Strong Mixing At the Cosmological Collider

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Outline

- What is the cosmological collider?

- What is the strong mixing regime?

- How can we study it?

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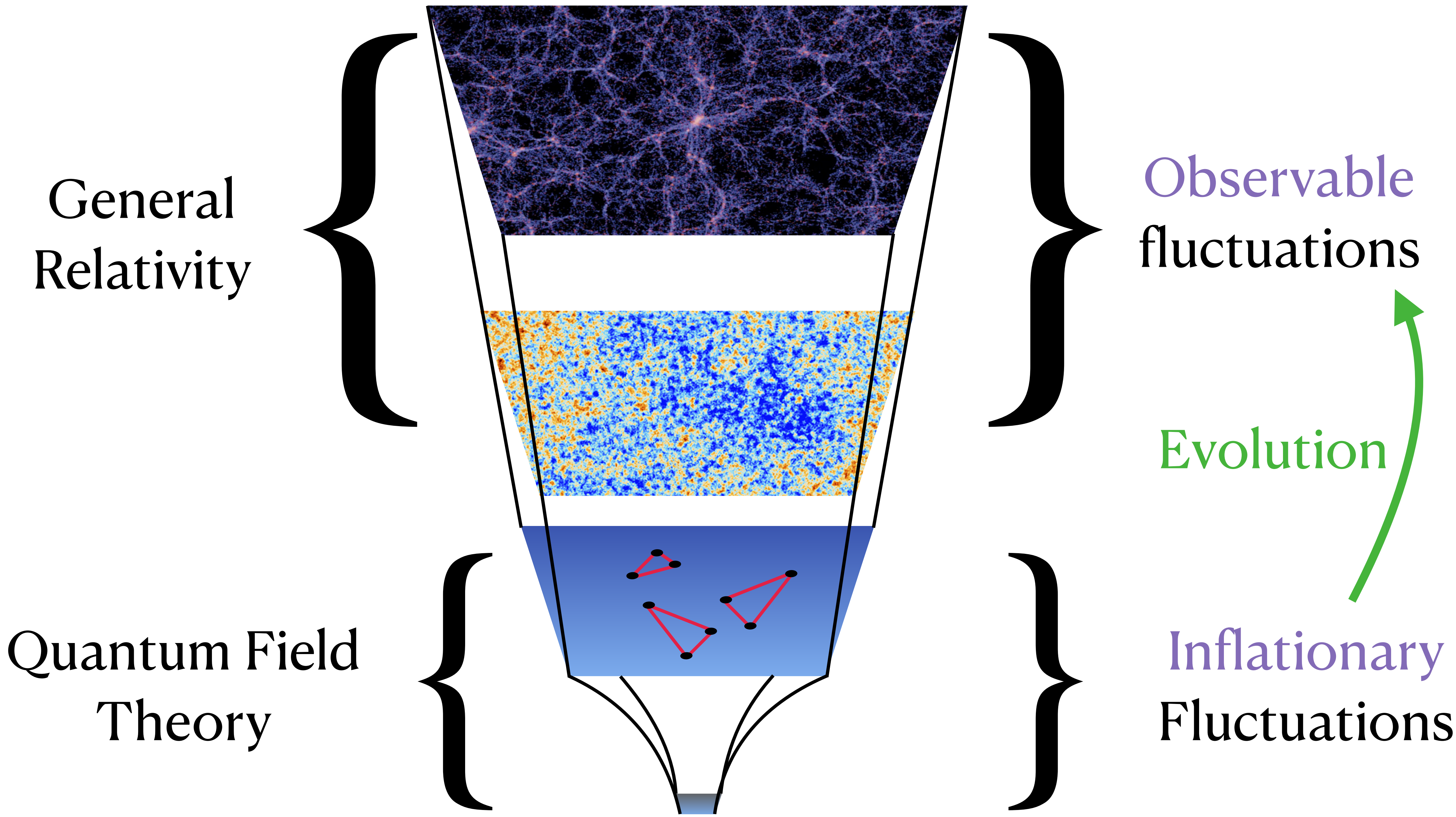
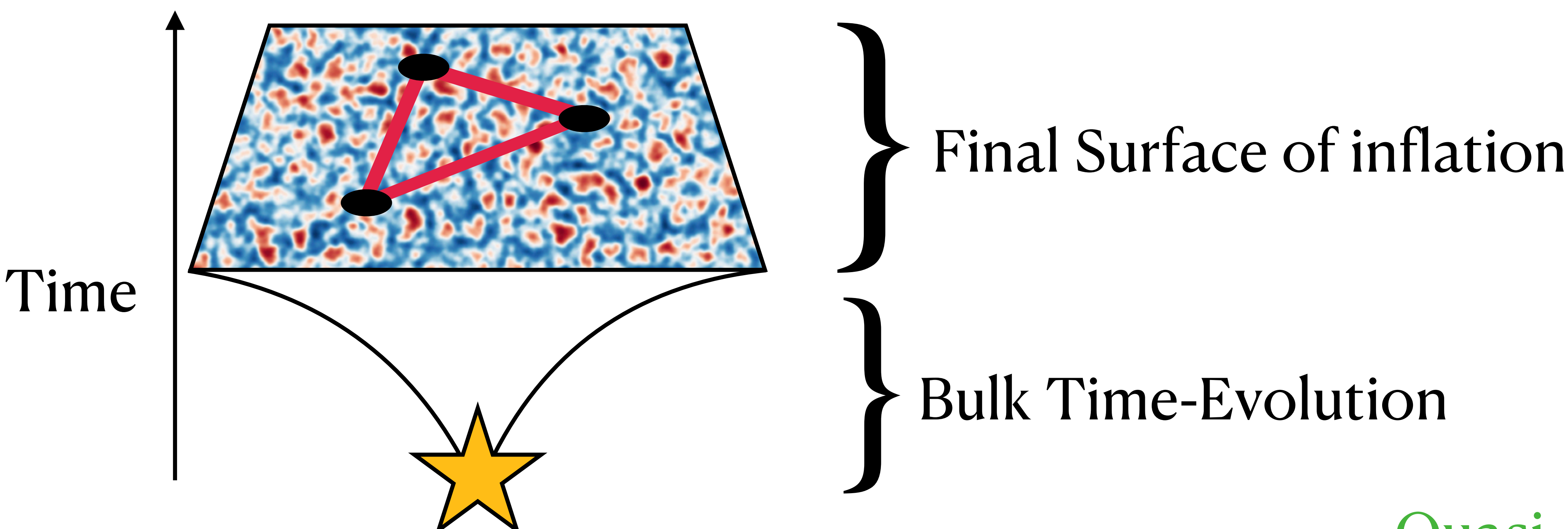
Inflation As Origine of Structure



Which Observable in Cosmology?

- We observe the primordial density fluctuations.



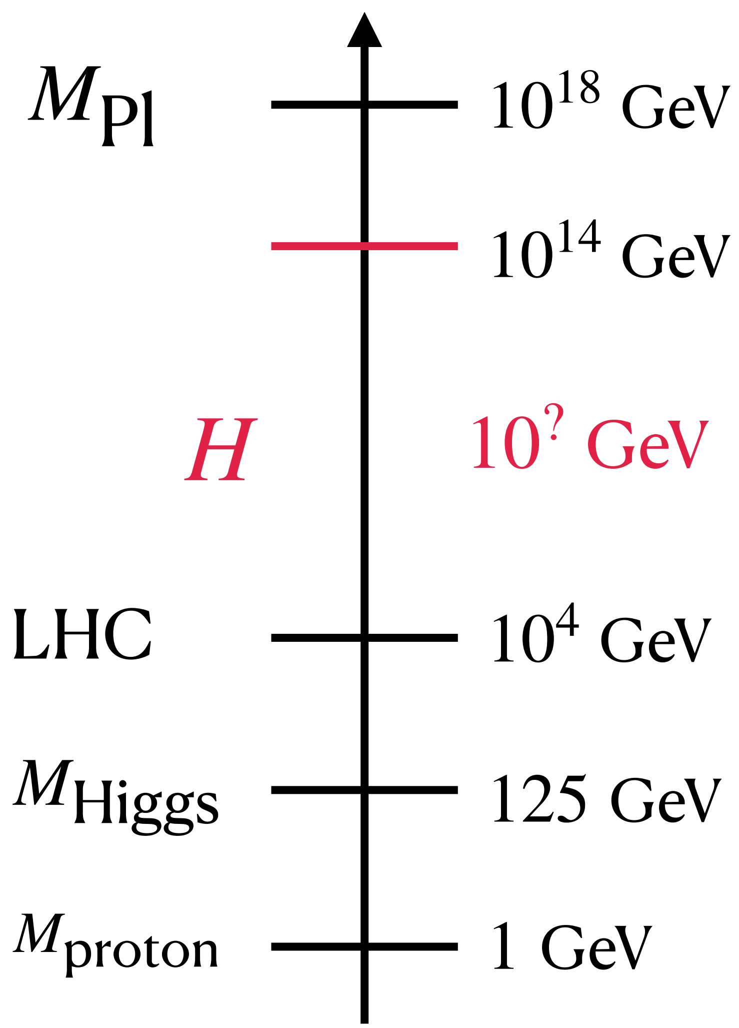
Quasi-Gaussian

- We predict their distribution $\mathbb{P}(\delta\rho_k)$
- The physics we want = encoded in the higher point correlators:

The Non-Gaussianities.

Energy Scales

- Inflationary physics = Very High energy scales.

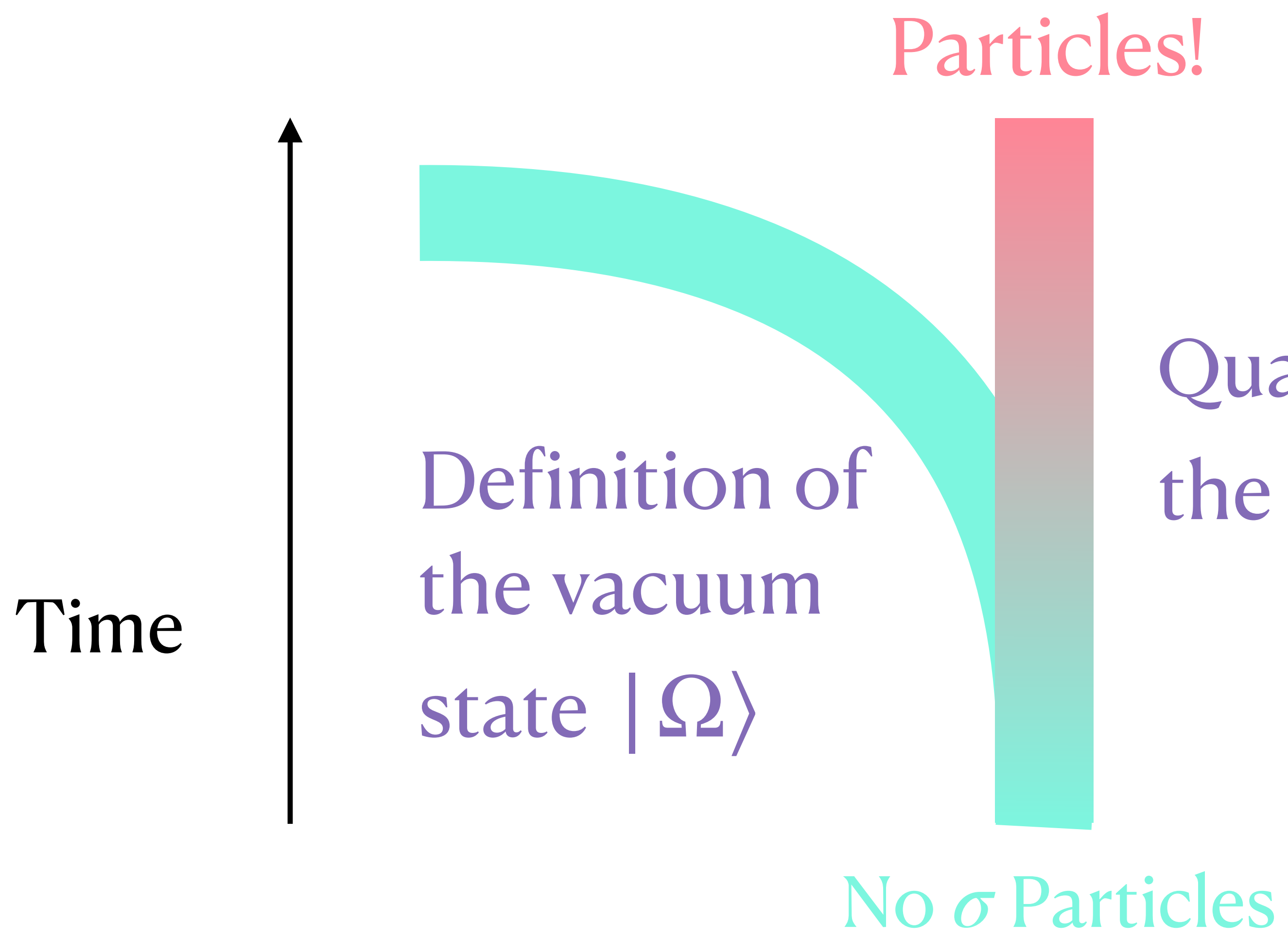


- PLANCK constraints: $H \lesssim 10^{14}$GeV
- **Energy Conservation**: we cannot produce on-shell particles heavier than 10^4GeV at the LHC.
- High-energy theories: often rely on the existence of very massive particles.

Idea: Use Inflation as a **Cosmological Collider**

Spontaneous Particle Production

- Expansion = time dependent background: No energy conservation.
- Massive particles are spontaneously produced!



- Initially, $|\Psi\rangle$ and $|\Omega\rangle$ coincide and they are driven away from each other by the expanding background.

Spontaneous Particle Production

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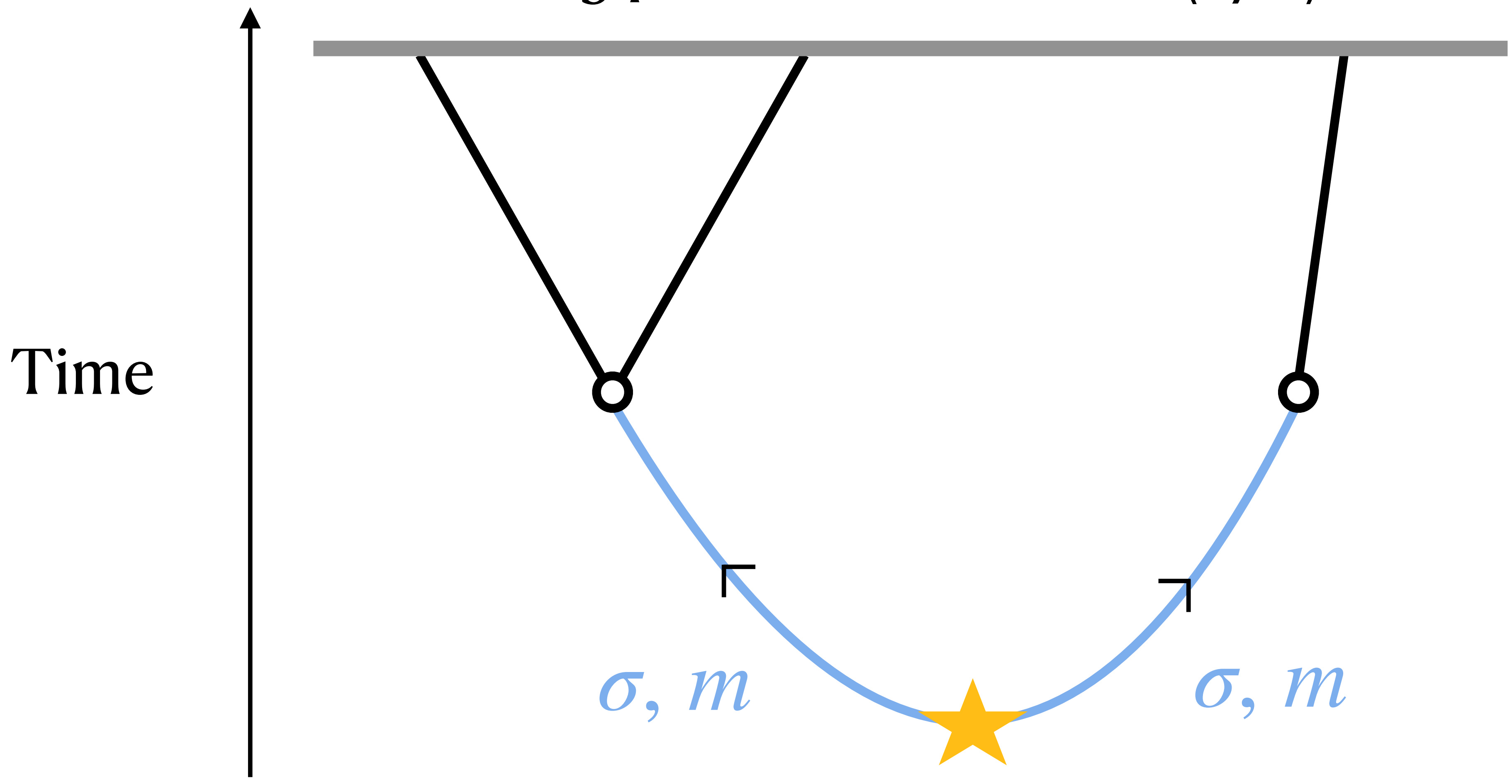
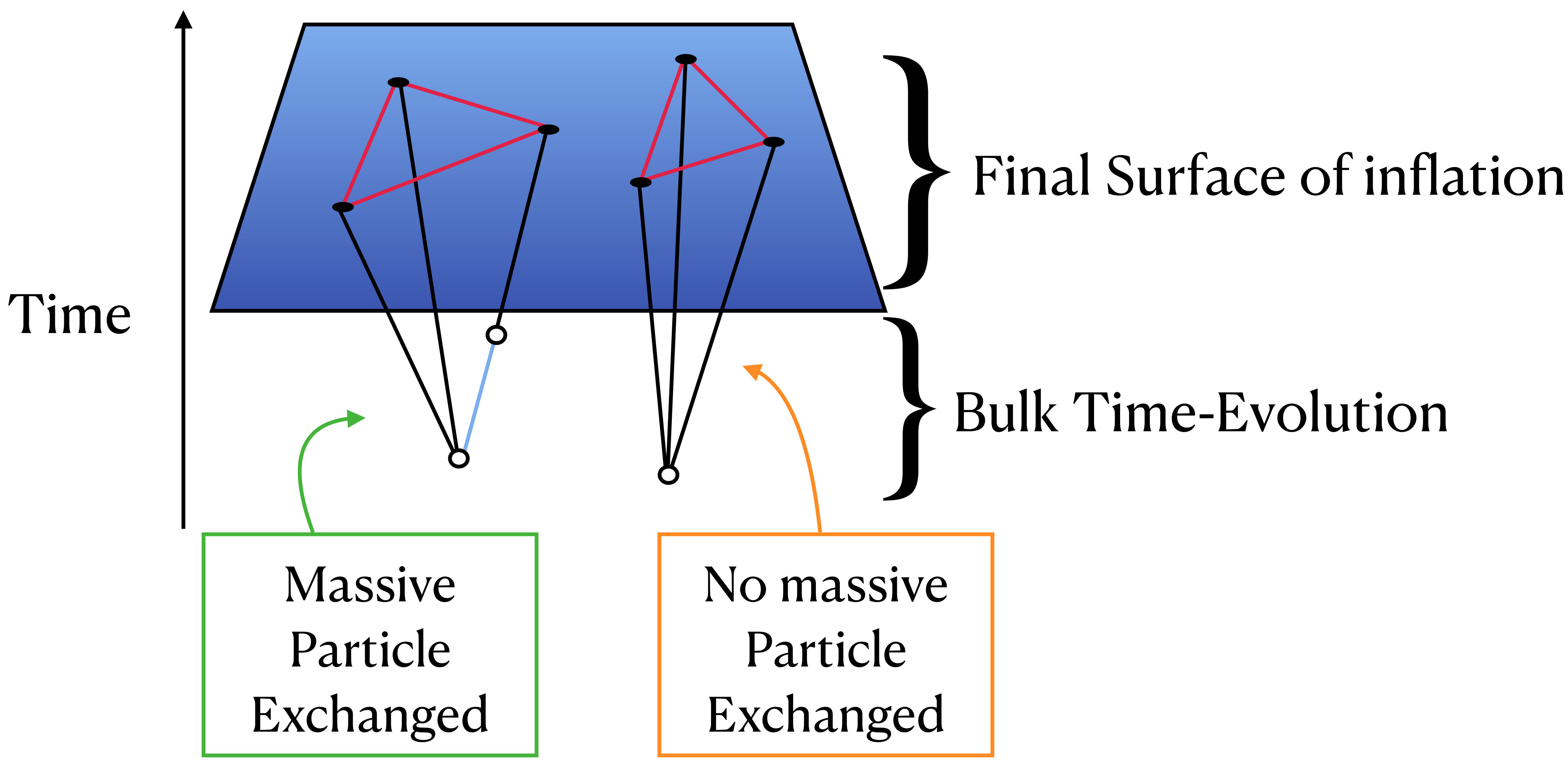
- Massive particles are **spontaneously** produced!
- Produced massive particles can decay into density fluctuations.

3 points correlations $\langle \delta\rho^3 \rangle$



Exchange Process in Inflation

- End of Inflation = Initial Condition for Large Scale Structures.
- Different process in the bulk leads to different correlations.



Cosmological Collider Signal

- Exchange of massive particles leads to oscillating behavior in the squeezed limit:

$$\langle \delta\rho^{k_1}\delta\rho^{k_2}\delta\rho^{k_3}\rangle \sim \left(\frac{k_3}{k_2}\right)^{1/2} e^{-\pi m/H}\cos(m/H\log(k_3/k_1)+\varphi)$$

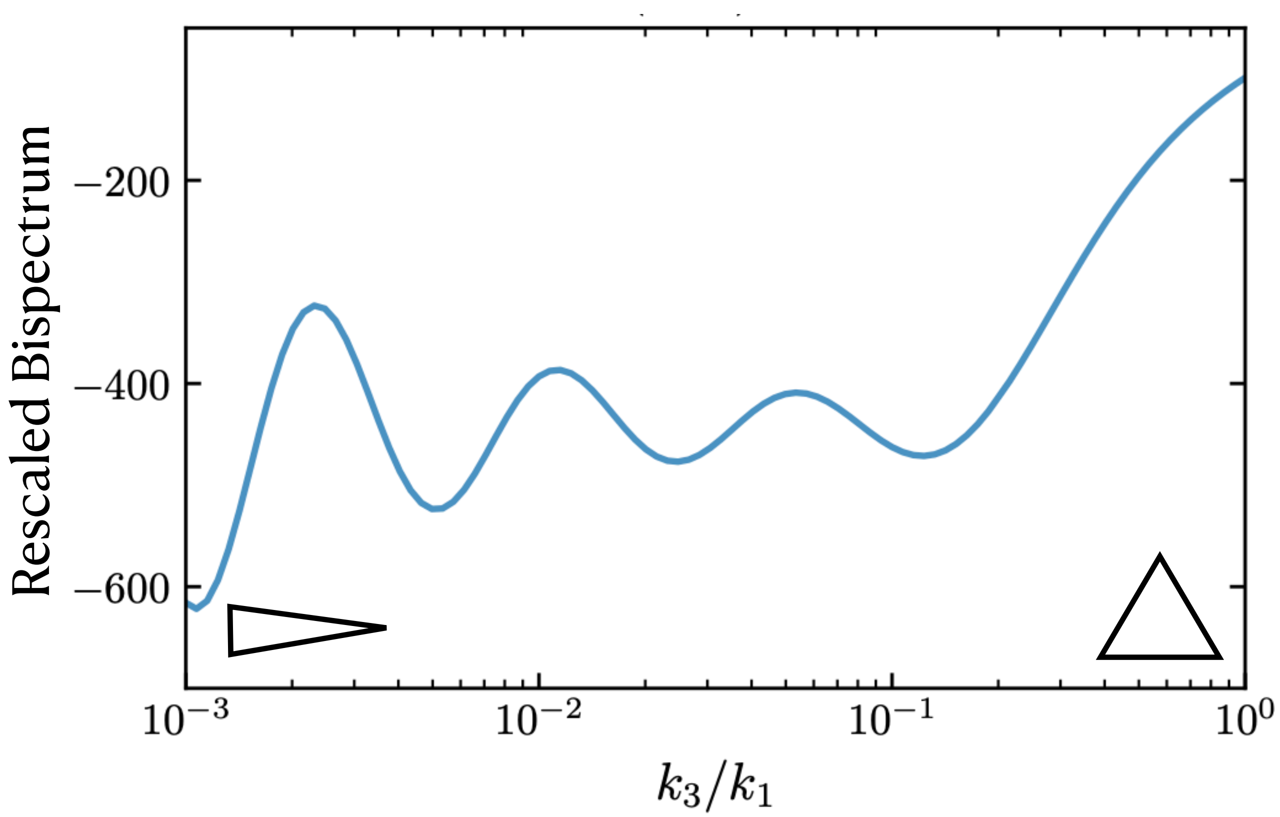


Figure from Werth, Pinol, Renaux-Petel, 2312.06559
Using CosmoFlow™

- Physically: property of massive field propagation if $m \gg H$:

$$\sigma'' - \frac{2}{\tau}\sigma' + \left(k^2 + \frac{m^2}{\tau^2 H^2}\right)\sigma = 0$$

$$\Longrightarrow \sigma \sim (k\tau)^{\frac{3}{2}\pm\Delta}, \Delta = \sqrt{\frac{9}{4} - \frac{m^2}{H^2}}$$

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Particle Physics In Inflation

- We can build a general theory of inflationary fluctuations.
- Inflation = scalar inflaton field + metric field.

Matter sector

$\pi \sim$ 1 degree of freedom

Einstein equations

Gravity sector

$\delta g_{\mu\nu} \sim$ 2 degree of freedom

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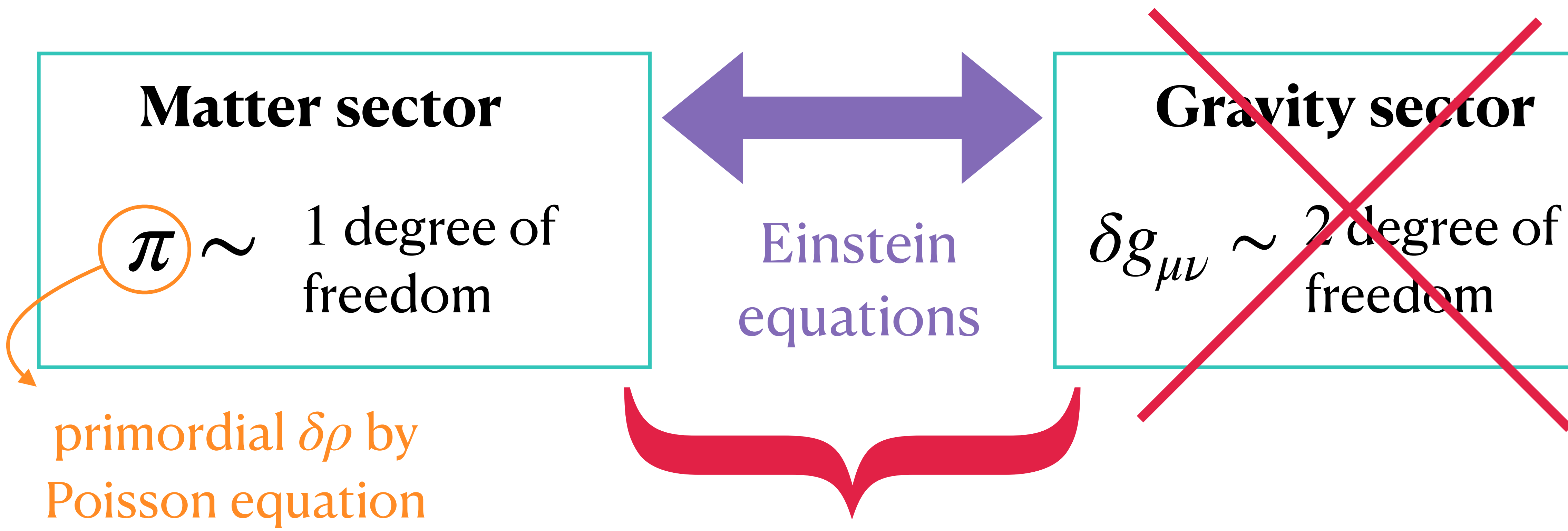

Decoupling in Slow Roll regime

- We can ignore the coupling with the tensor modes at the Leading order.

Effective Field Theory of Inflation

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$$\mathscr{L} = a^3 \left(\frac{\dot{\pi}^2}{2} + \frac{\dot{\sigma}^2}{2} - \frac{c_s^2}{2} \frac{(\nabla \pi)^2}{2a^2} - \frac{1}{2} \frac{(\nabla \sigma)^2}{2a^2} - \frac{m^2}{2} \sigma^2 + \rho \dot{\pi} \sigma - \frac{1}{2\Lambda a^2} (\nabla \pi)^2 \sigma \right)$$

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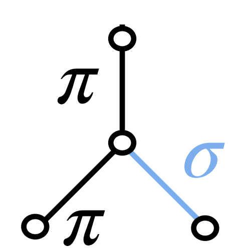
Cubic Coupling



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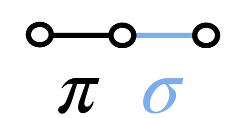
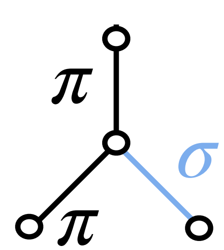
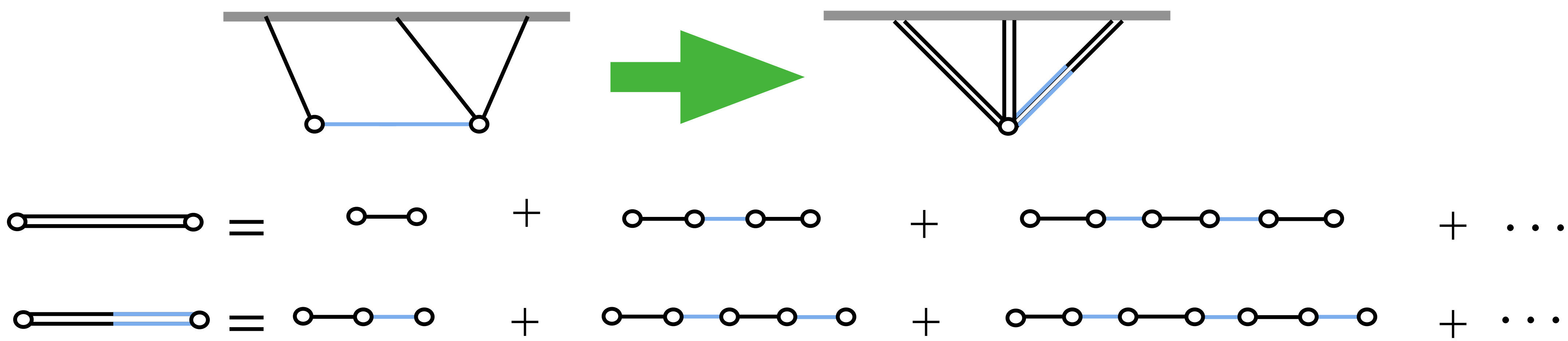
Mixing Term





Strong Mixing: Current Project

- Strong Mixing: $\rho \gg H$,  is giving a strong contribution.
- We cannot rely on the simpler diagrams!



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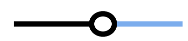
- Strong Mixing: $\rho \gg H$,  is giving a strong contribution.
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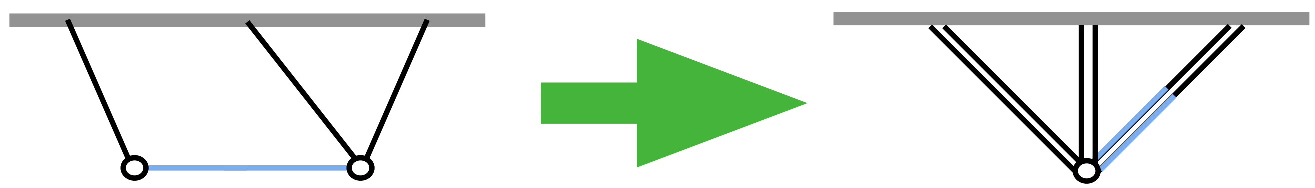
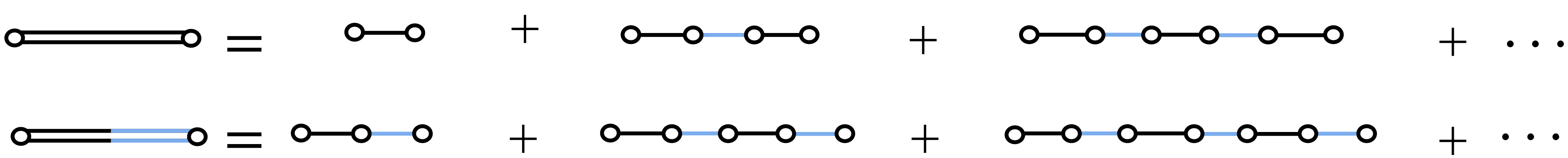


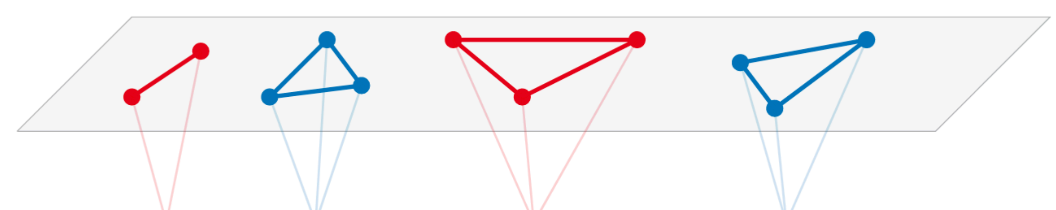


- Can be computed numerically:

  →Include mixing in the free Hamiltonian

CosmoFlow

Python Package for Cosmological Correlators



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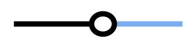
- Strong Mixing: $\rho \gg H$,  is giving a strong contribution.
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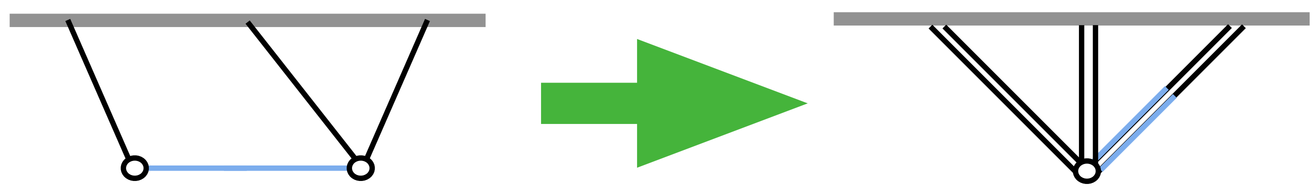

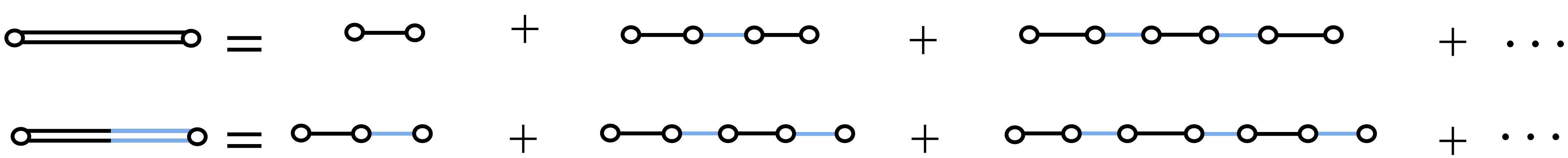

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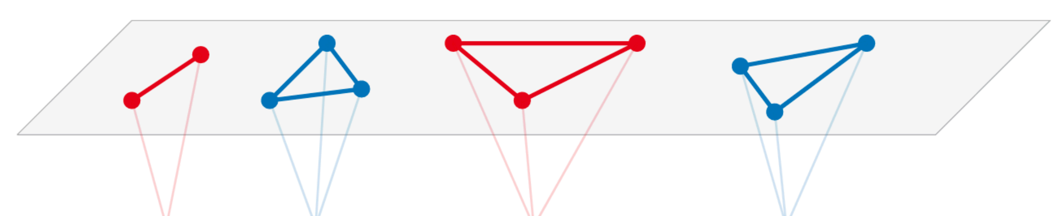

- It's solving fully coupled linear equations of motion: could it be done analytically?

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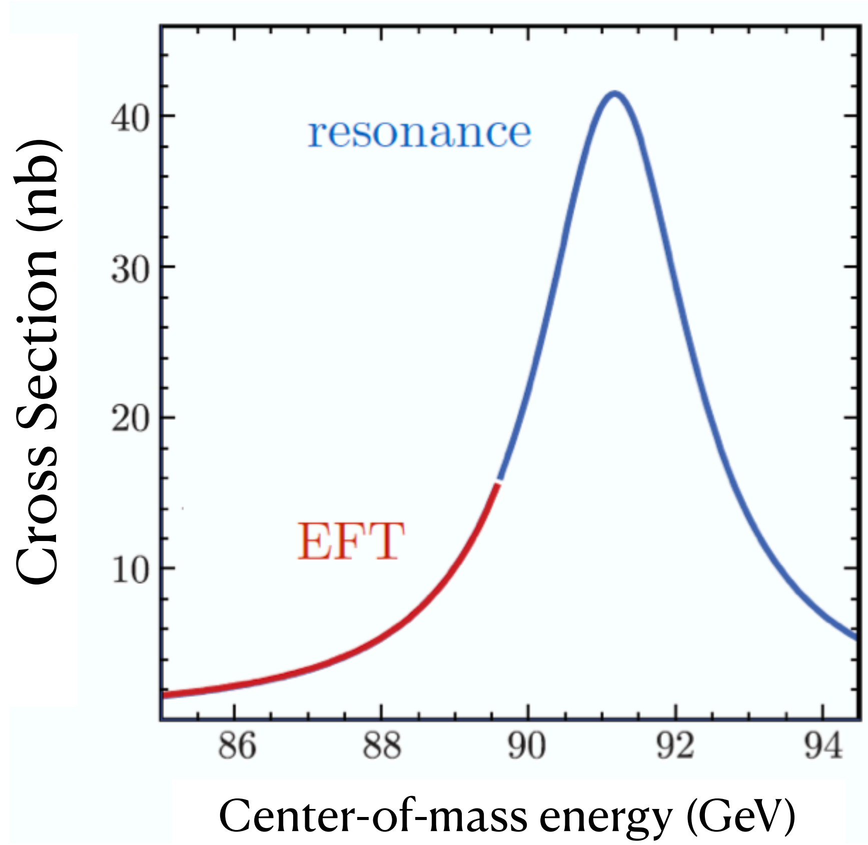
Effective Field Theory

- At low energy, any two-field system can be approximated by a single-field effective theory.



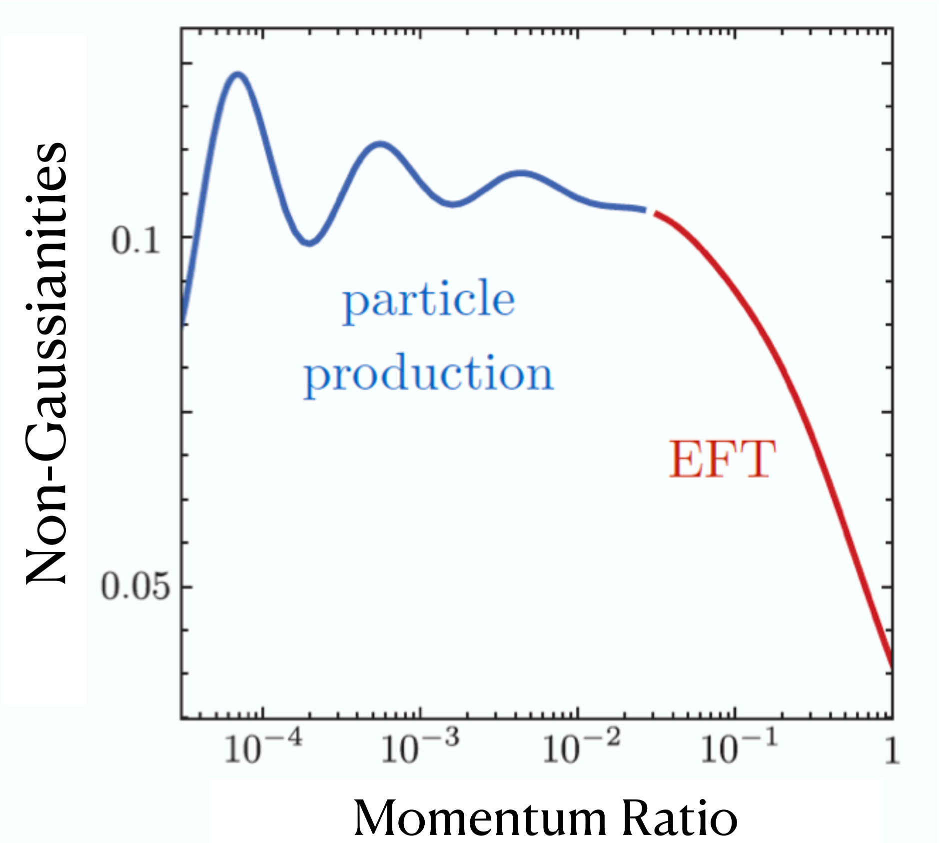


Figure from Arkani-Hamed and al. 1811.00024

Canonical Transformation

- We use the EFT to parametrize a field redefinition.

$$\begin{cases} X_1 = \pi_{\text{EFT}} + \delta\pi \\ X_2 = \sigma - \sigma_{\text{EFT}} \end{cases}$$

- Canonical Transformation: this should capture particle production.

- These variables allow one to know the interaction picture fields perturbatively:

$$\begin{cases} X_1 = X_1^{\text{Free}} + \delta X_1 \\ X_2 = X_2^{\text{Free}} + \delta X_2 \end{cases}$$

In our Concrete Example

- Original quadratic Hamiltonian:

$$\mathscr{H}_{(2)} = a^3\left(\frac{p_\pi^k p_\pi^{-k}}{2} + \frac{p_\sigma^k p_\sigma^{-k}}{2} + c_s^2\frac{k^2\pi^k\pi^{-k}}{2a^2} + \frac{k^2\sigma^k\sigma^{-k}}{2a^2} + \frac{\boxed{m^2+\rho^2}}{2}\sigma^k\sigma^{-k} - \rho\, p_\pi \sigma\right)$$

($m^2+\rho^2 \rightarrow m_{\text{eff}}^2$)

- Small c_s: consider the leading order in time derivative expansion:

Classical EOM approximate solution

$$\sigma_{\text{EFT}} \approx \frac{\rho}{k^2/a^2 + m_{\text{eff}}^2} p_\pi$$

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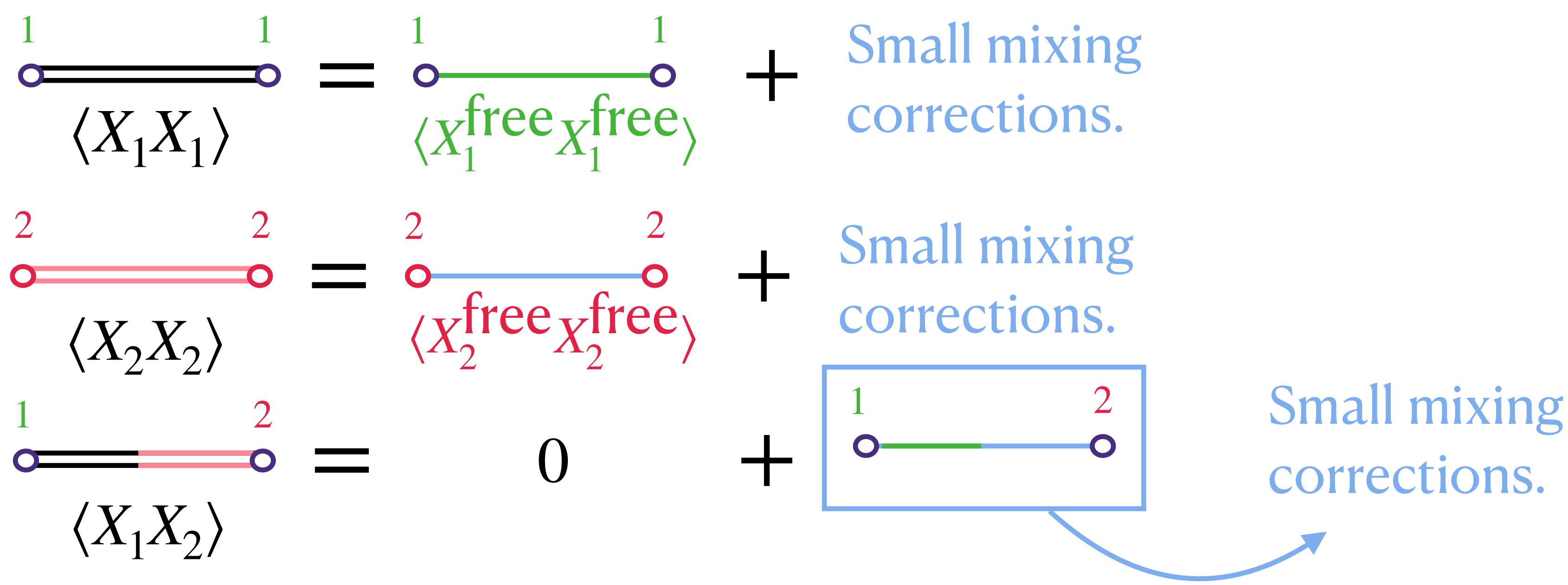
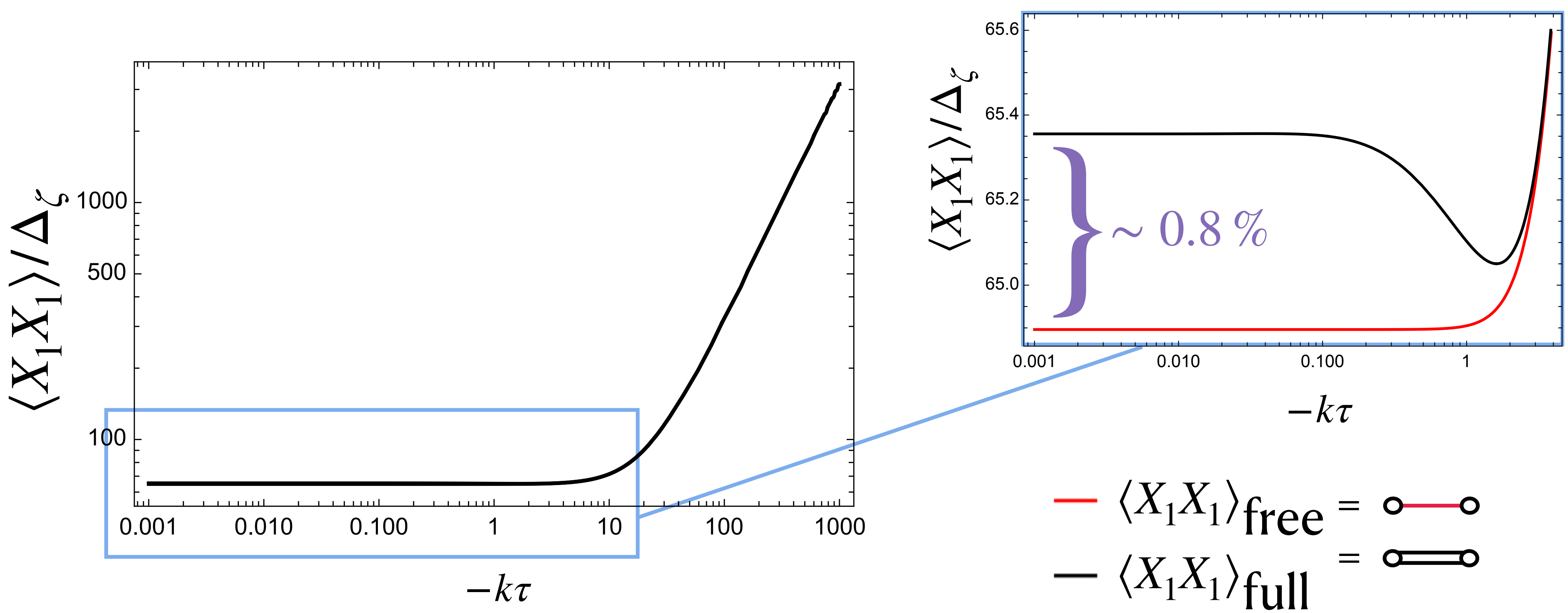
- Decay of the momenta at late time:

$$\langle X_1^n(t \to t_{\rm end})\rangle \to \langle \pi^n(t_{\rm end})\rangle$$

Perturbativity: 2 Points functions

- 2 Point functions:





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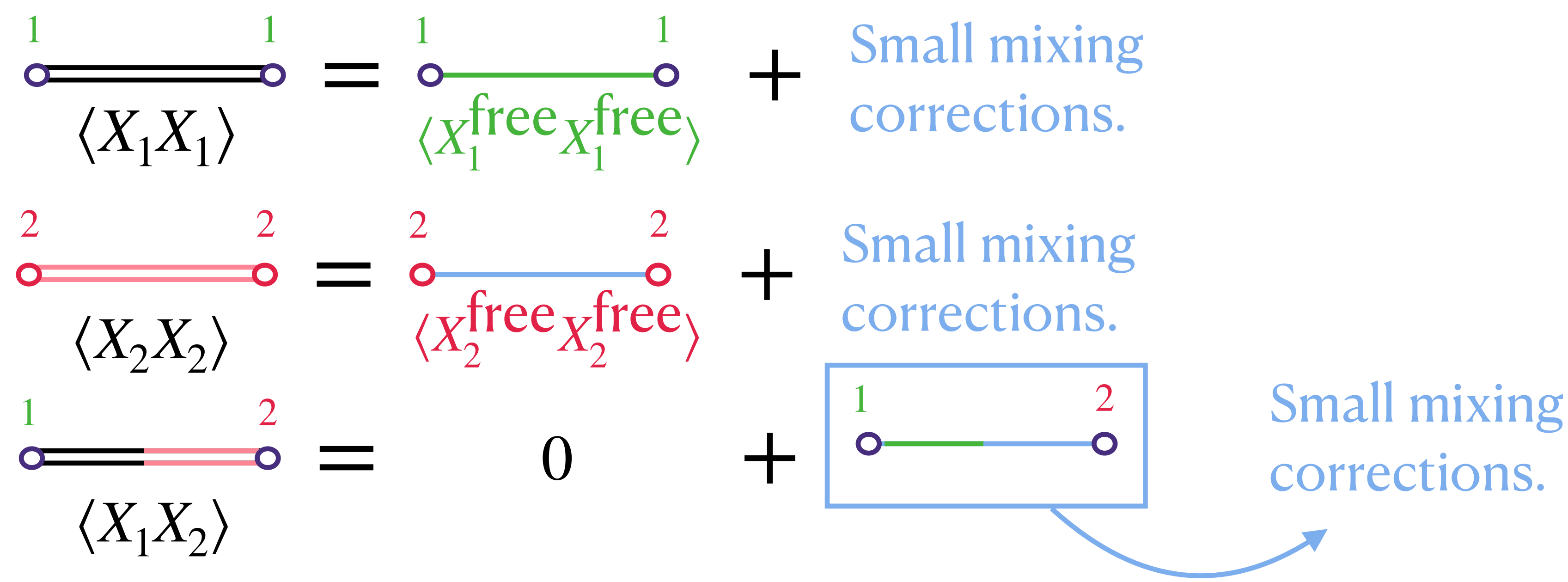

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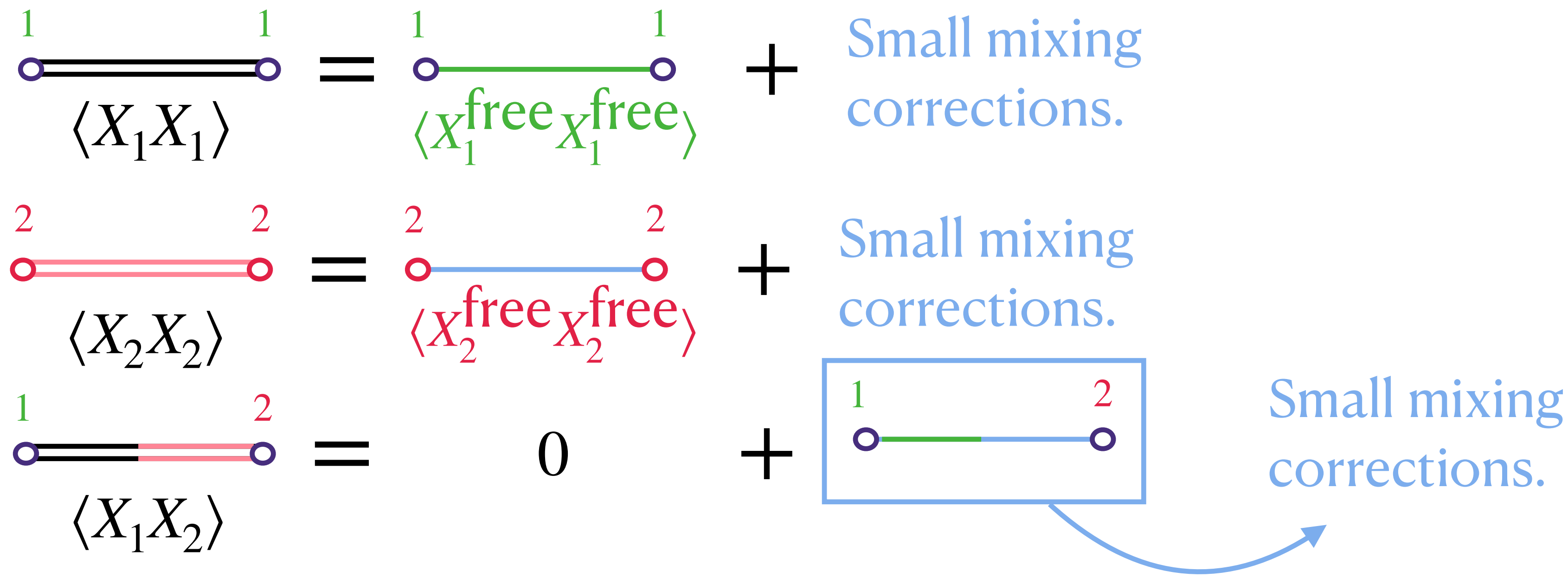

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- However, the mixing contribution is treated non-perturbatively.

Cubic Interaction

- Original Hamiltonian:

$$\mathcal{H}_I \supset -(2\pi)^3\delta^{(3)}\left(\sum \mathrm{k}_i\right)\frac{a}{2\Lambda}\mathrm{k}_1\cdot\mathrm{k}_2\pi^{k_1}\pi^{k_2}\sigma^{k_3}$$

- New Hamiltonian $\supset$ 6 interactions:

EFT Dominated | **Cosmic Collider Dominated**

$$\mathcal{K}_I \supset \quad \boxed{X_1X_1P_1}, \boxed{X_1X_1X_2},$$

$$\boxed{X_1P_1P_2,\ X_1X_2P_2,\ P_2P_2P_1,\ P_2P_2X_2}$$

Subleading contributions

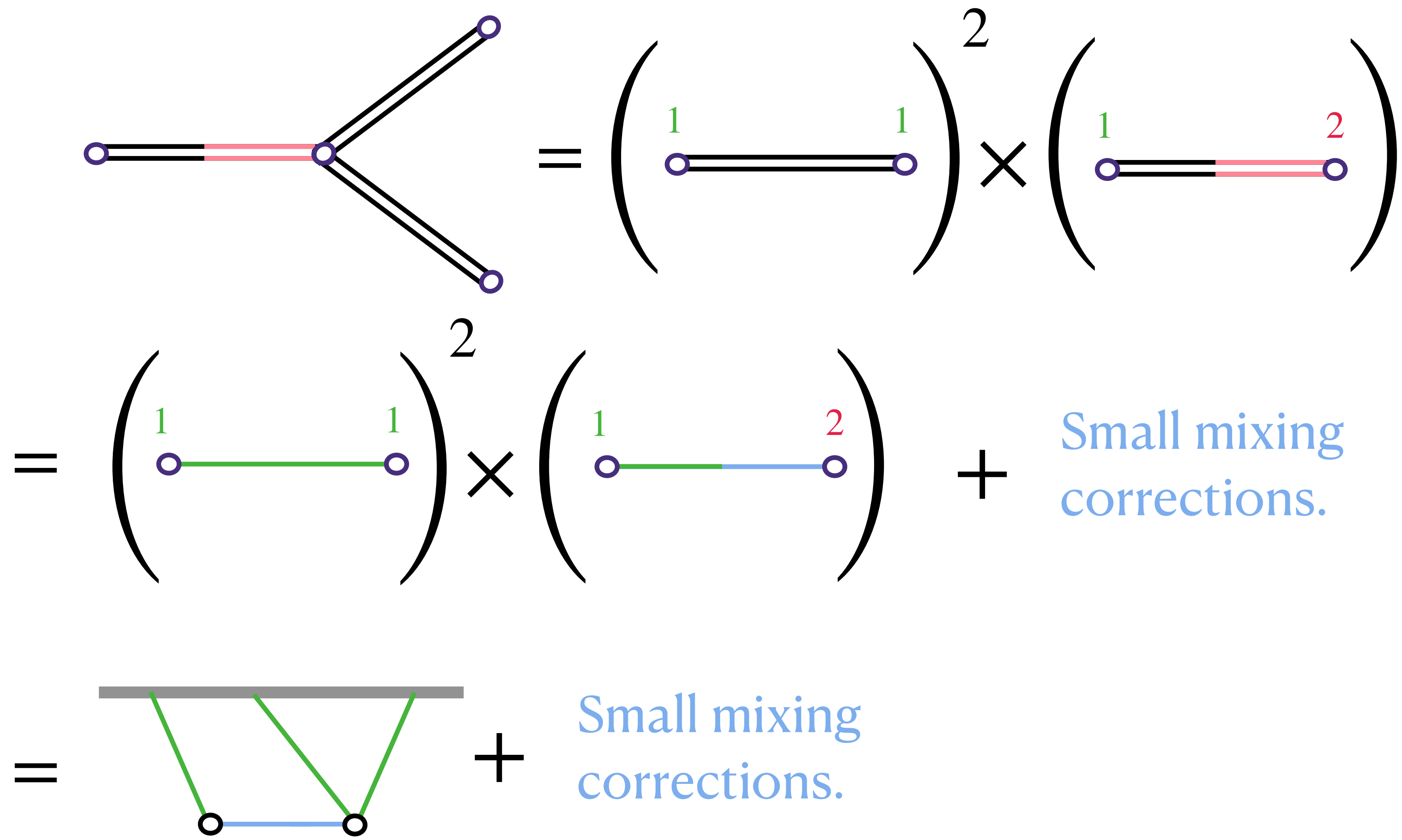
Single relevant interaction

$$\mathcal{K}_I^{\text{c.c.}} = -\frac{a}{2\Lambda}(2\pi)^3\delta^{(3)}\left(\sum \mathrm{k}_i\right)\times \mathrm{k}_1\cdot\mathrm{k}_2X_1^{k_1}X_1^{k_2}X_2^{k_3}$$

Perturbativity: 3 Points functions

- Perturbative knowledge of 2 points $\Longrightarrow$ 3 points by Wick Theorem:

$$\langle \pi\pi\pi \rangle_{X_1X_1X_2} =$$



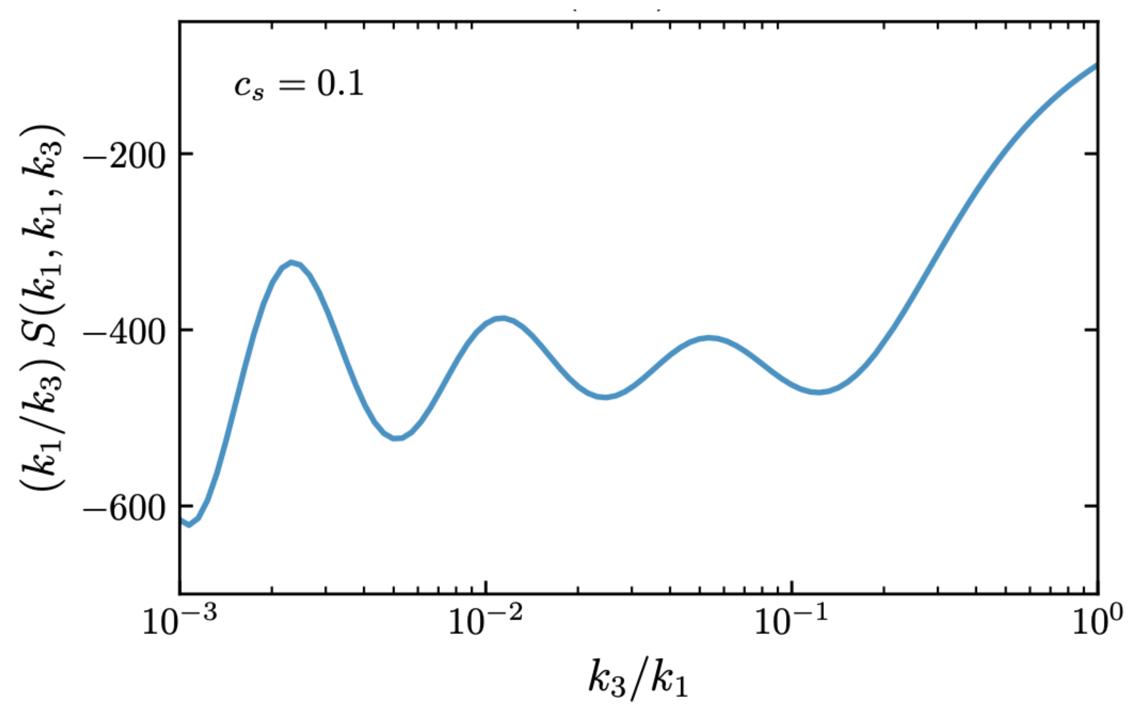
- Generic extension of the EFT techniques.

- Work in progress!

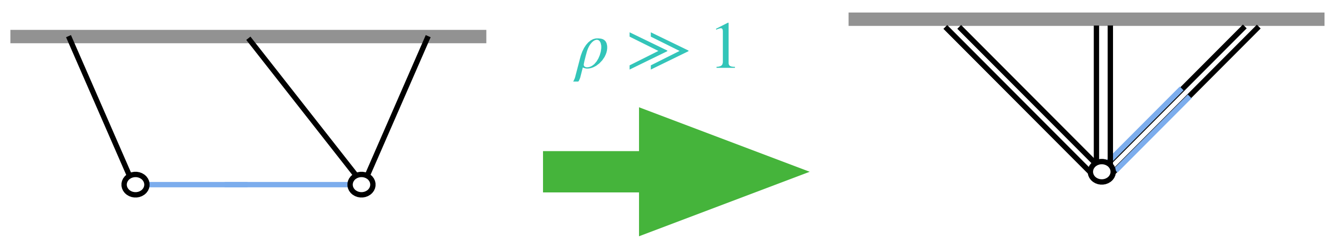
Conclusion

- Cosmological collider is a probe of the particle content during inflation:

$$\langle \pi^{k_1}\pi^{k_2}\pi^{k_3}\rangle \sim \left(\frac{k_3}{k_2}\right)^{1/2} e^{-\pi m/H}\cos(m/H\log(k_3/k_1)+\varphi)$$



- Strong Mixing regime to be understood analytically:



- Interaction picture fields can be approached using new variables motivated by an EFT description: