Revisiting the stochastic QCD axion window: departure from equilibrium during inflation

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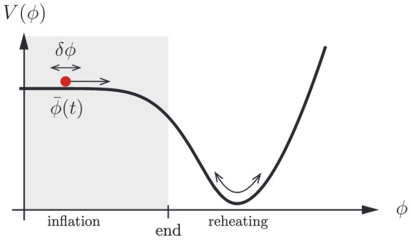
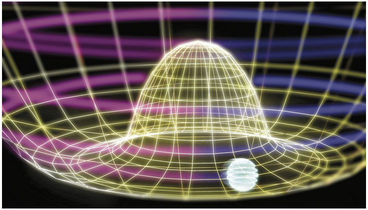
- JCAP05(2024)085
- Kenji Kadota, Shinji Mukohyama, Alireza Talebian, Vincent Vennin

Spectator QCD axion during Inflation





Background :

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\mathrm{d}V_{\mathrm{inf}}}{\mathrm{d}\phi} = 0$$

$$3M_{\mathrm{Pl}}^2 H^2 = V_{\mathrm{inf}} + \frac{\dot{\phi}^2}{2}$$

Spectator axion :

$$\ddot{\theta} + 3H\dot{\theta} + \frac{\mathrm{d}v_{\mathrm{ax}}}{\mathrm{d}\theta} = 0$$

$$v_{\mathrm{ax}}(\theta) = m^2(T)\left[1 - \cos(\theta)\right]$$

Goals and assumptions

Goals :

- Compute the distribution of the axion at the end of inflation under more realistic assumptions
- Derive constraints on f
- Derive constraints on the inflationary scenario

Hypothesis :

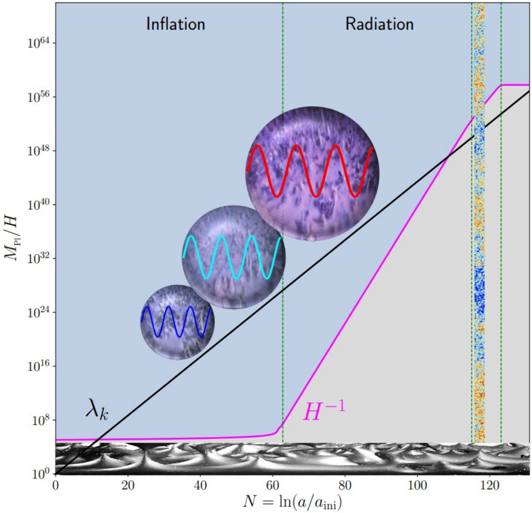
- $\Omega_{\rm a} = \Omega_{\rm DM}$
- The axion is a spectator field during inflation

$$h^2\Omega_{\rm DM} = b\theta_{\rm end}^2 \left[1 - \ln\left(1 - \frac{\theta_{\rm end}^2}{\pi^2}\right)\right]^{\beta} \left(\frac{f}{\hat{f}}\right)^{\beta}, \quad \text{Turner (1986)}$$

Constraints :

- Satisfy isocurvature bound

Stochastic inflation



Starobinsky (1982,1986)
Starobinsky,Yokoyama (1994)

Equation of motion in the SR limit

Slow-roll condition for the axion :

- $H(N) >> m$

Langevin equation :

$$\frac{\mathrm{d}\theta}{\mathrm{d}N} = -\frac{v_{\mathrm{ax}}'(\theta)}{3H^2(N)} + \frac{H(N)}{2\pi f}\xi(N)$$

$\xi(N)$: Gaussian white noise :

- $\langle \xi(N) \rangle = 0$
- $\langle \xi(N)\xi(N') \rangle = \delta(N - N')$

Fokker-Planck equation

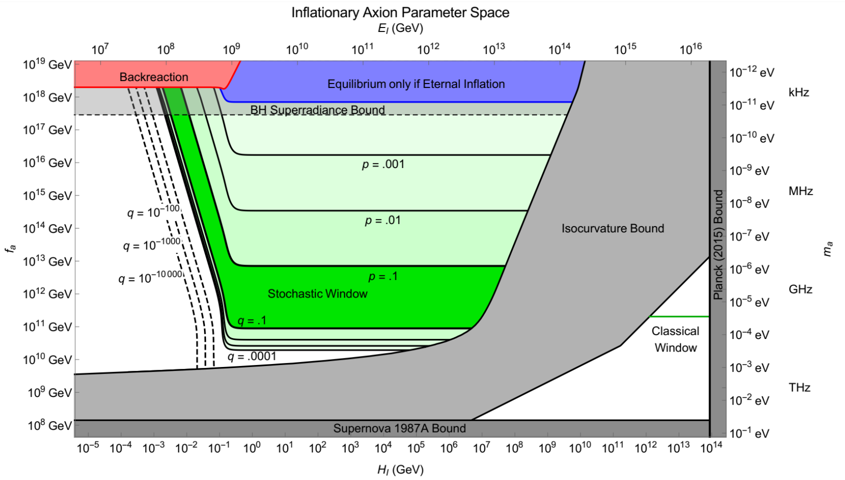
$$\frac{\partial P}{\partial N} = \frac{m\left(T\right)^2}{3H^2(N)}\frac{\partial}{\partial\theta}\left(\sin\left(\theta\right)P\right) + \frac{H^2(N)}{8\pi^2 f^2}\frac{\partial^2 P}{\partial\theta^2}$$

$$m\left(T\right) \sim \begin{cases} \frac{\Lambda^2}{f}\left(\frac{T_{\rm QCD}}{T}\right)^n & T > T_{\rm QCD}\,, \quad n = 4, \quad T_{\rm QCD} = 150 \text{ MeV} \\ \frac{\Lambda^2}{f} & T \leq T_{\rm QCD}\,, \quad \Lambda = 100 \text{ MeV} \end{cases}$$

If H is independent of time :

$$P_{\rm stat}(\theta) = \frac{\exp\left[\frac{8\pi^2 m^2 f^2}{3H^4}\cos(\theta)\right]}{2\pi I_0\left(\frac{8\pi^2 m^2 f^2}{3H^4}\right)}$$

Axion window



Graham, Scherlis (2018)

Assumptions

Reaching equilibrium requires :

- $N_{\rm eq} \ll N_H$ (Hardwick, Vennin, Byrnes, Torrado, Wands 2017)

$$N_H = \epsilon_1^{-1} = |\mathrm{d}\ln H/\mathrm{d}N|^{-1}$$

$$N_{\rm eq} = \begin{cases} 16\pi^4 \frac{f^2}{H^2} & \text{if} \quad H \gg \Lambda \\ \frac{H^2}{m^2} & \text{if} \quad H \ll \Lambda \end{cases}$$

If $H = 10^{-5}\mathrm{GeV}$ and $f = 10^{10}\mathrm{GeV}$:

$$N_{\rm eq} = 10^{34} e\text{-folds}$$

If $H = 10^{5}\mathrm{GeV}$ and $f = 10^{10}\mathrm{GeV}$:

$$N_{\rm eq} = 10^{13} e\text{-folds}$$

Beyond the equilibrium distribution

- A long phase of inflation requires large-field completion
- Ubiquitous correction : mass term

$$V(\phi) = \frac{m_\phi^2}{2}\phi^2$$
$$H(N) = H_{\mathrm{end}}\sqrt{1 + 2\left(N_{\mathrm{end}} - N\right)}$$

Hypothesis :

- Pre-inflationary scenario : $H_{\mathrm{end}} < f$

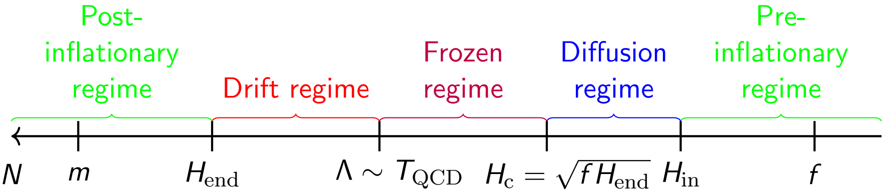
In quadratic inflation :

$$N_H = H^2 / H_{\mathrm{end}}^2$$

Equilibriation condition :

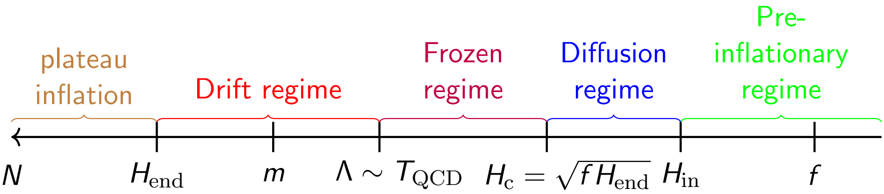
$$H >> H_{\mathrm{c}} \equiv \sqrt{f H_{\mathrm{end}}}$$

Quadratic inflation



$$H(N) = H_{\mathrm{in}} \sqrt{1 - 2N \left(\frac{H_{\mathrm{end}}}{H_{\mathrm{in}}} \right)^2}$$

Beyond slow-roll



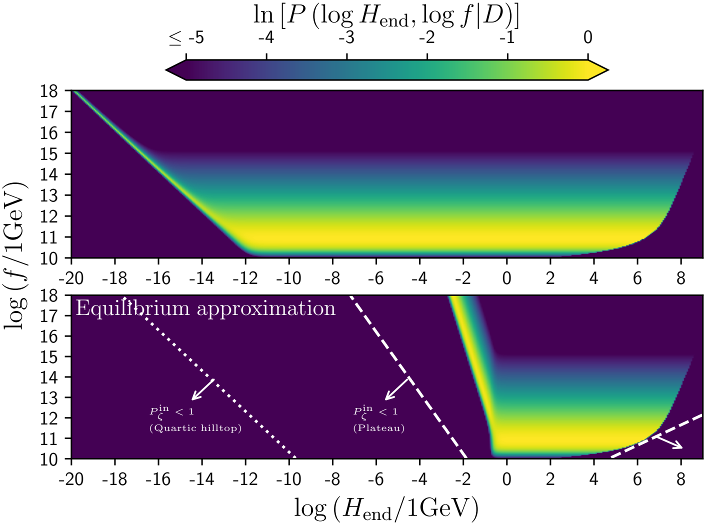
- Breakdown of slow-roll :

$$H(N) \approx m(N)$$

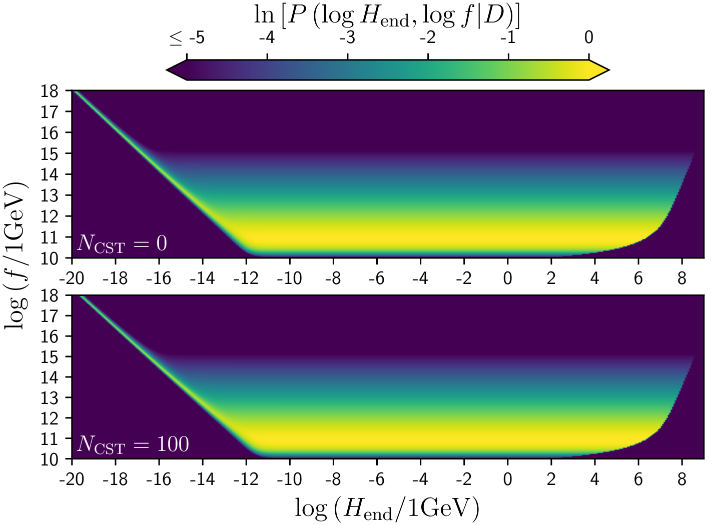
- Quantum diffusion is negligible when SR breaks down

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}N^2} + [3 - \epsilon_1(N)]\,\frac{\mathrm{d}\theta}{\mathrm{d}N} + \frac{m^2}{H^2(N)}\sin(\theta) = 0$$

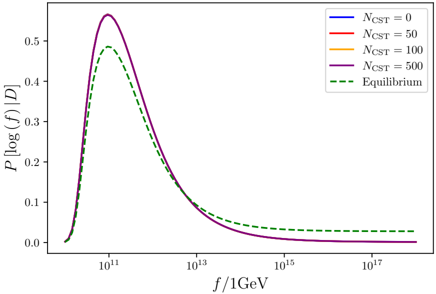
Equilibrium vs corrected

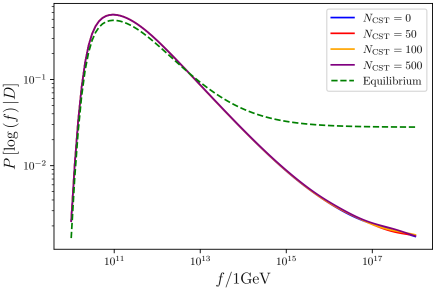


$N_{\mathrm{CST}} = 0$ vs $N_{\mathrm{CST}} = 100$

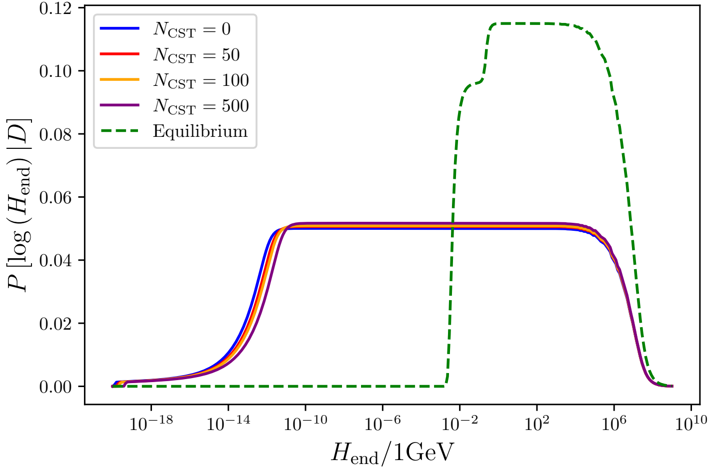


Marginalized constraints over Hend





Marginalized constraints over f



Conclusion

- Previous stochastic window

$$10^{10.4}\mathrm{GeV} \leq f \leq 10^{17.2}\mathrm{GeV}\,, \quad H_{\mathrm{end}} > 10^{-2.2}\mathrm{GeV}$$

- Corrected stochastic window with ubiquitous monomial corrections

$$10^{10.3}\mathrm{GeV} \leq f \leq 10^{14.1}\mathrm{GeV}\,, \quad H_{\mathrm{end}} > 10^{-13.8}\mathrm{GeV}$$

- Study the influence of a phase of plateau inflation
- Independent of the initial distribution if H_{in} is sufficiently large

Future directions

- Radial fluctuations
- ALP's