Astrophysics II

The astrophysical formation and evolution of EMRIs & massive black hole binaries

Note: Google slides presentation here: https://docs.google.com/presentation/d/1Z39snXemEwTzjMr2_dBko-TA4dlcnAJrWMkcR9iF6d8/edit?usp=sharing

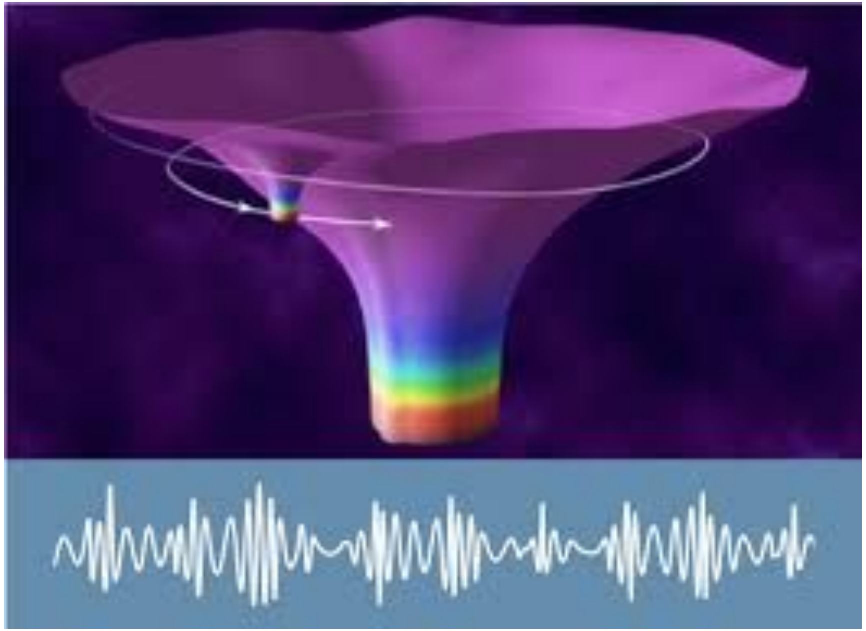
Elisa Bortolas

INAF – Osservatorio Astronomico di Padova

LISA school

Les Houches, France,
October 10th 2025

Extreme mass ratio inspirals

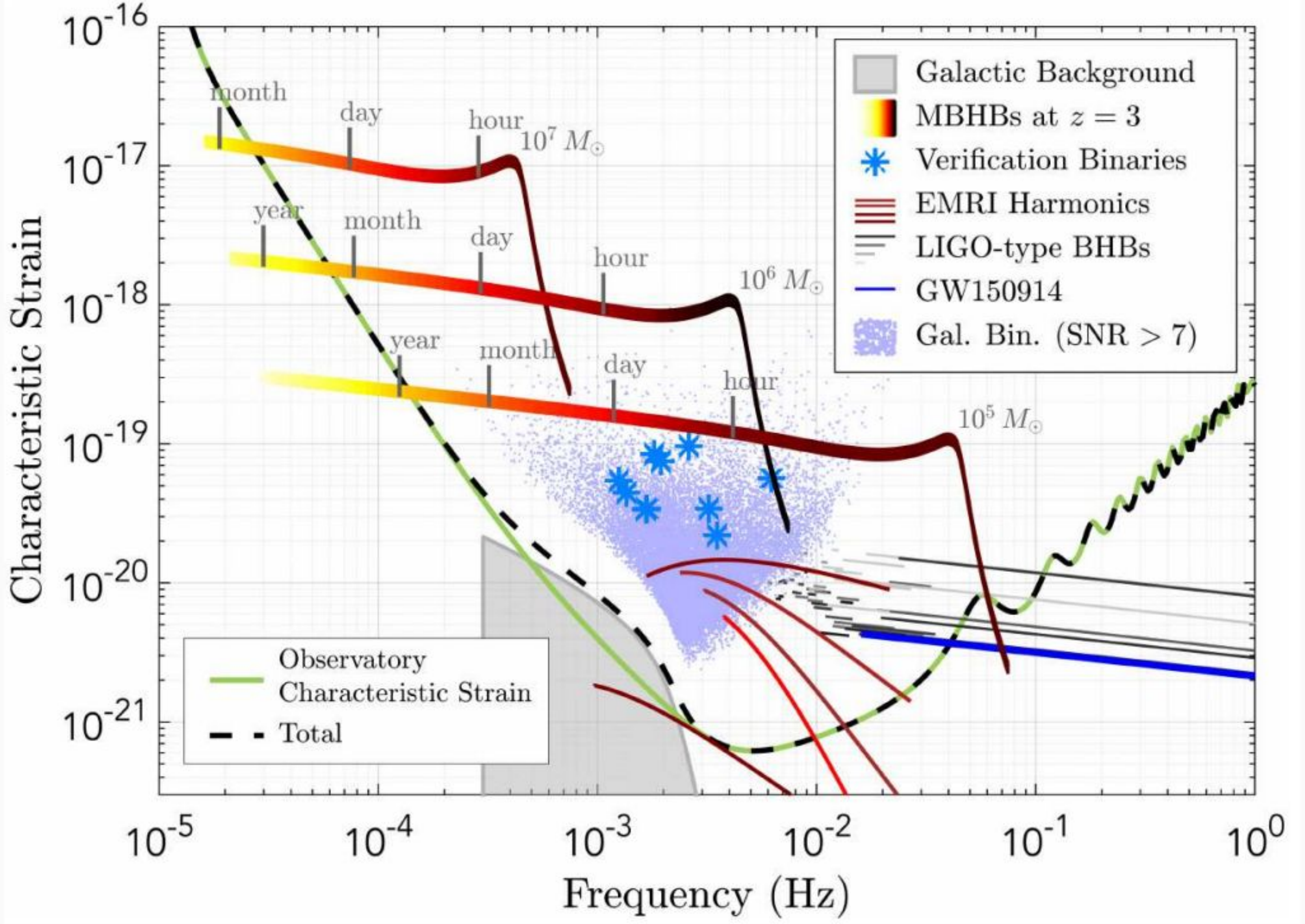


Gravitational-wave induced decay of a small compact object (stellar BH, neutron star, white dwarf...) onto a (super)massive black hole.

Mass ratio $q \sim 10^{-3} - 10^{-6}$ (perhaps even smaller)

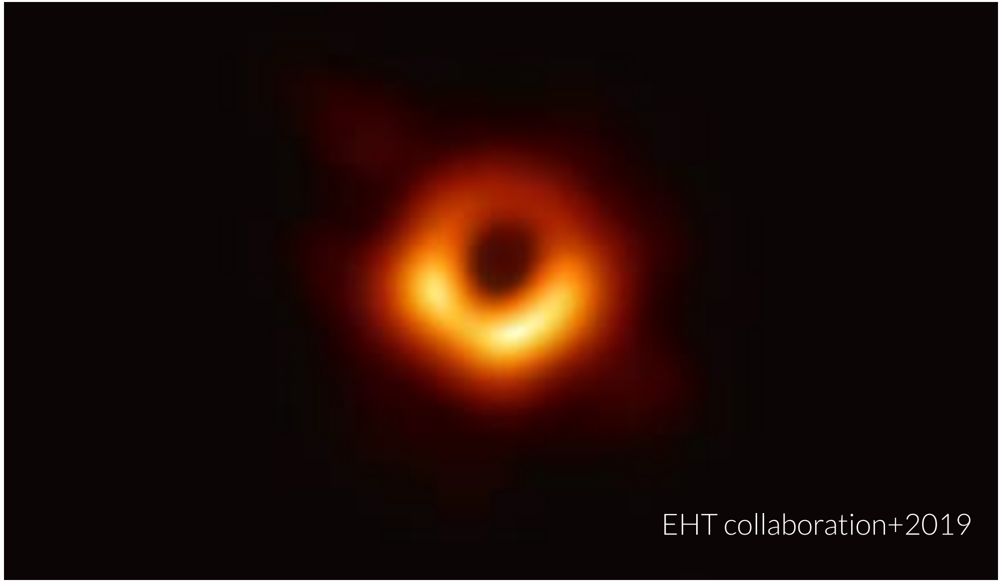
Observable by the LISA mission, $\sim 10^5$ orbits! (Amaro-Seoane+2017)

Will give us unprecedented information on the massive black hole masses, spins and host environment (+ GR tests) (Amaro-Seoane+2007, Gair+2013, Barausse+2014)

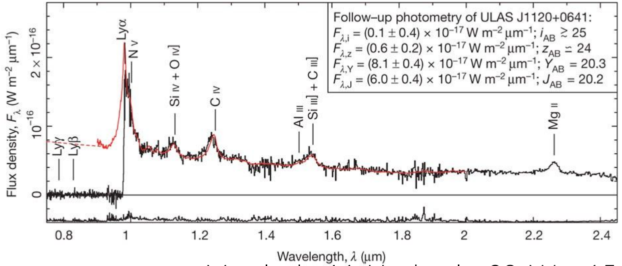


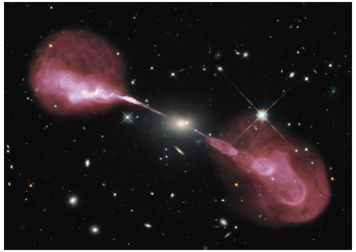
Astro white paper: Amaro-Seoane et al., 2022

Massive black holes & galaxy aggregation

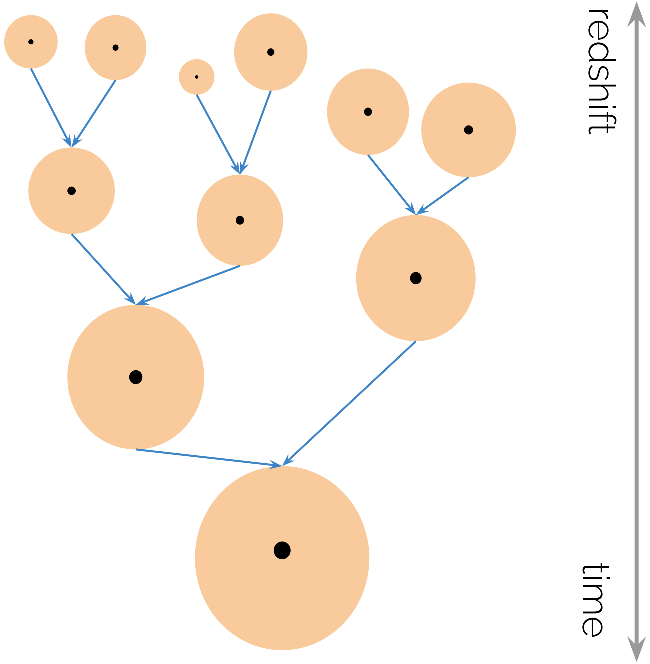


EHT collaboration+2019



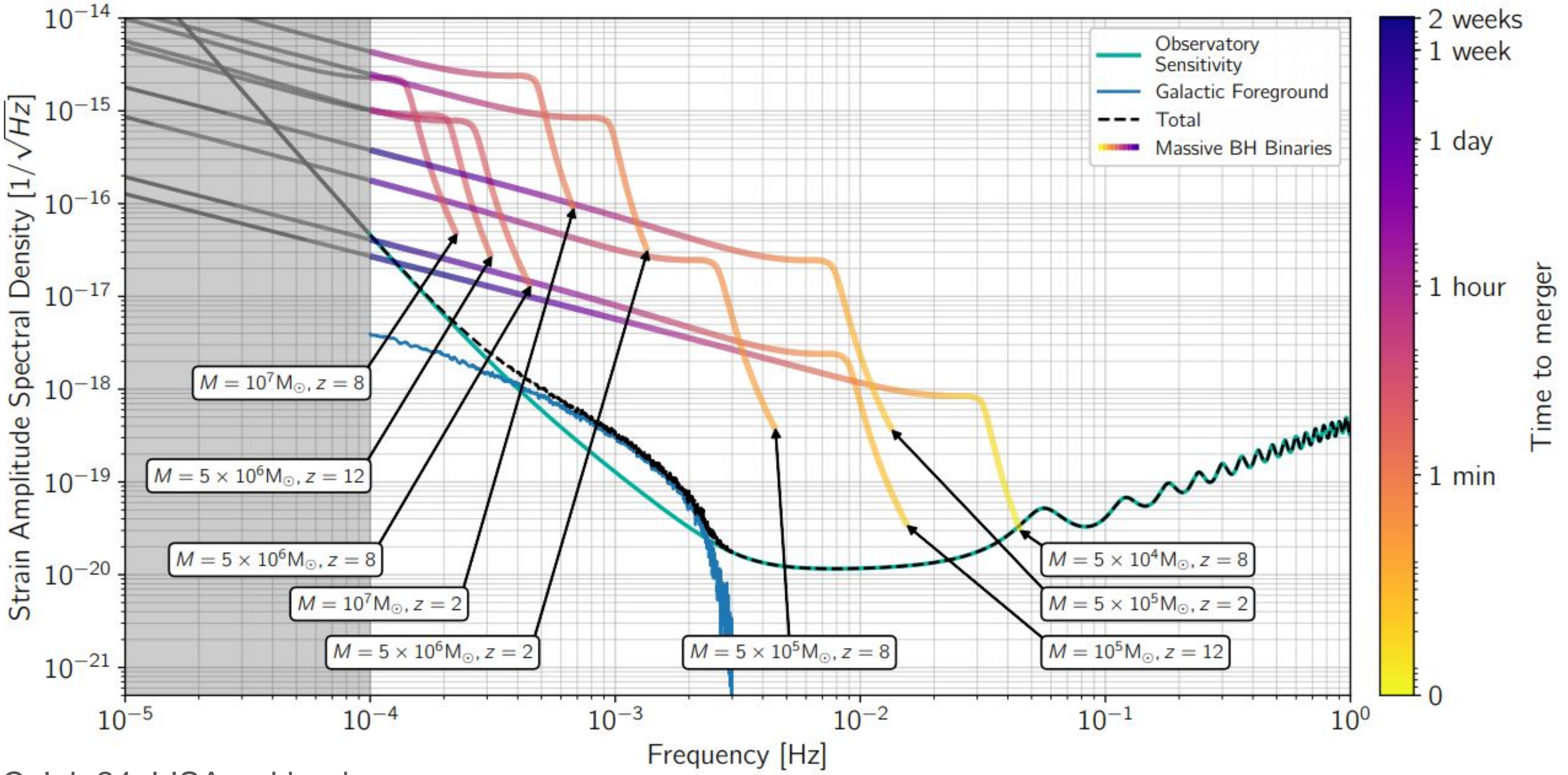


Mortlock+11, Haehnelt+98; Wu+15



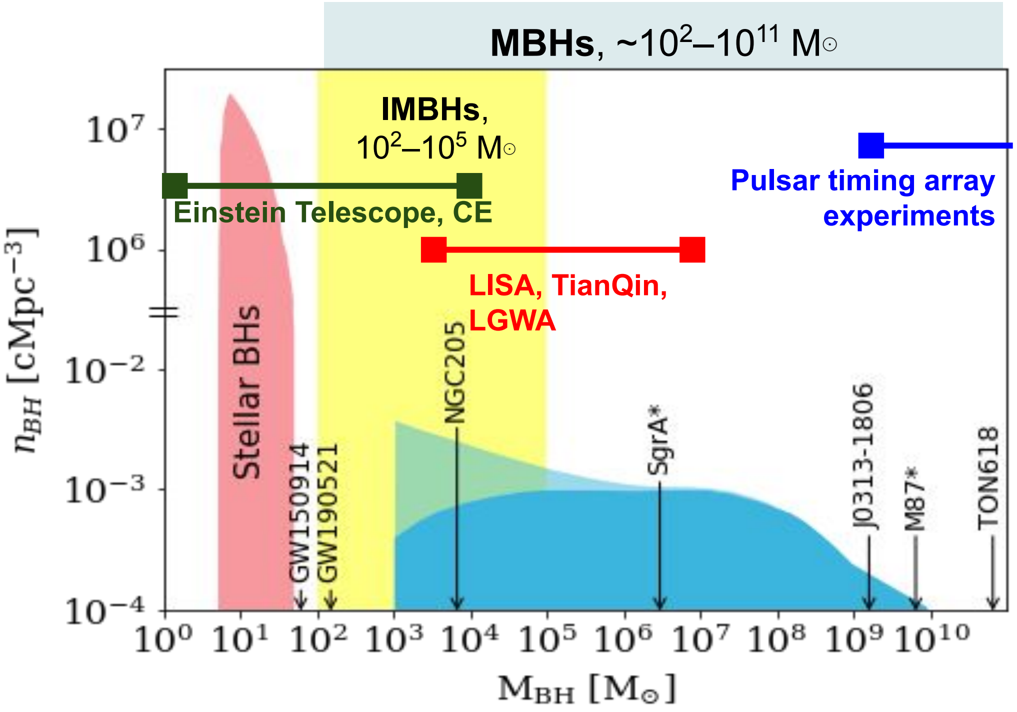
⇒ Formation of many massive black hole binaries across the cosmic times

Massive black hole binaries and LISA



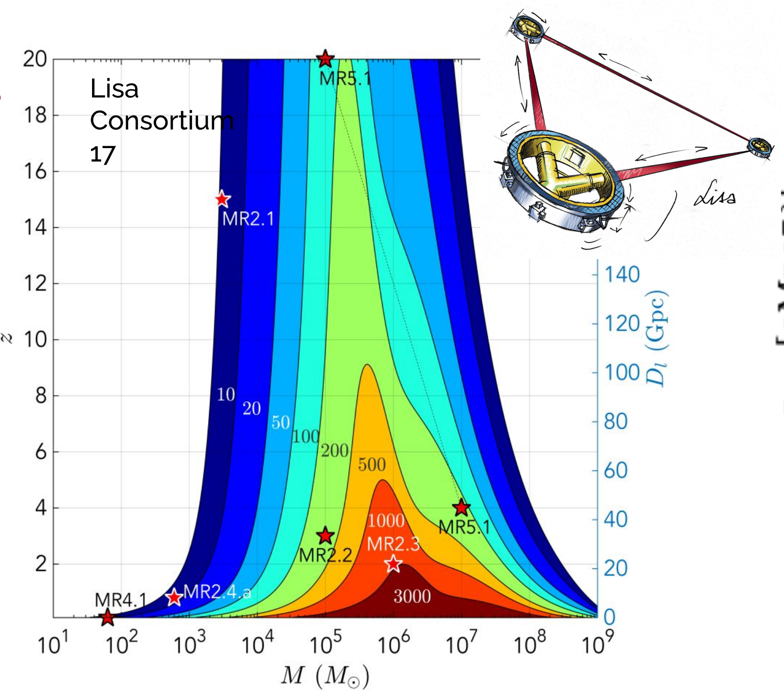
Colpi+24, LISA red book

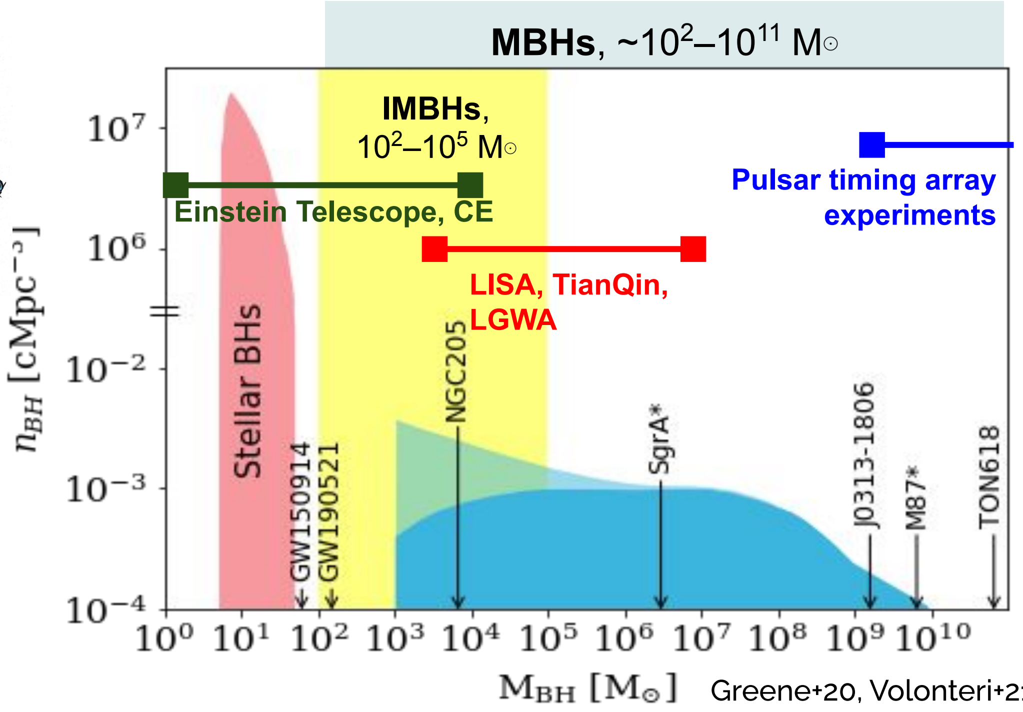
LISA in the context of MBHs history



LISA in the context of MBHs history

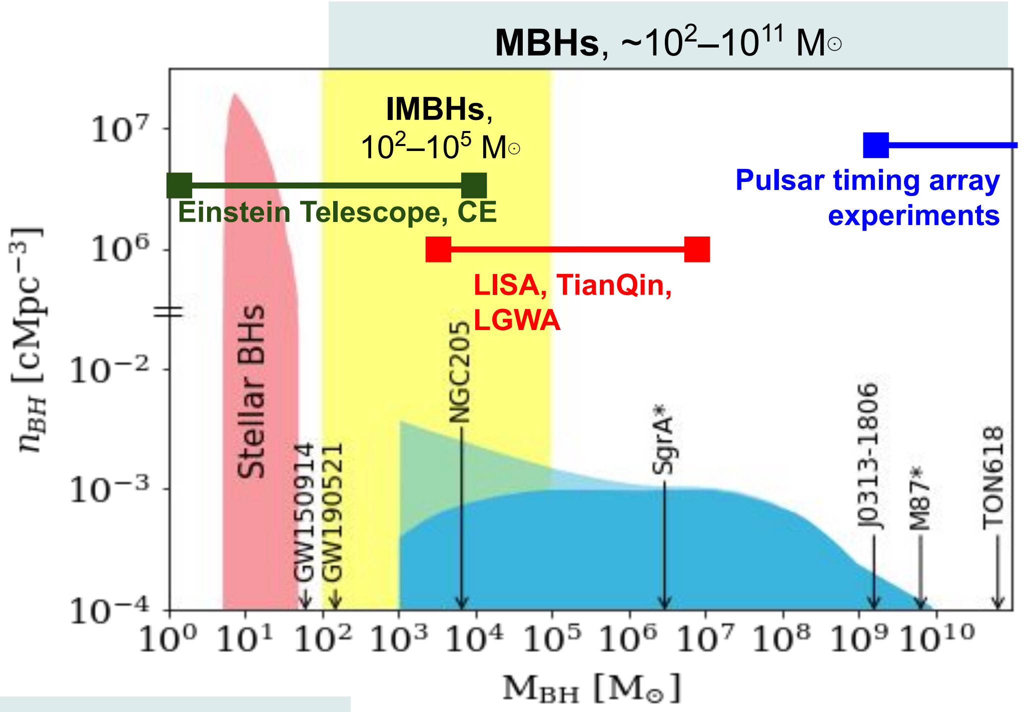
The LISA observatory





Greene+20, Volonteri+21

MBH binaries as gravitational wave sources



➢ **How did MBHs form?**
[gravitational runaway, pop-III, direct collapse…]

See e.g. Pacucci+17, Pacucci & Loeb20, Lupi+21, Terrazas+20, Habouzit+16, Regan+19…

➢ **How did they grow across the cosmic epochs?**
[gas accretion, accretion of stars (tidal disruption events/EMRIs), MBH-MBH mergers…]

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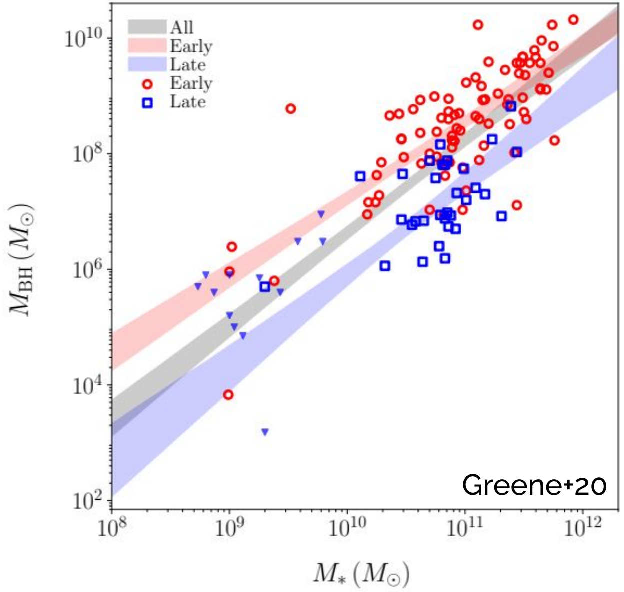
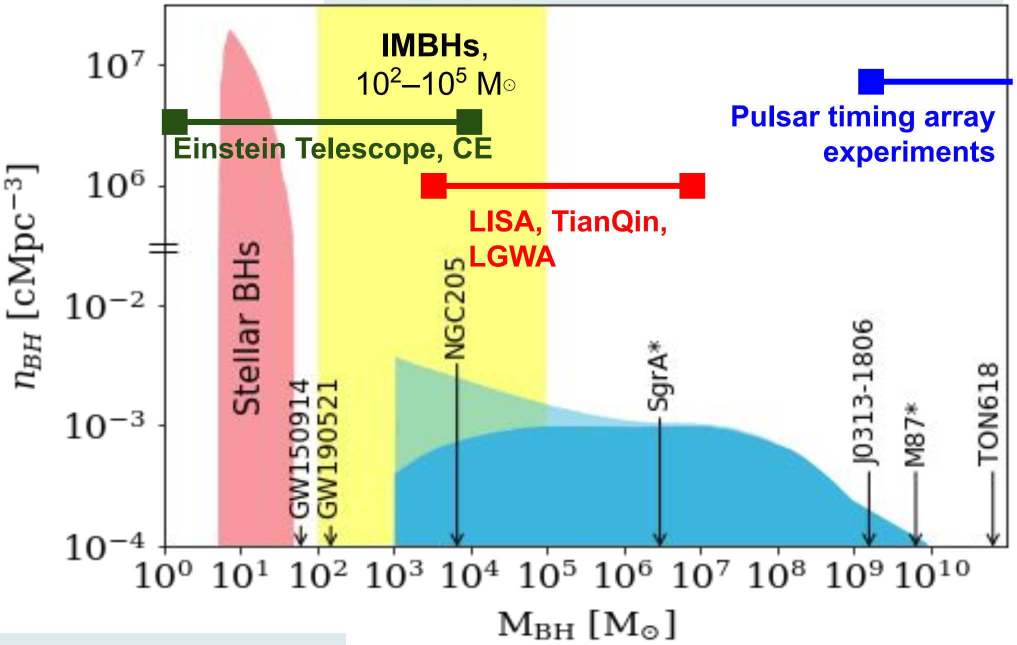


MBHs, ~10^2–10^{11} $M_\odot$



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[M-sigma, M-Mbulge… from AGN feedback?]

MBH binaries as gravitational wave sources

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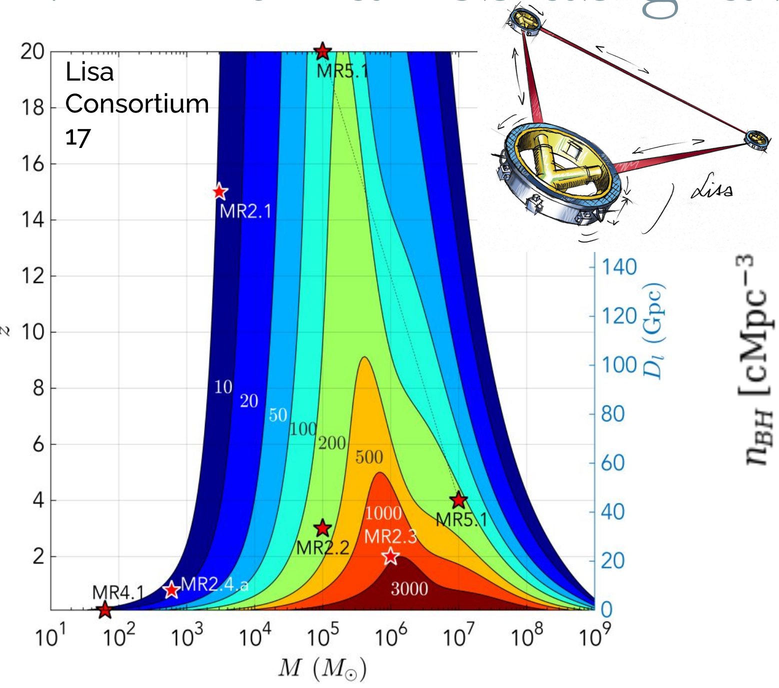

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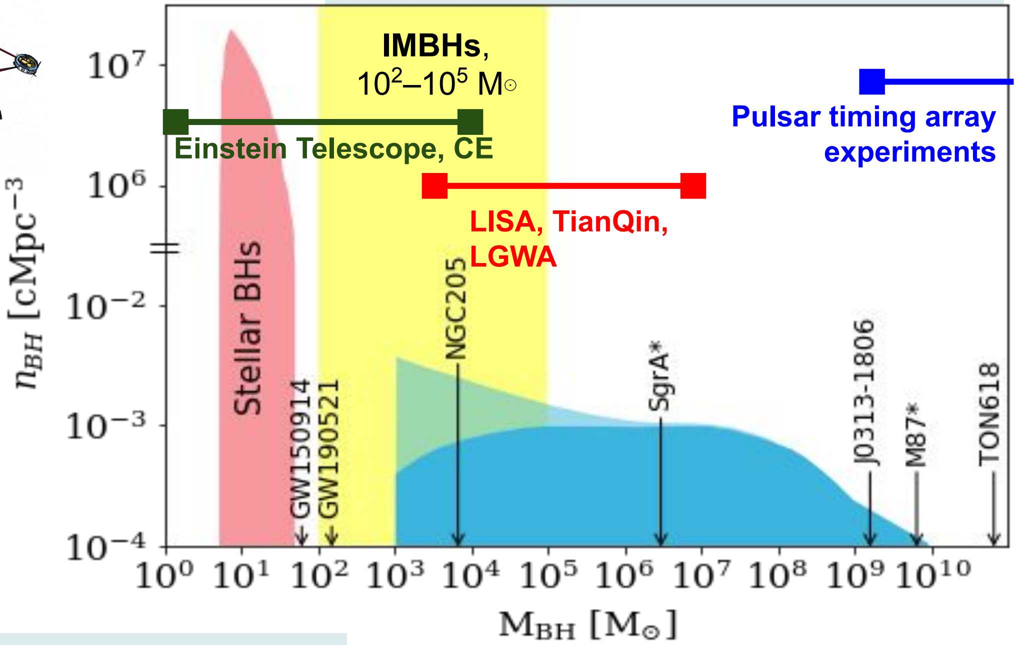

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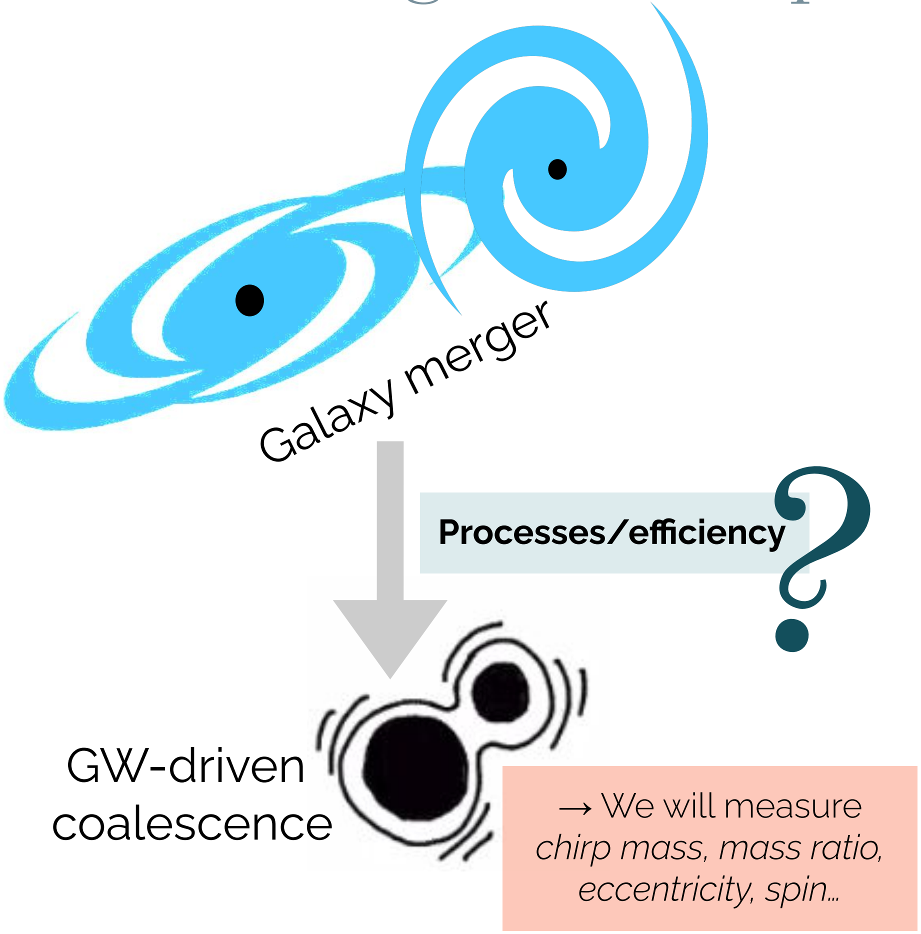
Predicting and interpreting LISA observations

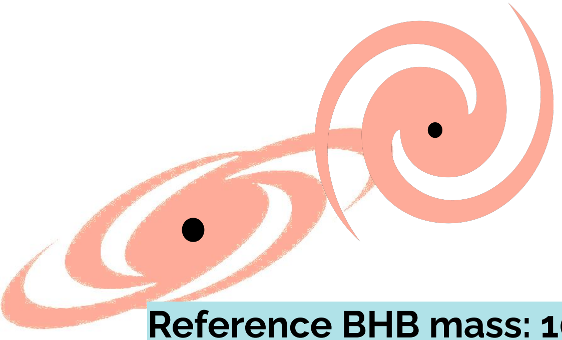


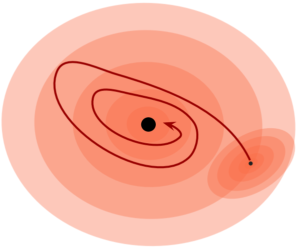
- **Time delay** from the galaxy merger to the MBH merger
- **Processes** involved in the shrinking & bottlenecks
- How do the **binary orbital properties** (separation, eccentricity…) change along the inspiral
- How does the **MBHs mass growth** & spin evolution proceed

The path to coalescence of massive BH binaries

Begelman, Blandford and Rees 1980

Galaxy merger





Dynamical friction

Reference BHB mass: 10^6 M☉

Reference BHB mass: 10^{10} M☉

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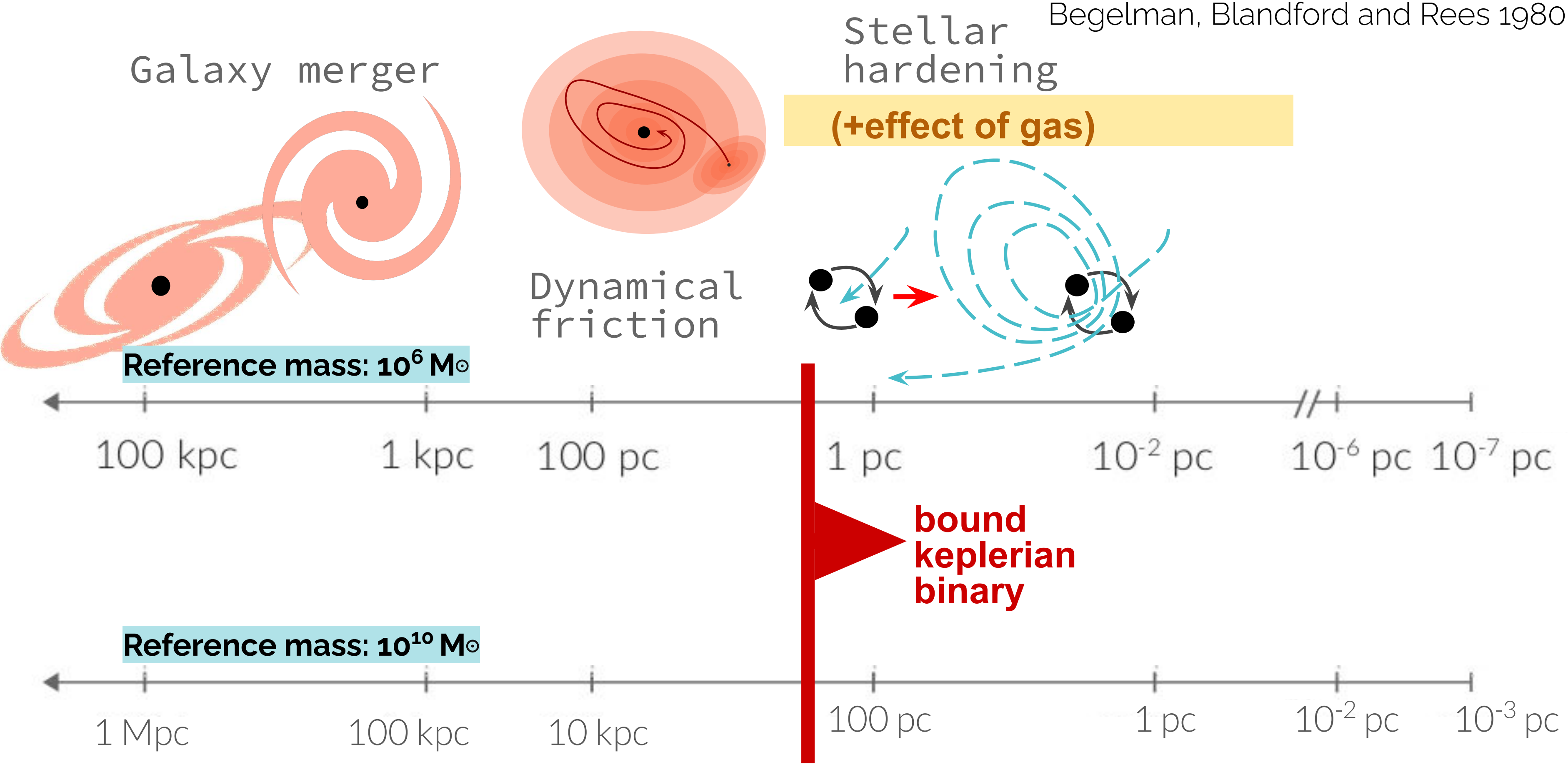

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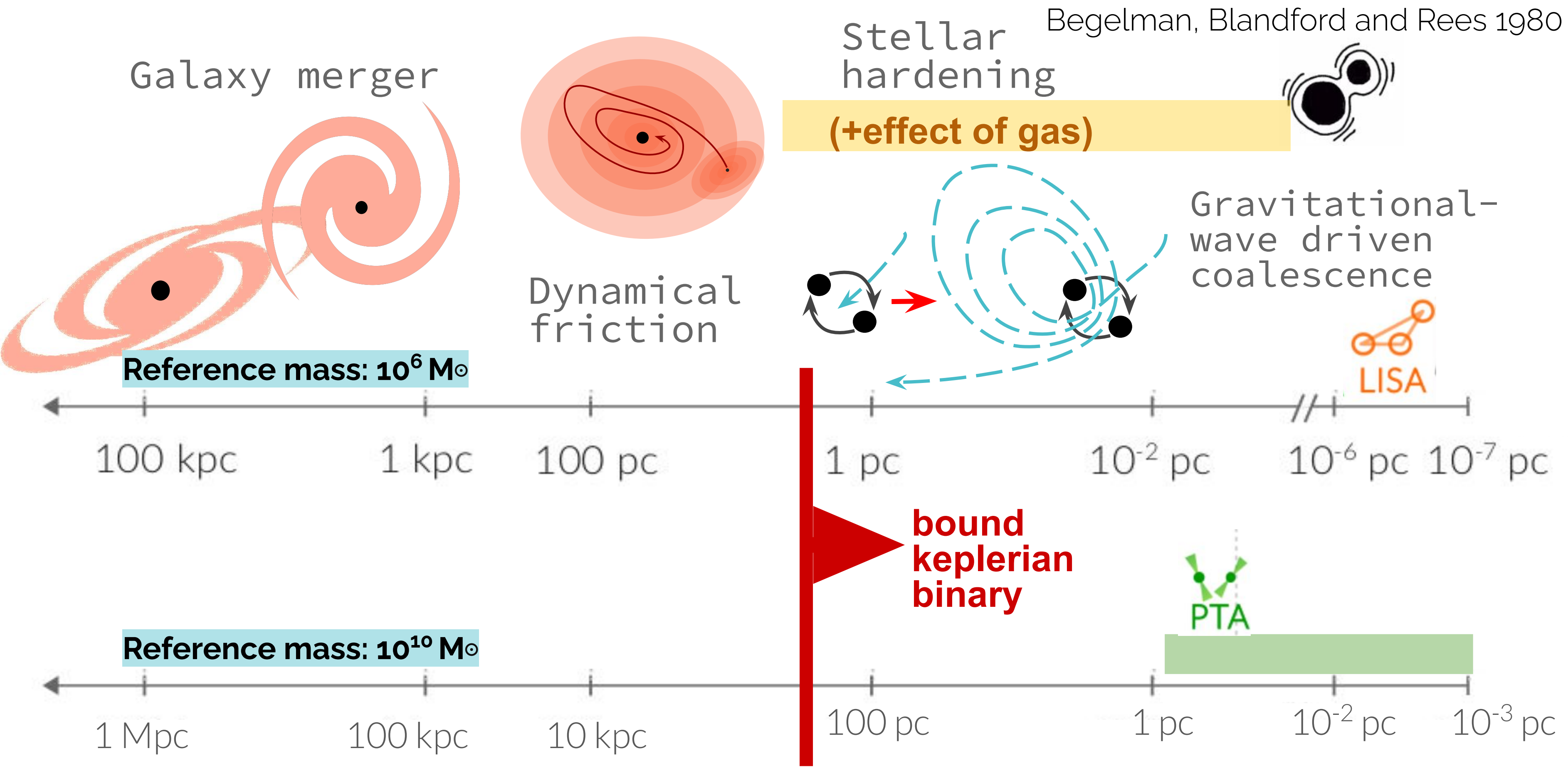
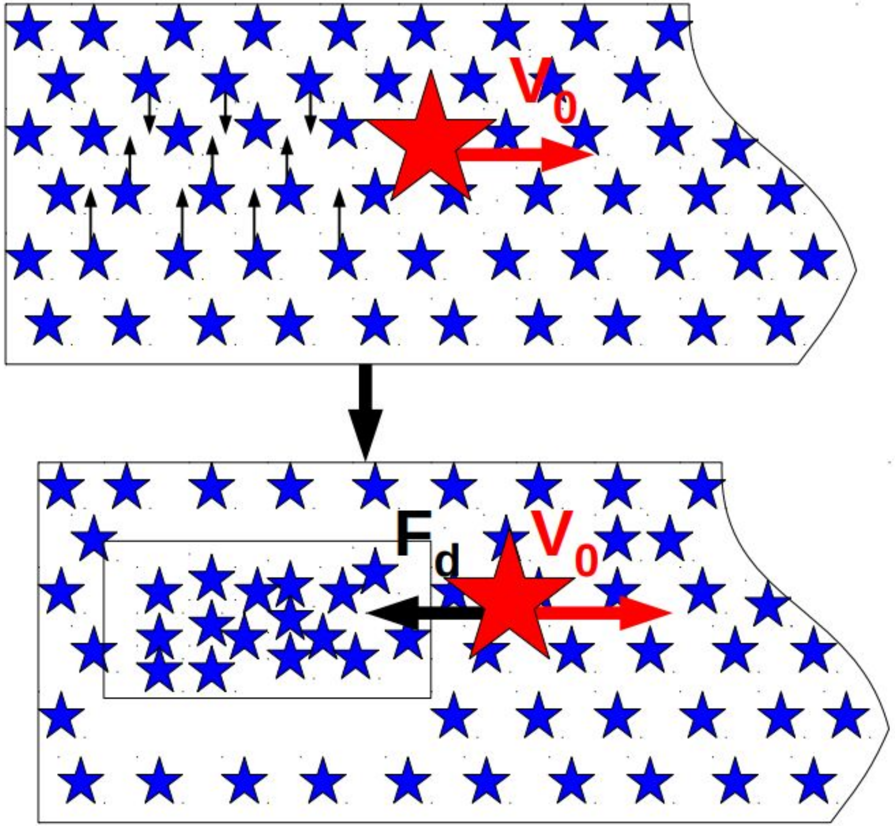


Dynamical friction – a qualitative picture



IMG credit: M. Mapelli

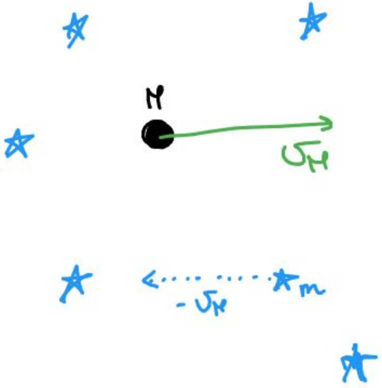
Chandrasekhar 1943

The heavy body M(SMBH) attracts the lighter particles (stars).
When the lighter particles approach M, M has already moved and leaves a local overdensity behind it.

The overdensity attracts the heavy body (with force Fd) and slows it down

NOTE: for a stellar background, this is a local, simplistic approximation of dynamical friction.

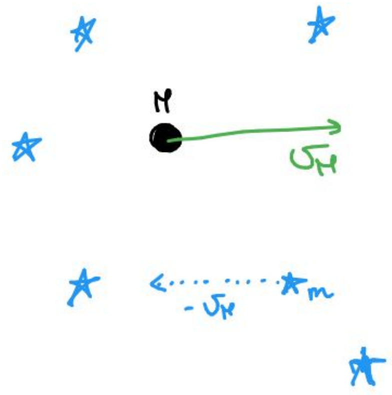
Dynamical friction



A perturber of mass M is embedded in a sea of stars with mass $m<<M$; the mass of the galaxy $M_g=Nm >> M$.
Let's assume a nearly constant stellar density; M is moving with velocity $\mathbf{v_M}$; while the stars have different velocities $\mathbf{v}$ and we set $\mathbf{v_o}=\mathbf{v}-\mathbf{v_M}$ relative velocity

In the reference frame of M:

Dynamical friction



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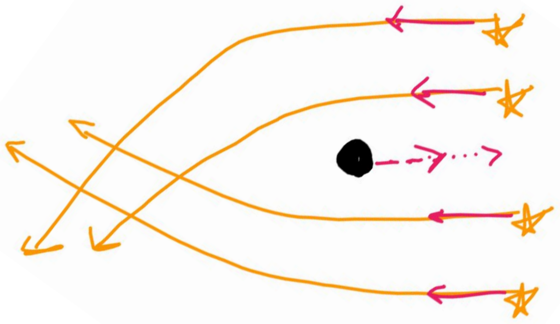
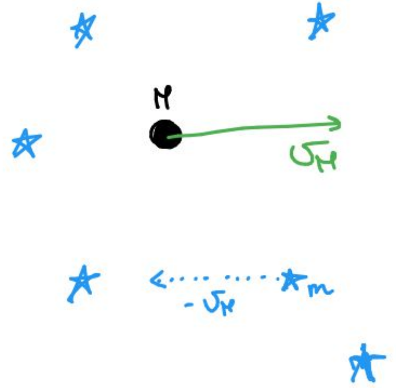
In the reference frame of M:

Many stars:



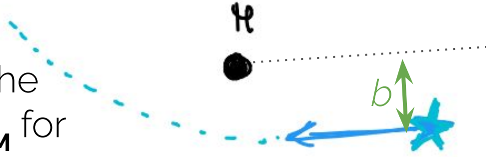
On average, the change in the velocity of M is negligible in the component perpendicular to $\mathbf{v_M}$, and is maximum in the parallel one → M slows down, loses angular momentum and inspirals

Dynamical friction



A perturber of mass *M* is embedded in a sea of stars with mass *m*<<*M*; the mass of the galaxy M_g=*Nm* >> *M*.
Let's assume a nearly constant stellar density; *M* is moving with velocity $\mathbf{v_M}$; while the stars have different velocities **v** and we set $\mathbf{v_o}=\mathbf{v}-\mathbf{v_M}$ relative velocity

The average change in the parallel component of $\mathbf{v_M}$ for *each stellar interaction* is



$$\langle|\delta v_{M\parallel}|\rangle = \frac{2m\bar{v}_0}{M+m}\left(1+\frac{b^2 v_0^4}{G^2(M+m)^2}\right)^{-1}$$

b is the impact parameter between the star and M at infinity

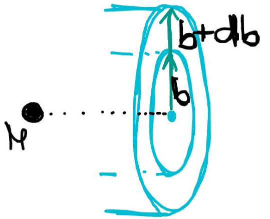
The change rate in the velocity (deceleration) due to stars with velocity **v** is going to be the rate at which those stars are encountered times the velocity change for all impact parameters *b*:

$$\left.\frac{d\bar{v}_M}{dt}\right|_{\bar{v}_0} = \bar{v}_0 f(\bar{v}) d^3\bar{v} \int_{b_{MIN}}^{b_{MAX}} \langle|\delta v_{M\parallel}|\rangle \cdot 2\pi b\, db$$

f(**v**) = number of stars in the phase space with velocity between **v**, **v**+d**v**`

Integrating over all *b*:

$$\left.\frac{d\bar{v}_M}{dt}\right|_{\bar{v}_0} = 2\pi G^2 m(m+M)\ln\left(1+\frac{b_{max}^2}{b_{min}^2}\right)\frac{\bar{v}-\bar{v}_M}{|\bar{v}-\bar{v}_M|} f(\bar{v})$$



$b_{max} \simeq R_{gal}$ $\quad b_{min} \simeq \frac{GM}{\sigma^2}$

$\ln\left(1+\frac{b_{max}^2}{b_{min}^2}\right) = \ln(1+\Lambda^2) \simeq 2\ln\Lambda$ ($\Lambda \gg 1$)

Integrating over all **v**:

$M \gg m$ $\quad \Lambda \neq \Lambda(\bar{v})$

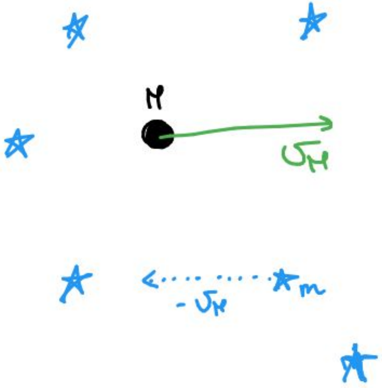
$$\frac{d\bar{v}_M}{dt} = 4\pi G^2 m M \ln\Lambda \int_{Vol} f(\bar{v})\frac{\bar{v}-\bar{v}_M}{|\bar{v}-\bar{v}_M|^3}$$

~ Newton's (/Gauss) theorem

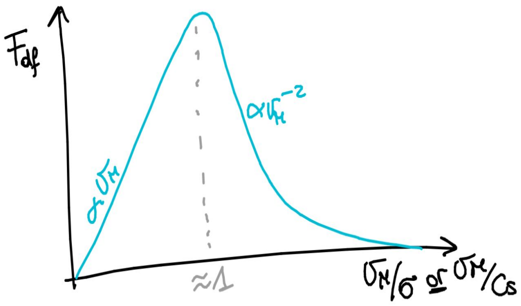
→ assuming isotropy in the velocity space:

$$\frac{d\bar{v}_M}{dt} = -16\pi^2 G^2 m M \ln\Lambda \frac{\bar{v}_M}{|\bar{v}_M|^3}\int_0^{v_M} f(v)\, v^2\, dv$$

Dynamical friction



$$\frac{d\vec{v}_M}{dt} = -16\pi^2 G^2 m M \ln\Lambda \frac{\vec{v}_M}{|v_M|^3}\int_0^{v_M} f(v)\, v^2\, dv$$



- Long-range force
- Linear in v_M for $v_M \to 0$
- Declines as v_M^{-2} for very large v_M
- The drag is maximum when v_M is close to the typical velocity of the stellar background
- It is only induced by stars slower than M (approximation!)

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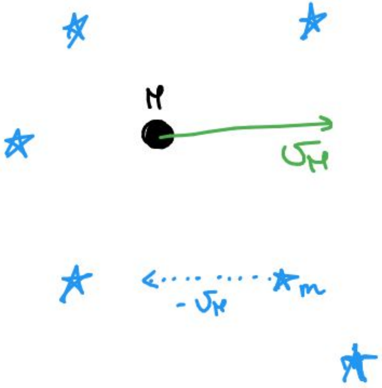

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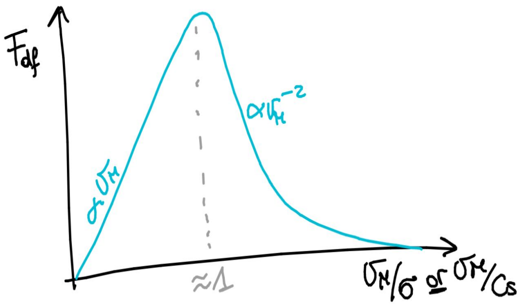

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In the case of **gaseous dynamical friction**, the result is similar, but the reference velocity is the speed of sound c_s of the gas, and the Mach number $\mathcal{M} = v_M/c_s$

$$\mathbf{F}_{\rm DF}^{\rm gas} = -4\pi \ln\left[\frac{b_{\rm max}}{b_{\rm min}}\frac{(\mathcal{M}^2-1)^{1/2}}{\mathcal{M}}\right] G^2 M_{\rm BH}^2 \rho_{\rm gas}\frac{\mathbf{V}}{V^3}, \quad \text{for} \quad \mathcal{M} > 1$$

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Ostriker 99

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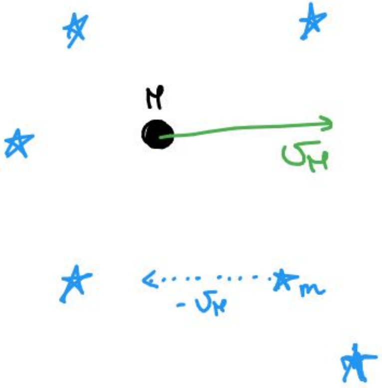

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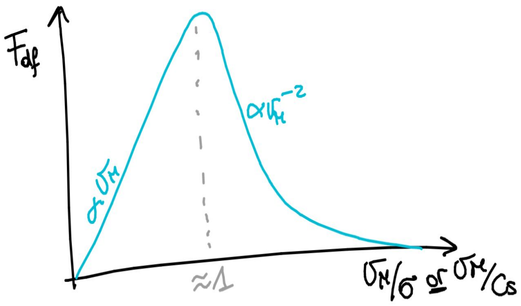

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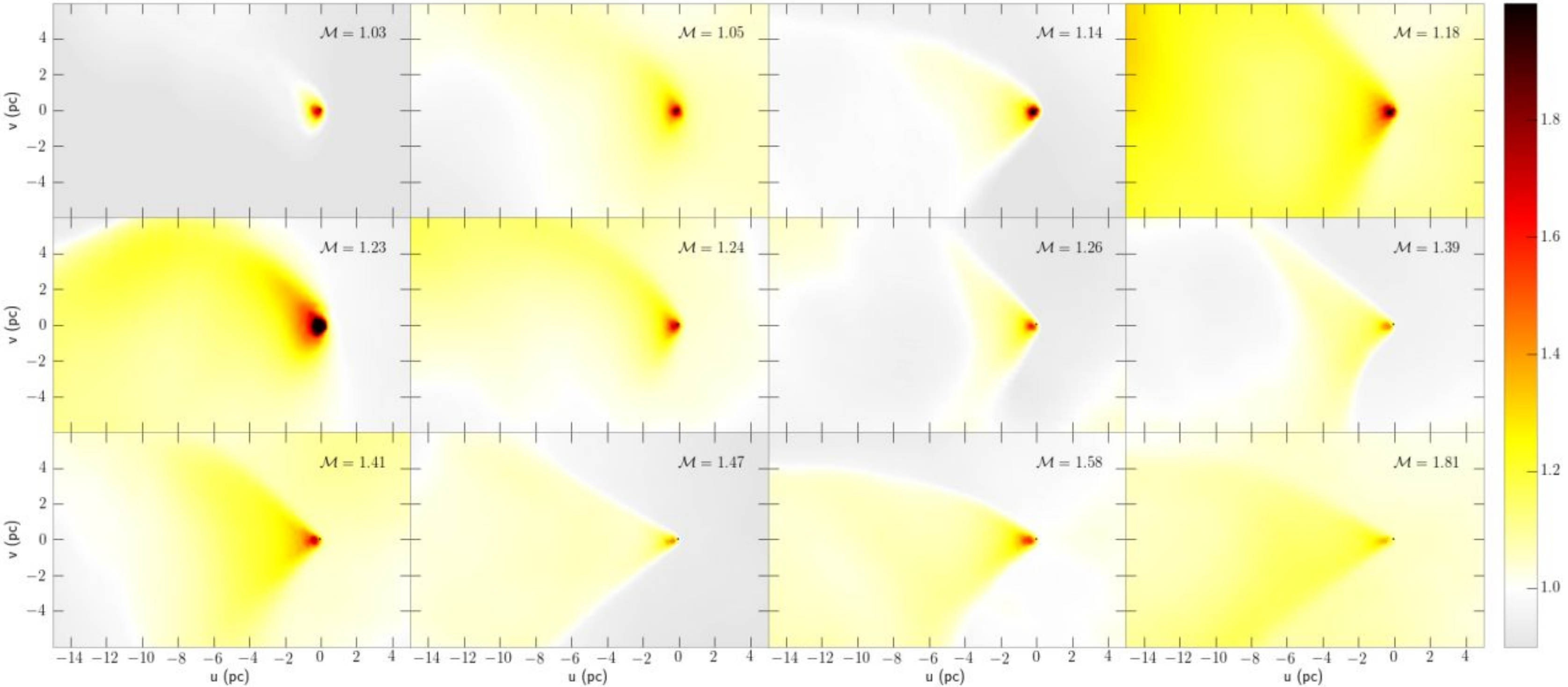
Gaseous dynamical friction and local vs global interpretation

Can be interpreted as a *local effect in the gaseous case* Chapon+11



The 'local' interpretation may be less accurate in the stellar/DM case. See Tremaine and Weinberg 1984; Weinberg 1986, 1989 for a global interpretation (global asymmetries triggered in the mass distribution of the host system give rise to global torques)

The dynamical friction timescale

Let's first introduce the
SINGULAR ISOTHERMAL SPHERE
Not too far from a realistic stellar distribution – Solution of Poisson's equations
σ = fixed value (param. of the model) – velc
dispersion of stars

$$\rho(r) = \frac{\sigma^2}{2\pi G r^2} \propto r^{-2}$$

$$M(r) = \int_0^r 4\pi r'^2 dr' \rho(r') = \frac{2\sigma^2 r}{G}$$

$$v_{circ} = \sqrt{\frac{GM(r)}{r}} = \sqrt{2}\,\sigma$$

$$f(v) = \frac{n_0 \exp\left(-\frac{v^2}{2\sigma^2}\right)}{(2\pi\sigma^3)^{3/2}}$$

Let's assume the perturber moves in a singular isothermal sphere with velocity = circular velocity of the system.

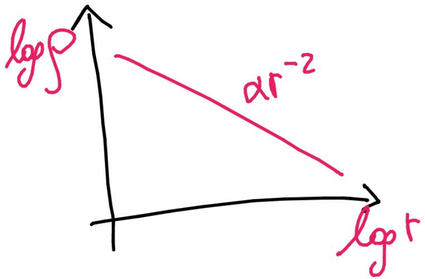
$$\frac{d\vec{v}_M}{dt} = -16\pi^2 G^2 m M \ln\Lambda \frac{\vec{v}_M}{|v_M|^3}\int_0^{v_M} f(v) v^2 dv$$

Here it becomes (Gauss integrals):

$$\frac{dv}{dt} = -\frac{4\pi \ln\Lambda\, G^2 M \rho}{v^2}\left(\mathrm{erf}(x) - \frac{2x e^{-x^2}}{\sqrt{\pi}}\right)$$

$$\text{if } v = v_c,\; x = 1$$

$$\frac{dv}{dt} = -0.428 \ln\Lambda \frac{GM}{r^2}$$



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Starting from $\frac{dv}{dt} = -0.428 \ln\Lambda \frac{GM}{r^2}$

we can write the torque acting on the mass M

$$\text{Torque} = \frac{dL}{dt} = M\left(r\frac{dv_M}{dt} + v_M\frac{dr}{dt}\right) \overset{set}{\equiv} r \times \left(M\frac{dv_M}{dt}\right)$$

This results in an estimate of the decay timescale!

$$\Rightarrow \int_0^{t_{DF}} dt = \int_{R_i}^{0} \frac{dr}{r}\, v_M \frac{r^2}{0.428 \ln\Lambda\, GM}$$

$$t_{\rm fric} = \frac{1.65}{\ln\Lambda}\frac{r_i^2\sigma}{GM} = \frac{19\,\rm Gyr}{\ln\Lambda}\left(\frac{r_i}{5\,\rm kpc}\right)^2 \frac{\sigma}{200\,\rm km\,s^{-1}}\frac{10^8 M_\odot}{M}$$

Some relevant caveats

Description

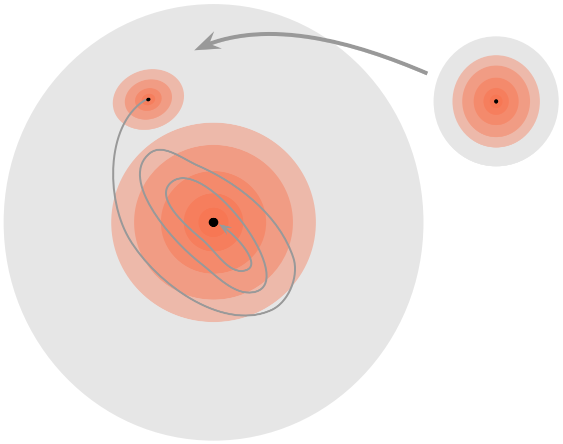
Chandrasekhar1943

$$\tau_{DF} \approx \frac{8\,\mathrm{Gyr}}{\ln\Lambda}\left(\frac{r}{\mathrm{kpc}}\right)^2 \frac{\sigma}{200\,\mathrm{km/s}} \frac{10^7 M_\odot}{M_{BH}}$$

Effective… but limited

Galactic tidal stripping must be taken into account,i.e. the initial M is the mass of the entire inspiralling galaxy,slowly stripped as it spirals in. In the end, the remaining mass can be the sole MBH mass →we should know M(t)

Colpi+99, Varisco+24



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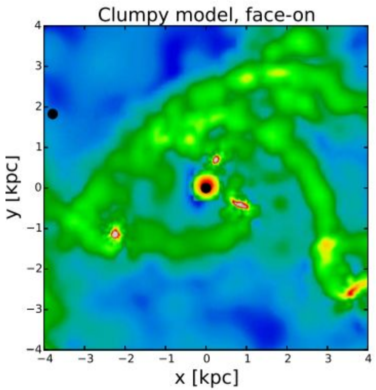
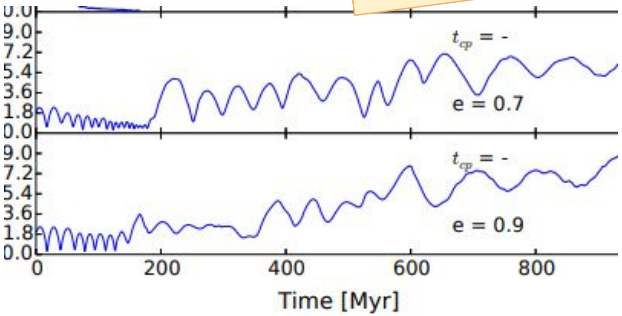
Colpi+99, Varisco+24

The effect of global asymmetries may significantly randomize the inspiral timescale (galaxy bar, massive stellar/gaseous clumps, galaxy flybys and galaxy harassment generate global torques that may be larger than the dynamical friction torque.

Bortolas+22, Tremmel+18, Pfister+19, Ricarte+21, Ma+21, Di Matteo+22

Clumpy galaxies





Tamburello+16

Some relevant caveats

Description

Chandrasekhar1943

$$\tau_{\rm DF} \approx \frac{8\,{\rm Gyr}}{\ln\Lambda}\left(\frac{r}{\rm kpc}\right)^2 \frac{\sigma}{200\,{\rm km/s}} \frac{10^7 M_\odot}{M_{\rm BH}}$$

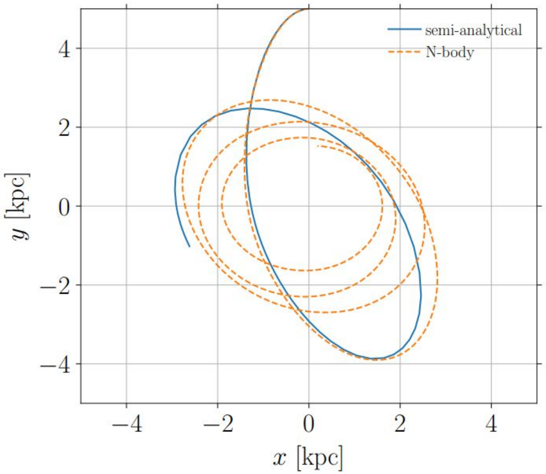
Effective… but limited

Galactic tidal stripping must be taken into account,i.e. the initial M is the mass of the entire inspiralling galaxy,slowly stripped as it spirals in. In the end, the remaining mass can be the sole MBH mass →we should know M(t)

Colpi+99, Varisco+24

The effect of global asymmetries may significantly randomize the inspiral timescale (galaxy bar, massive stellar/gaseous clumps, galaxy flybys and galaxy harassment generate global torques that may be larger than the dynamical friction torque.

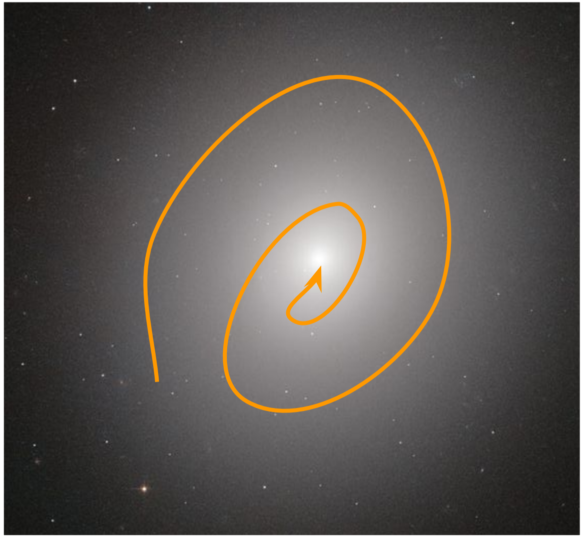
Bortolas+22, Tremmel+18, Pfister+19, Ricarte+21, Ma+21, Di Matteo+22



The very eccentric initial MBH orbit/the presence of a galaxy disc and its rotation may have a significant impact on the evolution of the inspiral and especially its final eccentricity

Bonetti,EB+20,21

Galaxies can have crazy morphologies!

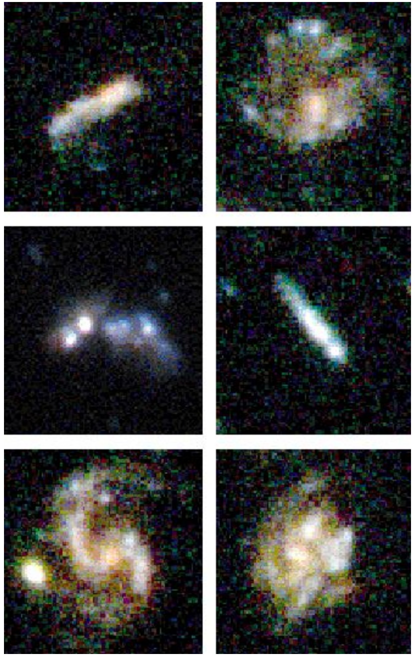


Spherical bowl of stars

vs

interacting/irregular

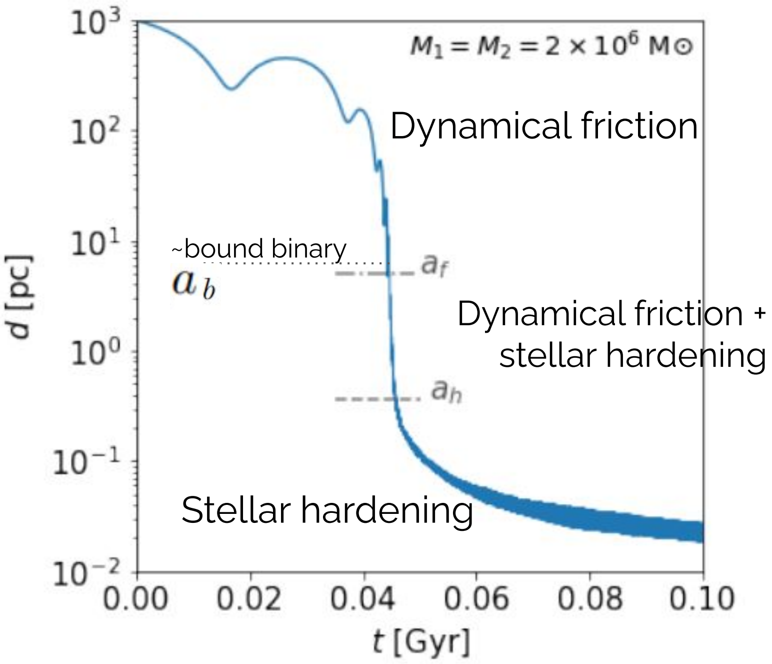
vs

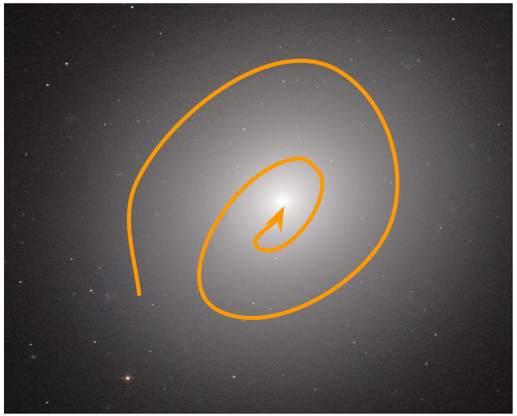


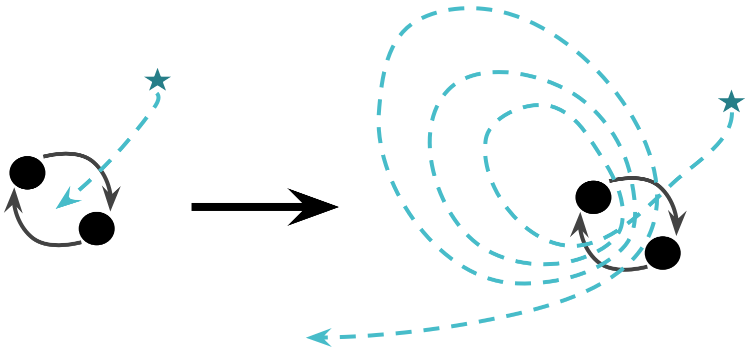
z>1

LISA!!

From large to small scale



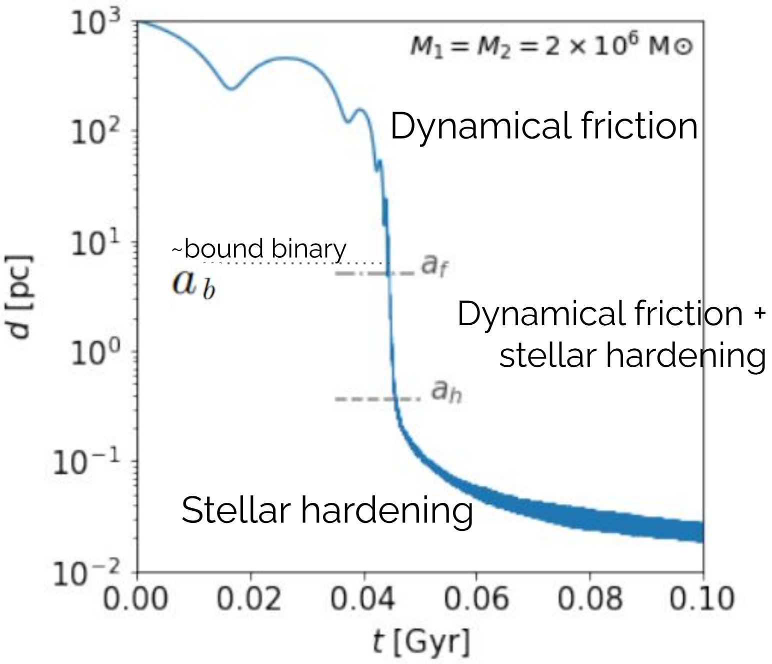




From large to small scale

$$M_1 + M_2 = M_b$$

$$q = M_2 / M_1 \leq 1$$



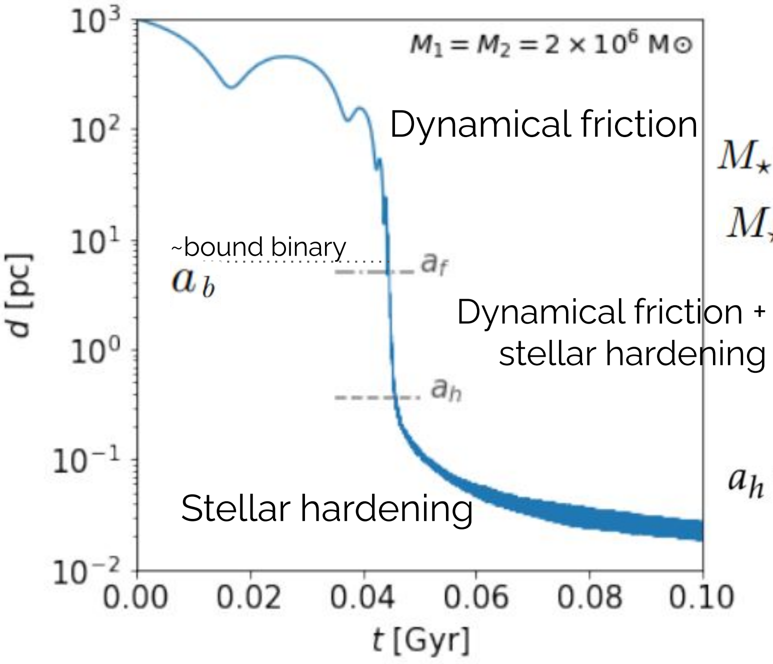
Relevant scales

- A bound binary forms when the mass in stars enclosed within the separation of the two MBHs roughly equals the mass of the two MBHs $$M_\star(a_b) = M_b$$
- Dynamical friction ceases to be the only shrinking mechanism at a_f $$M_\star(a_f) = M_2$$
- Between a_f and a_h, the binary evolves due to dynamical friction and stellar interactions of mostly bound stars, and it carves a core in the stellar distribution
- At a_h, the binary is said to be *hard* and it proceeds via stellar interactions; the evolution slows down $$a_h = \frac{GM_2}{4\sigma_*^2}$$

From large to small scale

$$M_1 + M_2 = M_b$$

$$q = M_2 / M_1 \leq 1$$



$$M_\star(a_b) = M_b$$

$$M_\star(a_f) = M_2$$

$$a_h = \frac{GM_2}{4\sigma_*^2}$$

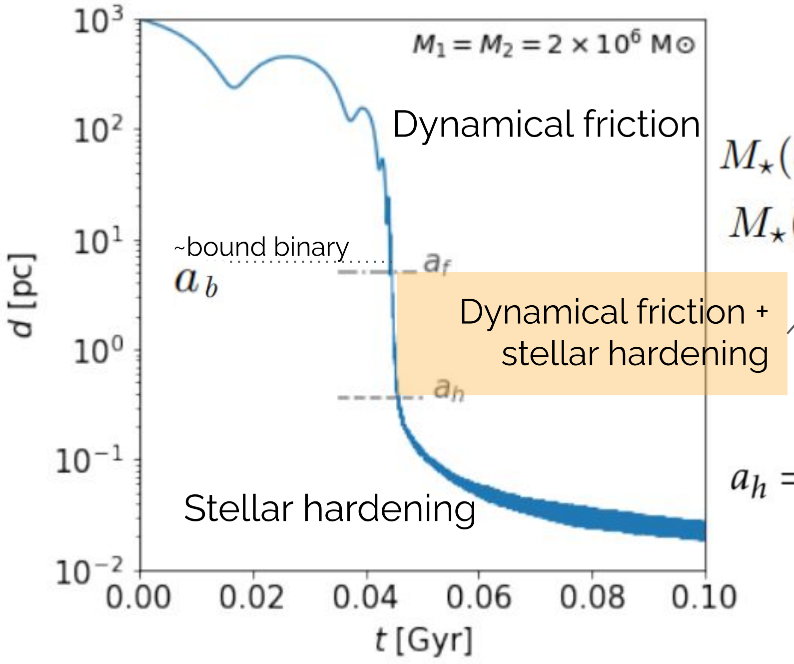
$$M(r) = \frac{2\sigma^2 r}{G} \Rightarrow r(M) = \frac{GM}{2\sigma^2} \Rightarrow r(2M_{bh}) = \frac{GM_{BH}}{\sigma^2} \equiv r_i$$

I.e. the influence radius of an MBH is equal to the radius enclosing twice the mass of the central object (in the assumption of a *singular isothermal sphere*).
It follows that, if *q* is not too far from 1, a_b and a_f are close to r_i.

Core excavation

$$M_1 + M_2 = M_b$$

$$q = M_2/M_1 \leq 1$$

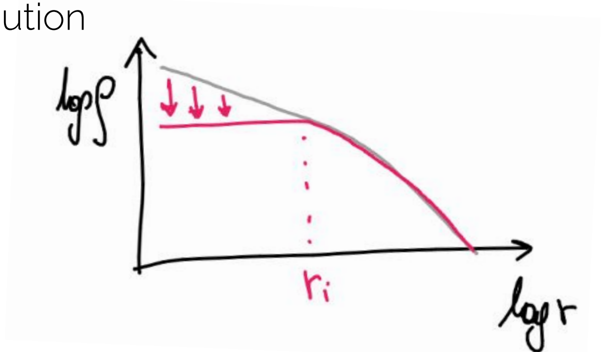


$$M_\star(a_b) = M_b$$

$$M_\star(a_f) = M_2$$

$$a_h = \frac{GM_2}{4\sigma_*^2}$$

In this phase, the binary scatters bound stars and carves a core in the stellar distribution



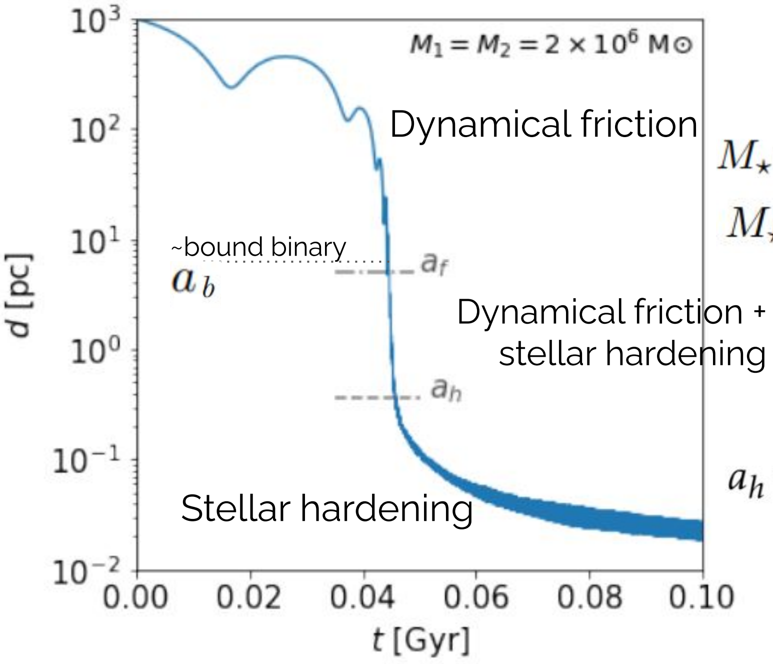
The energy injected in the stellar distribution is same order as the energy associated with stars within the binary influence sphere

$$\Delta E \sim \frac{GM_1M_2}{2}\left(\frac{1}{a_f} - \frac{1}{a_h}\right) \sim \frac{GM_1M_2}{2\frac{GM_2}{4\sigma^2}} \sim \frac{GM_1}{\sigma^2}$$

$$M_1 + M_2 = M_b$$

$$q = M_2/M_1 \leq 1$$

Stellar hardening



$$M_\star(a_b) = M_b$$

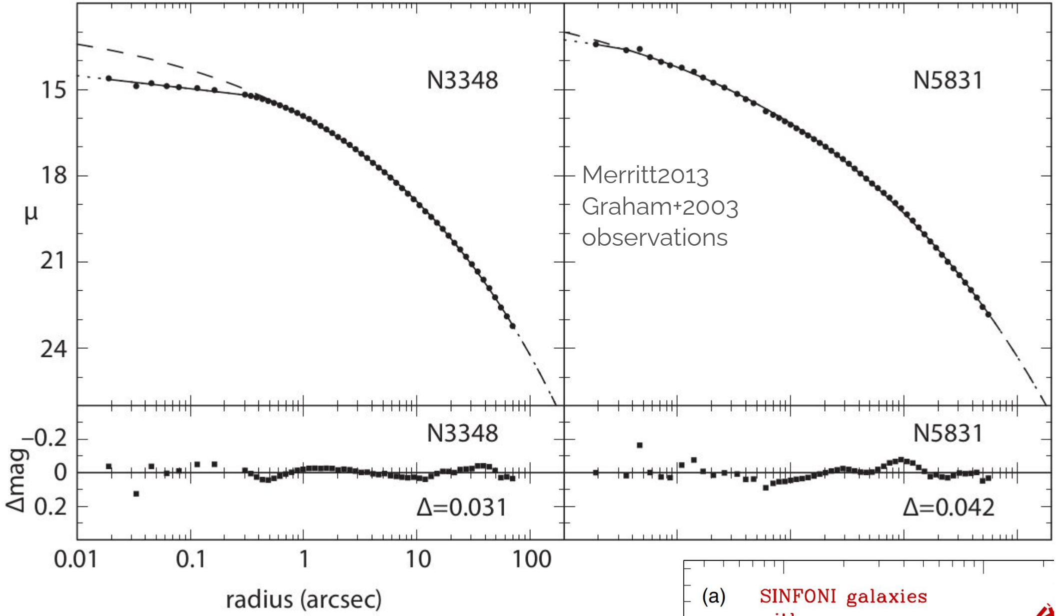
$$M_\star(a_f) = M_2$$

$$a_h = \frac{GM_2}{4\sigma_*^2}$$

Below a_h, dynamical friction is completely negligible and the binary tends to eject stars that come close enough to the binary:

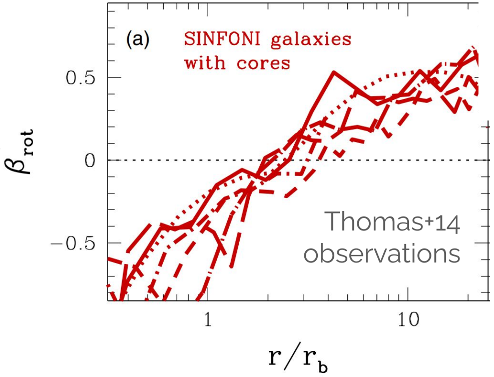
$$v_f = \sqrt{\frac{GM_2}{a_h}} \sim 2\sigma \sim v_{\rm escape}$$

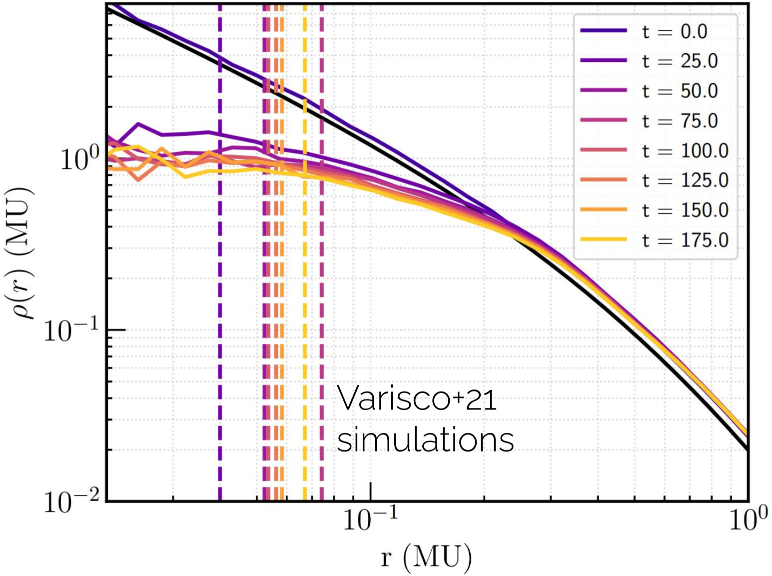
Core scouring



See also Lauer+07, Rusli+13, Dullo+15, Bonfini+15,16,18, Savorgnan+15,16,

Velocity anisotropy →





See also Gualandris+12, Rantala+18,19 , Bortolas+18, Dosopoulou+21,24

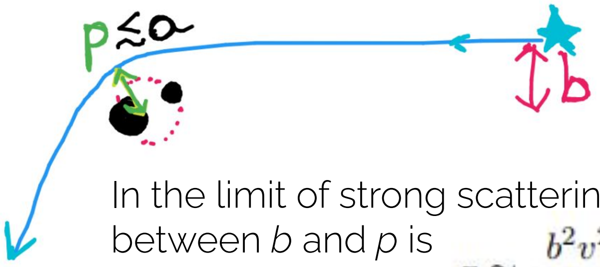
Stellar hardening

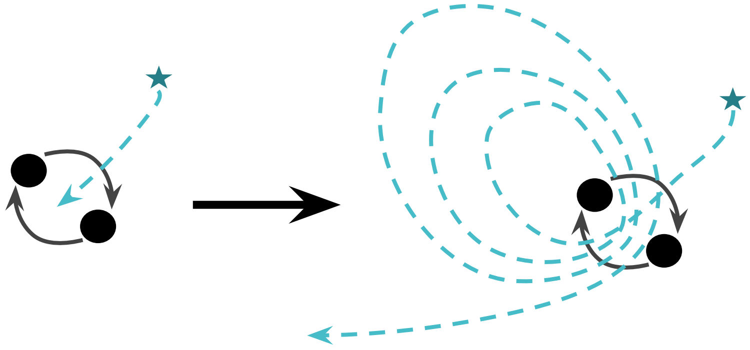
$$a < a_h = \frac{GM_2}{4\sigma_*^2}$$

$$M_1 + M_2 = M_b$$

$$q = M_2/M_1 \leq 1$$

Consider a star experiencing a close scattering with the binary: closest approach within the binary semimajor axis





In the limit of strong scatterings, the relation between b and p is

$$p \approx \frac{b^2 v^2}{2GM_b}$$

(*p*~*a*)

We can write the binary cross section for close encounters as $\Sigma_b = \pi b^2 = \frac{2\pi G M_b a}{v^2}$

→ the rate of events the binary experiences per unit time is: $dN/dt = \Sigma_b \sigma n$

(*n* is the number density of stars)

If $\boldsymbol{\sigma}$~v is the typical stellar velocity the encounter rate becomes

$$\frac{dN}{dt} = \frac{2\pi G M_b a n}{\sigma}$$

Stellar hardening

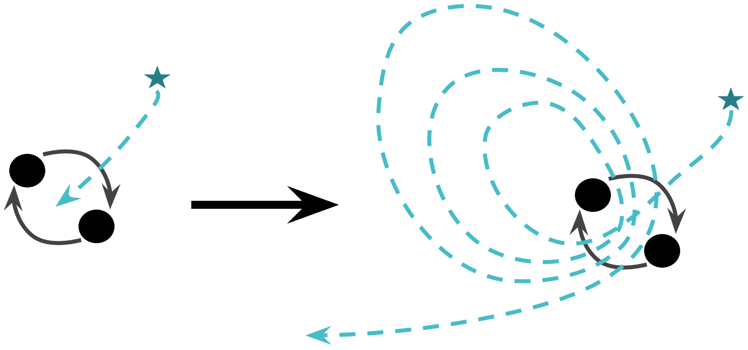
$$a < a_h = \frac{GM_2}{4\sigma_*^2}$$

$$M_1 + M_2 = M_b$$

$$q = M_2/M_1 \leq 1$$

Rate of binary–star interactions $\frac{dN}{dt} = \frac{2\pi G M_b a n}{\sigma}$

Binary energy change for each stellar interaction $\frac{\Delta E}{E} \approx -\frac{m_\star}{M_b}$



→Rate of average binary energy change

$(nm_\star = \rho)$

$$\frac{dE}{dt} = \Delta E \frac{dN}{dt} = 4\pi G \frac{\rho a}{\sigma} E$$

Knowing the energy of a binary $E = -\frac{GM_1M_2}{2a}$

we can write the energy change in its semimajor axis: $\frac{d}{dt}\frac{1}{a} \approx 4\pi \frac{G\rho}{\sigma} \equiv H \frac{G\rho}{\sigma}$

Stellar hardening

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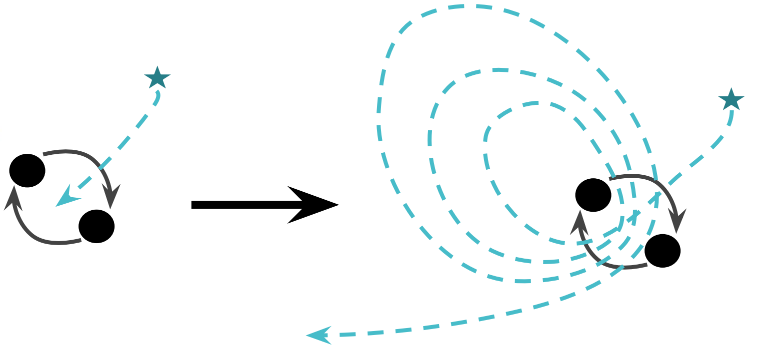


→Rate of average binary energy change

$(n m_\star = \rho)$

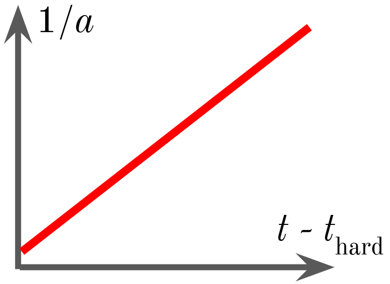
$$\frac{dE}{dt} = \Delta E \frac{dN}{dt} = 4\pi G \frac{\rho a}{\sigma} E$$

Knowing the energy of a binary $E = -\frac{G M_1 M_2}{2a}$

we can write the energy change in its semimajor axis:

$$\frac{d}{dt}\frac{1}{a} \approx 4\pi \frac{G\rho}{\sigma} \equiv H \frac{G\rho}{\sigma}$$

ρ and σ evaluated near r_i



HARD BINARIES HARDEN AT A CONSTANT RATE

(provided that the host properties remain ~ the same)

Stellar hardening

$$a < a_h = \frac{GM_2}{4\sigma_*^2}$$

$$M_1 + M_2 = M_b$$

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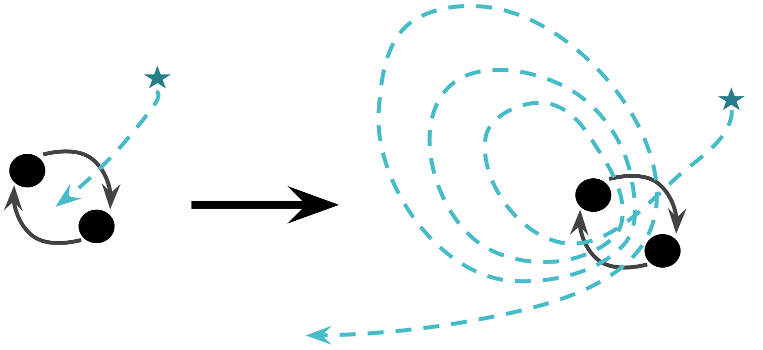

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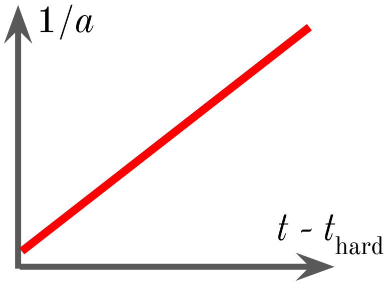


HARD BINARIES HARDEN AT A CONSTANT RATE

(provided that the host properties remain ~ the same)

$$t_h \sim \frac{\sigma}{H G \rho\, a_{\rm GW}} \approx 0.25\ {\rm Gyr} \left(\frac{18}{H}\right) \left(\frac{\rho}{10^4 M_\odot {\rm pc}^{-3}}\right)^{-1} \left(\frac{\sigma}{200\ {\rm km\ s}^{-1}}\right) \left(\frac{a_{\rm GW}}{1\ {\rm mpc}}\right)^{-1}$$

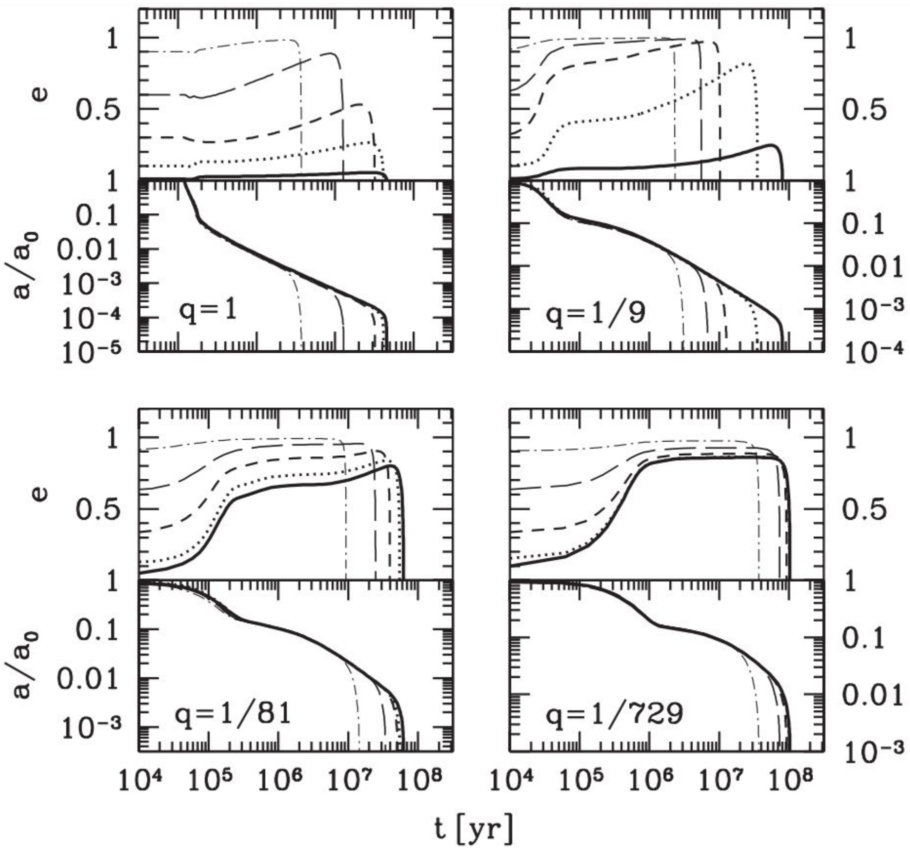
Stellar hardening

Sesana+06,07, Sesana10

$$\left.\frac{da}{dt}\right|_\star = -a^2\frac{HG\rho}{\sigma}$$

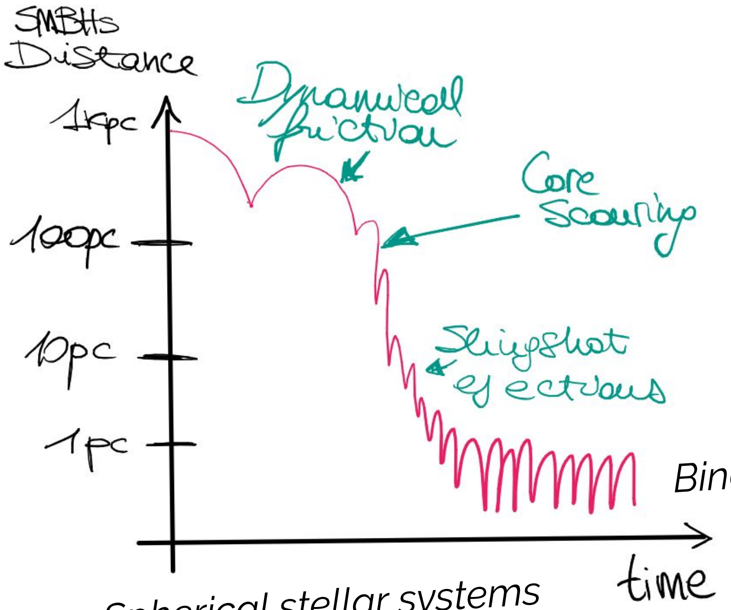
$$\left.\frac{de}{dt}\right|_\star = a\frac{HKG\rho}{\sigma}$$

H~15
K~ -0.2 – 0.2

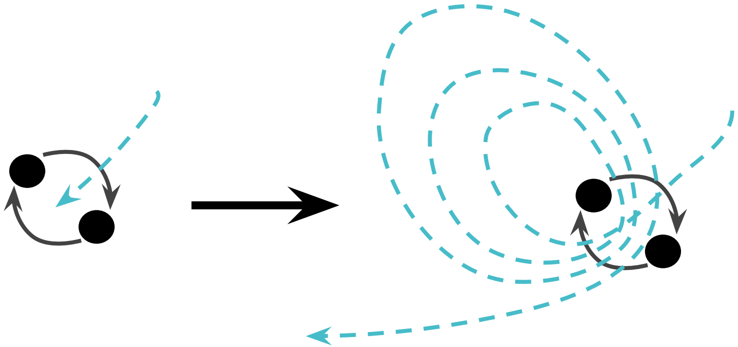


BINARIES TYPICALLY GROW THEIR ECCENTRICITY IN ISOTROPIC ENVIRONMENTS

The final parsec problem



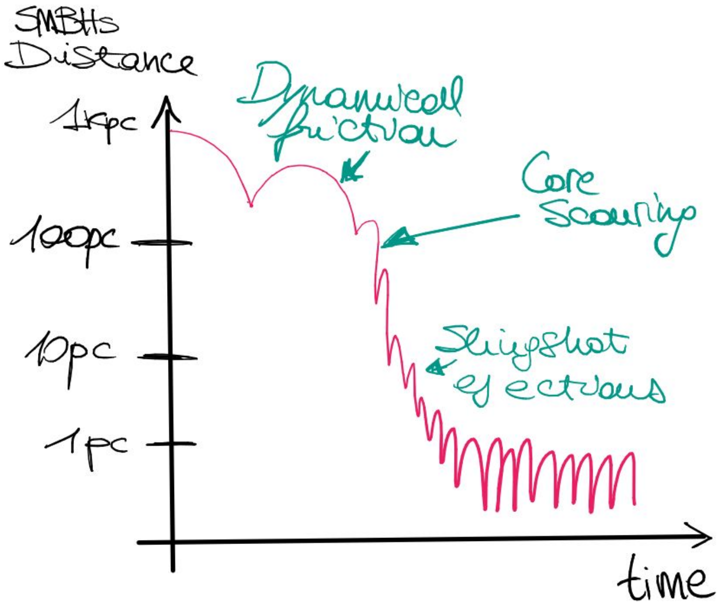
Spherical stellar systems



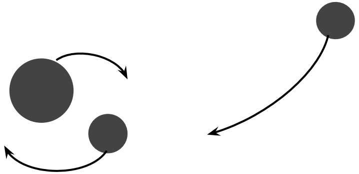
Binary stalling at 1 pc separation

The final parsec problem

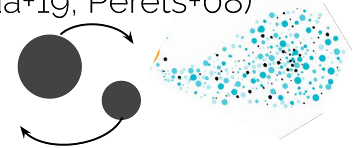
...and its solutions



- Third incoming massive black hole
  (eg Bonetti+18,19)



- The presence of massive perturbers
  (eg Bortolas+18, Arca-Sedda+19, Perets+08)

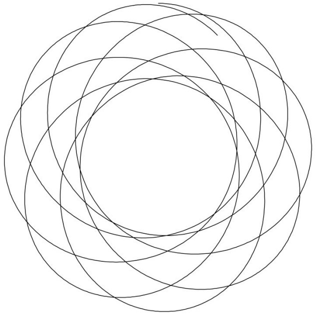


- Galaxy rotation
  (eg Varisco+21, Holley-Bockelman+15, Mirza+17)
- **<u>The presence of a gaseous disk</u>**

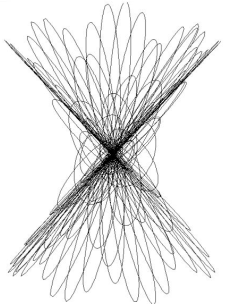
  (eg Escala+04,
  Dotti+06,
  Goicovic+16)

MBHs efficiently shrink via stellar interactions in **all realistic galaxies!**

*The **most generic solution to the final parsec problem***

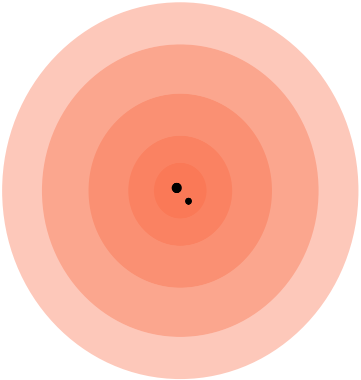


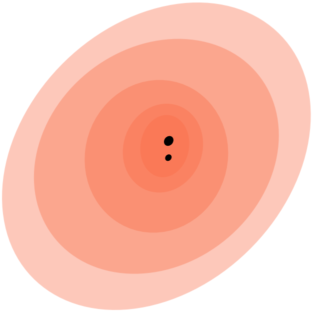
Perfectly spherical galaxy

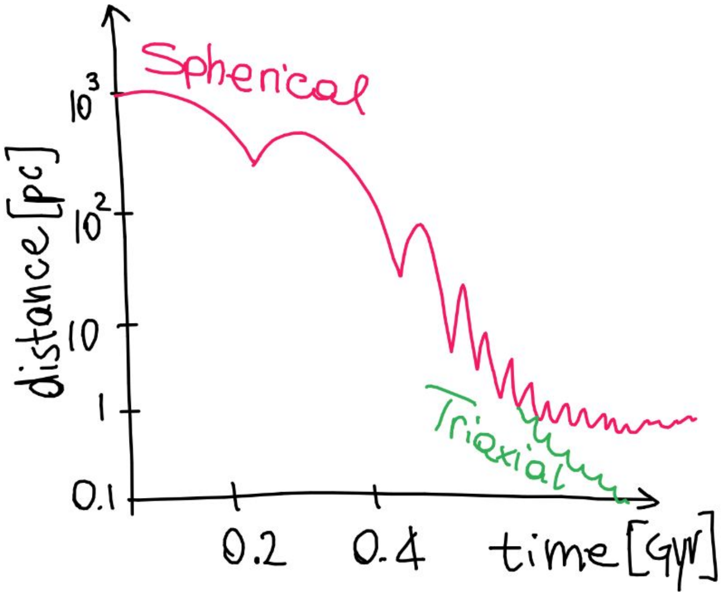


Triaxial galaxy

versus



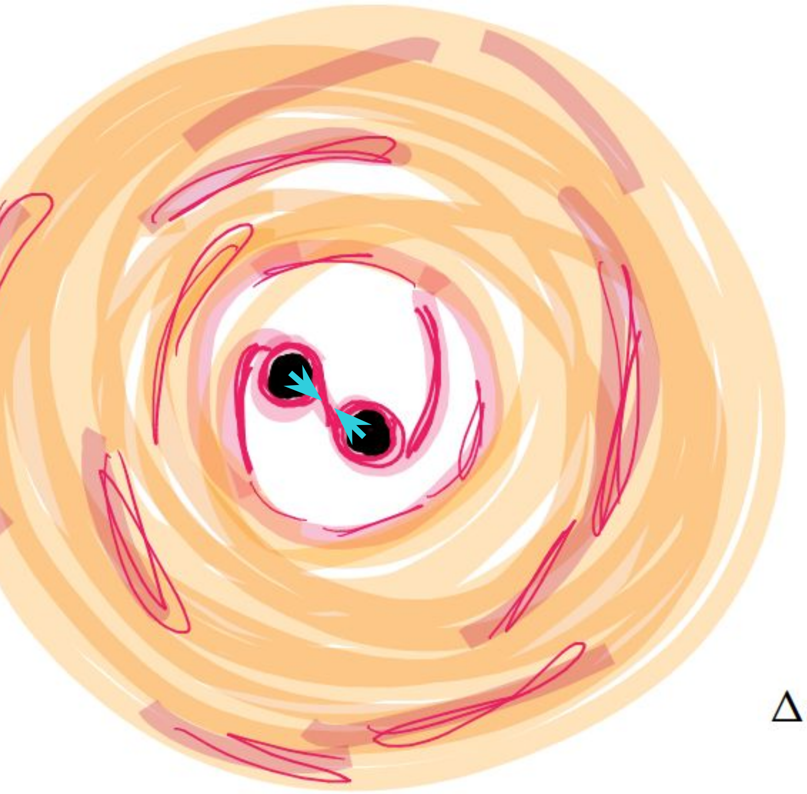




Berczik+06; Preto+11; Khan+11,16;
Gualandris+12,17; Vasiliev+15; Bortolas+16, 18

The key role of the environment: **small-scale inspiral**

What about the effect of gaseous disks?



Early studies suggest the disk promotes the binary shrinking

(Artymowicz&Lubow94,96, Escala+05, Dotti+07, Cuadra+09)

At the order of magnitude level (Dotti+15, see also Celoria+18):

$$\frac{da}{a} = -\frac{dM_{\text{gas}}}{M},$$

This implies that the binary shrinks by an e-fold by accreting its own mass in gas.
If the gas inflow is limited at the Eddington limit and the radiative efficiency $\epsilon = \sim 10^{-1}$

$$\Delta t_{\text{BHB}} \sim \ln\left(\frac{a_i}{a_c}\right) \frac{\mu \, \epsilon \, c^2}{2\sqrt{2}\, L_{\text{Edd}}} \sim 10^7 \frac{q}{(1+q)^2} \ln\left(\frac{a_i}{a_c}\right) \text{ yr}$$

The key role of the environment: **small-scale inspiral**

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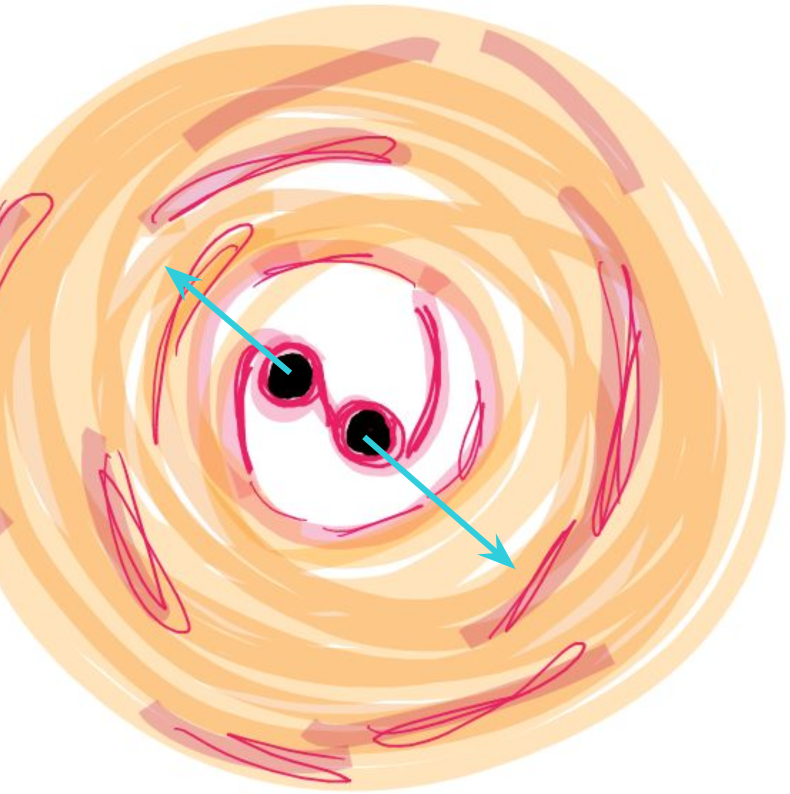


Early studies suggest the disk promotes the binary shrinking
(Artymowicz&Lubow94,96, Escala+05, Dotti+07, Cuadra+09)

More recent studies suggest that **the disk can expand the binary instead!**
(Moody+19, Munoz+19,20, Duffel+20, Tiede+20, Heath&Nixon20, Franchini+21, D'orazio&Duffel21+)

$$\dot{a}_{\rm gas} = 2.68\frac{\dot{m}}{m}a$$

But the delay due to this process is at most ~10^8 yr even in the most pessimistic scenarios, still shorter/of the order of the dynamical friction timescale (Bortolas+22)

The gravitational-wave inspiral phase

The evolution of the binary semimajor axis and eccentricity in the GW phase (Peters 1964)

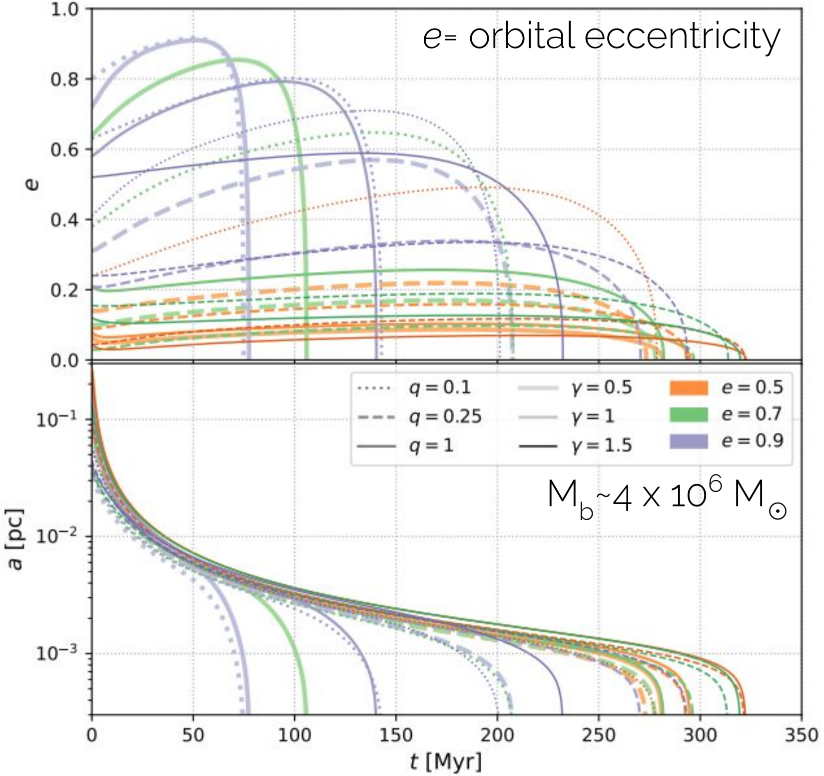
$$\left(\frac{\mathrm{d}a}{\mathrm{d}t}\right)_{GW} = -\frac{64}{5}\frac{G^3 M_1 M_2 (M_1+M_2)}{a^3 c^5} \cdot f(e);$$

$$\left(\frac{\mathrm{d}e}{\mathrm{d}t}\right)_{GW} = -\frac{304}{15}\frac{e\, G^3 M_1 M_2 (M_1+M_2)}{a^4 c^5 (1-e^2)^{5/2}}\left(1+\frac{121}{304}e^2\right)$$

$$f(e) = (1-e^2)^{-7/2}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)$$

$$e = 0.9 \implies f(e) \sim 1000!!$$

The gravitational-wave inspiral phase



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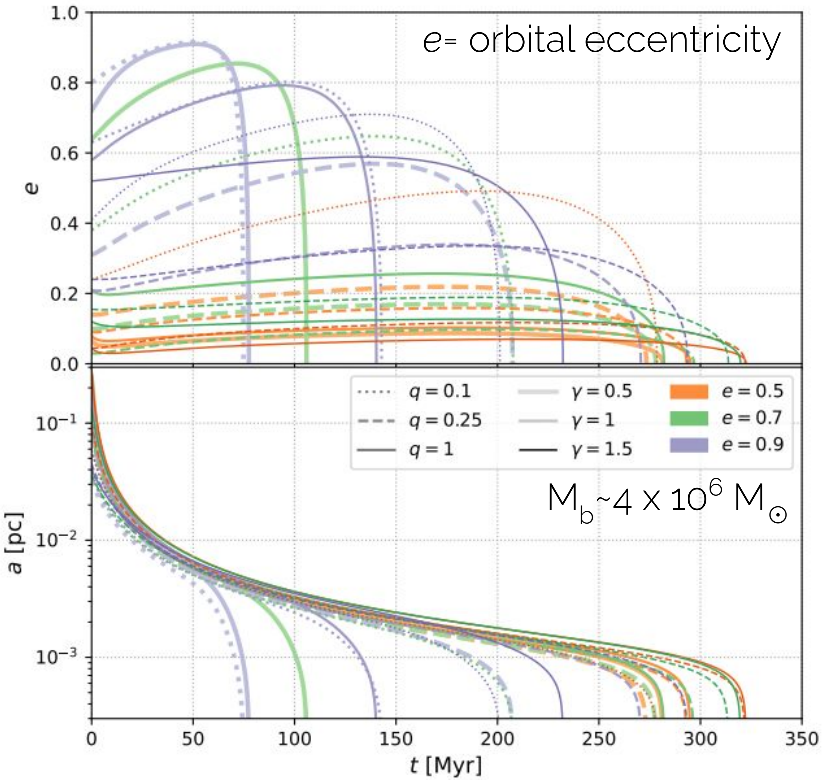

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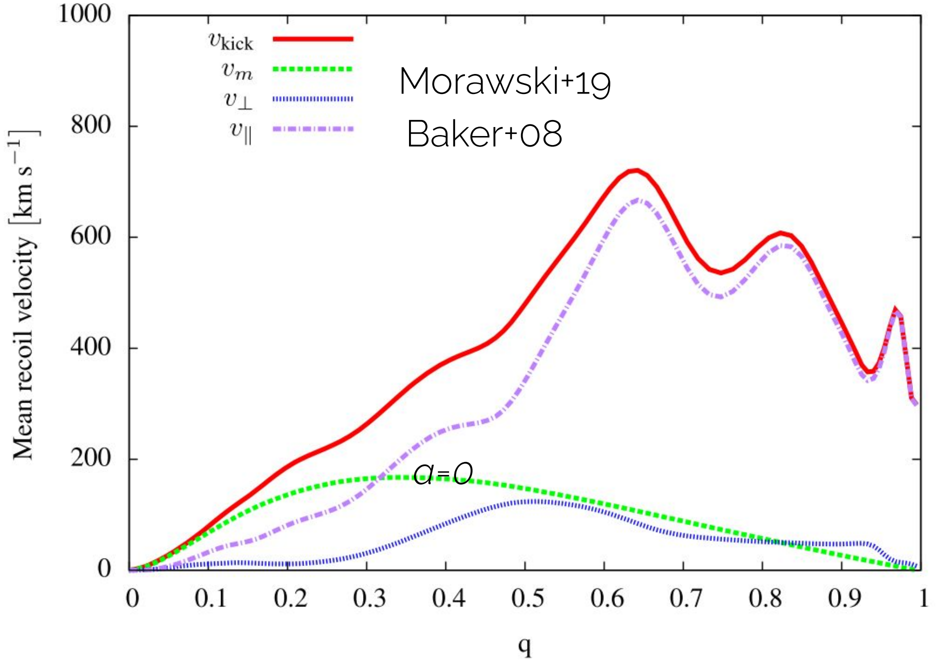
$$f(e) = (1-e^2)^{-7/2}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)$$

$$e = 0.9 \implies f(e) \sim 1000!!$$

$$t_{gw} = \frac{5}{256}\frac{c^5}{G^3}\frac{1}{f(e)}\frac{a^4}{\mu(M_1+M_2)} \approx$$

$$\approx \frac{580\ \mathrm{Gyr}}{f(e)}\left(\frac{a}{0.1\ \mathrm{pc}}\right)^4\left(\frac{\mu}{10^7\ \mathrm{M}_\odot}\right)^{-1}\left(\frac{M_1+M_2}{10^8 \mathrm{M}_\odot}\right)^{-2}$$

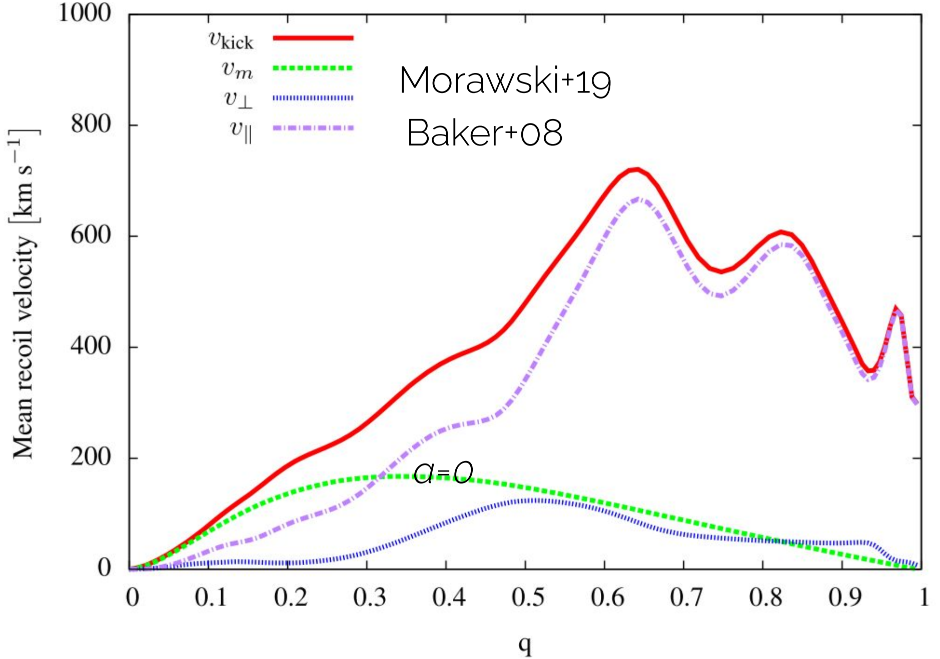
The gravitational wave kick

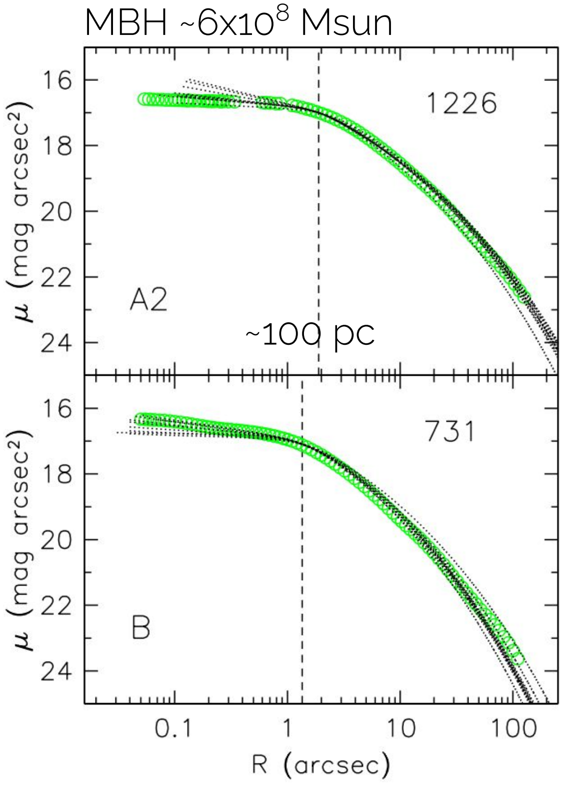


The magnitude of the kick is independent of the BH masses involved

The gravitational wave kick and the carving of huge cores

Gualandris & Merritt 18





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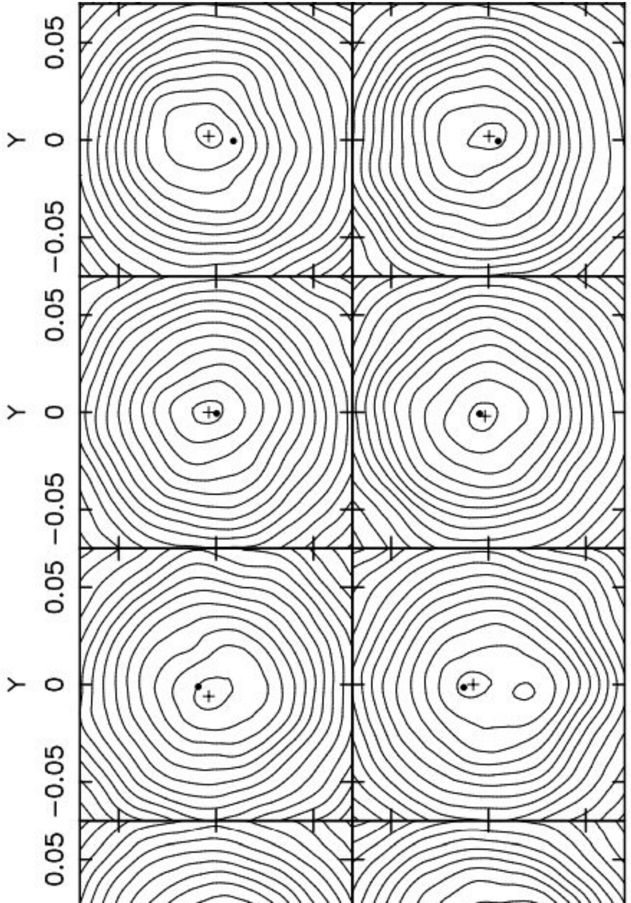
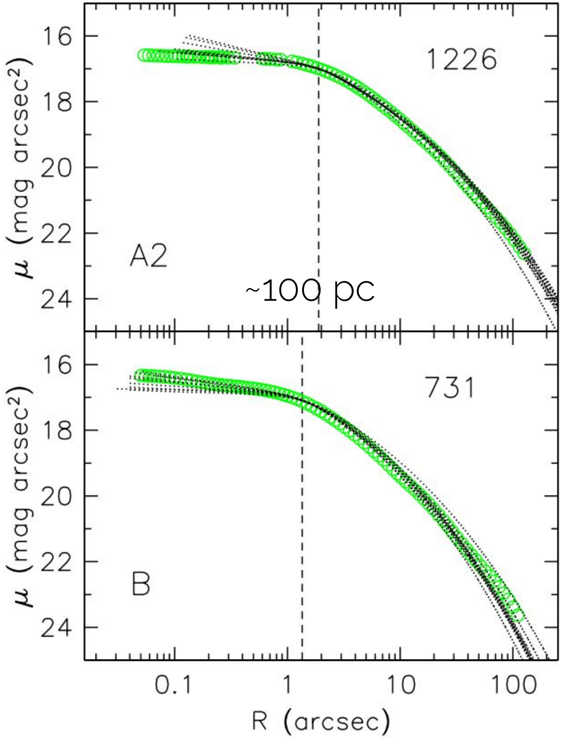


MBH ~6×10^{8} Msun



Putting everything together

The residence timescale

$$t_r \approx \frac{r}{\dot{r}} = \frac{dt}{dr} r = \frac{dt}{d\ln r}$$

Dynamical friction

$$\left.\frac{dr}{dt}\right|_{\rm DF} \approx -0.428 \frac{\ln \Lambda G M}{\sqrt{2}\sigma r}$$

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With the help of scaling relations…

$$\sigma \approx 155 \left(\frac{M}{10^8 M_\bullet} \right)^{1/4.38} \mathrm{km/s}$$

$$r_i \approx 35 \left(\frac{M}{10^8 M_\odot} \right)^{0.56} \mathrm{pc}$$

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Stellar hardening

$$\left.\frac{da}{dt}\right|_{\star} \approx -\frac{HG\rho}{\sigma} a^2 \implies \frac{a}{|\dot{a}|} \approx \frac{\sigma}{HG\rho a} \sim 0.1 \left(\frac{M}{10^8 M_\odot}\right)^{0.9} \left(\frac{a}{0.01\mathrm{pc}}\right)^{-1} \mathrm{Gyr}$$

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Gravitational-wave inspiral

$$\left.\frac{da}{dt}\right|_{\mathrm{GW}} \approx -\frac{64}{5} \frac{G^3 q M^3}{c^5 (1+q)^2 a^3} f(e) \implies \frac{a}{|\dot a|} \approx \frac{5}{64} \frac{c^5 (1+q)^2}{G^3 M^3 q f(e)} a^4 \sim \frac{0.2}{f(e)} \left(\frac{M}{10^8 M_\odot}\right)^{-3} \left(\frac{q}{0.2}\right)^{-1} \left(\frac{a}{0.01\mathrm{pc}}\right)^4 \mathrm{Gyr}$$

Putting everything together

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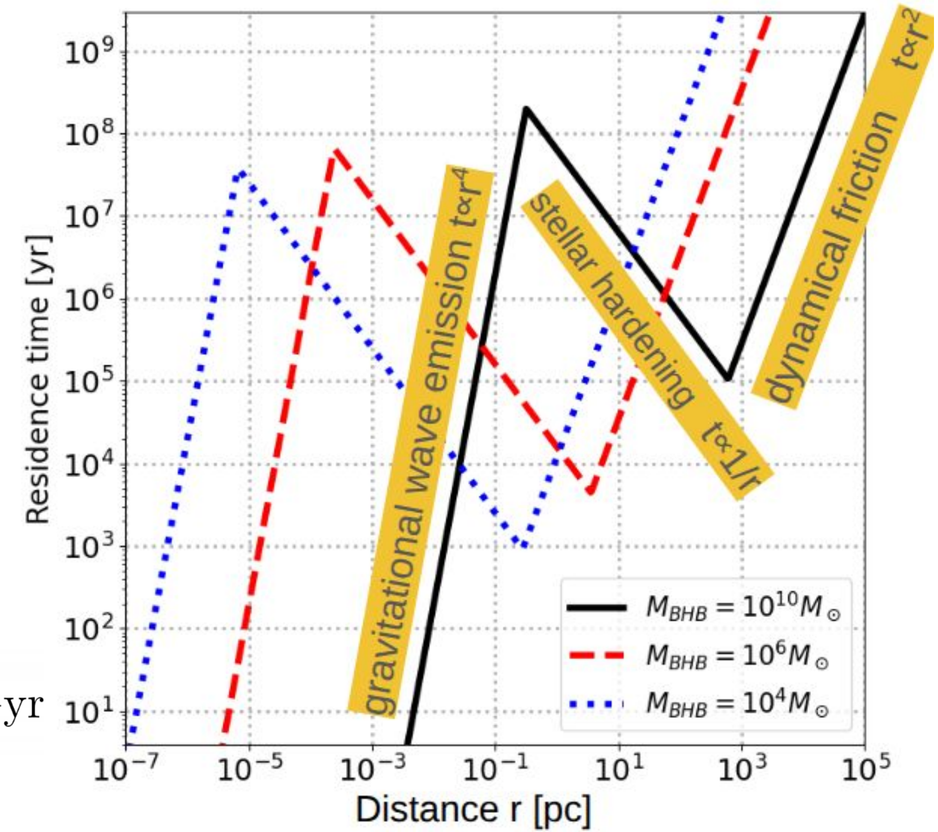
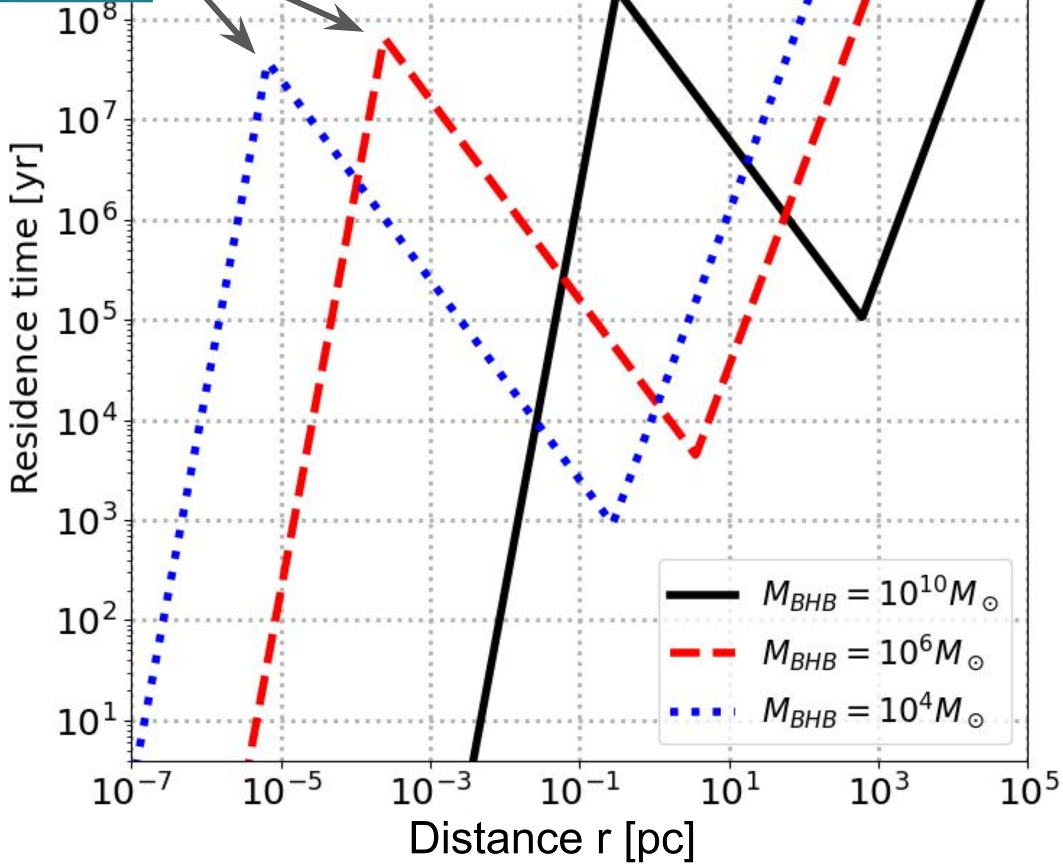


Typical evolutionary timescales

$$t_r \approx \frac{r}{\dot{r}} = \frac{dt}{dr} r = \frac{dt}{d \ln r}$$

Binaries spend most of their time at this transition



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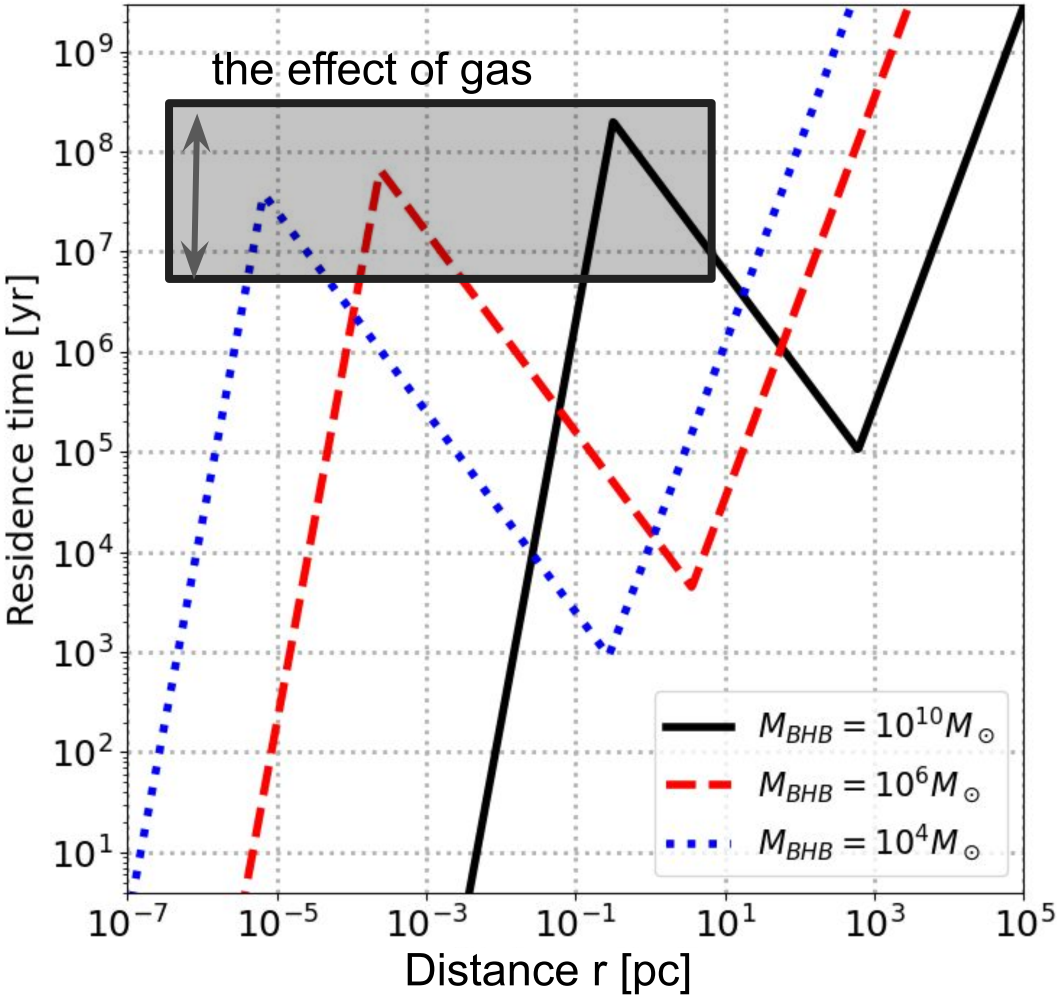

A hand-wavy estimate of the rate of massive black hole mergers

N = Number of galaxies in the Universe: $\sim 10^{11}$

R = Major galaxy mergers per each galaxy: ~1

T = Hubble time today: $\sim 10^{10}$ yr

Assuming the delay between galaxy merger and MBH merger is small enough

→ Binary merger rates = N x R / T **~ 10/yr**

To conclude

LISA will help us understand the formation and evolution of massive black holes from the onset of galaxy formation;

The inspiral of massive black hole binaries is a multi faceted process

- What are the effects of stochastic processes in the inspiral (es dynamical friction phase)?
- The features of the host stellar system are key to characterizing the inspiral of massive black holes: how can we model them properly?
- What happens when gas is around?
- How can we observe electromagnetically LISA mergers?

More broadly...

- How can we anticipate LISA merger rates? Where are intermediate-mass black holes? How do we find them prior to LISA? ...

The bright future of gravitational waves in the context of MBHs