Bohmian Mechanics and Bouncing Universes

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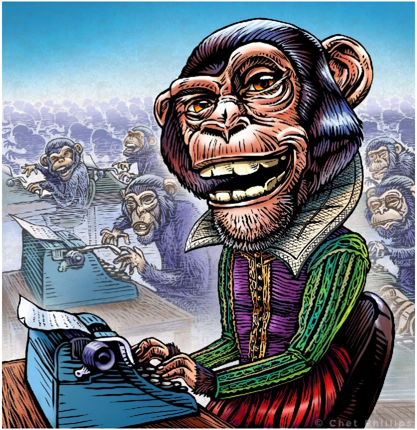
A Recreational Talk

Emile Borel’s Infinite Monkey Theorem



Think out of the box?

Outline

The subject of ’Quantum Trajectories’ has nowadays gained very good reputation through applications in various fields:

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- quantum chemistry (reactive scattering, electronic structure, ...),
- atomic physics (photoionisation, atomtronics, ...),
- high-dimensional systems (rare-gas, ...),
- classical & quantum optics,
- nanoelectronics (fast nanometer devices,...),
- ...

see e.g. *Quantum Dynamics With Trajectories*, *R.E. Wyatt*, (*Springer* 2000)
Applied Bohmian Mechanics, *X.Orials*, *J.Mompart* (*ed.*),(*Pan Stanford Pub.* 2012)
Quantum Potential,*I. Licatti*, *D.Fiscaletti* (*Springer* 2014)

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This seems at odds with all what we learned at school about quantum mechanics!!

☹?

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James Cushing (1994)

...some references:

1) The undevided universe
D. Bohm & B.J. Hiley ed. Routledge

2) Quantum Mechanics Historical contingency and the Copenhagen hegemony
J.T. Cushing ed. The University of Chicago Press

3) Speakable and unspeakable in quantum mechanics
J.S. Bell ed. Cambridge University Press

4) The Quantum theory of motion
P.R. Holland ed. Cambridge University Press

...+ the original papers.

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Outline

Two radically opposite attitudes:

A There is NO physical motivation whatsoever:

- after all, ordinary Quantum Mechanics describes very well atomic and nuclear physics.
- going relativistic and infinite number of degrees of freedom
  $\rightarrow$ Quantum Field Theories describe accurately the subatomic world: QED, QCD, SM, ..., BSM, just extensions!
  $\rightarrow$ Marriage with Gravity? just a matter of time, Canonical QG, Loop QG, Superstrings ···?

B There are several conceptual problems in ordinary Quantum Mechanics

- Fuzzy definition of the 'classical' measuring apparatus
- Measurement process and the postulate of the wave packet reduction
- Delayed-choice, Incompleteness, Non-locality,...
- Where is the observer of the Universe?

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- Bell even advocated it...(Bell's inequalities violated)

Outline

$$i\hbar\frac{\partial}{\partial t}\psi(\vec{x},t) = H\psi(\vec{x},t) = (-\frac{\hbar^2}{2m}\vec{\nabla}_{\vec{x}}^2 + V(\vec{x}))\psi(\vec{x},t) \tag{1}$$

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postulate a particle with momentum $\vec{p} \equiv m\vec{v} = \vec{\nabla}S(\vec{x},t)$

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postulate a particle with momentum $\vec{p} \equiv m\vec{v} = \vec{\nabla}S(\vec{x},t)$ define particle density $\rho \equiv R^2 = |\psi|^2$

$$\frac{\partial S}{\partial t} + \frac{(\vec{\nabla} S)^2}{2m} + V - \frac{\hbar^2}{2m}\frac{\vec{\nabla}^2 R}{R} = 0 \ \& \ \vec{v} = \frac{1}{m}\vec{\nabla} S$$

imply a *quantum mechanical* Newton's law:

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if the particle enters a space region where R is very small
$\rightarrow$ violent fluctuations of momentum $\vec{p}$ and energy E $\rightarrow$ in general very irregular and complicated trajectories resembling Brownian motion (Bohm '52)

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-b- the particle is guided by the wave through $\vec{v} \equiv \frac{d\vec{x}}{dt} = \frac{1}{m}\vec{\nabla} S_{|\vec{x}=\vec{x}(t)}$

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$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0$$
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- what happens if $P(\vec{x},t_0) \neq |\psi(\vec{x},t_0)|^2$??

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- define $f(\vec{X}, t)$ by $P(\vec{X}, t) = f(\vec{X}, t) \times |\psi(\vec{X}, t)|^2$
- use conservation of probability and Bohmian dynamics (-a- & -b-) to show $\frac{df}{dt} \equiv \frac{\partial f}{\partial t} + \dot{X} \cdot \nabla f = 0$
- N.B. f is taken on the particle trajectories, $f(\vec{X}(t), t) = cte$

Notion of 'quantum equilibrium'

- consider a system of N particles, with space coords $(X_1, X_2, ..., X_N) \equiv \vec{X}$
- define $f(\vec{X}, t)$ by $P(\vec{X}, t) = f(\vec{X}, t) \times |\psi(\vec{X}, t)|^2$
- use conservation of probability and Bohmian dynamics (-a- & -b-) to show $\frac{df}{dt} \equiv \frac{\partial f}{\partial t} + \dot{X} \cdot \nabla f = 0$
- N.B. f is taken on the particle trajectories, $f(\vec{X}(t), t) = cte$
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- $\overline{H}$ reaches its minimum $\Leftrightarrow \ \overline{f} = 1 \ \Leftrightarrow \overline{P(\vec{X}, t)} = \overline{|\psi|^2}$

$\rightarrow$ Quantum Mechanics as a classical statistical system in 'thermodynamic equilibrium'

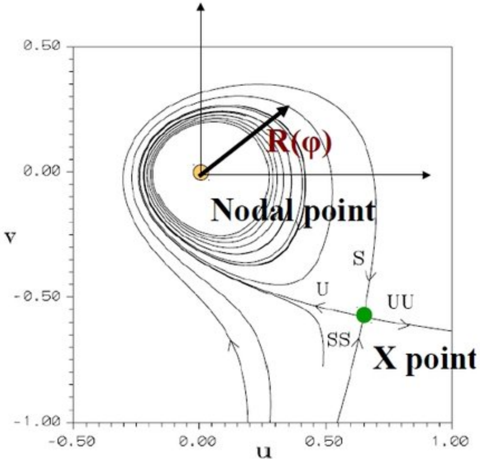
$\rightarrow$ Quantum Mechanics as a classical statistical system in ’thermodynamic equilibrium’

$\rightarrow$ interesting consequences

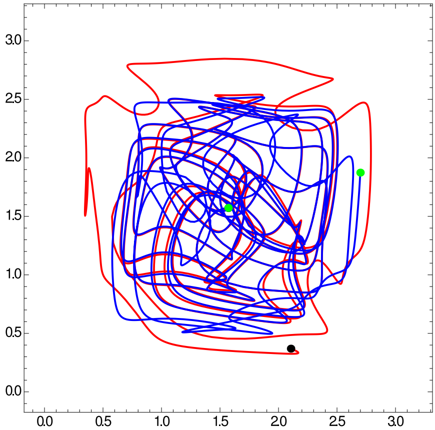
Outline

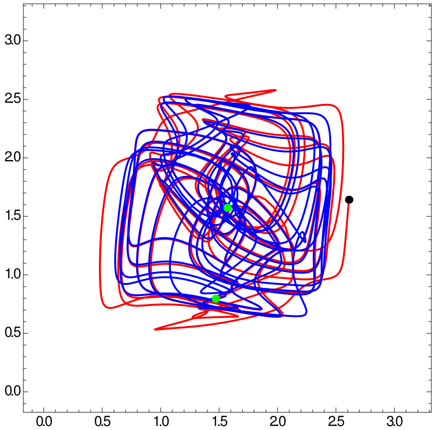
- Are Bohmian trajectories chaotic?
- An increased interest in recent years in 1-, 2-, or 3-particle systems in 2D and 3D boxes.
- The role of nodes (space-time points where $\psi(x, t) = 0$) and X-points (where the particle velocity in the frame of nodes $= 0$.)

review: *C.Efthymiopoulos et al*, *Annales de la Fondation de Broglie*, *Volume* 42 (2017)

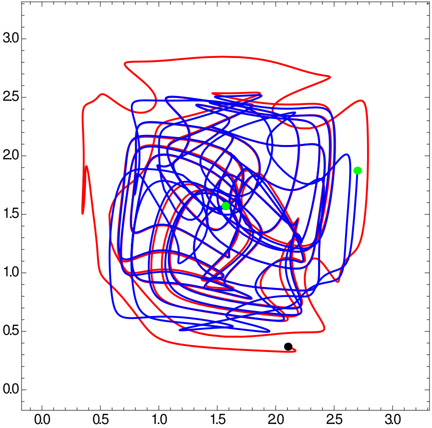

0.50
0.00
-0.50
-1.00
-0.50
0.00
0.50
1.00
v
u
R(φ)
Nodal point
S
U
UU
SS
X point

$$\Psi[x_1, x_2, t] = \frac{2}{\pi} \sum_{m,n=1}^{2} \sin(m x_1) \sin(n x_2) e^{i(\theta mn - E_{mn} t)}, \quad E_{mn} = \frac{1}{2}(m^2 + n^2)$$



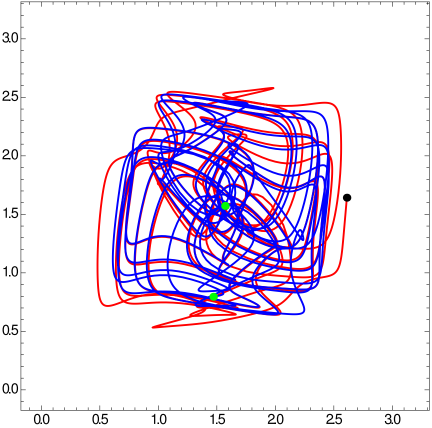


$$\Psi[x_1, x_2, t] = \frac{2}{\pi} \sum_{m,n=1}^{2} \sin(m x_1) \sin(n x_2) e^{i(\theta_{mn} - E_{mn} t)}, \quad E_{mn} = \frac{1}{2}(m^2 + n^2)$$



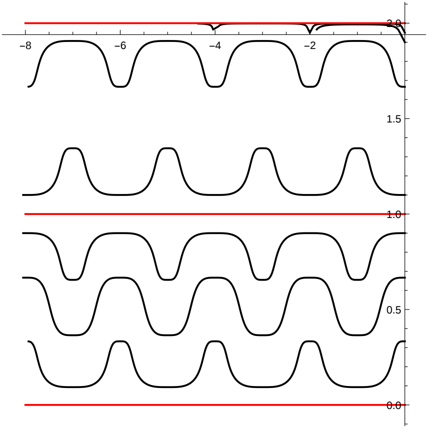
$\theta_{11}=1.1525988926093297, \ \theta_{12}=4.2775762116024665$

$\theta_{21}=2.1660329888555025, \ \theta_{22}=2.8960554218806349$



$\theta_{11}=1.2, \ \theta_{12}=4.3$

$\theta_{21}=2.2, \ \theta_{22}=2.9$



$$\left(-\hbar^2\frac{\nabla^2}{2m}+\lambda\delta(x_1-x_2)\right)\Psi^{\delta}_{n_1,n_2}(x_1,x_2)=E_{n_1,n_2}\Psi^{\delta}_{n_1,n_2}(x_1,x_2)$$

Outline

Valentini '91:

(1) P relaxes to $|\psi|^2$ at equilibrium in a gas of Bohmian particles

(2) proof not fully convincing: $\overline{H(t)} < \overline{H(0)}$ rather than $\frac{\overline{dH}}{dt} \leq 0$ (H-theorem)

(3) away from equilibrium ($P \neq |\psi|^2$) $\rightarrow$ violation of the uncertainty principle & violation of Lorentz invariance!

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- the problem in (2) is typical in stat. phys. whenever a specific kinetic equation is lacking (e.g. Boltzmann equation, Vlasov/Landau equation, etc.)
- A Bohmian gas kinetic equation is still missing: difficult to derive, due to the non-local potential! Boltzmann's 'molecular chaos' hyp. cannot be used straightforwardly. A careful BBGKY hierarchy approach is needed.

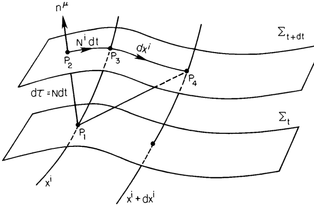
Such an equation would be very important $\rightarrow$ quantitative information about relaxation to ’quantum equilibrium’
AND quantitative information about *statistical fluctuations*, i.e. about possible Lorentz violation in relativistic (field) theories...

Outline

The wave function of the Universe?!

The ADM time + 3-space decomposition
n^μ unit vector normal to Σ_t,

N: the lapse function, N^i: the shift vector



(stolen form Kolb & Turner)

The wave function of the Universe?!

Einstein's equations ⇔ a dynamical system with constraints

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -(Ndt)^2 + h_{ij}(N^i dt + dx^i)(N^j dt + dx^j)$$

h_{ij}: induced 3D intrinsic metric on Σ_t

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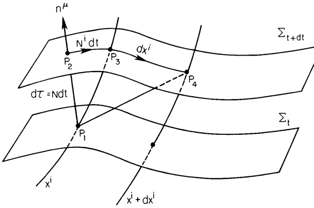

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$$S = \int_{\mathcal{M}} d^4x\mathcal{L} = \frac{1}{16\pi G}\int_{\mathcal{M}} d^4x\sqrt{-g}\left({}^{(4)}\mathcal{R} - 2\Lambda + \mathcal{L}_{matter}\right) + \frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^3x\sqrt{h}\,K^i_{\ i}$$

$K_{ij} = \frac{1}{2N}\left(N_{i|j} + N_{j|i} - \frac{\partial}{\partial t}h_{ij}\right)$, the *extrinsic* curvature of Σ_t

$\rightarrow {}^{(4)}\mathcal{R} = (K^i_{\ i})^2 - K_{ij}K^{ij} - {}^{(3)}\mathcal{R} \ + [4-\mathrm{div}]$

$\rightarrow$ express the gravitational part of S in terms of N, N_i, h_{ij} and their *first* derivatives.

$\rightarrow$ However, NO dependence on time derivative of N and N_i !

$\Longrightarrow \pi = \frac{\delta}{\delta\dot{N}}L = 0$ and $\pi^i = \frac{\delta}{\delta\dot{N}_i}L = 0 \leftarrow$ Primary constraints.

$\rightarrow \pi^{ij} = \frac{\delta}{\delta\dot{h}_{ij}}L$

$\rightarrow$ From $L = \int d^3x\mathcal{L}$ to $H = \int d^3x\mathcal{H}$

$H = \int d^3x\left(\pi\dot{N} + \pi^i\dot{N}_i + \pi^{ij}\dot{h}_{ij} - \mathcal{L}\right) = \int d^3x\left(N\mathcal{H} + N_i\mathcal{H}^i\right)$

$0 = \dot{\pi} = -\frac{\delta}{\delta N}H = -\mathcal{H}$ and $0 = \dot{\pi}^i = -\frac{\delta}{\delta N_i}H = -\mathcal{H}^i \leftarrow$ Secondary constraints.

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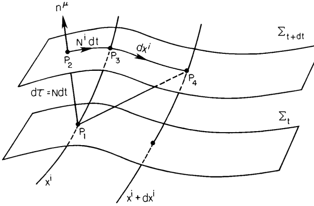


(stolen form Kolb & Turner)

The wave function of the Universe?!

$$\mathcal{H}(h_{ab}, \pi^{ab}) = 0$$

quantization: $\pi^{ab} \to -i\frac{\delta}{\delta h_{ab}}$

$$\downarrow$$

$$\left[\frac{G_{ijkl}}{(16\pi G)^2}\frac{\delta}{\delta h_{ij}}\frac{\delta}{\delta h_{kl}} + \frac{\sqrt{h}}{16\pi G}({}^{(3)}\mathcal{R} - 2\Lambda) - \mathcal{T}\right]\Psi[h_{mn}, \phi] = 0$$

Wheeler-deWitt Equation

$G_{ijkl} = \frac{1}{2\sqrt{h}}(h_{ik}h_{jl} + h_{il}h_{jk} - h_{ij}h_{kl})$, $\mathcal{T} = \sqrt{h}T^0_0$

Outline

Bohmian version in Minisuperspace & a Radiation-filled Universe

$\rightarrow$ reduce the infinite dimensional functional superspace in $\Psi[h_{mn}, \phi]$ by freezing some degrees of freedom $\rightarrow$ minisuperspaces!

$\rightarrow$ Freeze all degrees of freedom except for one...

i) for the geometry, assume FRW

and

ii) for matter, assume a perfect fluid

a radiation-filled Universe + conformal-time gauge ($N = a$) gives a very simple Hamiltonian, with one dynamical degree of freedom, the scale factor $a(t)$:

$$H = \frac{p_a^2}{24a} + 6ka, \; (c = 16\pi G = 1)$$

quantization

$$\downarrow$$

$$\hat{H} = -\frac{1}{24}\frac{d^2}{da^2} + 6ka$$

However, $a \geq 0$ & $\hat{H}$ self-adjoint $\rightarrow$ restrictions on Ψ, e.g. $\Psi(a = 0) = 0$;

Bohmian version in Minisuperspace & a Radiation-filled Universe

Solve for $\Psi \equiv R(a)e^{iS(a)/\hbar}$

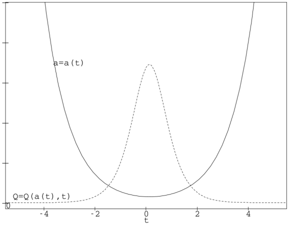
Bohmian guidance equation $\rightarrow \dot{p} = 12\dot{a} = \vec{\nabla}_a S(a)_{|a \rightarrow a(t)}$

$$a(t) = a_0\sqrt{b^2 \sin^2 t + (6 - B\tan t)^2 \cos^2 t}, \quad (k = 1)$$

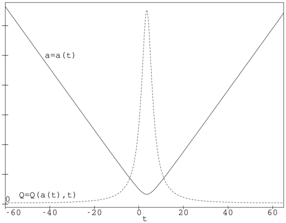
$$a(t) = a_0\sqrt{b^2 t^2 + (6 - Bt)^2}, \qquad \text{(k=0)}$$

$$a(t) = a_0\sqrt{b^2 sinh^2 t + (6 - B\, tanht)^2 cosh^2 t} \quad \text{(k=-1)}$$

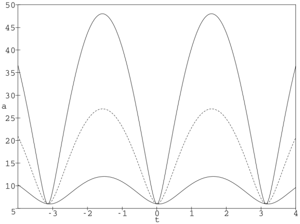
From J. Acacio de Barros N. Pinto-Neto, M. A. Sagioro-Leal Phys.Lett.A241,229 (1998)



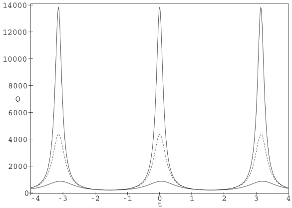
$k = -1$



$k = 0$



$k = +1$



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Conclusion & Outlook

- pilot-wave quantum mechanics, invented by de Broglie (’27), killed by Pauli (the same year), resurrected by Bohm (’52), and advocated by Bell and others.

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- an increaslingly large community (chem., phys.-chem., nonascience,...)is relying on its practical power to study quantum systems
- some communities still ignore it, as "useless or probably wrong".
- one can regard ordinary QM as a kind of effective ’low-energy’ theory of an underlying physics where indeterminism is NOT a fundamental feature!

- Outlook
  - can it be tested?
  - can it be an alternative road to new physics, in particular in high energy physics beyond the standard model?
  - can it be an unconventional path to ’Quantum’ Gravity $\rightarrow$ everything is deterministic at the fundamental level!?

The Quantum Pandora Box?

THANK YOU FOR YOUR ATTENTION