

# An Introduction to Late-time Cosmology with Gravitational Waves

Danny Laghi



LISA School for early-career scientists – 6–17 October 2025, Les Houches





# Standard sirens

The central question is then: **how to get the GW source redshift?**

Here we will consider three methods that are mostly used in current GW standard siren studies:

**1. Identification of a direct EM counterpart (“bright siren” method)**

**Requires an EM counterpart!**

**2. Using spectral features in the GW mass distribution (“spectral siren” method)**

**Requires assumptions about the GW source populations!**

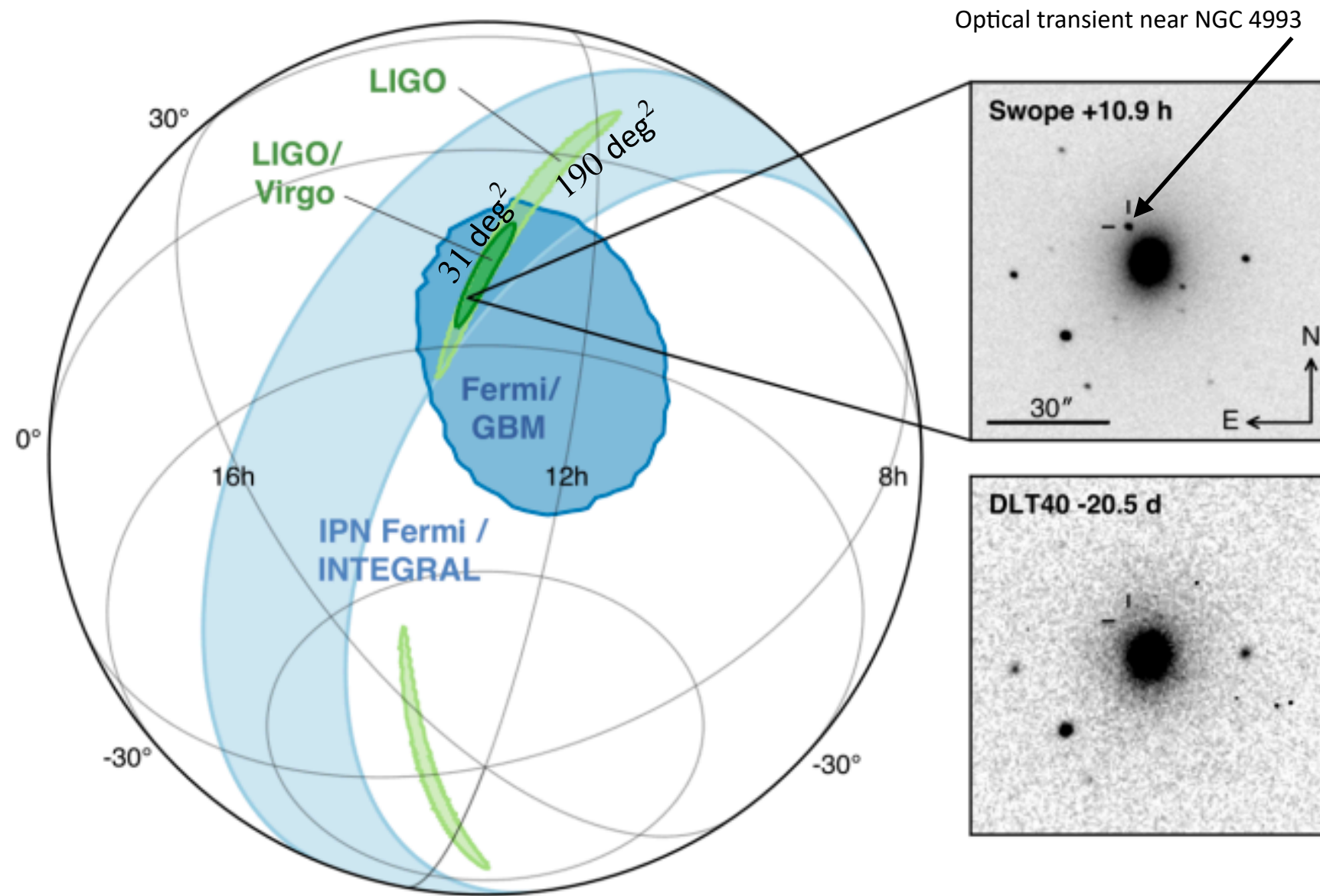
**3. Adding redshift information coming from potential host galaxies (“dark siren” method, or “galaxy catalog” method)**

**Requires a galaxy catalog!**

# Standard sirens

## 1. Identification of a direct EM counterpart (“bright siren” method)

**GW170817**

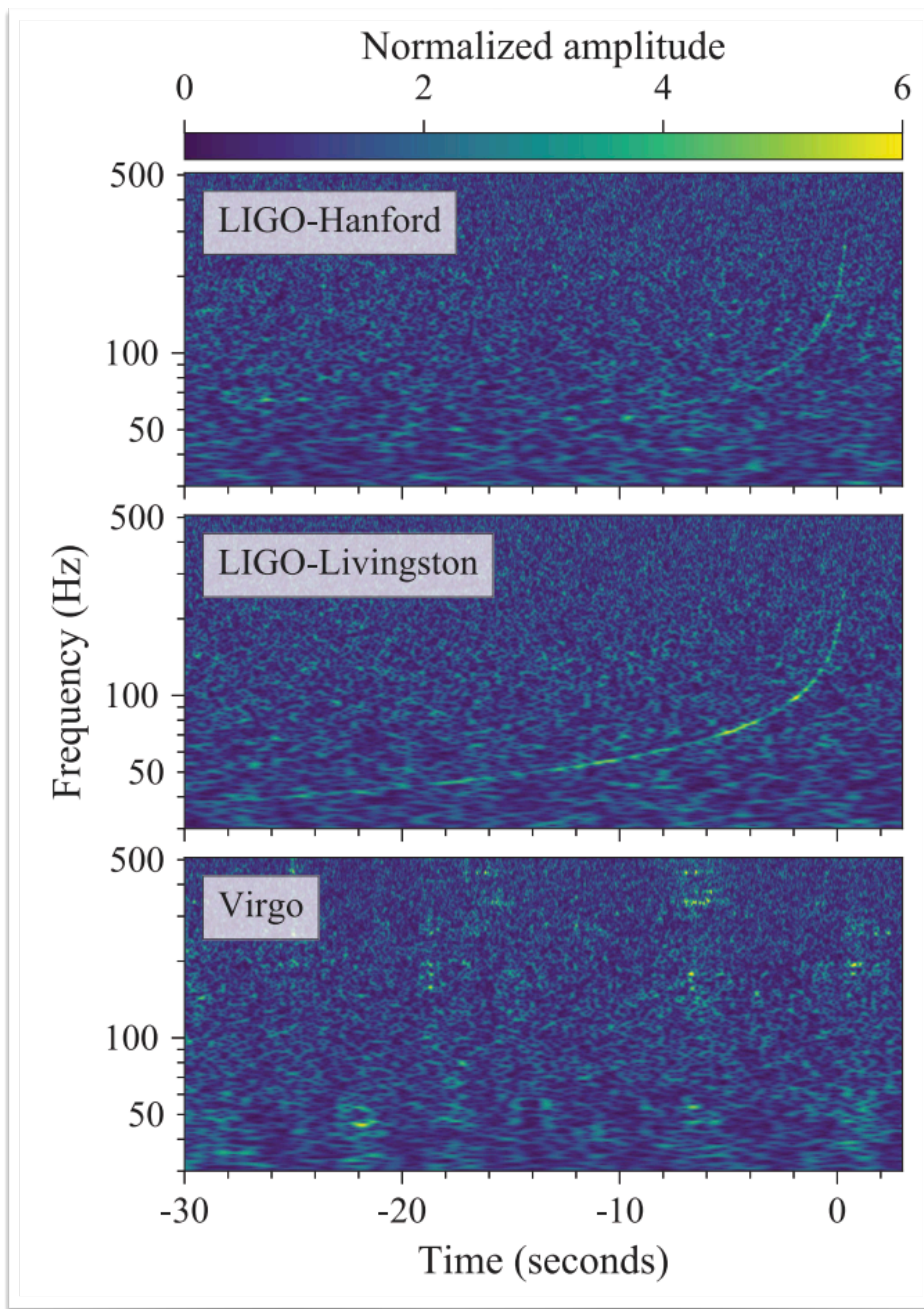


# Standard sirens

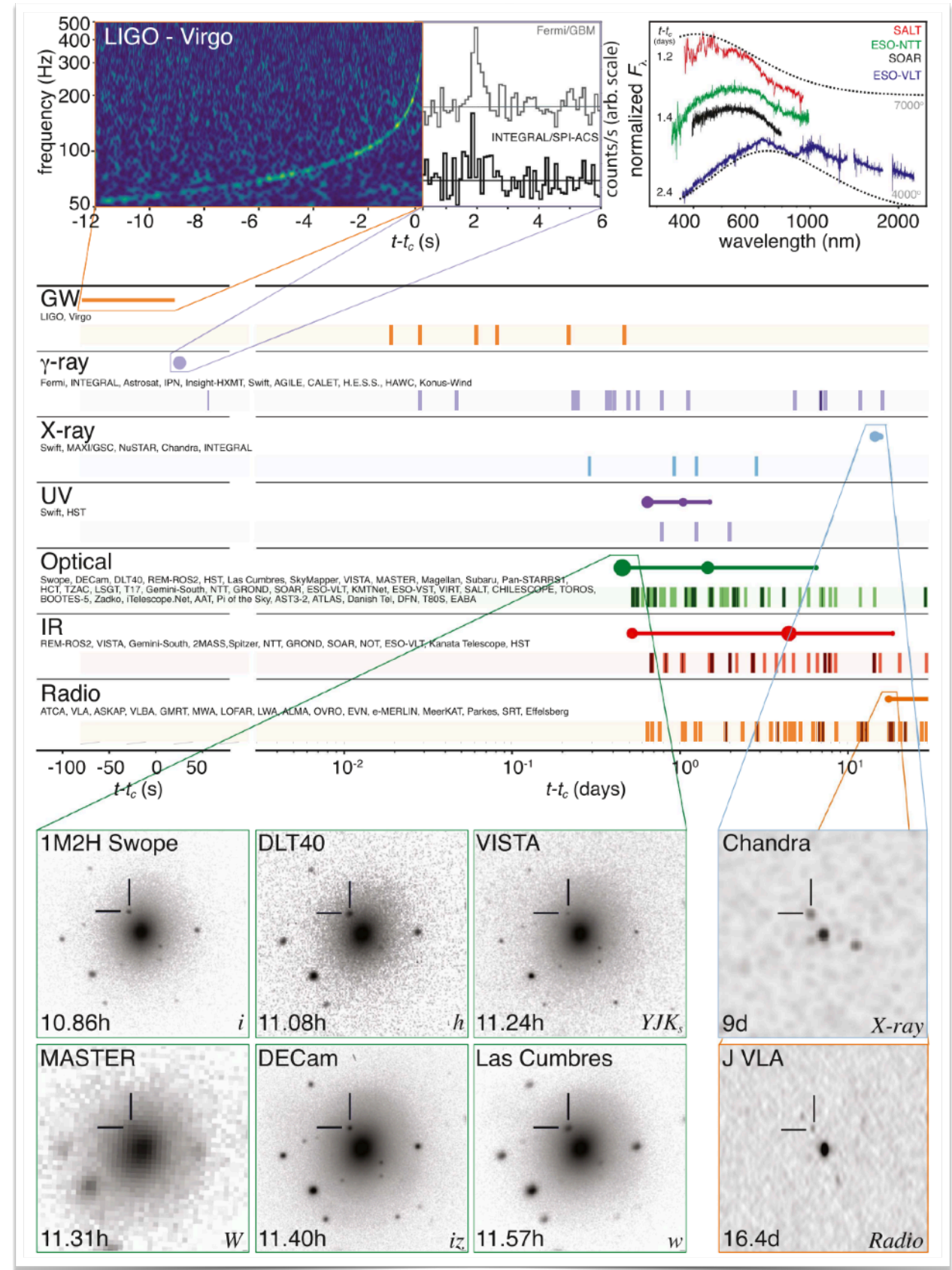
## 1. Identification of a direct EM counterpart (“bright siren” method)

The coincident GW-EM detection of GW170817 puts stringent **constraints on the speed of GWs**:

$$c_T = c_{-3 \times 10^{-15}}^{+7 \times 10^{-16}}$$



LVC, PRL (2017)



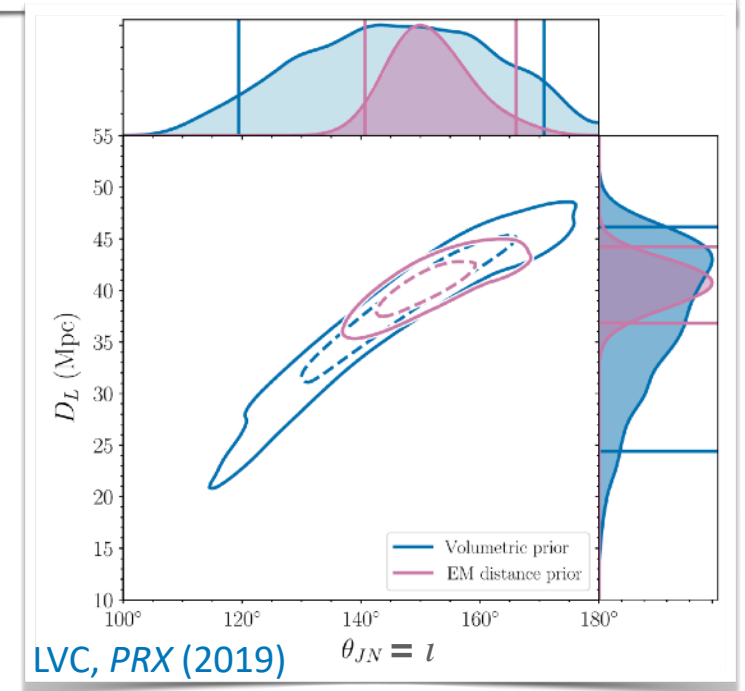
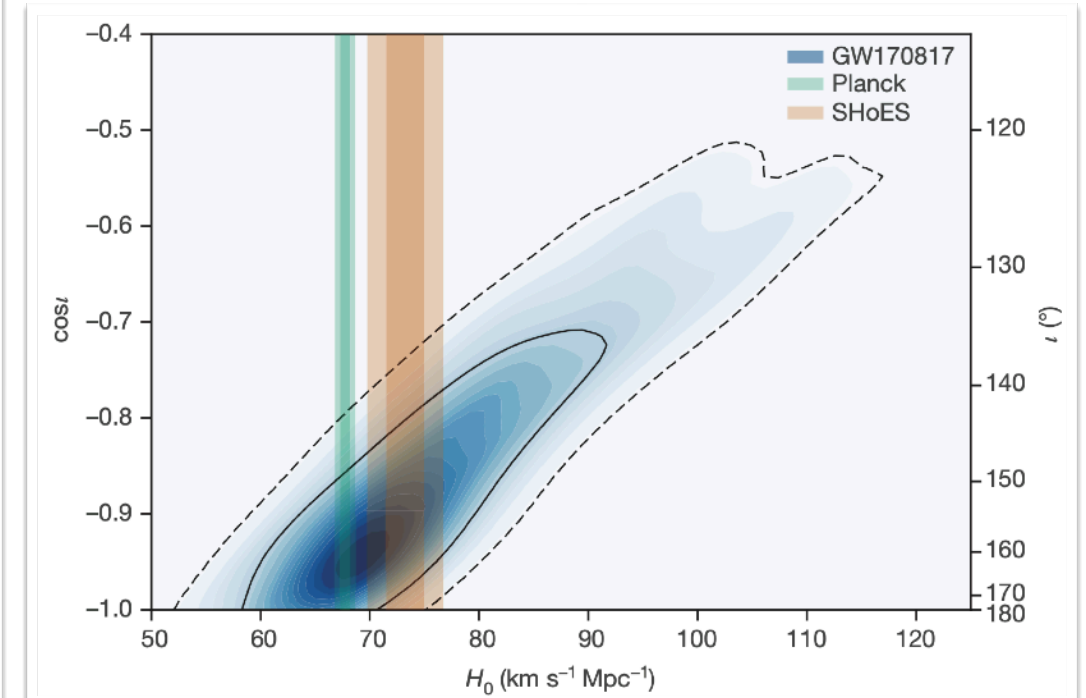
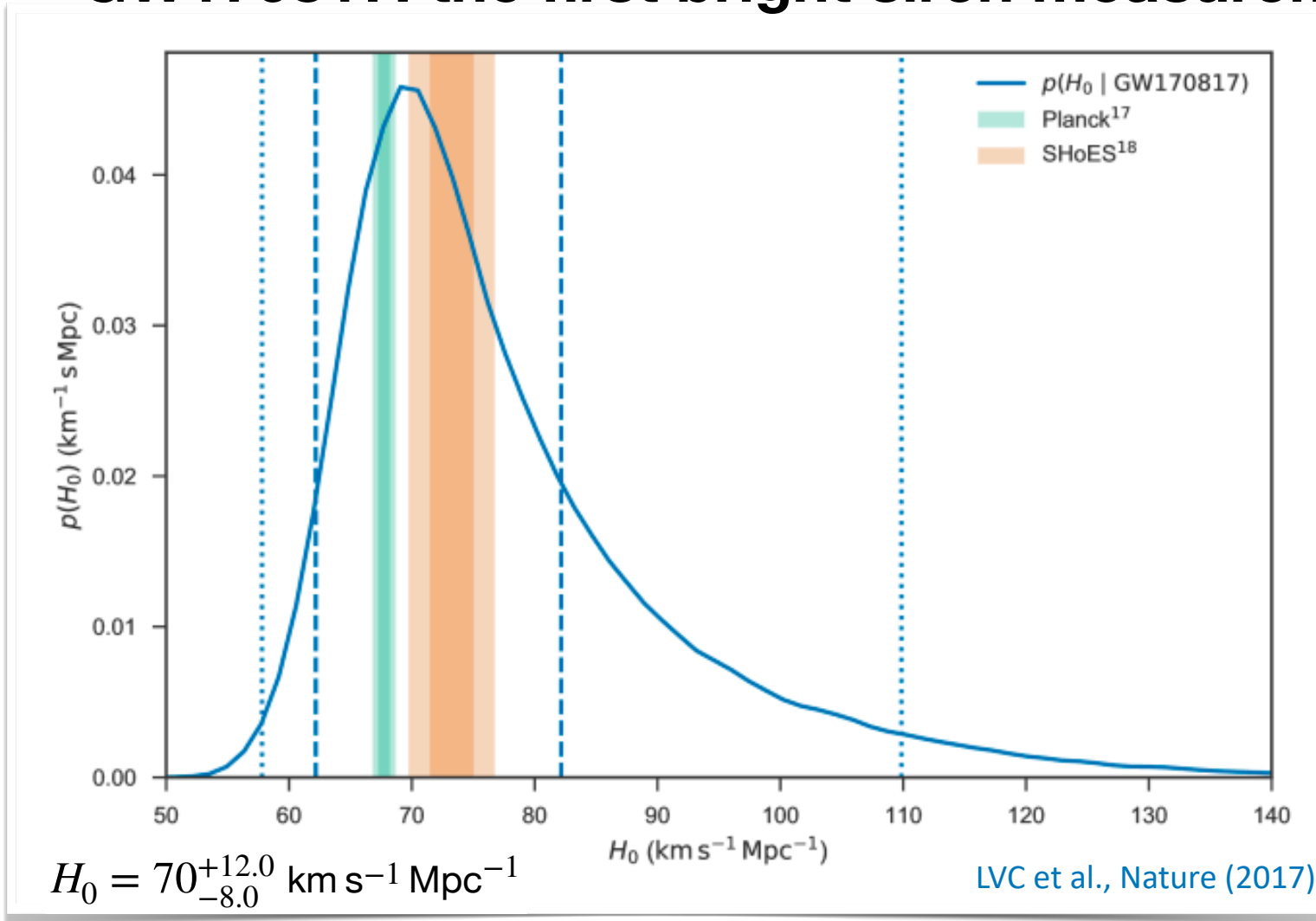
LVC et al., ApJL (2017)



# Standard sirens

## 1. Identification of a direct EM counterpart (“bright siren” method)

### GW170817: the first bright siren measurement of $H_0$



**Low- $z$  event ( $z \sim 0.01$ ):**  $D_L(z) \simeq \frac{cz}{H_0}$

**Results in agreement with EM constrains (CMB/SNIa)**

**Waiting for more EM counterparts to narrow down the  $H_0$  posterior...**

Chen et al., Nature (2018)

# Standard sirens

## 2. Using spectral features in the GW mass distribution (“spectral siren” method)

$$m_i^{\text{det}} = \left( 1 + z(D_L, H_0, \Omega_M) \right) m_i$$

From  
GWs

From  
phenomenological  
models

At a single-event level, the redshift is degenerate with the source-frame mass. However, at the population-level, features in the source-frame mass spectrum (e.g., sharp cutoff at high mass, or a peak at a specific mass) get **shifted** in the observed distribution



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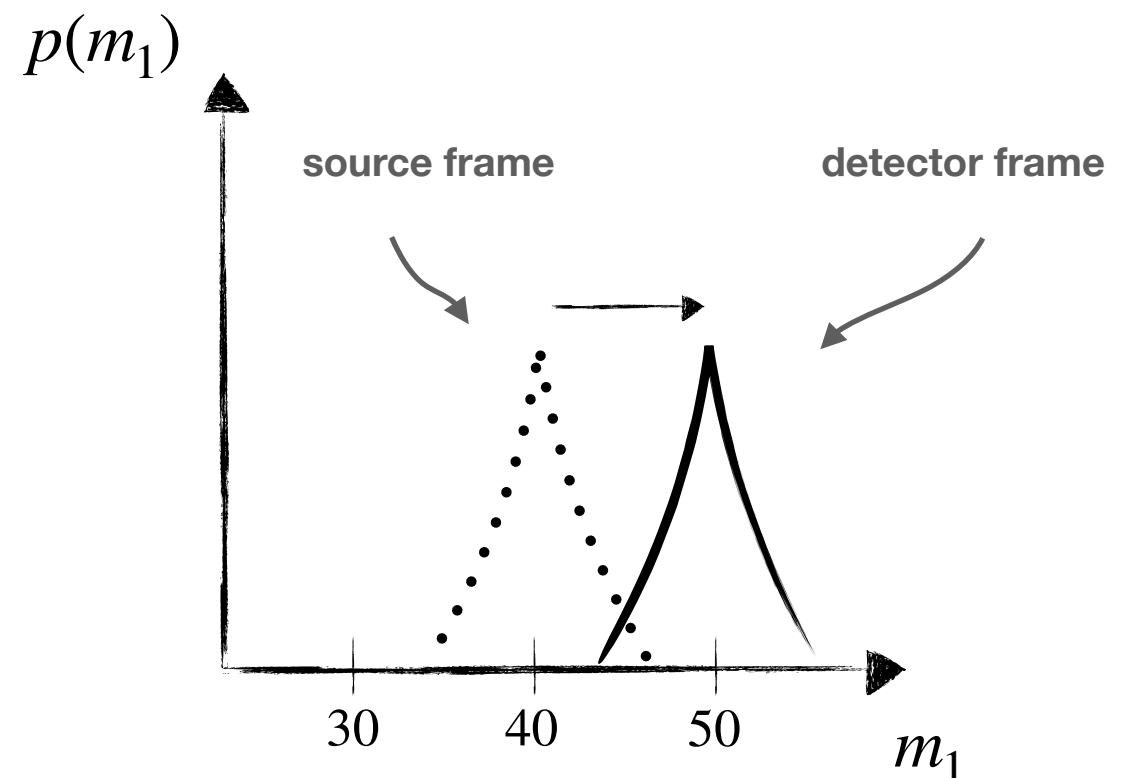
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We look for “landmarks” in the mass distribution and see how far they appear shifted.

By measuring how far these features move, we can estimate the average redshift



# Standard sirens

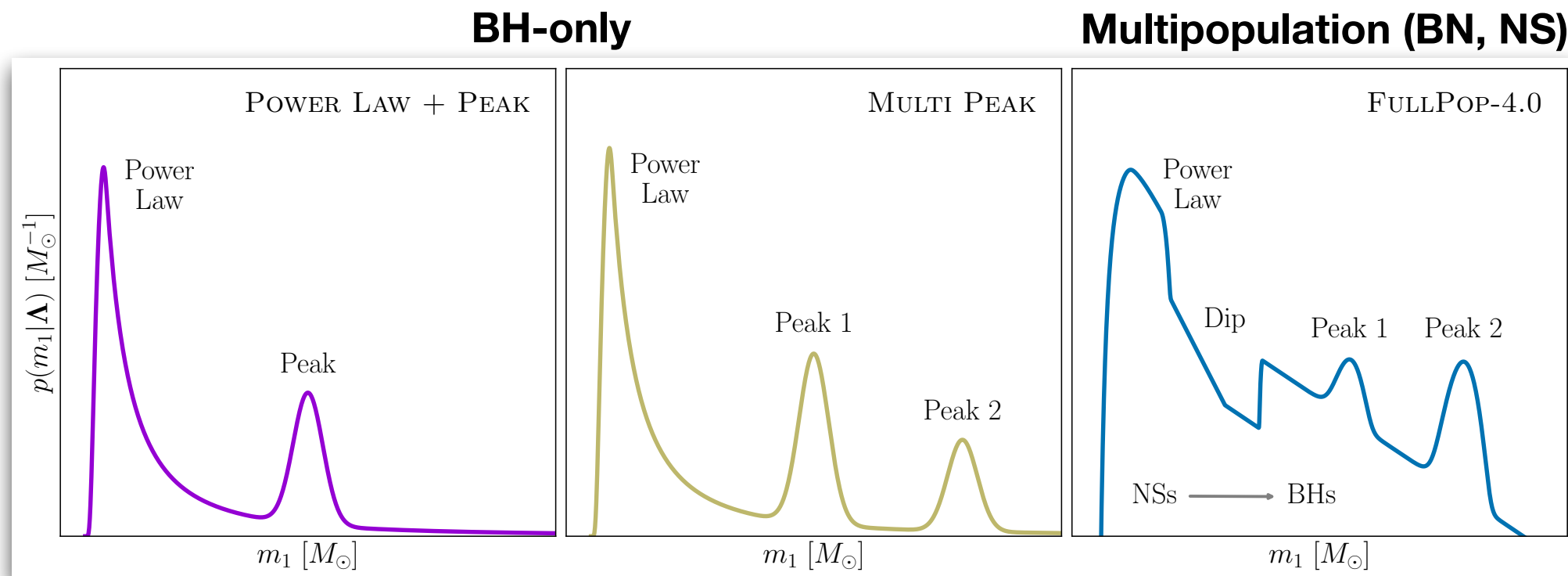
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LVK O4a cosmology paper, arXiv:2509.04348

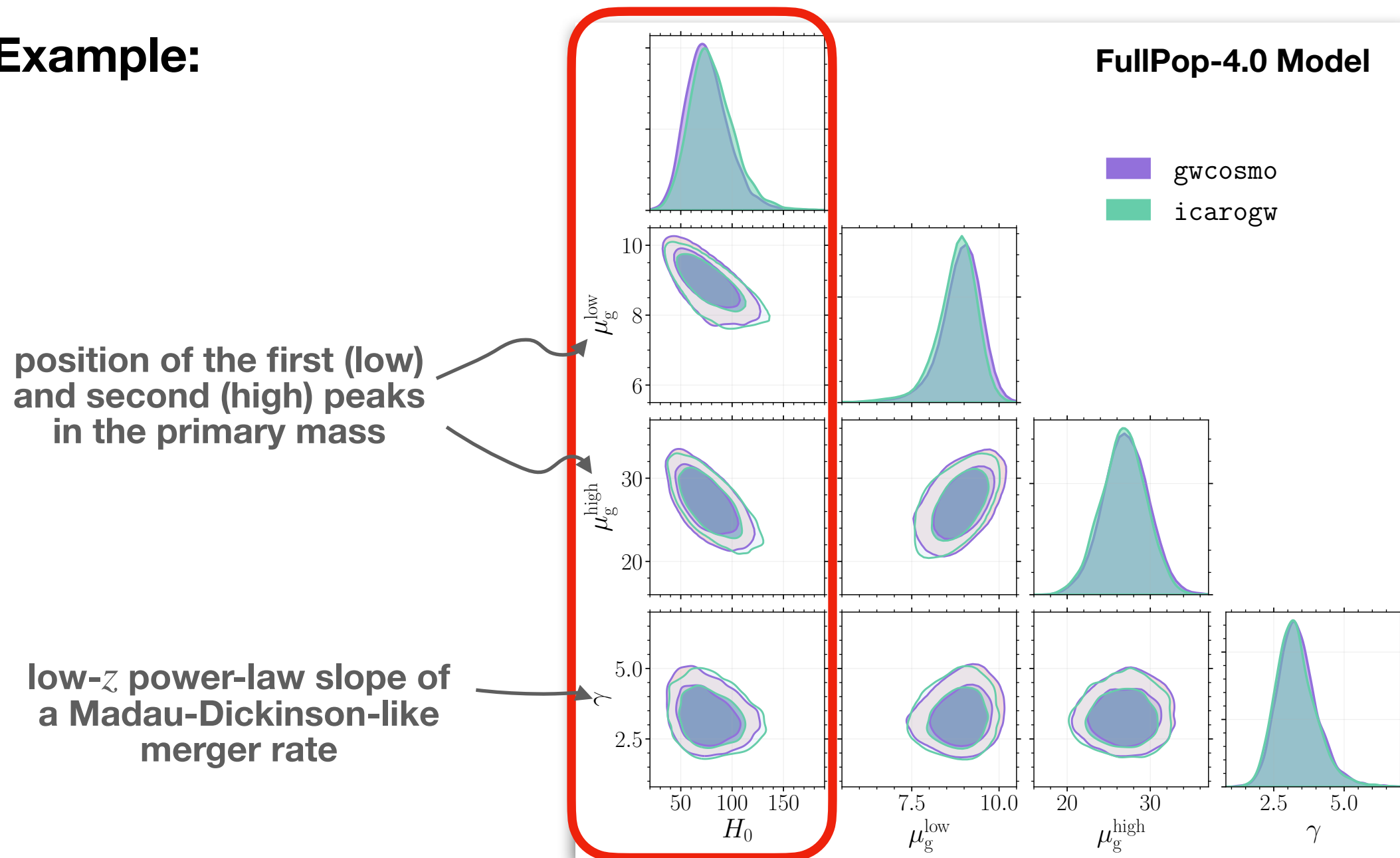


# Standard sirens

## 2. Using spectral features in the GW mass distribution (“spectral siren” method)

We sample both the cosmological and population parameters

Example:



Gray et al., 2020, 2022, 2023

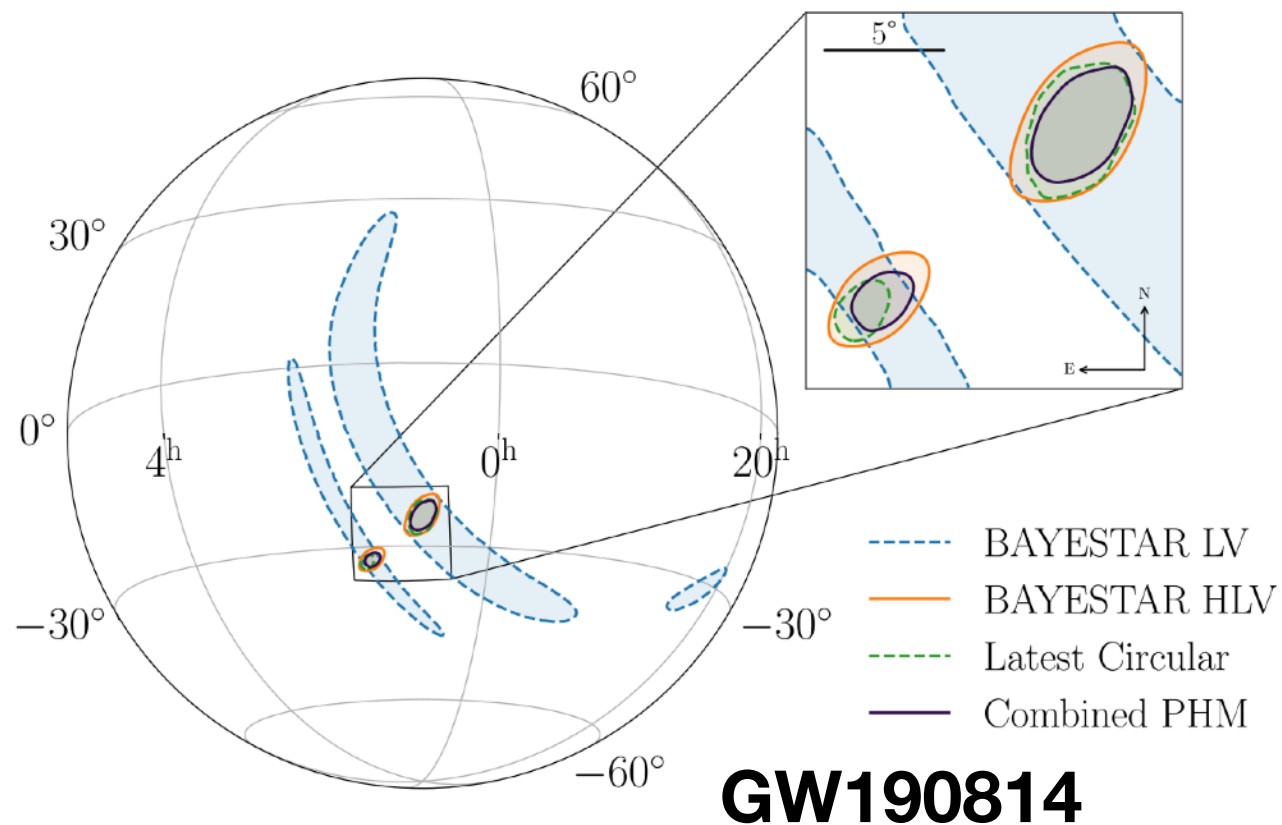
Mastrogiovanni et al., 2023, 2024

LVK O4a cosmology paper, arXiv:2509.04348

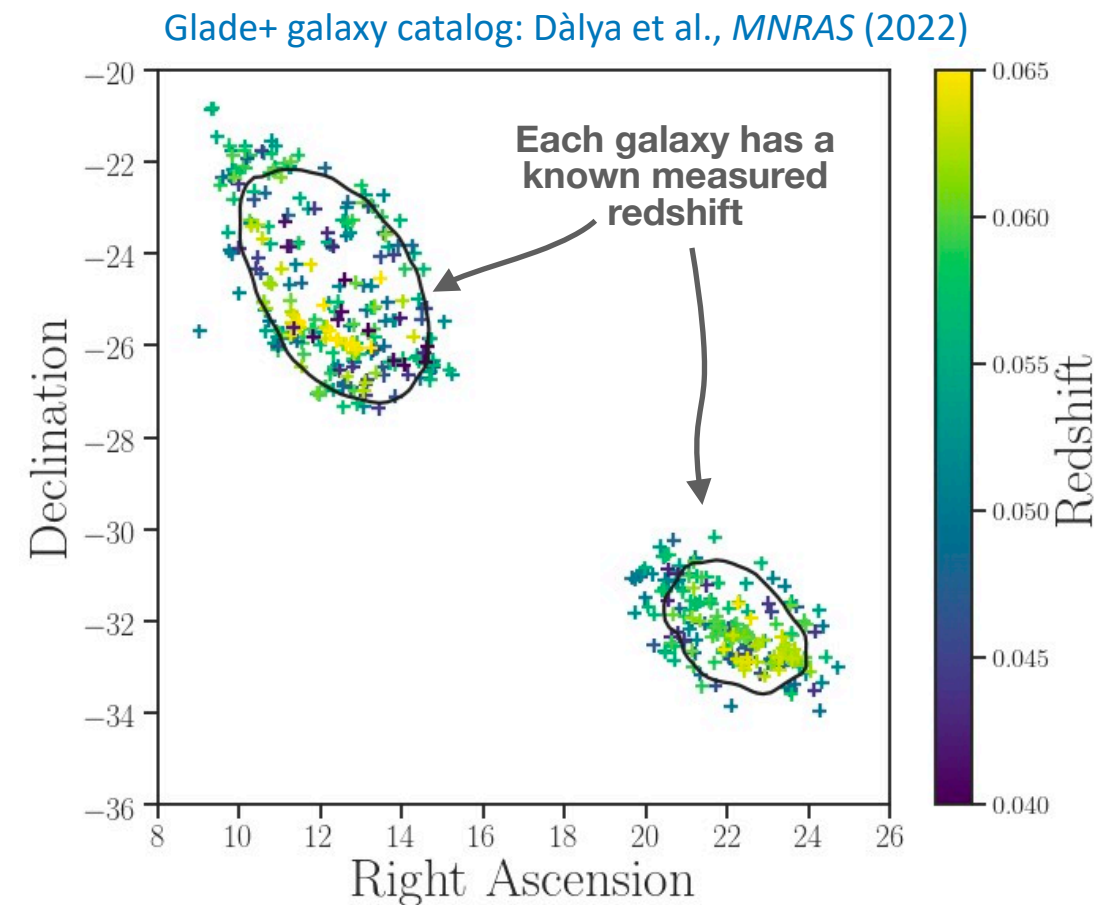
# Standard sirens

## 3. Adding redshift information coming from potential host galaxies (“dark siren” method, or “galaxy catalog” method)

To account for the uncertainty as to which galaxy is the true host, we can marginalize over the redshifts of the galaxies that are compatible with the GW sky location



LVK, ApJL (2020)



LIGO-Virgo, LIGO Document P2000227-v4

Other first applications of this method: GW170817 (Fishbach et al., ApJL 2019), GW170814 (DES, LVC et al., ApJL 2019)

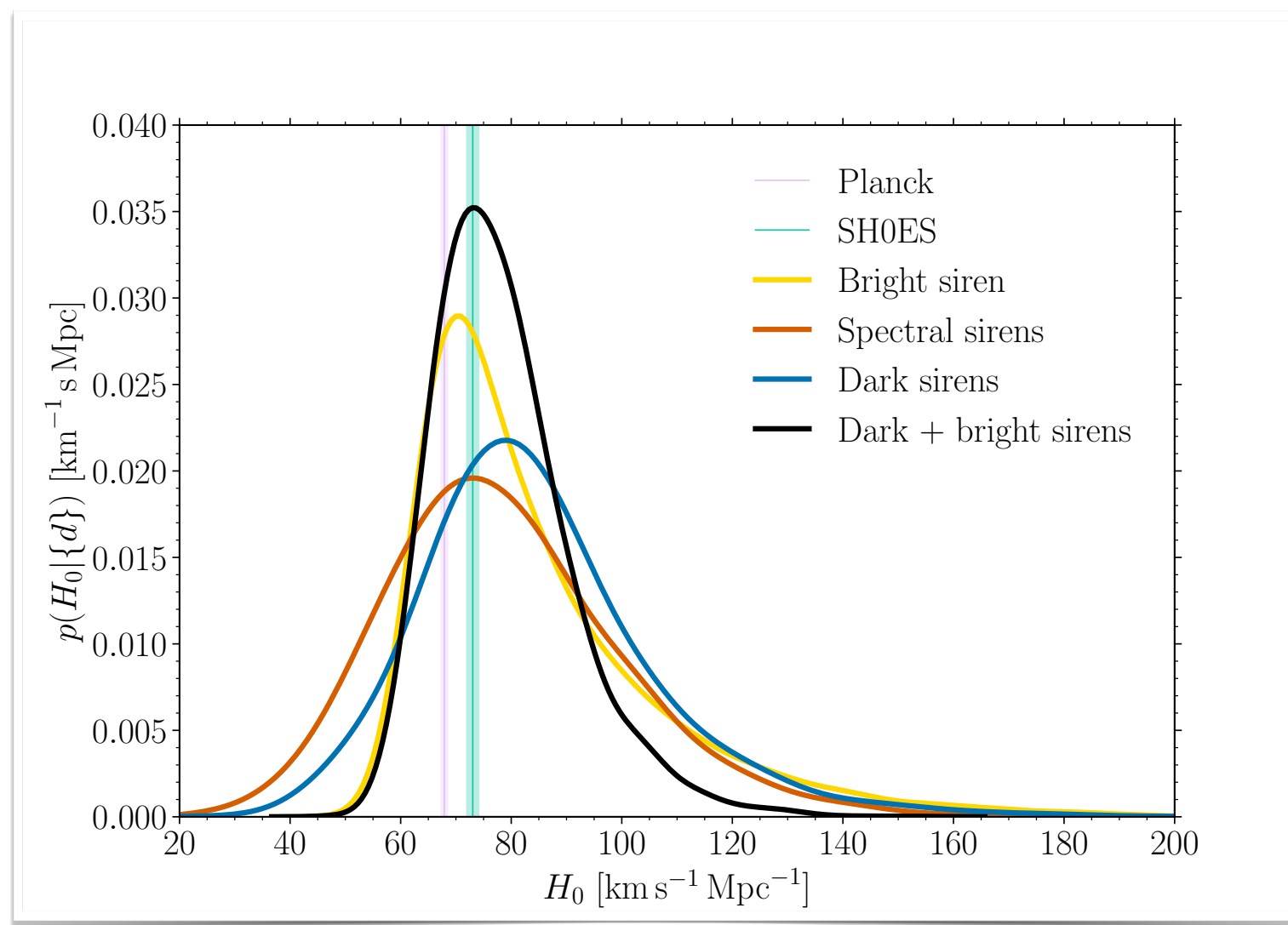


# Standard sirens

## 3. Adding redshift information coming from potential host galaxies (“dark siren” method, or “galaxy catalog” method)

By combining the information from many GW events, the contributions from the true host galaxies will grow since they will all share the same  $H_0$ . Contributions from the others will statistically average out, leading to a constraint on cosmological parameters.

### Example:



$$H_0 = 76.4^{+23.0}_{-18.1} \text{ km s}^{-1} \text{ Mpc}^{-1}$$
$$H_0 = 81.6^{+21.5}_{-15.9} \text{ km s}^{-1} \text{ Mpc}^{-1}$$
$$H_0 = 76.6^{+13.0}_{-9.5} \text{ km s}^{-1} \text{ Mpc}^{-1}$$

(median + 68.3% CI)

LVK O4a cosmology paper, arXiv:2509.04348

# Standard sirens: methods

**How are these analyses done in practice? We work within the framework of hierarchical Bayesian inference in the presence of selection effects.**

Mandel et al, *MNRAS* (2019)

Vitale et al., Chapter 42 in *Handbook of Gravitational Wave Astronomy*, Springer (2022) - arXiv:2007.05579



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**For the spectral and dark siren methods, the observed GW sample is modeled as resulting from an inhomogeneous Poisson process (constant rate in detector-frame time) in the presence of selection effects. The cosmological  $\Lambda_c$  and population  $\Lambda_p$  parameters are called hyperparameters.**

$$p(\Lambda | \{x\}) \propto p(\Lambda) L(\{x\} | \Lambda)$$

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Hyperposterior of  $\Lambda = \{\Lambda_c, \Lambda_p\}$   
given some observations  $\{x_i\}_{i=1}^{N_{GW}}$

Hyperprior  
for  $\Lambda$

(Hierarchical) likelihood  
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$$L(\{x\} | \Lambda) \propto e^{-N_{exp}} \prod_i^{N_{GW}} T_{obs} \int d\theta_i L(x_i | \theta_i) \frac{dN_{CBC}}{d\theta_i dt}(\theta_i, \Lambda) \frac{1}{1+z}$$



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Observation time in detector-frame

GW event likelihood function

Compact binary coalescence (CBC) event rate in source-frame time

conversion factor from  $t_0$  to  $t = t_s$

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Observation time in detector-frame      GW event likelihood function      Compact binary coalescence (CBC) event rate in source-frame time      conversion factor from  $t_0$  to  $t = t_s$

$$N_{exp} = T_{obs} \int d\theta P_{det}(\theta; \Lambda) \frac{dN_{CBC}}{d\theta dt}(\theta, \Lambda) \frac{1}{1+z}$$

Probability of detecting a GW source of parameters  $\theta$  given  $\Lambda$

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Probability of detecting a GW source of parameters  $\theta$  given  $\Lambda$

What is this term?



# Standard sirens: methods

The compact-binary-coalescence merger rate is parametrized in terms of redshift and source-frame time.

For a **spectral siren analysis**:

$$\frac{dN_{CBC}}{d\vec{m}d\theta d\Omega dz dt} \propto \psi_{MD}(z; \Lambda_p) p_{pop}(\vec{m}, \theta | \Lambda_p) \frac{dV_c}{dz d\Omega}(z, \Lambda_c)$$

$\vec{m} = \{m_1, m_2\}$   
binary component masses

other GW parameters,  
e.g., spins

sky position

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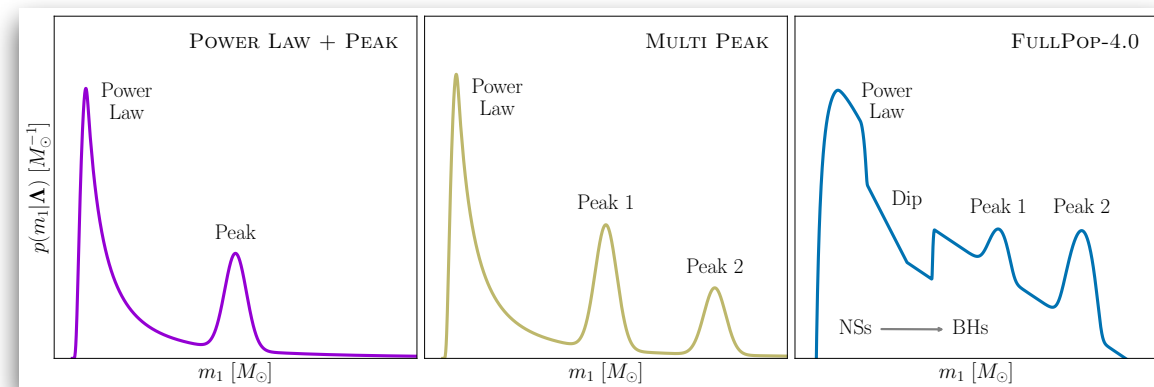
CBC merger rate as a function of  $z$ , usually parametrized as a Madau-Dickinson distribution

comoving volume factor

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For a **spectral siren analysis**:



PDF describing the distribution of GW sources in masses  $\vec{m}$  and other GW parameters  $\theta$

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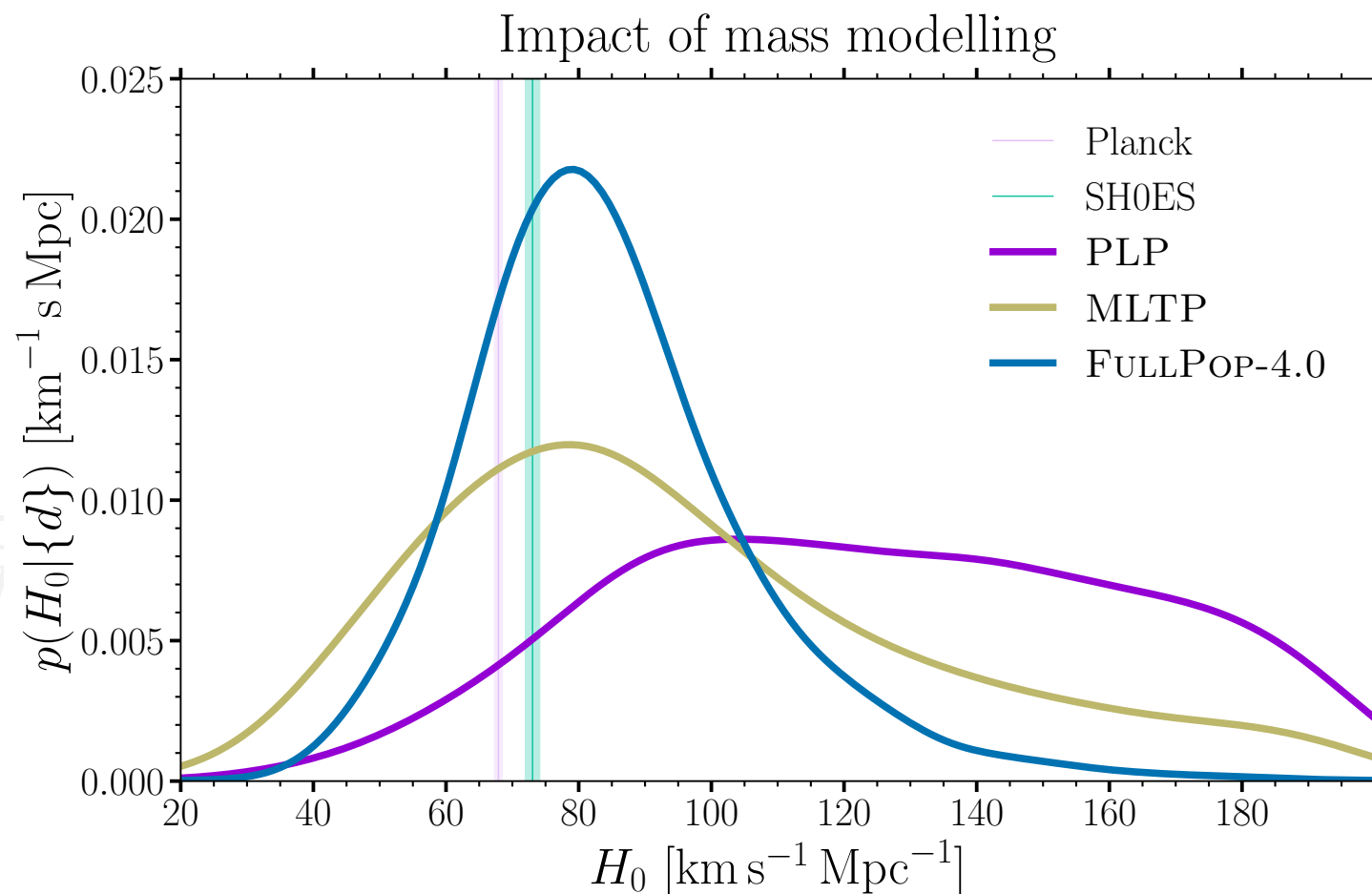
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# Standard sirens: methods

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For a **spectral siren analysis**:

**Main limitation: systematics  
due to the choice of a  
population model**



**LVK O4a cosmology paper, arXiv:2509.04348**

Dickinson distribution



# Standard sirens: methods

The compact-binary-coalescence merger rate is parametrized in terms of redshift and source-frame time.

For a **dark siren analysis**, it's more complex! We have to build a pixelated “line-of-sight” redshift prior:

“In-catalog” term  
(accounts for the galaxies  
in your galaxy catalog)

$$\frac{dN_{CBC}}{d\vec{m}d\theta d\Omega dz dt} \propto \psi_{MD}(z; \Lambda_p) p_{pop}(\vec{m}, \theta | \Lambda_p) \left[ \frac{1}{\Delta\Omega} \sum_j^{N_{gal}(\Omega)} \left( \frac{L}{L_*} \right)^\epsilon p(z | z_j, \sigma_{z,j}) + \int_{M_{thr}(z, m_{thr}(\Omega))}^{M_{max}} dM \text{Sch}(M; \Lambda_p) \left( \frac{L}{L_*} \right)^\epsilon \frac{dV_c}{dz d\Omega} \right]$$

Absolute magnitude      Apparent magnitude      “K-corrections”

$$M = m + 5 - 5 \log D_L(z, \Lambda_c) - K_{corr}$$

Absolute luminosity

$$\frac{L}{L_*} = 10^{0.4(M_* - M)}$$

“Out-of-catalog” term  
(corrects for the incompleteness  
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Diagram annotations:

- Number of galaxies in a given sky direction**: points to  $N_{gal}(\Omega)$
- Absolute luminosity of the galaxy over a reference galaxy luminosity**: points to  $\left( \frac{L}{L_*} \right)^\epsilon$
- Luminosity weight  $\epsilon = \{0,1\}$** : points to  $\epsilon$
- Galaxy redshift uncertainty model**: points to  $p(z | z_j, \sigma_{z,j})$
- pixel area**: points to  $\Delta\Omega$
- Threshold magnitude above which we are missing galaxies due to flux-limited surveys**: points to  $M_{thr}(z, m_{thr}(\Omega))$
- Schechter (luminosity) function giving the number density of galaxies as a function of their absolute magnitudes**: points to  $\text{Sch}(M; \Lambda_p)$
- Characteristic luminosity**: points to  $L_*$

**Absolute magnitude**      **Apparent magnitude**      **“K-corrections”**

$$M = m + 5 - 5 \log D_L(z, \Lambda_c) - K_{corr}$$

**Absolute luminosity**

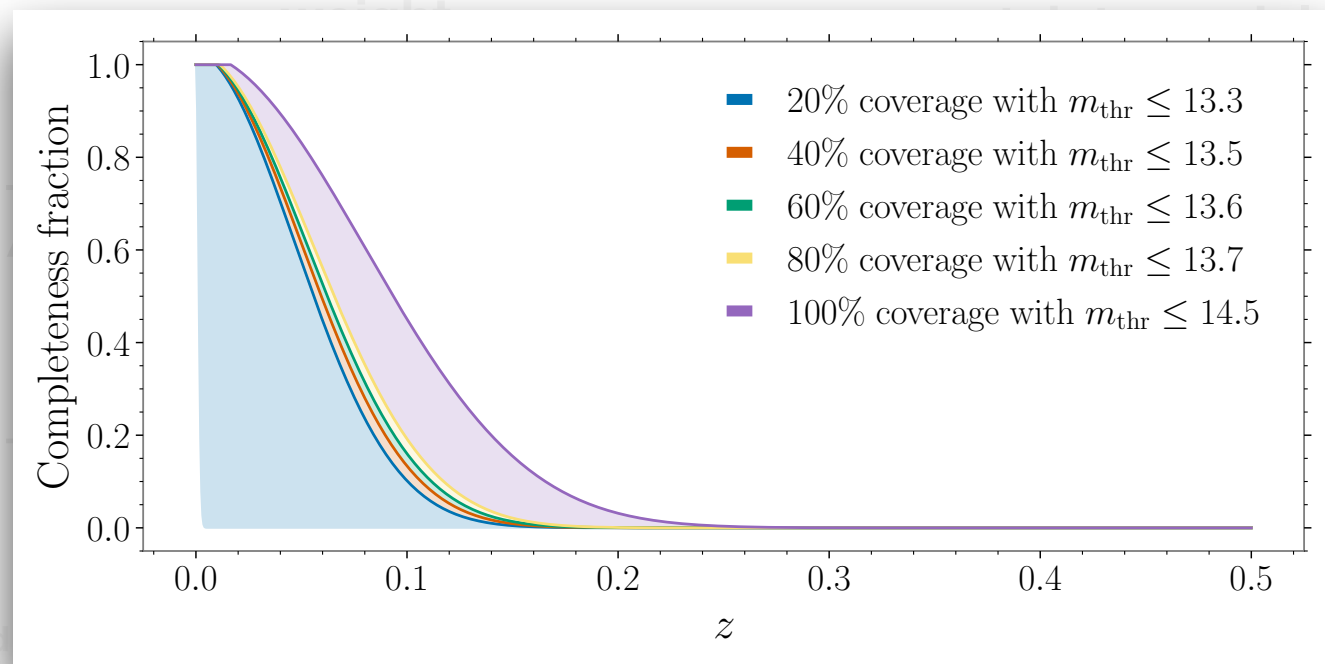
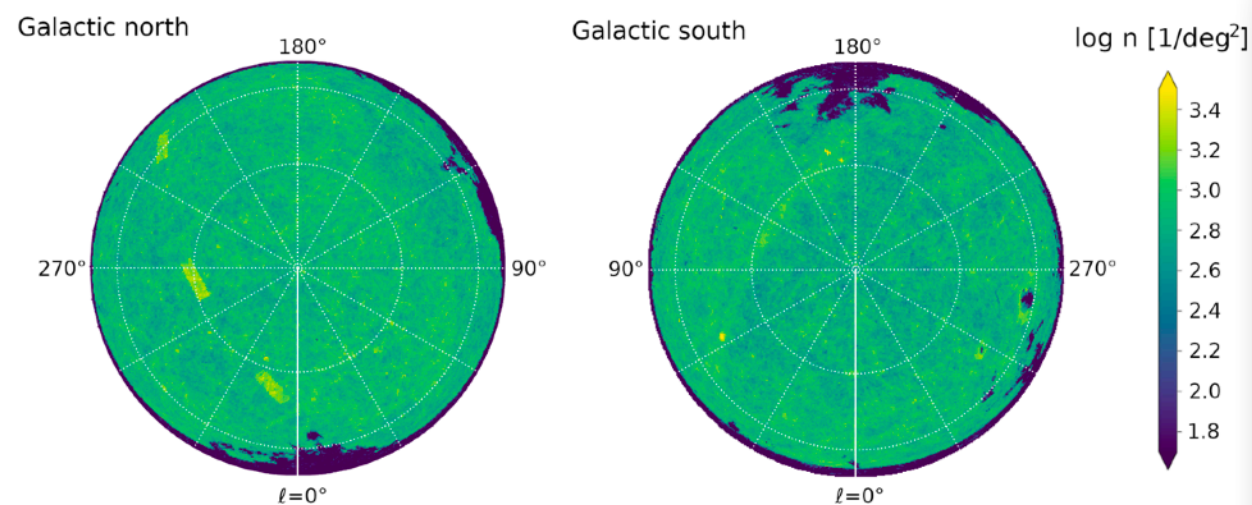
$$\frac{L}{L_*} = 10^{0.4(M_* - M)}$$

$$\text{Sch}(M, \Lambda_p) dM = \Phi(L, \Lambda_p) dL$$

# Standard sirens: methods

**GLADE+**: Galaxy List for the Advanced Detector Era +

Collection of **6 different surveys** (~22.5M galaxy: GWGD, 2MPZ, 2MASS XSC, HyperLEDA, WISExSCOSPZ; ~750k quasars: SDSS-DR16Q)



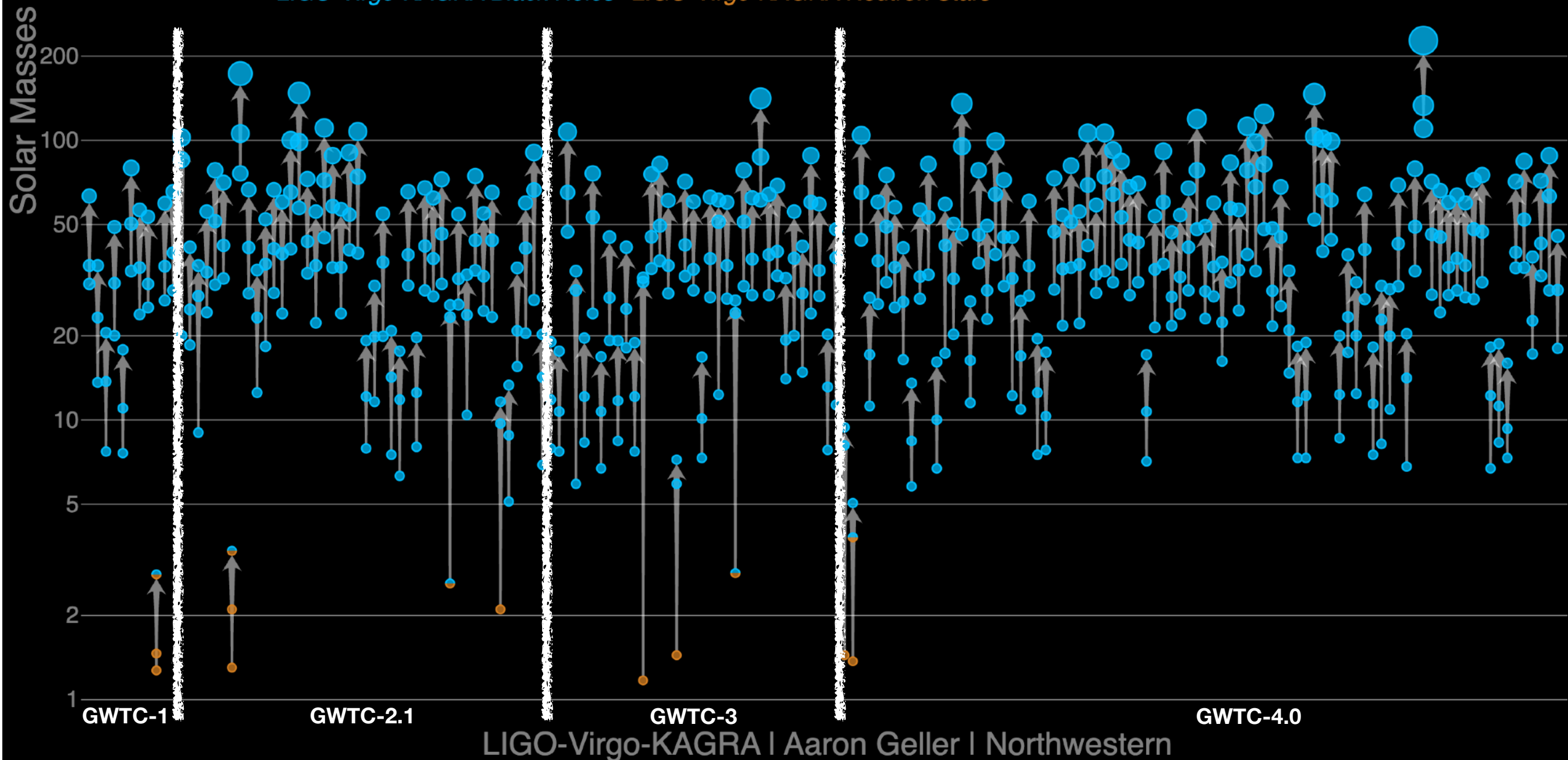
G. Dály et al., MNRAS 514 1 (2022)

LVK O4a cosmology paper, [arXiv:2509.04348](https://arxiv.org/abs/2509.04348)

**Main limitation: incompleteness of the galaxy catalog used**

## Masses in the Stellar Graveyard

*LIGO-Virgo-KAGRA Black Holes* *LIGO-Virgo-KAGRA Neutron Stars*





# GWTC-4.0 is here!

The methods we have seen have been applied to the latest GW detections

## New cosmology paper with GWTC-4.0!

[LVK Collaboration, [arXiv:2509.04348](https://arxiv.org/abs/2509.04348)]

### GWTC-4.0: Constraints on the Cosmic Expansion Rate and Modified Gravitational-wave Propagation

THE LIGO SCIENTIFIC COLLABORATION, THE VIRGO COLLABORATION, AND THE KAGRA COLLABORATION

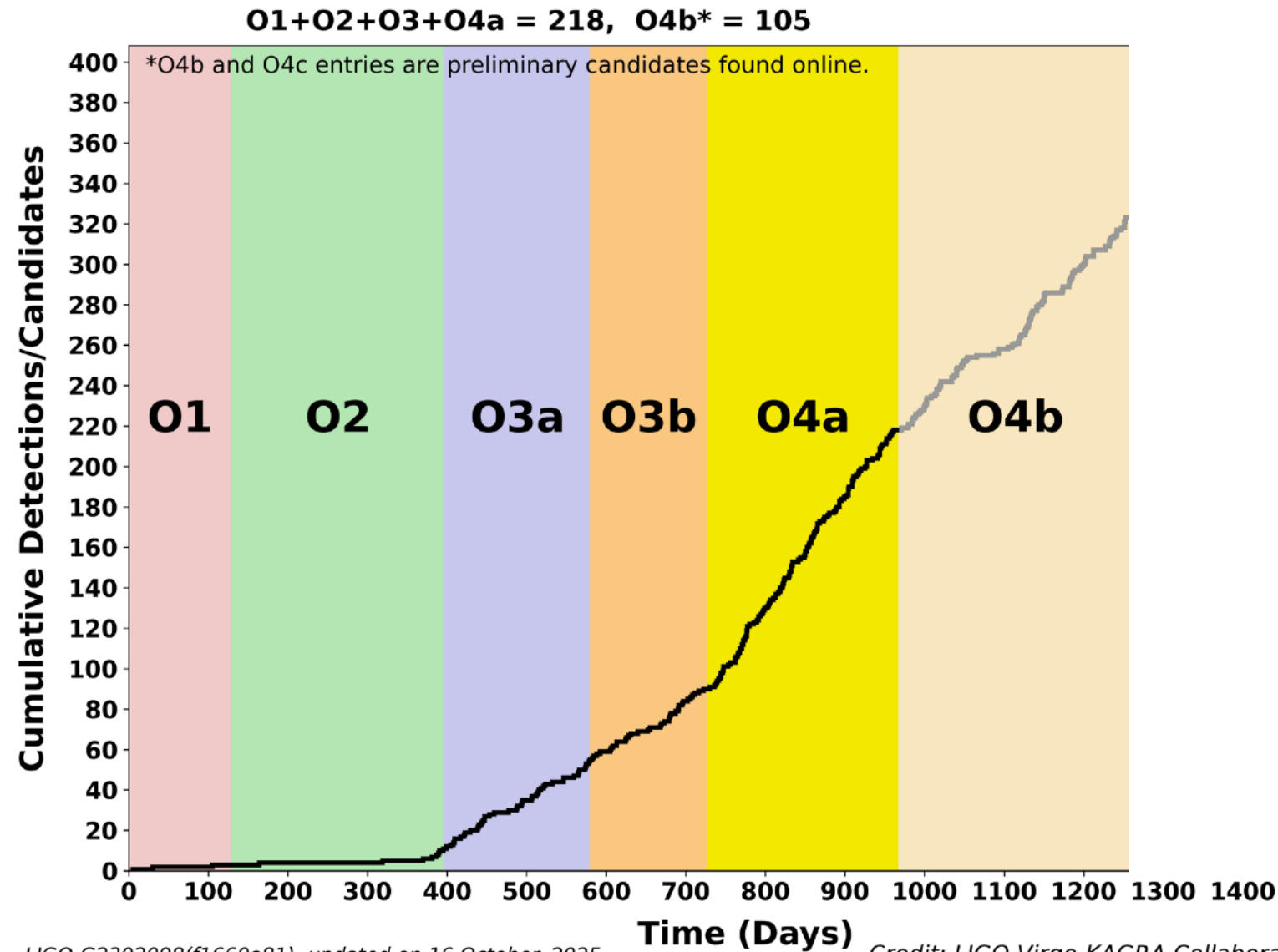
#### ABSTRACT

We analyze data from 142 of the 218 gravitational-wave (GW) sources in the fourth LIGO–Virgo–KAGRA Collaboration (LVK) Gravitational-Wave Transient Catalog (GWTC-4.0) to estimate the Hubble constant  $H_0$  jointly with the population properties of merging compact binaries. We measure the luminosity distance and redshifted masses of GW sources directly; in contrast, we infer GW source redshifts statistically through i) location of features in the compact object mass spectrum and merger rate evolution, and ii) identifying potential host galaxies in the GW localization volume. Probing the relationship between source luminosity distances and redshifts obtained in this way yields constraints on cosmological parameters. We also constrain parameterized deviations from general relativity which affect GW propagation, specifically those modifying the dependence of a GW signal on the source luminosity distance. Assuming our fiducial model for the source-frame mass distribution and using GW candidates detected up to the end of the fourth observing run (O4a), together with the GLADE+ all-sky galaxy catalog, we estimate  $H_0 = 76.6^{+13.0}_{-9.5} (76.6^{+25.2}_{-14.0}) \text{ km s}^{-1} \text{ Mpc}^{-1}$ . This value is reported as a median with 68.3% (90%) symmetric credible interval, and includes combination with the  $H_0$  measurement from GW170817 and its electromagnetic counterpart. Using a parametrization of modified GW propagation in terms of the magnitude parameter  $\Xi_0$ , we estimate  $\Xi_0 = 1.2^{+0.8}_{-0.4} (1.2^{+2.4}_{-0.5})$ , where  $\Xi_0 = 1$  recovers the behavior of general relativity.

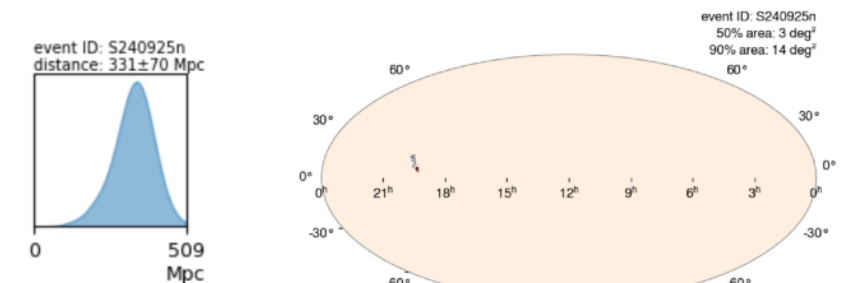
# LIGO-Virgo-KAGRA prospects

**O4b: Virgo joined the network  $\Rightarrow$  improved sky localization!**

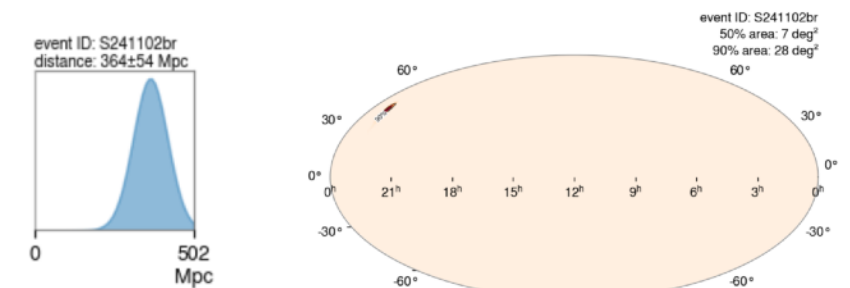
**A few preliminary candidates in O4b**



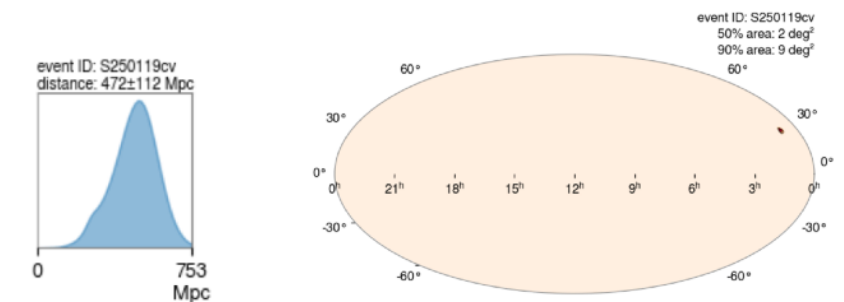
Credit: LIGO-Virgo-KAGRA Collaboration



<https://gracedb.ligo.org/superevents/S240925n>



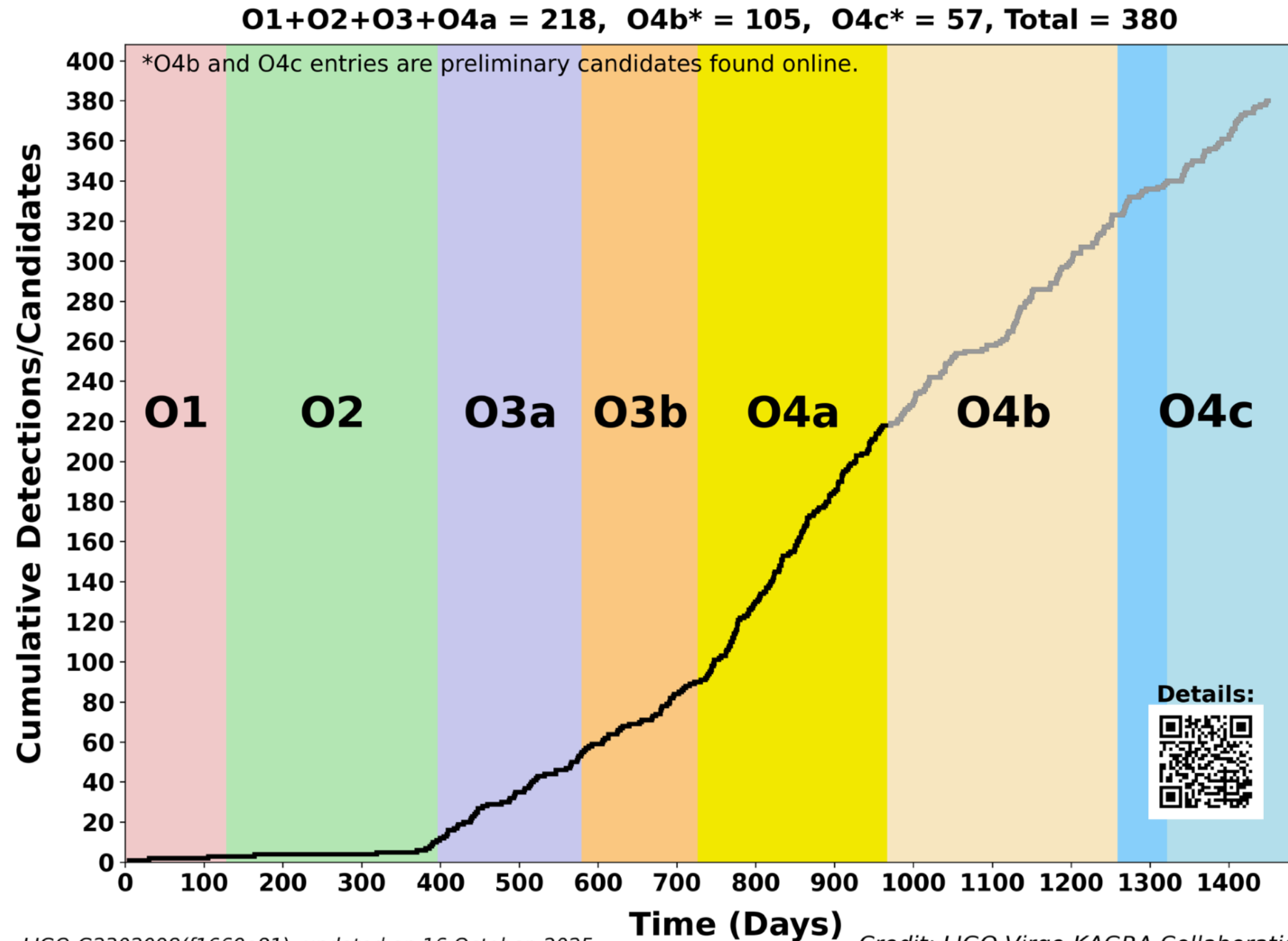
<https://gracedb.ligo.org/superevents/S241102br>



<https://gracedb.ligo.org/superevents/S250119cv>

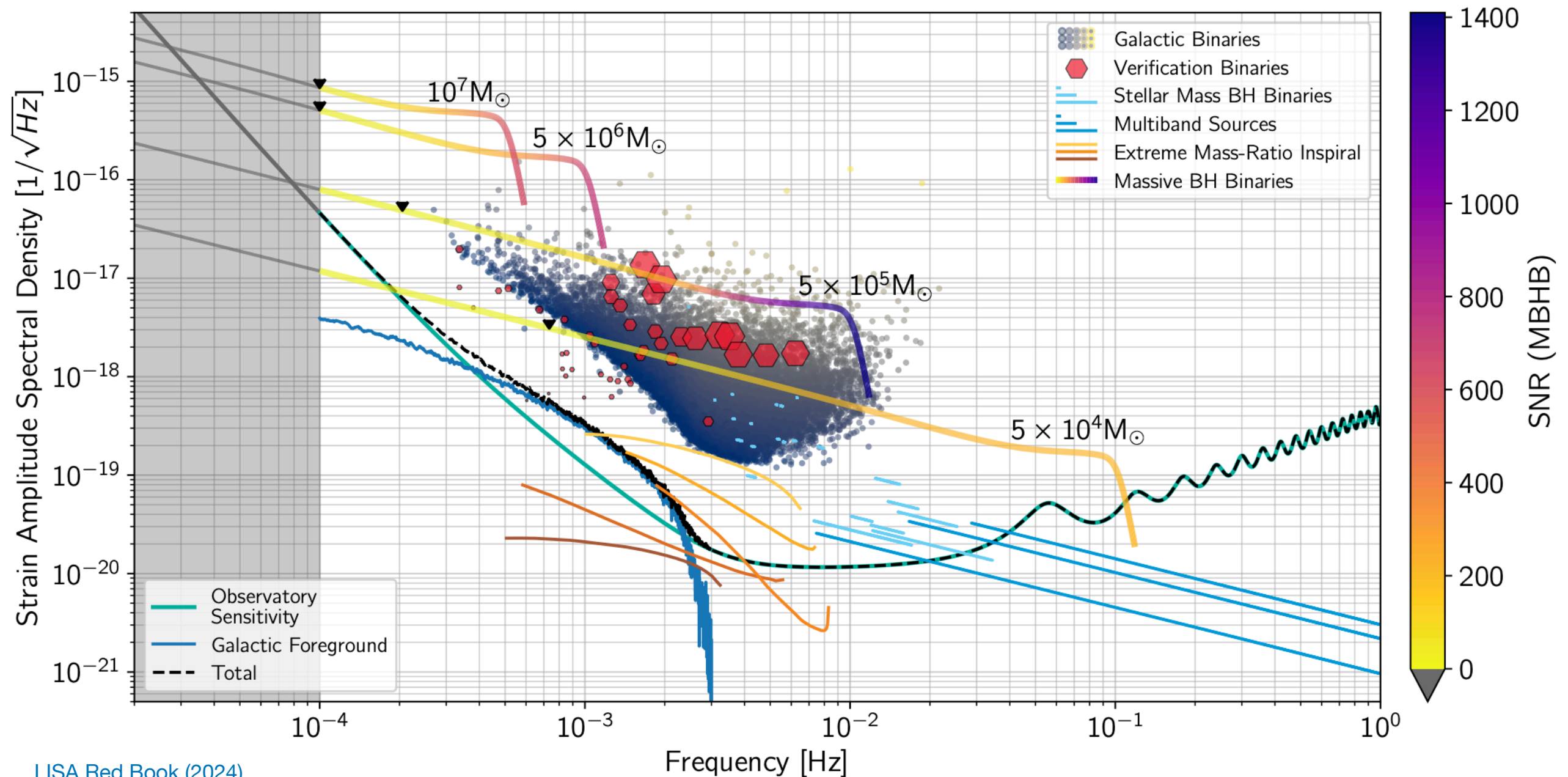
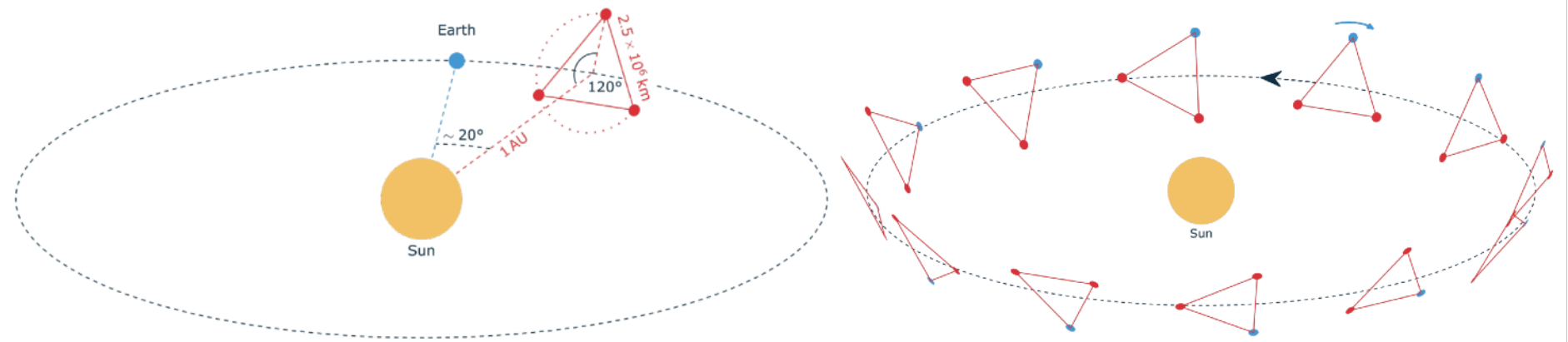
# LIGO-Virgo-KAGRA prospects

**O4c: current total of 380 alerts, but ~1 month of observation still to go!**



# Prospects with LISA

Review on cosmology with LISA: LISA CosWG, Liv Rev. Rel. (2023) -  
arXiv:2204.05434

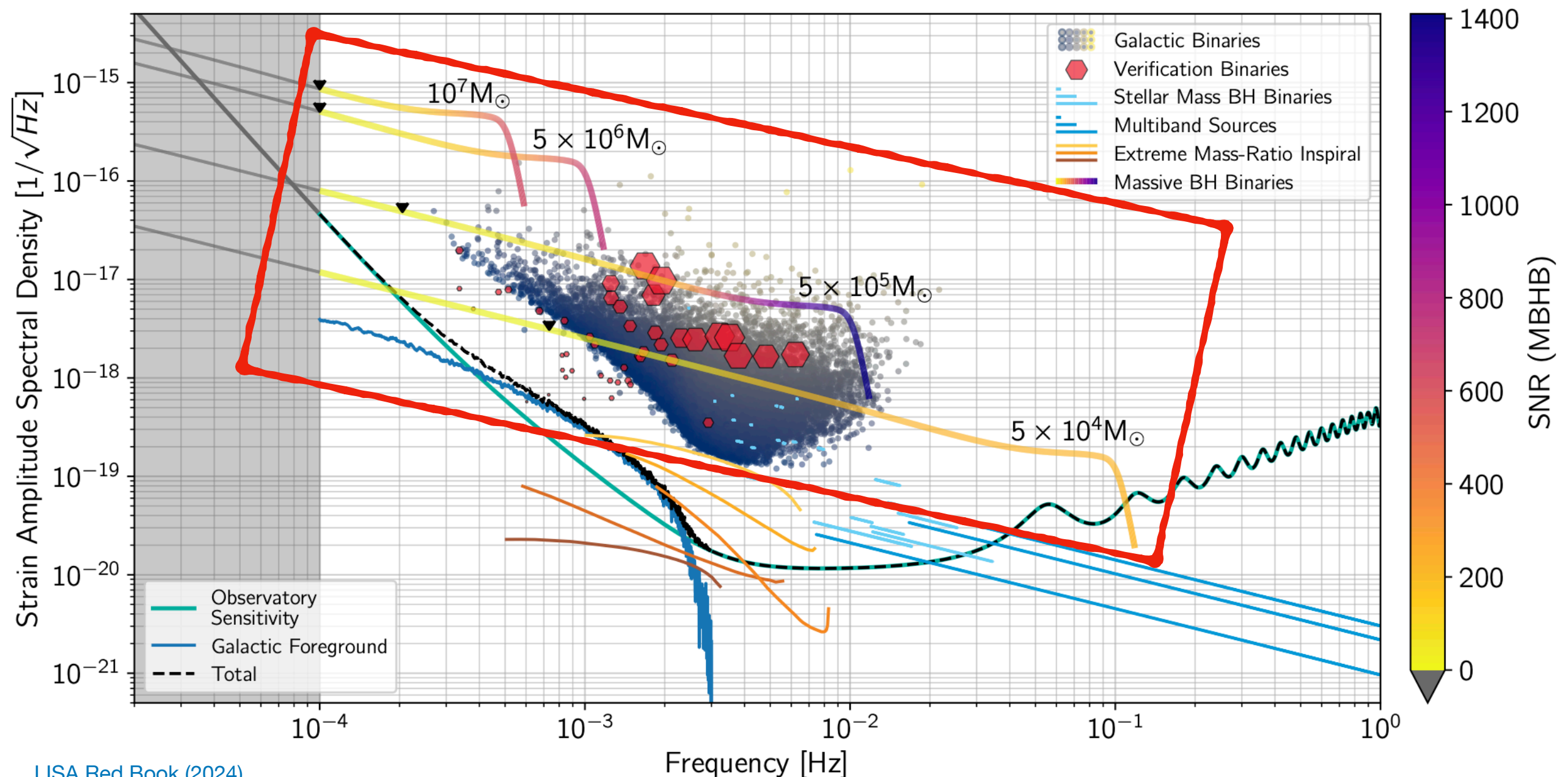




# MBHBs as bright sirens

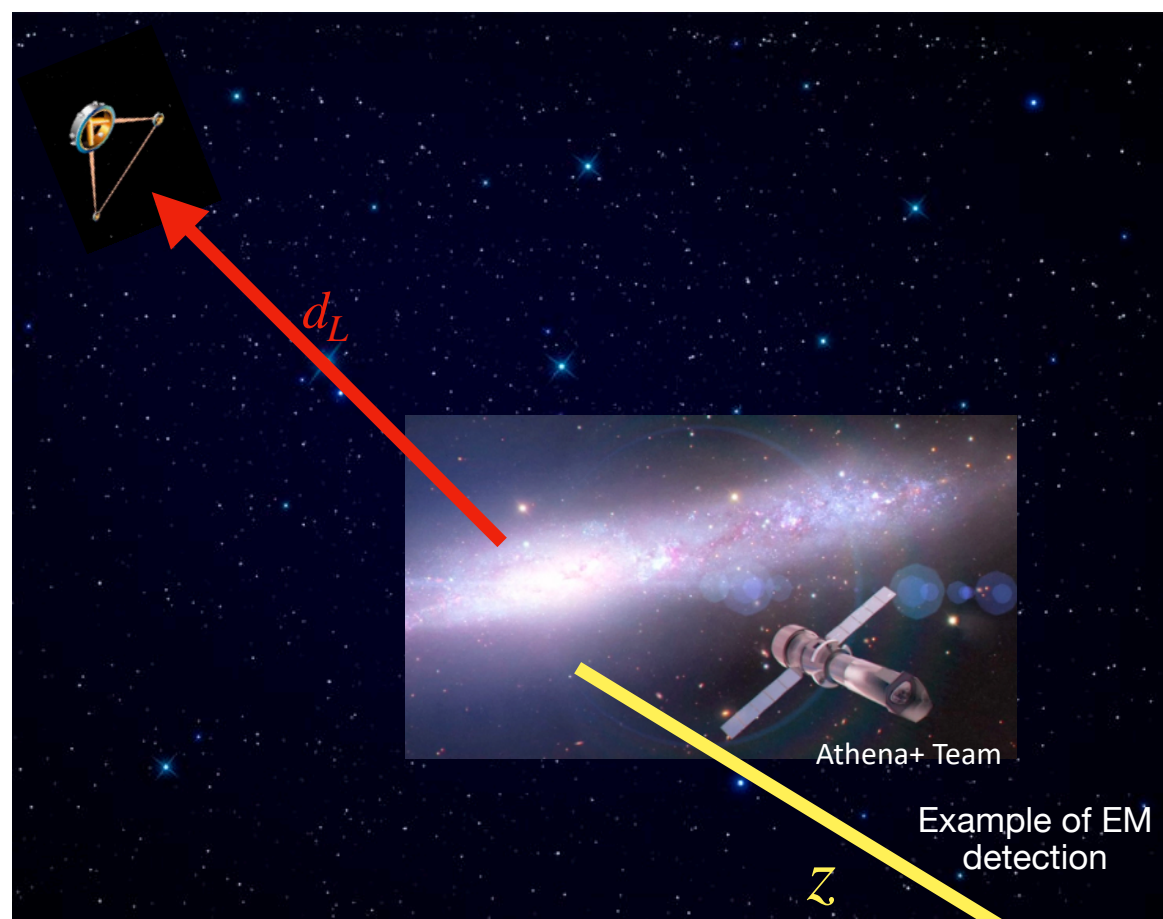
Here we will look at two promising classes of LISA standard sirens:

## 1. Massive Black Hole Binaries (MBHBs) as bright sirens



# MBHBs as bright sirens

Assuming we can identify the EM counterpart and obtain the redshift:



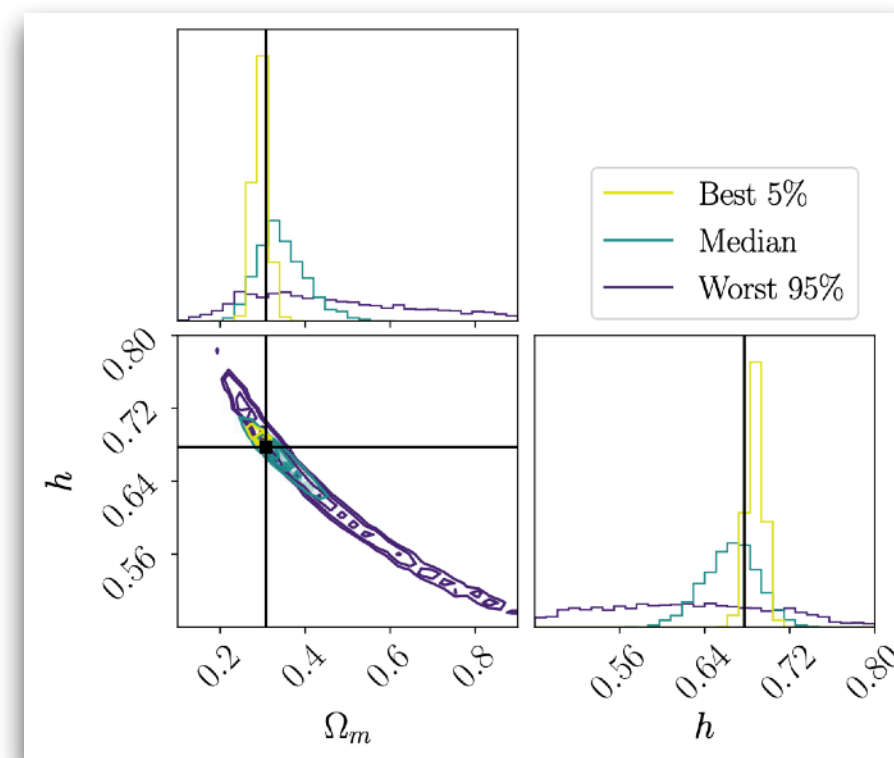
**$O(1-5)$  bright sirens per year, up to  $z \lesssim 7$**

**Assuming good sky localization ( $\Delta D_L/D_L \lesssim 0.1$  and  $\Delta\Omega < 10 \text{ deg}^2$ ):**

**$O(1 - 5)$  bright sirens per year**

**$H_0$  at few-%!**

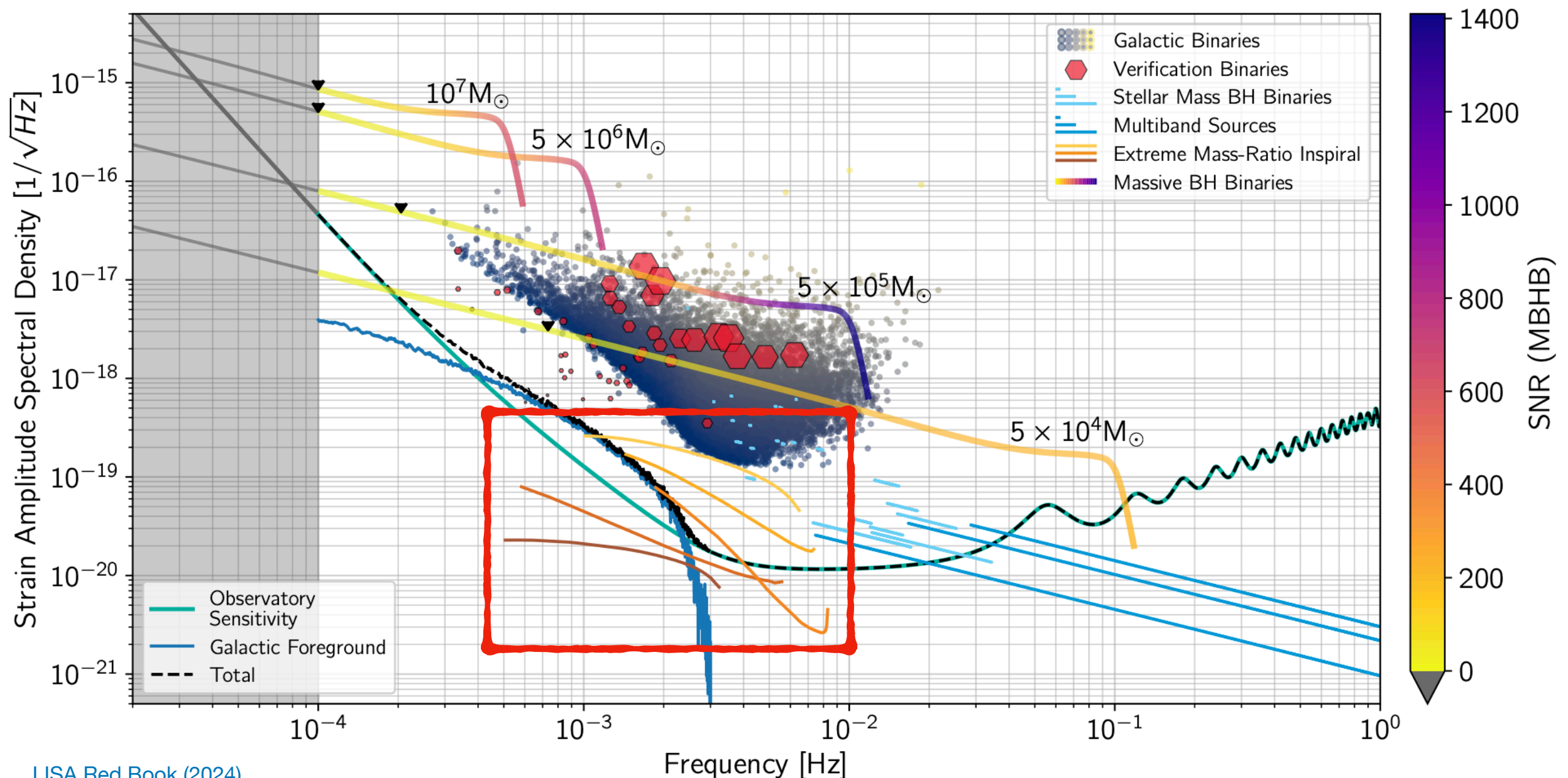
**Precise high- $z$  cosmography**



# EMRIs as dark sirens

Here we will look at two promising classes of LISA standard sirens:

## 2. Extreme Mass-Ratio Inspirals (EMRIs) as dark sirens





# EMRIs as dark sirens

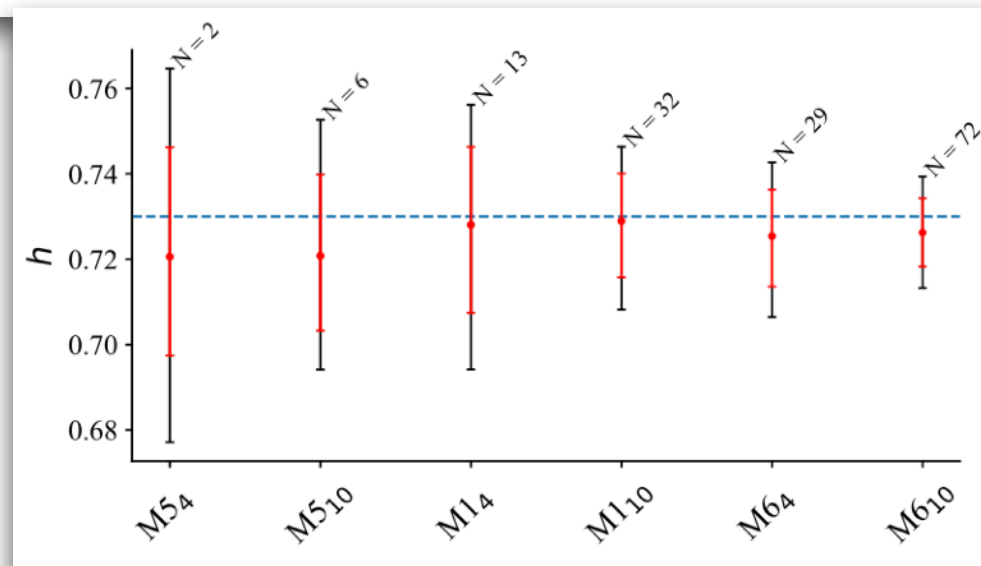
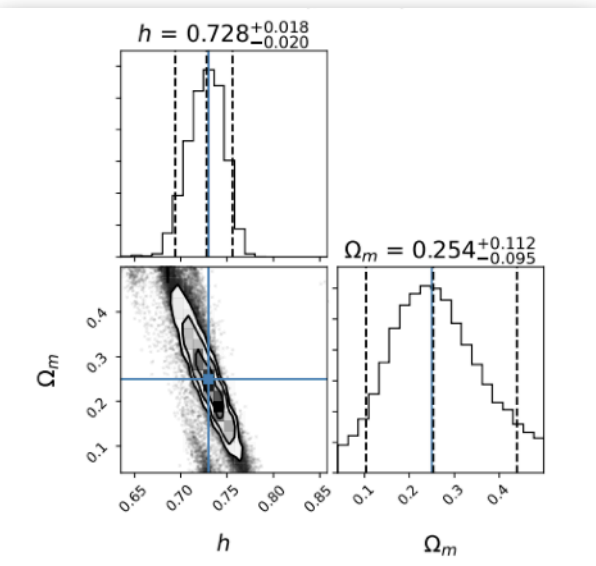
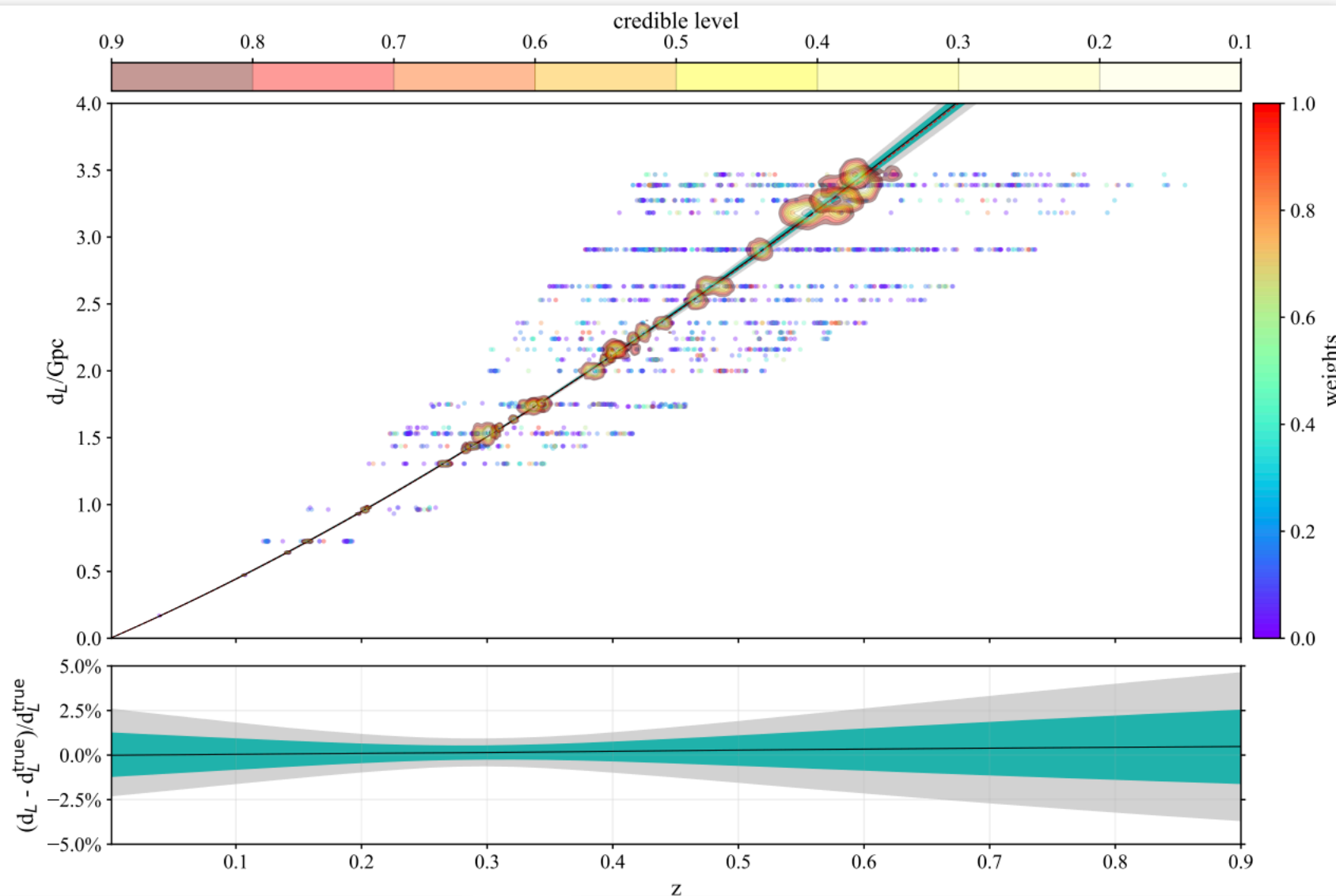
**We can cross-match the EMRI sky location with galaxy catalogs**

MacLeod, Hogan (2009), PRD (2008)

**$O(1-1000)$  detections  
per year, up to  $z \lesssim 4$**

**No EM counterpart  
expected, but very  
good sky localization  
( $\Delta D_L / D_L \lesssim 0.1$  and  
 $\Delta \Omega < 2 \text{ deg}^2$ ):**

**$O(1 - 100)$  standard  
sirens per year**



**$H_0$  at 1-6%!**

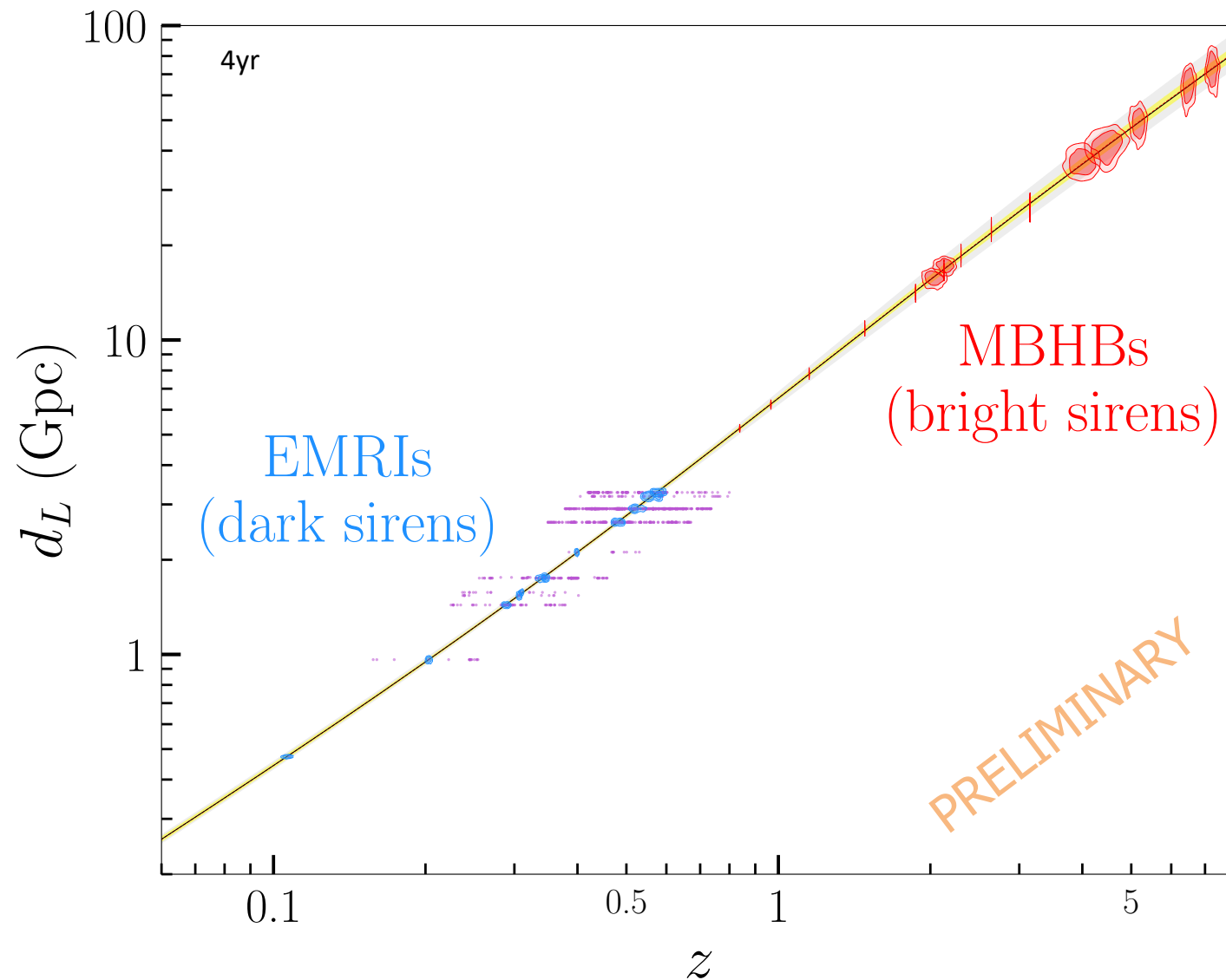
**$w_0$  at 5-10%**

DL et al., MNRAS (2021)



# LISA bright and dark sirens

**We can constrain the expansion history of the universe by combining different LISA standard sirens**



DL, Tamanini, et al., in preparation

$H_0$  at  $\lesssim 1\%$

$\Omega_m$  at  $< 10\%$

