How to create a horizon in the lab
and
the route to measure entanglement in experiments

Quantum Optics group
Laboratoire Kastler Brossel, Paris

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TUG Annecy 05/11/2024

Today's talk

The propagation of waves in nonlinear media may be controlled to engineer situations where the waves propagate as though they were on an effectively curved geometry, like around a black hole or in an inflating universe. This enables the experimental study of field theories on curved geometries.

Controlled propagation of waves → effective geometry → linearised excitations
(engineered nonlinearity) (curvature) (quantum field)

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Spatial change in geometry: black hole

Temporal change in geometry: cosmology

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Controlled propagation of waves → effective geometry → linearised excitations
(engineered nonlinearity) (curvature) (quantum field)

Spatial change in geometry: black hole

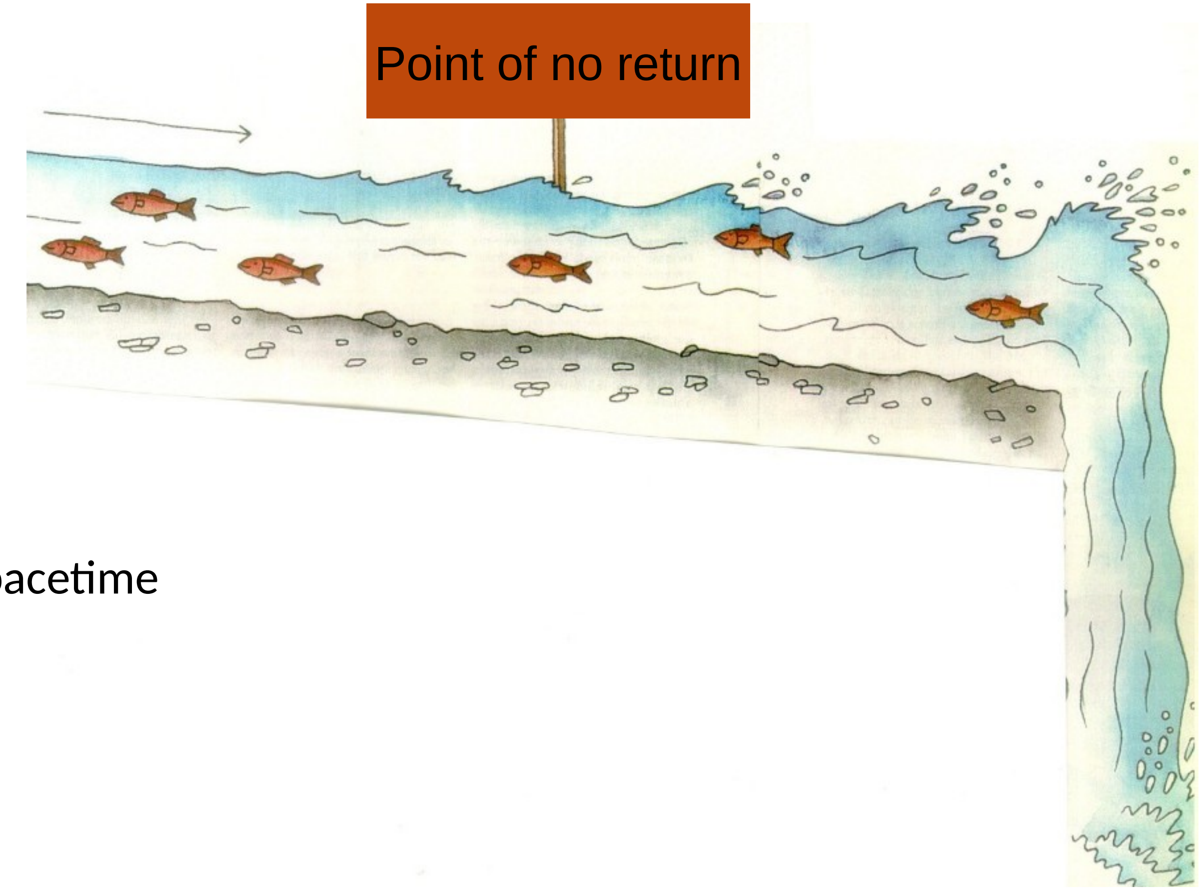
Temporal change in geometry: cosmology

Spontaneous emission from the vacuum

Correlated/entangled waves

Dynamical instabilities

Spacetime curvature



General Relativity identifies gravity with curvature of spacetime

Point of no return = event horizon

Foundations of analogue gravity

In a quantum fluid $\psi = \sqrt{n}e^{-i(\omega t + \phi(r))}$

Fluid velocity $\mathrm{v}_0 = \frac{\hbar}{m}\nabla\phi_0$ Speed of sound $c_s \propto \sqrt{\frac{g n_0}{m}}$

m – mass
g – interaction constant
n_0 – mean field density

Wave eq for collective excitations of quantum fluid $\psi = (\psi_0 + \psi_1)e^{-i(\omega t + \phi(r))}$

$$-\partial_t\left(\frac{n_0}{c_s^2}(\partial_t n_1 + v_0\nabla n_1)\right) + \nabla\left(n_0\nabla n_1 - \frac{n_0 v_0}{c_s^2}\partial_t n_1 + v_0\nabla n_1\right) = 0$$

Relativistic form of wave eq for collective excitations: $|\eta|^{-1/2}\,\partial_\mu\left(\sqrt{|\eta|}\eta^{\mu\nu}\partial_\nu\psi_1\right) = 0$

with $$\eta_{\mu\nu} = \begin{pmatrix} -(c_s^2 - \boldsymbol{v_0}^2) & -v_o^x & -v_o^y \\ -v_o^x & 1 & 0 \\ -v_o^y & 0 & 1 \end{pmatrix}$$

Surface gravity

$$\kappa = \frac{1}{2c_s(x)}\frac{d}{dx}[v_0^2(x) - c_s^2(x)]|_{x_H}$$

Motion of collective excitations in inhomogeneous fluid flow ↔ scalar field on curved spacetime

Control parameters: ***v*₀, *c*ₛ**

Unruh *PRL* **46** 1351 (1981), Visser *Class Quant Grav* **15** 1767 (1998)

In a (quantum) fluid

Fluid velocity $\mathrm{v}_0 = \frac{\hbar}{m}\nabla\phi_0$ Speed of sound $\mathrm{c}_s \propto \sqrt{\frac{gn_0}{m}}$

m – mass
g – interaction constant
n_0 – mean field density

Possible geometries with $\eta_{\mu\nu} = \begin{pmatrix} -(c_s^2 - \boldsymbol{v_0}^2) & -v_o^x & -v_o^y \\ -v_o^x & 1 & 0 \\ -v_o^y & 0 & 1 \end{pmatrix}$

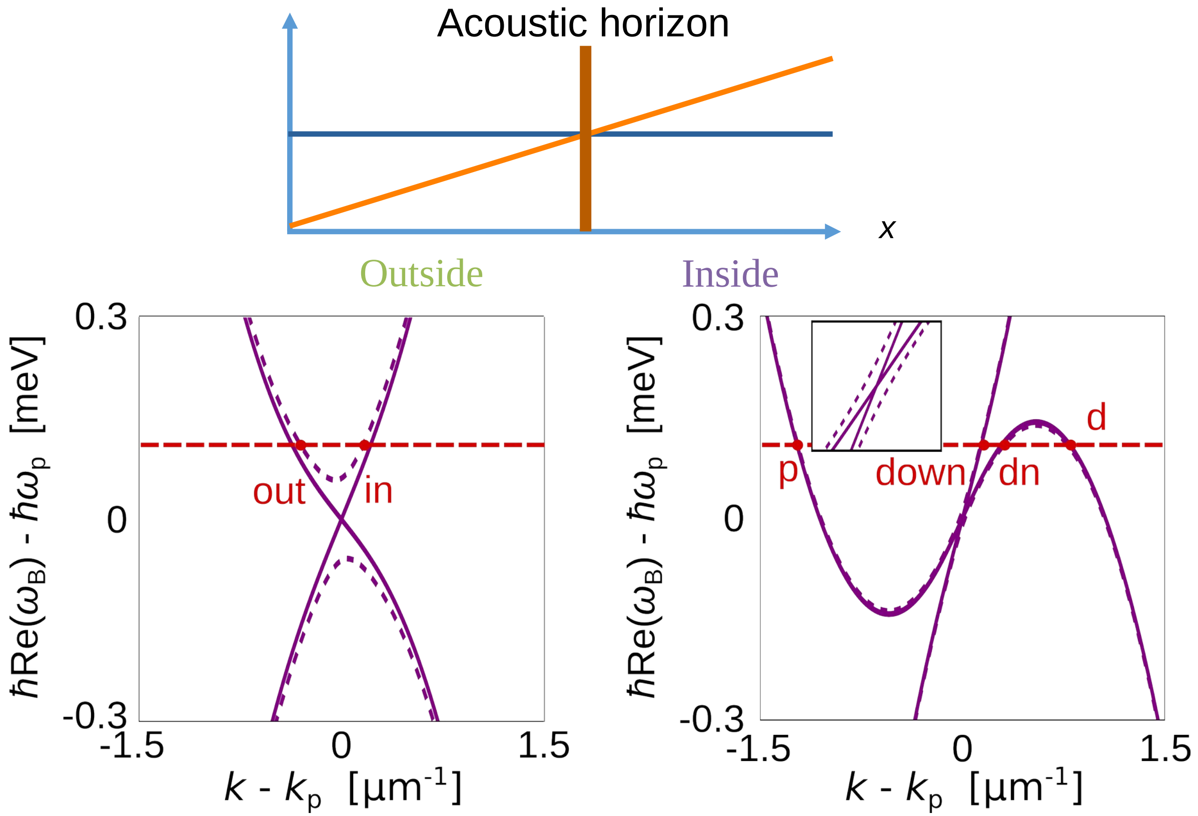
(i) transsonic flow along 1 spatial dimension → stationary 1D spacetime
Horizon where $\mathrm{v}_0 = c_s$

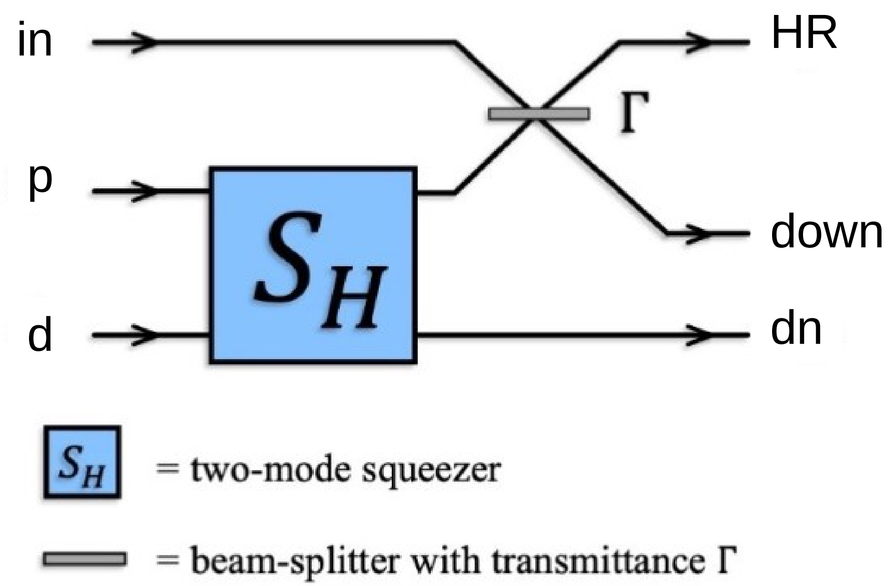
(ii) radially transsonic flow in 2 spatial dimensions → stationary spherically symmetric 2D spacetime
Horizon where $\mathrm{v}_r = c_s$

(iii) radially and azimuthally transsonic flow in 2 spatial dimensions → stationary rotating spacetime
Horizon where $\mathrm{v}_r = c_s$
Ergosurface where $|\boldsymbol{v_0}| = c_s$

Almeida *et al.* EPJH **48** 15 (2023)

Theory: Jacquet *et al* in prep 2024 + EPJD **76** 152 (2022),
Exp: Falque *et al* arXiv:2311.01392





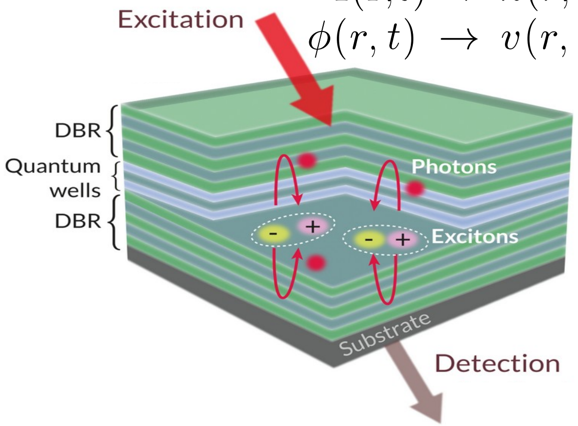
S_H controlled by surface gravity

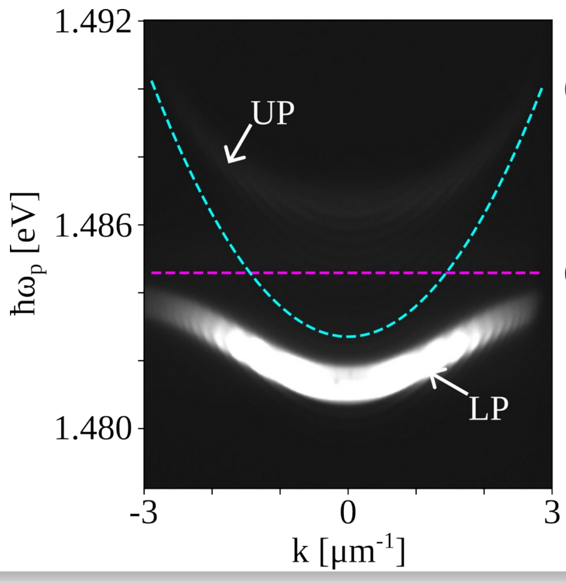
$$\kappa = \frac{1}{2c_s(x)}\frac{d}{dx}\left[v_0^2(x) - c_s^2(x)\right]\Big|_{x_H}$$

Agullo *et al* "Event horizons are tunable factories of quantum entanglement" Int. Jour. Mod. Phys. D **31** 2242008 (2022)

Microcavity polaritons

$$\mathrm{I}(\mathrm{r},\mathrm{t}) \rightarrow n(r,t)$$
$$\phi(r,t) \rightarrow v(r,t)$$





cavity photon
→ effective mass

exciton
→ interactions

Polaritons = photons dressed with material excitations that live in the cavity plane

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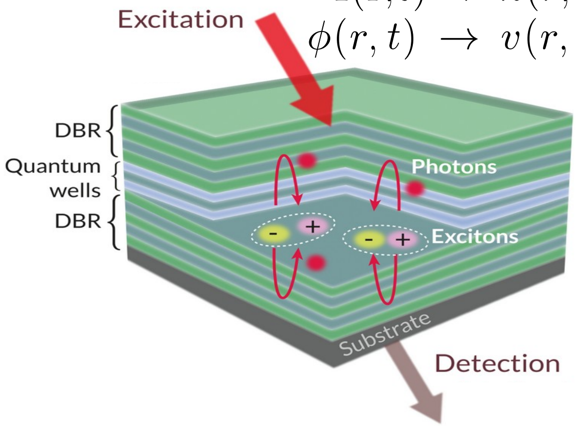

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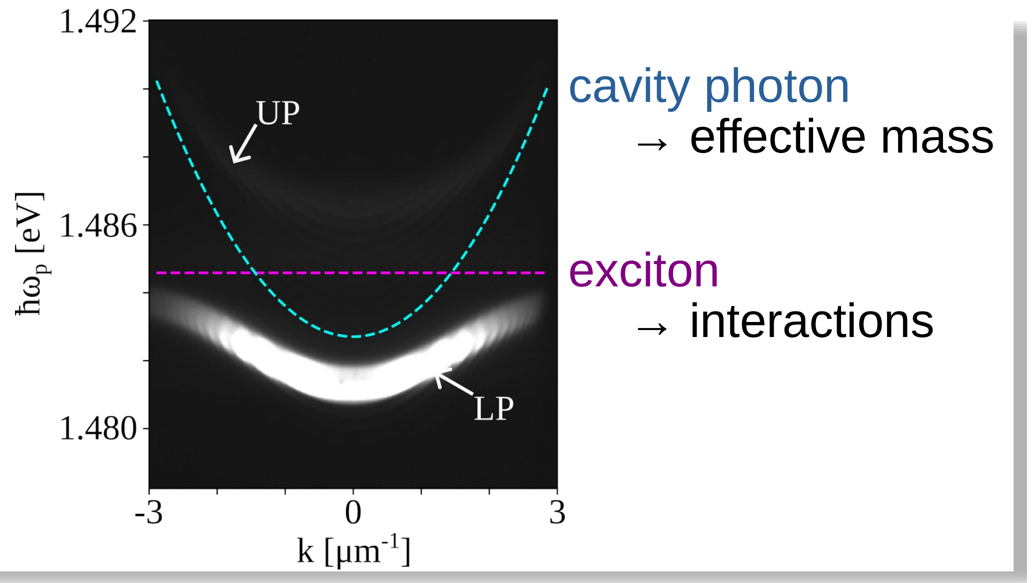


Dynamics in the cavity plane described by Gross-Pitaevskii (Nonlinear Schrödinger) equation:

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2\nabla^2}{2m^*_{LP}} + g\overbrace{n}^{|\psi|^2}\right)\psi - \frac{i\hbar\gamma}{2}\psi + P(r,t)$$

Driven-dissipative dynamics $\rightarrow$ Out-of-equilibrium system

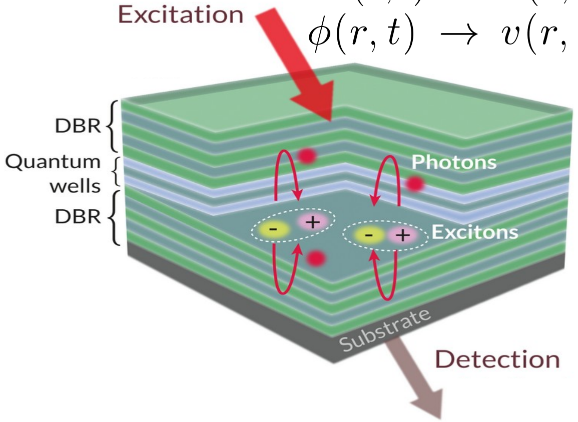
g polariton-polariton interaction constant

γ Losses

P pump

Microcavity polaritons

Carusotto and Ciutti RMP **85** 299 (2013)



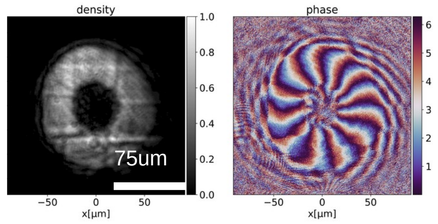
$\mathrm{I(r,t)} \rightarrow n(r,t)$

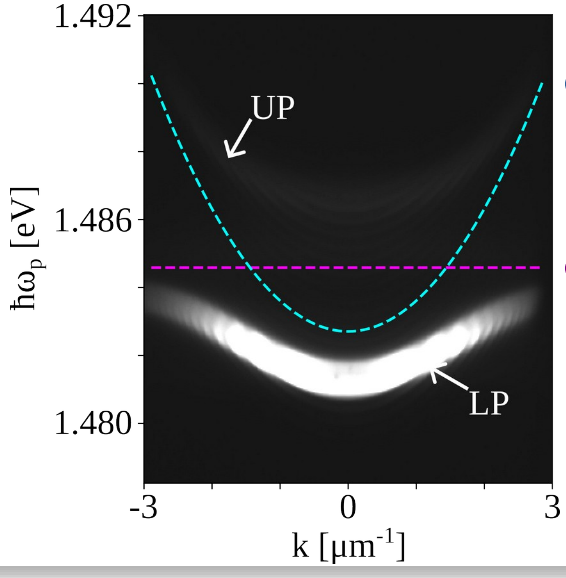
$\phi(r,t) \rightarrow v(r,t)$

SLM to control phase and mode of pump

Imaging photons leaking out of the cavity

$\mathrm{n(r,t)} \rightarrow I(r,t) \quad \mathrm{v(r,t)} \rightarrow \phi(r,t)$





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Polaritons = photons dressed with material excitations that live in the cavity plane

Dynamics in the cavity plane described by Gross-Pitaevskii (Nonlinear Schrödinger) equation:

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2\nabla^2}{2m^*_{LP}} + gn\right)\psi - \frac{i\hbar\gamma}{2}\psi + P(r,t)$$

$|\psi|^2$

Driven-dissipative dynamics $\rightarrow$ Out-of-equilibrium system

g polariton-polariton interaction constant

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P pump

Our sample: DBR GaAs, QW InGaAs, Q = 3000, T=4K, $\hbar\gamma/2 = 90\mu eV$

Bogoliubov spectrum out of equilibrium

Falque K *et al.*, arXiv:2311.01392

GPE: $$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2\nabla^2}{2m^*_{LP}} + gn\right)\psi - \frac{i\hbar\gamma}{2}\psi + P(r,t)$$

Linearise GPE around steady-state solution $\psi = (\sqrt{n_0} + e^{-\imath\gamma/2}\psi_1)e^{-i(\omega_p t+\phi_p r)}$

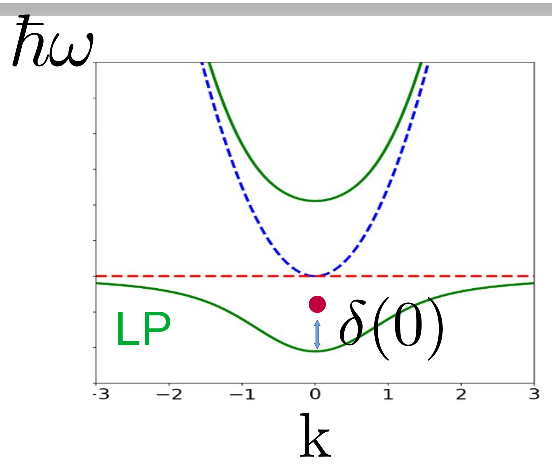
→ Bogoliubov – de Gennes dynamics for ψ_1

WKB dispersion relation $$\omega^{\pm}(\delta k) = \pm\sqrt{(\alpha^2k^4 + (k^2+m^2_{det})c_s^2} - \imath\frac{\gamma}{2}$$

higher order derivatives — pump-dependent mass — spectral linewidth

Quasi-resonant photon injection

$$\delta(k_p) = \omega_p - \omega_0 - \frac{\hbar k_p^2}{2m}$$
$$\delta(0) > \frac{\sqrt{3}}{2}\gamma$$



Pump-dependent mass

$$\boxed{\mathrm{m}_{det} \propto \delta(0) - gn_0}$$

$$\left[\frac{1}{\sqrt{|\eta|}}\partial_\mu\sqrt{|\eta|}\eta^{\mu\nu}\partial_\nu - \frac{(m_{\rm det})^2}{\hbar^2}\right]\psi_1 = 0$$

Falque K *et al.*, arXiv:2311.01392

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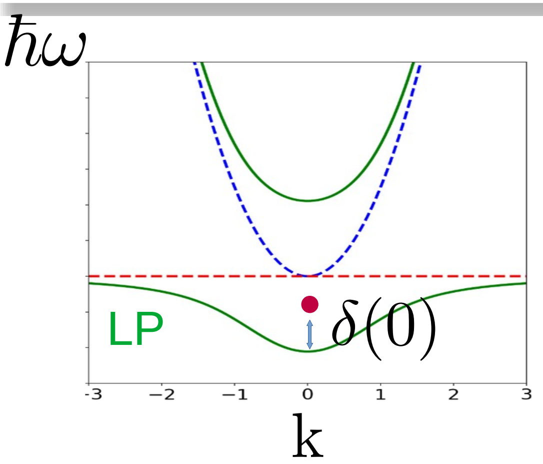
→ Bogoliubov – de Gennes dynamics for ψ_1

WKB dispersion relation $$\omega^{\pm}(\delta k) = \pm\sqrt{(\underbrace{\alpha^2 k^4}_{\text{nonlinearities}} + (k^2 + \underbrace{m^2_{det}}_{\text{pump-dependent mass}})c_s^2} - \underbrace{\imath\frac{\gamma}{2}}_{\text{spectral linewidth}}$$

nonlinearities pump-dependent mass spectral linewidth



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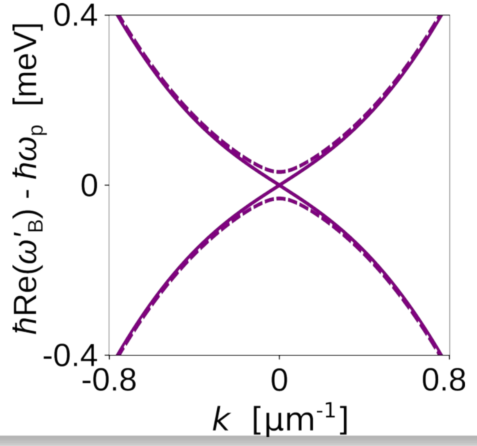


Norm of Bogoliubov excitations

Jacquet MJ *et al.*, EPJD **76** 2022
Falque K *et al.*, arXiv:2311.01392

Expansion of acoustic field in terms of excitations

$$\psi_1 = \int d\omega (f_\omega \hat{a}_\omega + f_\omega^* \hat{a}_\omega^\dagger)$$

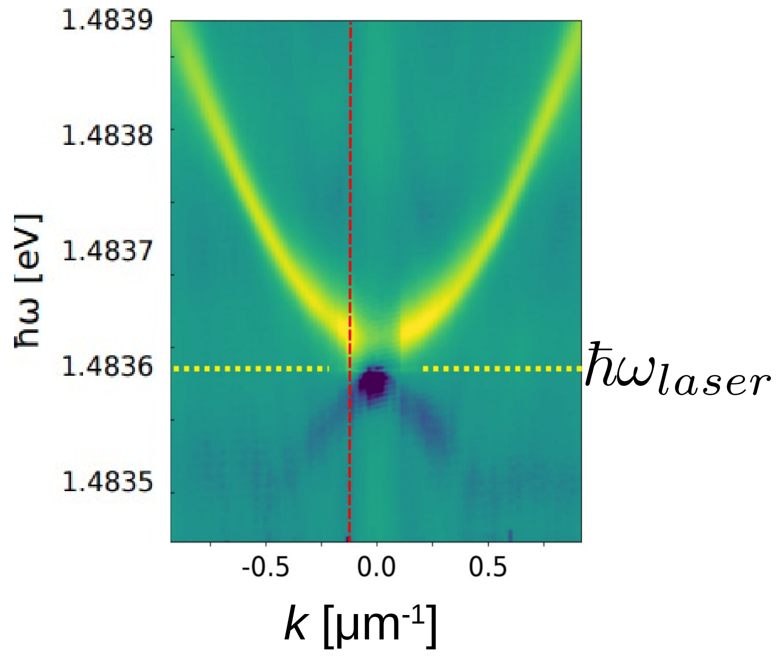
In fluid rest frame, excitations have frequencies

$$\omega^\pm(\delta k) = \pm\sqrt{(\alpha^2 k^4 + k^2 + m_{det}^2 c_s^2} - i\frac{\gamma}{2}$$

Norm of excitations = Noether charge

$$\mathrm{Q(f_\omega)} = i \int dx (f_\omega^* \partial_t f_\omega - \partial_t f_\omega^* f_\omega)$$

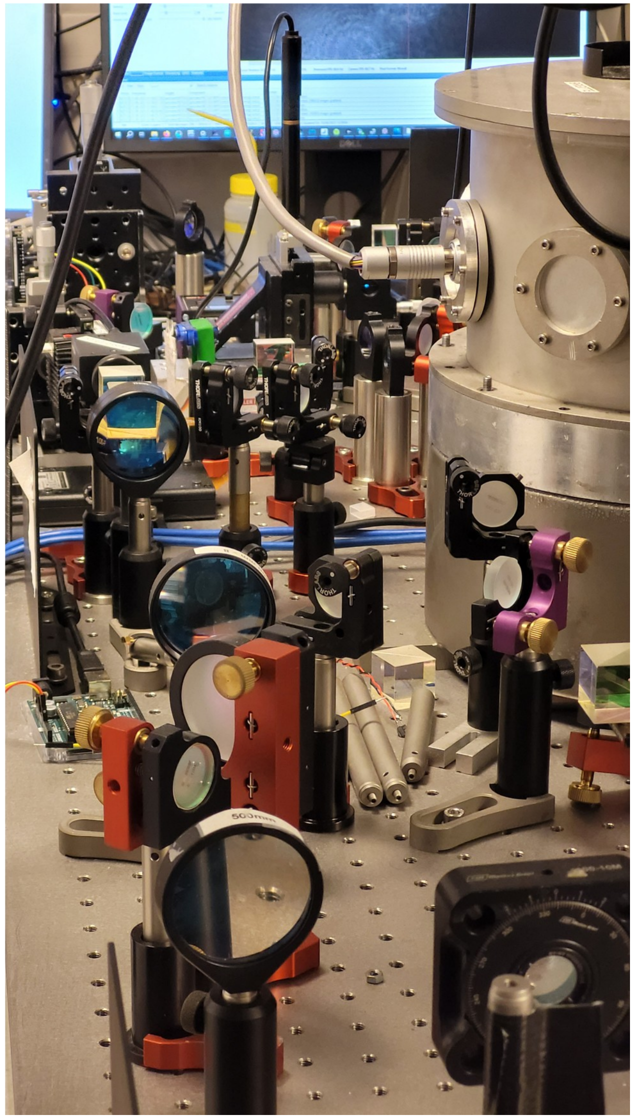
Dispersion relation in fluid rest frame

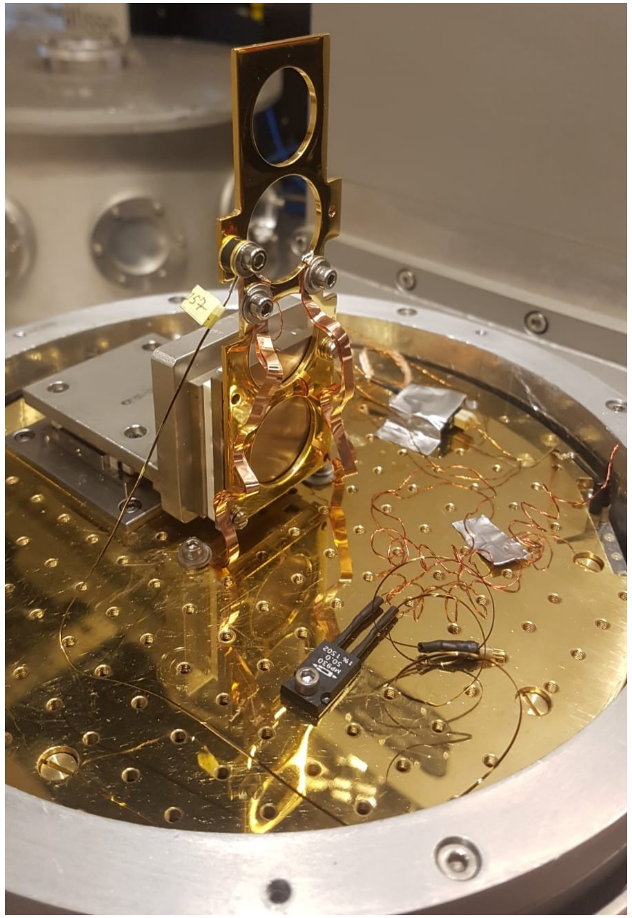


In fluid rest frame:

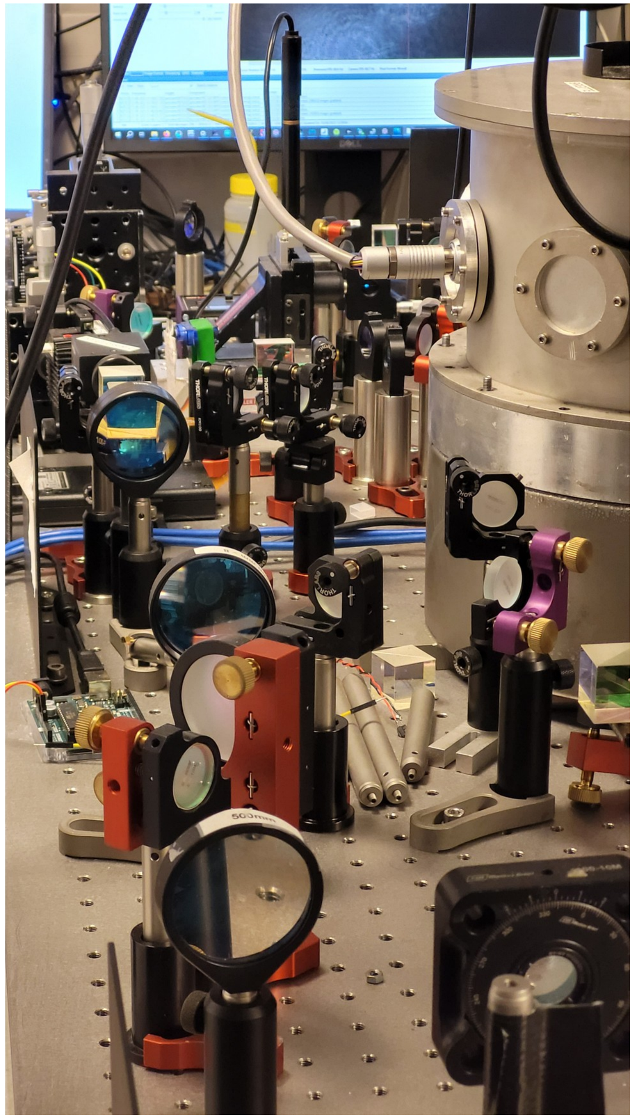
$\omega > \omega_{laser}$ positive-norm mode
$\omega < \omega_{laser}$ negative-norm mode

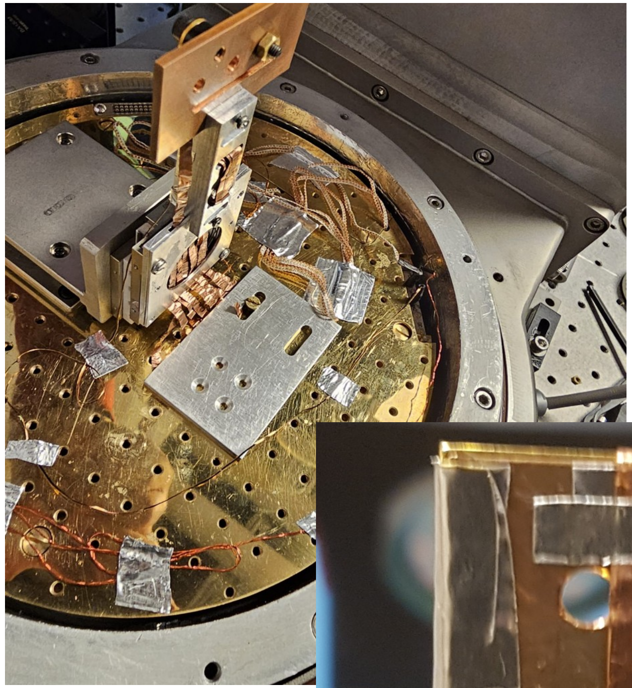
Experimental scheme

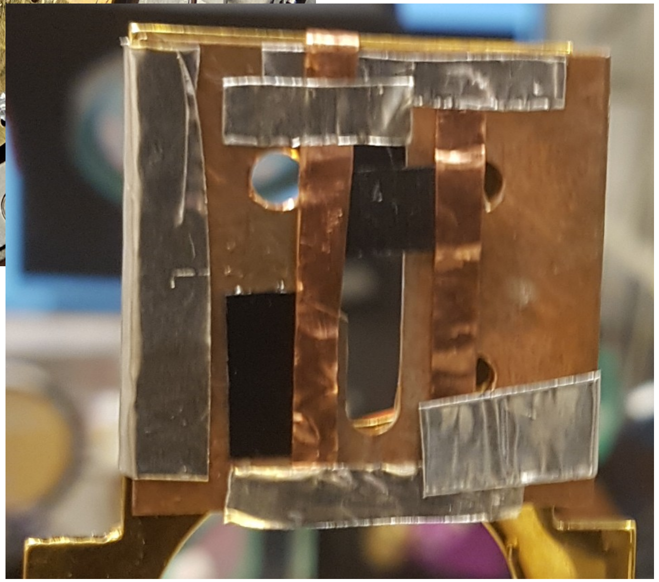




Experimental scheme



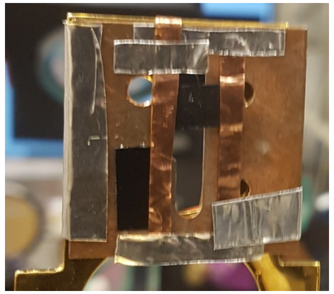


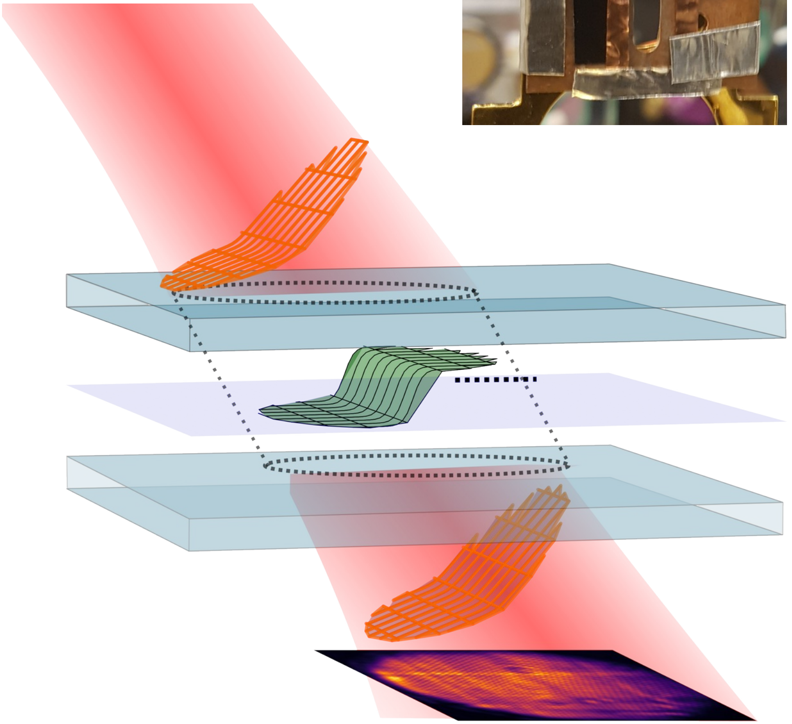


How to create the effective spacetime?

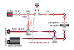
Falque K *et al.*, arXiv:2311.01392

$$\mathrm{F}_p(r,t) = e^{\frac{-r^2}{w(r)}} e^{\theta_p(r)} e^{-\imath\omega_p t}$$



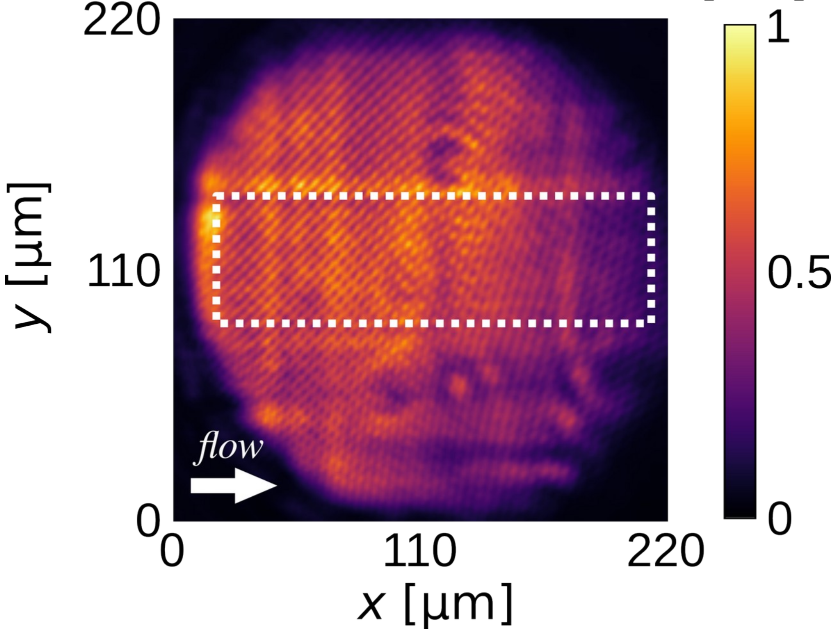


w(r)=180μm

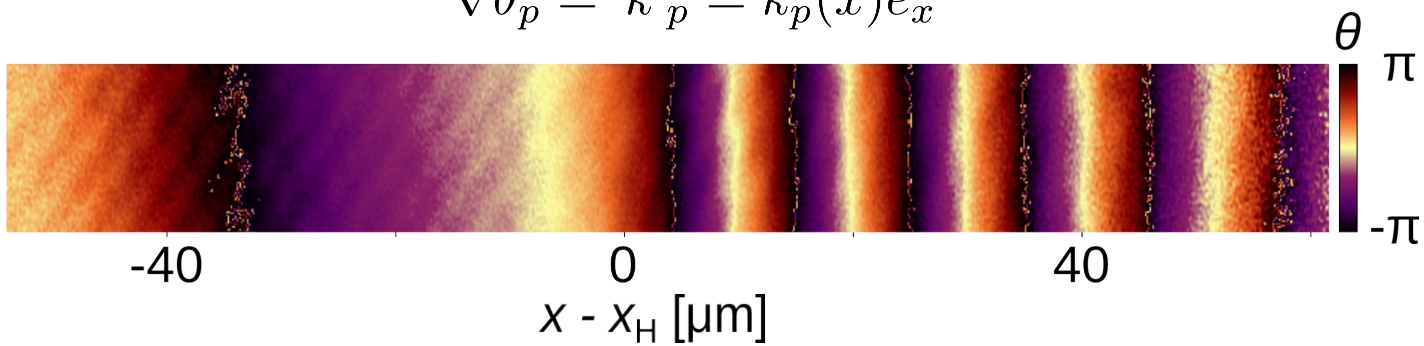


The effective spacetime

Output:



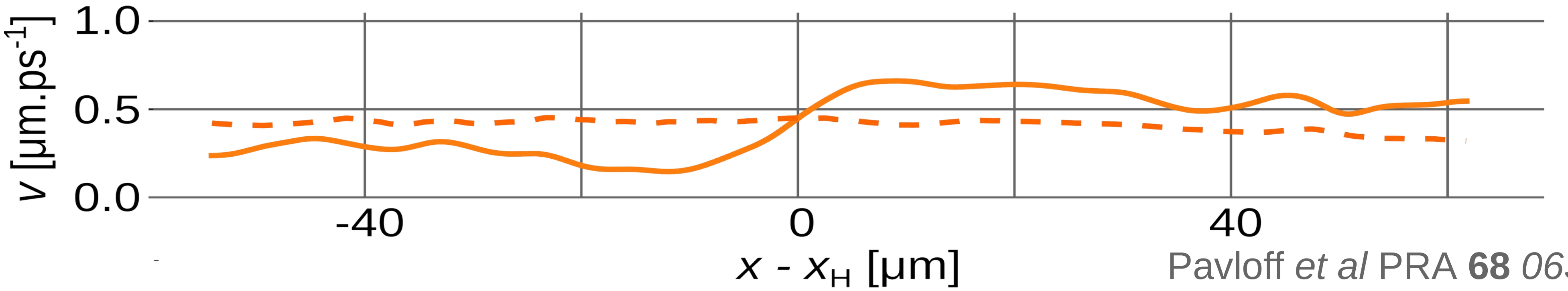
$$\nabla\theta_p = \overrightarrow{k}_p = k_p(x)\vec{e}_x$$



$$\mathrm{v}_0(x) = \frac{\hbar}{m^*}\partial_x\theta_p(x)$$

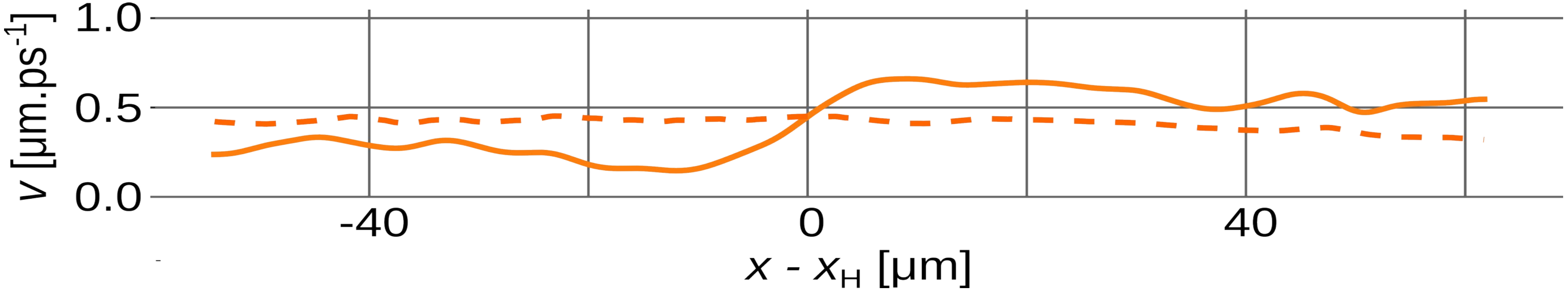
$$\mathrm{v}_0(x) = \frac{v_\mathrm{d}-v_\mathrm{u}}{2}\, tanh\left(\frac{x-x_\mathrm{H}}{w_\mathrm{H}}\right) + \frac{v_\mathrm{d}+v_\mathrm{u}}{2}$$

$$\mathrm{c}_s(x) = \sqrt{\frac{gn_0(x)}{m}}$$



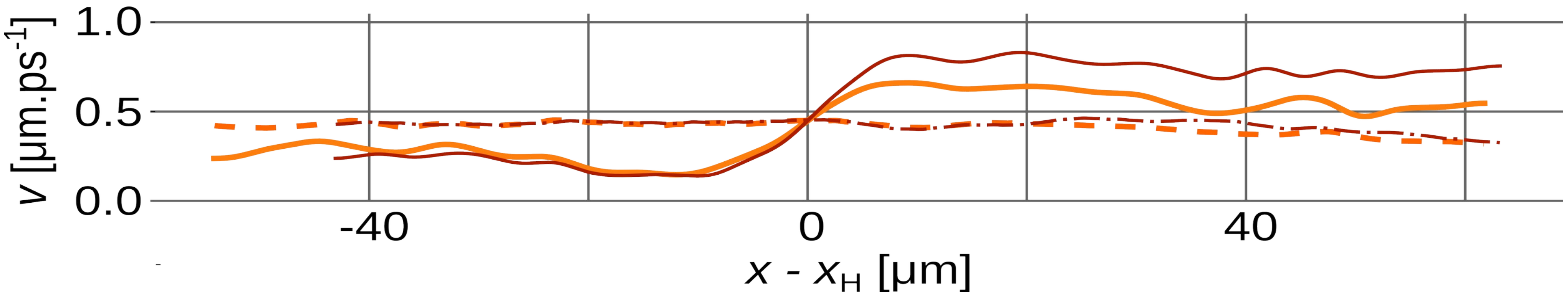
Pavloff *et al* PRA **68** *063608 (2003)*

Falque K *et al.*, arXiv:2311.01392



	$v_u = 0.27 \mu m.ps^{-1}$	$v_d = 0.53 \mu m.ps^{-1}$
	$c_s = 0.4 \mu m.ps^{-1}$	$c_s = 0.4 \mu m.ps^{-1}$
$M = v/c_s$	0.6	1.4

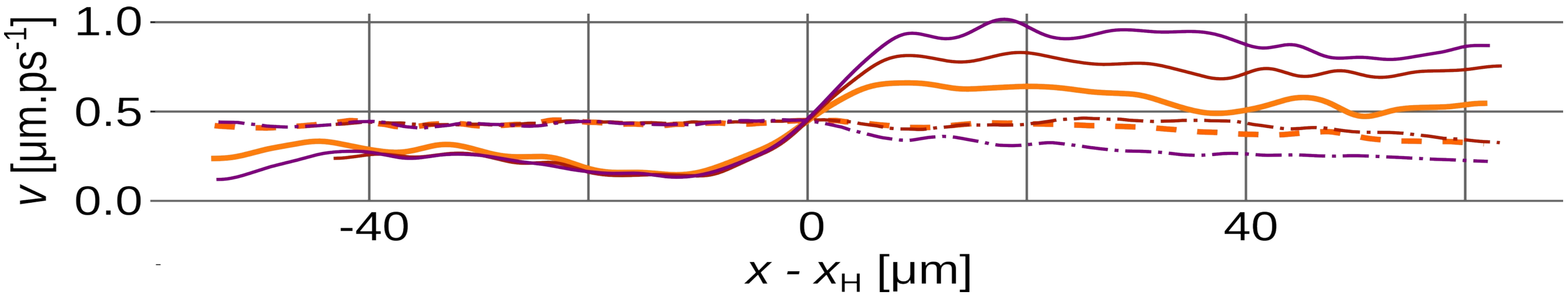
Falque K *et al.*, arXiv:2311.01392



$\mathrm{M=v/c}_s$	0.6	1.4
$\mathrm{M=v/c}_s$	0.6	1.9

$$\mathrm{v}_0(x) = \frac{v_\mathrm{d}-v_\mathrm{u}}{2}\, tanh(\frac{x-x_\mathrm{H}}{w_\mathrm{H}}) + \frac{v_\mathrm{d}+v_\mathrm{u}}{2}$$

Tuning the effective spacetime

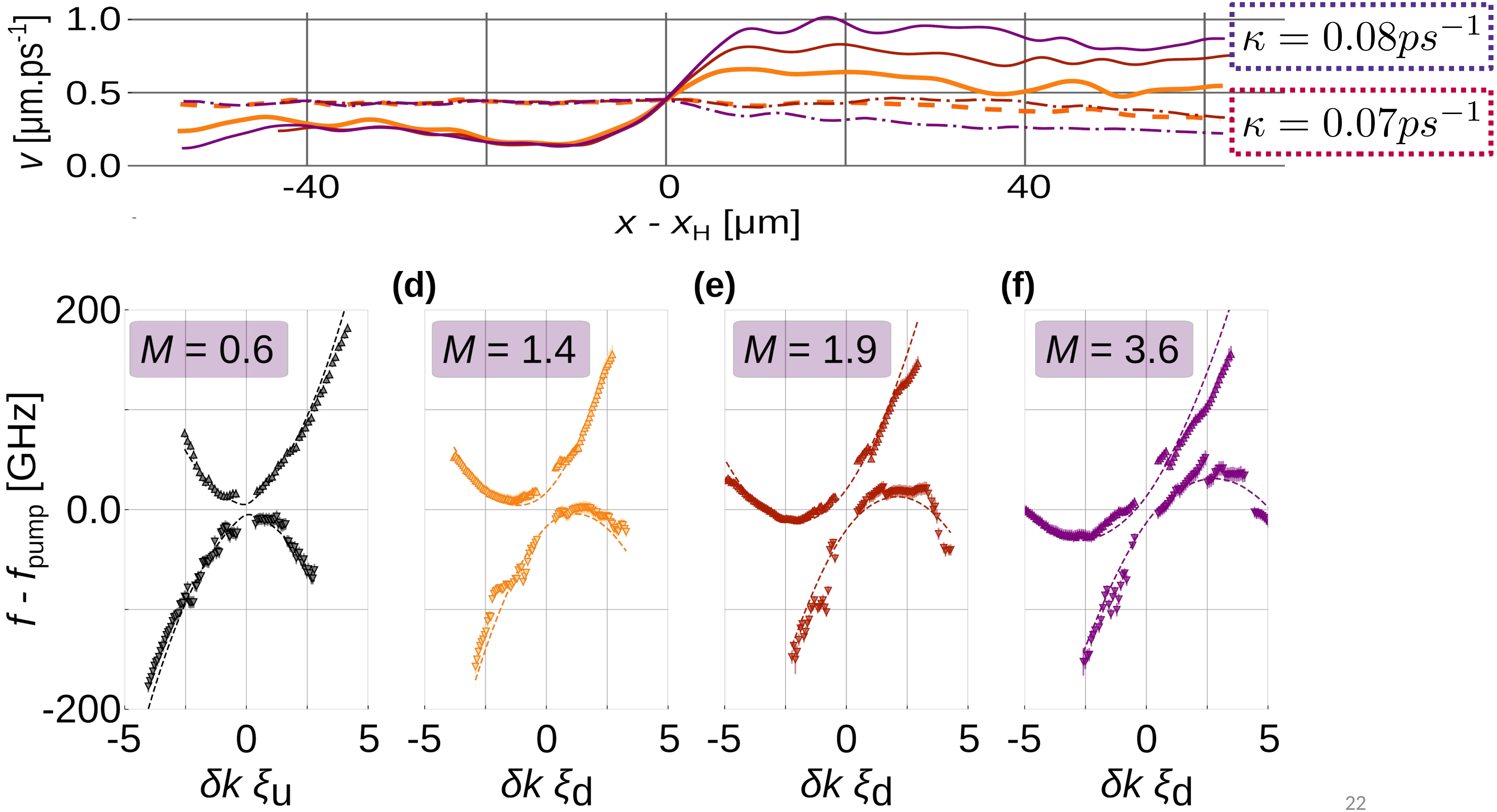


$\mathrm{M{=}v/c}_s$	0.6		1.4
$\mathrm{M{=}v/c}_s$	0.6	$0.07\mathrm{ps}^{-1}$	1.9
$\mathrm{M{=}v/c}_s$	0.6	$0.08\mathrm{ps}^{-1}$	3.6

Strength of emission controlled by

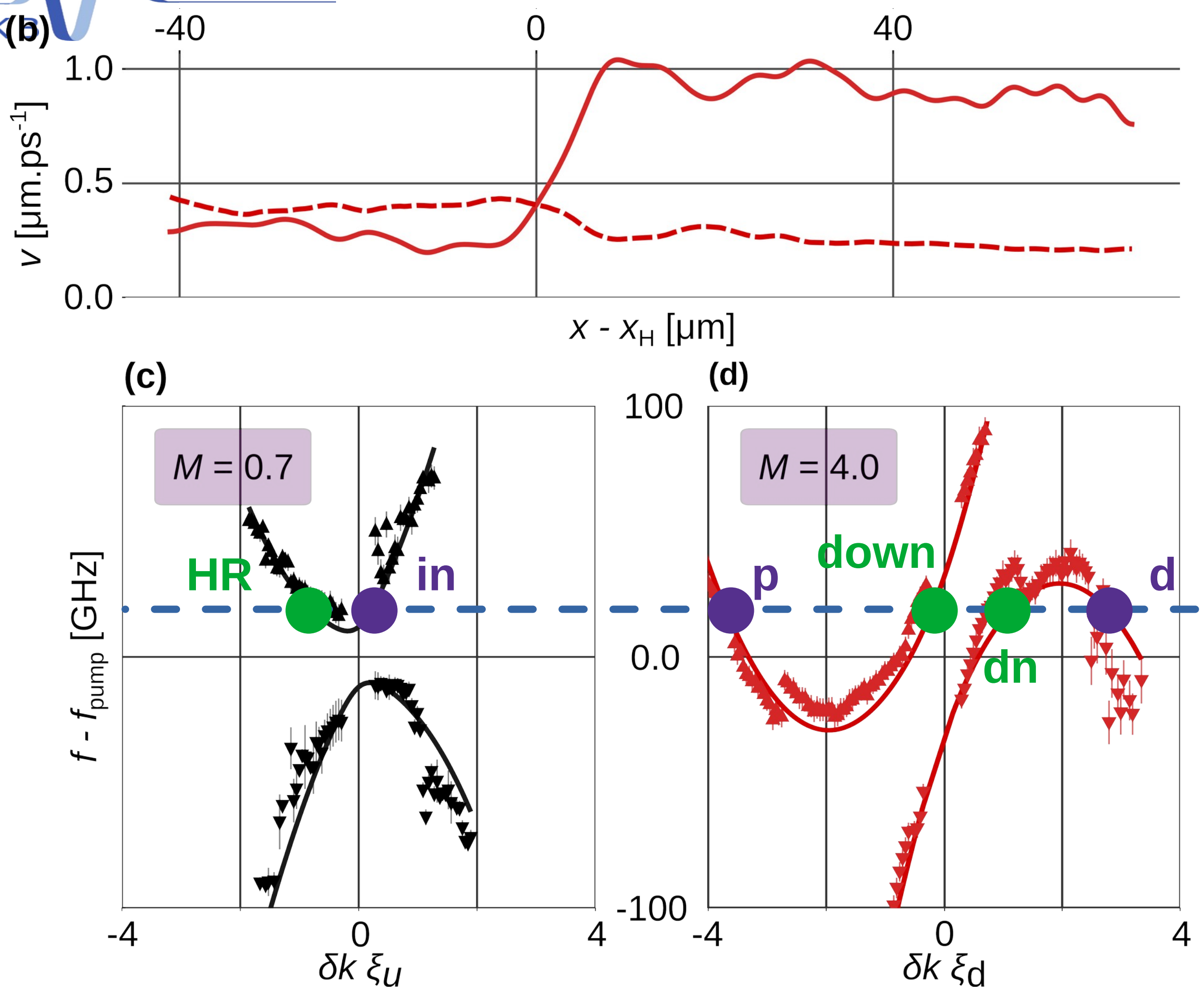
$$\kappa = \frac{1}{2c_s(x)}\frac{d}{dx}[v_0^2(x) - c_s^2(x)]|_{x_H}$$

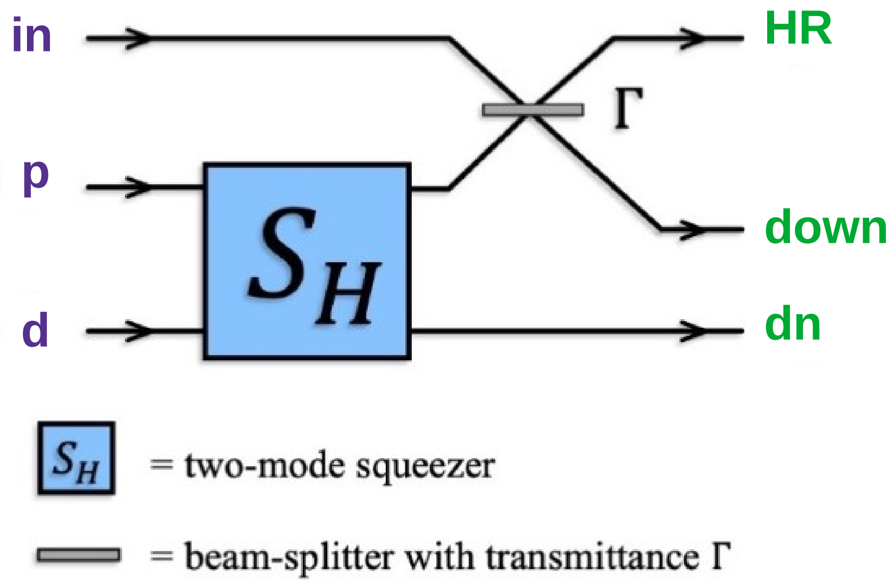
Bogoliubov spectrum in transsonic fluid



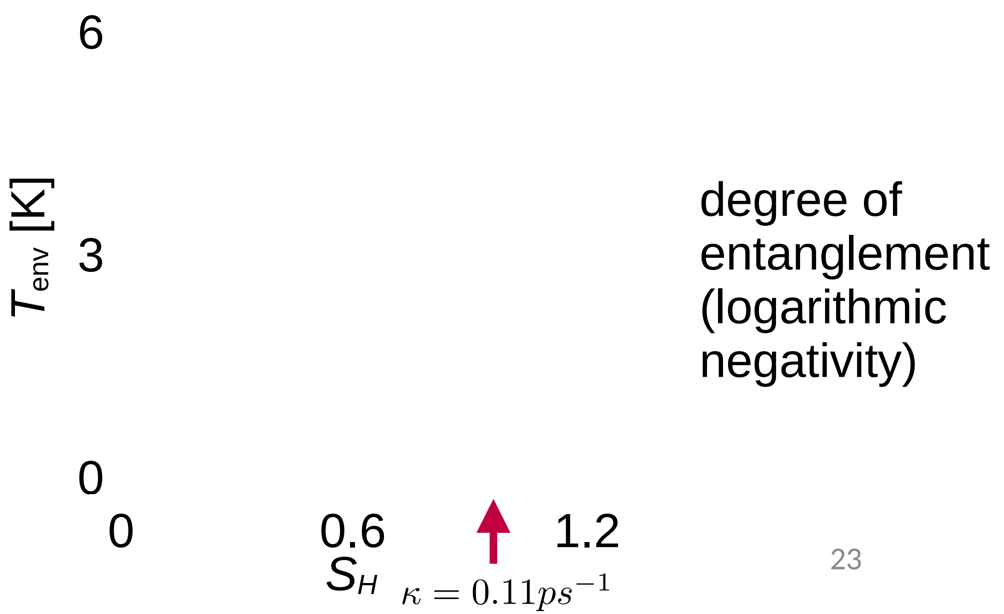
Entanglement by the Hawking effect

Theory: Jacquet *et al* in prep 2024
Exp: Falque *et al* arXiv:2311.01392





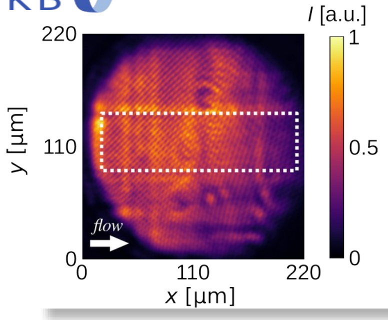
S_H controlled by surface gravity $\kappa = 0.11ps^{-1}$

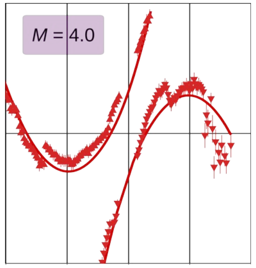


Direction of propagation: group velocity $\partial\omega/\partial k$

Hawking effect due to scattering on stationary potential

Today

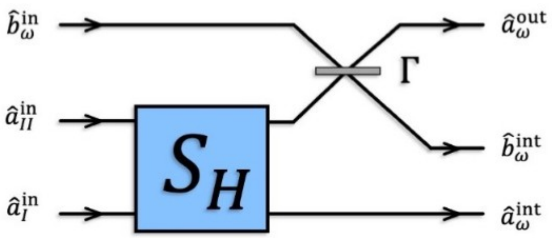




Experiments with polaritons

- High-resolution method to measure spectrum — PRL **129** 103601 2022, PRB **107** 174507 2023
- All-optical control of curvature
  - tunable surface gravity $\kappa \rightarrow$ observe two-mode squeezing
  - Measurement of spectrum $\rightarrow$ QFT

Experiment arXiv:2311.01392
Theory: EPJD **76** 152 2022
PRL **130** 111501 2023

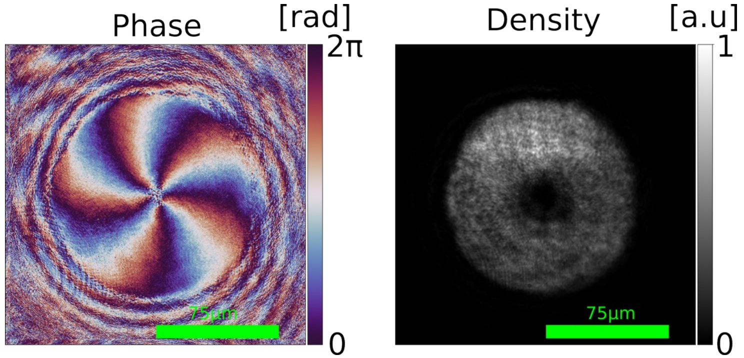


Quantum optics experiments

- Measure phase and density $\rightarrow$ access full field statistics and dynamics
- Homodyne detection to enhance signal strength and measure quantum correlations
- Enhance strength of emission and degree of entanglement by probing with squeezed state

I Agullo *et al* PRL **128** 091301 2022
arXiv:2307.06215

Where do we go from here?



Entanglement in rotating geometries?

Theory: PRD **109** 105024 2024

Winter school analogue gravity/cosmology in Benasque 7th - 17th January 2026