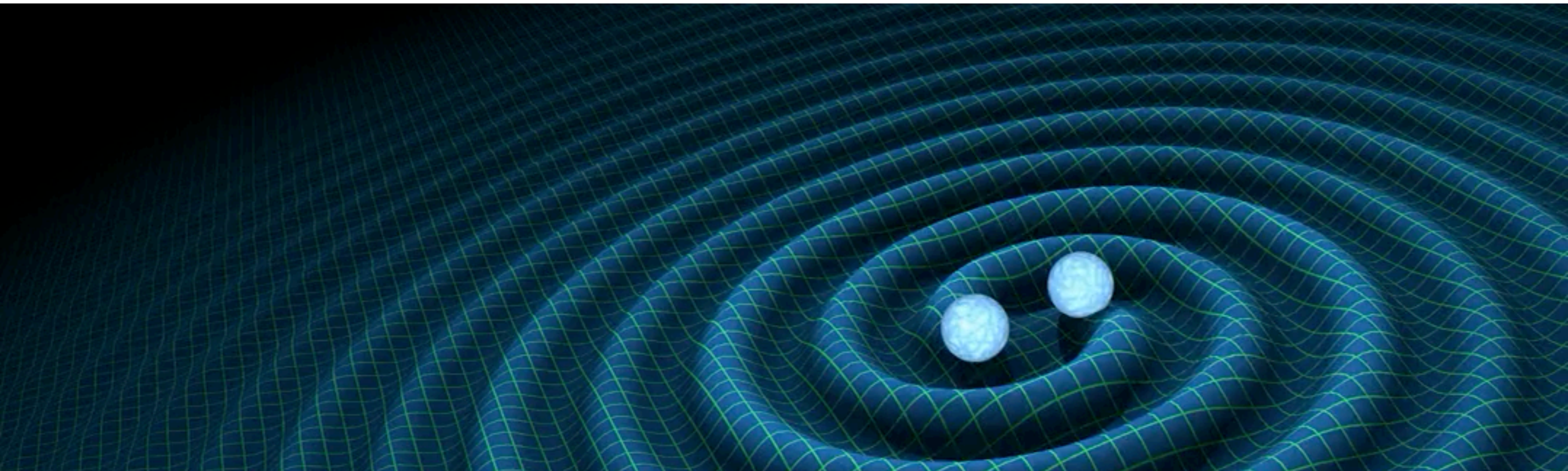
An Introduction to Late-time Cosmology with Gravitational Waves



Danny Laghi

Universität Zürich UZH

LIGO VIRGO KAGRA

LISA CONSORTIUM

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ÉCOLE DE PHYSIQUE DES HOUCHES

FLRW in a nutshell

On the Gpc scale, the Universe is to a first approximation isotropic and homogeneous, and is described by the Friedmann-Lemaître-Robertson-Walker (FLRW) metric in comoving coordinates (t, r, θ, ϕ):

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + \underbrace{a^2(t)}_{\text{scale factor}}\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\phi^2\right]$$

$$k = \begin{cases} -1 & \text{open universe} \\ 0 & \text{flat universe} \\ +1 & \text{closed universe} \end{cases}$$

Consider two points having comoving coordinates $(0,\theta,\phi)$ and (r,θ,ϕ). Their physical spatial distance is given by the proper spatial distance $\mathrm{d}r_\mathrm{p}^2 = g_{ij}\mathrm{d}x^i\mathrm{d}x^j$:

$$r_\mathrm{p} = a(t)\int_0^r \frac{\mathrm{d}r}{(1-kr^2)^{1/2}} \quad \Rightarrow \quad r_\mathrm{p} = a(t)r \quad \textbf{(flat universe)}$$

Because of the expanding universe, an observer at $r=0$ receiving GW/EM signals from a source located at r will experience time dilation:

$$\mathrm{d}t_\mathrm{o} = (1+z)\mathrm{d}t_\mathrm{s} \qquad f^{(\mathrm{o})} = f^{(\mathrm{s})}/(1+z)$$

[Maggiore, GW Vol 1 (2008)]

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Exercise

Derive $dt_\mathrm{o} = (1+z)dt_\mathrm{s}$ **starting from the condition** $ds^2 = 0$

GW waveform in a nutshell

Consider the GWs emitted by a Newtonian compact binary system in circular orbit (quadrupole formula, $v \ll c$, weak gravitational field). In the local wave zone, i.e., a region far from the source but where the expansion of the universe is negligible, the two GW polarizations are given by:

$$h_+(t_\mathrm{s}) = \frac{4}{a(t_\mathrm{e})r}\left(\frac{G\mathcal{M}_c}{c^2}\right)^{5/3}\left(\frac{\pi f_\mathrm{gw}^{(\mathrm{s})}(t_\mathrm{s}^\mathrm{ret})}{c}\right)^{2/3}\frac{1+\cos^2\iota}{2}\cos\left[2\pi\int^{t_\mathrm{s}^\mathrm{ret}}\mathrm{d}t'_\mathrm{s}f_\mathrm{gw}^{(\mathrm{s})}(t'_\mathrm{s})\right]$$

$$h_\times(t_\mathrm{s}) = \frac{4}{a(t_\mathrm{e})r}\left(\frac{G\mathcal{M}_c}{c^2}\right)^{5/3}\left(\frac{\pi f_\mathrm{gw}^{(\mathrm{s})}(t_\mathrm{s}^\mathrm{ret})}{c}\right)^{2/3}\cos\iota\sin\left[2\pi\int^{t_\mathrm{s}^\mathrm{ret}}\mathrm{d}t'_\mathrm{s}f_\mathrm{gw}^{(\mathrm{s})}(t'_\mathrm{s})\right]$$

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time measured by the clock of the source

$$M_c = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$$

chirp mass

$$t_\mathrm{s}^\mathrm{ret} = t_\mathrm{s} - r/c$$

angle between normal to the orbit and line-of-sight

$$\iota = \begin{cases} 0 & \text{“face-on”} \\ \pi/2 & \text{“edge-on”} \\ \pi & \text{“face-off”} \end{cases}$$

where the (evolving) GW frequency is given by:

$$f_\mathrm{gw}^{(\mathrm{s})}(\tau_\mathrm{s}) = \frac{1}{\pi}\left(\frac{5}{256}\frac{1}{\tau_\mathrm{s}}\right)^{3/8}\left(\frac{GM_c}{c^3}\right)^{-5/8}$$

$\tau_\mathrm{s} = t_\mathrm{s,c} - t_\mathrm{s}$ **time to coalescence**

GW waveform in a nutshell

If the source is at cosmological distance, we have to account for the fact that the scale factor has evolved during the signal propagation. This just amounts to replacing the scale factor at time of emission with its value today

$$h_+(t_s) = \frac{4}{a(t_o)r}\left(\frac{GM_c}{c^2}\right)^{5/3}\left(\frac{\pi f_{gw}^{(s)}(t_s^{ret})}{c}\right)^{2/3}\frac{1+\cos^2\iota}{2}\cos\left[2\pi\int^{t_s^{ret}} dt_s' f_{gw}^{(s)}(t_s')\right]$$

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proper distance at the
time of observation

$$r_p(t_o) = a(t_o)r$$

But the frequency is expressed in source-frame quantities…

GW waveform in a nutshell

In terms of time and frequency measured by the observer, the $+$ polarization is (the same applies to the $\times$ polarization):

$$h_+(t_\mathrm{o}) = \frac{4}{a(t_\mathrm{o})r}(1+z)^{2/3}\left(\frac{GM_c}{c^2}\right)^{5/3}\left(\frac{\pi f_\mathrm{gw}^{(\mathrm{o})}(t_\mathrm{o}^\mathrm{ret})}{c}\right)^{2/3}\frac{1+\cos^2\iota}{2}\cos\left[2\pi\int^{t_\mathrm{o}^\mathrm{ret}}\mathrm{d}t'_\mathrm{o}\, f_\mathrm{gw}^{(\mathrm{o})}(t'_\mathrm{o})\right]$$

The gravitational phasing does not change (redshift factors simplify!)

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$$= \frac{4}{(1+z)a(t_\mathrm{o})r}(1+z)^{5/3}\left(\frac{GM_c}{c^2}\right)^{5/3}\left(\frac{\pi f_\mathrm{gw}^{(\mathrm{o})}(t_\mathrm{o}^\mathrm{ret})}{c}\right)^{2/3}\frac{1+\cos^2\iota}{2}\cos\left[2\pi\int^{t_\mathrm{o}^\mathrm{ret}}\mathrm{d}t'_\mathrm{o}f_\mathrm{gw}^{(\mathrm{o})}(t'_\mathrm{o})\right]$$

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$$= \frac{4}{D_\mathrm{L}(z)}\left(\frac{G\mathcal{M}_c}{c^2}\right)^{5/3}\left(\frac{\pi f_\mathrm{gw}^{(\mathrm{o})}(t_\mathrm{o}^\mathrm{ret})}{c}\right)^{2/3}\frac{1+\cos^2\iota}{2}\cos\left[2\pi\int^{t_\mathrm{o}^\mathrm{ret}} \mathrm{d}t'_\mathrm{o} f_\mathrm{gw}^{(\mathrm{o})}(t'_\mathrm{o})\right]$$

$D_\mathrm{L}(z) = (1+z)a(t_\mathrm{o})r$

luminosity distance

$\mathcal{M}_c = (1+z)M_c$

“redshifted” chirp mass

GW waveform in a nutshell

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$$D_\mathrm{L}(z) = (1+z)a(t_\mathrm{o})r$$

luminosity distance

$$\mathscr{M}_c = (1+z)M_c$$

"redshifted" chirp mass

Exercise

Check that $f_\mathrm{gw}^{(\mathrm{o})}(t_\mathrm{o}^\mathrm{ret}) = \frac{1}{\pi}\left(\frac{5}{256}\frac{1}{\tau_\mathrm{o}}\right)^{3/8}\left(\frac{G\mathscr{M}_c}{c^3}\right)^{-5/8}$ **where** $\tau_\mathrm{o} = (1+z)\tau_\mathrm{s}$

GW waveform in a nutshell

By analyzing how quickly the phase evolves (i.e., the changes in frequency), one can infer the chirp mass:

Exercise

Check that $$\frac{\mathrm{d}f_{\mathrm{gw}}^{(\mathrm{o})}}{\mathrm{d}t} = \frac{96}{5}\pi^{8/3}\left(\frac{G\mathscr{M}_c}{c^3}\right)^{5/3} f_{\mathrm{gw}}^{(\mathrm{o})\,11/3}$$

By measuring both polarization amplitudes, we can constrain the inclination angle:

$$\frac{h_+(t_\mathrm{o})}{h_\times(t_\mathrm{o})} \propto \frac{1+\cos^2\iota}{2\cos\iota}$$

Thus we can read off the luminosity distance from the waveform amplitude:

$$h_+(t_\mathrm{o}) = \frac{4}{D_\mathrm{L}}\left(\frac{G\mathscr{M}_c}{c^2}\right)^{5/3}\left(\frac{\pi f_{\mathrm{gw}}^{(\mathrm{o})}(t_\mathrm{o}^{\mathrm{ret}})}{c}\right)^{2/3}\frac{1+\cos^2\iota}{2}\cos\left[2\pi\int^{t_\mathrm{o}^{\mathrm{ret}}}\mathrm{d}t'_\mathrm{o} f_{\mathrm{gw}}^{(\mathrm{o})}(t'_\mathrm{o})\right]$$

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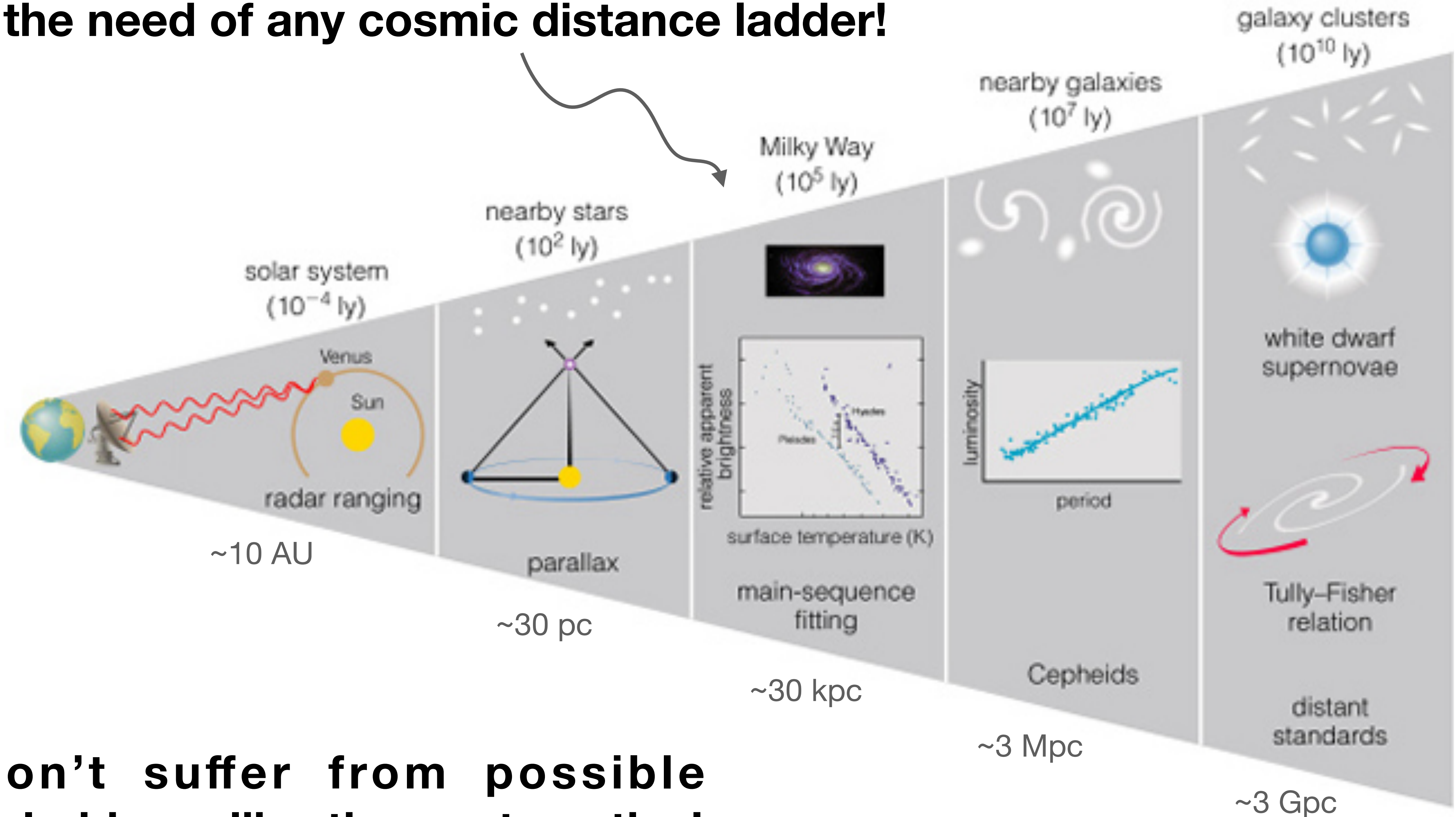
Standard sirens

In practice, we measure a combination of the polarizations weighted by the detector response. We perform GW parameter estimation simultaneously over all the waveform parameters.

As a result, we have direct access to the luminosity distance to the source without the need of any cosmic distance ladder!



GWs don't suffer from possible distance ladder calibration systematics!

Standard sirens

$$h_+(t_o) = \frac{4}{D_L}\left(\frac{G\mathcal{M}_c}{c^2}\right)^{5/3}\left(\frac{\pi f_{gw}^{(o)}(t_o^{ret})}{c}\right)^{2/3}\frac{1+\cos^2\iota}{2}\cos\left[2\pi\int^{t_o^{ret}} dt_o' f_{gw}^{(o)}(t_o')\right]$$

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We can make advantage of the luminosity distance-redshift relation to extract the cosmological parameters:

$$D_L(z) = \frac{c(1+z)}{H_0}\int_0^z \frac{dz'}{\sqrt{\Omega_M(1+z)^3 + \Omega_\Lambda(1+z)^{3(1+w(z))}}}$$

(flat LCDM)

$$H(z) = H_0\sqrt{\Omega_M(1+z)^3 + \Omega_\Lambda(1+z)^{3(1+w(z))}}$$

Hubble constant

fraction of matter density today

dark energy equation of state

Standard sirens

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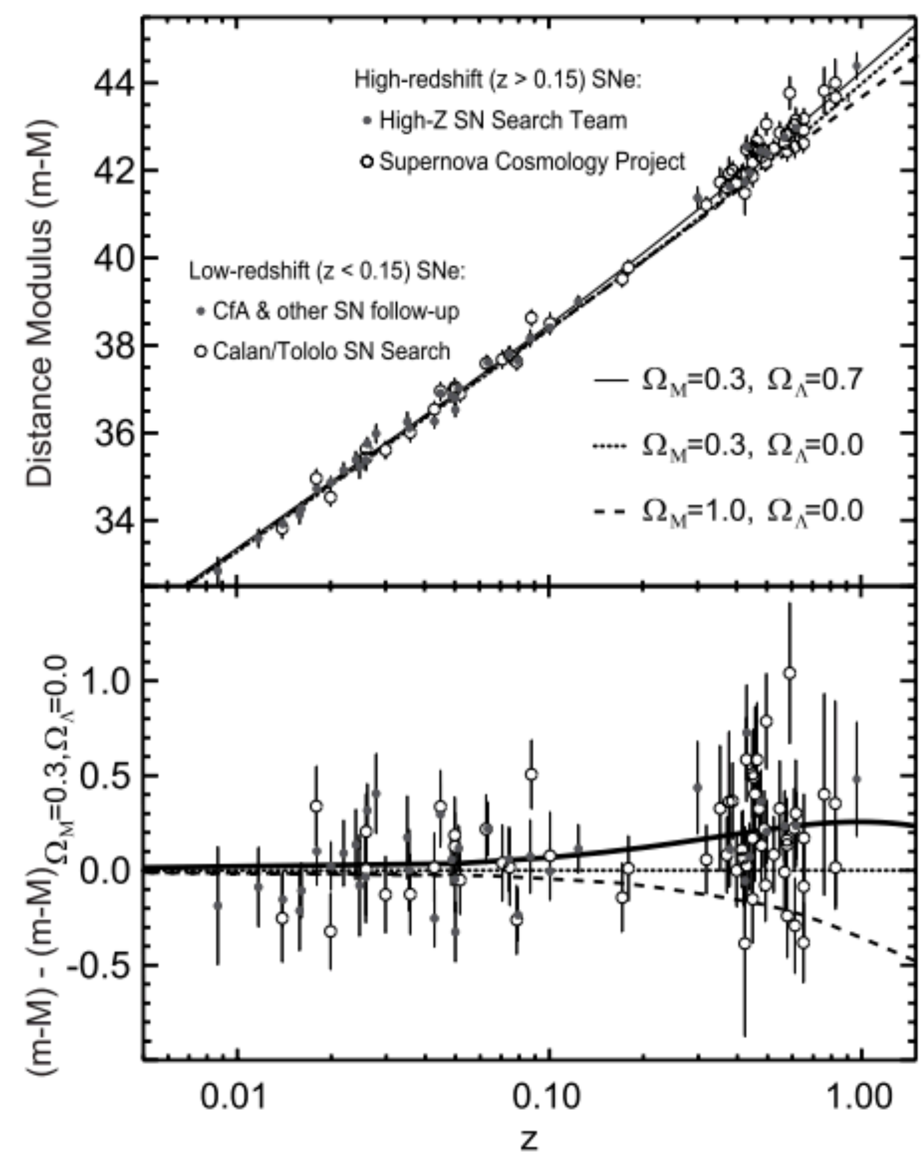
The idea is similar to what is done with Type-Ia Supernovae, which are called *standard candles*

Similarly, we call GWs standard sirens (since they are more akin to sound waves)

But there is one missing ingredient…

In order to fit the $D_L - z$ relation and do cosmology, we also need to know the source redshift, not obtainable from the waveform

Krolak, Schutz, 1987



Perlmutter, Schmidt (2003)

Standard sirens

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    Requires an EM counterpart!

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2. **Using spectral features in the GW mass distribution ("spectral siren" method)**

    Requires assumptions about the GW source populations!

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3. **Adding redshift information coming from potential host galaxies ("dark siren" method, or "galaxy catalog" method)**

   Requires a galaxy catalog!