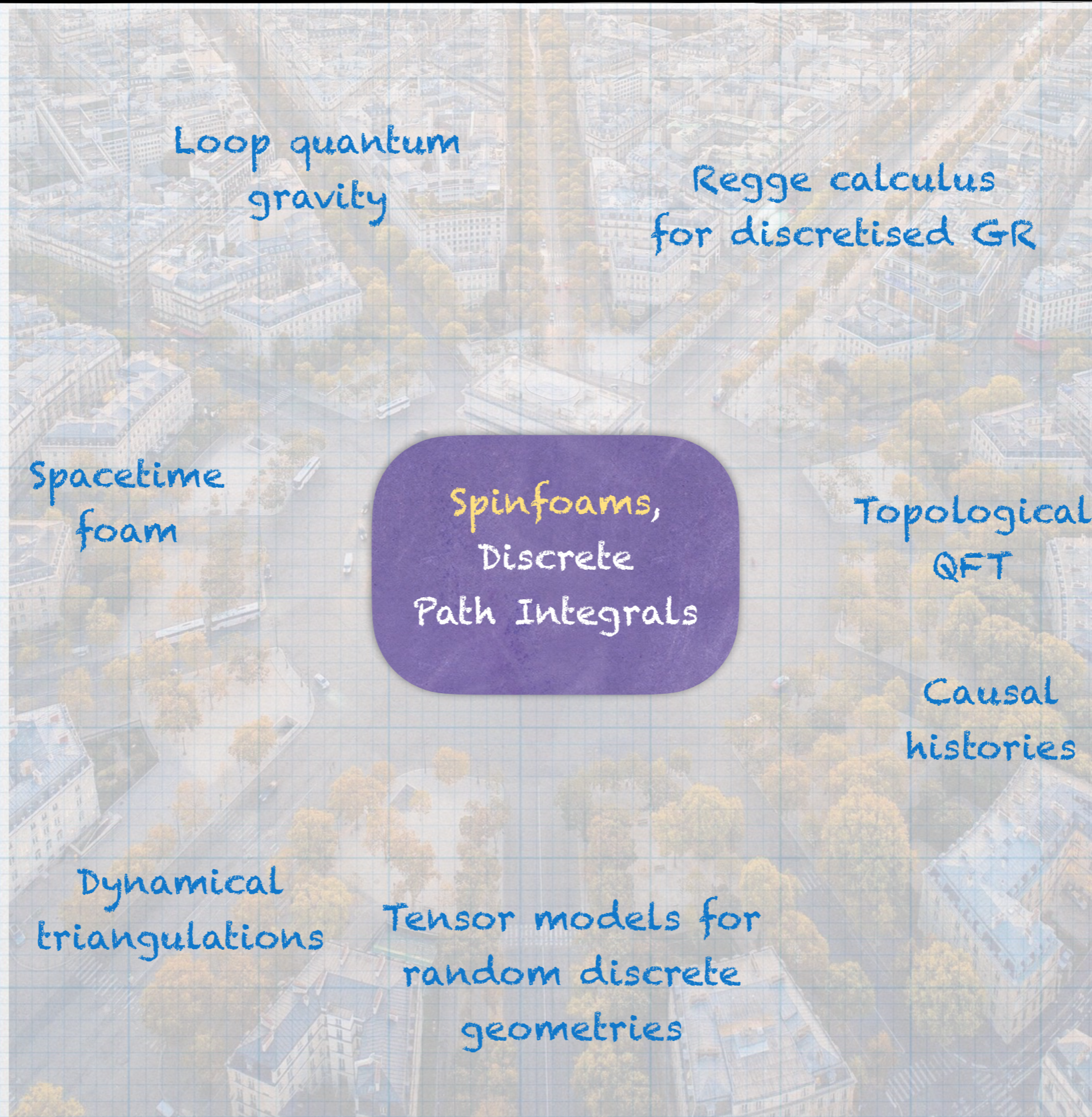
Condensate Cosmology

My local Quantum Gravity map



Spinfoams,
Discrete
Path Integrals

Spinfoams at a Crossroad



The way of the Spinfoam

a framework for a quantum gravity path integral

↳ Why a path integral ?

The way of the Spinfoam

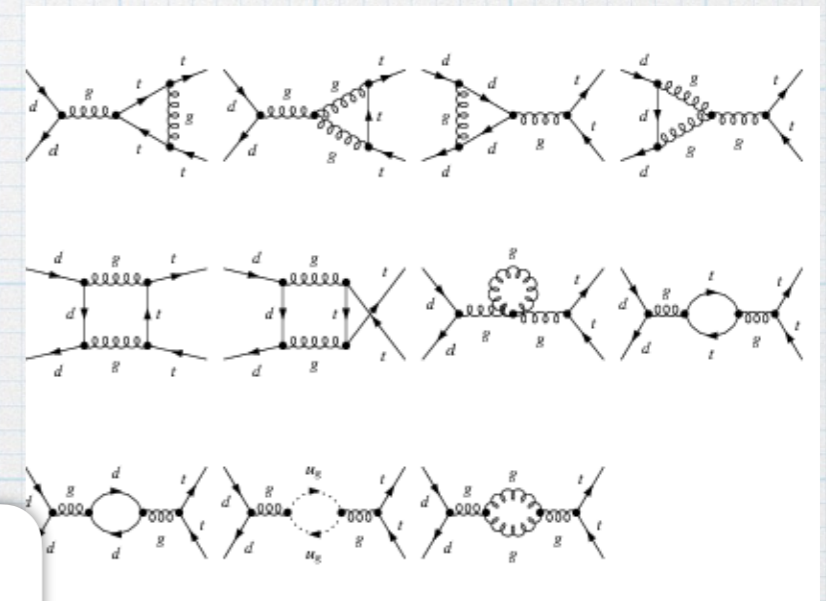
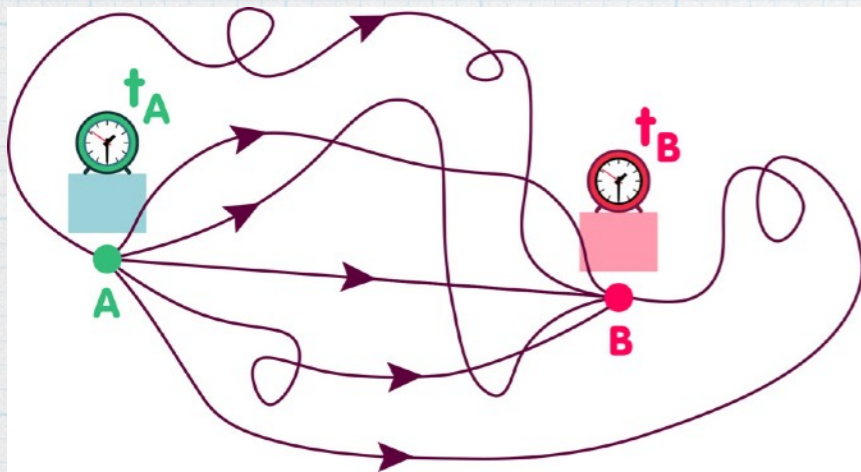
a framework for a quantum gravity path integral

↳ Why a path integral ?

Intuitive and effective framework for QM

Clear understanding of quantum → classical

Operational formulation of QFT amplitudes and renormalisation



$$\int [D\varphi] e^{\frac{i}{\hbar} S[\varphi]} \sim \sum_{\{\varphi_0\}} \mu[\varphi_0] e^{\frac{i}{\hbar} S[\varphi_0]}$$

The way of the Spinfoam

a framework for a quantum gravity path integral

↳ What path integral ?

The way of the Spinfoam

a framework for a quantum gravity path integral

↳ What path integral? Quantum superposition of quantum probability amplitudes for quantized geometries

$$\int_{g|_{\partial=h}} [Dg] e^{-\frac{i}{\hbar} \int_{\mathcal{M}_{4d}} \mathcal{L}_{EH}[g]} \longrightarrow \int_{Qg|_{\partial=\psi}} \mathcal{D}\mu[Qg] \mathcal{A}_{QM}[Qg]$$

Quantum boundary condition

Quantum measure

Quantum geometries

Quantum amplitudes

- Background independent (bridge between backgrounds)
- Symmetry under (quantum) diffeomorphisms
- Recover manifolds and GR (and QFT) in suitable limits

The way of the Spinfoam

a framework for a quantum gravity path integral

↳ How to define a path integral ? Discretize it !

The way of the Spinfoam

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Consistent with intuition of
universal minimal length scale



$$\frac{\hbar c}{\delta l} \leq E \leq \frac{c^4 \delta l}{G}$$

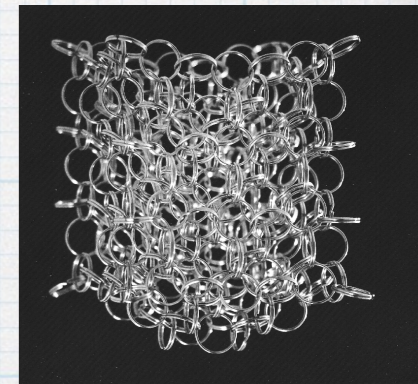
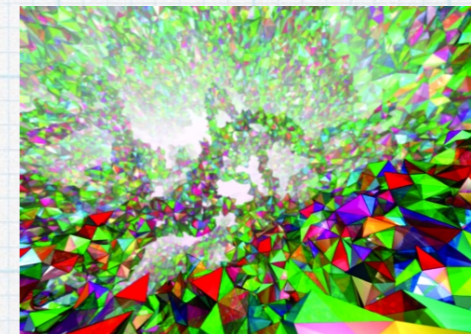
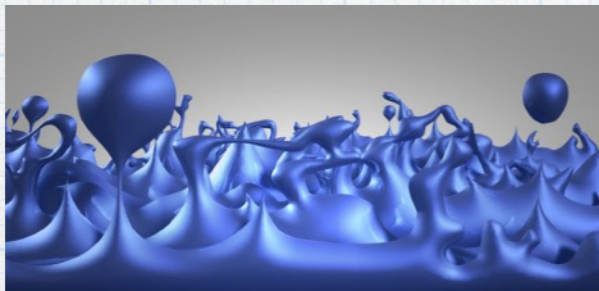
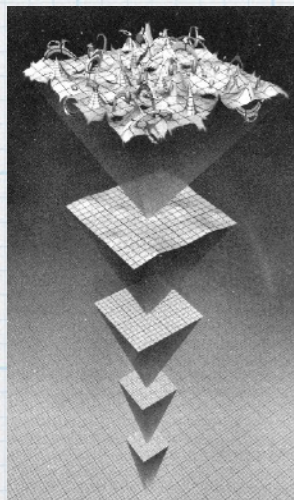
↓

$$\delta l \geq \sqrt{\frac{\hbar G}{c^3}} = l_{Planck}$$

The way of the Spinfoam

a framework for a quantum gravity path integral

↳ How to define a path integral ? Discretize it !



Space-time
foam

Spin network
dynamics

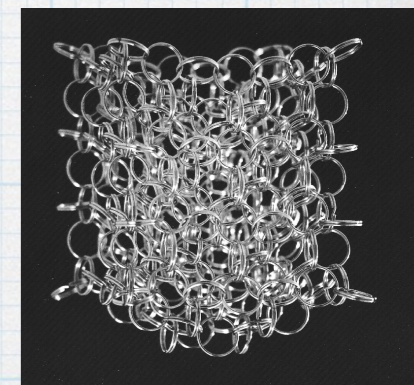
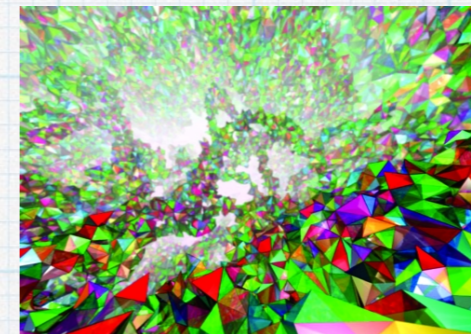
SPIN FOAM

Named by John C. Baez (1999)

The way of the Spinfoam

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↳ How to define a path integral ? Discretize it !



Loop Quantum
Gravity

SPIN FOAM

Baez 99



History formalism for Loop Quantum Gravity

↳ Loop Quantum Gravity :

Canonical Quantization of General Relativity as a Gauge Field Theory

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Canonical Quantization of General Relativity as a Gauge Field Theory

No need for modified gravity,
or specific matter content

Identification of d.o.f.s,
Straightforward quantization
respecting the symmetries

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A conservative,
yet versatile,
approach

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Cartan formulation

Identification of d.o.f.s,
Straightforward quantization
respecting the symmetries

$$S[e_\mu^I, \omega_\mu^{IJ}] = \int_{\mathcal{M}} \epsilon_{IJKL} e^I \wedge e^J \wedge F^{KL}[\omega]$$

Tetrad gives
local vector basis

$$g_{\mu\nu} = e_\mu^I e_{I\nu}$$

Connection gives
transport of tetrad

A conservative,
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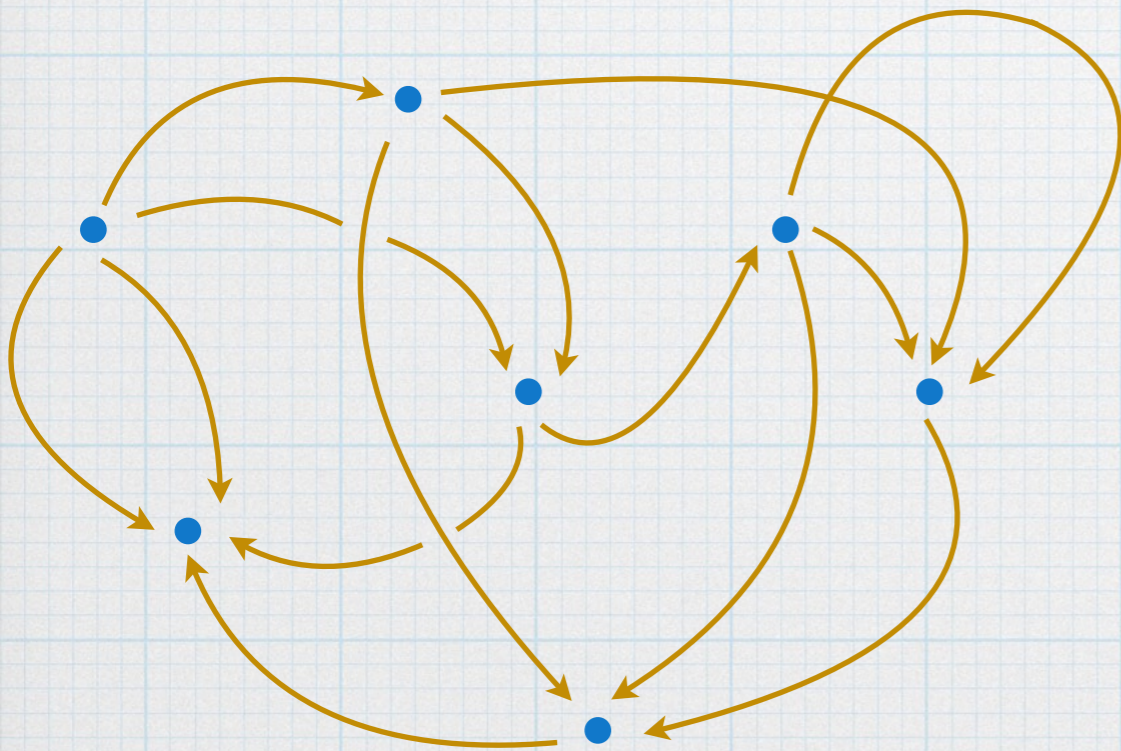
- Invariant under 4d Diffeomorphisms
- Invariant under local Lorentz transformations

History formalism for Loop Quantum Gravity

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Canonical Quantization of General Relativity as a Gauge Field Theory

- Look at evolution of spatial slice in time
- Quantum states as in lattice gauge theory : **Networks of connection**

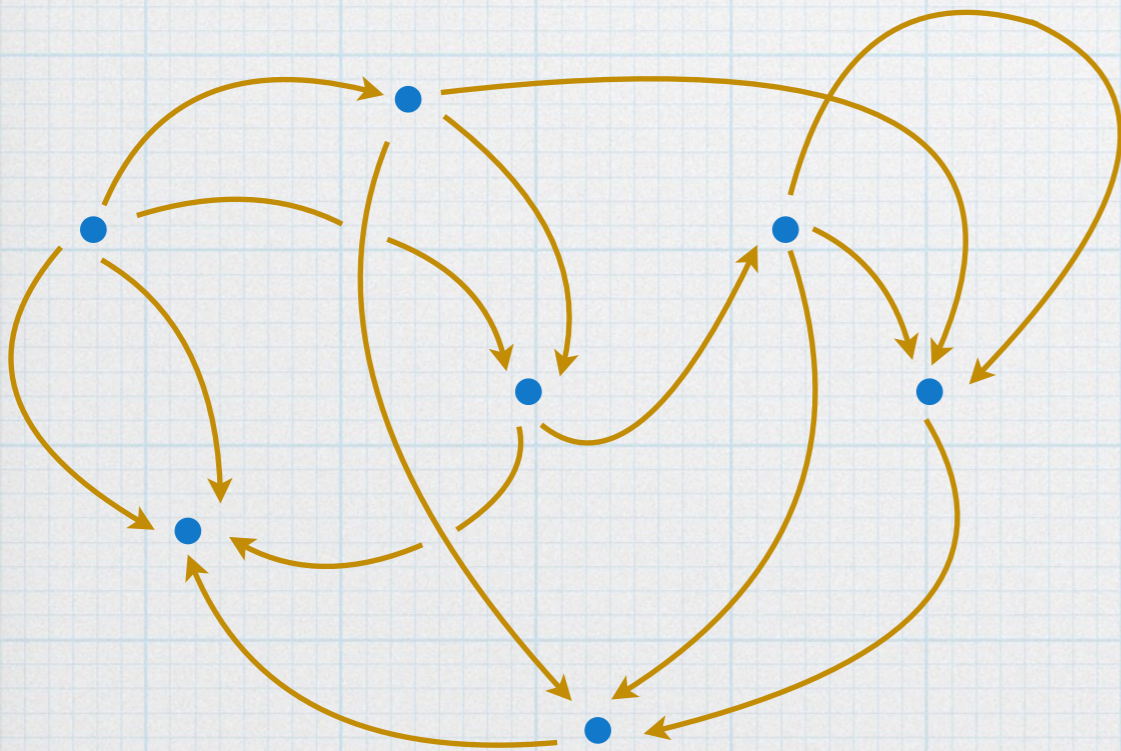


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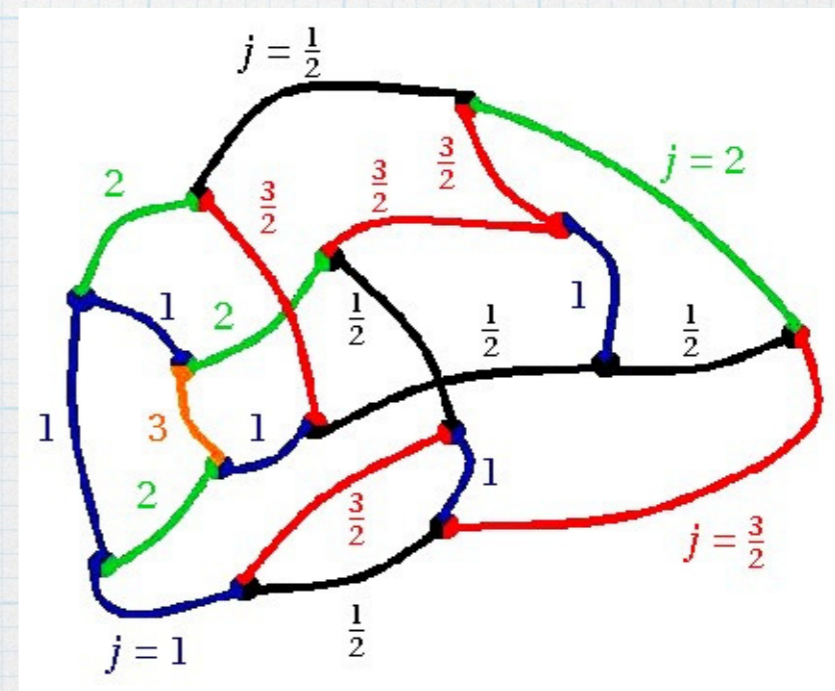
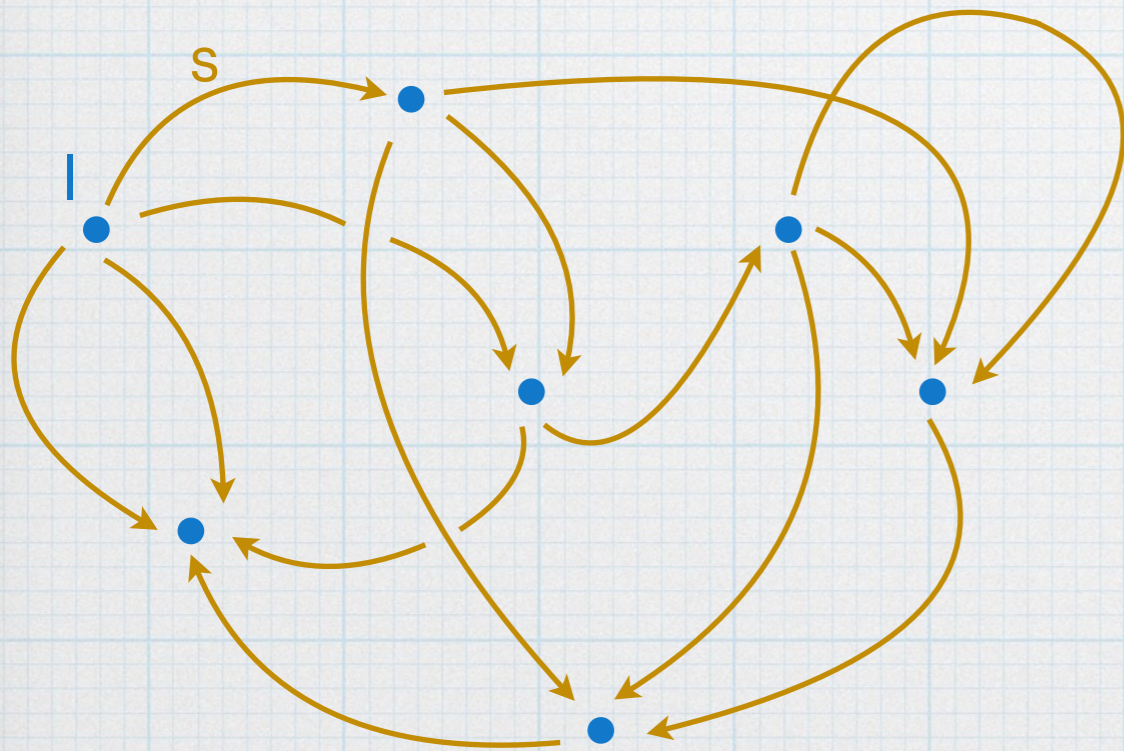
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Spin Networks



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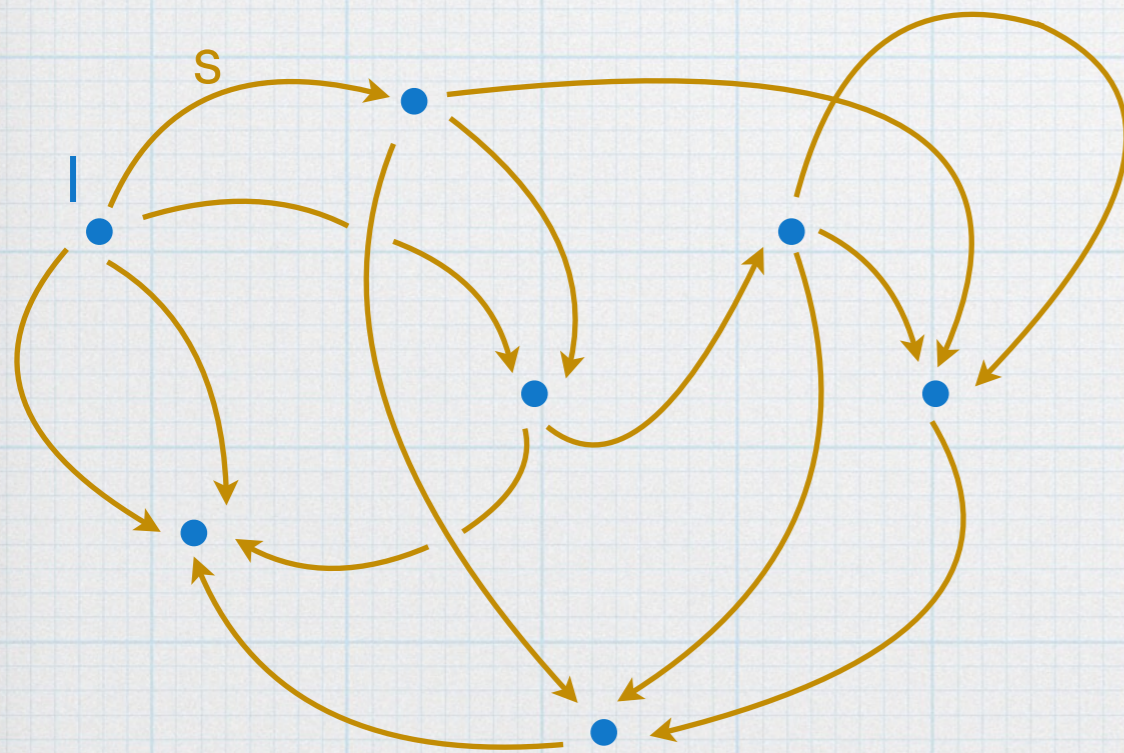
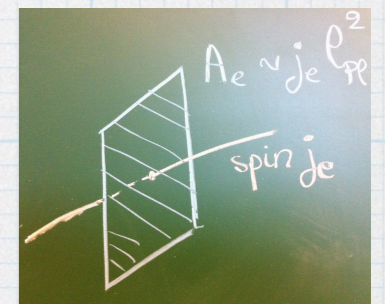
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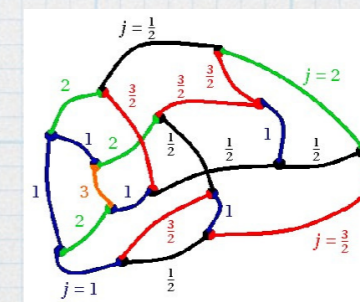
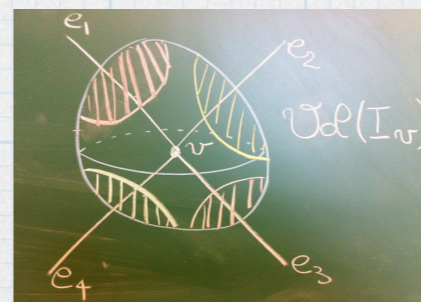
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Spin Networks

Quantum numbers with geometrical interpretation :



- **Spins** give discrete spectra of 2d areas
- **Intertwiners** give discrete spectra of 3d volumes



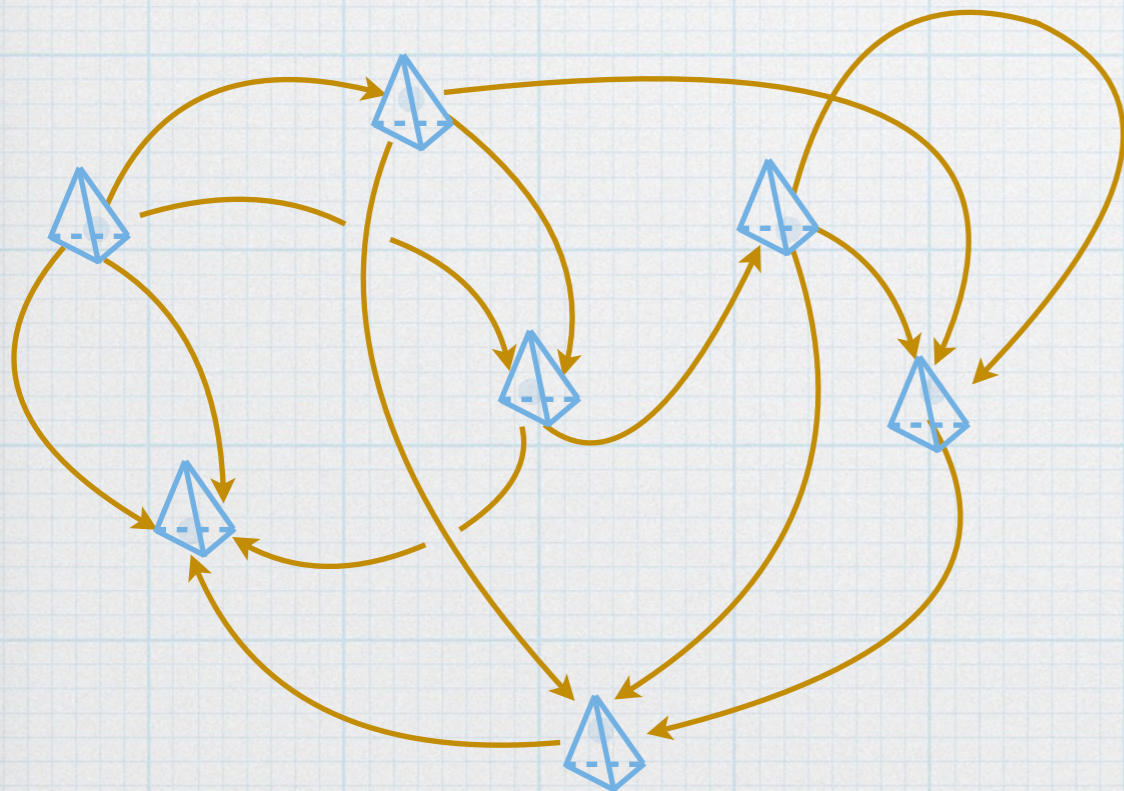
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Quantum volumes glued by quanta of area !

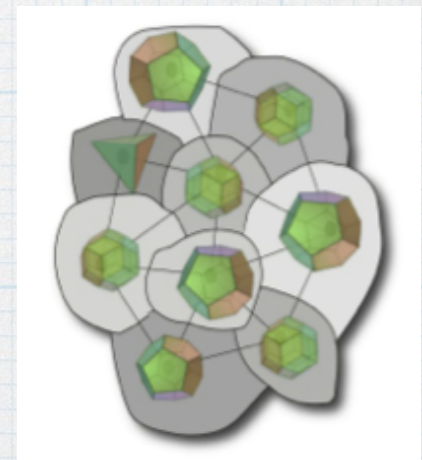
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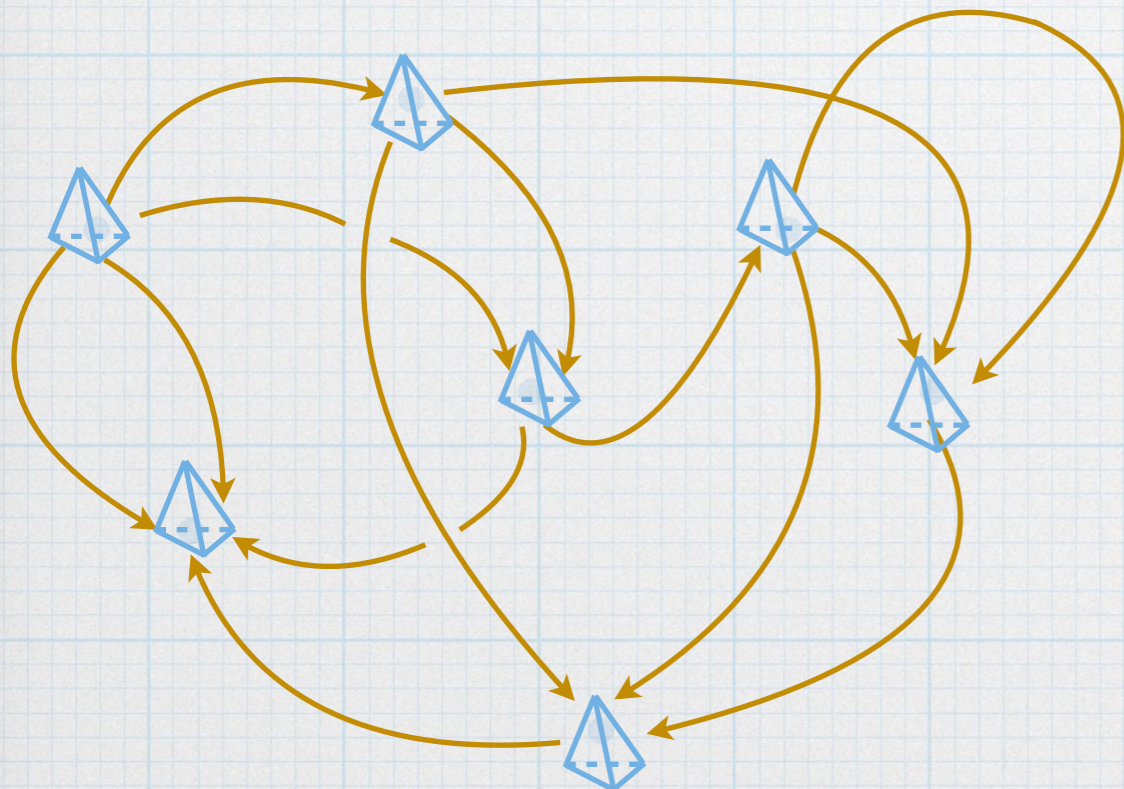
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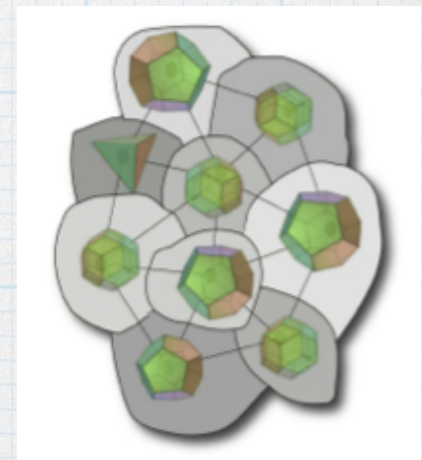
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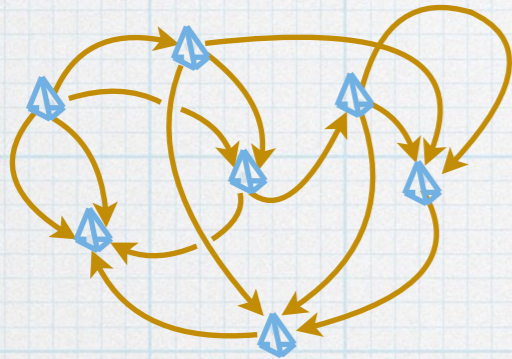
Quantum volumes glued by quanta of area !

What about dynamics ?

History formalism for Loop Quantum Gravity

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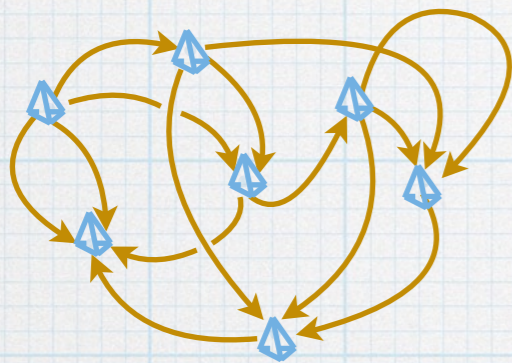
Spin Networks describe quantized transport on 3d slice
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↳ Spinfoam : Defines transition amplitudes between spin networks

History formalism for Loop Quantum Gravity

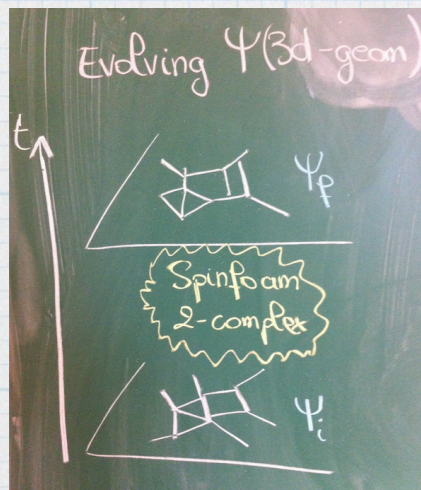
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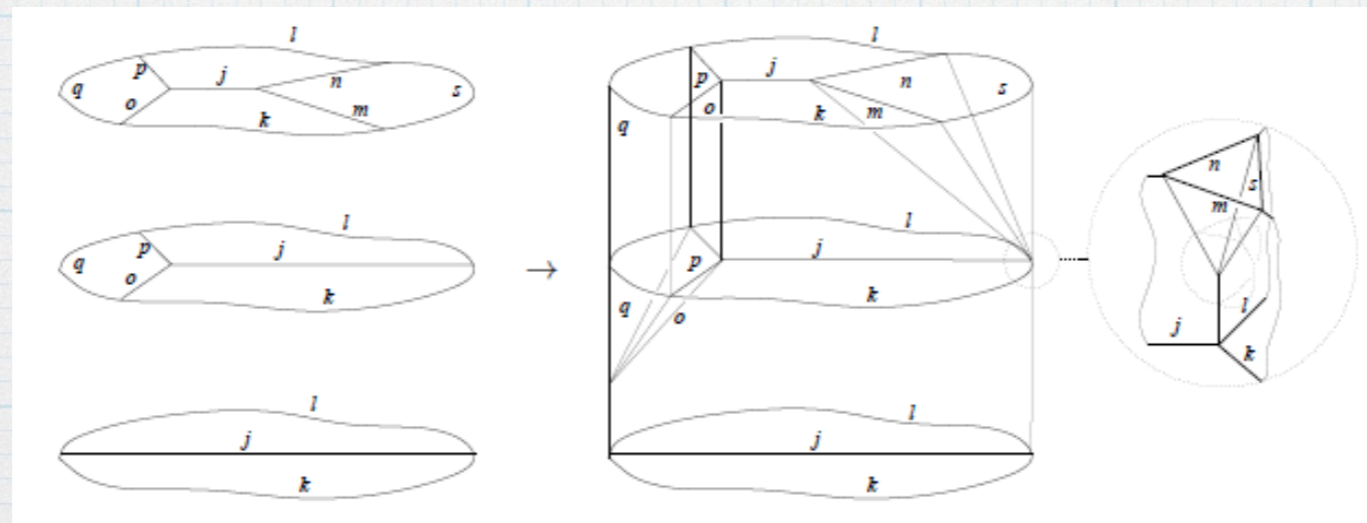


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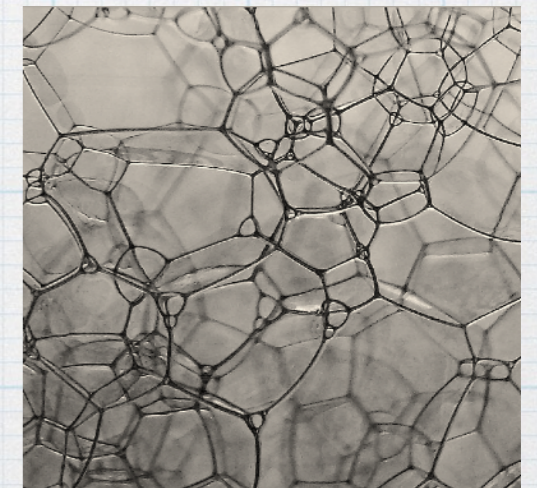
↳ Spinfoam : Defines transition amplitudes between spin networks



Links evolve along surfaces.
Nodes evolve along edges.



Vertices, where graph changes,
are space-time events

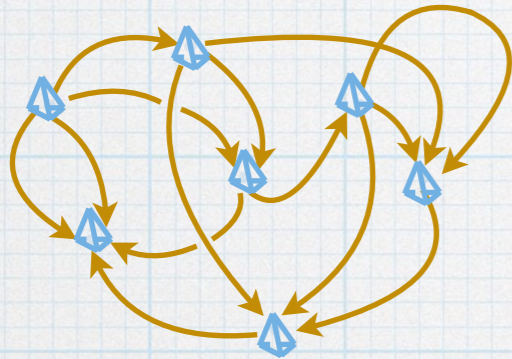


Space-time from
Bubbles of Spin

History formalism for Loop Quantum Gravity

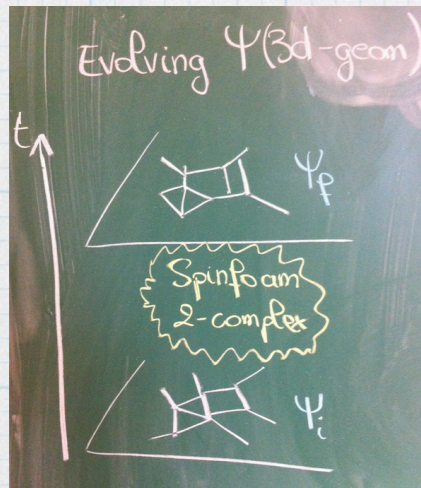
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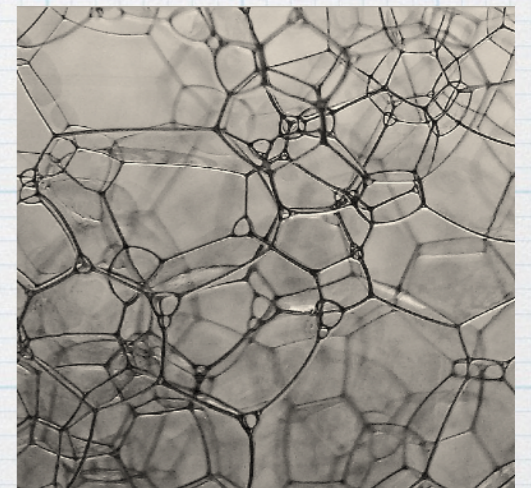


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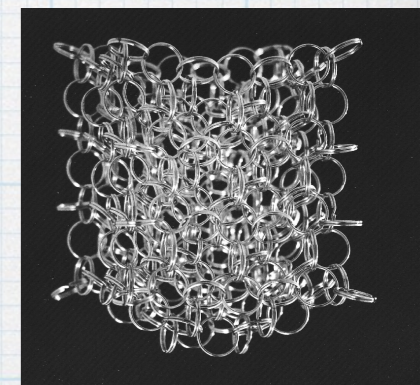
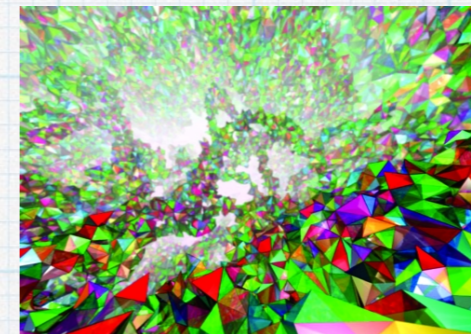
by assigning local probability amplitude to each space-time event, i.e. spinfoam vertex



The way of the Spinfoam

a framework for a quantum gravity path integral

↳ How to define a path integral ? Discretize it !



Loop Quantum
Gravity

SPIN FOAM

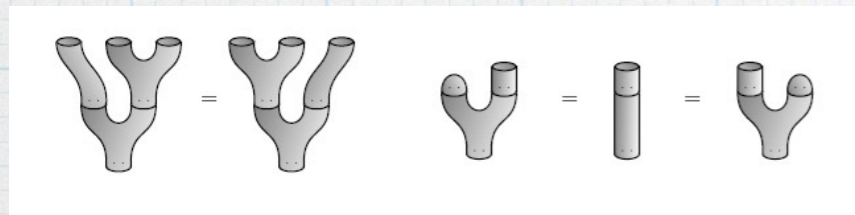
Baez 99



The way of the Spinfoam

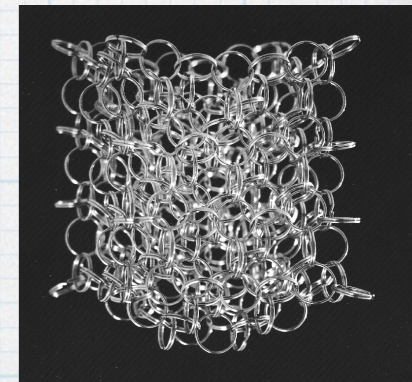
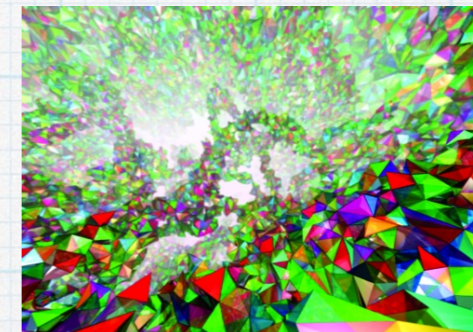
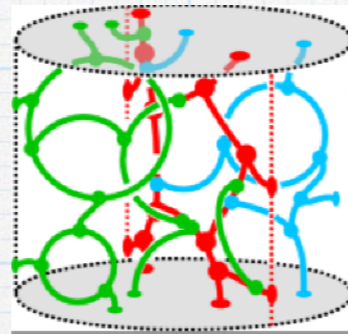
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Barrett-Crane 97,99
Baez-Barrett 99

TQFT,
State-sums



Loop Quantum
Gravity

Reisenberger-Rovelli 96
Markpoulou-Smolin 97

SPIN FOAM

Baez 99

2 complementary methods to construct
Spinfoam models

Discrete path integral for BF theory as a TQFT

↳ Why TQFT? Local field theory with no local d.o.f.s.



↳ Full theory exactly captured by discretization

T for Topological



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Convergence of methods : topological invariants, Atiyah axioms, BRST quantisation, ...

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$$S[e, \omega] = \int \star(e \wedge e) \wedge F[\omega]$$

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$$S[B, \omega] = \int B \wedge F[\omega]$$

$$B = \star(e \wedge e)$$

↳ Simplicity constraints
on bivector field

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- Then impose constraints on path integral (like filling a sea of defects)

Discrete path integral for BF theory as a TQFT

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... Spinfoams !

The EPRL spinfoam vertex

↳ Why TQFT ? Local field theory with no local d.o.f.s.

↳ Full theory exactly captured by discretization

↳ Gravity as a quasi-TQFT : $S[B, \omega] = \int B \wedge F[\omega] + \text{constraints on } B$

↳ Let's spinfoam it !

The EPRL spinfoam vertex

↳ Why TQFT ? Local field theory with no local d.o.f.s.

↳ Full theory exactly captured by discretization

↳ Gravity as a quasi-TQFT : $S[B, \omega] = \int B \wedge F[\omega] + \text{constraints on } B$

↳ Let's spinfoam it !

BF discretized on
space-time triangulation

- space-time from 4-simplices glued together
- B field discretized on 2-cells
- Connection discretized across 3-cells
- action discretized on 4-cells

The EPRL spinfoam vertex

↳ Why TQFT ? Local field theory with no local d.o.f.s.

↳ Full theory exactly captured by discretization

↳ Gravity as a quasi-TQFT : $S[B, \omega] = \int B \wedge F[\omega] + \text{constraints on } B$

↳ Let's spinfoam it !

BF discretized on
space-time triangulation

quantization

Spinfoam

- space-time from 4-simplices glued together
- B field discretized on 2-cells
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- Quantum 4-simplices glued by quantum tetrahedra
- Lorentz Spins on 2-cells
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Crane-Yetter (4d BF)
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Engle-Pereira-Rovelli-Livine

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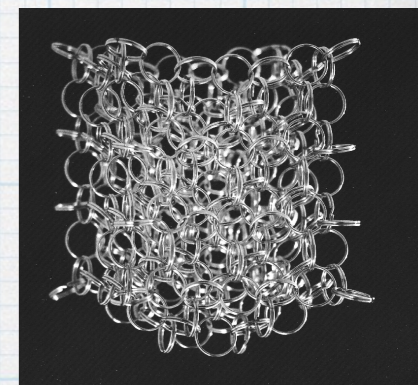
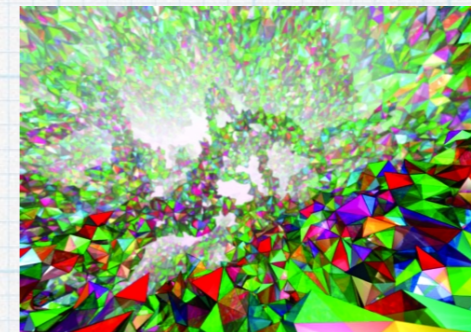
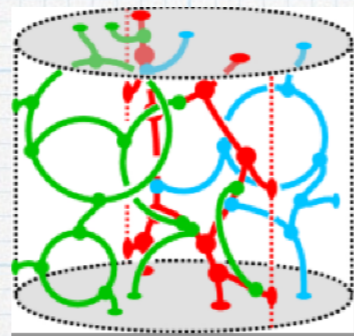
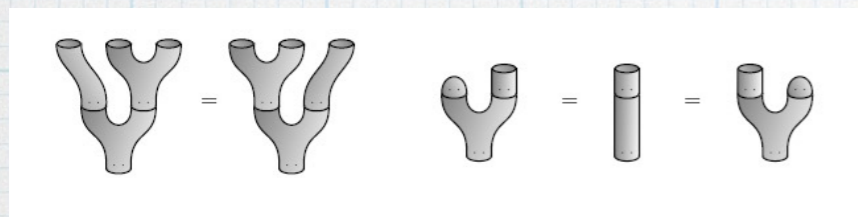
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Looks like a discretization of GR ... like Regge calculus ?

The way of the Spinfoam

a framework for a quantum gravity path integral

↳ How to define a path integral ? Discretize it !



Barrett-Crane 97,99
Baez-Barrett 99

**TQFT,
State-sums**

↑
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SPIN FOAM
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**Loop Quantum
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Reisenberger-Rovelli 96
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3 complementary perspectives
on Spinfoams

Quantized Regge calculus

↳ Let us look at a single spinfoam vertex

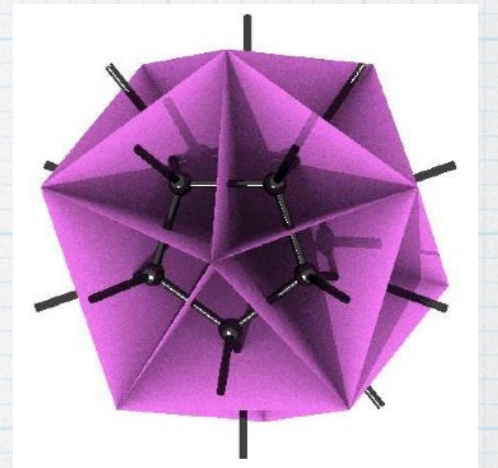
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= quantization of 4-simplex in FLAT SPACE-TIME

because of
equivalence
principle

= spinfoam fluctuations around flat space-time



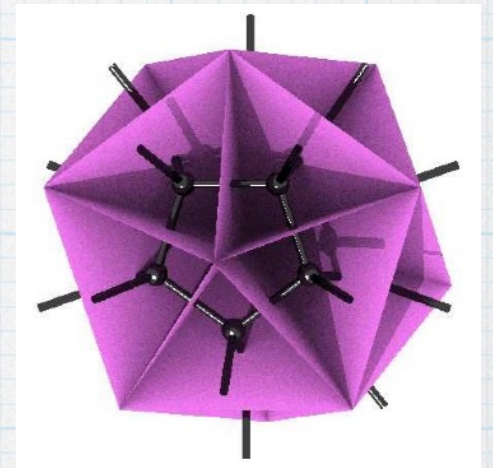
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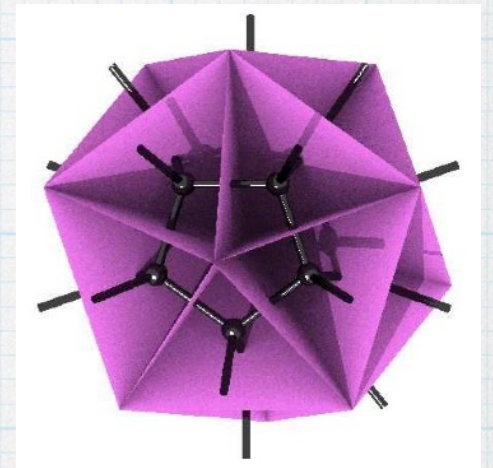
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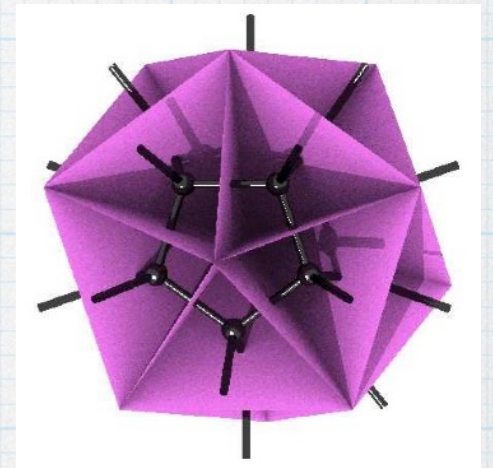
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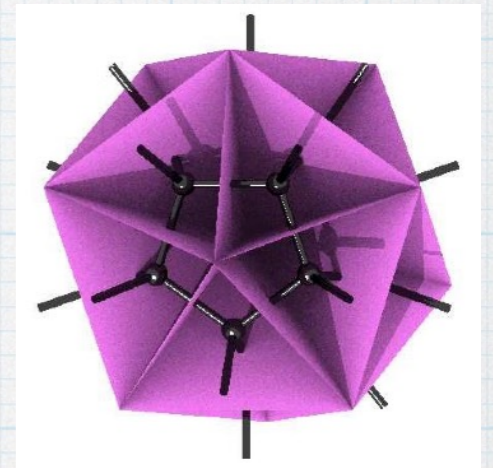
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Rovelli 05
L-Speziale 06
Bianchi-R-S 06
Alesci 08
B-Ding 11

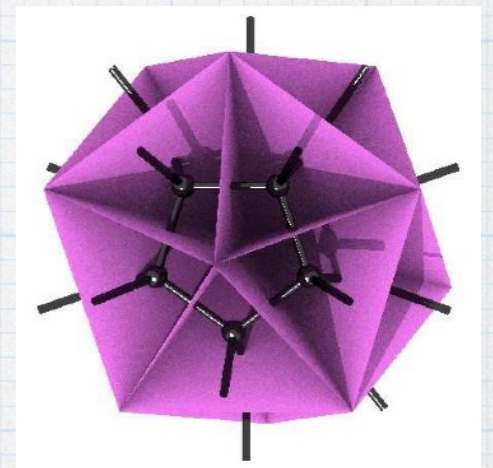
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\hookrightarrow QG corrections ?

LO

Quadratic Area Regge calculus,
Classical space-time solutions

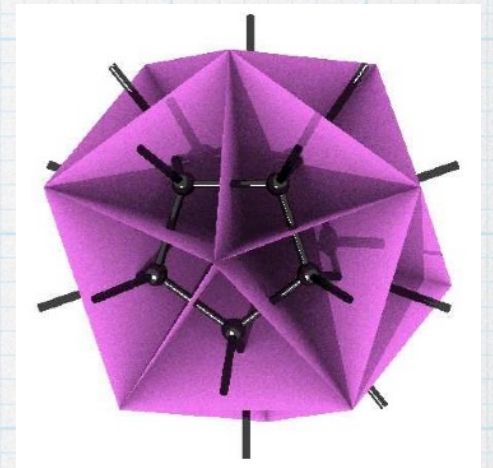
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NLO

Area Regge path integral,
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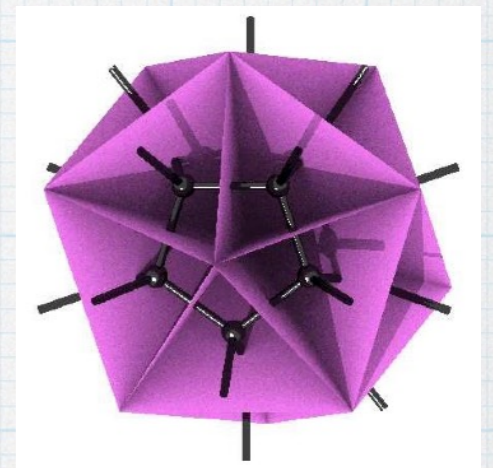
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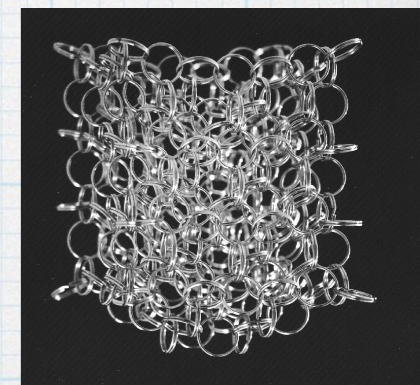
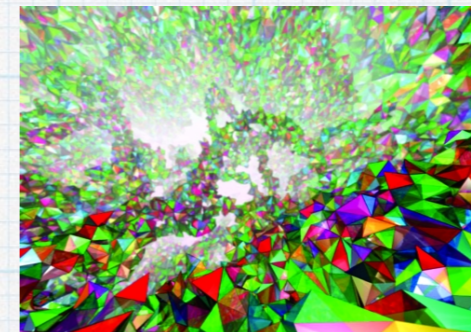
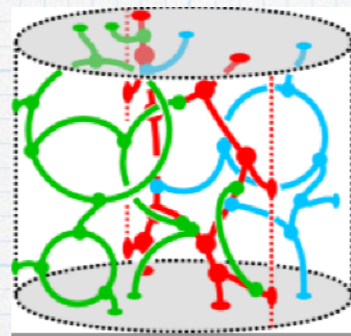
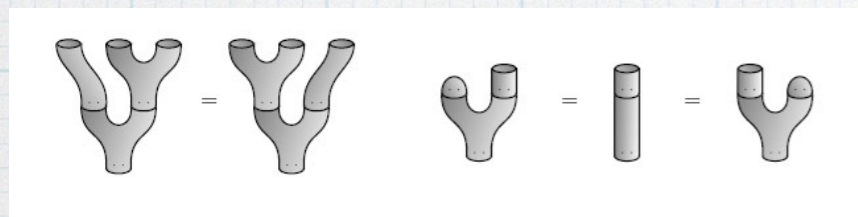
NNLO

Discrete area/vol spectrum
SF corrections to Regge

The way of the Spinfoam

a framework for a quantum gravity path integral

↳ How to define a path integral ? Discretize it !



Barrett-Crane 97,99
Baez-Barrett 99

**TQFT,
State-sums**

↑
Ponzano-Regge
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↓

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SPIN FOAM
Baez 99

**Loop Quantum
Gravity**

Reisenberger-Rovelli 96
Markpoulou-Smolín 97

**3 complementary perspectives
on Spinfoams**

3d Spinfoams : the proof-of-concept

↳ The Ponzano-Regge model :

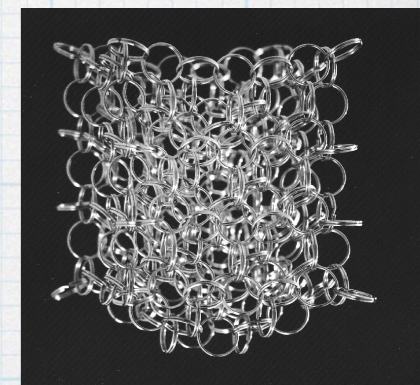
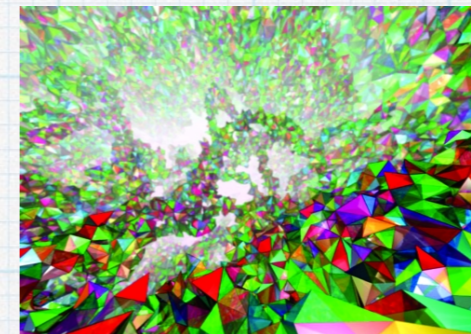
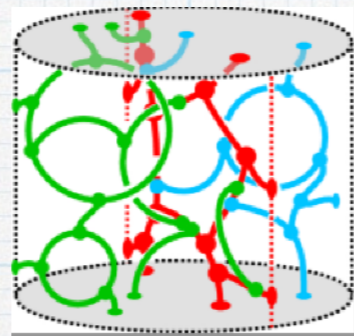
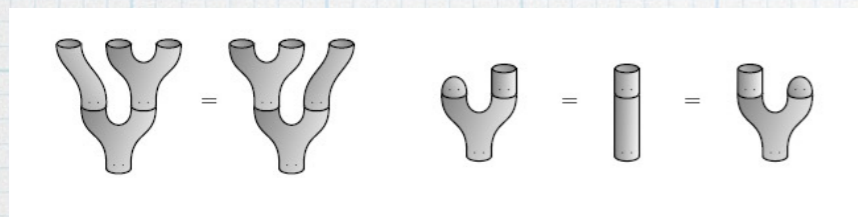
- ↳ Discrete path integral:
Spinfoams Prescribes a probability amplitude to every 3d triangulation
- ↳ Planck scale geometry:
À la LQG Length, area, volume, angle operators with quantized spectrum
- ↳ Topological Invariant:
TQFT Inv under +/- edges & vertices, encoded in Biedenharn-Elliott identity
- ↳ Finite Amplitudes:
Operational theory matches Ray-Singer torsion, knot invs, and Chern-Simons quantization
- ↳ Extensions:
Versatile formalism E/L signature, topological defects, q-deformation, supergravities
- ↳ Bulk/Boundary:
Interface with Phy Stat Exact holographic duality with inhomogeneous 2d Ising
- ↳ Matter coupling
NCG Effective Non-Commutative Geometry for matter coupled to 3d QG



The way of the Spinfoam

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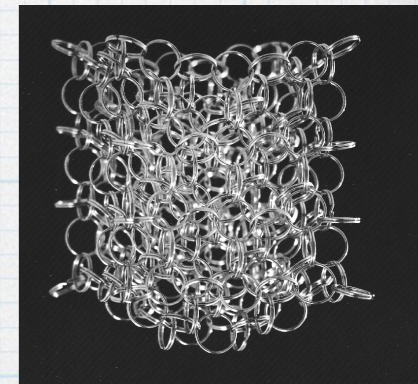
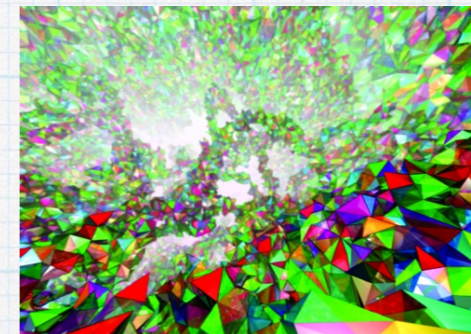
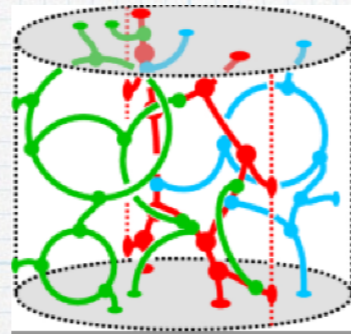
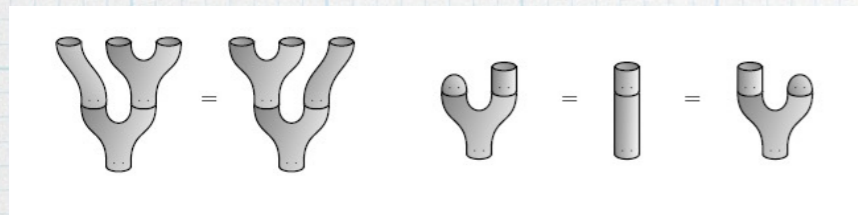
Reisenberger-Rovelli 96
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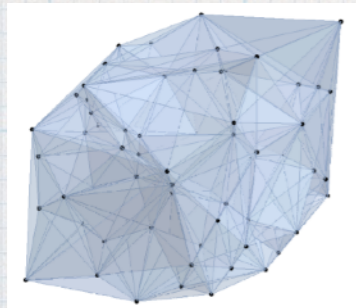


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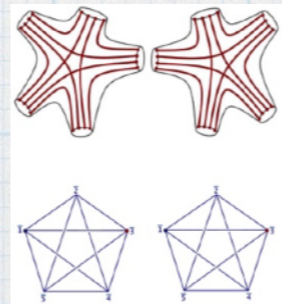
SPIN FOAM

Baez 99

**Group Field Theory,
Tensor Models**

**Quantized
Regge calculus**

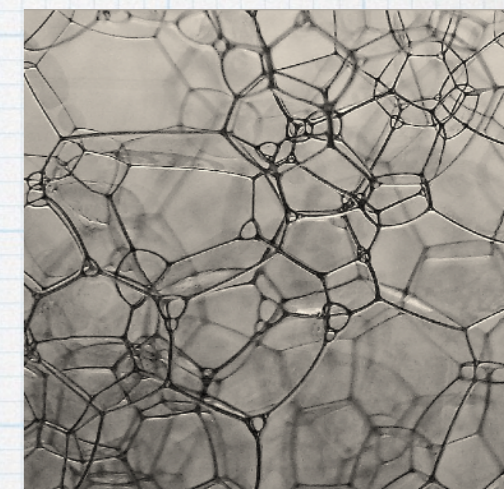
Rovelli 93
Barrett-Williams 98



Boulatov 93
DePietri-Freidel-Krasnov-Rovelli 99
Reisenberger-Rovelli 00 Freidel 05

Non-perturbative spinfoams: Group Field Theory

↪ How to sum over bulk geometries ?



Non-perturbative spinfoams: Group Field Theory

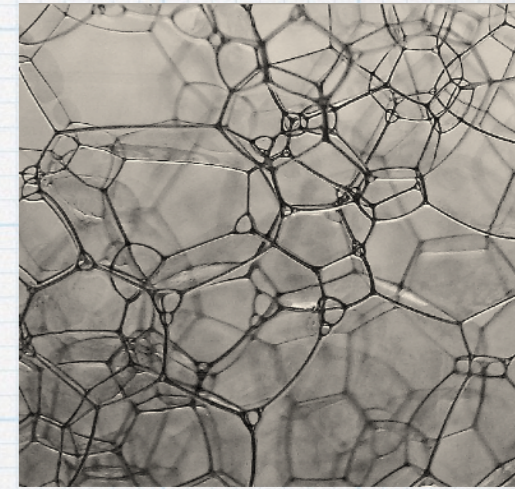
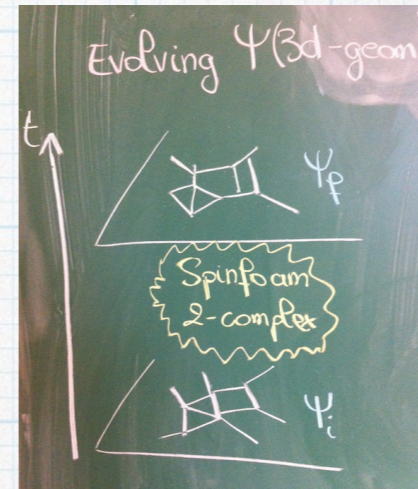
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Define a "auxiliary" field theory such that

Feynman diagrams

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Spinfoam amplitudes for 4d triangulations



Non-perturbative spinfoams: Group Field Theory

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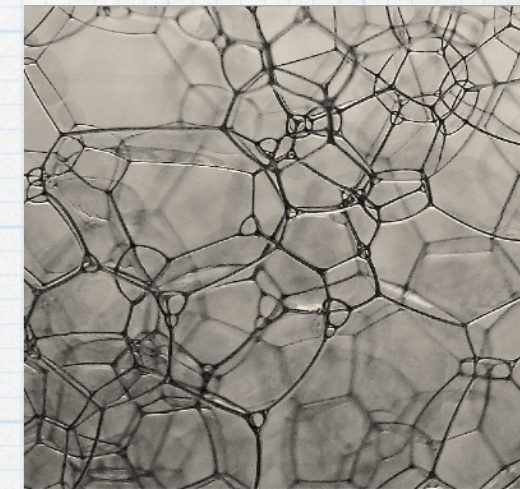
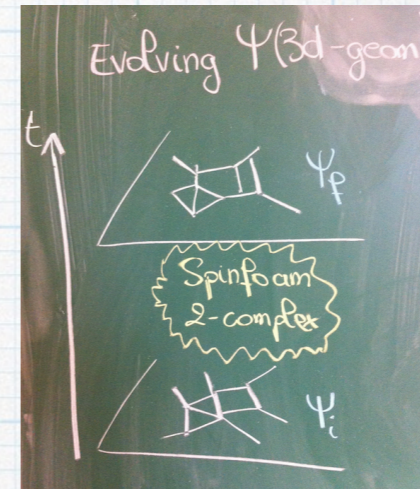
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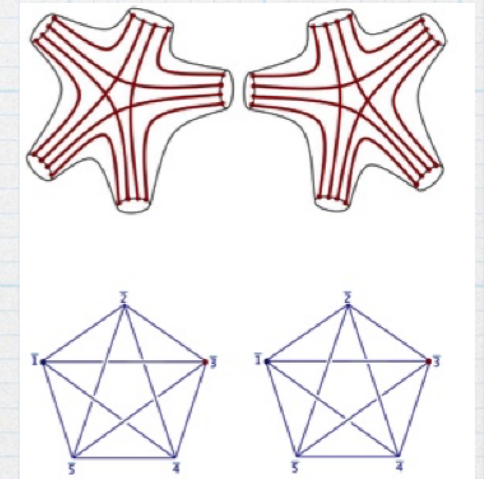
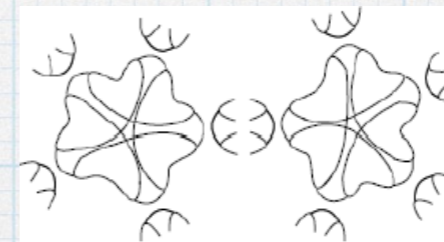
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$$S[\varphi] = \frac{1}{2} \int [dg]^4 \varphi(g_i)^2 - \frac{\lambda}{5!} \int [dh]^{20} \mathcal{V}(h_{ab}) \varphi^{\otimes 5}(h_{ab})$$



Field = quantum tetrahedra



Self-Interaction of tetrahedra

=

Quantum 4-simplex

Non-perturbative spinfoams: Group Field Theory

↪ How to sum over bulk geometries ?

Define a "auxiliary" field theory such that

Feynman diagrams

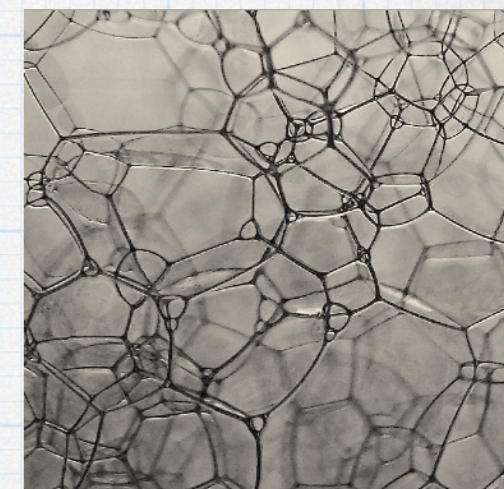
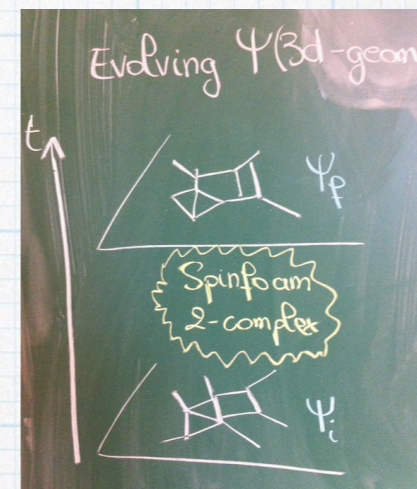
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Spinfoam amplitudes for 4d triangulations

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Group Field Theory :

- Extension of matrix and tensor models
- Non-local field theory on group manifold
- Admits quantum group symmetries (NCQFT)

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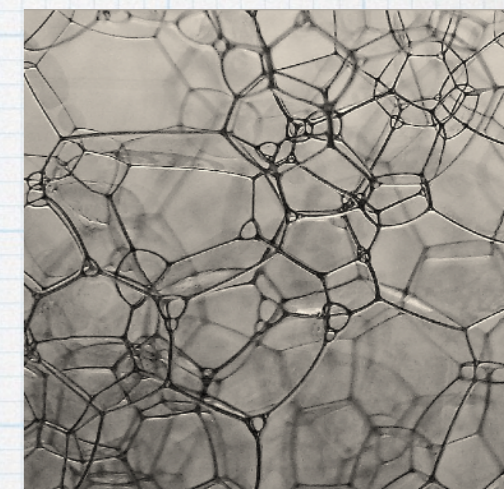
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Group Field Theory :

- Extension of matrix and tensor models
- Non-local field theory on group manifold
- Admits quantum group symmetries (NCQFT)
- Possible to couple matter

[Fairbairn-L, Freidel-Oriti, Girelli-L-Oriti,...]

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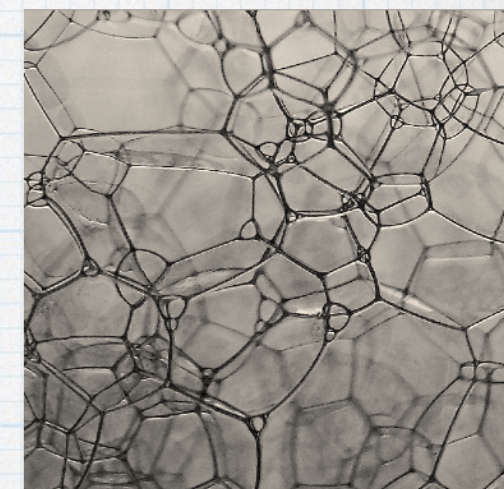
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Group Field Theory ...

A useful tool ?

Can model large ensembles of quantum 4-simplices

e.g. by mean field methods



Condensate cosmology proposal [Oriti]

The future of Spinfoam quantum gravity

Some directions :

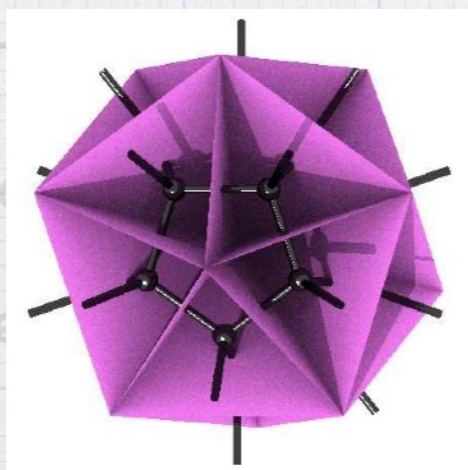
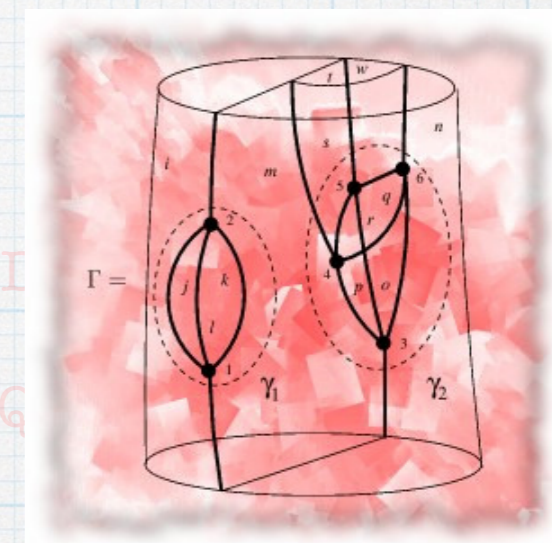
- Coarse-graining by Effective Spinfoams (area-metric gravity) [Dittrich, ...]
- Renormalization by Analytical Computations & Numerics [Han, Qu, Dona, ...]
- More on Group Field Theory [Oriti, ...]
- Holographic dualities, edge modes & boundary sym alg [Livine, Freidel, ...]
- Going null ? [Speziale, Wieland, ...]
- Black Hole-White Hole stable Planck scale superposition [Rovelli, Vidotto, ...]
- More on cosmology and effective corrections to GR [Borissova, Qu, ...]
- More on matter

The future of Spinfoam quantum gravity

Some directions :

Merçi!

- Coarse-grained (metric gravity) [I...]
- Renormalization Numerics [Han, G...]
- Moduli
- Holographic duality, edge modes, boundary sym alg [Livine, Freidel, ...]
- Going null ? [Speziale, Wieland, ...]
- Black Hole-Whiskers, Planck scale superposition
- More on cosmology, quantum corrections to GR [B...]
- More on matter



TUG Workshop - Annecy 2024

the Spinfoam Framework for Quantum Gravity

Etera Livine

Laboratoire de Physique LP ENSL & CNRS

