Cosmic Inflation at the Crossroads

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Annecy, 6/11/2024

Outline

Qualitative Inflation
- Triggering and stopping acceleration
- Cosmological perturbation

Quantitative inflation
- Encyclopædia Inflationaris: Opiparous Edition
- Primordial power spectra
- Reheating consistent model predictions

Bayesian inference
- Machine-learning an effective likelihood
- Marginalized posteriors
- Computing bayesian evidences
- Bayes factors for all models
- Information gain on the reheating

Predictions in model space
- Running of the spectral index
- Model space vs slow-roll space
- Reheating energy density and equation of state

Conclusion

J. Martin, CR and V. Vennin: arXiv:2404.10647, arXiv:2404.15089, arXiv:1303.3787v4

Qualitative Inflation

Cosmic Inflation

- A predictive, testable and tested early universe paradigm
  - ✦ Accelerated expansion of the universe at $E_{\mathrm{inf}} > \mathrm{MeV}$ (BBN)
  - ✦ Addresses some unexplainable features of the Friedmann-Lemaître model
- For the simplest incarnation of inflation...
  - ✦ Historically introduced to dilute monopoles formed at GUT
  - ✦ Flatness of the spatial sections ($\Omega_{\mathrm{K}} = 0.0009 \pm 0.0018$)
  - ✦ Statistical isotropy of the observable universe (horizon problem)
  - ✦ Origin of CMB and LSS (quantum fluctuations)
  - ✦ Gaussianities of the cosmological perturbations ($f_{\mathrm{NL}} < -0.9 \pm 5$)
  - ✦ Adiabaticity of the cosmological perturbations (isocurv. $< 1\%$)
  - ✦ Almost scale invariance ($n_{\mathrm{S}} = 0.9649 \pm 0.004$)

Triggering and stopping acceleration

- The simplest way: single-field inflation

$$S = \int \mathrm{d}x^4 \sqrt{-g}\left[\frac{1}{2\kappa^2} R + \mathcal{L}(\phi)\right] \quad \text{with} \quad \mathcal{L}(\phi) = -\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)$$

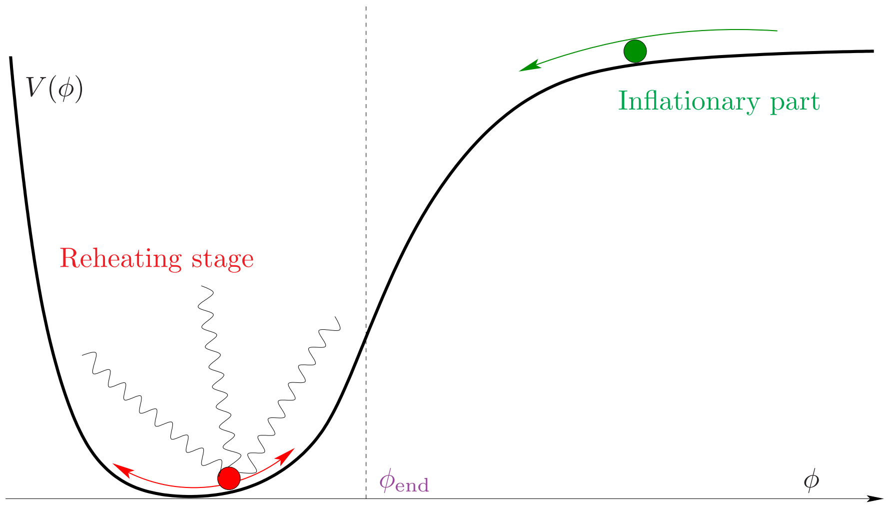
- Inflation occurs in the plateau and is followed by a reheating era

  - Friedmann-Lemaître

$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V\right)$$

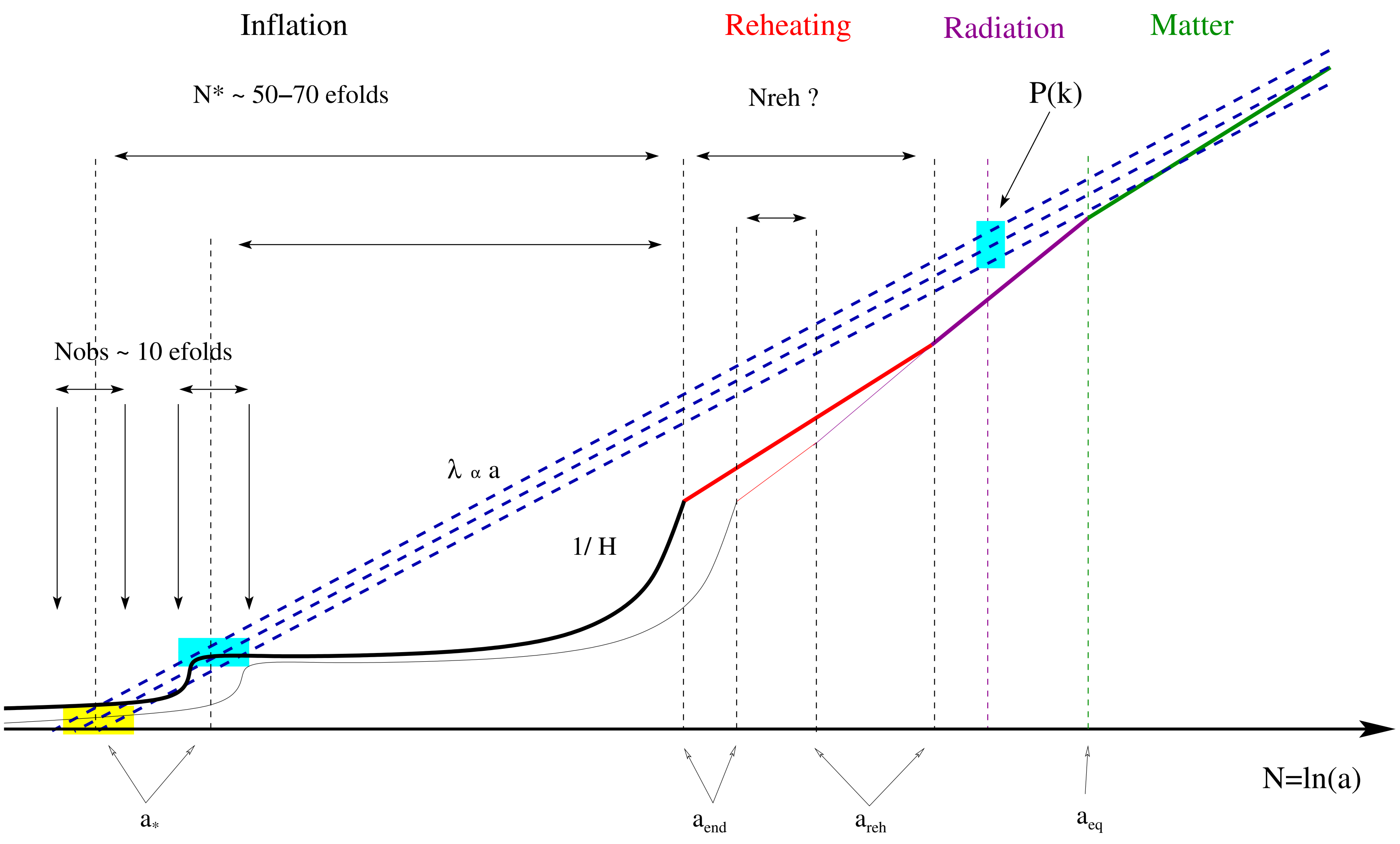
$$\frac{\ddot{a}}{a} = -\frac{1}{3}\left(\dot{\phi}^2 - V\right)$$

  - $H \simeq \text{Constant} \to a \propto e^{Ht}$



- The reheating stage: everything after $\phi_{\rm end}$ till radiation domination

Cosmological perturbations of quantum origin



- Model testing: reheating effects must be included!

Redshift at which inflation ends

- Depends on the redshift of reheating

$$1 + z_{\rm end} = \frac{a_0}{a_{\rm end}} = \frac{a_{\rm reh}}{a_{\rm end}}(1 + z_{\rm reh}) = \frac{a_{\rm reh}}{a_{\rm end}} \left(\frac{\rho_{\rm reh}}{\tilde{\rho}_\gamma}\right)^{1/4} = \frac{1}{R_{\rm rad}} \left(\frac{\rho_{\rm end}}{\tilde{\rho}_\gamma}\right)^{1/4}$$

  - ✦ The reheating parameter $R_{\rm rad} \equiv \dfrac{a_{\rm end}}{a_{\rm reh}} \left(\dfrac{\rho_{\rm end}}{\rho_{\rm reh}}\right)^{1/4}$
  - ✦ Encodes any observable deviations from a radiation-like or instantaneous reheating $R_{\rm rad} = 1$

- $R_{\rm rad}$ can be expressed in terms of $(\rho_{\rm reh}, \overline{w}_{\rm reh})$ or $(\Delta N_{\rm reh}, \overline{w}_{\rm reh})$

$$\ln R_{\rm rad} = \frac{\Delta N_{\rm reh}}{4}(3\overline{w}_{\rm reh} - 1) = \frac{1 - 3\overline{w}_{\rm reh}}{12(1 + \overline{w}_{\rm reh})} \ln\left(\frac{\rho_{\rm reh}}{\rho_{\rm end}}\right)$$

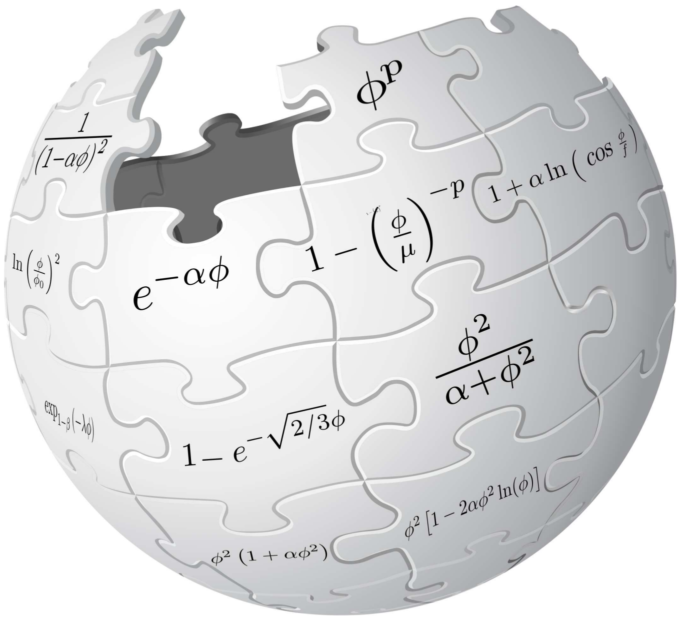
- Scary Astrophysics (early universe)

$$10^{10} < z_{\rm end} < 10^{28}, \qquad -46 < \ln R_{\rm rad} < 15$$

Quantitative inflation

Encyclopædia Inflationaris: Opiparous Edition

- New version (published in PDU 10/2024) (arXiv:1303.3787v4)
  - Deals with accurate slow-roll predictions for 287 models
  - Comes with a public runtime library ASPIC



- Computes the Hubble-flow functions from the model parameters $\boldsymbol{\theta}_{\mathrm{inf}}$

$$(\boldsymbol{\theta}_{\mathrm{inf}}, R_{\mathrm{rad}}) \longrightarrow \texttt{ASPIC} \longrightarrow \epsilon_i(\boldsymbol{\theta}_{\mathrm{inf}}, R_{\mathrm{rad}}) \equiv \frac{\mathrm{d}\ln|\epsilon_{i-1}|}{\mathrm{d}\ln a}, \quad \epsilon_0 \propto \frac{1}{H}$$

Primordial power spectra

- Uniquely determined from the Hubble-flow functions

$$
\begin{aligned}
\mathcal{P}_\zeta = \frac{H_*^2}{8\pi^2 M_{\mathrm{Pl}}^2 \epsilon_{1*}} \Bigg\{ & 1 - 2(1+C)\epsilon_{1*} - C\epsilon_{2*} + \left(\frac{\pi^2}{2} - 3 + 2C + 2C^2\right)\epsilon_{1*}^2 \\
& + \left(\frac{7\pi^2}{12} - 6 - C + C^2\right)\epsilon_{1*}\epsilon_{2*} + \left(\frac{\pi^2}{8} - 1 + \frac{C^2}{2}\right)\epsilon_{2*}^2 + \left(\frac{\pi^2}{24} - \frac{C^2}{2}\right)\epsilon_{2*}\epsilon_{3*} \\
& + \left[-2\epsilon_{1*} - \epsilon_{2*} + (2+4C)\epsilon_{1*}^2 + (-1+2C)\epsilon_{1*}\epsilon_{2*} + C\epsilon_{2*}^2 - C\epsilon_{2*}\epsilon_{3*}\right] \ln\left(\frac{k}{k_*}\right) \\
& + \left[2\epsilon_{1*}^2 + \epsilon_{1*}\epsilon_{2*} + \frac{1}{2}\epsilon_{2*}^2 - \frac{1}{2}\epsilon_{2*}\epsilon_{3*}\right] \ln^2\left(\frac{k}{k_*}\right) + \cdots \Bigg\},
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{P}_h = \frac{2H_*^2}{\pi^2 M_{\mathrm{Pl}}^2} \Bigg\{ & 1 - 2(1+C)\epsilon_{1*} + \left[-3 + \frac{\pi^2}{2} + 2C + 2C^2\right]\epsilon_{1*}^2 + \left[-2 + \frac{\pi^2}{12} - 2C - C^2\right]\epsilon_{1*}\epsilon_{2*} \\
& + \left[-2\epsilon_{1*} + (2+4C)\epsilon_{1*}^2 + (-2-2C)\epsilon_{1*}\epsilon_{2*}\right] \ln\left(\frac{k}{k_*}\right) + \left(2\epsilon_{1*}^2 - \epsilon_{1*}\epsilon_{1*}\right) \ln^2\left(\frac{k}{k_*}\right) + \cdots \Bigg\}
\end{aligned}
$$

- Currently known at third order: arXiv:2205.12608 (involves ϵ_{4*})

Qualitative Inflation

Quantitative inflation

❖ Encyclopædia Inflationaris: Opiparous Edition

❖ Primordial power spectra

❖ Reheating consistent model predictions

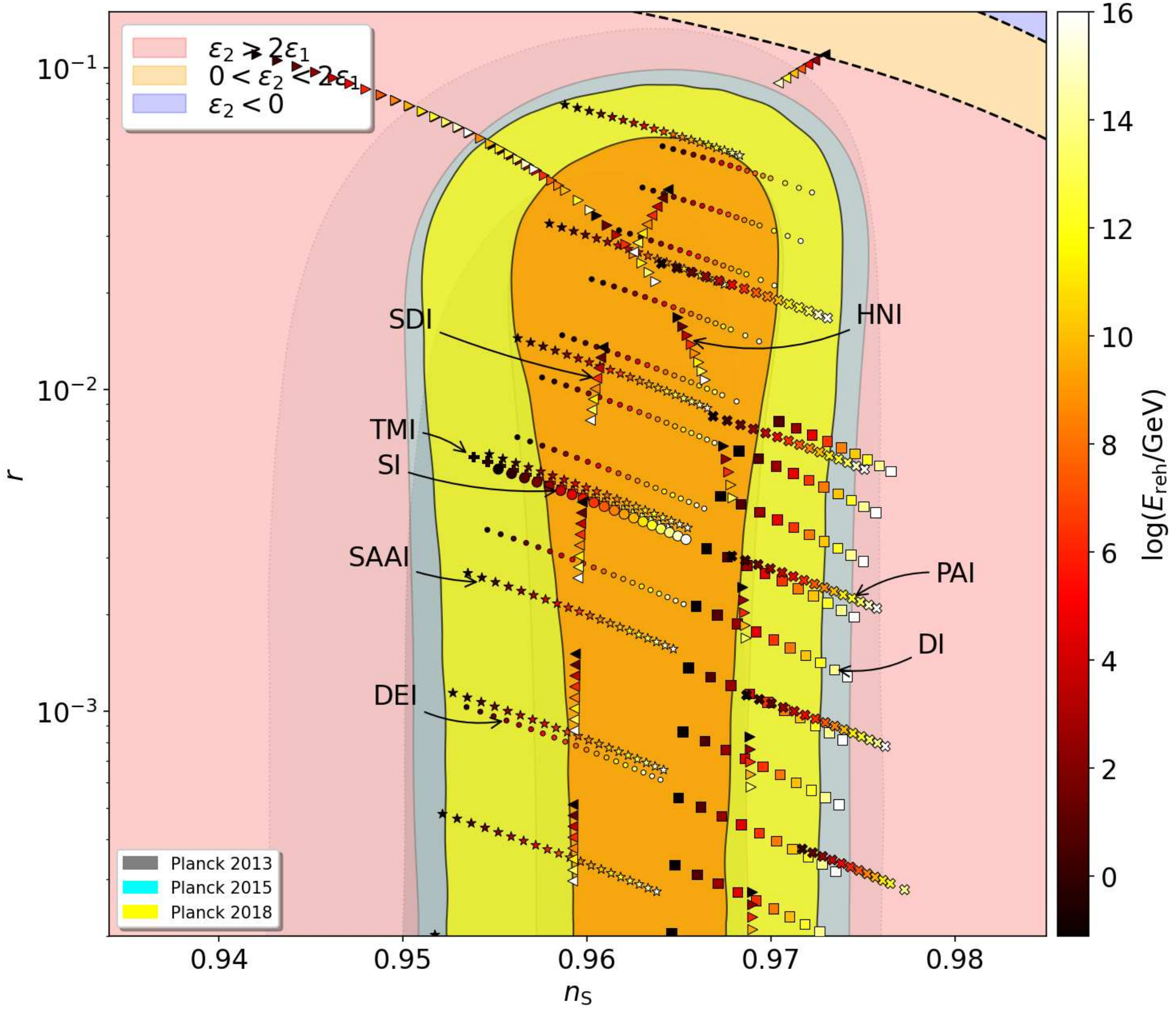
Bayesian inference

Predictions in model space

Conclusion

Reheating consistent model predictions

- Quick check for which reheating history a model is compatible with the data



Bayesian inference

Machine-learning an effective likelihood

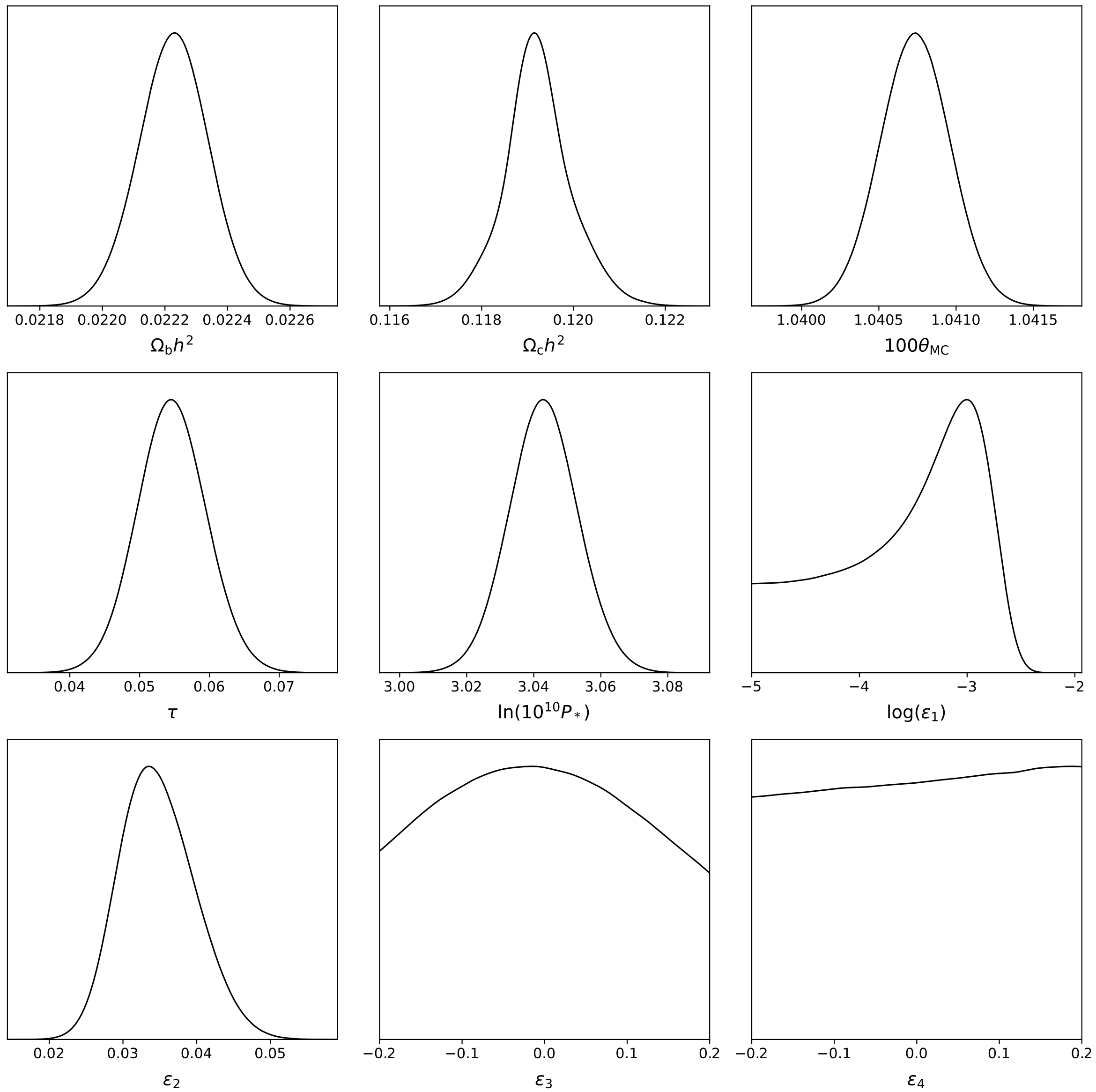
- To speed-up data analysis for 287 models

$$\mathcal{L}_{\mathrm{eff}}(\boldsymbol{D}|P_*,\varepsilon_1,\varepsilon_2,\varepsilon_3) \propto \int P(\boldsymbol{D}|\boldsymbol{\theta}_{\mathrm{s}},P_*,\varepsilon_1,\varepsilon_2,\varepsilon_3,\varepsilon_4)\,\pi(\varepsilon_4)\,\pi(\boldsymbol{\theta}_{\mathrm{s}})\,\mathrm{d}\varepsilon_4\mathrm{d}\boldsymbol{\theta}_{\mathrm{s}}.$$

- The full likelihood has 55 parameters and is built upon
  - Planck 2020 post-legacy TT, TE, EE data (`PR4/NPIPE` maps)
  - Large scale EE polarization (`lowE`)
  - BICEP/Keck B-mode 2018 (arXiv:2110.00483)
  - Small scale TE and EE from SPT-3G (arXiv:2103.13618)
  - Baryon Acoustic Oscillations (SDSS collaboration)
- MCMC exploration of the 55 dimensions
  - `COSMOMC` up to 25 million samples ($R-1<10^{-3}$)
  - `GetDist` marginalization in 4D: $P_*,\varepsilon_1,\varepsilon_2,\varepsilon_3$
- Basic machine learning: 1 hidden layer with 300 nodes

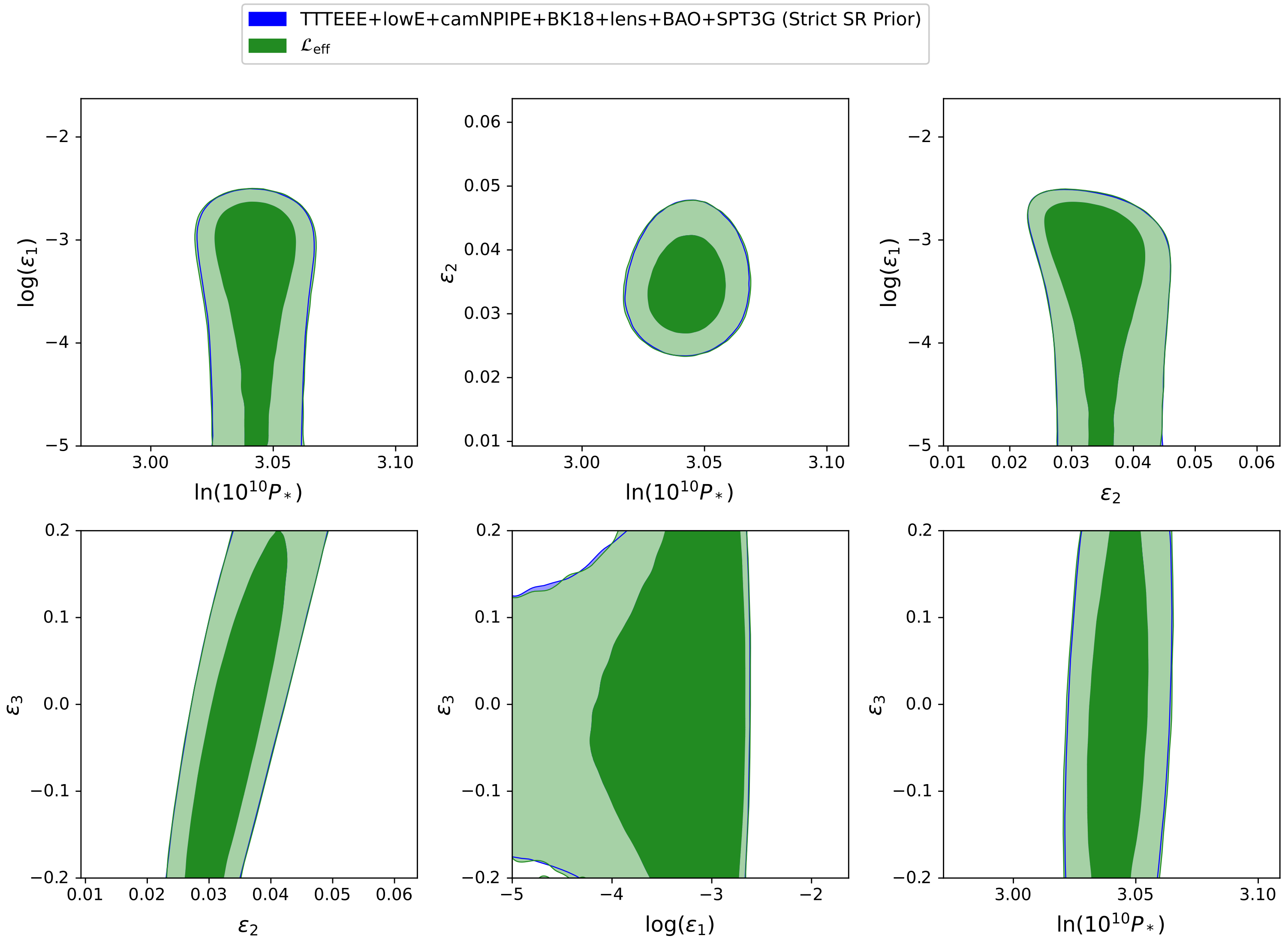
Marginalized posteriors

- Cosmo and Hubble-flow parameters



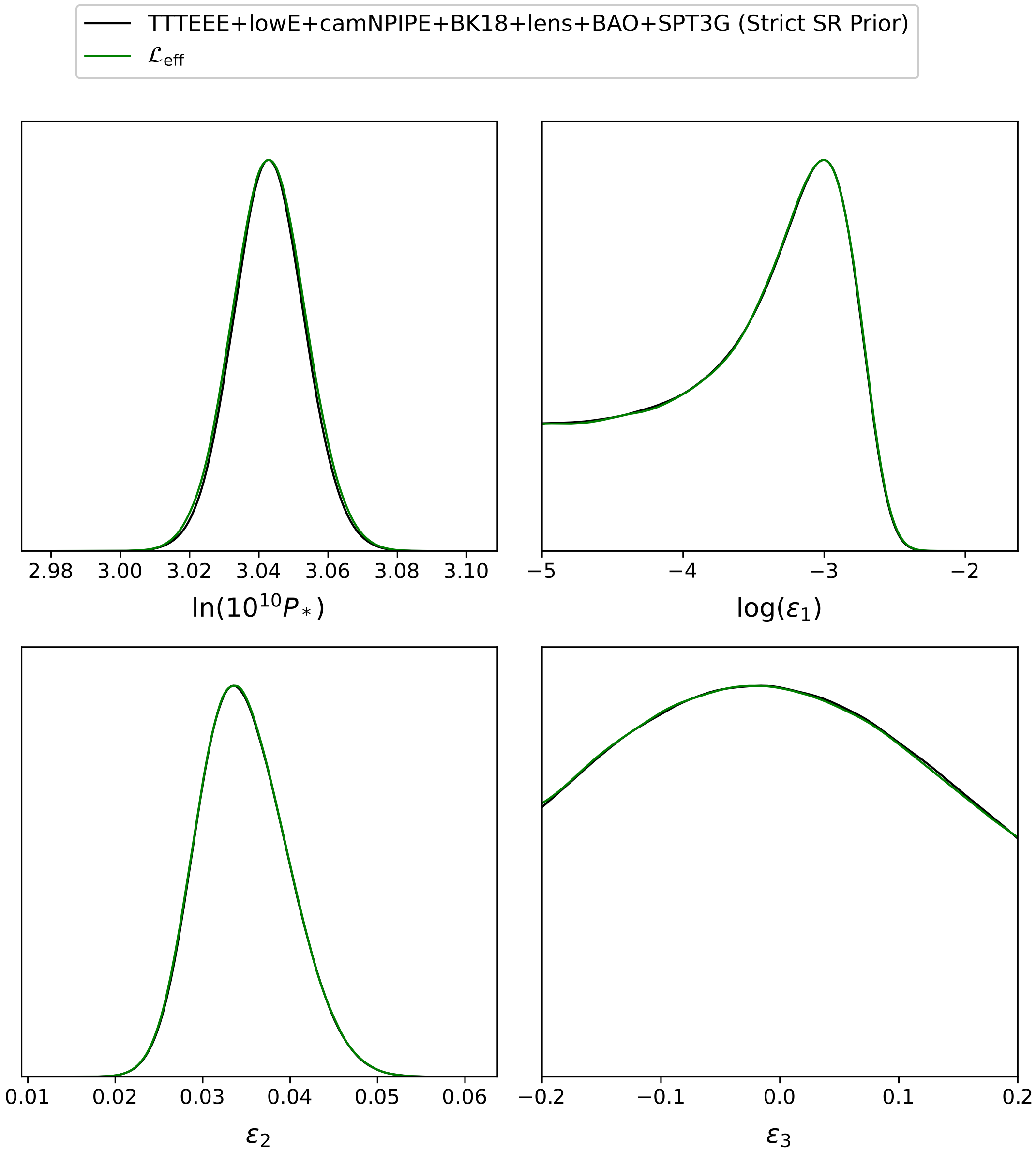
Marginalized posteriors

- Effective likelihood vs exact: 2D



Marginalized posteriors

- Effective likelihood vs exact: 1D



Computing bayesian evidences

- Probability of a model $\mathcal{M}$ to explain the data $\boldsymbol{D}$

$$P(\mathcal{M}|\boldsymbol{D}) = \frac{\mathcal{E}(\boldsymbol{D}|\mathcal{M})\, P(\mathcal{M})}{P(\boldsymbol{D})}$$

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- Bayesian evidence

$$\mathcal{E}(\boldsymbol{D}|\mathcal{M}) \propto \int \mathcal{L}_{\mathrm{eff}}(\boldsymbol{D}|P_*, \varepsilon_1, \varepsilon_2, \varepsilon_3)\, \pi(\boldsymbol{\theta}_{\mathrm{inf}}, R_{\mathrm{rad}})\, \mathrm{d}\boldsymbol{\theta}_{\mathrm{inf}}\, \mathrm{d}R_{\mathrm{rad}}$$

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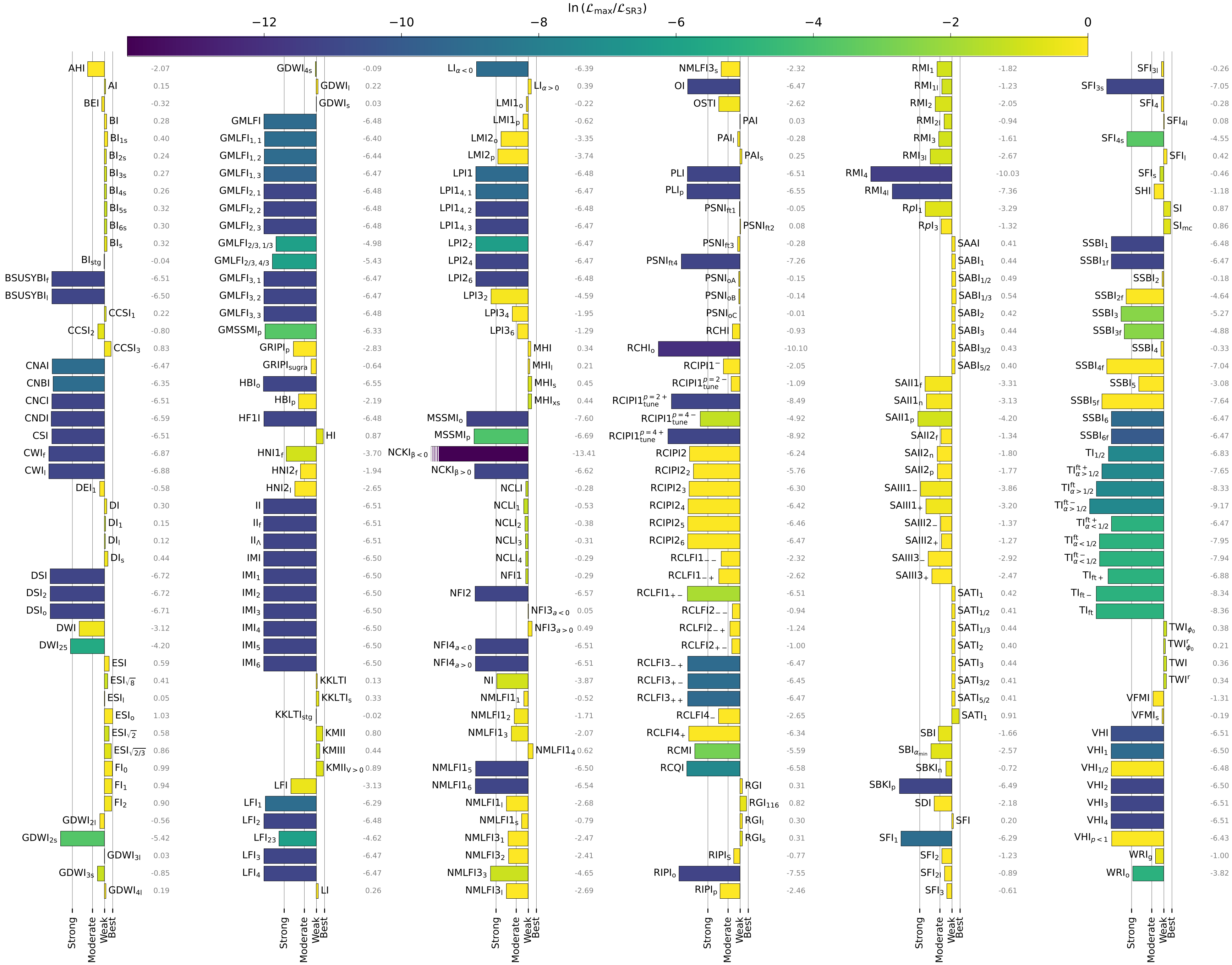
- Computed with `BAYASPIC` running over 287 models
  - ✦ $\texttt{BAYASPIC} \equiv \texttt{ASPIC} + \texttt{PolyChord} + \mathcal{L}_{\mathrm{eff}}$
  - ✦ A few cpu-hours per model $\mathcal{M}$ (but up to 2 days for some)
- $1\,\mathrm{TB}$ of data output (nested chains, posteriors, plots,. . .)

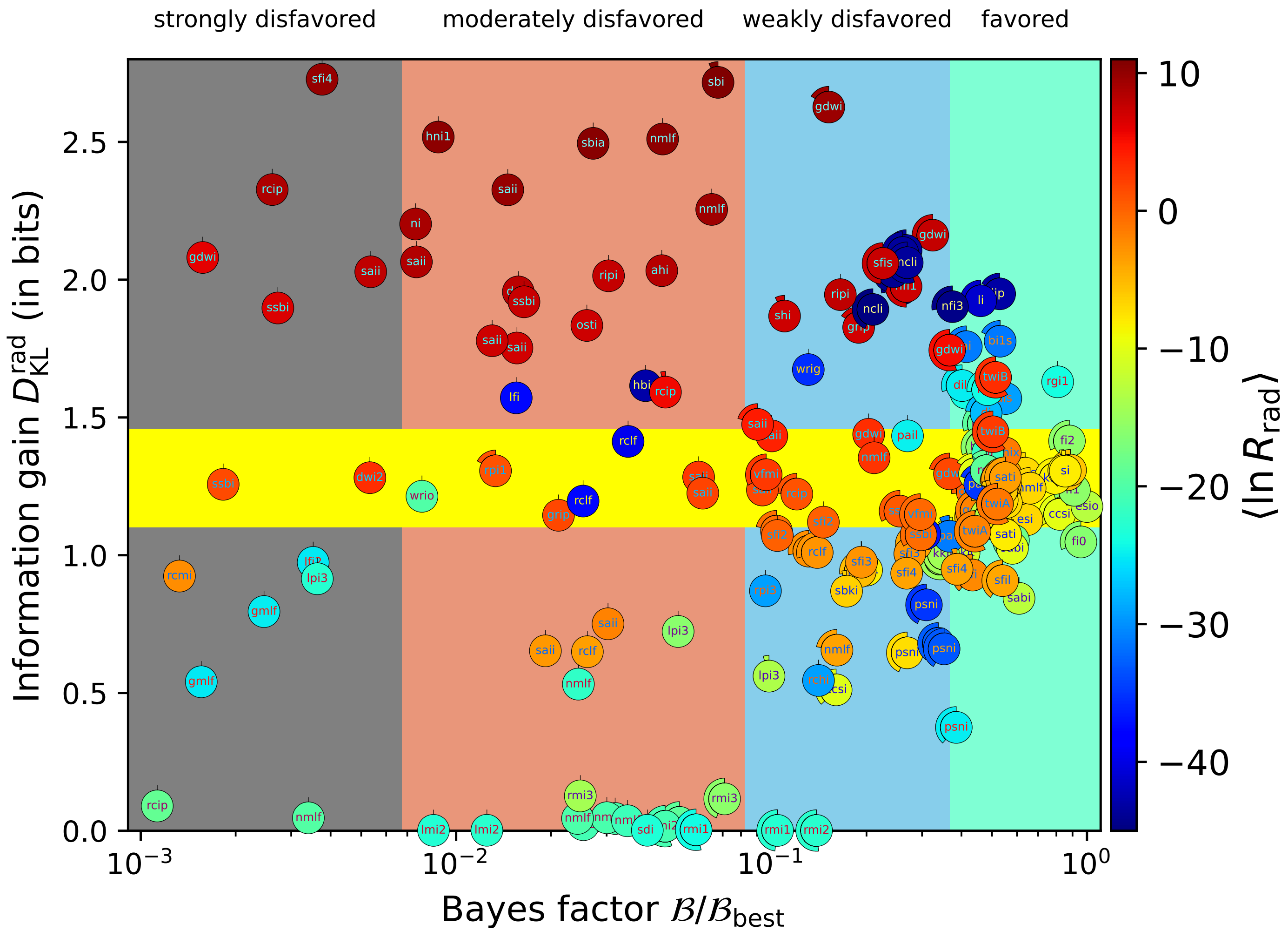
Bayes factors for all models



Information gain on the reheating

- Kullback-Leibler divergence between the prior and posterior

$$D_{\mathrm{KL}}^{\mathrm{rad}} = \int P(\ln R_{\mathrm{rad}}|\boldsymbol{D}) \ln\left[\frac{P(\ln R_{\mathrm{rad}}|\boldsymbol{D})}{\pi(\ln R_{\mathrm{rad}})}\right] \mathrm{d}\ln R_{\mathrm{rad}},$$



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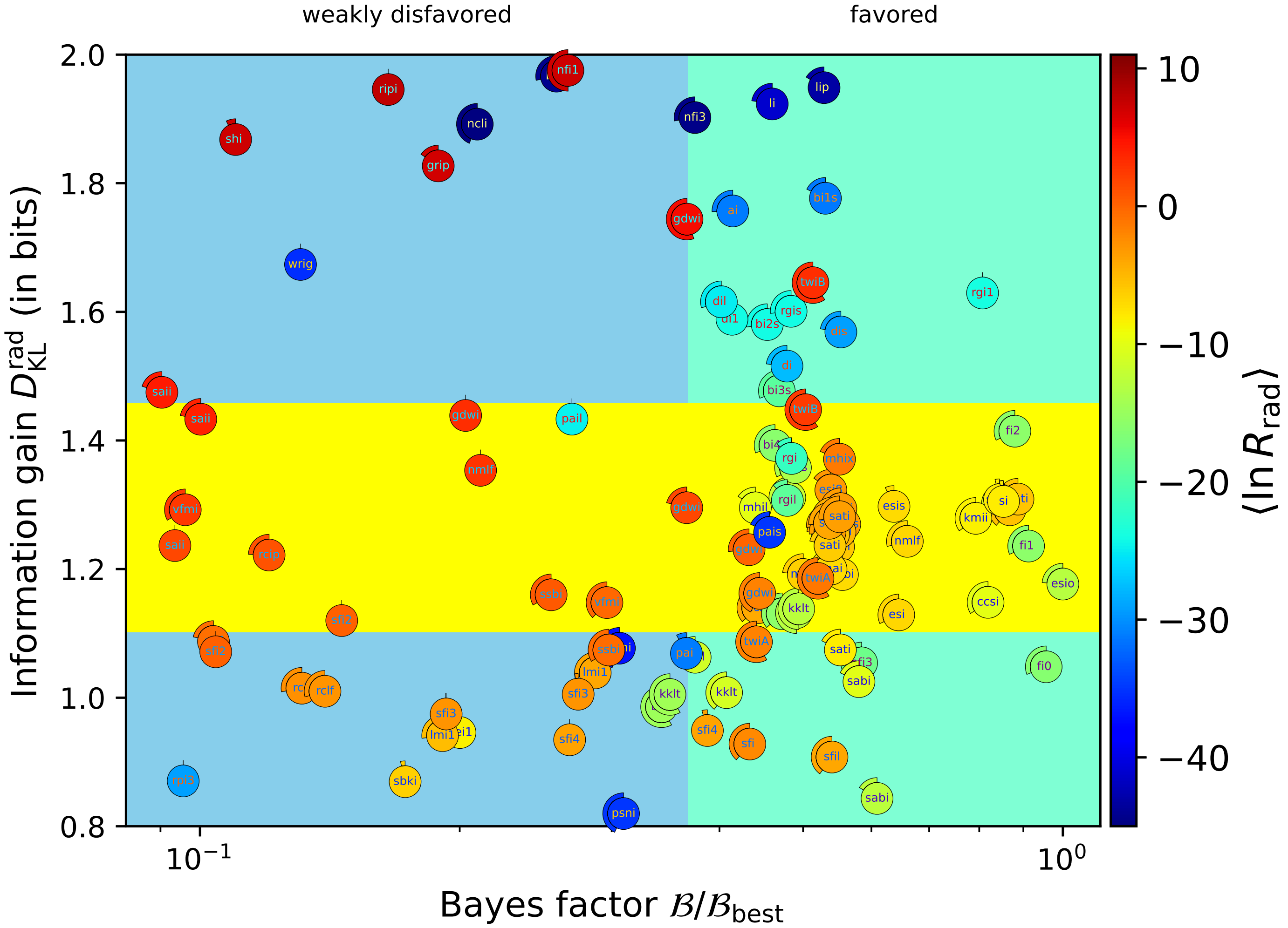

Predictions in model space

Running of the spectral index

- Within the space of the single-field models $\mathcal{I}_{\mathrm{mod}} \equiv \{\mathcal{M}_i\}$
- Posterior probability of the running $\alpha_{\mathrm{S}} \simeq -\epsilon_{2*}\left(2\epsilon_{1*} + \epsilon_{3*}\right)$

$$P(\alpha_{\mathrm{S}}|\boldsymbol{D}, \mathcal{I}_{\mathrm{mod}}) = \sum_i P(\alpha_{\mathrm{S}}|\boldsymbol{D}, \mathcal{M}_i)\, P(\mathcal{M}_i|\boldsymbol{D})$$

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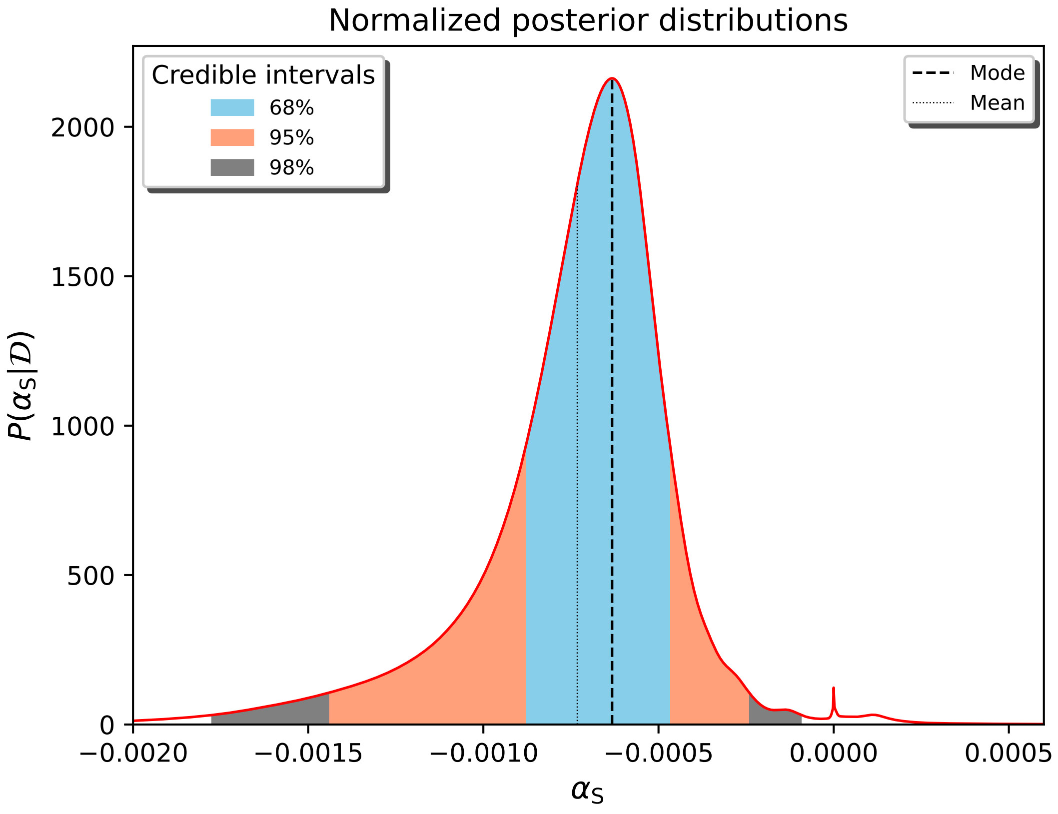

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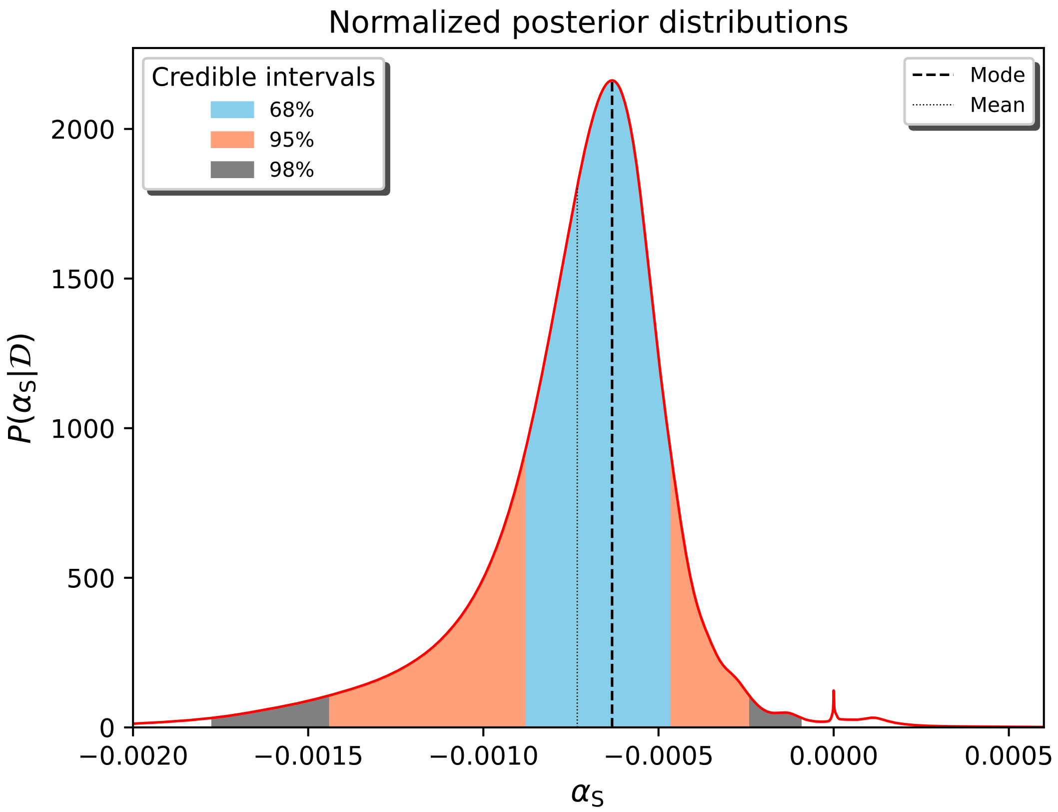

- Credible interval: $-1.8 \times 10^{-3} < \alpha_{\mathrm{S}} < -9.1 \times 10^{-5} \quad (98\%)$

Model space vs slow-roll space

- Non-trivial prediction coming from both theoretical prior + data
- The sign of α_{s} remains undetermined otherwise

Model space vs slow-roll space

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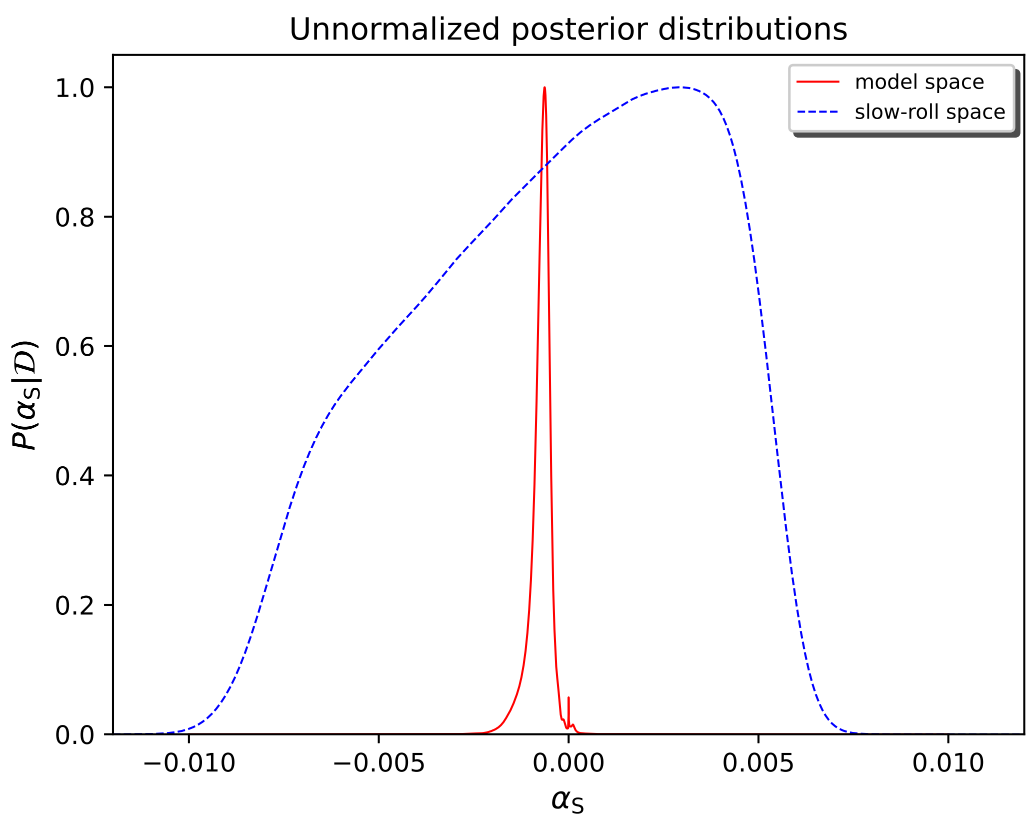
  - ✦ Posterior by assuming "just slow-roll" (no model) + data



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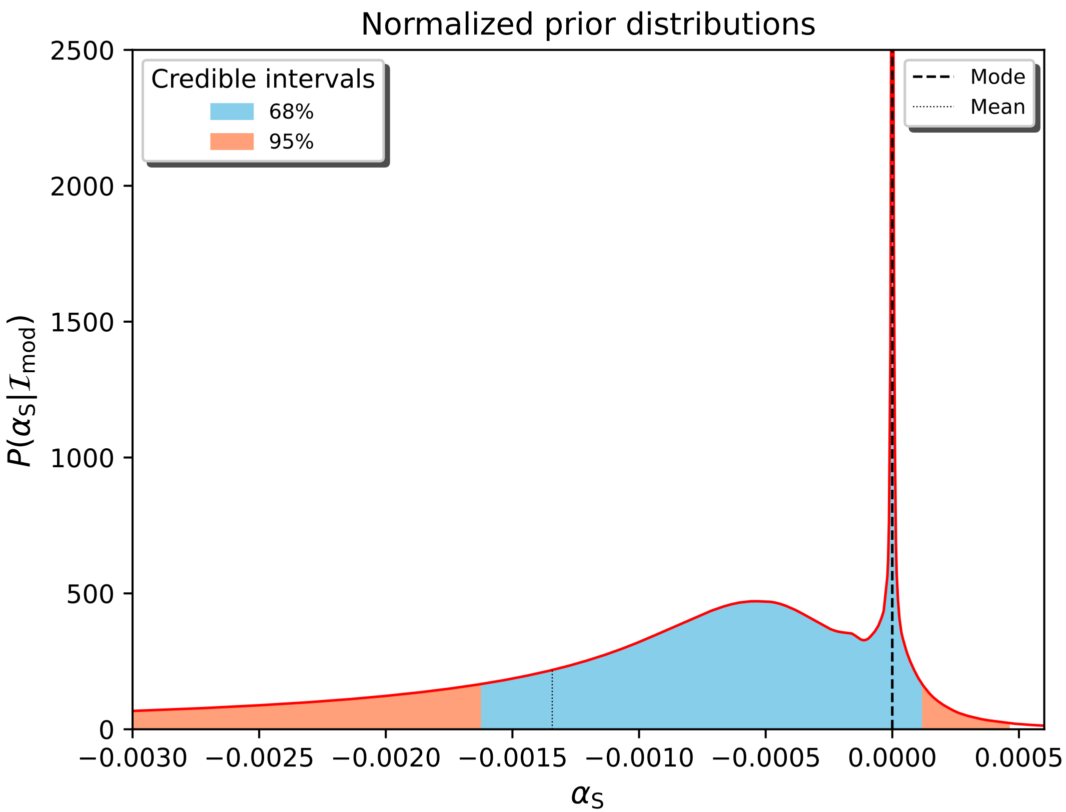
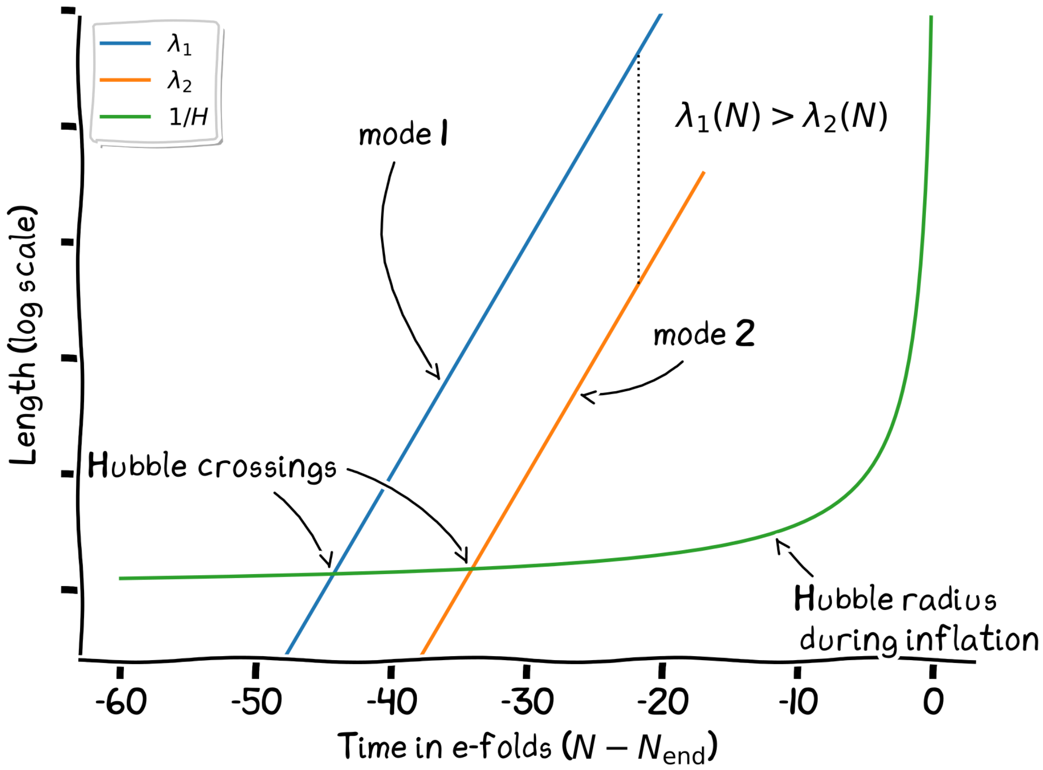
  - ✦ Model space prior (no data input)



Model space vs slow-roll space

- Non-trivial prediction coming from both theoretical prior + data
- The sign of α_s remains undetermined otherwise
- The field evolution in single-field models creates a correlation between the sign of $n_s - 1$ and the sign of the running α_s!



✦ Models with $n_s \lesssim 1$ have all an accelerated Hubble radius in the observable window $\Longrightarrow$ $\alpha_s < 0$

Reheating energy density and equation of state

- Posterior probability for $(\ln \rho_{\rm reh}, \overline{w}_{\rm reh})$

$$P(\ln \rho_{\rm reh}, \overline{w}_{\rm reh} | \boldsymbol{D}, \mathcal{I}_{\rm mod}) = \sum_i P(\ln \rho_{\rm reh}, \overline{w}_{\rm reh} | \boldsymbol{D}, \mathcal{M}_i)\, P(\mathcal{M}_i | \boldsymbol{D})$$

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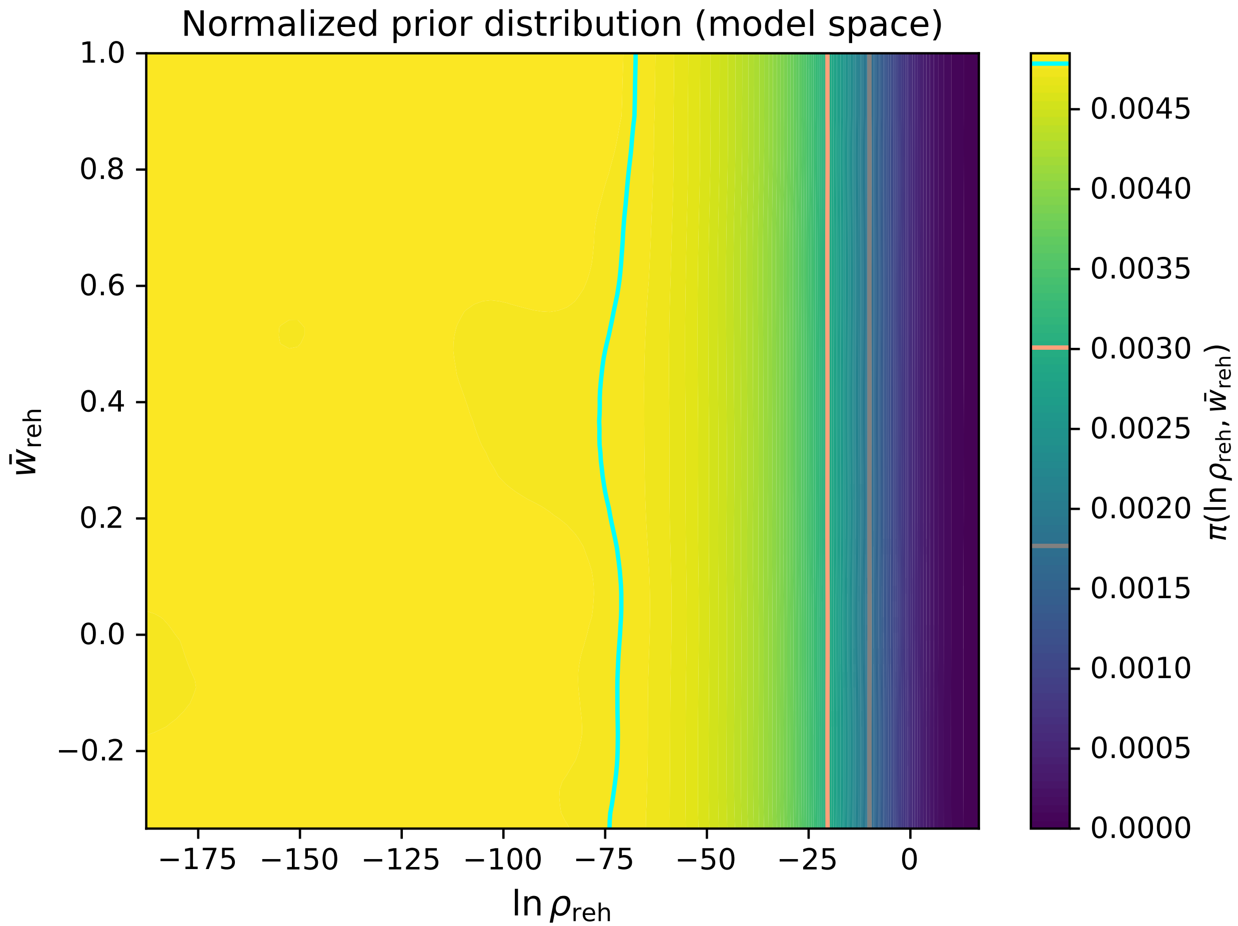

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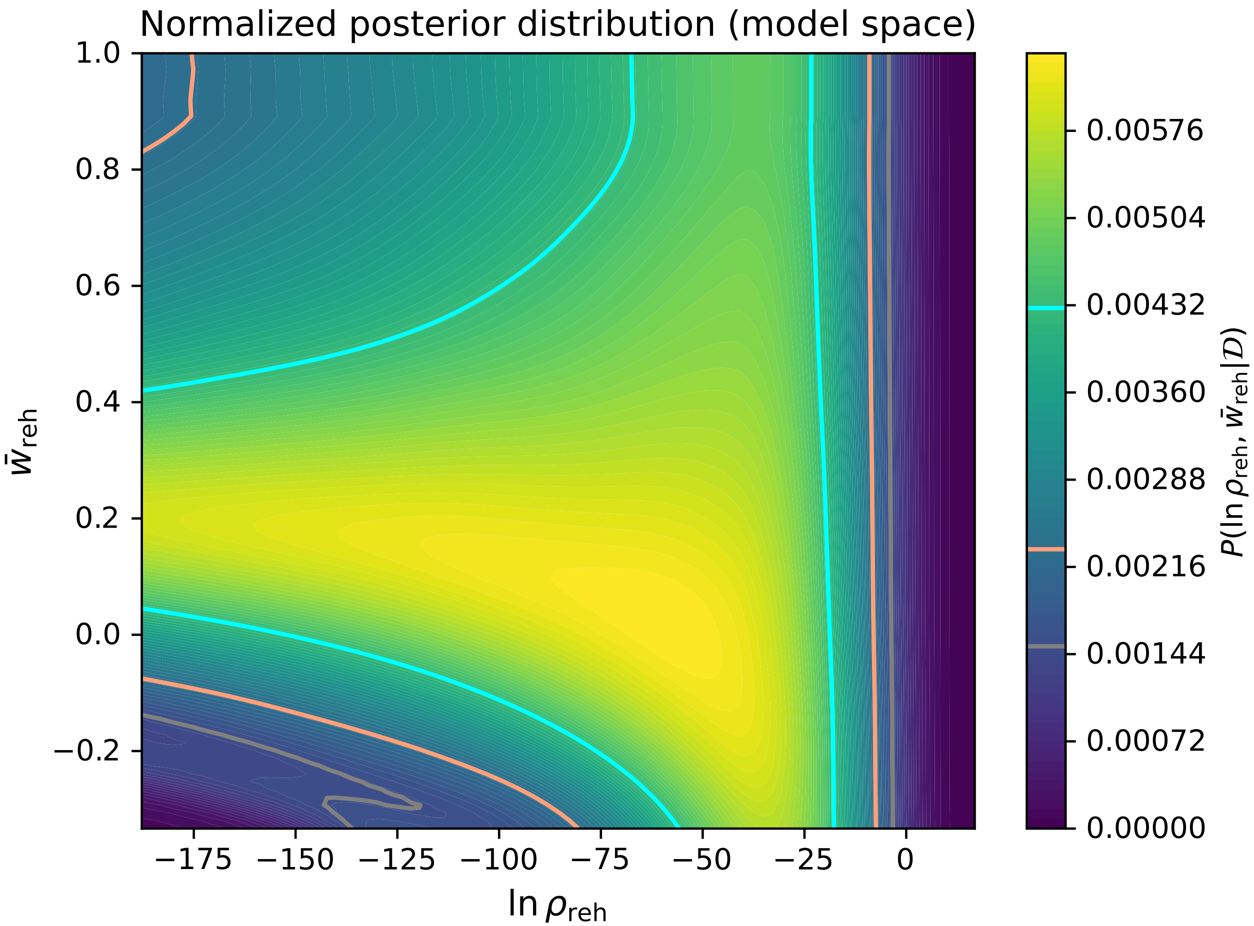
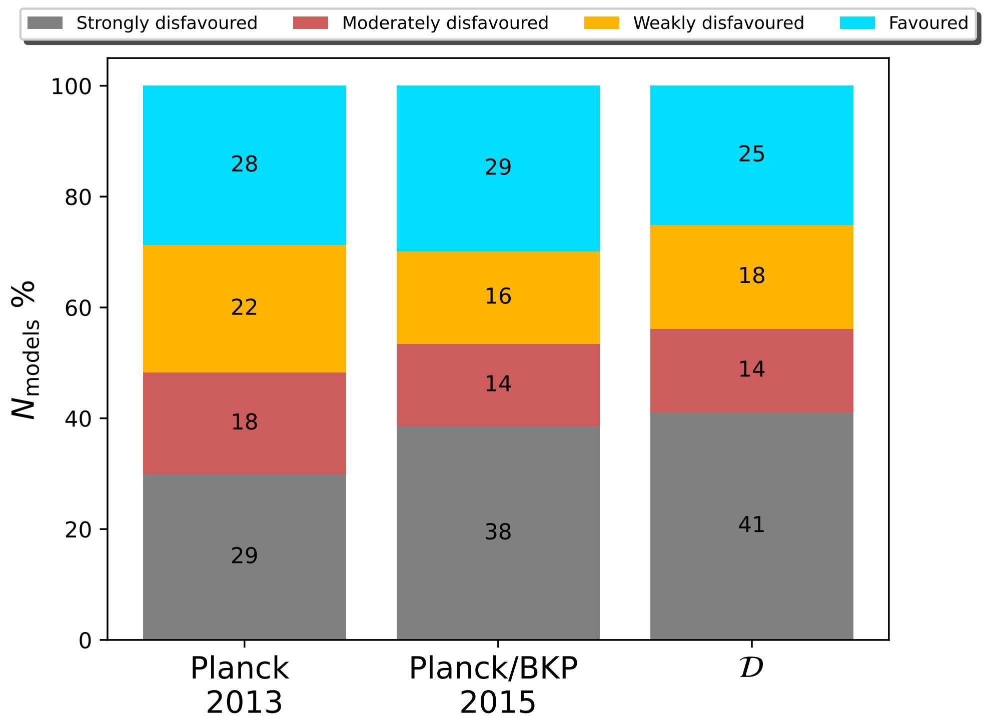


Conclusion

- Bayesian data analysis in model space $\mathcal{I}_{\mathrm{mod}}$
  - Enforces model consistency + new insights on the reheating era
  - Predicts: $\langle \alpha_{\mathrm{S}} \rangle = -7.3 \times 10^{-4}$
- Data constraining power is winning against theoretical proposals



- Looking forward to the Euclid, LSS & CMB-S4 data!