Quintessence: analytical results

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Théorie, Univers et Gravitation - TUG

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LAPTh, Annecy

Introduction

Topic: **Dark Energy**

Our universe is expanding + the expansion is currently **accelerating**

Accelerated expansion due to dark energy

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Very timely question because of **observations**!!

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'24 ΛCDM: ✓

Dyn. dark energy: ✓

Settled very soon?!

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Difficulties characterized by (Strong) de Sitter swampland conjecture

Still attempts / work in progress

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⟶ today: **Quintessence** (model-independent manner)

Consider 4d **cosmological model**: $\int \mathrm{d}^4x \sqrt{|g_4|} \left(\frac{M_p^2}{2} \mathcal{R}_4 - \frac{1}{2} g_{ij}\, \partial_\mu \varphi^i \partial^\mu \varphi^j - V(\varphi^k) \ + \mathcal{L}_{m,r} \right)$

(multifield, minimally coupled, no direct coupling to matter, radiation)

$(k=0)$

+ solutions with FLRW metric: $\mathrm{d}s_4^2 = -\mathrm{d}t^2 + a^2(t) \left(\frac{\mathrm{d}r^2}{1-kr^2} + r^2 \mathrm{d}\Omega_2^2 \right) \quad + \varphi^i(t) \quad (\dot{\varphi}^i \equiv \partial_t \varphi^i)$

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Interested in **« realistic » solutions** (tuning I.C.): **radiation, matter, dark energy domination phase**

$\Omega_n = \rho_n/(3H^2) \qquad 0 \leq \Omega_n \leq 1 \qquad \Omega_r, \Omega_m, \Omega_\varphi$

Today: $\Omega_{r0} = 0.0001 \ , \ \Omega_{m0} = 0.3149 \ , \ \Omega_{\varphi 0} = 0.6850$

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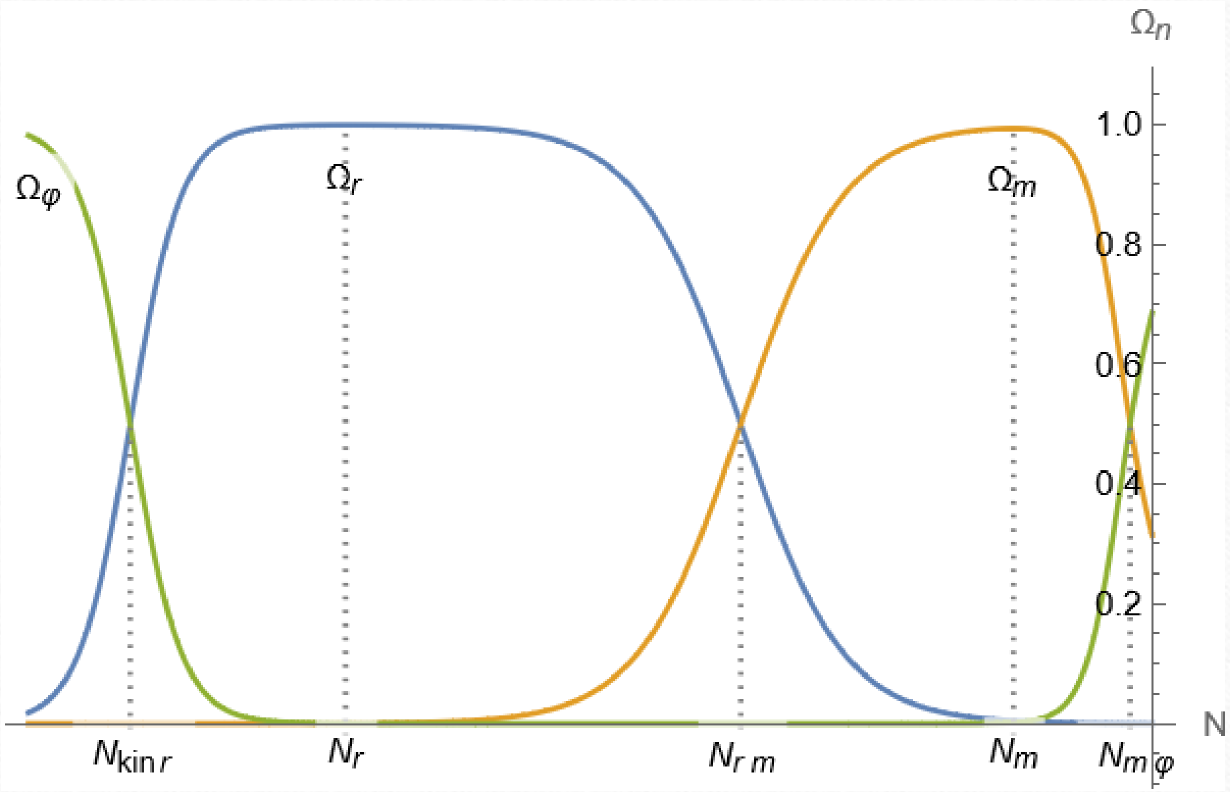

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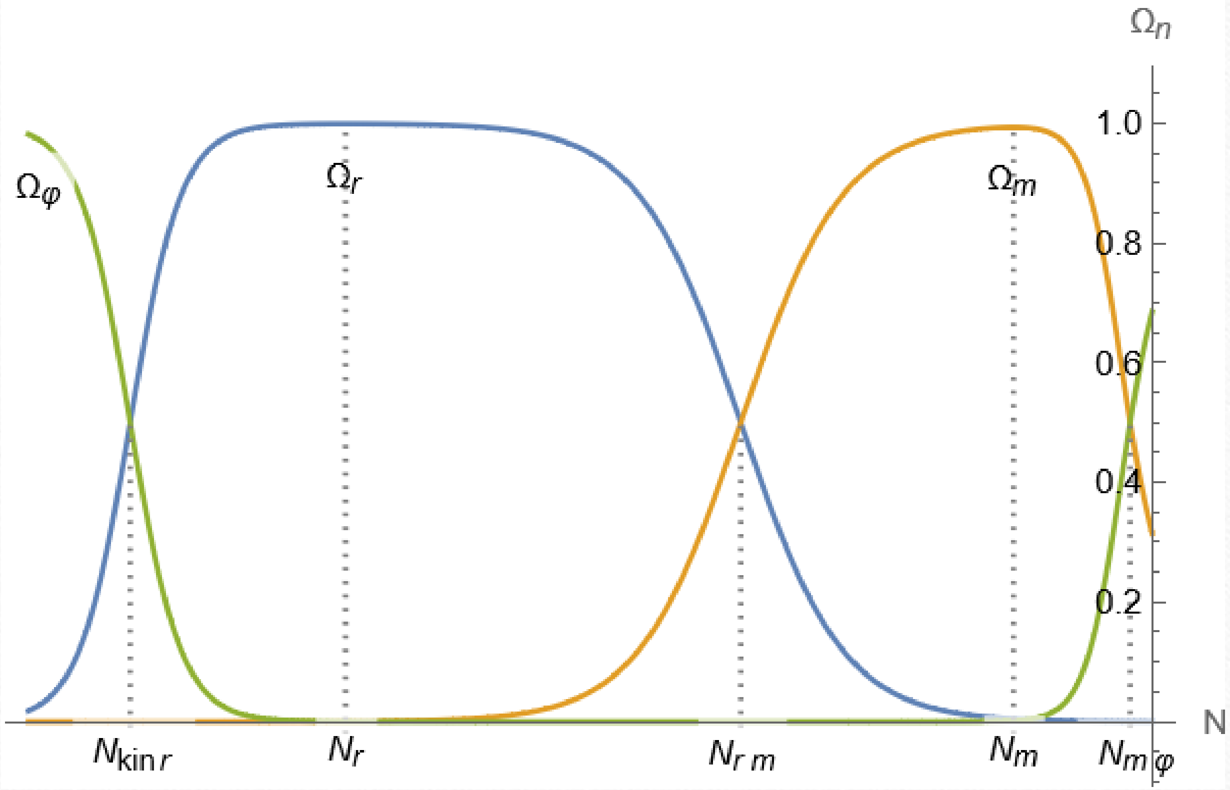

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kinetic energy dominated

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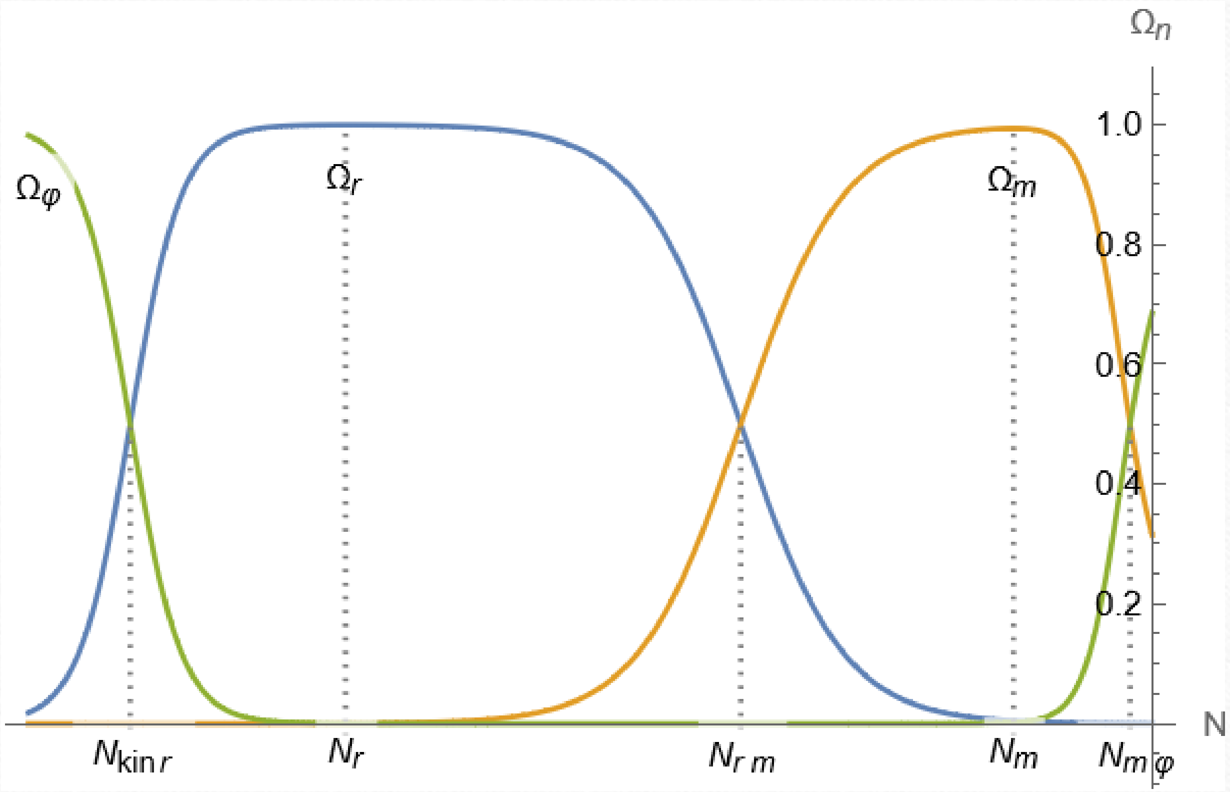

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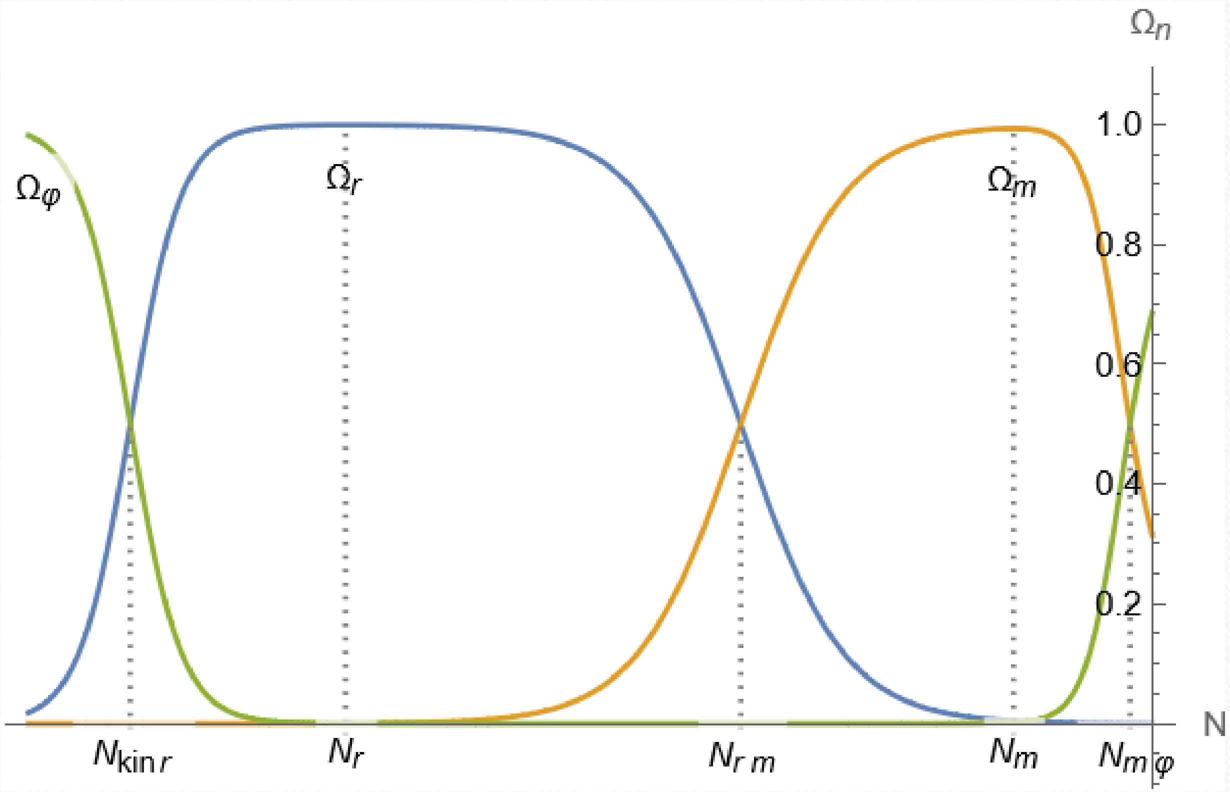

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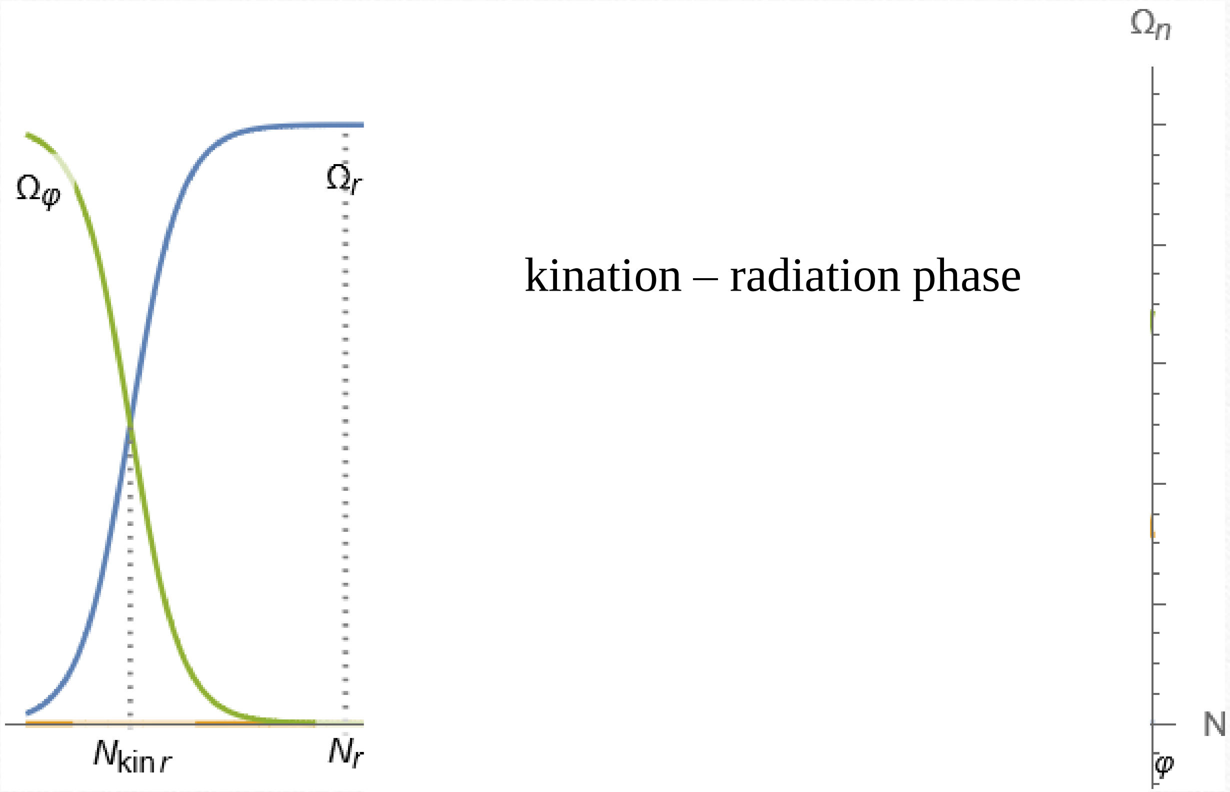

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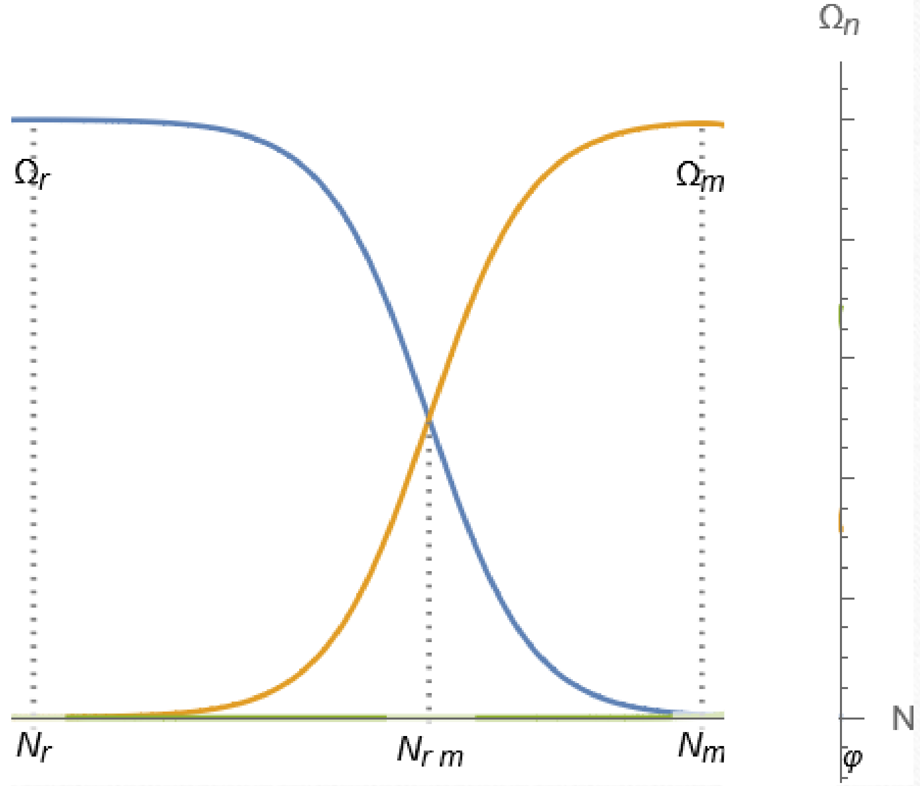

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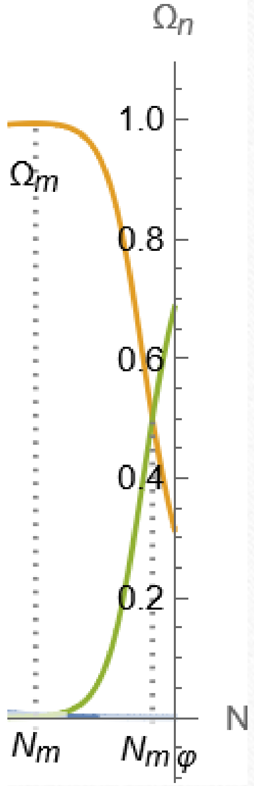
matter – dark energy phase



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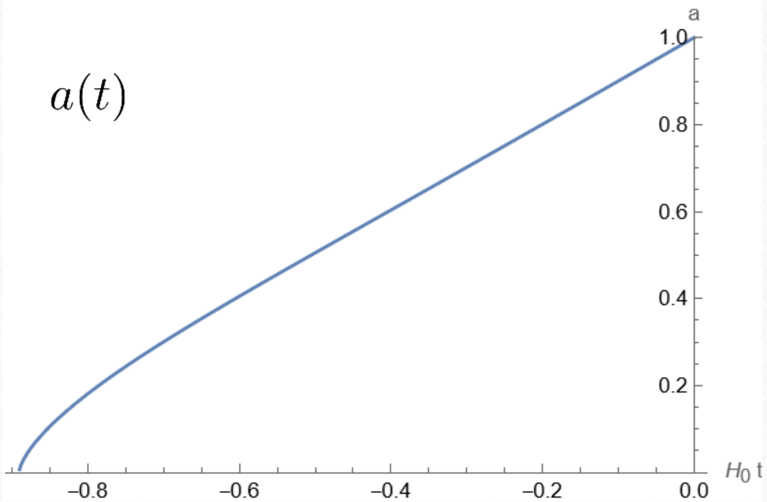
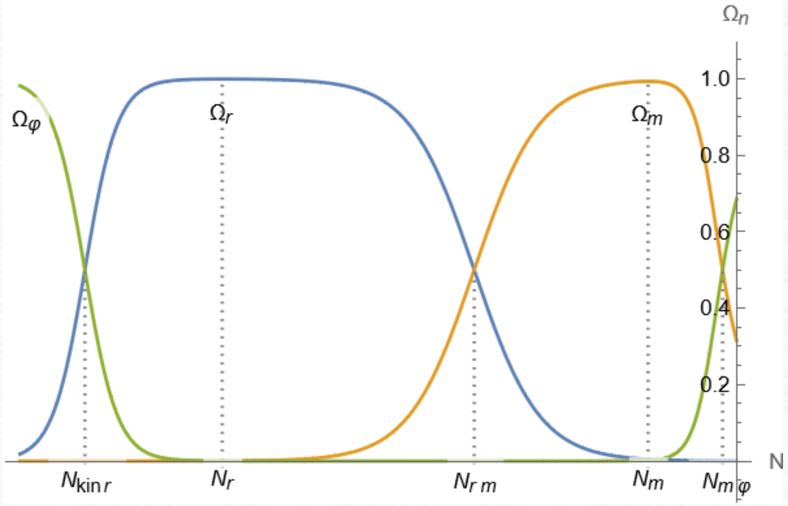
(numerical) **solution example**:





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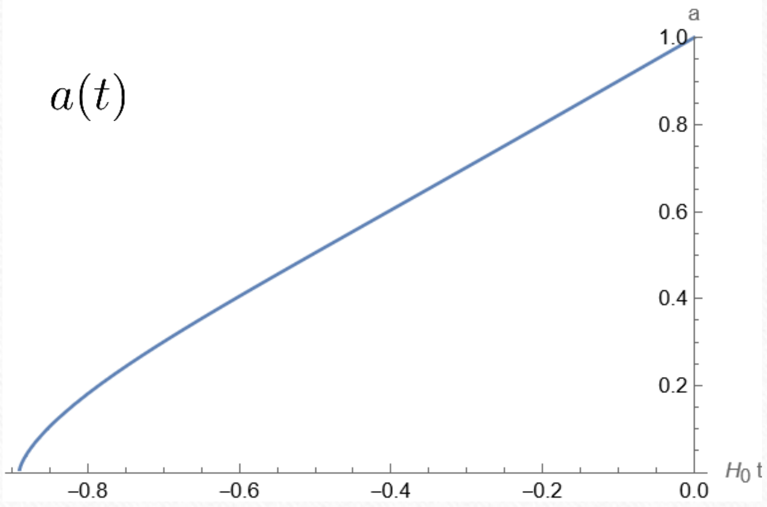

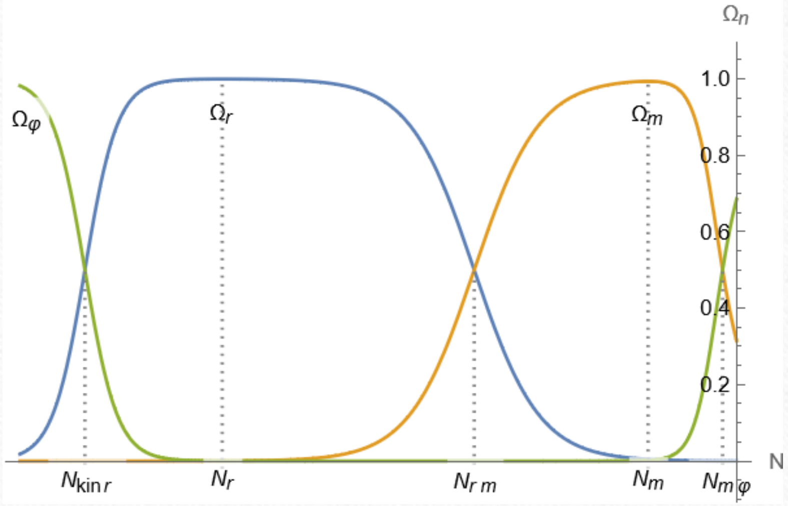

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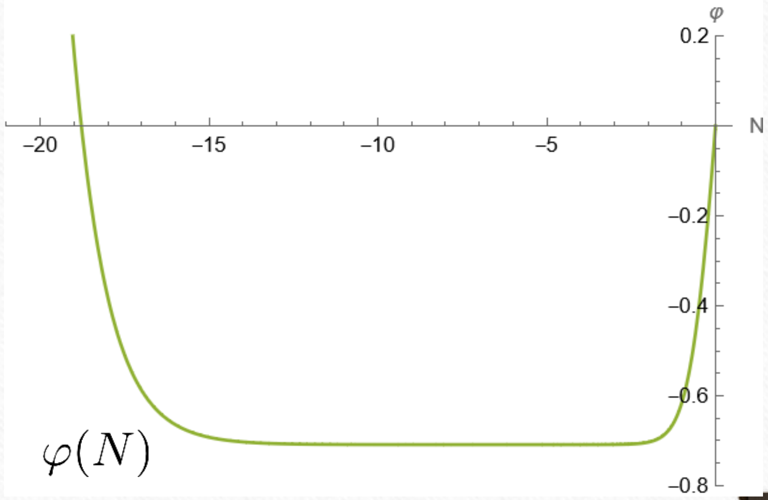

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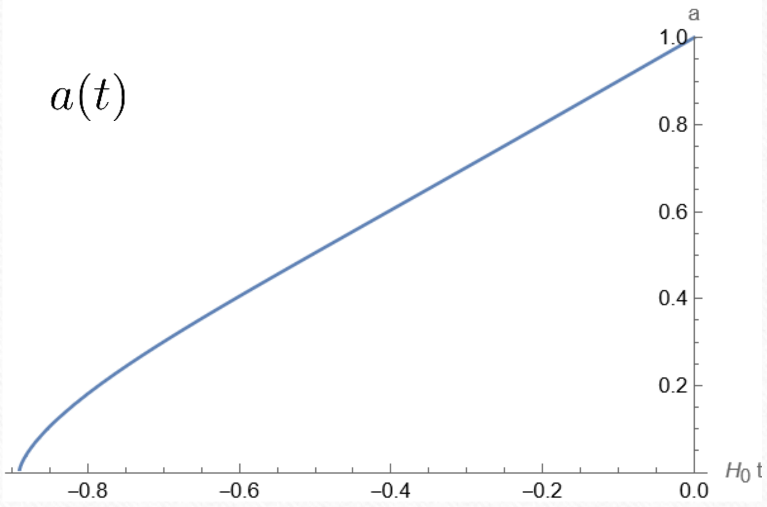

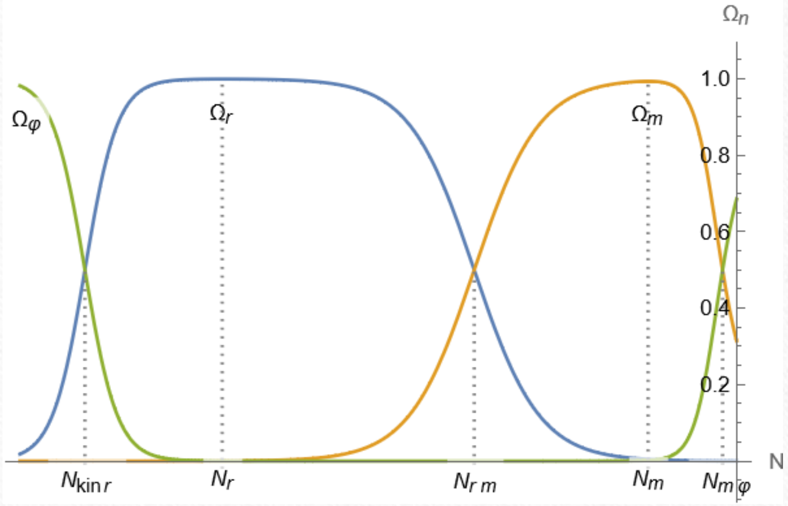

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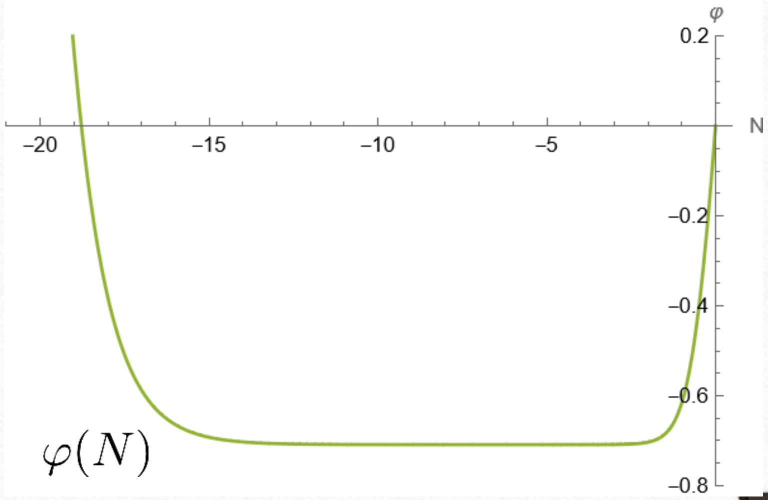

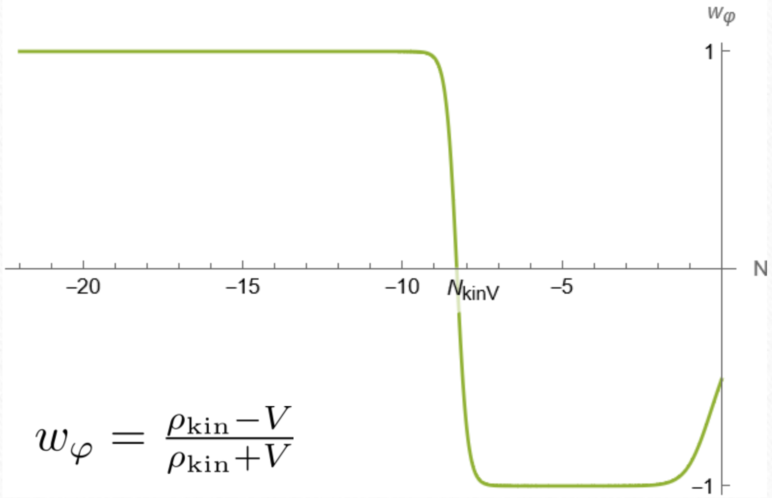

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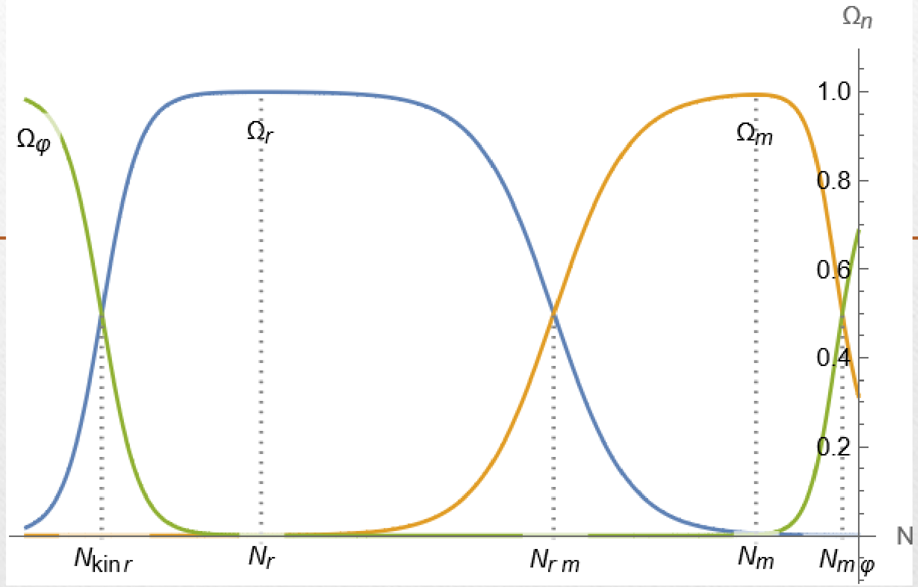
Transition **moment**: $N_{\text{kin}V}$

Model-independent analytical results describing such solutions

I. Kination – radiation phase



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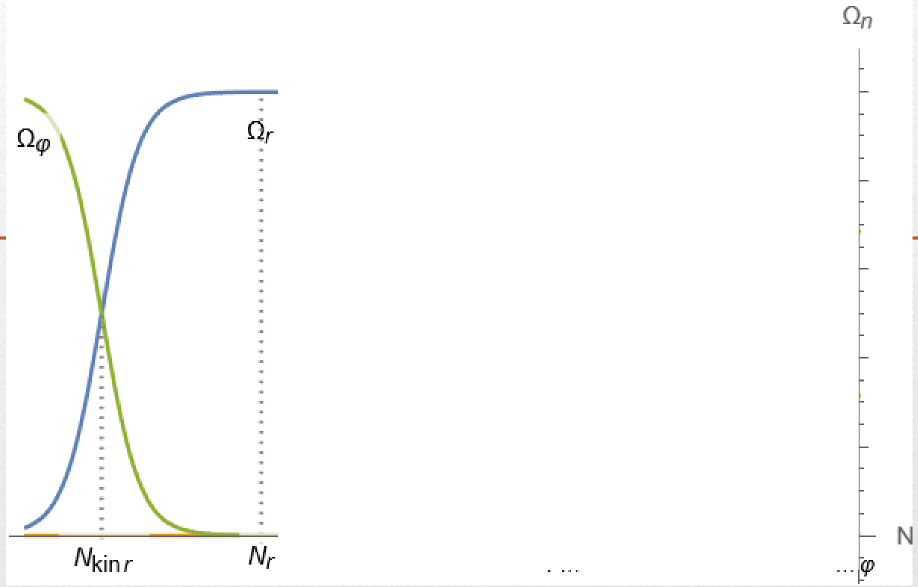


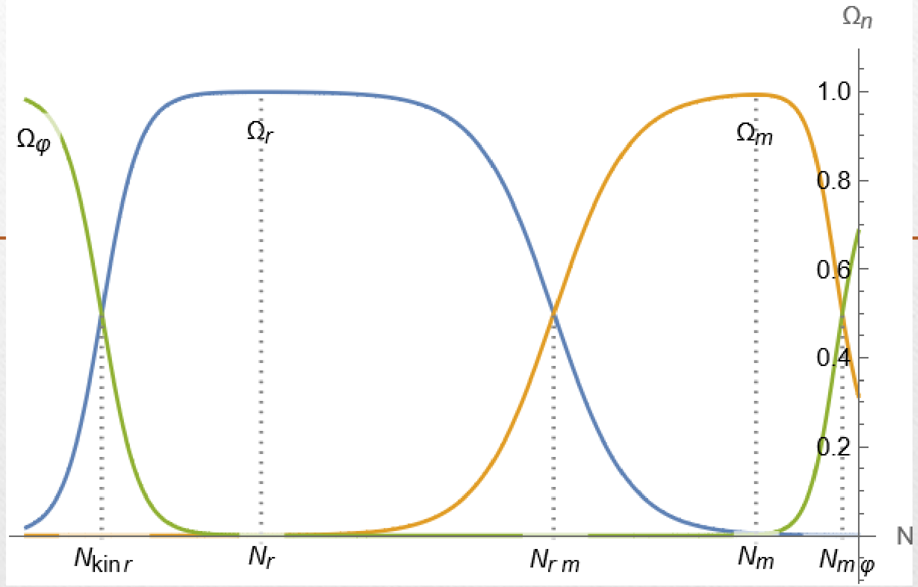
Analytical solutions…

Condition for kination to happen: **initial speed** and **negligible potential**

$$Early\ kination\ phase \quad \Rightarrow \quad V(\varphi) \ll e^{-\sqrt{6}\,\varphi}\ at\ early\ times$$

(large negative field)

II. Radiation - matter phase



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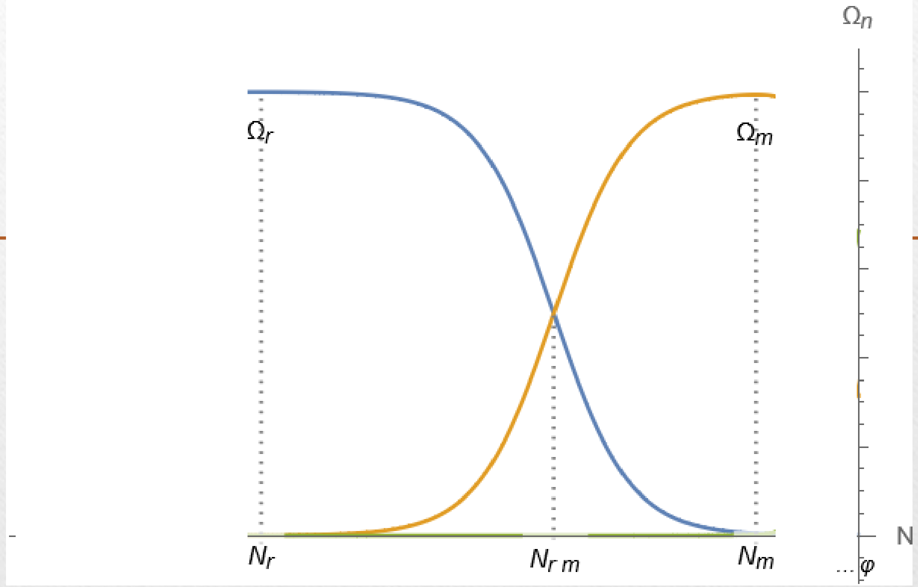

Analytical solutions: $a(t),\ \varphi^i(t)$

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$a(t)$ $\quad \Omega_\varphi \ll \Omega_r + \Omega_m \longrightarrow V = 0,\ \varphi^i = \text{constant}$

Solve 1st Friedmann equation, get solution $t(a)$

We show it can be inverted $\longrightarrow a(t)$

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$$T = \frac{\sqrt{3}}{2}\frac{\rho_{m0}^2}{\rho_{r0}^{3/2}}\, t\ ,\ \ X = \frac{\rho_{m0}}{\rho_{r0}}\, a$$

- $T \in (0,2)$: $\;X = 2\cos\left(\frac{1}{3}\arccos\left(1 - \frac{1}{2}T(4-T)\right) + \frac{4\pi}{3}\right) + 1$
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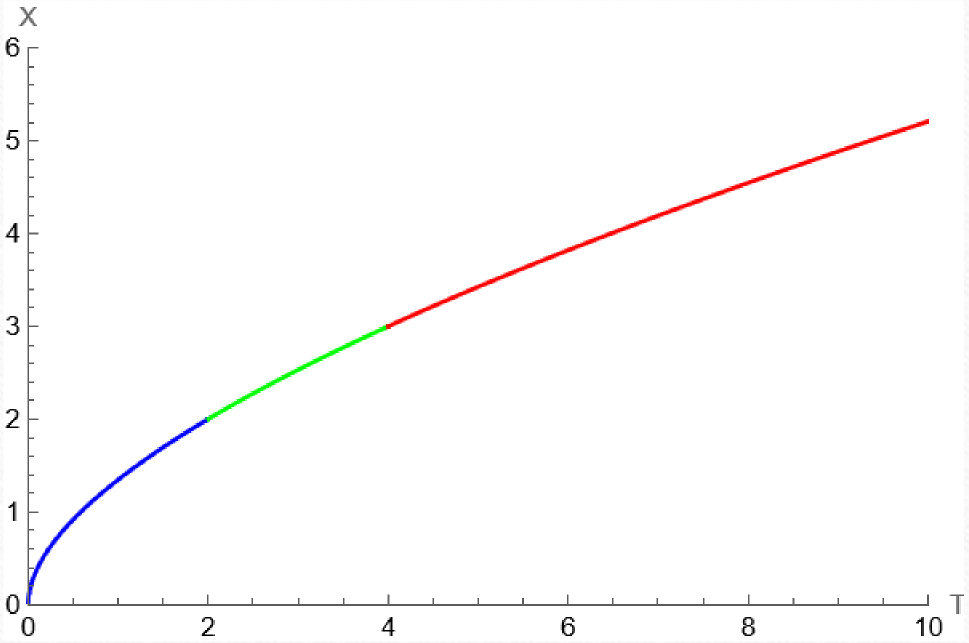
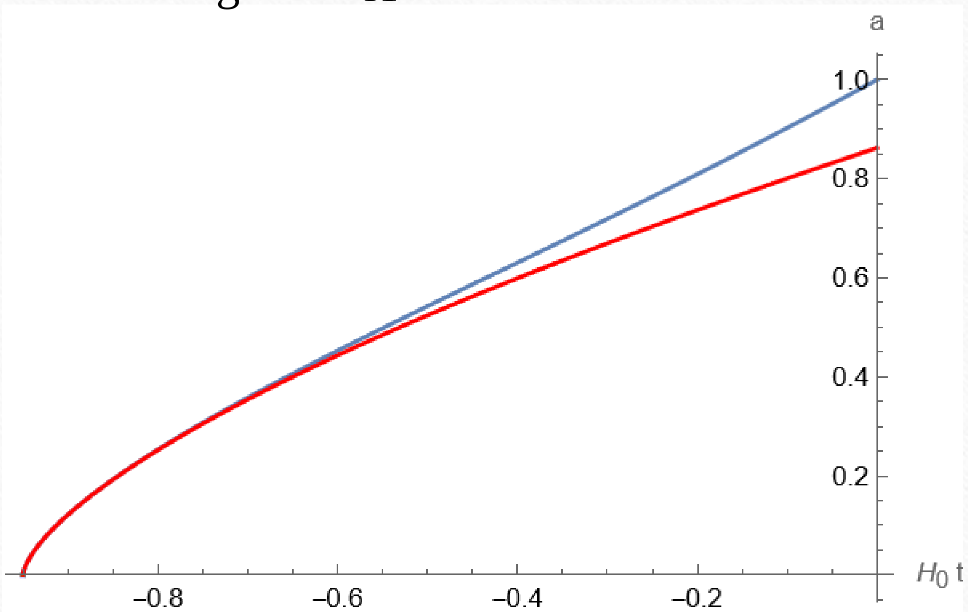


Matching with ΛCDM:



(approximate) analytical solution: $\varphi^i(t)$

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Solve field equation of motion with $H(t) = \frac{\dot{a}(t)}{a(t)} \longrightarrow \varphi^i(t),\, \varphi^i(N)$

Phase: 2 parts

Part 1: $\varphi^i = \varphi^i_{0rm} \mp c^i_k \sqrt{6}\, e^{N_{\mathrm{kin}\,r} - N}$

Part 2: $\varphi^i = \varphi^i_{0rm} - \frac{2}{9}\, \frac{\partial_i V}{V}\Big|_{0rm} e^{4(N_{m\,\Lambda} - N_{m\,q})}\; e^{3(N - N_{m\varphi\,\Lambda})}$

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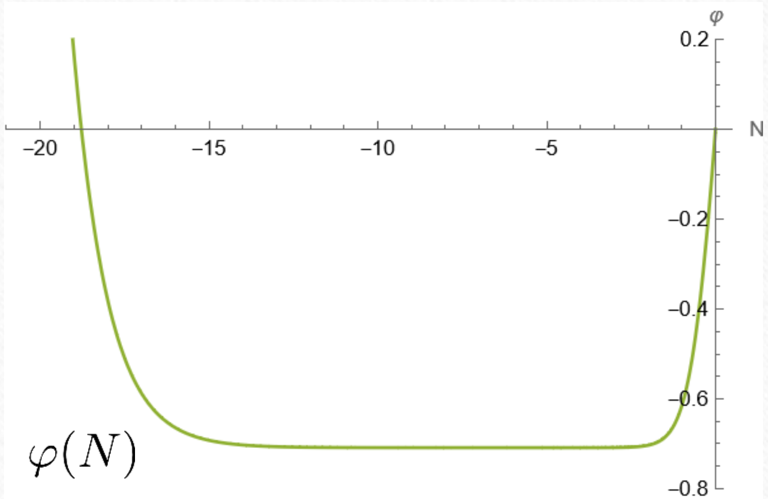
$$\longrightarrow \quad \Delta\varphi = \sqrt{6}\, e^{N_{\mathrm{kin}r} - N_r} - \frac{2}{9} \left.\frac{\partial_\varphi V}{V}\right|_{0rm} e^{4(N_{m\,\Lambda} - N_{m\,q})}\, e^{3(N_{m\,q} - N_{m\varphi\,\Lambda})}$$

$\Delta\varphi \lesssim 10^{-2}$

Field(s) frozen on slope due to Hubble friction



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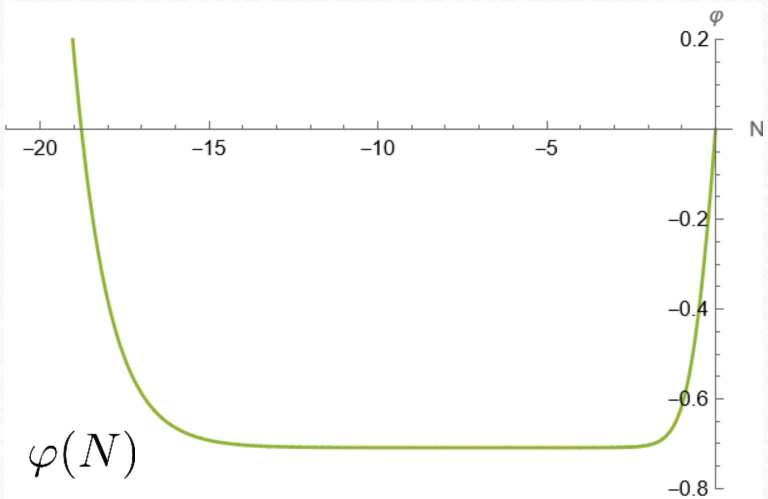

$\longrightarrow$ **String theory** model building:

no need of moduli stabilisation??

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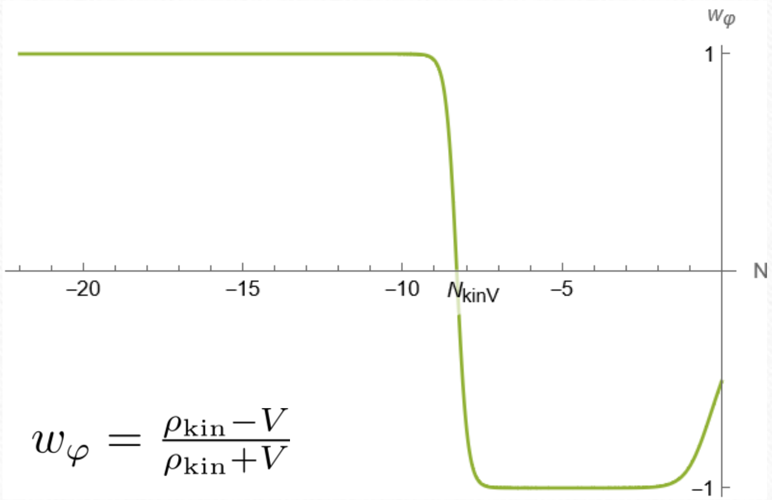

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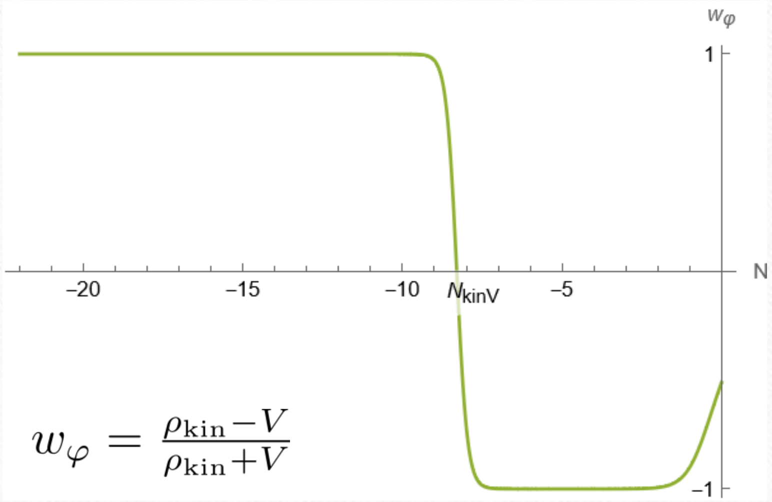

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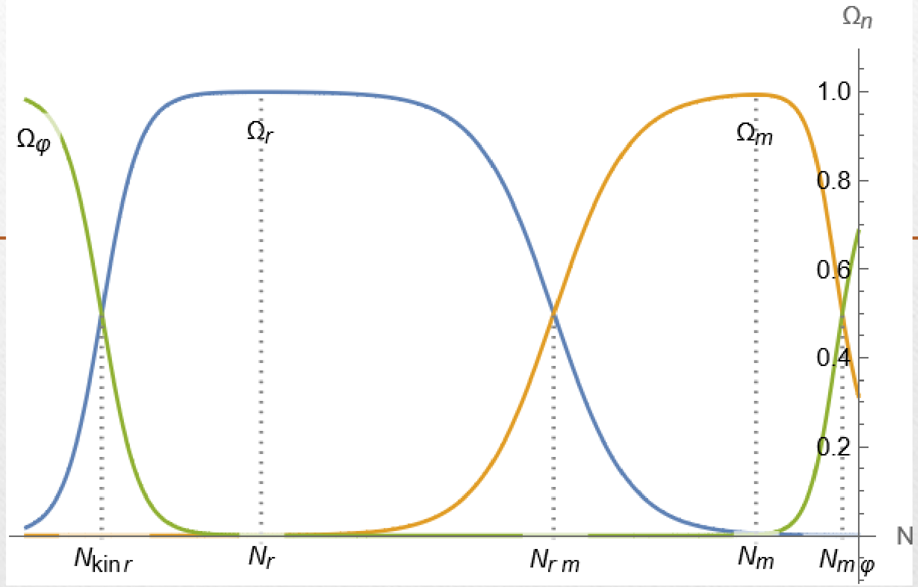
$$\approx \frac{1}{6}\,(2N_{\mathrm{kin}r} + N_{rm}) + \mathrm{corr.}$$

$z \geq 3000$ ⟶ **Observational target?**

III. Matter – dark energy phase



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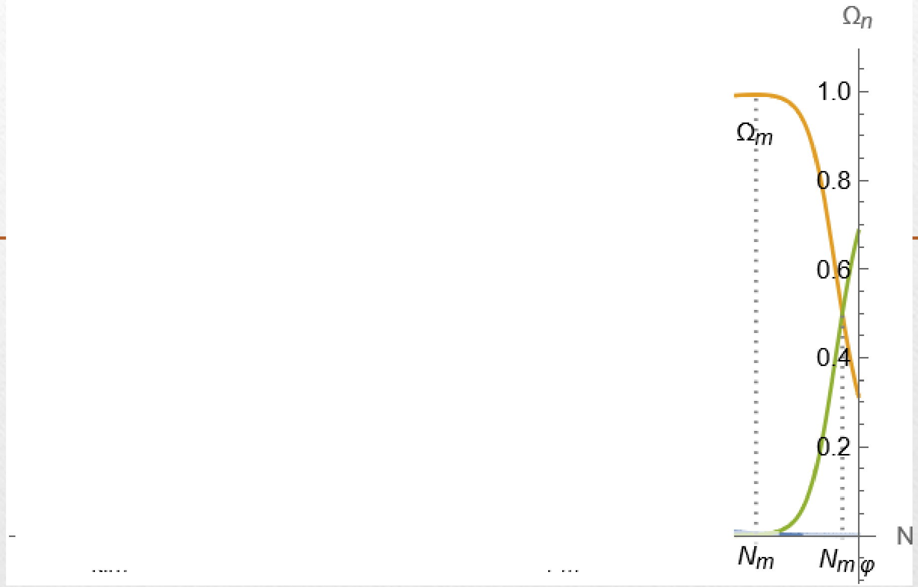

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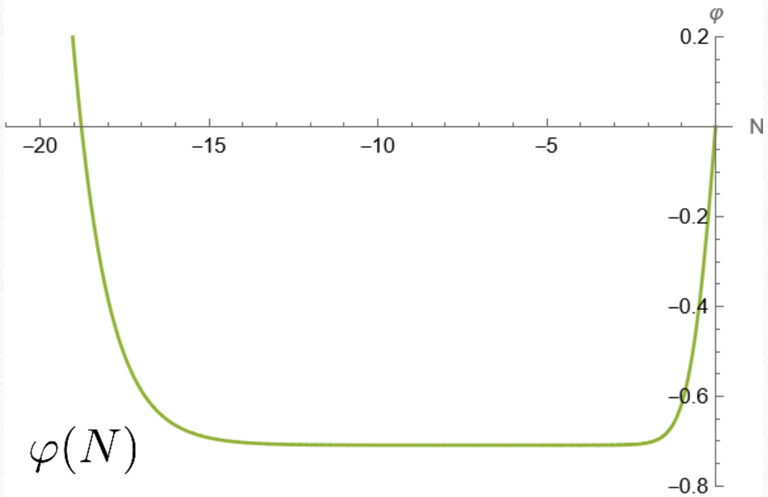

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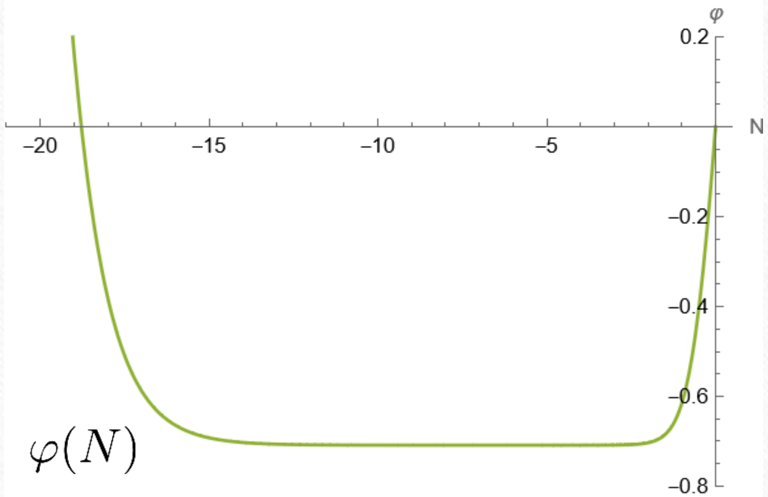
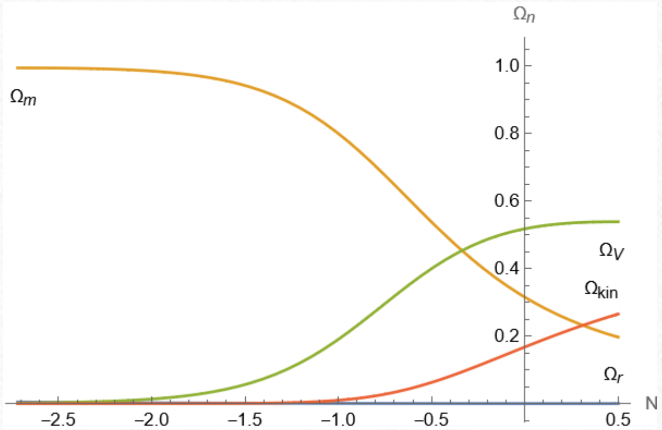
How much is $\Delta\varphi$? Prove (with reasonable assumptions) that sub-Planckian: $\Delta\varphi \leq 1$



$$\Delta\varphi = \int_{N_m}^{0} \sqrt{6\Omega_{\rm kin}}\, {\rm d}N$$



$\Omega_{\rm kin}$ is growing and is bounded

Dynamics is model-dependent: fields get unfrozen, roll down potential

Analytical solutions difficult, coupled equations $a(t),\ \varphi^i(t)$

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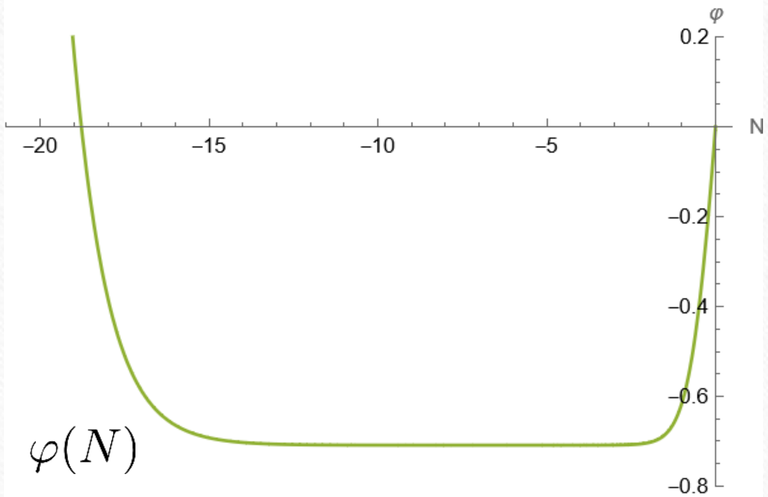

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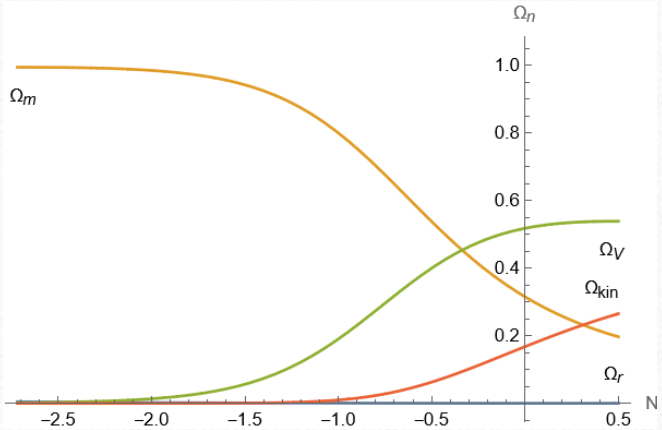


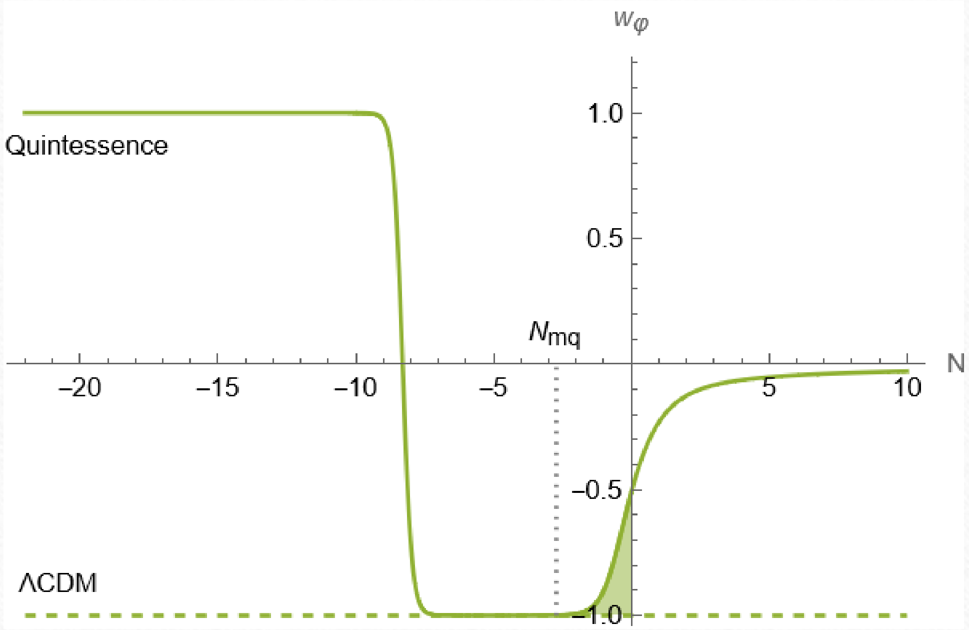
$\Omega_{\rm kin}$ is growing and is bounded

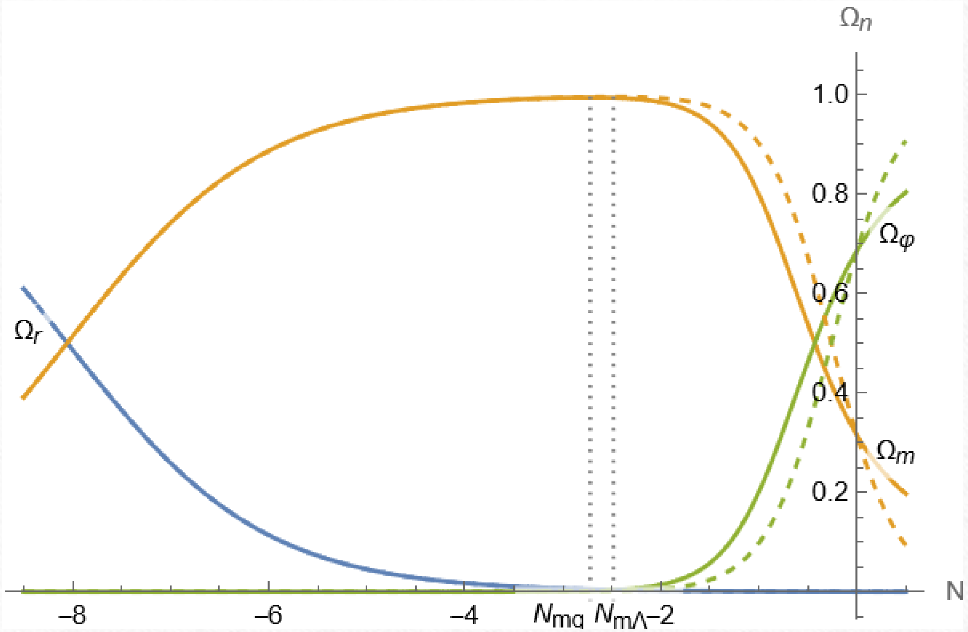
$\Delta\varphi \leq 1 \longrightarrow$ control on **effective theory** (swampland distance conj., string theory)

+ variation of coupling constants?

Deviation from ΛCDM:

$$\int_{N_{m\,q}}^{0} (w_\varphi + 1)\, \mathrm{d}N = \frac{4}{3}\left(N_{m\,\Lambda} - N_{m\,q}\right)$$





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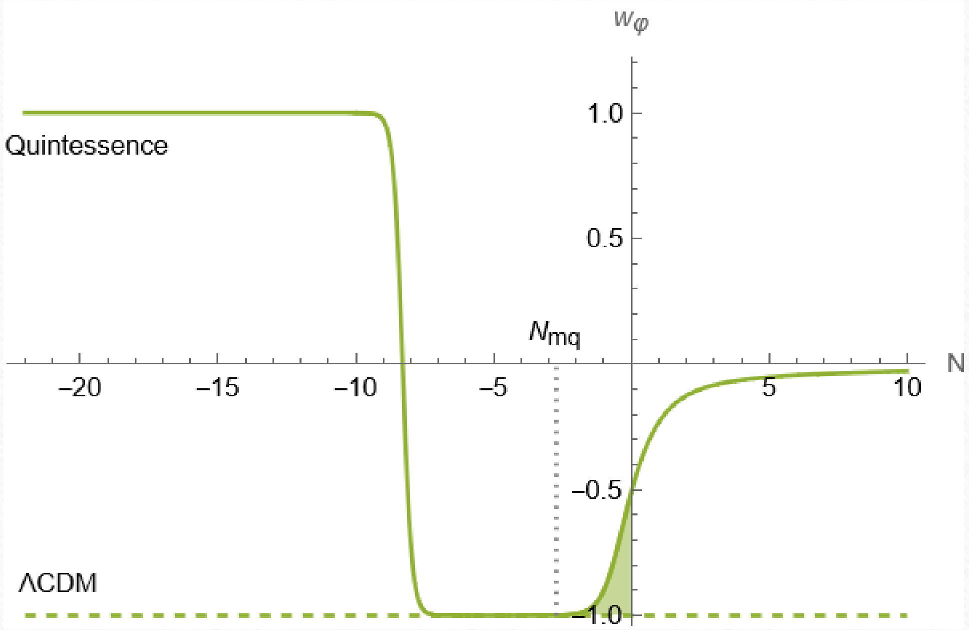

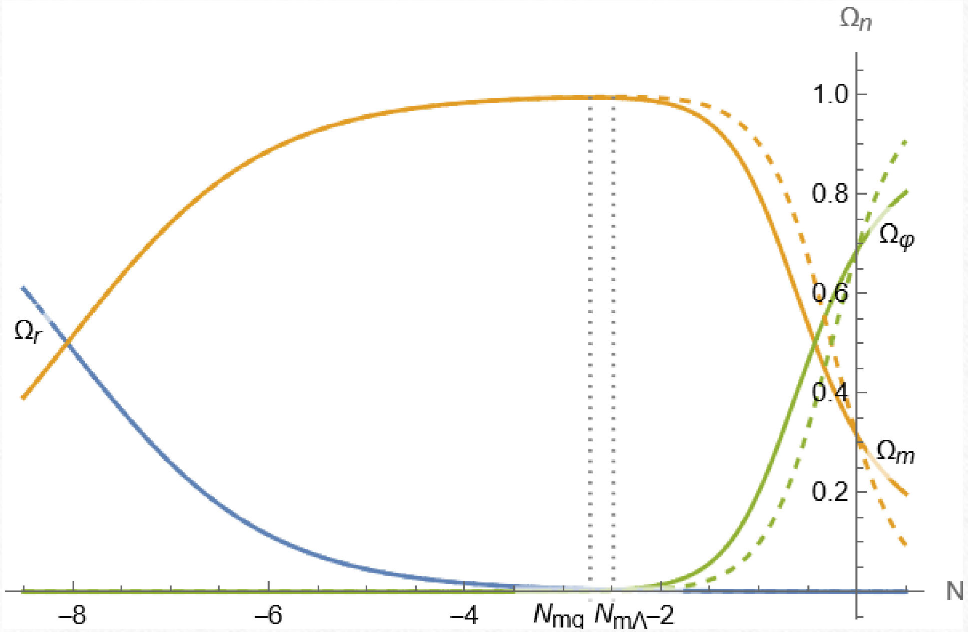

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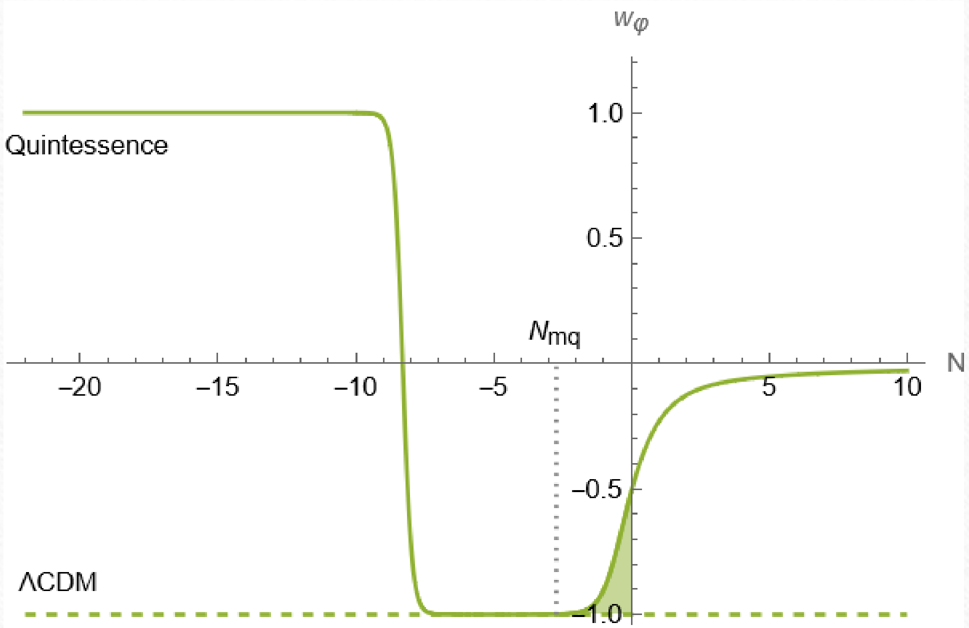

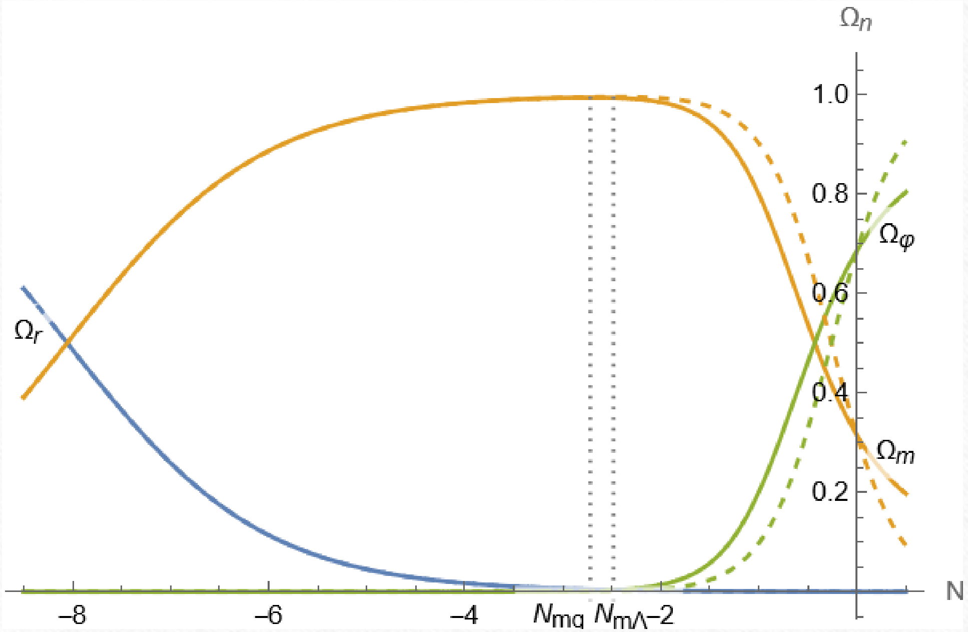

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Observational target: evolution of Ω_m , model independently?

Relation useful to normalise measurements?

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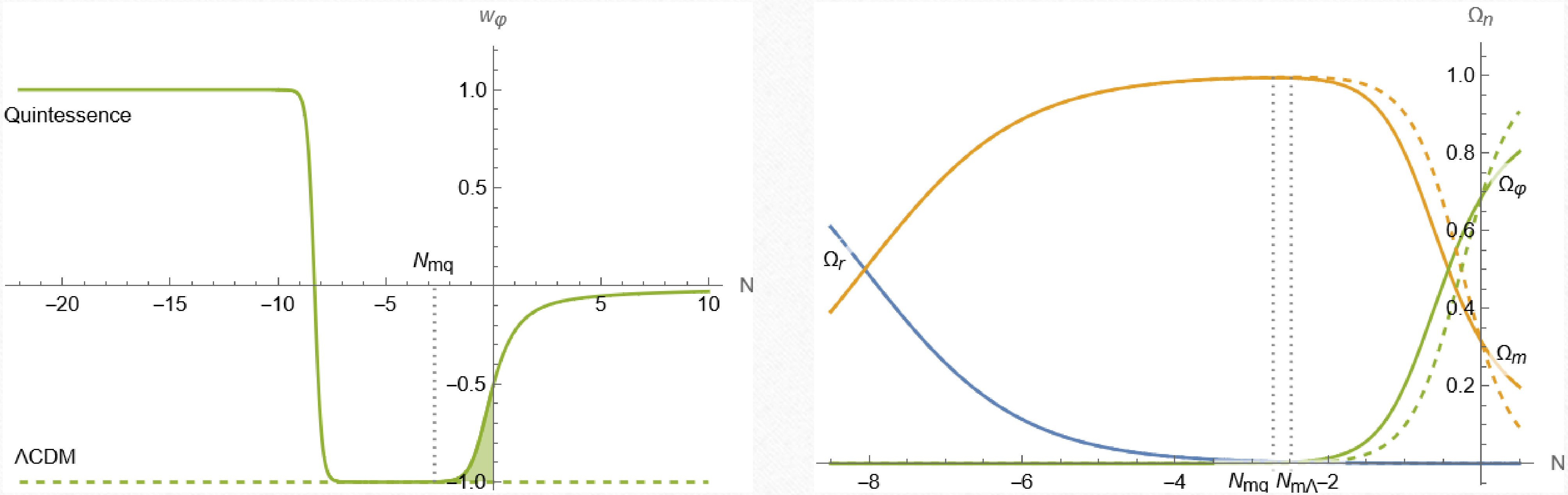

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Observational target: evolution of Ω_m , model independently?

Relation useful to normalise measurements?

+ discuss CPL parametrisation

+ phantom behaviour (DESI…)

Get bounds: $w_0 > \ldots$

$w_a < \ldots$

Summary

Quintessence models + realistic cosmological solutions

Model – independent analytical results / solutions, among which:

- Freezing of fields during radiation – matter phase
- Transition $w_\varphi : +1 \to -1$
- Sub-Planckian $\Delta\varphi \leq 1$ during matter – dark energy phase
- Recent deviation from ΛCDM: $\int_{N_{m\,q}}^{0} (w_\varphi + 1)\,\mathrm{d}N = \frac{4}{3}\,(N_{m\,\Lambda} - N_{m\,q})$

⟶ **Observational target**: evolution of Ω_m

- And more…

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Thank you for your attention!