Dark Matter Halo Dynamics in 2D Vlasov simulations

A self-similar approach

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Presentation Overview

1. **Introduction**

    Context

    Discrepancies

2. Methodology

    Numerical simulations

    Theoretical model

3. Data analysis

    Particle Trajectories

    Mass and Density profiles

4. Conclusions

What do we already know about it?

Numerical Model

Single stream flow | Multi stream flow | $t \rightarrow$

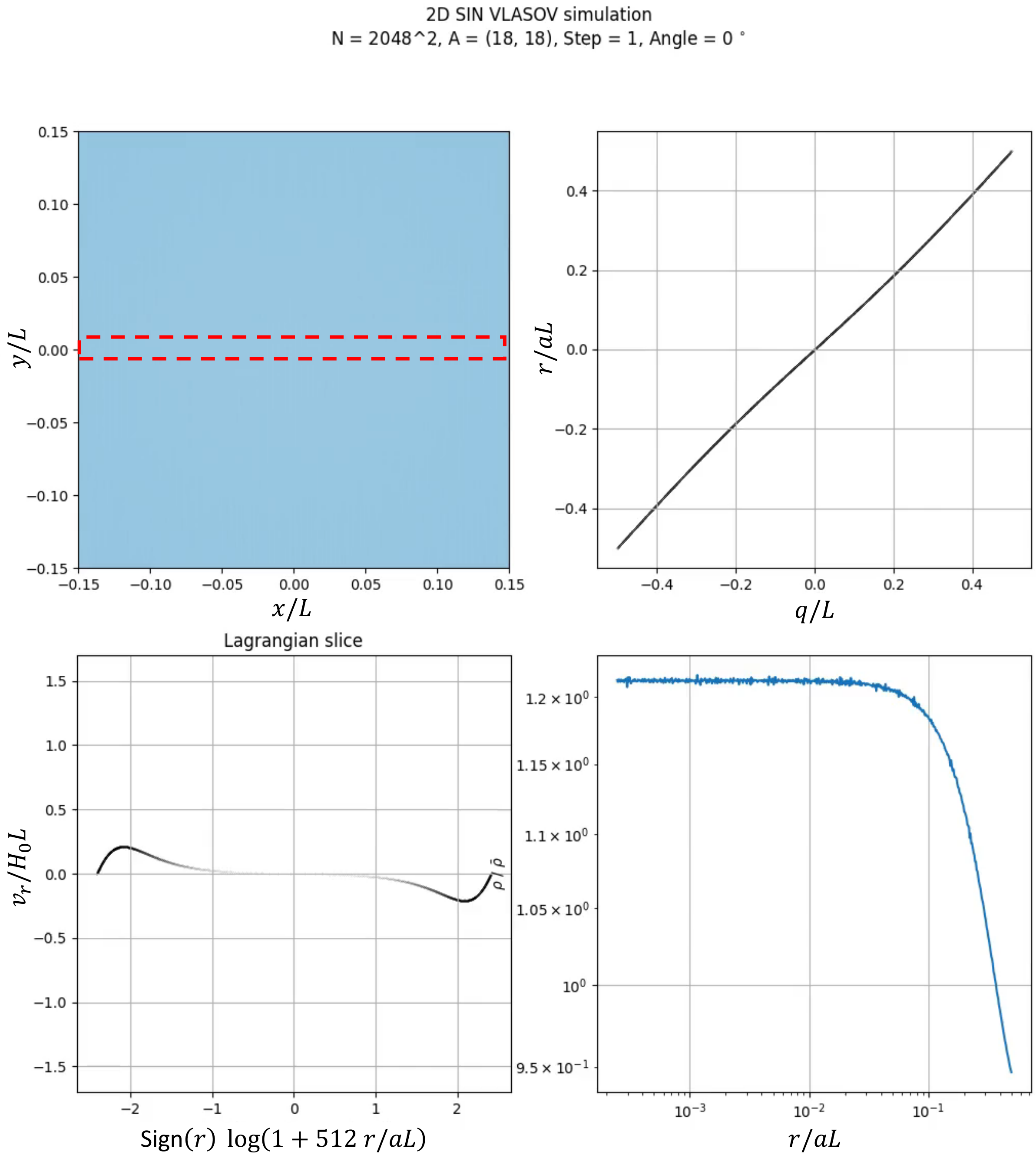
Init. cond. End of Inflation — Hubble Flow — Turn-around — Shell crossing — Violent relaxation — Cuspy profile $\rho \propto r^{-1.5}$ — Accretion & Mergers — Universal NFW Profile

Analytical Models

Lagrangian Perturbation Theory	Post-collapse Pert. Theory	Self-similarity
Zeldovich 1970 $\vec{x}(\vec{q},t) = \vec{q} + \vec{\Psi}(\vec{q},t)$ $\vec{\Psi} = \Sigma_{n=1}^{\infty} \vec{\Psi}^n$ • Valid till $\vec{\Psi}$ remains single valued, small	*Taruya, Colombi 2017* $\vec{x} = \Sigma_{n,m} \ \partial_{\vec{q}^n,t^m} \vec{x}^{LPT} \Big\rvert_{\vec{q}_0,t_0} (\vec{q}-\vec{q}_0)^n (t-t_0)^m$ • Valid around the neighborhood of $\vec{q}_0$ upto a few shell-crossings	*Fillmore & Goldreich 1984* *Bertschinger 1985* $f(\lambda_1 \vec{r}, \lambda_2 t) = \lambda_3 f(\vec{r},t)$ • **All halo particles trace the same trajectory if scaled characteristically** • Valid as long as there are no other scales

A 2D monolithic CDM Halo

- Initial singlestream flow → shell crossing at snap 9 → multistream
- Appearance of caustics
  - Extrema $r - q$ curve
  - Folds in phase-space
  - Spikes density profile
  - Splashback radius
- Self-similarity is very well evident from the phase-space spirals

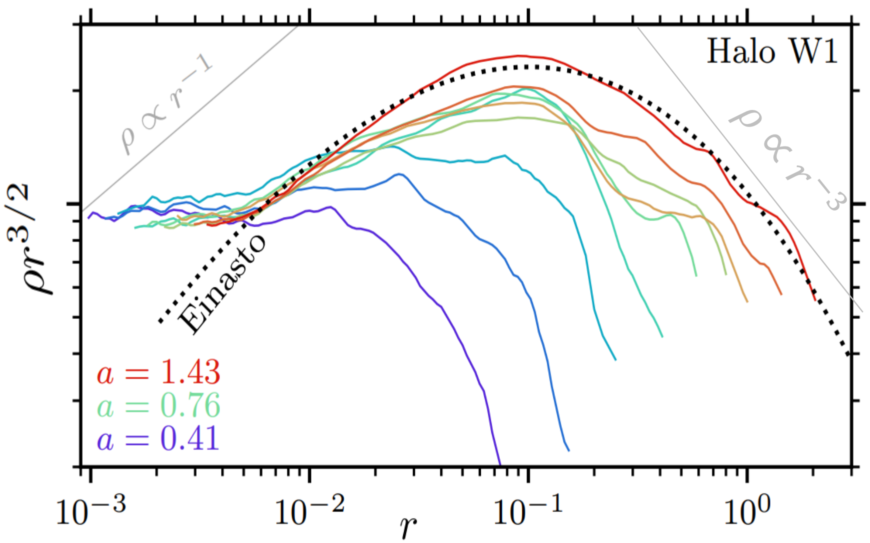


Where are the gaps in our understanding?

Numerical Simulations

Primordial CDM halos: $\rho \propto r^{-1.5}$ *[Delos 2022]*

Accretion, mergers → universal NFW profile *[Navarro 1997]*: $\rho \propto r^{-1}$ center, $\rho \propto r^{-3}$ fall-off.



Theory

Fillmore 1984, Bertschinger 1985 (purely radial dynamics): $\rho \propto r^{-2.25}$

CDM halos seeded by gaussian fields → elliptical collapses, transverse motion.

Despite phase-space being evidently self-similar, it is not reflected in the density profile!

Goals of my project:

1. What is the extent of self-similarity in halo dynamics?
2. Where does it deviate and what causes it?
3. What can we infer about CDM halos in actual 3D cosmologies?

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Vlasov Simulations in 2D

- Numerically, CDM is modelled as a fluid:
  - Non-relativistic
  - Collisionless
  - Negligible velocity dispersion
  - Self-gravitating
- Its phase space distribution f obeys the Vlasov-Poisson equations:

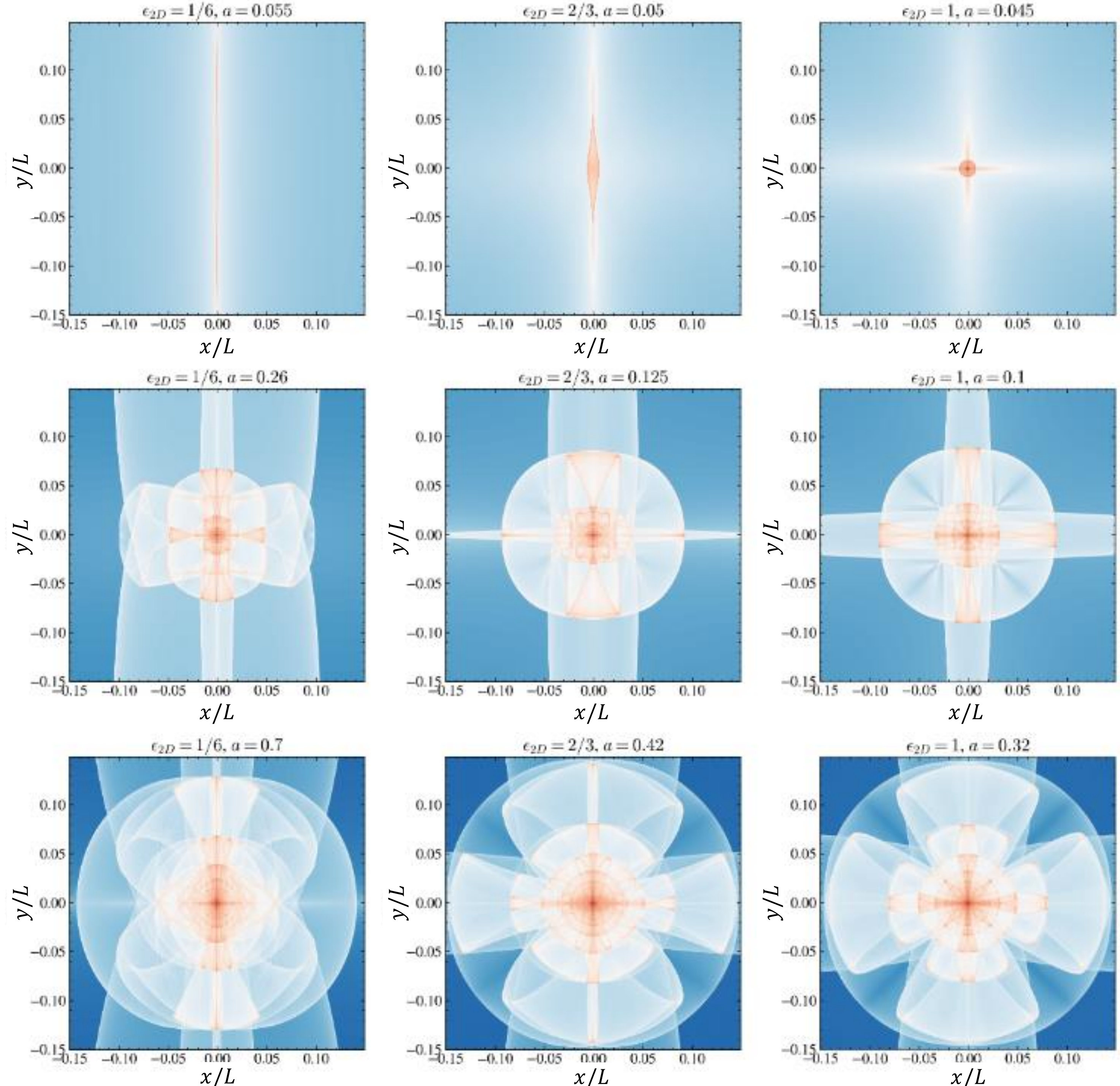
$$\frac{\partial f}{\partial t} + \vec{u} \cdot \nabla_{\vec{r}} f - \nabla_{\vec{r}} \phi \cdot \nabla_{\vec{u}} f = 0 \quad ; \quad \Delta_{\vec{r}} \phi = 4\pi G \rho$$

- ColDICE *[Sousbie, Colombi 2016]* - f is a 2D(or 3D) sheet in 4D(or 6D) in phase-space, vertices are evolved as per lagrangian equations of motion:

$$f(\vec{r}, \vec{u}, t_i) = \rho_i(\vec{r}) \delta_D(\vec{u} - \vec{u}_i)$$

- We study 3 highly symmetric cases in $\Omega_\mathrm{M} = 1$ universe, where the initial displacement field is composed of crossed sin-waves: $\Psi_\mathrm{i} \sim \epsilon_i \sin\left(\frac{2\pi}{L} q_i\right)$

Designation	$\epsilon_{2D} = \epsilon_y / \epsilon_x$	$a_{sc,x}$	$a_{sc,y}$
Quasi-1D (Q1D)	1/6	0.053	0.14
Anisotropy (ANI)	2/3	0.045	0.055
Symmetric (SYM)	1	0.041	0.041



$\epsilon_{2D} = 1/6$, $a = 0.055$
$\epsilon_{2D} = 2/3$, $a = 0.05$
$\epsilon_{2D} = 1$, $a = 0.045$
$\epsilon_{2D} = 1/6$, $a = 0.26$
$\epsilon_{2D} = 2/3$, $a = 0.125$
$\epsilon_{2D} = 1$, $a = 0.1$
$\epsilon_{2D} = 1/6$, $a = 0.7$
$\epsilon_{2D} = 2/3$, $a = 0.42$
$\epsilon_{2D} = 1$, $a = 0.32$
x/L
y/L
−0.15
−0.10
−0.05
0.00
0.05
0.10

Fillmore & Goldreich's self-similarity

- Purely radial motion of spherical shells around an initial perturbation $\delta \equiv \delta M_i(r)/M_i(r) = (M_i(r)/M_0)^{-\epsilon}$ in matter dominated era.

  Model parameters $\begin{cases} \epsilon \text{ : mass-accretion rate} \\ M_0 \text{ : scale of initial perturbation} \rightarrow \text{turnaround} \end{cases}$

- For a shell initially enclosing mass M_i, spherical collapse model:

$$\frac{r_{ta}}{r_i} = C_r \left(\frac{M_i}{M_0}\right)^{\epsilon} \qquad \frac{t_{ta}}{t_i} = C_t^{3/2} \left(\frac{M_i}{M_0}\right)^{3\epsilon/2} \quad ; \quad C_r, C_t = 0.74, 1.39$$

- Position, time and mass are rescaled w.r.t turnaround scales:

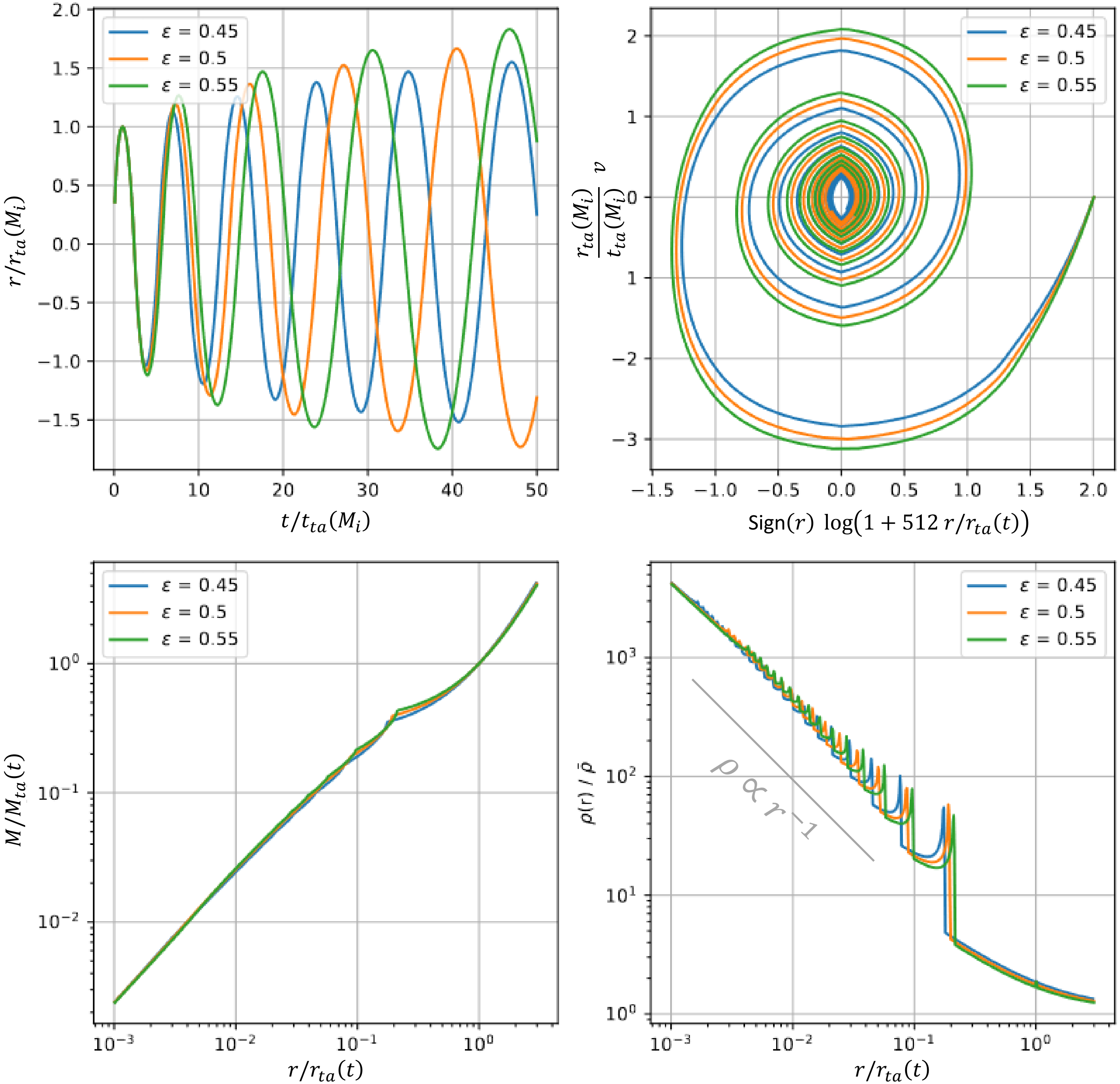
$$\lambda = r/r_{ta} \quad \tau = t/t_{ta} \quad \mathcal{M}(r/r_{ta}) = M(r,t)/M_{ta}(t)$$

- Newtonian equations of gravity in terms of rescaled variables:

$$\frac{d^2\lambda}{d\tau^2} = \frac{\lambda}{9\tau^2} - \frac{1}{3}\left(\frac{C_r}{C_t}\right)^2 \frac{\tau^{\frac{2}{3}\left(\frac{1}{\epsilon}-1\right)}}{\lambda} \mathcal{M}\left(\frac{\lambda}{\Lambda}\right)$$

$$\mathcal{M}\left(\frac{\lambda}{\Lambda}\right) = \frac{2}{3\epsilon}\int_1^{\infty} \frac{d\tau'}{\tau'^{1+\frac{2}{3\epsilon}}} H\left[\frac{\lambda}{\Lambda}(\tau) - \frac{\lambda}{\Lambda}(\tau')\right]; \quad \Lambda(\tau) = \tau^{\frac{2}{3}\left(1+\frac{2}{3\epsilon}\right)}$$

- At turnaround $\tau = 1,\ \lambda = 1$ and $d\lambda/d\tau = 0 \rightarrow$ No dependence on t_i, r_i, M_i



ε = 0.45
ε = 0.5
ε = 0.55
r/r_ta(M_i)
t/t_ta(M_i)
r_ta(M_i)/t_ta(M_i) v
Sign(r) log(1 + 512 r/r_ta(t))
M/M_ta(t)
r/r_ta(t)
ρ(r) / ρ̄
ρ ∝ r^-1

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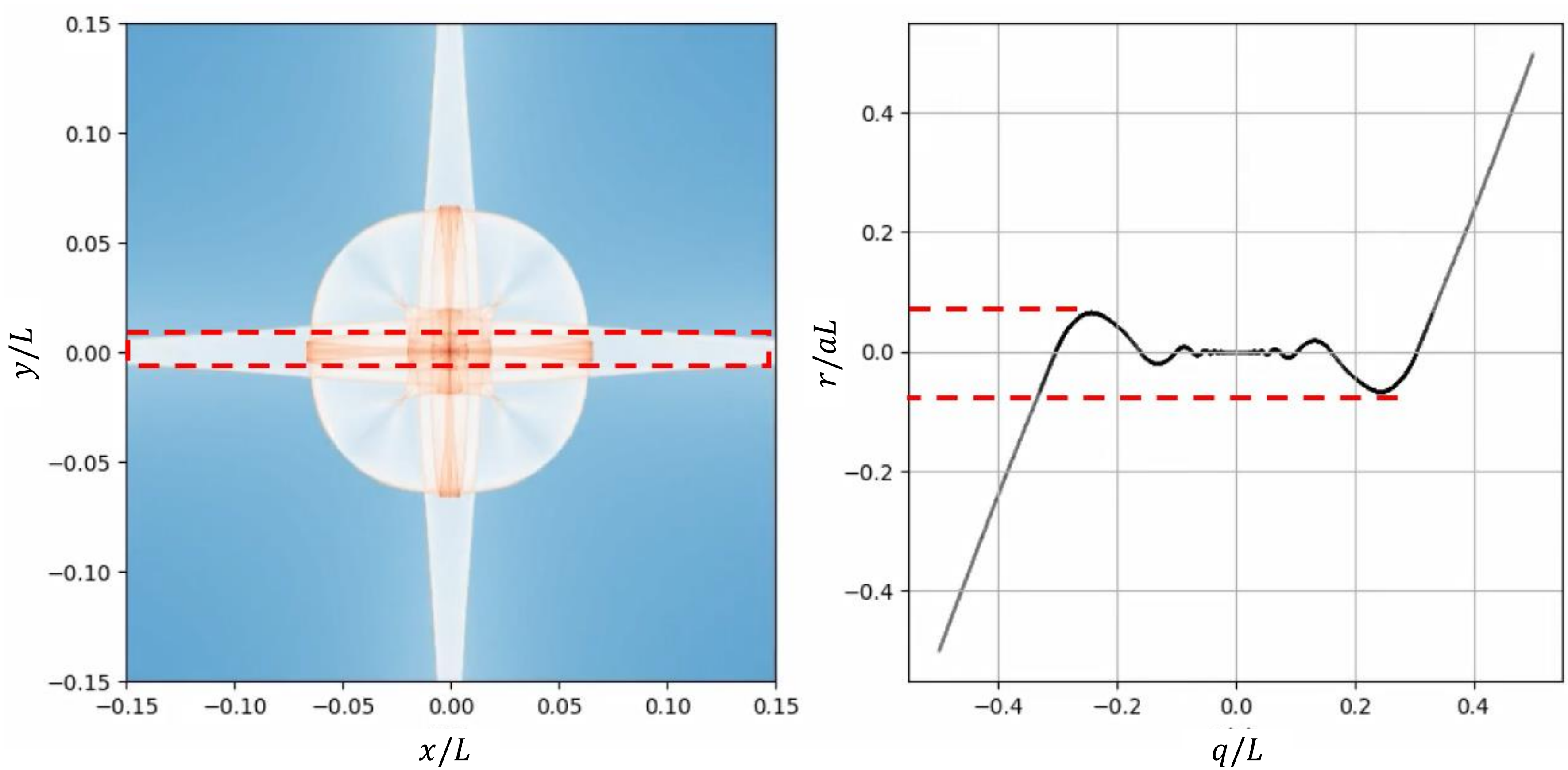
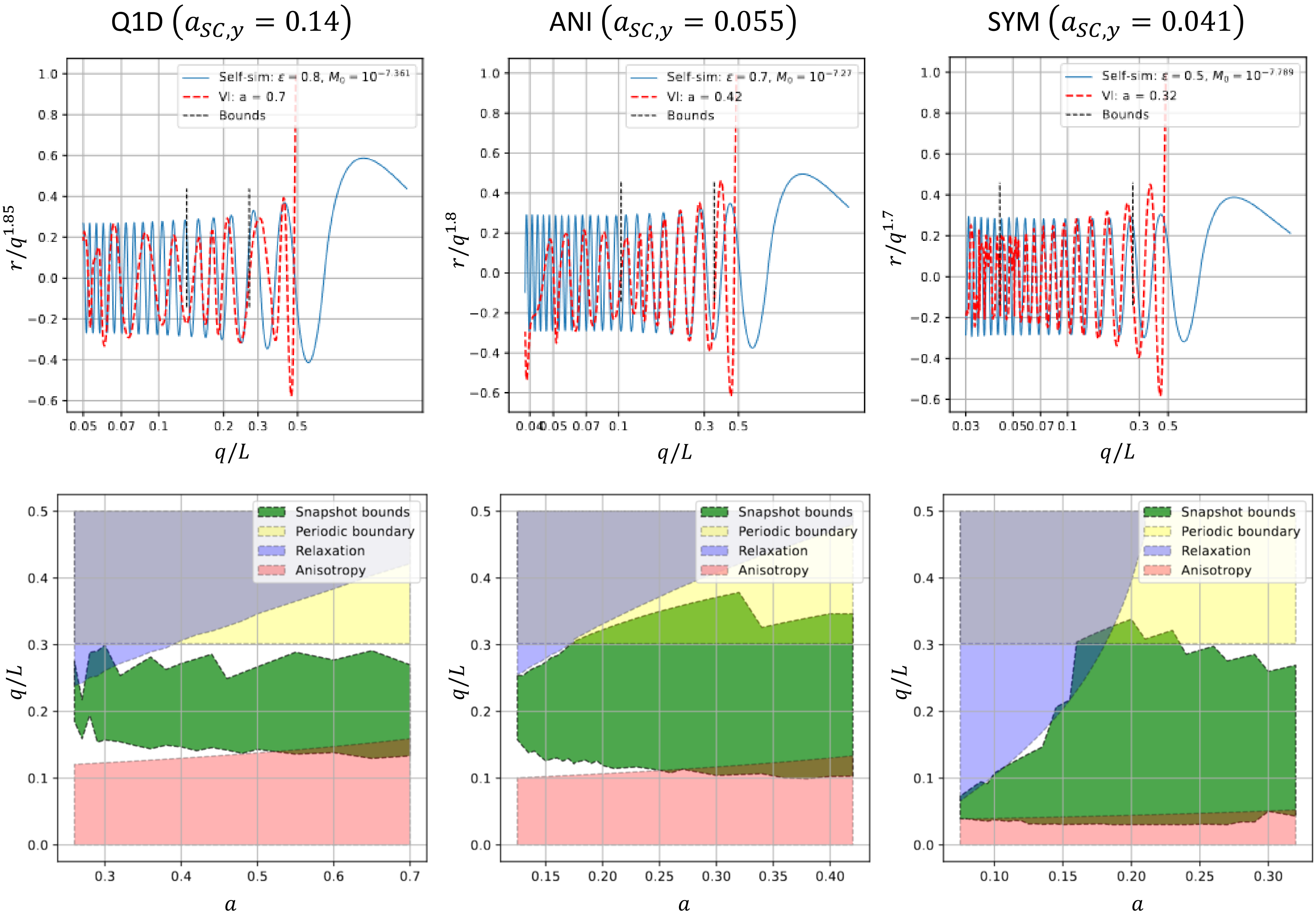
Particle Trajectories

The variable $\tau = C_t^{-3/2}\left(\frac{t}{t_i}\right)\left(\frac{M_0}{M_i}\right)^{3/2}$ can be interpreted in two ways:

1. If M_i is fixed → proxy for time. Limited no. of snapshots ~ 50-60
2. If t is fixed → proxy for shells. Grid resolution = 2048

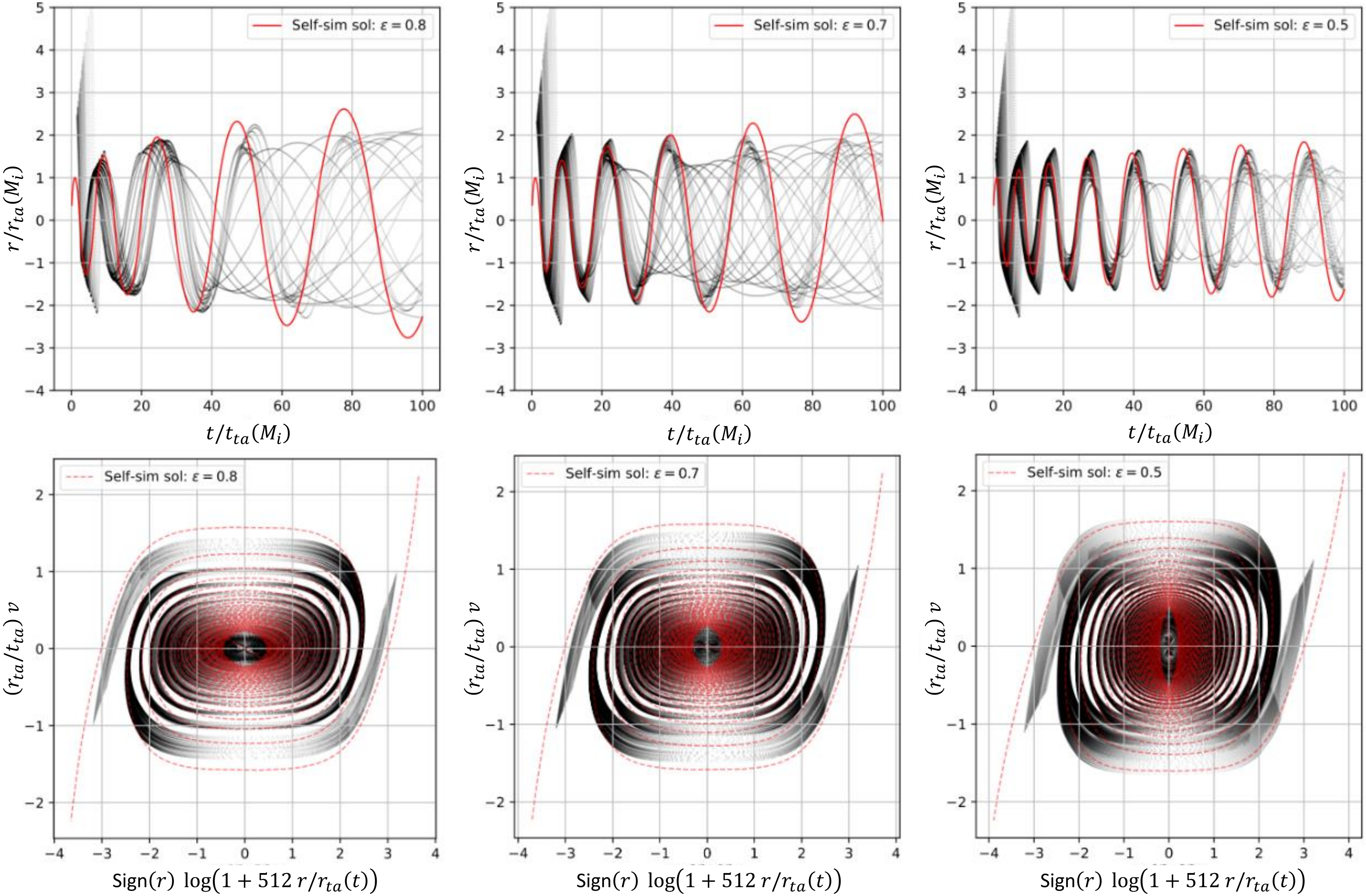



Q1D $\left(a_{SC,y} = 0.14\right)$
ANI $\left(a_{SC,y} = 0.055\right)$
SYM $\left(a_{SC,y} = 0.041\right)$
Self-sim: ε = 0.8, $M_0 = 10^{-7.361}$
VI: a = 0.7
Bounds
Self-sim: ε = 0.7, $M_0 = 10^{-7.27}$
VI: a = 0.42
Self-sim: ε = 0.5, $M_0 = 10^{-7.789}$
VI: a = 0.32
$r/q^{1.85}$
$r/q^{1.8}$
$r/q^{1.7}$
q/L
Snapshot bounds
Periodic boundary
Relaxation
Anisotropy
a

Particle Trajectories

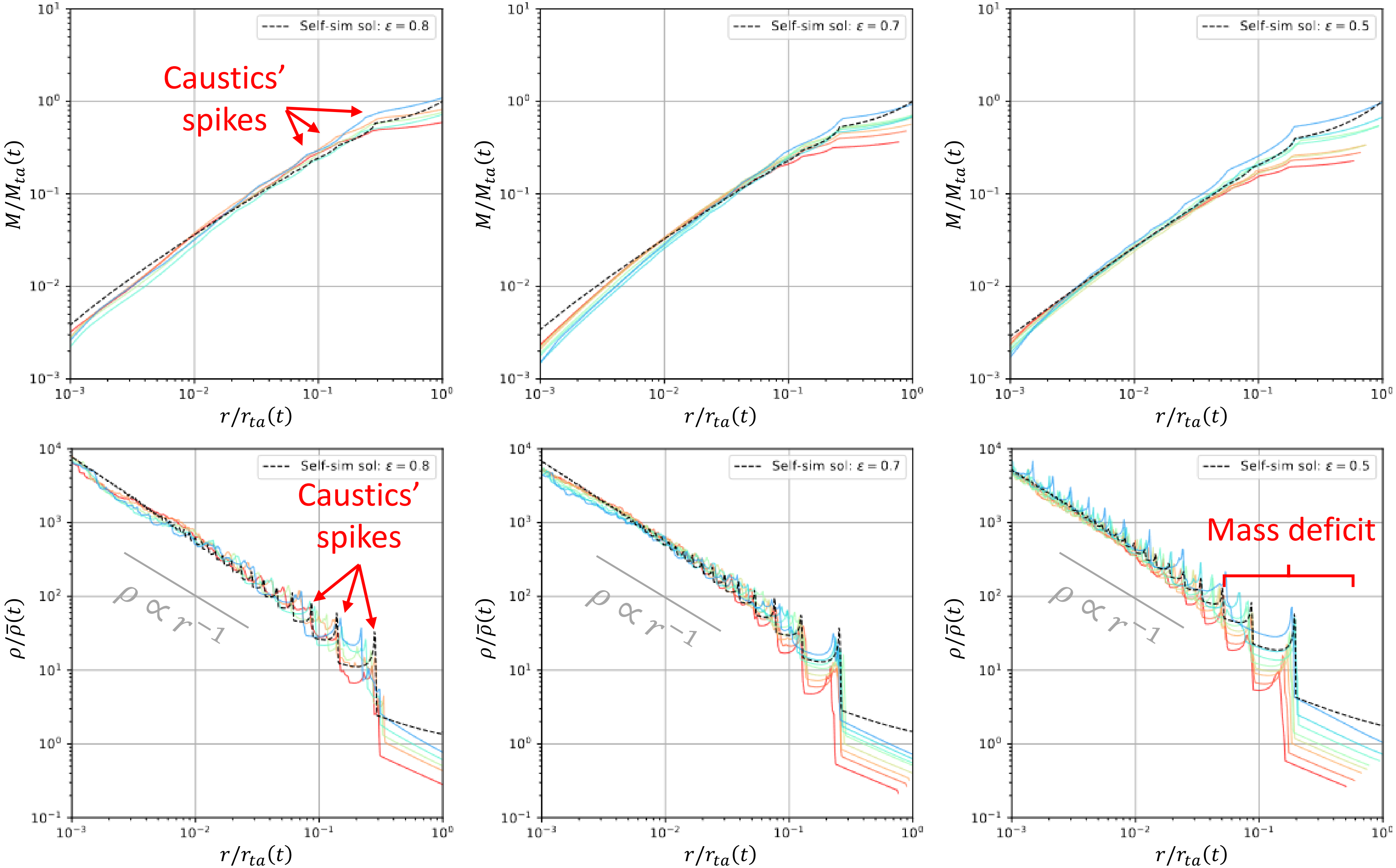
Reasons behind the deviations:

- Shell crossing → relaxation → power law density profile → convergence to self-similar behavior. So, the build up of prompt cusp during violent relaxation cannot be explained by self-similar dynamics.
- The period of relaxation for a particle is roughly 1-2 oscillations → observed in all the 3 cases.
  Additionally, erroneous forces due to halo image arising from periodic boundaries prevent infall of particles close to the boundaries $q \gtrsim 0.3$
- The trajectories deviate again eventually → no. of oscillations we can follow using the fits: Q1D < ANI < SYM. Thus, extent of agreement correlated with the degree of anisotropy and transverse motion in the simulations: Q1D > ANI > SYM.
  Typically, dynamics in halo exterior → radial and interior → transverse. Once the amplitude of oscillations decrease down to the transverse motion dominated interior, we see deviations from FG model → purely radial orbits.



Self-sim sol: ε = 0.8
Self-sim sol: ε = 0.7
Self-sim sol: ε = 0.5
$r/r_{ta}(M_i)$
$t/t_{ta}(M_i)$
$(r_{ta}/t_{ta})\ v$
$\text{Sign}(r)\ \log(1 + 512\ r/r_{ta}(t))$

Mass and Density profiles



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Did we meet our goals?

Extent of self-similarity?

With a narrow range of (M_0, ϵ), we could track ($\leq$ 10% error) 30-60% halo particles for $\geq$ 3-4 oscillations → Self-similarity is quite powerful!

Deviations?

Initial deviation → violent relaxation, build-up of a power-law profile. Particles typically take 1-2 oscillations to relax.

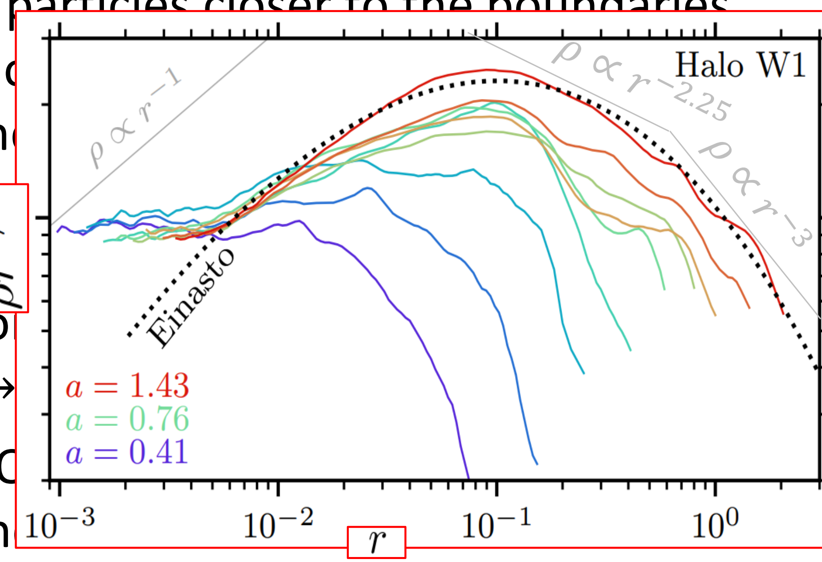
Periodic boundaries slow down the infall of particles closer to the boundaries

Transverse motion in halo interior → deviatio

Mass deficit in simulations → dip in mass an

Implications on 3D CDM halos seed

1. The dynamics during relaxation and pro
   cannot be explained by self-similarity →
2. CDM halos seeded from gaussian IC → C
   model by including elliptical collapse an
3. Even CDM halos have limited mass to accrete → self-similar infall $\rho \propto r^{-2.25}$ which dips to $\rho \propto r^{-3}$ in halo exterior → NFW profile!



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Collaborators : Shohei Saga, Atsushi Taruya (YITP, Japan)

References

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"As our island of knowledge grows, so does the shore of our ignorance..."

- J. A. Wheeler

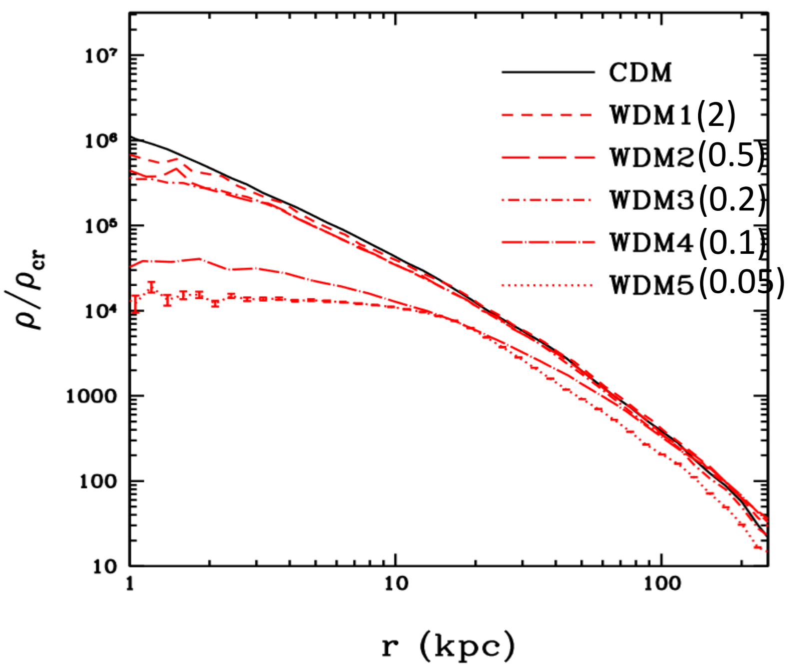
THANK YOU!

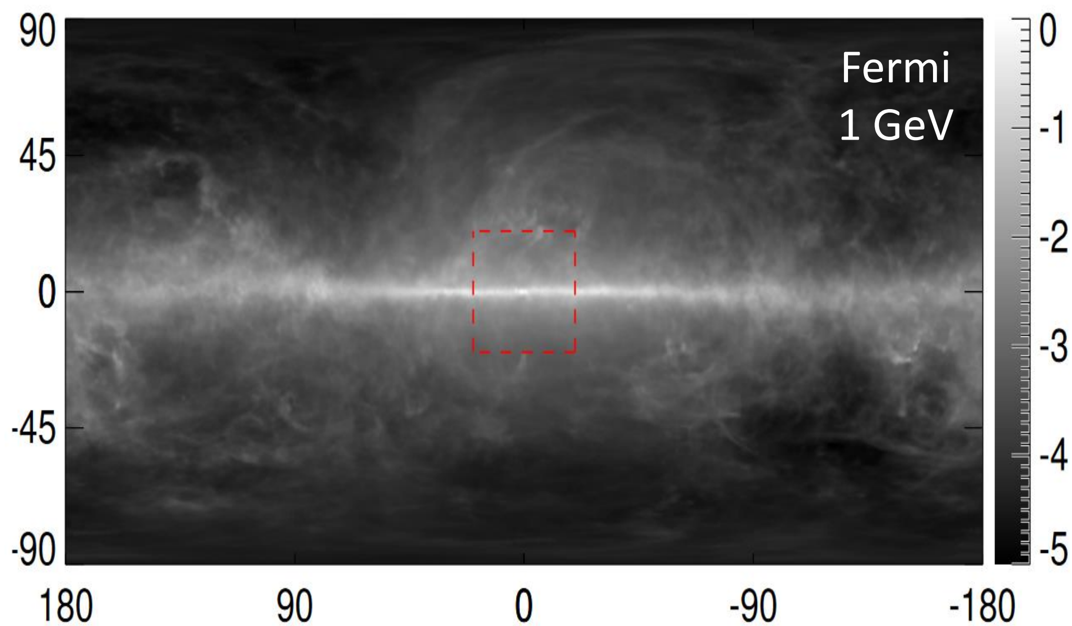
Why do we study dark matter dynamics?

1. Large Scale Structure
   Dark matter $\sim 84\ \%$ of the total mass of the Universe [Planck 2018] .
   Structures $\gtrsim 10^2\ \mathrm{h}^{-1}$ Mpc → clustering of dark matter

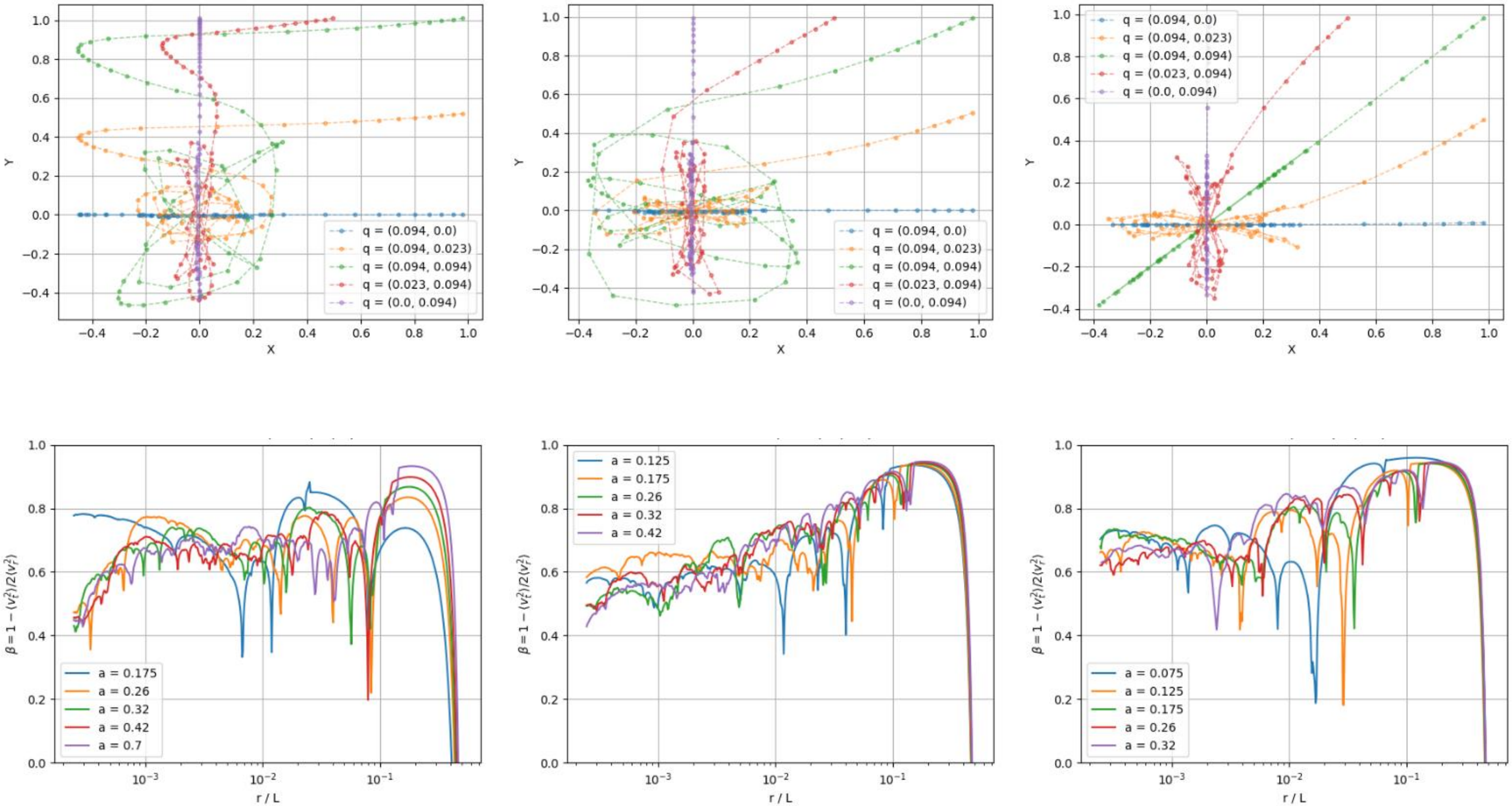
2. Modelling Halos ← My PhD
   DM Halos are the basic units of cosmological structures.
   Particle properties(mass, cross-section) → dynamics → halo structure

3. Modelling observables:
   Annihilation signal of WIMPs [Slatyer 2016] → indirect DM detection!





Transverse motion and Anisotropy



Parameter distribution

- From the residues, the best-fit ϵ:
  Q1D (0.75-0.85) > ANI (0.65-0.75) > SYM (0.45-0.55)
  Slower infall along y-axis → oscillation freq ↓
- Even within a simulation, the best-fit ϵ ↑ with time
  Deficit of infalling mass → oscillation freq ↓
- Theoretically, one (M_0, ϵ) per halo. But, our simulations are not exact depictions → expect a dist. for (M_0, ϵ)
  Spread in the best-fit params → 10-20% relative spread in initial $\delta = (M_i(r)/M_0)^{-\epsilon}$ assumed in theory.