

Covariant Cosmography with the Expansion Rate Fluctuation Field

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Théorie Univers et Gravitation, 5-7 Nov 2024, LAPTh Annecy

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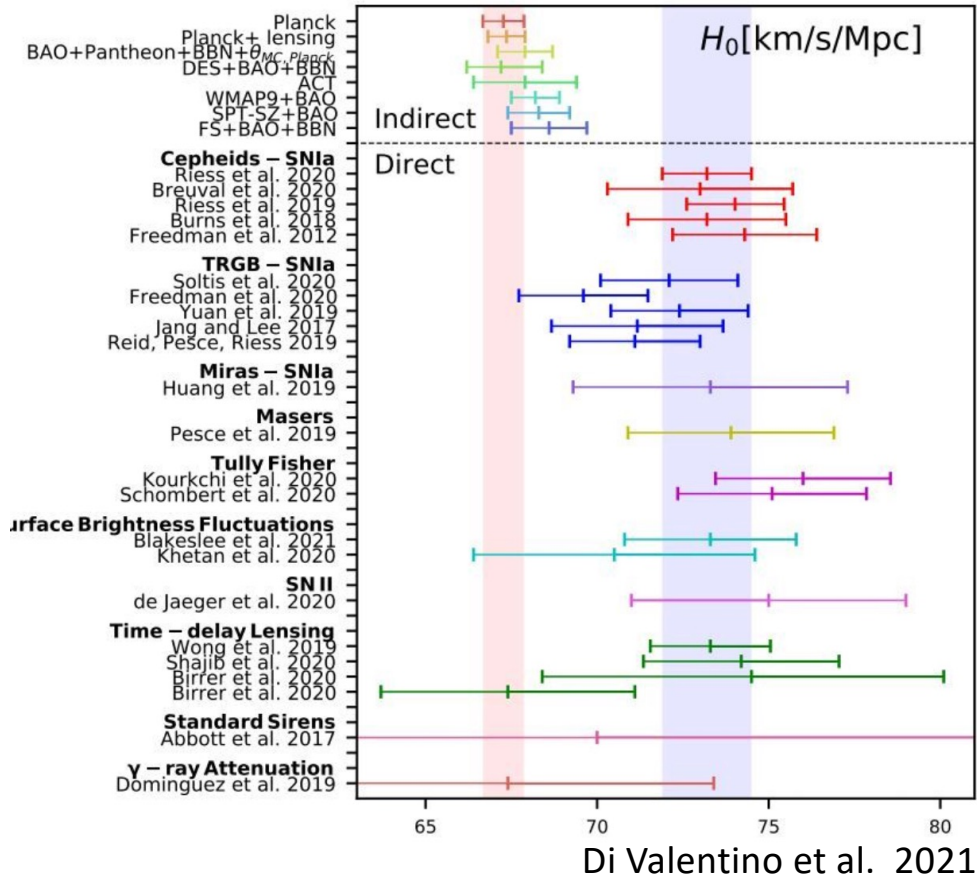
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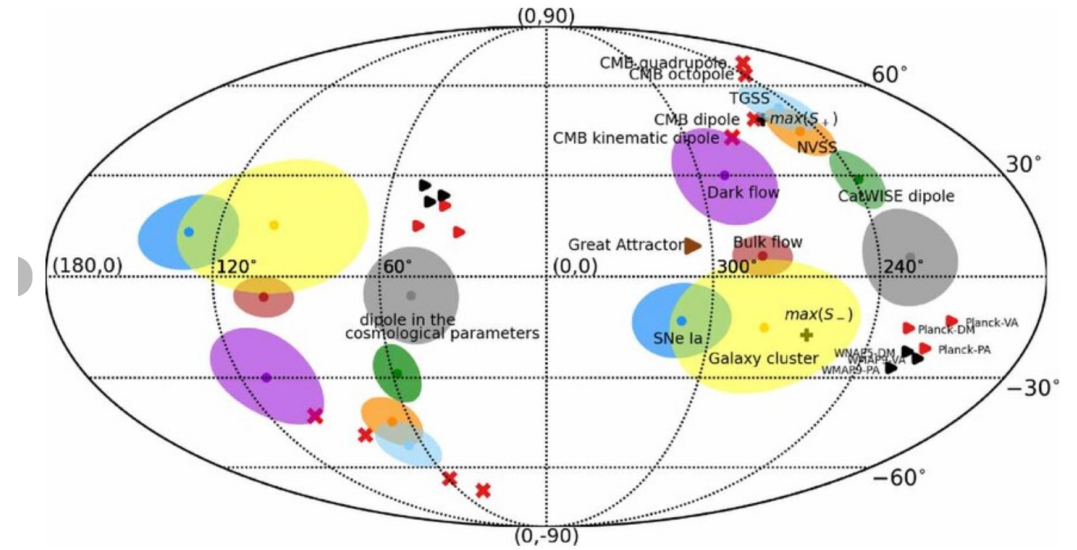
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Anomalies in cosmology

Hubble tension



Dipole tension



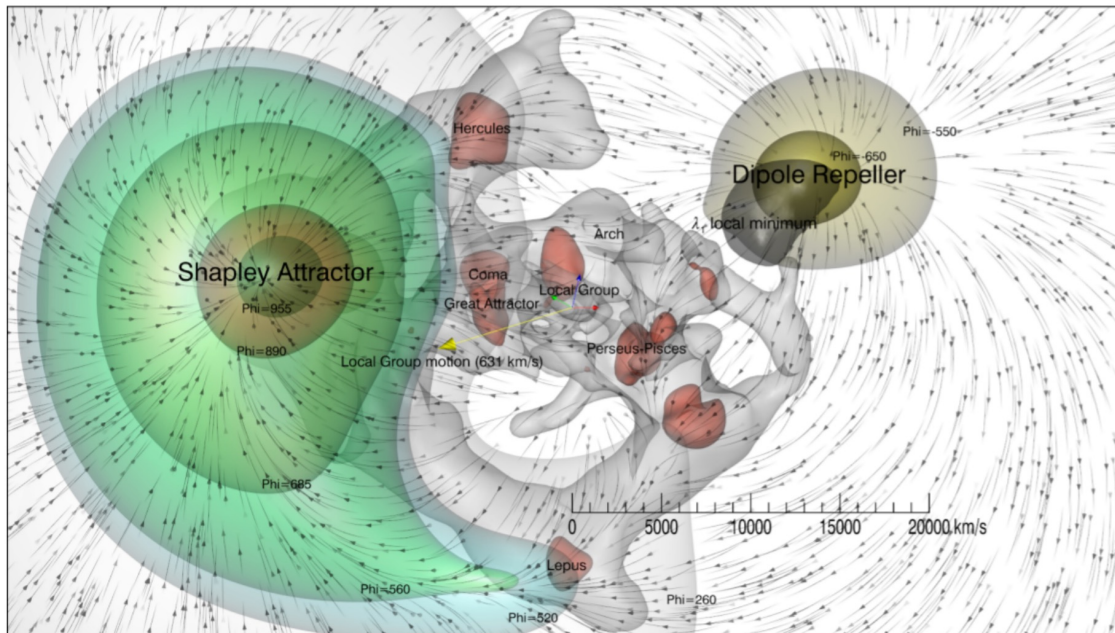
Aluri et al. 2023

Departures from the Cosmological Principle?

Deviations from the Cosmological Principle : traditional approach

Background + Perturbation split of the FLRW models

Peculiar velocities



Hoffman et al 2017

CAVEATS

- it is a linear order approximation scheme (no large perturbations, irrotationality....)
- it is vectorial (spherical harmonic analysis is tricky...)
- It is model dependent (EFE + FRW)



**OUR
BRAND
MANIFESTO**

Analyze fluctuations in the expansion rate **nonperturbatively**, without relying on peculiar velocities, and in a **model-independent way**, without any prior reference to a metric

Characterize the expansion rate on local cosmic scales ($z < 0.1$) more meaningfully than using the Hubble parameter of the Standard Model alone.

This program involves two critical steps.

- First, we need to **identify and classify deviations from isotropy** in the observed redshift-distance relation in a robust and unbiased way.
- Next, we need to **interpret** these angular distortions by relating them to physical quantities that provide insights into the geometry of local spacetime: the **covariant cosmographic parameters**

Optimizing the Detection of Angular Anisotropies in the Hubble diagram

The expansion rate fluctuation field η

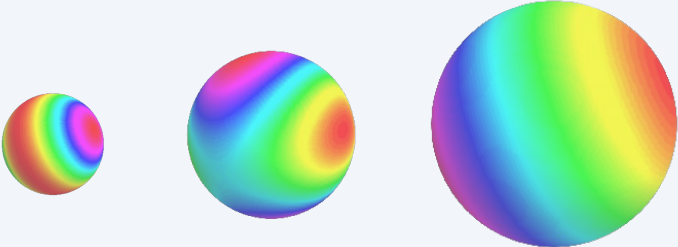
Kalbouneh, Marinoni & Bel *Phys. Rev. D* **107**, 023507 (2023)
Kalbouneh, Marinoni & Maartens, *JCAP* (arxiv 2401.12291)

$$\eta(z, \mathbf{n}) \equiv \log\left(\frac{z}{d_L(z, \mathbf{n})}\right) - \mathcal{M}$$
$$\mathcal{M} \equiv \int_S \log\left(\frac{z}{d_L(z, \mathbf{n})}\right) d\Omega$$

Spherical shell Redshift-independent distances Line of sight

$$\eta(0, \mathbf{n}) \propto \frac{\delta H(\mathbf{n})}{H}$$

Fluctuations estimated on all-sky shells of different sizes



A non-zero value of η indicates that the expansion rate has a more complex structure than that predicted by the CP

Optimizing the Detection of Angular Anisotropies in the Hubble diagram

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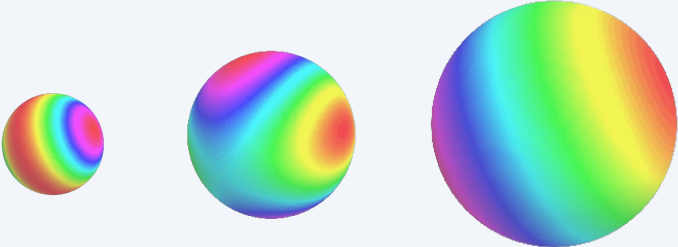
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Statistical properties

The estimator is a r.v. with Gaussian distribution.

The estimator is statistically unbiased.

The condition $\langle \eta \rangle = 0$ ensures independence from the chosen calibration of distances and the chosen distance units.

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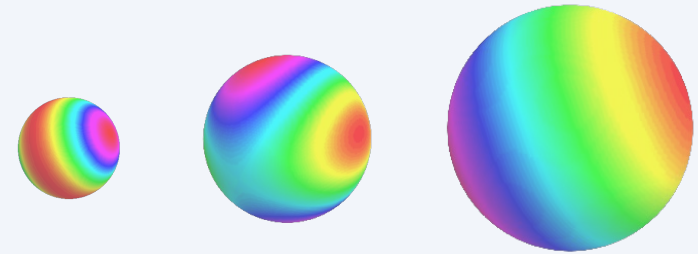
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Fluctuations estimated on all-sky shells of different sizes

SH analysis

$$\eta(\theta, \phi, z) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m}(z) Y_{\ell m}(\theta, \phi)$$

$$a_{\ell m}(z) \equiv \int_0^{2\pi} \int_0^{\pi} \eta(\theta, \phi, z) Y_{\ell m}^*(\theta, \phi) \sin \theta d\theta d\phi$$

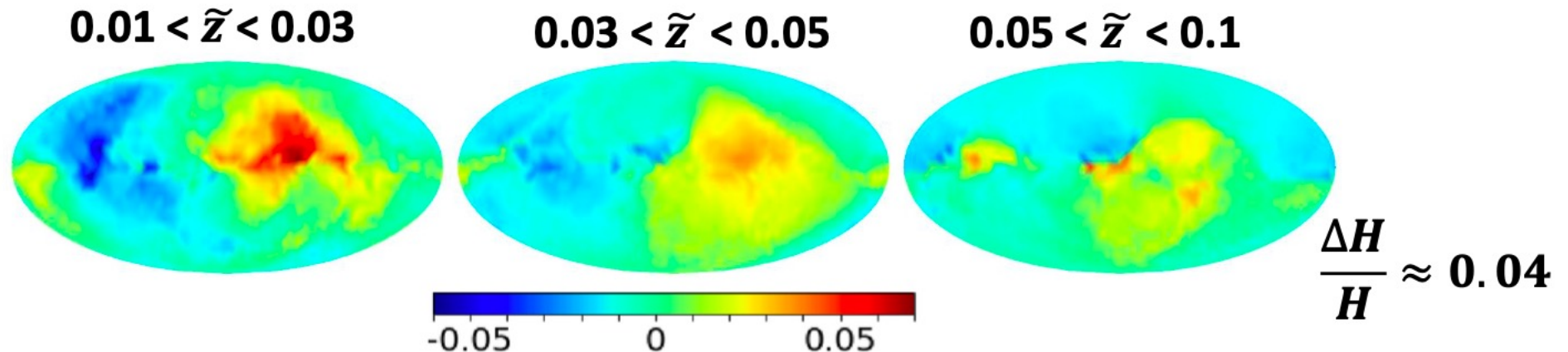
- Contributions from different angular scales are statistically independent
- Statistical likelihood analyses are minimally affected by correlations between different modes
- The classification of anisotropies is straightforward, facilitating the identification of potential symmetries in the metric fluctuations.

RESULTS : analysis of the CosmicFlows4 catalog (Tully et al 2022)

(56000 galaxies up to $z=0.1$)

Kalbouneh, Marinoni & Bel
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All-sky map of the expansion rate fluctuation field η
as measured by an observer at rest in the CMB frame
(galactic coordinates)

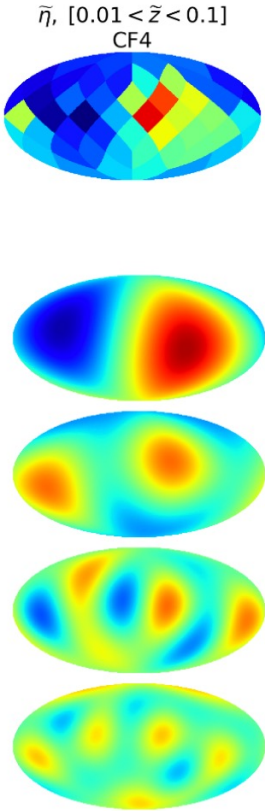


RESULTS : analysis of the CosmicFlows4 catalog (Tully et al 2022)

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CF4 galaxies (48pix)
~56000 objects

Multipoles of the expansion rate fluctuations



Strong S/N for dipole, quadrupole, octupole and hexadecapole. (The random nature of the signal in an LCDM model is excluded)

Maxima all aligned in the same direction ($l \sim 300^\circ, b \sim 5^\circ$)

Axialsymmetry of the lower multipoles

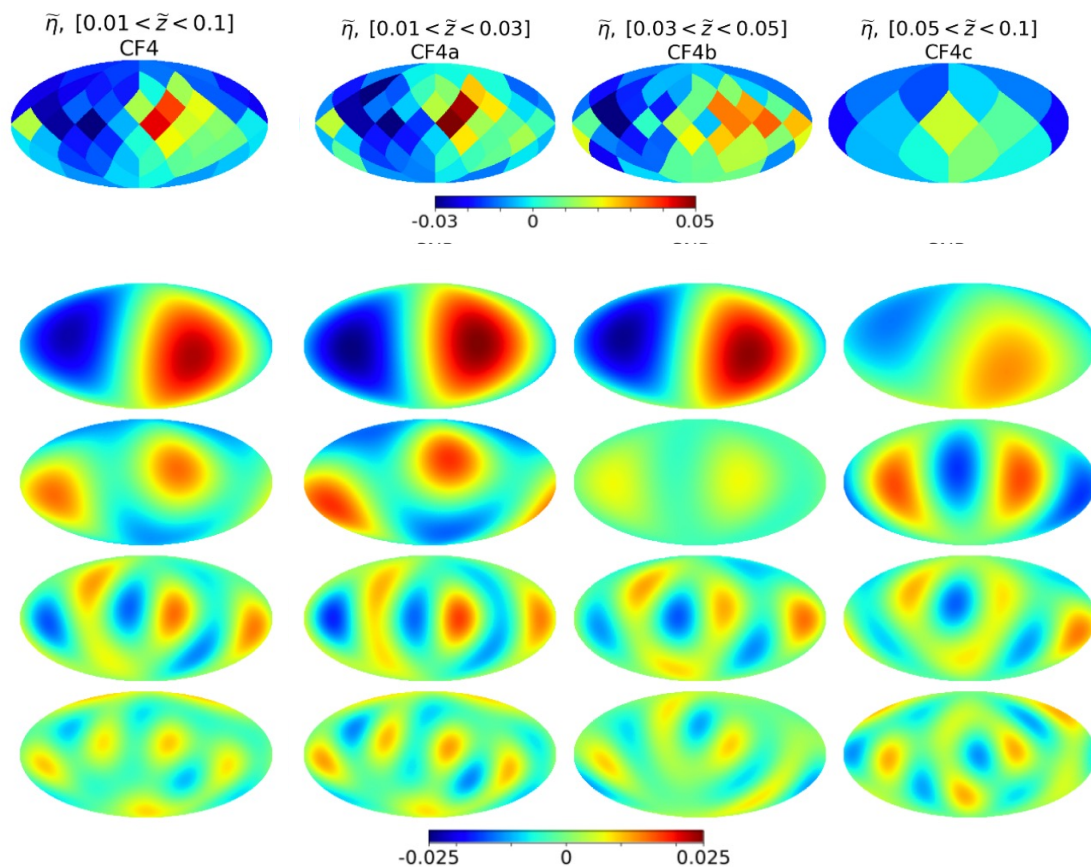
Confirm and extend previous results we obtained by analysing the CF3 sample

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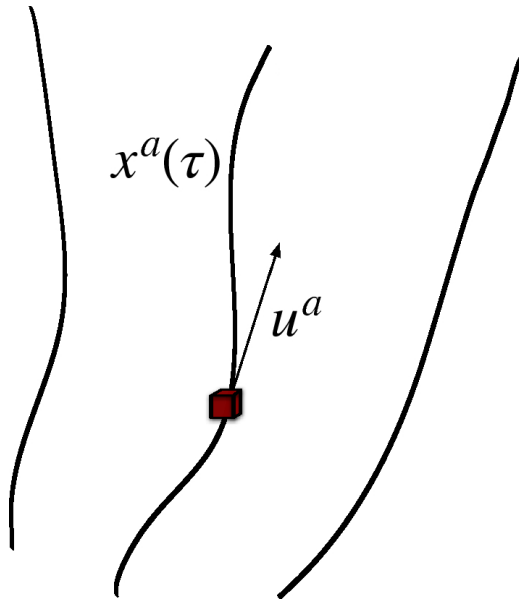
Same stable multipole configuration :

- In all three redshift shells studied (up to $z=0.1$)
- Regardless of the distance indicator used (TF, $D_n-\sigma$, SNIa standard candles)

Physical Interpretation of Expansion Rate Anisotropies

Covariant Cosmography

Constrain the geometry of matter geodesics around generic observers in generic spacetimes



Fluid model : observers have their own four-velocity u^a and proper time. Their worldlines are assigned by averaging over many nearby galaxies.

Observables: the redshift z and the angular diameter distance D are well defined functions along each light-like geodesic

$$1 + z \equiv \frac{(k_b u_m^b)_s}{(k_b u_m^b)_o}$$

$$d_A \equiv \sqrt{\frac{dA}{d\Omega}}$$

Model-independent
Fully unperturbative

This idea dates back to the seminal work [Kristian & Sachs 1966 The Astrophysical Journal 143 \(1966\) 379](#) which we extended and refined in [R. Maartens et al \(arXiv: 2312.09875\)](#) and [B. Kalbouneh et al. \(arXiv: 2408.04333\)](#)

Covariant Cosmography in a nutshell

Expansion of η (i.e. luminosity distance) as a function of redshift

$$\eta(z, \mathbf{n}) = \log \mathbb{H}(\mathbf{n}) - \frac{1 - \mathbb{Q}(\mathbf{n})}{2 \ln 10} z + \frac{7 - \mathbb{Q}(\mathbf{n})[10 + 9\mathbb{Q}(\mathbf{n})] + 4[\mathbb{J}(\mathbf{n}) - \mathbb{R}(\mathbf{n})]}{24 \ln 10} z^2 - \mathcal{M} + \mathcal{O}(z^3).$$

This approach yields covariant cosmographic parameters, a set of line-of-sight dependent functions that characterize the metric structure in the neighbourhood of the matter observer.

$$\mathbb{H} \doteq K^\mu K^\nu \Theta_{\mu\nu},$$

$$\mathbb{Q} \doteq -3 + \frac{K^\mu K^\nu K^\alpha \nabla_\alpha \Theta_{\mu\nu}}{\mathbb{H}^2},$$

$$\mathbb{R} \doteq 1 + \mathbb{Q} - \frac{K^\mu K^\nu R_{\mu\nu}}{2\mathbb{H}^2},$$

$$\mathbb{J} \doteq -10\mathbb{Q} - 15 + \frac{K^\mu K^\nu K^\alpha K^\beta \nabla_\alpha \nabla_\beta \Theta_{\mu\nu}}{\mathbb{H}^3}$$

Dust matter expansion tensor

$\Theta_{\mu\nu} = \nabla_\mu u_\nu$

Observer's line of sight

$K^\mu \propto u^\mu + n^\mu$

(Past-pointing and normalised)
photon 4-momentum

Hasse & Perlick 1999,
Clarkson 2000,
Clarkson & Maartens 2010
Umeh 2013,
Heinesen 2021,
Maartens et al. 2023
Kalbouneh et al. 2024

I Covariant Cosmography has Finite Number of d.o.f.

e.g. Heinesen 2021, Kalbouneh et al. 2024

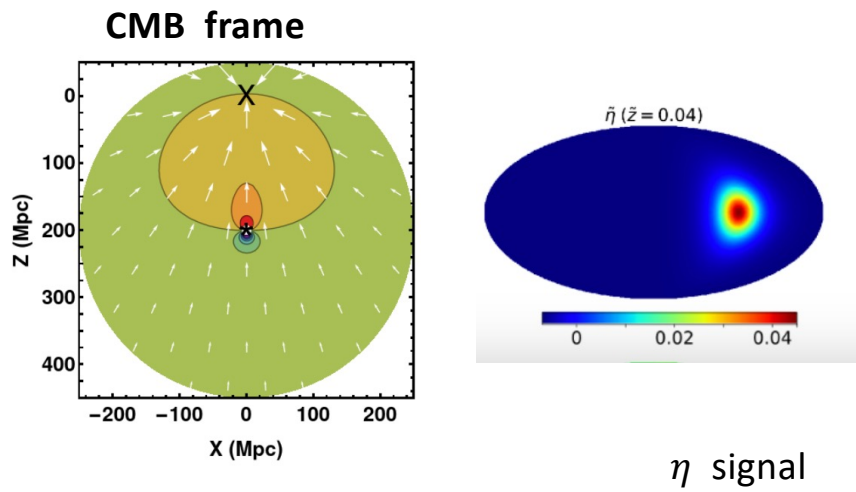
The degrees of freedom for each covariant cosmographic parameter are **finite** and associated with their **mutipoles**. By fitting a reasonably small set of **multipoles** to data, we can in principle fully reconstruct the functional form of the covariant cosmographic parameters.

Fitting Parameters if the cosmographic expansion is truncated to $o(z^3)$

Covarian Cosmographic Parameters	General case (including 4-acceleration) d.o.f.	Axial Symmetric Ansatz (and geodesic motion) d.o.f.	Dominant multipoles (approximation) d.o.f
\mathbb{H}	9	2	$l=0,2$
\mathbb{Q}	16	4	$l=(0),1,3$
$\mathbb{J} - \mathbb{R}$	25	5	$l=0,2,4$
TOTAL	50	11	8

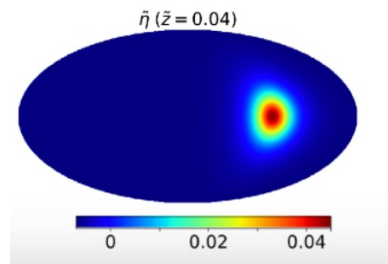
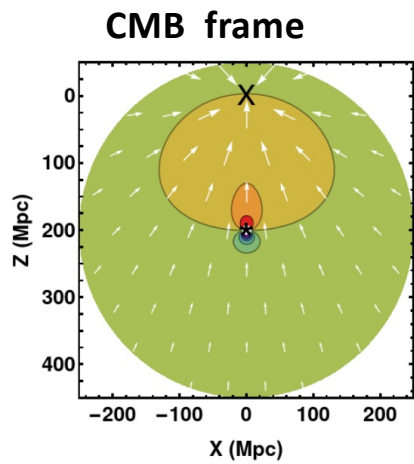
II Each multipole of η contains specific physical information

Kalbouneh Marinoni & Maartens 2023 arxiv 2401.12291

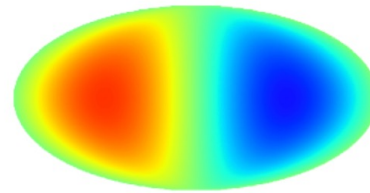


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η signal

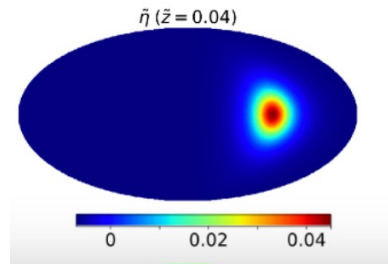
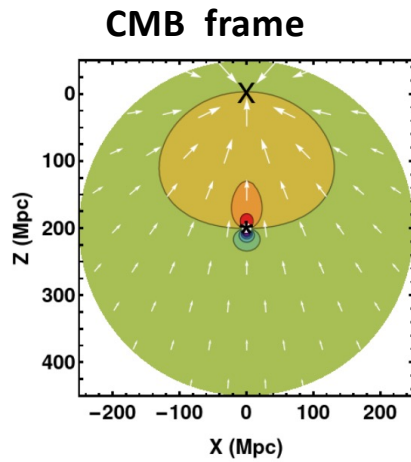


dipole

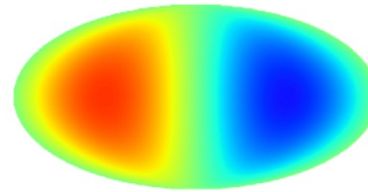
$$\eta_1(\bar{z}) = \frac{v_o(1 + \bar{z})}{\bar{z} \ln 10} + \frac{Q_1}{2 \ln 10} \bar{z} + \sigma(\bar{z}^2, v^2)$$

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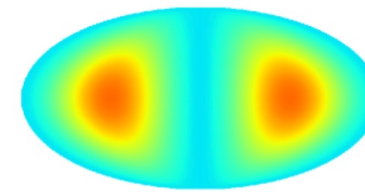
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η signal



dipole

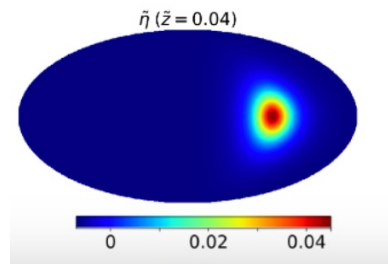
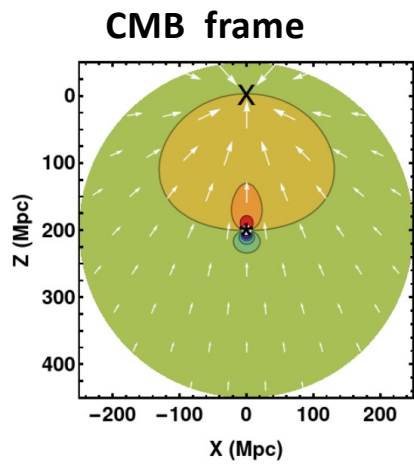


quadrupole

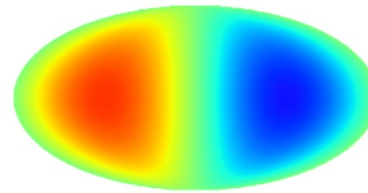
$$\eta_2(\bar{z}) = \frac{H_2}{H_0 \ln 10} + \frac{Q_2}{2 \ln 10} \bar{z} + \sigma(\bar{z}^2, v^2)$$

II Each multipole of η contains specific physical information

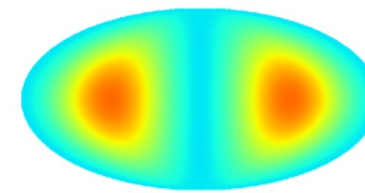
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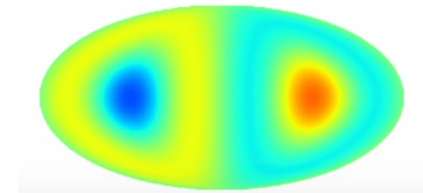
η signal



dipole



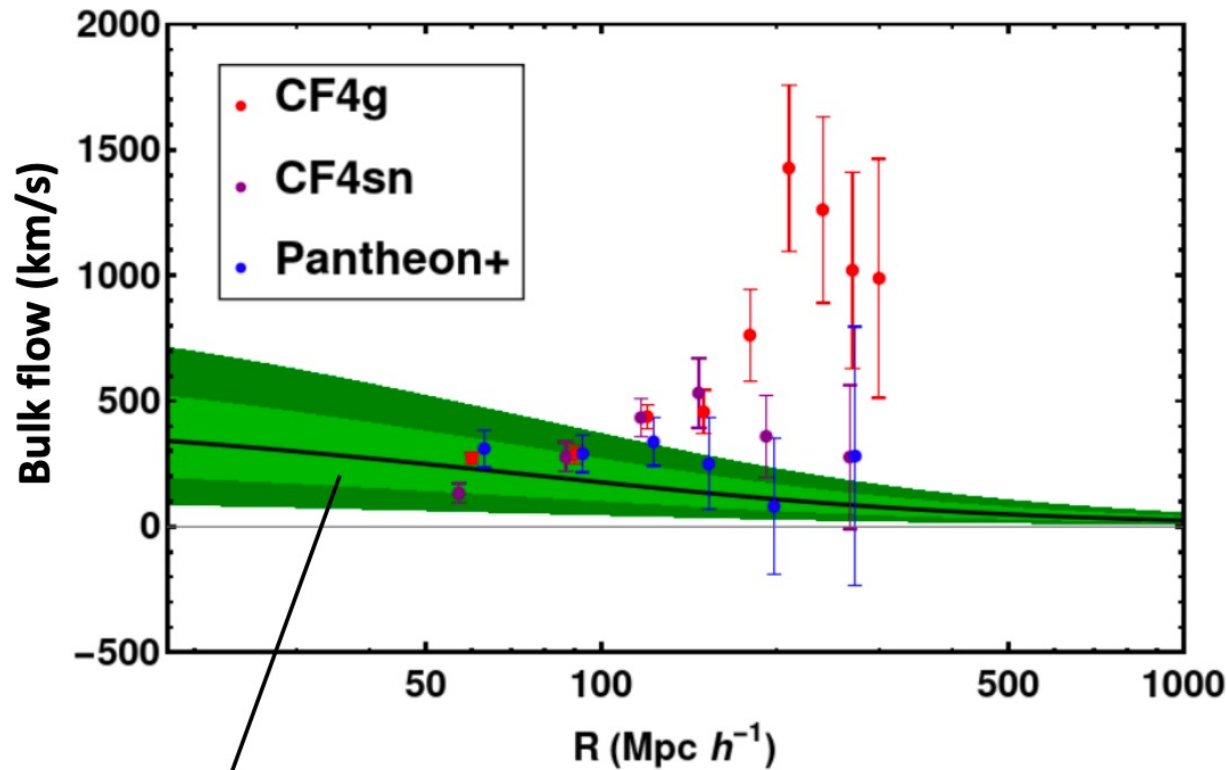
quadrupole



octupole

$$\eta_3(\bar{z}) = \frac{Q_3}{2 \ln 10} \bar{z} + \sigma(\bar{z}^2, v^2)$$

Bulk motion of the local universe? (a model dependent analysis of expansion anisotropies)



ΛCDM prediction

In the standard model (Λ CDM) the dipole of η is related to the notion of "bulk velocity" of a sphere of radius R

$$\tilde{\eta}_1 \approx v_b / (z \ln 10)$$

We confirm the tension between the bulk expected in Λ CDM and the measured one (e.g. Watkins et al. 2023, Whitford et al 2023)

However the bulk is a poor model. Multipoles higher than the dipole contribute to the observed anisotropies

The probability of having in an LCDM model such a strong dipole and quadrupole in the farthest redshift shell ($z \sim 0.1$) is low: 2% and 3% respectively !

Conclusions

The expansion rate fluctuation field η , a scalar Gaussian observable, allows for the unbiased identification and classification of deviations from isotropy in the observed redshift-distance relation, facilitating straightforward signal interpretation.

- **A multipolar universe: there are more things in heaven...than just a dipole:** CF4 data (galaxy and SNIa) reveal a significant quadrupolar component (about half the amplitude of the dipole) and evidence (~ 2 SNR) of an octupolar component with similar amplitude of quadrupole.

- **The local universe ($z < 0.1$) does not exhibit chaotic irregularities or complex fluctuations; rather, it displays a symmetric pattern of inhomogeneity:** quadrupole and octupole aligned with the dipole and all show an axially symmetric structure.

-The cosmic expansion rate is systematically and coherently perturbed on a scale exceeding $\sim 400h^{-1}\text{Mpc}$. It is no longer just a matter of anomalous bulk flows. It is **the whole multipolar structure of the expansion field around us that persists unchanged at least until $z = 0.1$** the highest redshift investigated.

Perspectives

Increase the precision on the estimation of the CC parameters using the larger and deeper samples that will soon be available (ZTF, LSST, DESI)

Identify which mass density field might generate the polarity and alignment observed in the multipoles of the expansion rate field.

Establish a mapping between Cosmological Constant (CC) parameters and Standard Model parameters to assess the impact of anisotropies on the estimation of the standard Hubble parameter."