HOW TO FORM MBHs

(1807.11489
2510.04599)

REES 1978 → START FROM A GAS CLOUD AND THE COLLAPSE VIA DIFFERENT ASTROPHYSICAL PROCESSES

JEANS MASS

DESCRIBE THE CONDITION FOR COLLAPSE ASSUMING THE BALANCE BETWEEN PRESSURE & GRAVITY

FROM VIRIAL THEOREM:

$2K + U = 0$ FOR A STABLE & GRAVITATIONALLY BOUND SYSTEM

FOR COLLAPSE: $|U| > 2K$

FOR A SELF-GRAVITATING SPHERE OF RADIUS R

$$U_{GRAV} = -\int_0^R \frac{GM}{r} 4\pi r^2 \rho \, dr = -\frac{16\pi^2 G \rho^2 R^5}{15} \qquad \left(M = \frac{4}{3}\pi r^3 \rho\right)$$

$$E_{KIN} = \frac{3}{2} N k_B T = \qquad (N: \text{TOTAL NUMBER OF GAS PARTICLES})$$

$$= \frac{3}{2} \frac{M}{\mu m_H} k_B T \qquad \left(\mu: \text{MEAN MOLECULAR WEIGHT}, \ \mu = 1 \text{ FOR ATOMIC HYDROGEN}\right)$$

COMPARE THE TWO USING THAT $R = \left(\frac{3M}{4\pi\rho}\right)^{1/3}$

$$\Rightarrow M > \boxed{\left(\frac{5 k_B T}{G \mu m_H}\right)^{3/2} \left(\frac{3}{4\pi\rho}\right)^{1/2} = M_J} \qquad (\rho = \mu m_u n_{GAS})$$

$$\boxed{M_J = 500 \left(\frac{T}{200\,K}\right)^{3/2} \left(\frac{n_{GAS}}{10^4\,cm^{-3}}\right)^{-1/2} M_\odot}$$

PROPER DERIVATION SEC. 26.1.1 OF "STELLAR STRUCTURE & EVOLUTION" KIPPENHAM EULER + CONTINUITY + POISSON

THREE MAIN FORMATION CHANNELS

- POP III STARS (LIGHT SEEDS) $\sim 10 - 10^3 M_\odot$: ABSENCE OF METAL DISFAVOUR FRAGMENTATION
  → STARS OF $\sim 10^3 M_\odot$ CAN FORM
  AT LOW METALLICITY, THERE IS A LINEAR RELATION BETWEEN MASS OF THE STAR & MASS OF THE REMNANT

TWO CONSIDERATIONS:

1) $T_{GAS} \sim 200$ IN MINIHALOS ($10^5 - 10^6$) → LARGE M_J → LARGE INITIAL MASS
   (AT $z = 0$, HYDROGEN IN STAR-FORMING REGION HAS $T_{GAS} \sim 10 - 20$ K)
2) LOW METALLICITY → WEAK PULSATIONS / WINDS → LESS MASS-LOSS FOR THE STAR

PROBLEMS:

1) FRAGMENTATION: MULTIPLE PopIII STARS FORM
   => INDIVIDUAL MASS IS REDUCED

2) WANDERING BHs: $\sim 100\ M_\odot$ NOT MASSIVE ENOUGH TO SET AT THE CENTRE OF GALAXY

- RUNAWAY CLUSTER MERGERS: MERGERS IN STAR CLUSTERS
  (MEDIUM-WEIGHT SEEDS)
  $\sim 10^3 - 10^4\ M_\odot$

  MASS SEGREGATION:
  LIGHT STARS ARE EJECTED
  ⇓
  LESS KINETIC ENERGY
  ⇓
  THE CLUSTER CONTRACTS AND BECOMES DENSER
  ⇓
  MORE STELLAR INTERACTIONS

- DIRECT COLLAPSE: BROAD CLASS. MAIN IDEA: GAS IS SUPPLIED AT THE CENTRE OF AN HALO WITH ↳ $(\sim 10^7 - 10^8\ M_\odot)$
  (HEAVY SEEDS)
  $\sim 10^4 - 10^6\ M_\odot$
  $\dot{M} \sim 0.01 - 1\ M_\odot/yr$

  - 2 WAYS TO CREATE PROTOSTARS SO MASSIVE:

  1) STRONG LW RADIATION IN LYMAN-WERNER BAND TO PREVENT THE FORMATION OF H_2
     BACKGROUND LW IS NOT ENOUGH => NEED NEARBY LW SOURCES
     ⇓
     "SYNCHRONIZED HALOS"

  2) DYNAMICAL HEATING: HALO GROWS RAPIDLY IN MASS
     ⇓
     TEMPERATURE INCREASES
     ⇓
     STAR FORMATION IS PREVENTED

  - DYNAMICAL INSTABILITIES: GALAXIES MERGERS, BARS (LESS LIKELY)

(PROTOSTAR := EARLY STAGE OF A STAR WITH $T_{CORE} \sim 10^3 - 10^4\ K$
=> NOT HOT ENOUGH FOR FUSION)

(LYMAN-WERNER := UV PHOTONS WITH $11.2 < \frac{h\nu}{eV} < 13.6$
THEY EXCITE MOLECULAR H_2 HYDROGEN → GO BACK TO GROUND STATE
↘ PHOTO-DISSOCIATION ($H_2 + H$))

PRIMORDIAL BHs / COSMIC STRINGS:

PBHs: $1g - 10^5\ M_\odot$
CONSTRAINTS FROM CMB / GLOBULAR CLUSTERS / X-RAY BACKGROUND

STRINGS: ??? STRINGS CAN ACCRETE MATTER & COLLAPSE IN A BH
PREDICTION OF STRING THEORY

NATURE IS (LIKELY) CONTINUOUS

BHs GROW # WITH 3 CHANNELS:

1) GAS → AGN
2) MERGER → GWs
3) STARS → TDE

EDDINGTON LIMIT

DESCRIBE THE BALANCE BETWEEN GRAVIT. FORCE (INWARDS) AND RADIATION FORCE (OUTWARDS)

SPHERICALLY DISTRIBUTED CLOUD OF FULLY IONIZED HYDROGEN

$F = \frac{L}{4\pi r^2}$ r: RADIAL DISTANCE

INTERACTION VIA THOMSON SCATTERING $\ll mc^2$

$L = \frac{dE}{dt}$ => F IS THE ENERGY FLUX

FOR A PHOTON, THE MOMENTUM $p = \frac{E}{c}$

=> THE MOMENTUM FLUX

$$P_{FLUX} = \frac{F}{c} = \frac{L/c}{4\pi r^2}$$

THE FORCE PRODUCED ON A SINGLE ELECTRON IS

$$F_{RAD} = P_{FLUX}\, \sigma_T = \frac{L/c}{4\pi r^2}\sigma_T \qquad \sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 = 6.62\, e{-}25\ cm^2$$

(is (MASS OF THE PROTON WOULD GIVE A $\sigma \sim 10^{-6}$ SMALLER))

THE GRAVITATIONAL FORCE IS

$$F_{GRAV} = \frac{GM}{r^2}(m_e + m_p) \approx \frac{GMm_p}{r^2}$$

(M: TOTAL MASS IN r)

EQUATING $F_{RAD} = F_{GRAV}$ =>

$$\boxed{L_{EDD} = \frac{4\pi G m_p c}{\sigma_T} M = 1.26 \cdot 10^{38}\left(\frac{M}{M_\odot}\right)\frac{erg}{s}}$$

IF $M_{BH} \sim 10^8 M_\odot$ => $L_{EDD} \sim 10^{46}\ erg/s$

IF WE DEFINE THE MASS ACCRETION RATE $\dot{M}_{ACC} = \frac{dM}{dt}$ =>

=> THE LUMINOSITY IS DEFINED AS

$$\boxed{L = \eta \dot{M}_{ACC} c^2}$$

η := "RADIATIVE EFFICIENCY": FRACTION OF THE ACCRETED MATTER THAT IS RADIATED AWAY

QUANTIFY η

GAS ELEMENT IN WIDE INITIAL ORBIT => $E_i = 0$

AT THE INNER EDGE OF THE CIRCUMBINARY DISK (CIRCULAR) => $E_{FIN} = -\frac{GmM}{2R_{EDGE}}$

$\frac{1}{2}mv^2 - \frac{GmM}{R_{EDGE}} = \frac{1}{2}m\frac{GM}{R_{EDGE}} - \frac{GmM}{R_{EDGE}} = -\frac{GmM}{2R_{EDGE}}$

$$\Delta E \equiv \frac{GmM}{2R_{ISCO}} \Rightarrow L = \frac{dE}{dt} = \frac{GmM}{2R_{ISCO}}$$

$$R_{ISCO} \equiv \beta\left(\frac{2GM}{c^2}\right)$$

$$\Rightarrow L = \frac{1}{4\beta}\dot{m}c^2 \Rightarrow \boxed{\eta = \frac{1}{4\beta}}$$

ISCO FOR $a=0$

$R_{ISCO} = \frac{6GM}{c^2} \Rightarrow \beta = 3 \Rightarrow \eta = 0.08$ (0.057)

FULL GR

ISCO FOR $a=1$

$R_{ISCO} = \frac{GM}{c^2} \Rightarrow \beta = \frac{1}{2} \Rightarrow \eta = 0.5$ (0.42)

THE MBH MASS GROWS AS

$$\dot{M} = (1-\eta)\dot{M}_{ACC} = \frac{1-\eta}{\eta} f_{EDD} \frac{M}{t_{EDD}} \qquad \left(t_{EDD} = \frac{Mc^2}{L_{EDD}} = \frac{\sigma_T c}{4\pi G m_p} = 0.45\ Gyr\right)$$

$$\Rightarrow M(t) = M_0 \exp\left(\frac{1-\eta}{\eta} f_{EDD} \frac{t-t_0}{t_{EDD}}\right)$$

M_0: INITIAL MASS AT TIME t_0

$\Rightarrow$ IF $a=0$, $\eta = 0.057 \Rightarrow M = M_0\, e^{\frac{t}{3\times10^7 yr}}$

IF $a=1$, $\eta = 0.42 \Rightarrow M = M_0\, e^{\frac{t}{3\times10^8 yr}}$

$t_{HUBBLE} \sim 1.4\times10^{10}\ yr$

$\Rightarrow$ 50-TO-500 e-FOLDS IN MASS (EXPONENTIAL GROWTH)

PROBLEM

CURRENT QSO RECORD: $M_{BH} \sim 10^9 M_\odot$ $z = 7.54$

$z = 7.54 \rightarrow t_{COSMIC} \approx 0.7\ Gyr$

$\Rightarrow$ IF $a=0$ $\frac{7\times10^8}{3\times10^7} = 23$ e-FOLDS $\approx 10^{10}$

$\Rightarrow$ IF $a=1$ $\frac{7\times10^8}{3\times10^8} = 2.3 \approx 10$

REMARKS:

1) ACCRETION TENDS TO SPIN UP
2) PROBABLY NOT FROM EDDINGTON ACCRETION
3) CHAOTIC ACCRETION

DISK EMISSION

SHAKURA - SUNYAEV MODEL (1973)
(SEC. 5.6 OF "ACCRETION POWER IN ASTROPHYSICS", J. FRANK)

1) "STEADY-STATE": DISK'S PROPERTIES DO NOT CHANGE IN TIME (I.E. IN EQUILIBRIUM)

2) "GEOMETRICALLY THIN": $H \ll R$ (DISK IS FLATTENED DUE TO ROTATION)

3) "OPTICALLY THICK": PHOTONS ARE ABSORBED & RE-EMITTED MANY TIMES BEFORE ESCAPING (RADIATION & MATTER ARE IN EQUILIBRIUM ⇓ BLACKBODY SPECTRUM)

$$T \simeq 2\times10^{7}\left(\frac{M}{M_\odot}\right)^{-1/4}\left(\frac{r}{r_s}\right)^{-3/4} K$$

WITH $r_s = \frac{2GM}{c^2}$

FOR $M = 1 M_\odot \Rightarrow T \approx 2e7\ K \Rightarrow$ X-RAY

FOR $M = 10^8 M_\odot \Rightarrow T \approx 3e5\ K \Rightarrow$ UV-BAND

REMARKS:

1) FROM THE BH CORONA, UV PHOTONS ARE INVERSE COMPTON SCATTERED AGAINST FREE ELECTRONS TO PRODUCE X-RAYS
(HIGH-ENERGY ELECTRONS, LOW-ENERGY PHOTONS)

2) DUST TORUS RE-PROCESS PHOTONS TO INFRARED

FEEDBACK MECHANISM

FEEDBACK WAS INTRODUCED IN STELLAR PHYSICS ⇒ THE SNe HEATS THE GAS PREVENTING STAR FORMATION

GAS IS ACCRETED BY MBH → RADIATION IS PRODUCED → GAS STOPS ACCREATING → MBH IS QUIESCENT → GAS IS ACCRETED BY MBH

M-σ RELATION

MBH ACCRETION AFFECTS THE HOST GALAXY. ENERGETIC ARGUMENT:

$$L = \eta \dot{M}_{acc} c^2$$

$$\Rightarrow \dot{M}_\bullet = (1-\eta)\dot{M}_{acc} \Rightarrow \frac{M_\bullet}{T} = (1-\eta)\dot{M}_{acc}$$

($M_\bullet$: TOTAL MASS ACCUMULATED; T: TOTAL ACCRETION TIME)

THE BINDING ENERGY OF THE WHOLE GALAXY IS $E_G \approx \underbrace{M}_{\text{MASS OF THE GALAXY}} \underbrace{\sigma_*^2}_{\text{VELOCITY DISPERSION}}$

IF WE TAKE THE RATIO

$$\frac{E}{E_G} = \frac{\eta}{1-\eta} \frac{M_\bullet}{M} \left(\frac{\sigma_*}{c}\right)^{-2} \approx 250 \left(\frac{\sigma_*}{200\ \mathrm{km/s}}\right)^2 \left(\frac{M_\bullet}{10^{-3} M}\right)$$

=> MBH EJECTS PLENTY OF ENERGY

SINGULAR ISOTHERMAL SPHERE

TO DESCRIBE SELF-GRAVITATING SYSTEMS WITH CONSTANT VELOCITY DISPERSION (BOTH STARS & GAS) => $\sigma^2 = \text{CONST.}$

PHASE SPACE DISTRIBUTION FUNCTION IS GAUSSIAN AS

$$\rho(v,\sigma) = \frac{1}{(2\pi\sigma^2)^{3/2}} e^{-\left(\frac{v^2}{2} + \phi\right)\sigma^2}$$

($\frac{v^2}{2} + \phi$ → TOTAL ENERGY PER UNIT MASS; ϕ: POTENT. ENERGY PER UNIT MASS)

MASS DENSITY OF THE DISTRIBUTION

$$\rho_{MASS} = \int_0^\infty \int_0^{2\pi} \int_0^{\pi} \rho(v,\sigma)\, v^2 \operatorname{sen}(\theta)\, dv\, d\theta\, d\phi = e^{-\phi/\sigma^2}$$

FROM POISSON EQUATION

$$\nabla^2 \phi = 4\pi G \rho_{MASS}$$

IN SPHERICAL COORDINATES

$$\frac{1}{r^2} \frac{d}{dr}\left(r^2 \frac{d\phi}{dr}\right) = 4\pi G \rho_{MASS}$$

$$\frac{1}{r^2} \frac{d}{dr}\left(r^2 \frac{d\ln \rho_{MASS}}{dr}\right) = 4\pi G \rho_{MASS}$$

ASSUME $\rho_{MASS} \sim \frac{1}{r^2}$

$$\Rightarrow \boxed{\rho_{MASS}(r) = \frac{1}{2\pi G} \frac{\sigma^2}{r^2}}$$

THE TOTAL MASS IN A RADIUS r IS

$$M(r) = 4\pi \int_0^r \rho(r')\, r'^2\, dr' = \frac{2\sigma^2 r}{G}$$

1) DIVERGENT FOR $r \to 0$ (FINITE-DENSITY CORE)
2) INFINITE MASS FOR $r \to \infty$ (CUTOFF AT LARGE RADII)

FOR MW BULGE => $\rho(r) \sim r^{-1.8}$

DERIVATION OF M-σ

THE RADIATION TRANSFERS MOMENTUM TO THE FLUID. ASSUMING A CONSTANT OUTFLOW VELOCITY, THE MOMENTUM OF THE OUTFLOW CAN BE WRITTEN AS

$$M_{OUT}(r) v_{OUT} = \frac{L_{ESS}}{c} t$$

$$M_{OUT} = f_G \underbrace{M(r)}_{\text{TOTAL MASS IN RADIUS } r}$$

$$f_G \frac{2\sigma^2}{G} r v_{OUT} = \frac{L_{ESS}}{c} t$$

IF $\frac{r}{t} \approx v_{OUT}$

$$\Rightarrow v_{OUT}^2 = \frac{G L_{ESS}}{2c} \frac{1}{f_G \sigma^2}$$

IF THE OUTFLOW IS NOT BOUND TO MBH $\Rightarrow$ IT CAN ESCAPE
(I.E. RADIATION HAS TRANSFERRED ENOUGH MOMENTUM TO THE GAS TO MAKE IT ESCAPES THE MBH)

THIS HAPPEN WHEN $v_{OUT} = \sigma$

$$\Rightarrow \boxed{M_{BH} = \left(\frac{\sigma_T / m_p}{2\pi G^2}\right) f_G \sigma^4 \approx 1.8 \times 10^8 \left(\frac{\sigma}{200\ km/s}\right)^4 M_\odot}$$

- IN AGREEMENT WITH OBSERVATIONS
- NOTE THAT THE BULGE IS MADE MOSTLY OF STAR

SOLTAN'S ARGUMENT

THE MASS DENSITY OF MBH IS COMPARABLE TO THE ACCRETED MASS DENSITY INFERRED FROM AGN LUMINOSITIES
THE BH MASS DENSITY ACCRETED DURING AGN PHASE IS

$$\rho_{QSO}(z) = \int_z^\infty \frac{dt}{dz} dz \int_0^\infty \frac{(1-\epsilon) L}{\epsilon c^2} \phi(L, z) dL$$

$$\phi(L, z) = \frac{\#\,AGN}{dL\, dV}$$

$$\rho_{QSO} \approx 4.5 \times 10^5 \left(\frac{0.1}{\epsilon}\right) \frac{M_\odot}{Mpc^3}$$

THE MASS DENSITY OF MBH CAN BE ESTIMATED BY M-M_* RELATION

$$\rho_{BH} \approx 3\text{-}5 \times 10^5 \frac{M_\odot}{Mpc^3} \Rightarrow \boxed{\text{MBH GROWS MOSTLY VIA ACCRETION}}$$