Relativity of the event: examples in JT gravity

A. Taskov[1] F. Nitti[2] F. Piazza[1]

[1]CPT Marseille

[2]APC Paris

2402.01847

Gauge Fixing

- QG $\to$ observables defined asymptotically
-
-
-

Gauge Fixing

- QG $\rightarrow$ observables defined asymptotically
- local analysis: gauge choices

$$ds^2 = \Omega^2(-dt^2 + dz^2)$$
$$ds^2 = -dt'^2 + f^2\ dz'^2$$

-
-

Gauge Fixing

- QG $\rightarrow$ observables defined asymptotically
- local analysis: gauge choices

$$ds^2 = \Omega^2(-dt^2 + dz^2)$$
$$ds^2 = -dt'^2 + f^2 \, dz'^2$$

- coord. change

$$x' = x'(x; \Omega)$$

-

Gauge Fixing

- QG $\rightarrow$ observables defined asymptotically
- local analysis: gauge choices

$$ds^2 = \Omega^2(-dt^2 + dz^2)$$
$$ds^2 = -dt'^2 + f^2\ dz'^2$$

- coord. change

$$x' = x'(x;\Omega)$$

- superposition of geometry

$$|\psi\rangle = \sum_i \psi_i \,|\Omega_i\rangle \rightarrow \mathrm{Var}(\hat{x}') \neq 0$$

Gauge Fixing

- fix gauge with observers

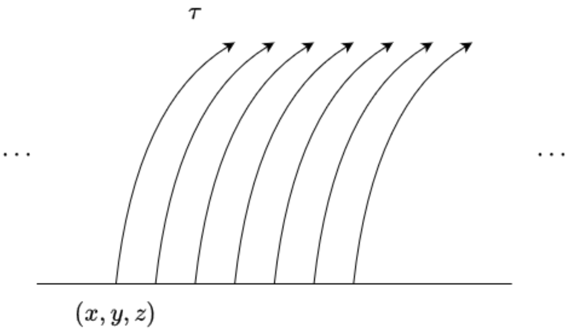


Figure 1: Alice coordinates. Alice is in free fall.

Gauge Fixing

- fix gauge with observers
- position label = observer, time label = proper time

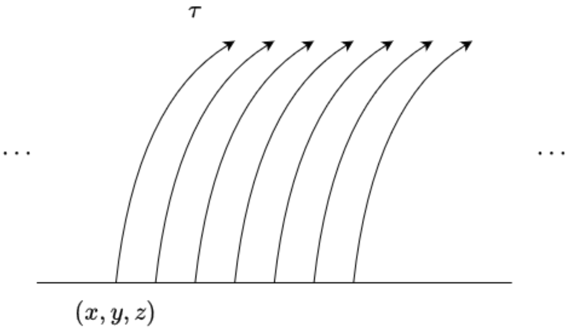


Figure 2: Alice coordinates. Alice is in free fall.

Gauge Fixing

- fix gauge with observers
- position label = observer, time label = proper time

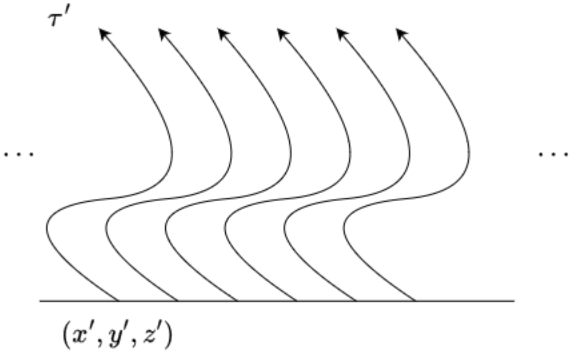


Figure 3: Bob coordinates. Bob has rocket boosters.

Gauge Fixing

- Consider a single Alice

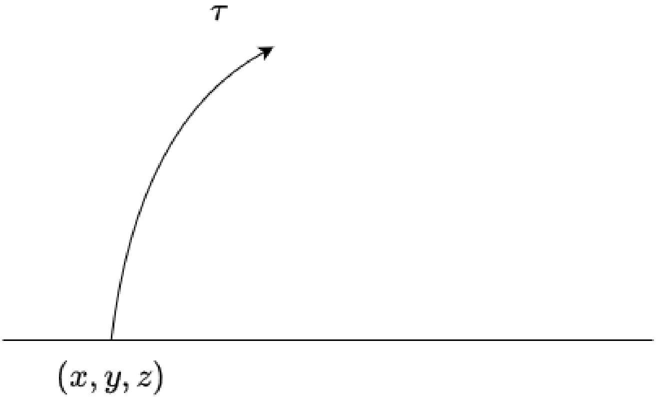


Figure 4: Alice labels a point (x, y, z, τ)

Gauge Fixing

- Consider a single Alice
- What is Bob's label for Alice's point?

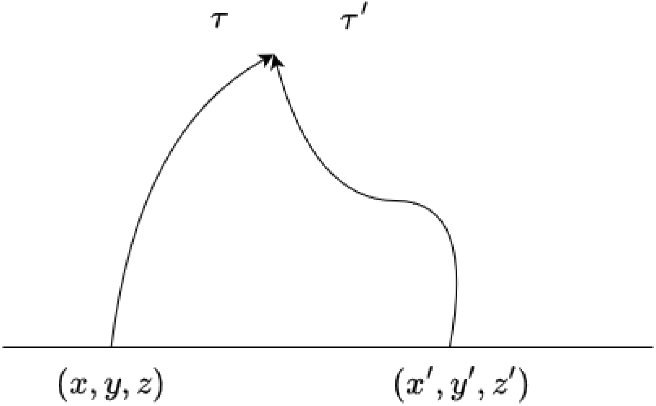


Figure 5: Bob labels Alice's point (x', y', z', τ')

Gauge Fixing

- Consider a single Alice
- What is Bob's label for Alice's point?
- Quantum mechanically...

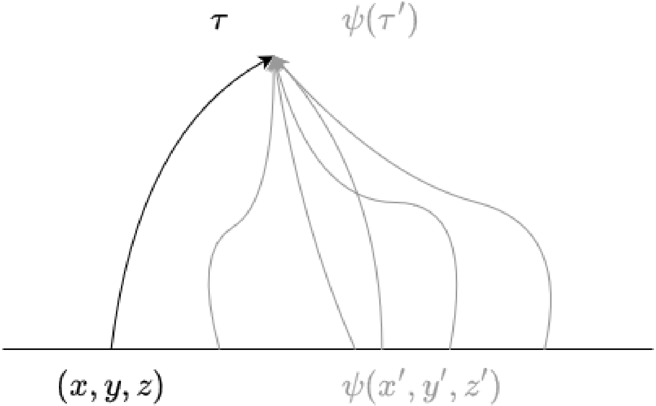


Figure 6: Spacetime in a superposition: Bob's label choice is probabilistic

Strategy

- Solvable quantum gravity theory
- Quantum state
- Observer coordinate systems (“localization“)

JT Gravity

- JT gravity path integral

$$\int \mathcal{D}g\mathcal{D}\phi \; e^{\frac{i}{16\pi G}\int d^2x\sqrt{-g}(R+2)\phi+\frac{i}{8\pi G}\int dx\sqrt{-\gamma}(K-1)\phi}$$

-
-

JT Gravity

- JT gravity path integral

$$\int \mathcal{D}g\mathcal{D}\phi \; e^{\frac{i}{16\pi G}\int d^2x\sqrt{-g}(R+2)\phi+\frac{i}{8\pi G}\int dx\sqrt{-\gamma}(K-1)\phi}$$

$$= \int \mathcal{D}g \; \delta(R+2) \; e^{\frac{i}{8\pi G}\int dx\sqrt{-\gamma}(K-1)\phi}$$

-
-

JT Gravity

- JT gravity path integral

$$\int \mathcal{D}g\mathcal{D}\phi \; e^{\frac{i}{16\pi G}\int d^2x\sqrt{-g}(R+2)\phi+\frac{i}{8\pi G}\int dx\sqrt{-\gamma}(K-1)\phi}$$

$$= \int \mathcal{D}g \; \delta(R+2) \; e^{\frac{i}{8\pi G}\int dx\sqrt{-\gamma}(K-1)\phi}$$

- dilaton $\rightarrow$ symmetry breaking
-

JT Gravity

- JT gravity path integral

$$\int \mathcal{D}g\mathcal{D}\phi \; e^{\frac{i}{16\pi G}\int d^2x\sqrt{-g}(R+2)\phi + \frac{i}{8\pi G}\int dx\sqrt{-\gamma}(K-1)\phi}$$

$$= \int \mathcal{D}g \; \delta(R+2) \; e^{\frac{i}{8\pi G}\int dx\sqrt{-\gamma}(K-1)\phi}$$

- dilaton $\rightarrow$ symmetry breaking
- Poincaré coordinates (T,Z), boundary time t

$$ds^2 = \frac{-dT^2 + dZ^2}{Z^2} \qquad T|_{\rm bdy} = T_b(t)$$

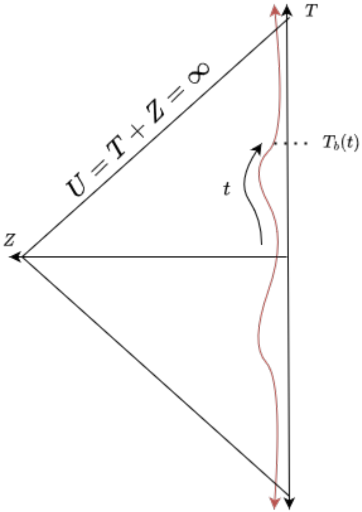
JT Gravity

$$\int \prod_t \frac{dT_b(t)}{T_b(t)} \; e^{-\frac{i\phi_b}{32\pi G}\int dt\{T_b(t),t\}}$$

$$\{T_b(t),t\} := \frac{\dddot{T}_b}{\dot{T}_b} - \frac{3}{2}\left(\frac{\ddot{T}_b}{\dot{T}_b}\right)^2$$

JT Gravity

- For visualization, introduce a regulator $\delta \to 0$
$$T|_\delta = T_b(t)$$
-
-

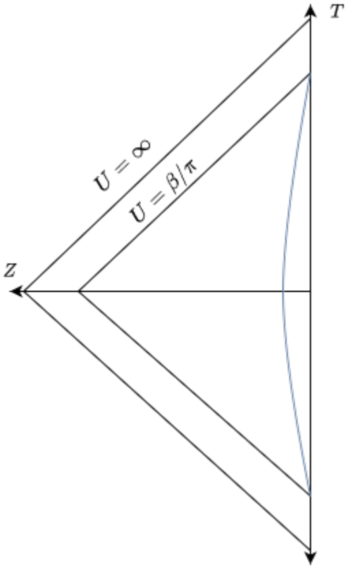


JT Gravity

- For visualization, introduce a regulator $\delta \to 0$
$$T|_\delta = T_b(t)$$

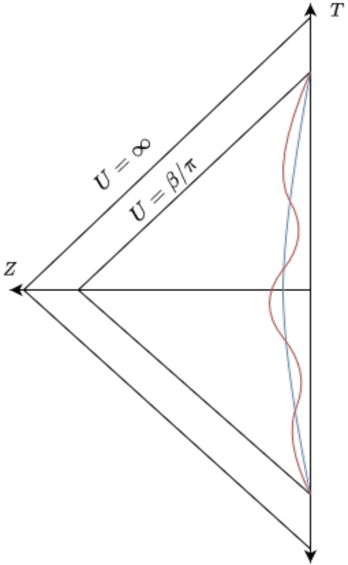
- Classical black hole solution
$$T_b(t) = \frac{\beta}{\pi} \tanh\left(\frac{\pi t}{\beta}\right)$$

-



JT Gravity

- For visualization, introduce a regulator $\delta \to 0$
$$T|_\delta = T_b(t)$$
- Classical black hole solution
$$T_b(t) = \frac{\beta}{\pi} \tanh\left(\frac{\pi t}{\beta}\right)$$
- Quantum parameter
$$\sqrt{\beta G/\phi_b}$$



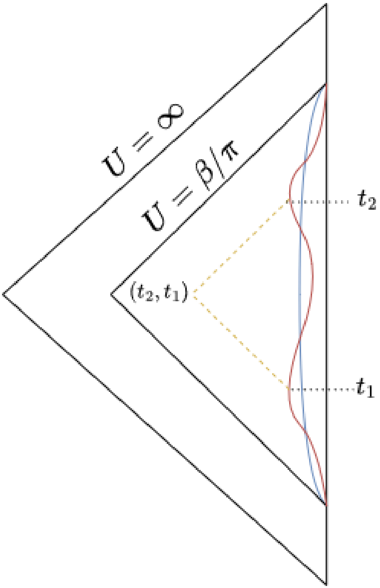
BMV Localization

- Blommaert, Mertens, Verschelde localization (1902.11194)

$$U = T + Z = T_b(t_2)$$

$$V = T - Z = T_b(t_1)$$

-



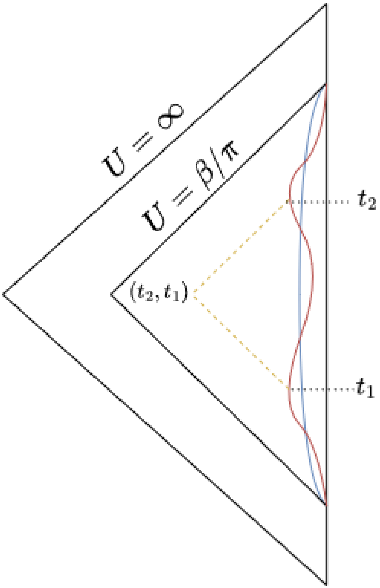
BMV Localization

- Blommaert, Mertens, Verschelde localization (1902.11194)

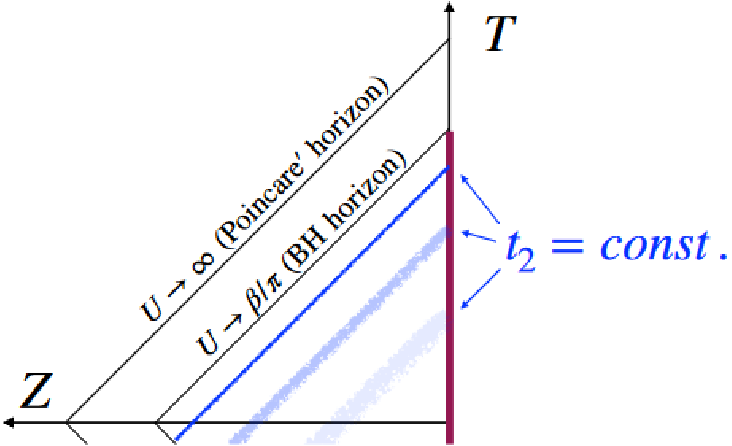
$$U = T + Z = T_b(t_2)$$
$$V = T - Z = T_b(t_1)$$

- $t_2 = \text{const.}$ surface in (U, V)?

$$\text{Var}(U) \propto \frac{\beta^3 G}{\phi_b \left(\cosh \frac{\pi t_2}{\beta}\right)^4} \xrightarrow{t_2 \to \infty} 0$$

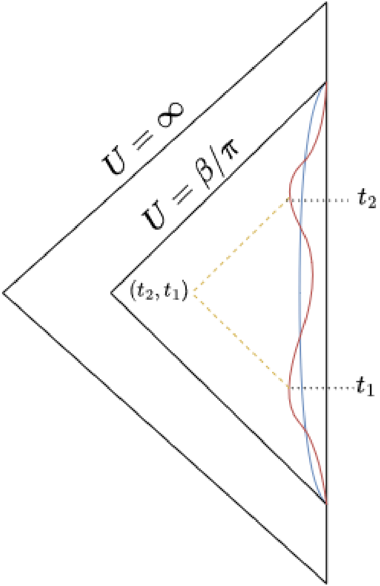


BMV Localization



BMV Localization

- Horizon smearing?
-



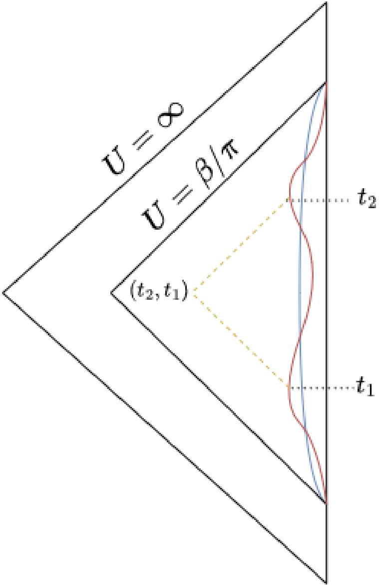
BMV Localization

- Horizon smearing?

  $\langle T_b \rangle |_{t_2 \to \infty} = \beta/\pi$

  $\mathrm{Var}(T_b)|_{t_2 \to \infty} = 0$

-



BMV Localization

- Horizon smearing?

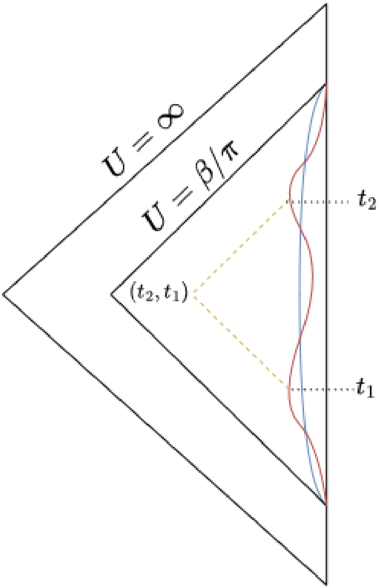
$$\langle T_b \rangle |_{t_2 \to \infty} = \beta/\pi$$

$$\text{Var}(T_b)|_{t_2 \to \infty} = 0$$

- Even more generally:

$$ds^2 = \frac{-4dUdV}{(U-V)^2} dUdV$$

$$= \frac{-4\dot{T}_b(t_1)\dot{T}_b(t_2)}{(T_b(t_1) - T_b(t_2))^2} dt_1 dt_2$$



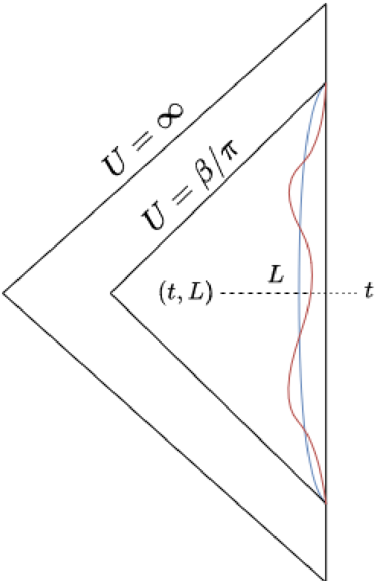
Spacelike Localization

- Spacelike localization

$$T = T_b(t)$$

$$Z = Z_b(t)e^L$$

-
-



Spacelike Localization

- Spacelike localization

$$T = T_b(t)$$

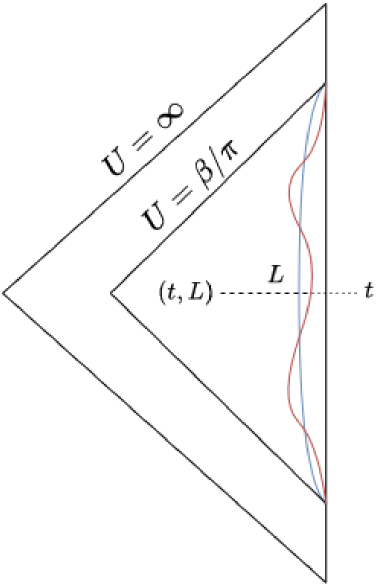
$$Z = Z_b(t)e^L$$

- Is the horizon sharp?

$$L_{\text{hor}} = \log\left(\frac{\frac{\beta}{\pi} - T_b}{\dot{T}_b}\right) - \log\delta$$

$$\text{Var}(L_{\text{hor}}) \propto \frac{\beta}{\phi_b}\left(a + \frac{1}{(1+e^{2\pi t/\beta})^2}\right)$$

-



Spacelike Localization

- Spacelike localization

$$T = T_b(t)$$

$$Z = Z_b(t)e^L$$

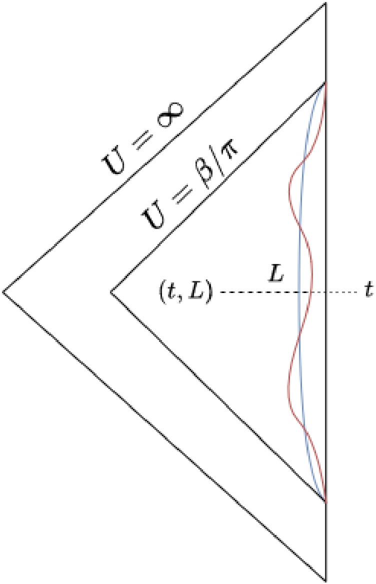
- Is the horizon sharp?

$$L_{\text{hor}} = \log\left(\frac{\frac{\beta}{\pi} - T_b}{\dot{T}_b}\right) - \log\delta$$

$$\text{Var}(L_{\text{hor}}) \propto \frac{\beta G}{\phi_b}\left(a + \frac{1}{(1+e^{2\pi t/\beta})^2}\right)$$

- Infinitesimal causal structure

$$\frac{dL}{dt} = -\left(\frac{\ddot{T}_b}{\dot{T}_b} \pm e^{-L}\right)$$



Conclusion

- Summary and consequences
  - horizon is not sharp in all localizations
  - change of gauge is probabilistic
  - local analysis $\rightarrow$ be careful about gauge changes!
-

Conclusion

- Summary and consequences
  - horizon is not sharp in all localizations
  - change of gauge is probabilistic
  - local analysis $\rightarrow$ be careful about gauge changes!
- Remarks and extensions
  - more physical frames?
  - relation to operator dressing?
  - non-commuting coordinates? (JT spacelike coordinates...)

Thank you :)

buffer frame

Black hole in JT

$$T_{b,BH}(t) = \frac{\beta}{\pi} \tanh\left(\frac{\pi t}{\beta}\right)$$

$$Z_{b,BH}(t) = \frac{\delta}{\cosh(\pi t/\beta)^2}$$

$$T_b(t) = T_{b,BH}(t + \epsilon(t))$$

$$\begin{aligned} I &= -\frac{\phi_b}{32\pi G} \int dt \{ T_{b,BH}(t + \epsilon(t)), t \} \\ &= -\frac{\phi_b}{16\pi G} \int dt \left(\ddot{\epsilon}^2 + \frac{4\pi}{\beta} \dot{\epsilon}^2 \right) \end{aligned}$$

JT Gravity

$$I = \frac{1}{16\pi G}\int d^2x\sqrt{-g}(R+2)\phi + \frac{1}{8\pi G}\int dx\sqrt{-\gamma}(K-1)\phi$$

$$(t,z) \qquad g_{tt} = -\frac{1}{z^2} + \dots \qquad \phi = \frac{\phi_b}{z} + \dots$$

$$(T,Z) \qquad ds^2 = \frac{-dT^2 + dZ^2}{Z^2}$$

$$z = \delta \qquad (T_b(t), Z_b(t)) \qquad \delta \to 0$$

$$g_{tt} = -\frac{1}{z^2} \quad \to \quad Z_b(t) = \delta\, T_b'(t) + \dots$$

JT Gravity

$$\int \mathcal{D}g\mathcal{D}\phi \; e^{\frac{i}{16\pi G}\int d^2x\sqrt{-g}(R+2)\phi+\frac{i}{8\pi G}\int dx\sqrt{-\gamma}(K-1)\phi}$$

$$= \int \mathcal{D}g \; \delta(R+2) \; e^{\frac{i}{8\pi G}\int dx\sqrt{-\gamma}(K-1)\phi}$$

$$= \int \prod_t \frac{dT_b(t)}{T_b(t)} \; e^{-\frac{i\phi_b}{32\pi G}\int dt\{T_b(t),t\}}$$

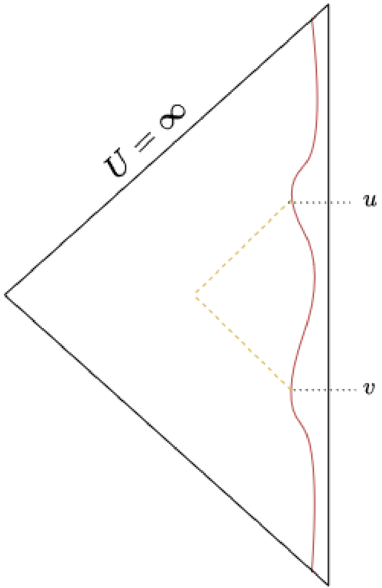
$$\{T_b(t),t\} = \frac{T_b'''}{T_b'} - \frac{3}{2}\left(\frac{T_b''}{T_b'}\right)^2$$

BMV Localization

- Blommaert, Mertens, Verschelde localization

$$U = T + Z = T_b(u)$$

$$V = T - Z = T_b(v)$$



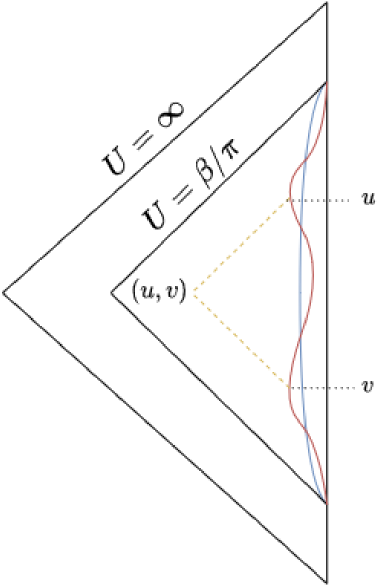
BMV Localization

- Blommaert, Mertens, Verschelde localization

$$U = T + Z = T_{b,BH}(u + \epsilon(u))$$

$$V = T - Z = T_{b,BH}(v + \epsilon(v))$$

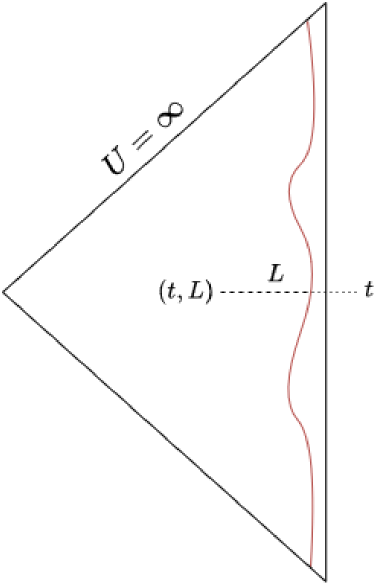
$$\mathrm{Var}(T_b) \propto \frac{\beta^3}{\phi_b \left(\cosh \frac{\pi u}{\beta}\right)^4} \xrightarrow{u\to\infty} 0$$



Spacelike Localization

- Spacelike localization

$$T = T_b(t)$$
$$Z = Z_b(t)e^L$$



Localizations in JT

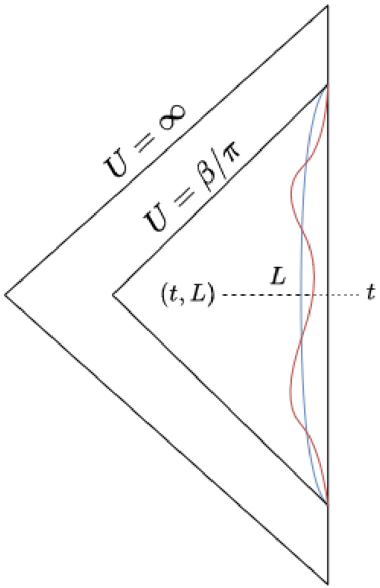
- Spacelike localization

$$T = T_{b,BH}(t+\epsilon(t))$$

$$Z = Z_{b,BH}(t+\epsilon(t))e^{L}$$

$$L_{\text{hor}} = \log\left(\frac{\frac{\beta}{\pi} - T_b}{\dot{T}_b}\right) - \log\delta$$

$$\text{Var}(L_{\text{hor}}) \propto \frac{\beta}{\phi_b}\left(a + \frac{1}{(1+e^{2\pi t/\beta})^2}\right)$$

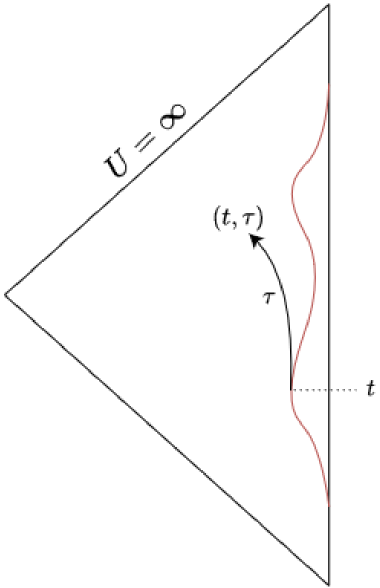


Localizations in JT

- Timelike localization

$$T = T_b(t) + Z_b(t)\tan\tau$$

$$Z = \frac{Z_b(t)}{\cos\tau}$$

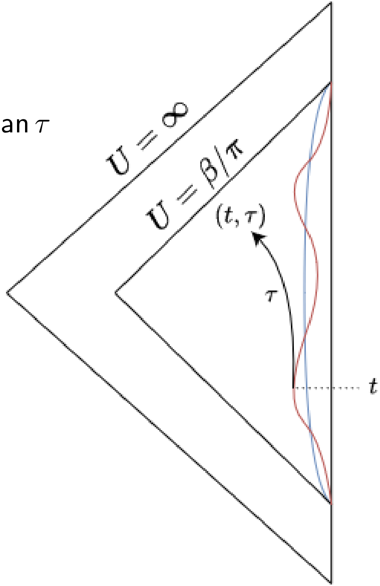


Localizations in JT

- Timelike localization

$$T = T_{b,BH}(t+\epsilon(t))+Z_{b,BH}(t+\epsilon(t))\tan\tau$$

$$Z = \frac{Z_{b,BH}(t+\epsilon(t))}{\cos\tau}$$



Localizations in JT

- Timelike localization

$$T = T_{b,BH}(t+\epsilon(t))+Z_{b,BH}(t+\epsilon(t))\tan\tau$$

$$Z = \frac{Z_{b,BH}(t+\epsilon(t))}{\cos\tau}$$

$$\tau_{\text{hor}} = \cos^{-1}\frac{2Z_b(\beta/\pi - T_b)}{Z_b^2 + (\beta/\pi - T_b)^2}$$

$$\text{Var}(\tau_{\text{hor}}) \propto \frac{\delta^2\beta}{\phi_b\gamma^4}\left(\beta^2 e^{-4\pi t/\beta} + b\gamma^2\right)$$

$$\gamma = \frac{\beta}{2\pi}\left(1 + e^{-2\pi t/\beta}\right)$$