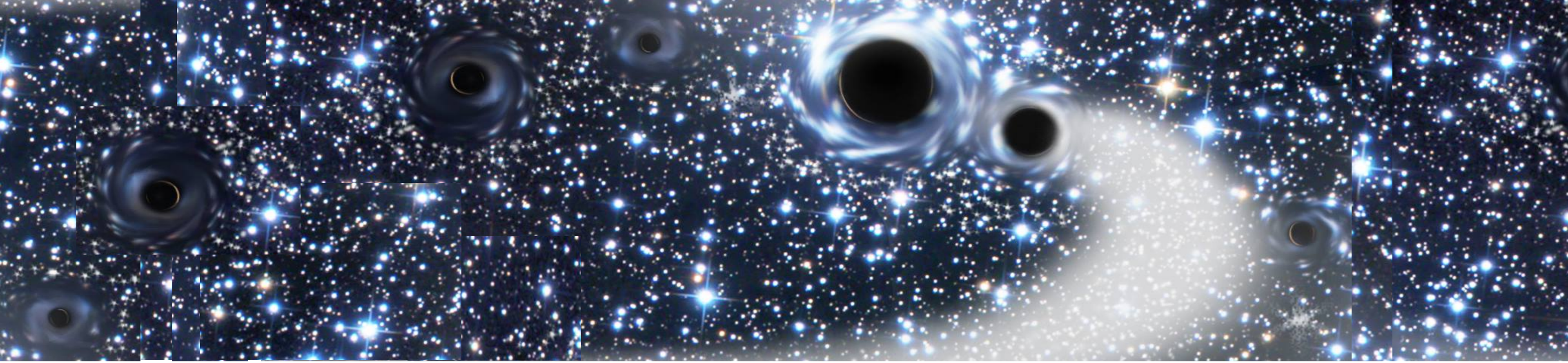


Formation pathways of GW sources

Gravitational Wave Astrophysics

Manuel Arca Sedda

GRAN SASSO SCIENCE INSTITUTE

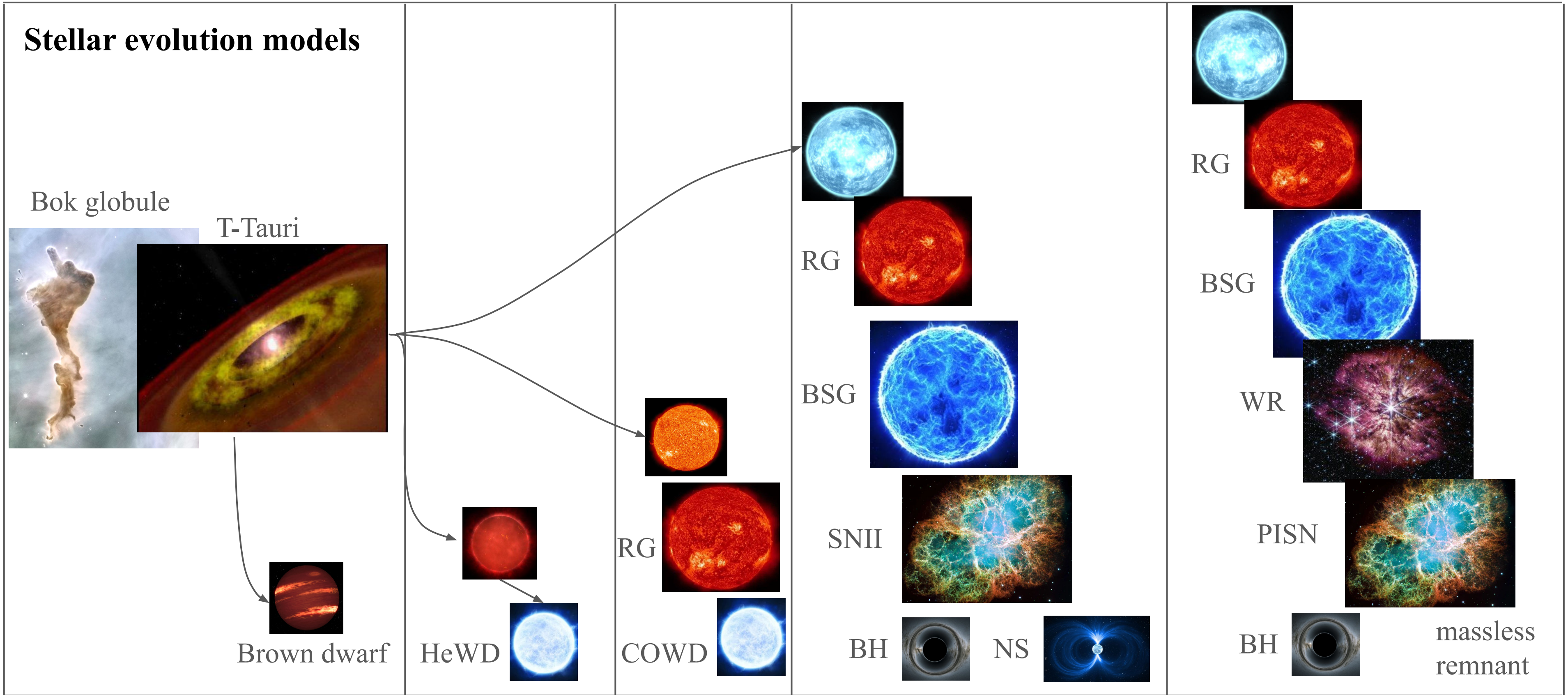
What will we do in the 1.5hrs? Crash course on

1. **Compact objects from the death of single stars**

2. **Binary stellar evolution**

3. **Dynamics of compact objects**

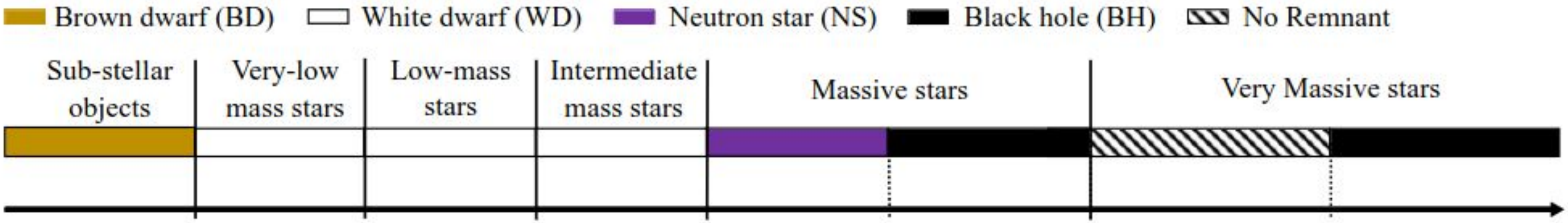
Stellar evolution models



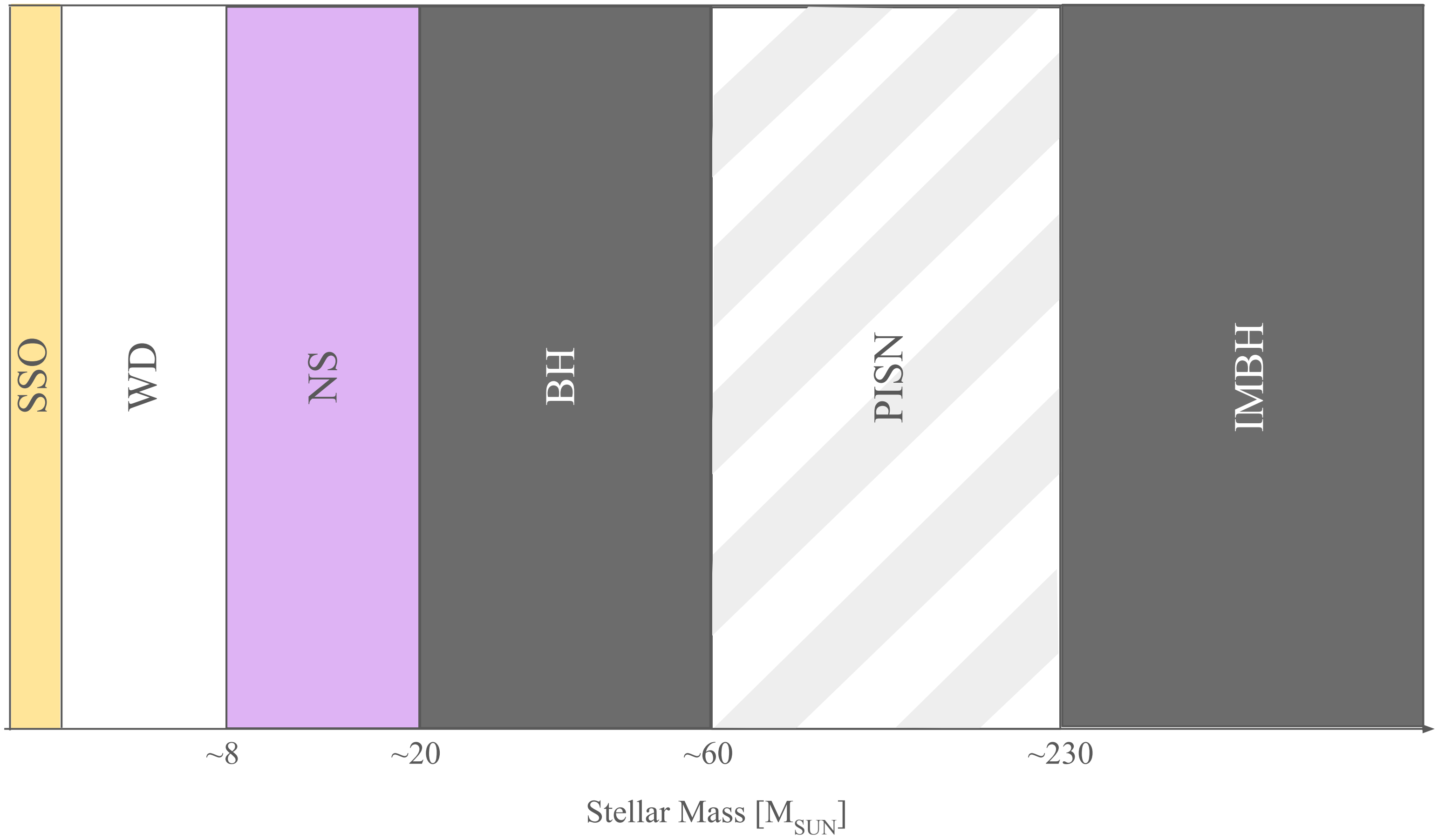
$< 0.1\ M_{SUN}$	$< 0.5\ M_{SUN}$	$< 6\text{-}8\ M_{SUN}$	$< 20\text{-}40\ M_{SUN}$	$> 40\ M_{SUN}$
H not ignited	He not ignited	C not ignited	SNII	PPISN & PISN

Stellar evolution models

Stellar remnant:



M_{ZAMS}	$0.08\ M_\odot$	$0.8\ M_\odot$	$2\ M_\odot$	$8-10\ M_\odot$	$25\ M_\odot$	$110\ M_\odot$	$230\ M_\odot$
τ_{MS}	> 100 Gyr	~ 20 Gyr	~ 1 Gyr	~ 35 Myr	~ 7 Myr	~ 3 Myr	~ 2.5 Myr
τ_{CHeB}	–	–	~ 280 Myr	~ 3.5 Myr	~ 0.5 Myr	~ 0.32 Myr	~ 0.28 Myr
τ_{end}	–	–	~ 25 Myr	~ 0.3 Myr	~ 0.01 Myr	~ 0.004 Myr	~ 0.003 Myr


SSO
WD
NS
BH
PISN
IMBH
~8
~20
~60
~230
Stellar Mass [M_SUN]

How many?

The fraction of stars with mass between m_1 and m_2

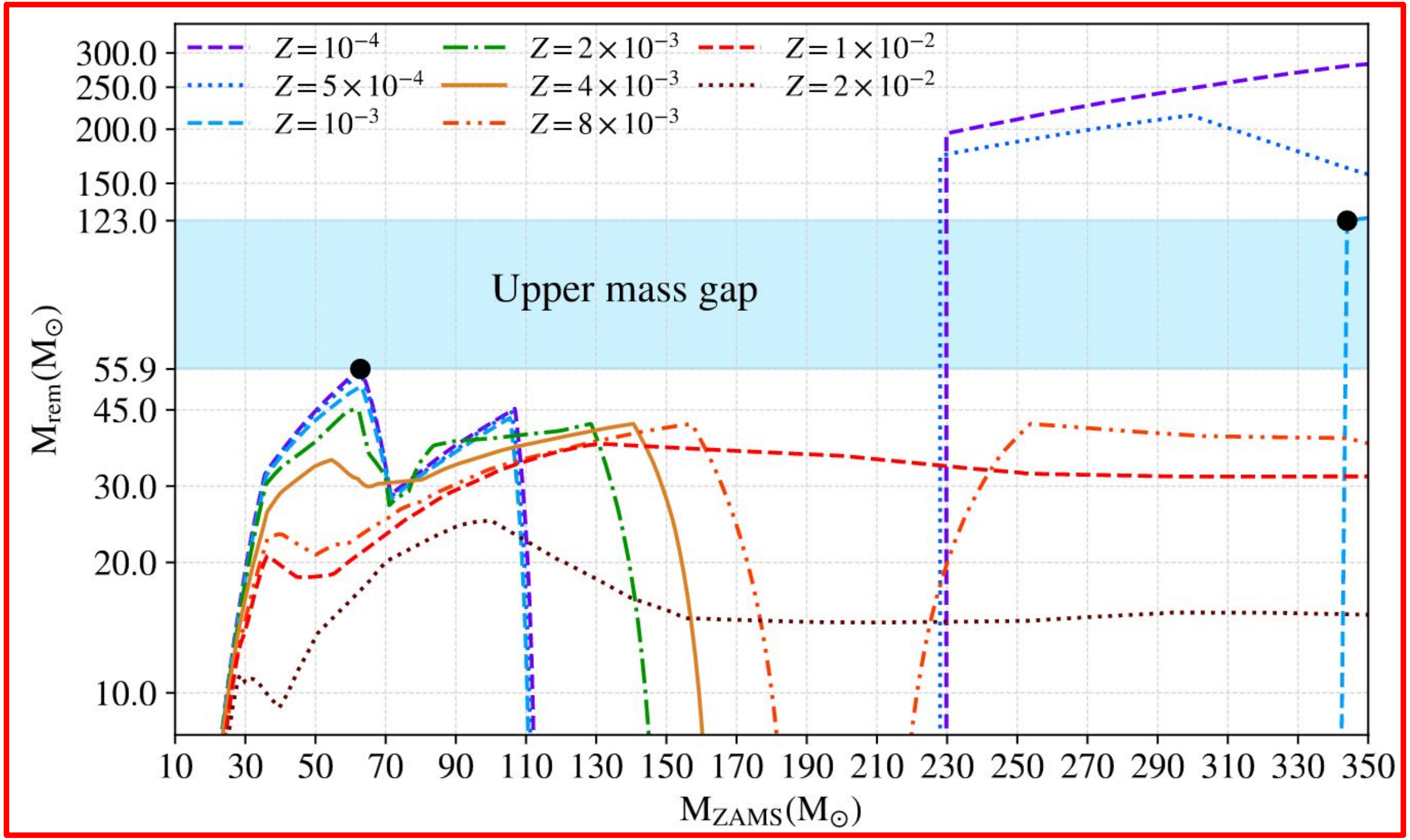
$$p(m_1, m_2) = \frac{\int_{m_1}^{m_2} \phi(m)dm}{\int_{m_{min}}^{m_{max}} \phi(m)dm}$$

Assuming a power-law initial mass function $\phi(m) \propto m^{-\alpha}$

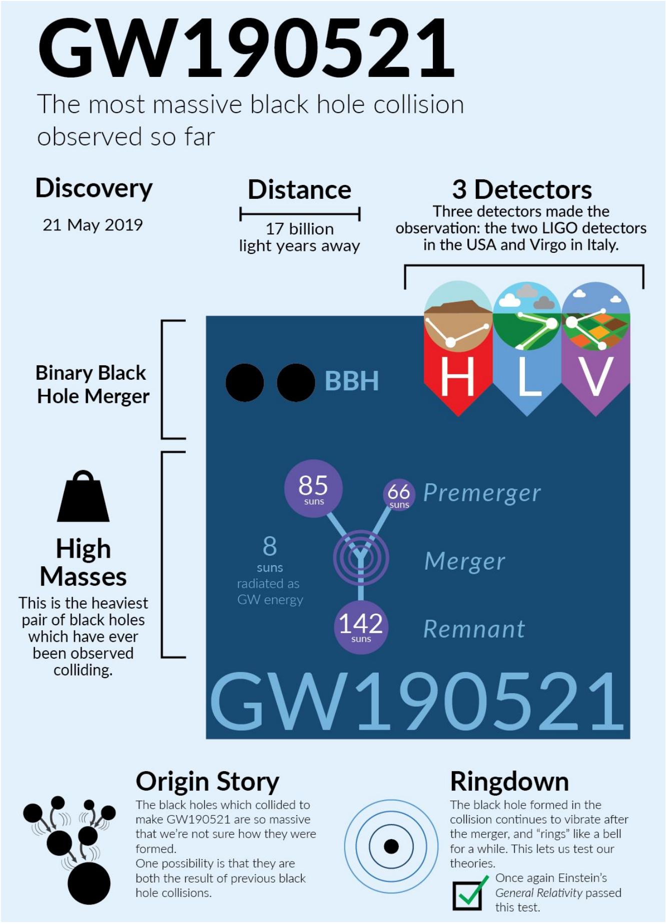
$$p(m_1, m_2) = \frac{m_1^{1-\alpha} - m_2^{1-\alpha}}{m_{max}^{1-\alpha} - m_{min}^{1-\alpha}},$$ and $\alpha = 2.35$ (Salpeter), $m_{min} = 0.08\ M_{SUN}$, $m_{max} = 300\ M_{SUN}$

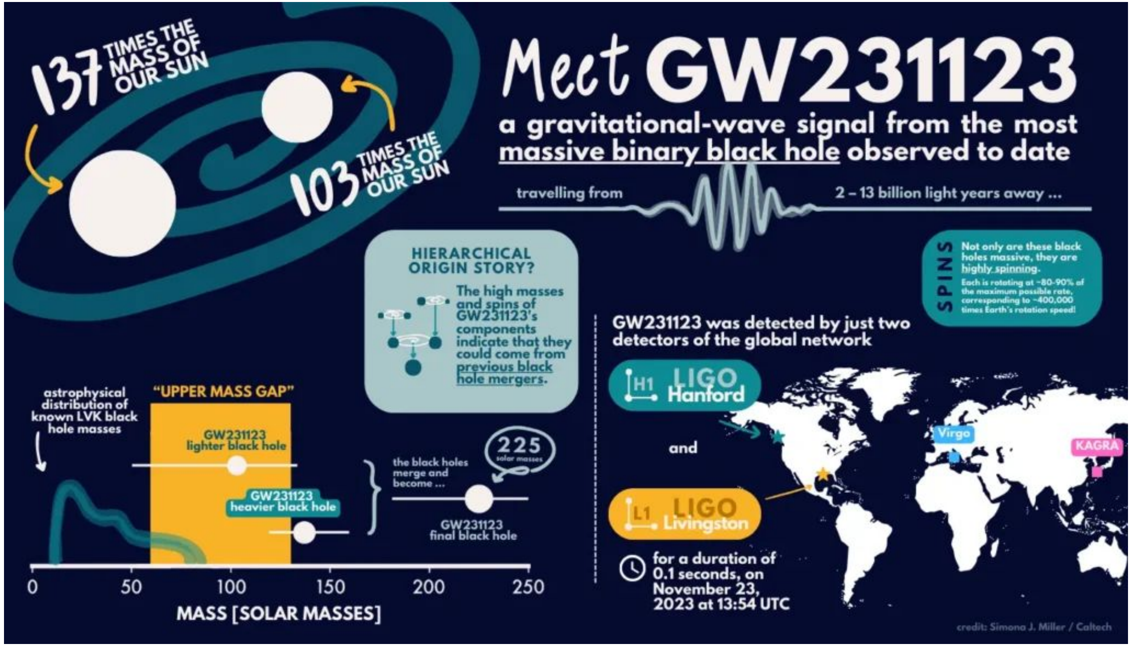
Type	> Hubble	WD	NS	BH	PISN	IMBH
Mass	$<0.8M_{SUN}$	$0.17\text{-}1.3M_{SUN}$	$1.3\text{-}2(?)M_{SUN}$	$3(?)\text{-}50M_{SUN}$	-	$>100M_{SUN}$
Occurrence rate	95.5%	4%	0.1%	~0.05%	~0.004%	~0.0006%

What are their masses?



Why some masses?



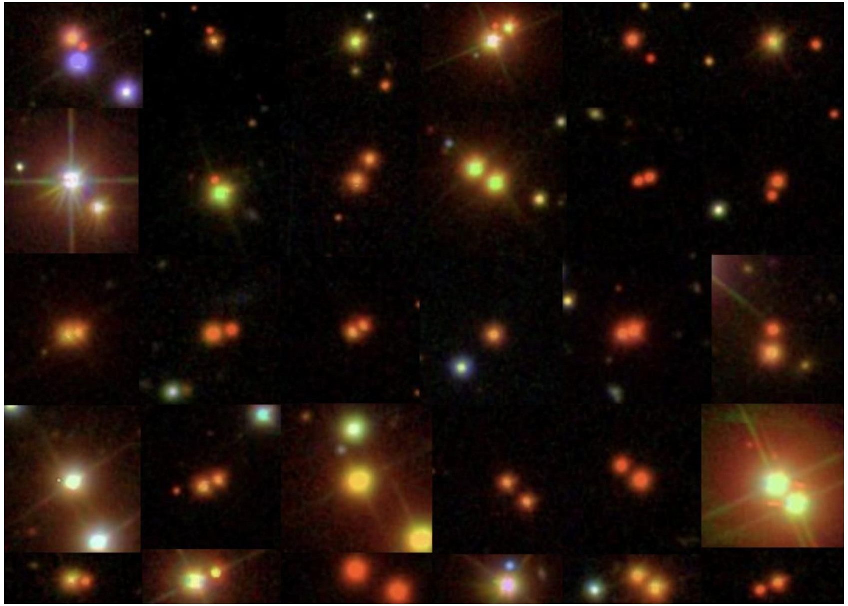


Binary evolution: the route to the formation of compact binaries

What are the formation channels of neutron star and black hole binaries observed through their GW emission?

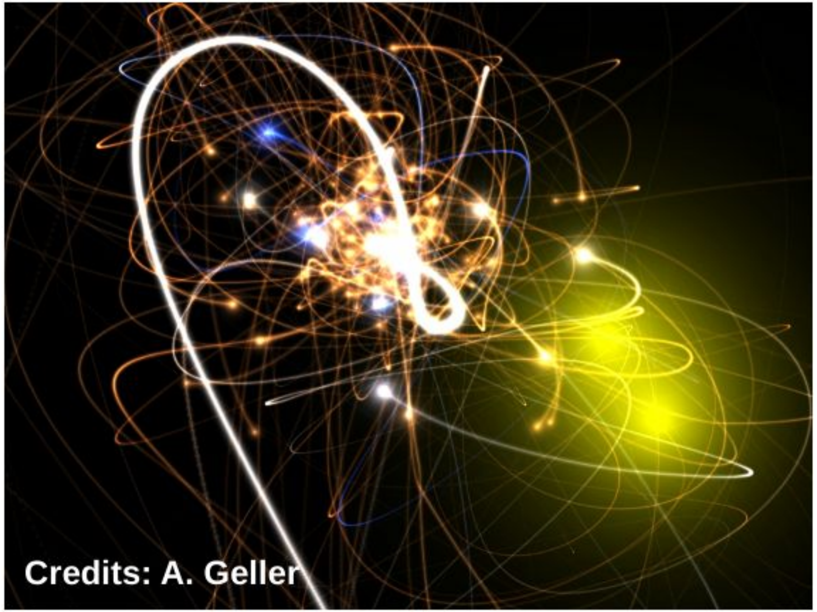
Isolated binary evolution

Two stars born together in the same cloud evolve into a merging binary solely via stellar evolution processes*



Dynamical formation

Two initially unbind BHs (or their progenitors) find each other inside a star cluster and merge via hardening*



*and GW emission

Isolated binary evolution

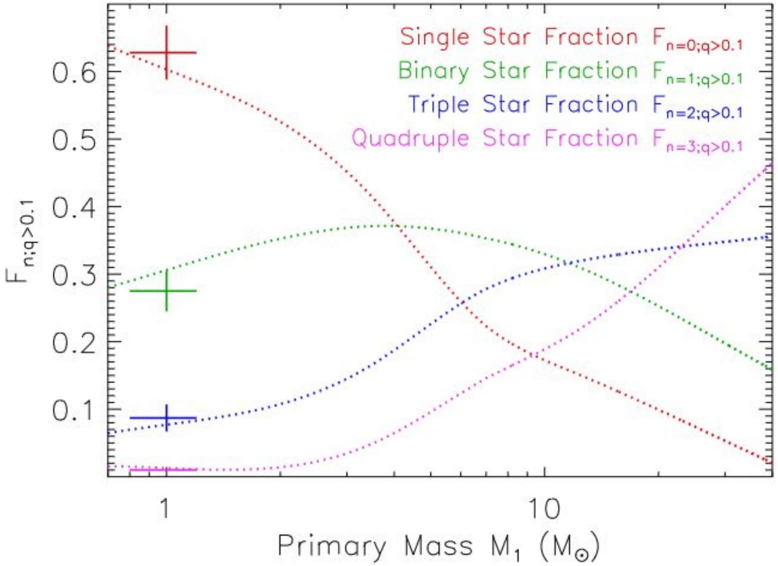
How do we model binary stars?

Primary mass

$$\phi(m) \propto m^{-\alpha}$$

$$\begin{aligned}
\alpha_0 &= +0.3 \pm 0.7, \quad 0.01 \le m/\mathrm{M}_\odot < 0.08,\\
\alpha_1 &= +1.3 \pm 0.5, \quad 0.08 \le m/\mathrm{M}_\odot < 0.50,\\
\alpha_2 &= +2.3 \pm 0.3, \quad 0.50 \le m/\mathrm{M}_\odot < 1.00,\\
\alpha_3 &= +2.3 \pm 0.7, \quad 1.00 \le m/\mathrm{M}_\odot,
\end{aligned}$$

Multiples



Isolated binary evolution

How do we model binary stars?

Primary mass

$$\phi(m) \propto m^{-\alpha}$$

Semimajor axis?

a) Logarithmic flat $\ln(R_1 + R_2) < \ln a < \ln a_{MAX}$
b) Guassian/Maxwellian?
c) Power-law?

Eccentricity?

a) Uniform
b) $p(e) \sim e^2$ (Jeans19, Ambartsumian37)
c) $p(e) \sim e^{-0.42}$

Secondary mass?

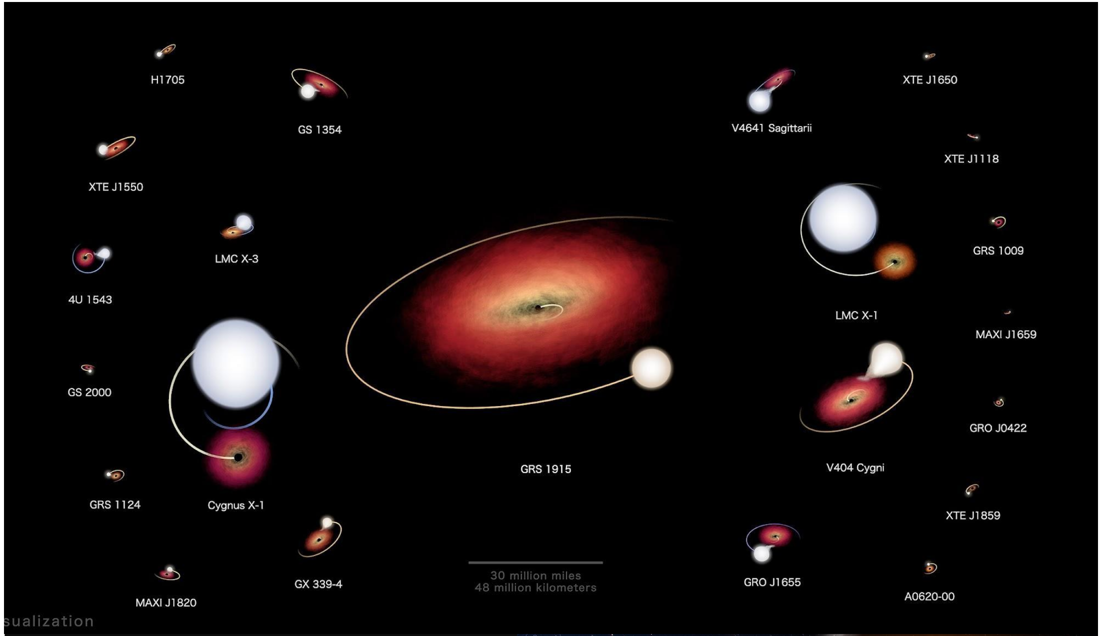
a) Random mass-ratio (q = 0.1-1) if $m_1 > 5\ M_{SUN}$
b) Random pairing if $m_1 < 5\ M_{SUN}$
c) $p(q) \sim q^{0.1}$ (Sana+12)

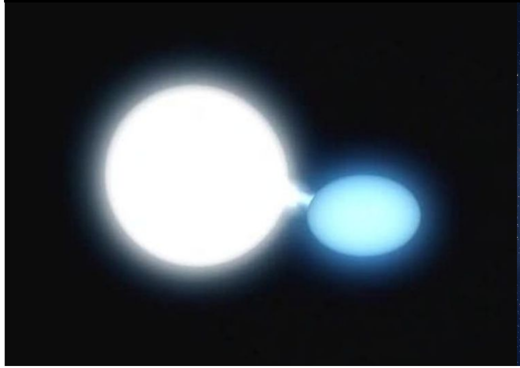
Orbital period?

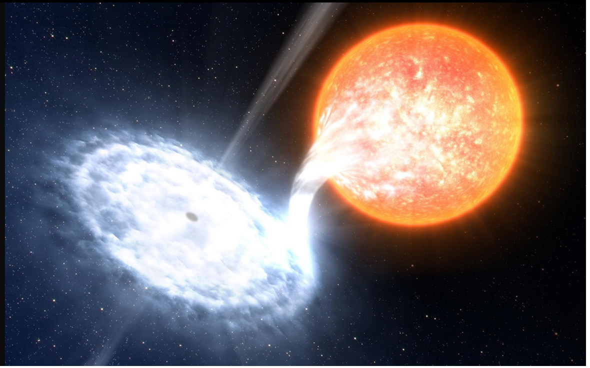
a) $p(P) \sim P^{-0.55}$ $[P = \log(T_{orb}/\mathrm{day})]$

Isolated binary evolution

1. **Wind Mass Transfer**
2. **Roche Lobe Overflow (RLO)**
3. **Common Envelope (CE)**







Isolated binary evolution
RLO and CE

The potential generated by a binary can be represented through equipotential surfaces. Points of intersection of equipotential surfaces mark points of equilibrium (stable/unstable)

L1 is the minimum contact equipotential surface, where the gravitational pull of m1 and m2 balance each other.

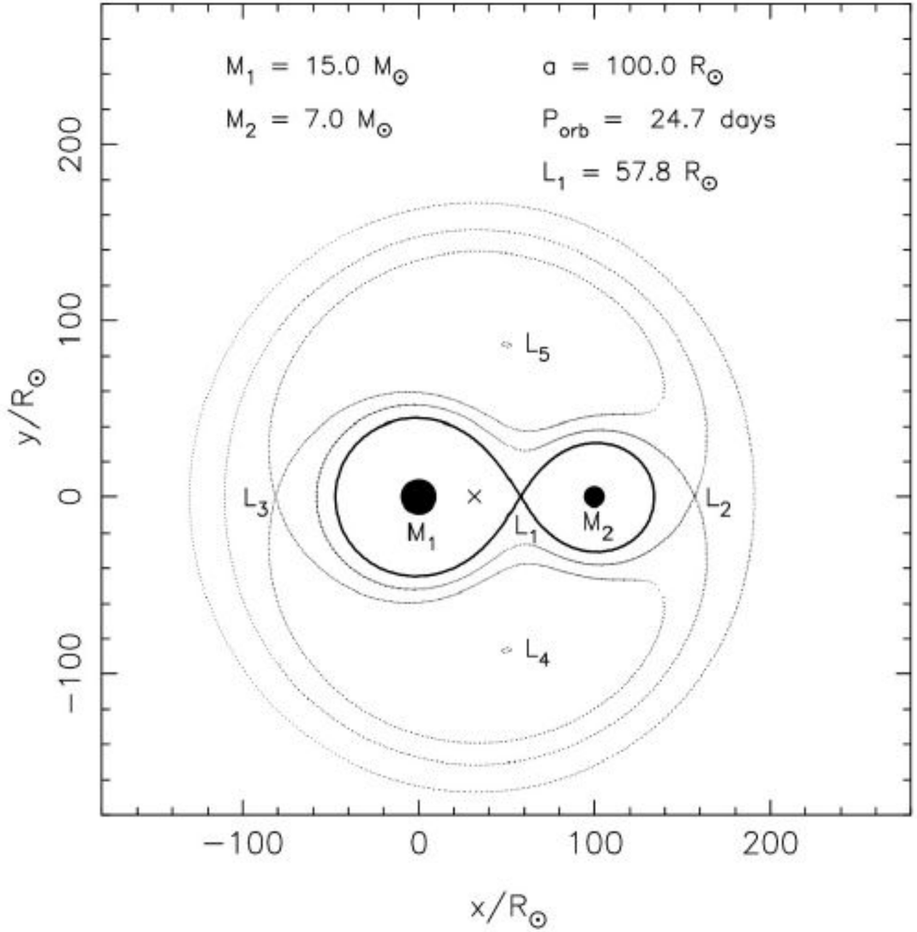
The two region embodying the stars are called Roche lobes.

If a star fills its Roche lobe, matter flows without energy change through L1, causing mass transfer.

This generally happens when

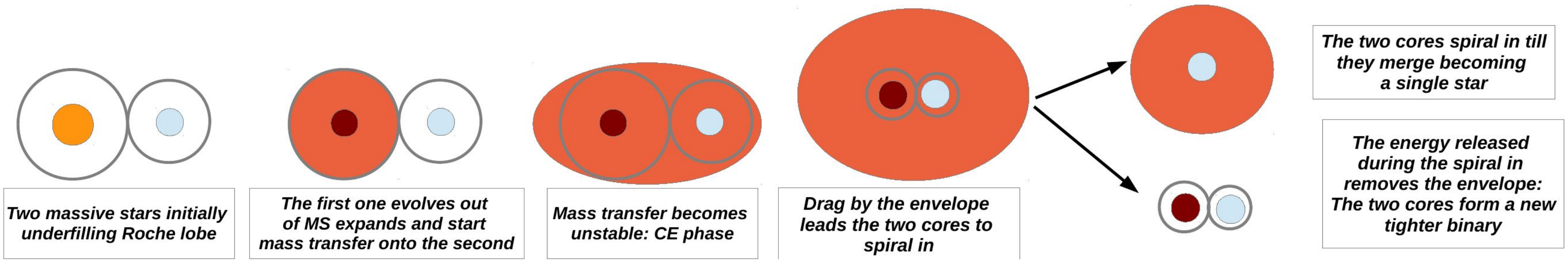
$$\frac{r_1}{a} = \frac{0.49\, q^{2/3}}{0.6\, q^{2/3} + \ln\left(1 + q^{1/3}\right)}$$

Eggleton 1983



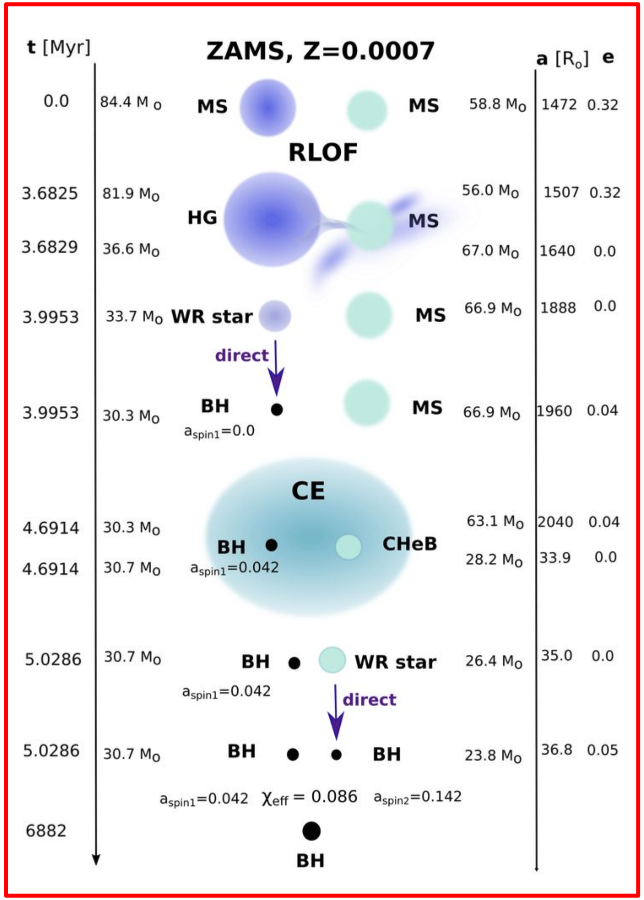
Isolated binary evolution
RLO and CE

- Donor loses mass and its structure adjusts trying to reach a new equilibrium
- If the timescale over which RL change is > than that for hydrostatic & thermal equilibrium: stable mass transfer (SMT)
- Otherwise: common envelope (CE)

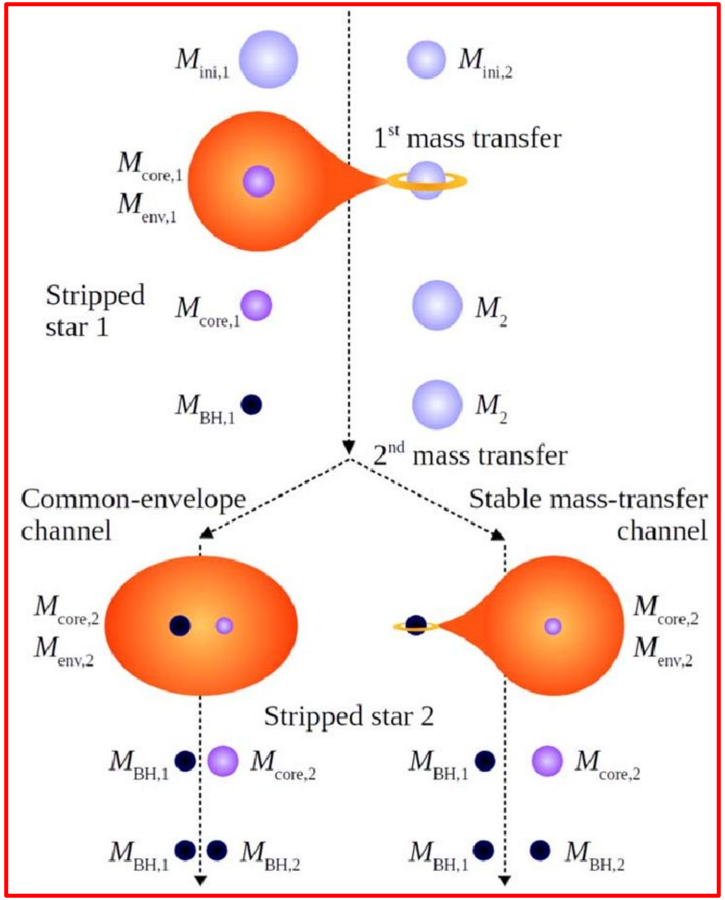


Isolated binary evolution

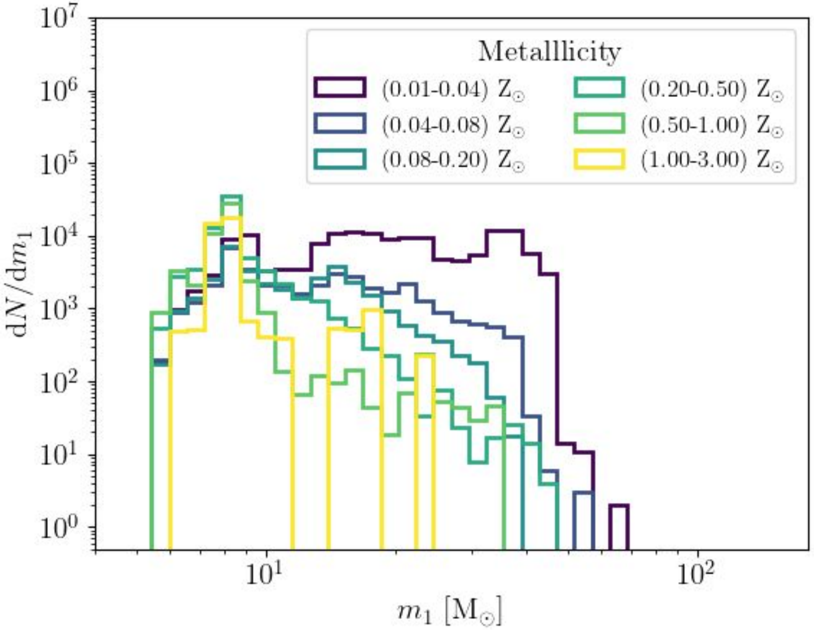
Belczynski+(2020)



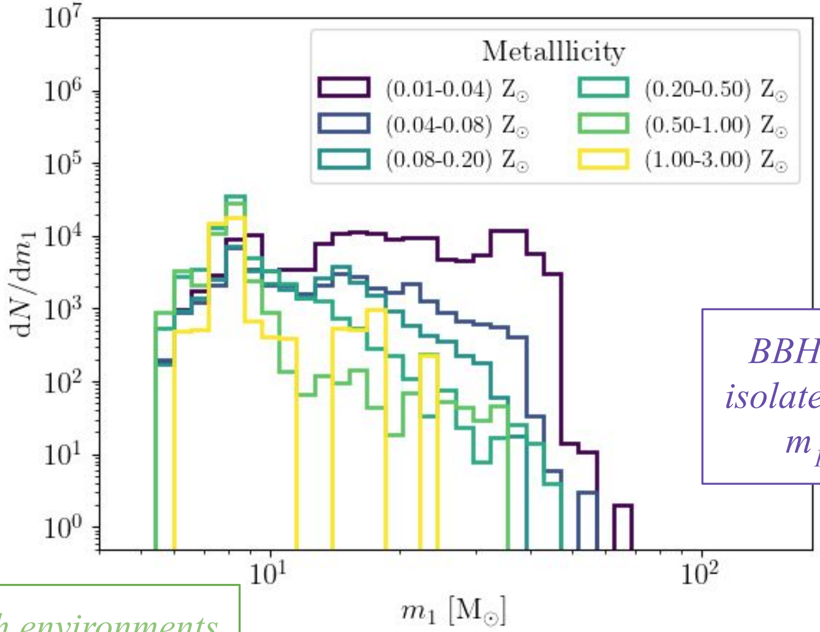
Schneider+(2023)



Isolated binary evolution



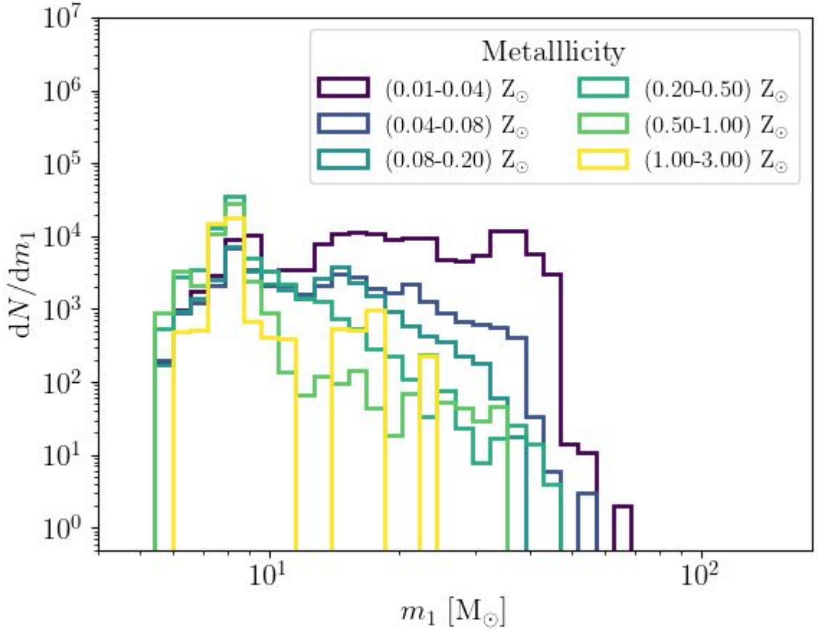
Isolated binary evolution



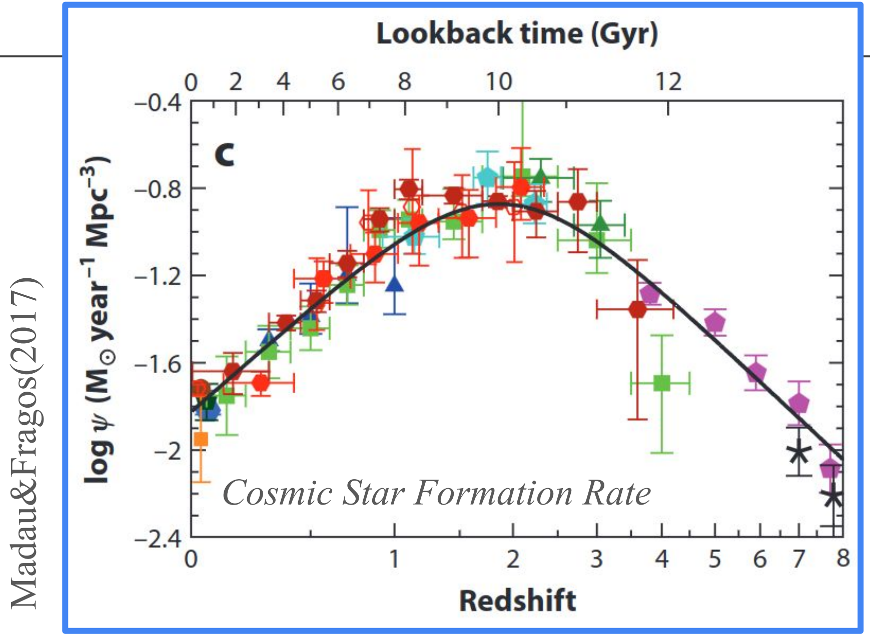
BBH mergers from isolated binaries have $m_1 < 70\ M_{SUN}$

Metal rich environments host lighter BHs

Isolated binary evolution



This shows us just what's the BBH population at different metallicity, what about the cosmic evolution?



$$\frac{dP}{d\log_{10}(Z)}(z) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(\log_{10}(Z)-\mu(z))^2}{2\sigma^2}\right)$$

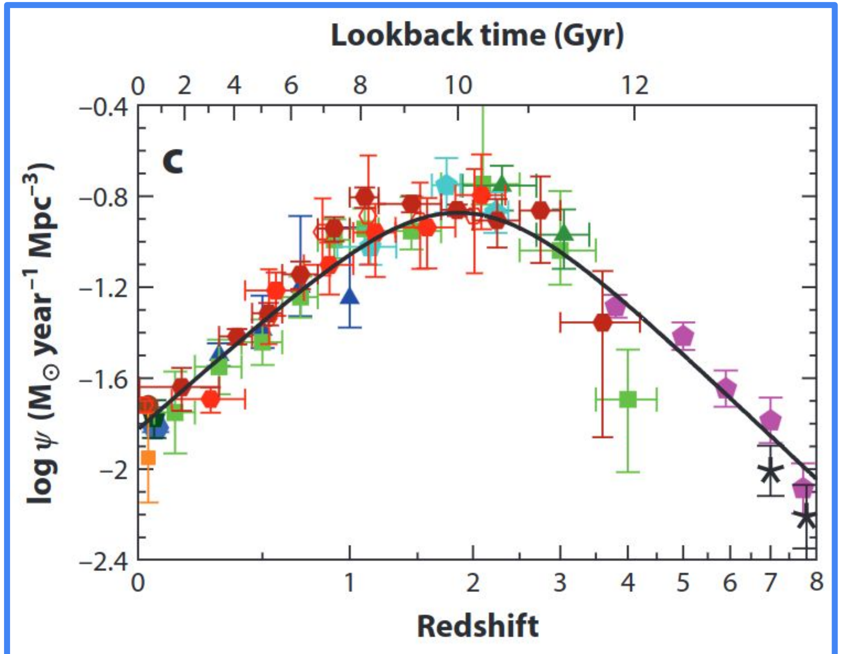
$$\mu = \log_{10}\left(\bar{Z}(z)\right) - \frac{\ln(10)}{2}\sigma^2$$

$$\bar{Z}(z) = Z_\odot \cdot 10^{0.153-0.074\cdot z^{1.34}}.$$

Isolated binary evolution

Merger rate

$$\mathcal{R}(z) = \int_{z_{\max}}^{z} \left[\int_{Z_{\min}}^{Z_{\max}} \mathrm{SFRD}(z', Z)\, \mathcal{F}(z', z, Z)\, \mathrm{d}Z \right] \frac{\mathrm{d}t(z')}{\mathrm{d}z'} \mathrm{d}z'$$

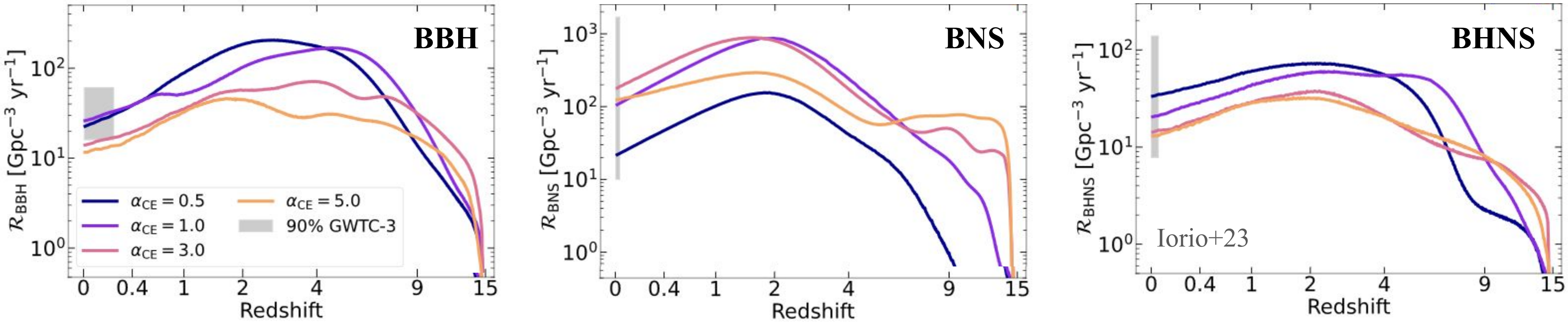


$$\frac{\mathrm{d}t(z')}{\mathrm{d}z'} = [H_0(1+z')]^{-1}\,[(1+z')^3\Omega_M + \Omega_\Lambda]^{-1/2}$$

$$\mathcal{F}(z', z, Z) = \frac{1}{M_{\mathrm{pop}}(Z)} \frac{\mathrm{d}\mathcal{N}(z', z, Z)}{\mathrm{d}t(z)}$$

This term depends on the stellar evolution model, binary distribution, cosmic metallicity distribution …

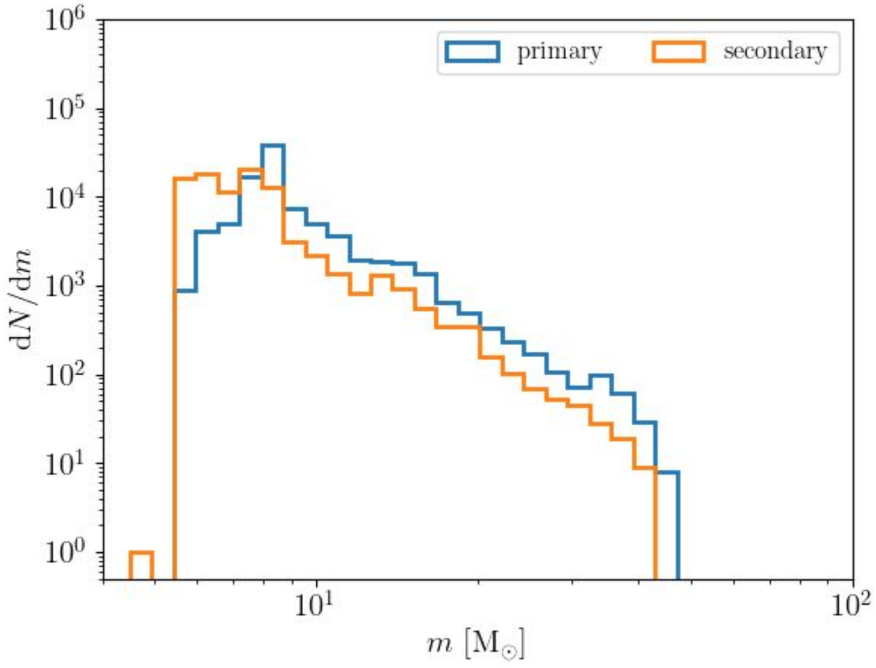
Isolated binary evolution

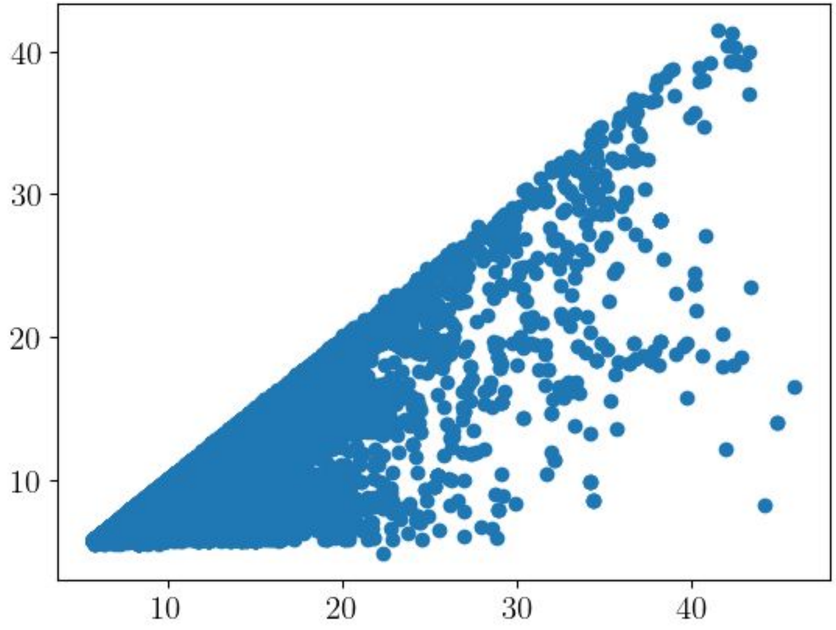


Integrating R(z) over the cosmological volume returns the number of mergers per year [O(10^5/yr) for typical models]

Isolated binary evolution

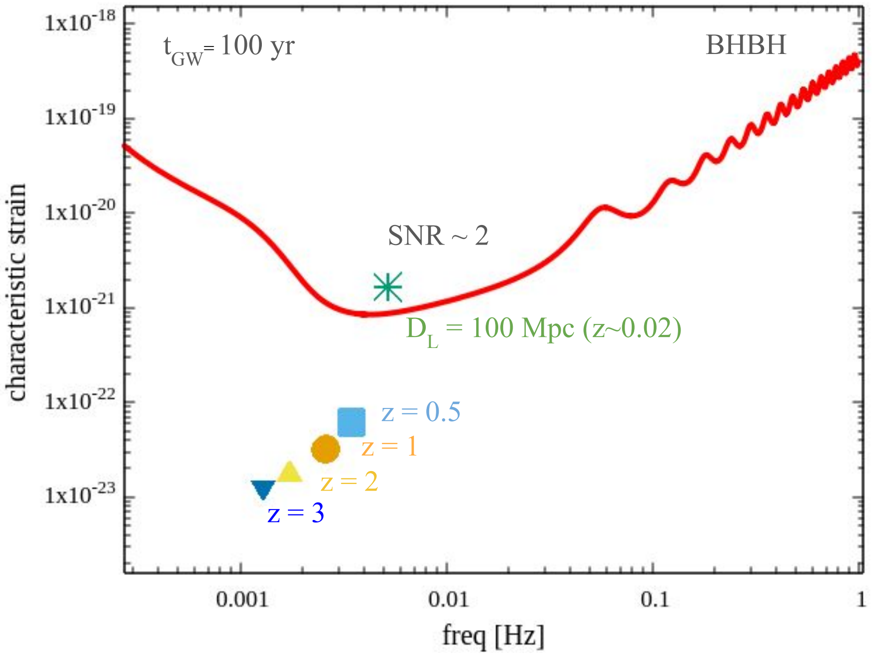
Integrating R(z) over the cosmological volume returns the number of mergers per year [O(10^5/yr) for typical models]



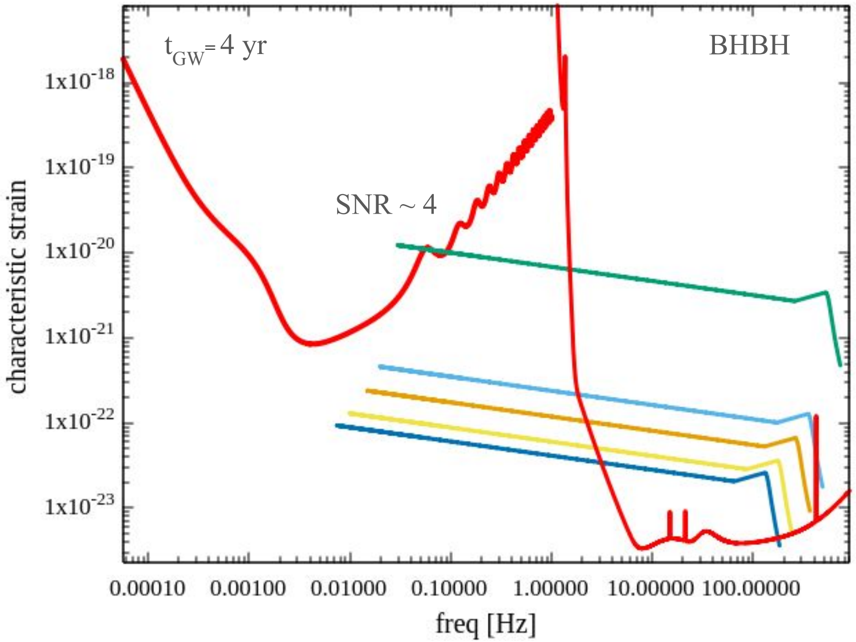


Isolated binary evolution

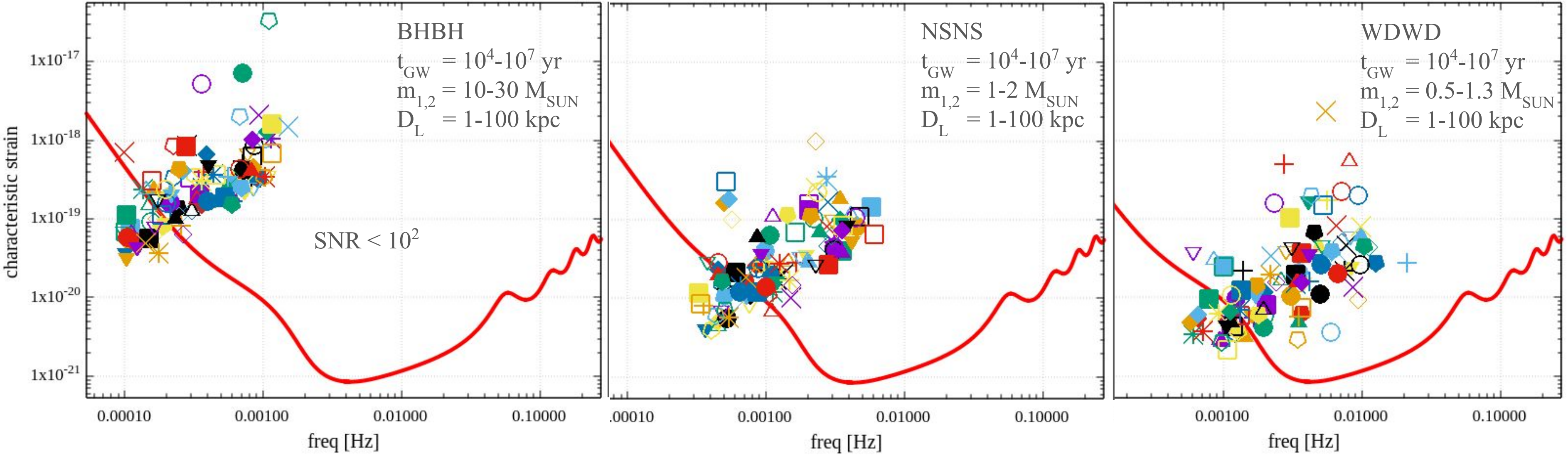
$$h_c \propto D_L^{-1}$$



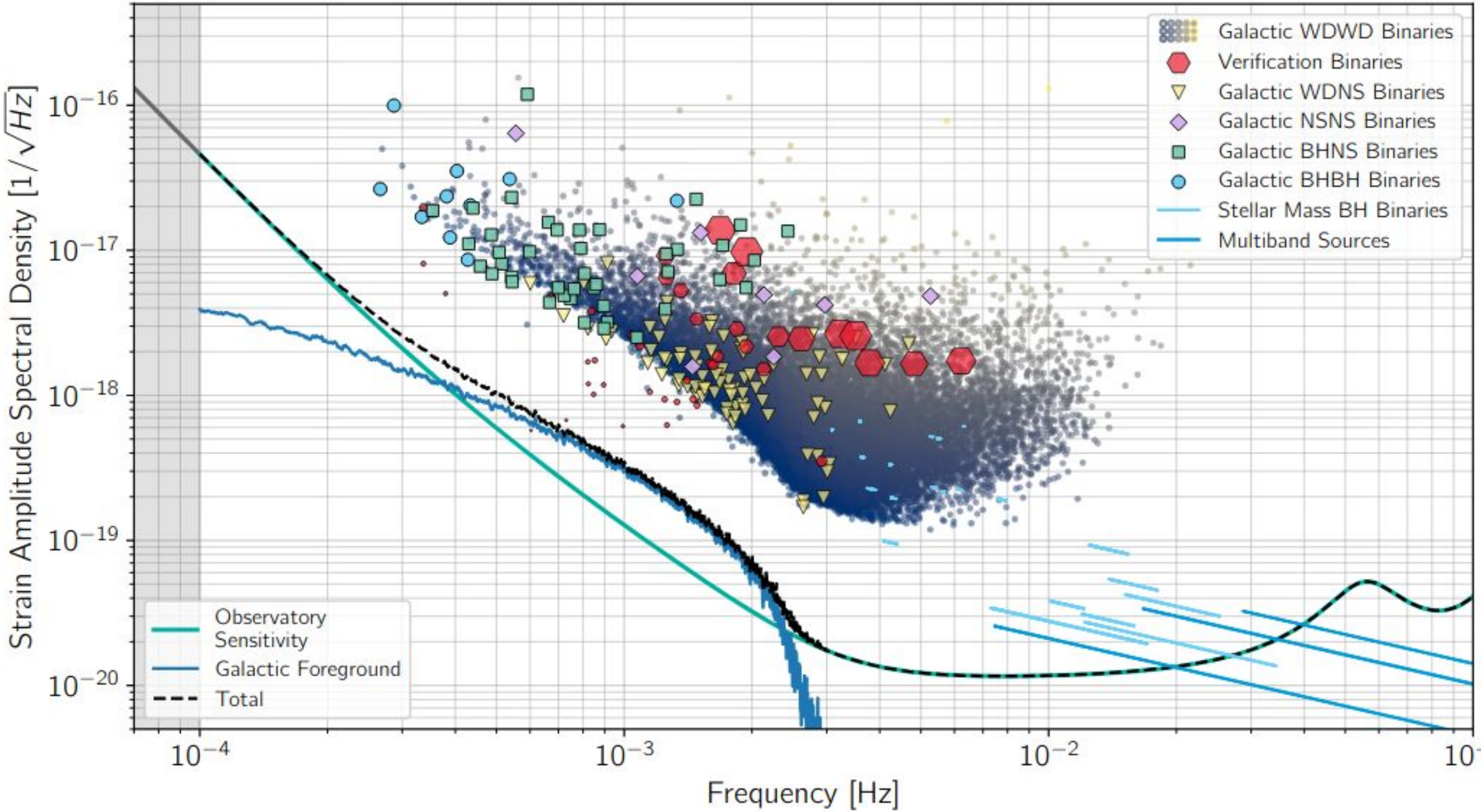
$$h_c \propto f^{-1/6} D_L^{-1}$$



Isolated binary evolution

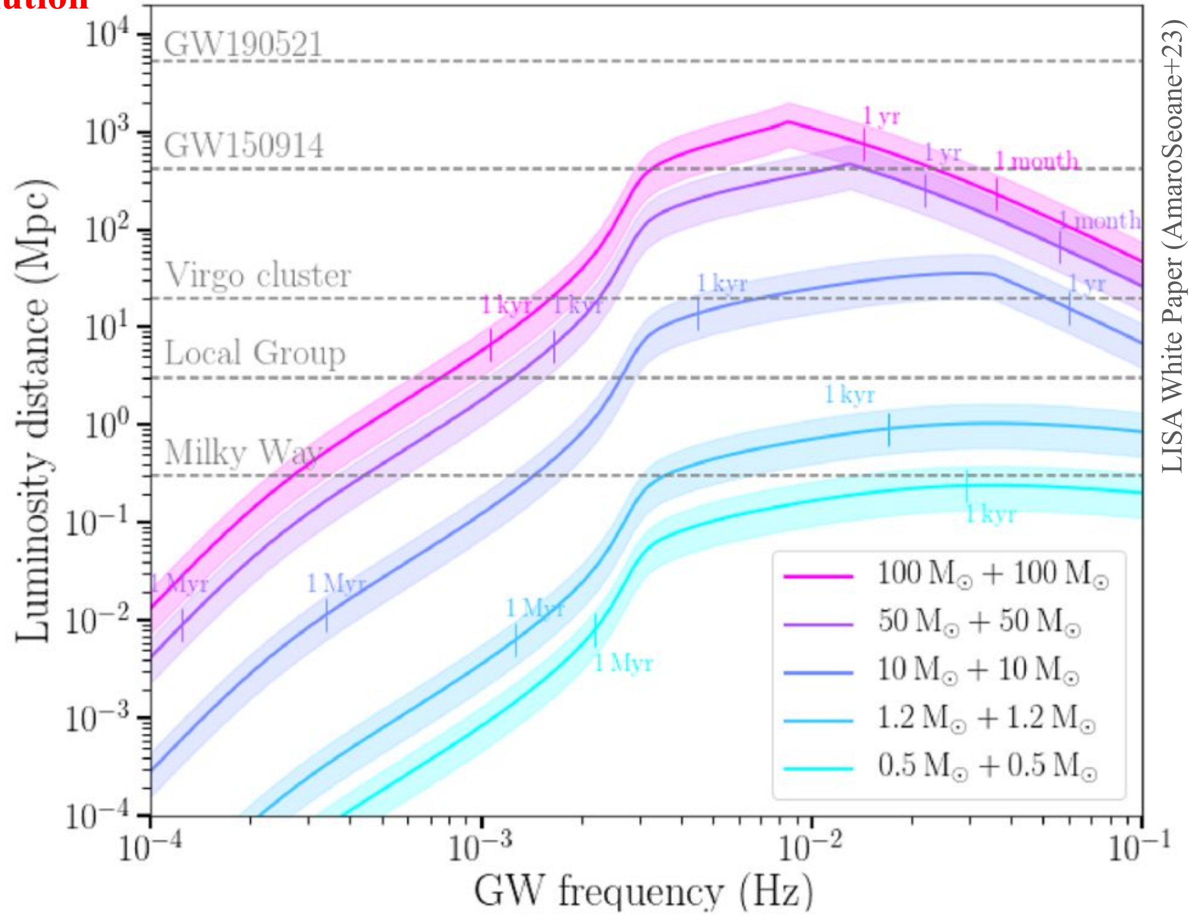


Isolated binary evolution



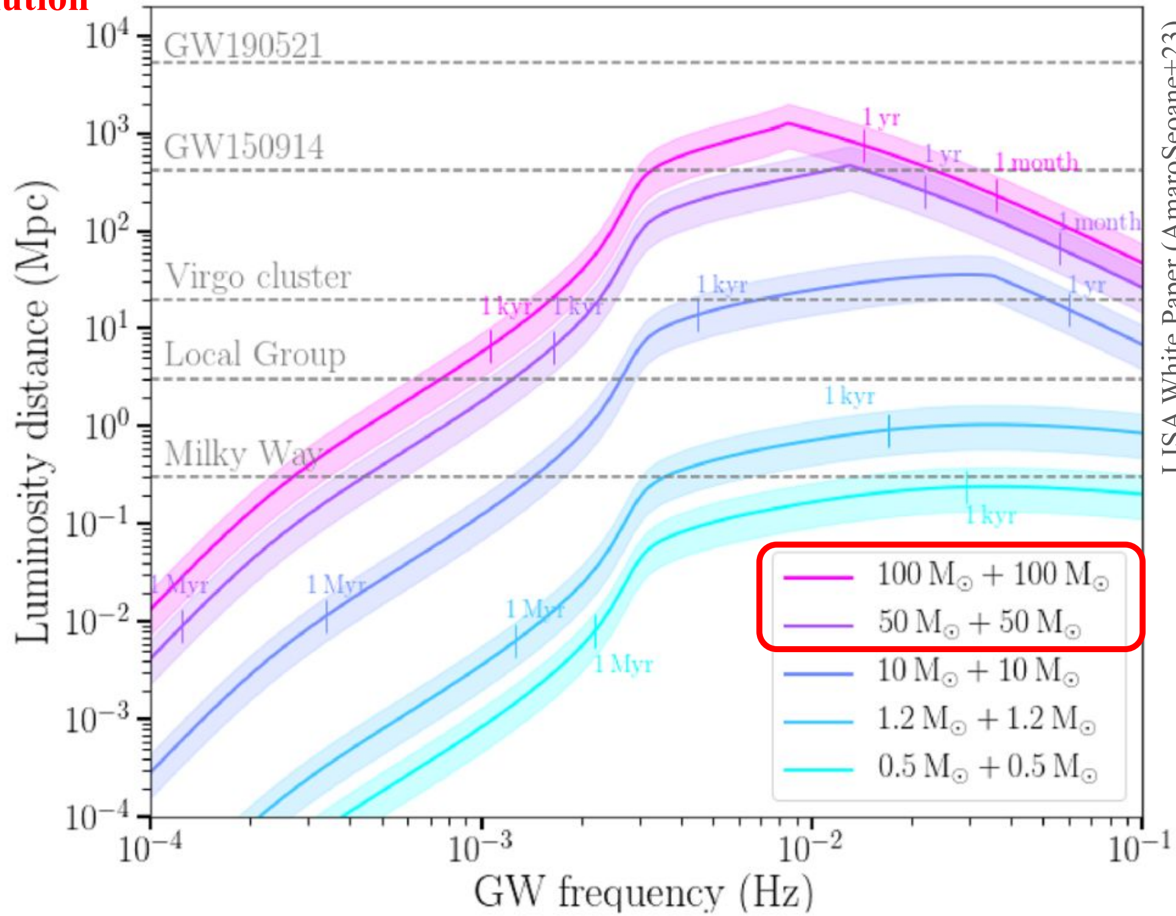
LISA Definition Study Report (Colpi+24)

Isolated binary evolution



LISA White Paper (AmaroSeoane+23)

Isolated binary evolution



LISA White Paper (AmaroSeoane+23)

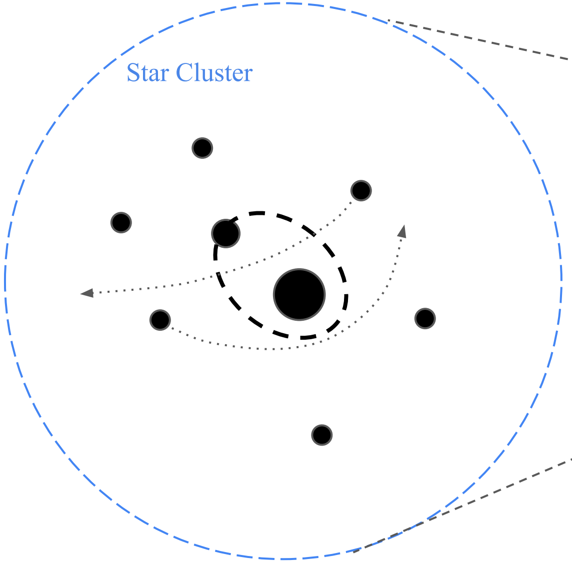
Dynamical formation

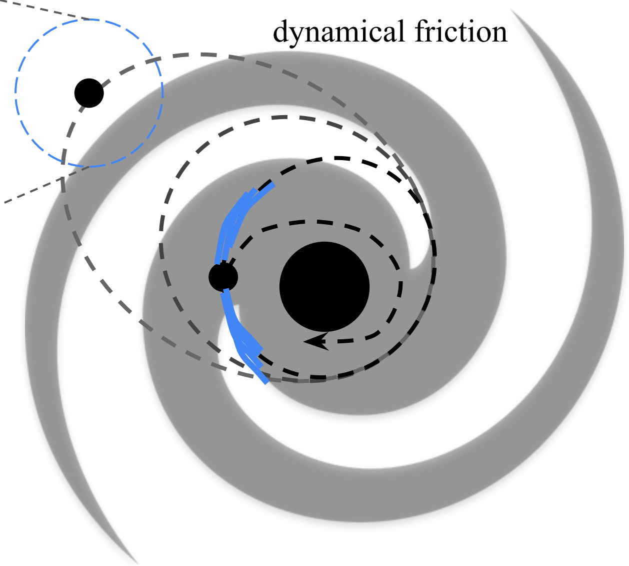
BBHs

- Three-body interactions and binary–single/binary–binary (Downing+09, Banerjee+10, Ziosi et al 2014, Rodriguez+16,17,19, Askar+17, Banerjee17,19,20, Fragione+18, Antonini+18,19, Di Carlo+19, Arca Sedda et al 2024, Paiella+25, …)

Heavy IMRIs: IMBH + SMBH (Amaro-Seoane+22)

- Minor galaxy mergers (?)
- Nuclear clusters (Ebisuzaki+01, Portegies-Zwart+06, Arca Sedda&Gualandris18, Arca Sedda&Capuzzo-Dolcetta19)





Light IMRIs: IMBH + compact object (Amaro-Seoane+07)

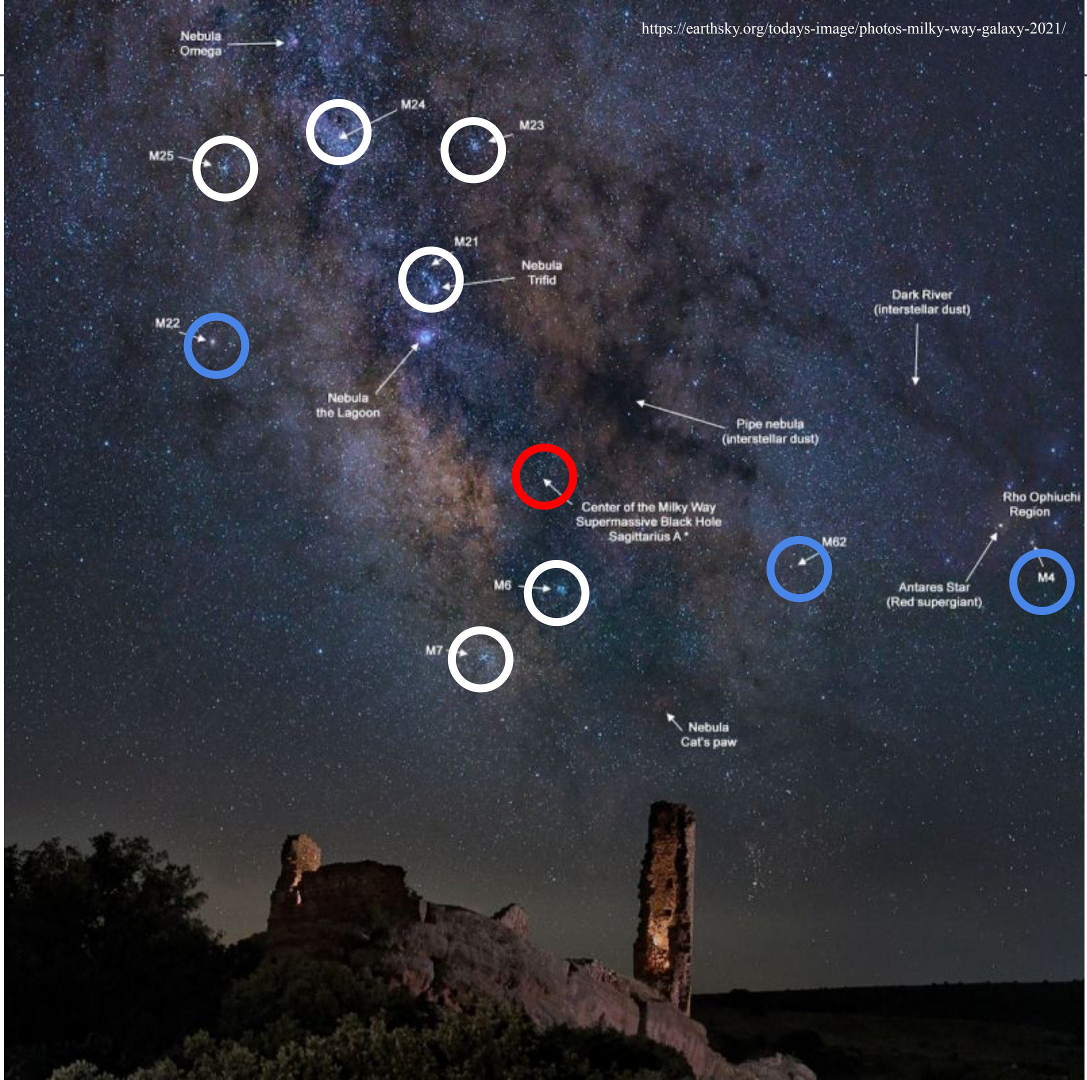
- IMBH mass ?
- compact object: BHs (2.7%), WDs (16%), NSs (0.2%)

(Arca Sedda+19, see also Konstantinidis+13, MacLeod+16)

Dynamical formation

Galaxy's zoology - many star cluster flavours:

- Open Clusters
- Young Massive Clusters
- Globular Clusters
- Nuclear Clusters



Open Clusters (OCs):

- $N > 1{,}100$ known OCs in MW (likely $> 10^{4\text{-}5}$)

- Cluster properties

  $M_{OC} \sim 10^2\text{-}10^4\ M_{SUN}$

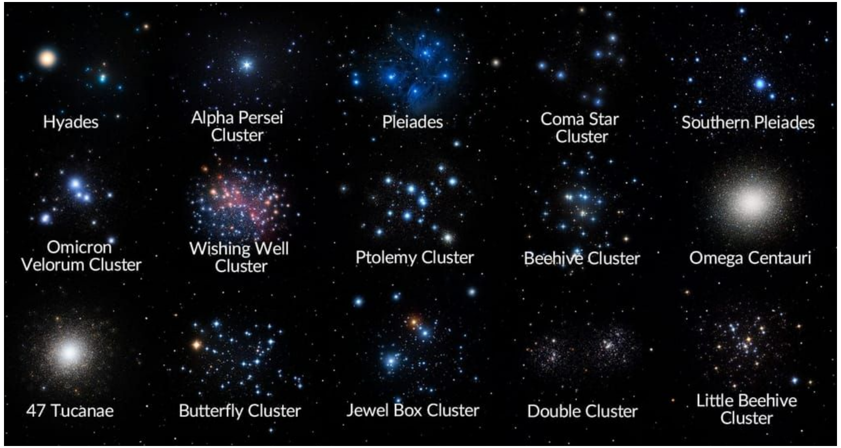
  $R_{HM,OC} \sim 2$ pc

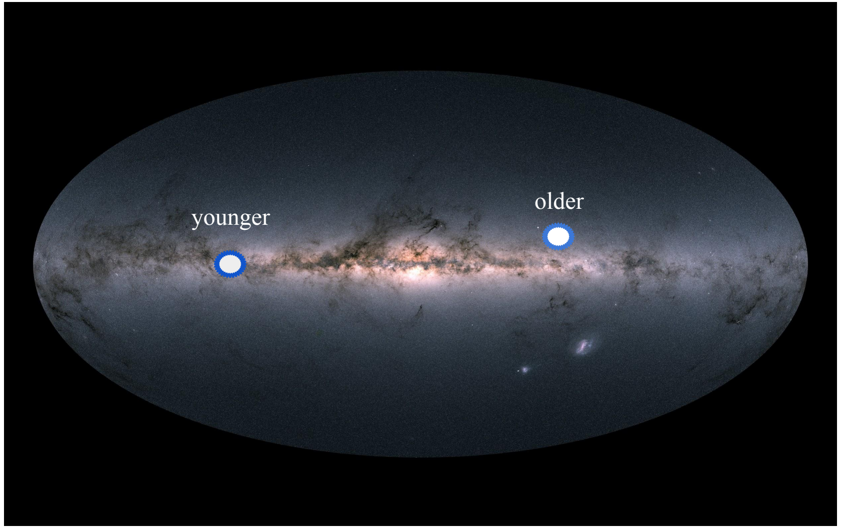
  $\sigma_{OC} \sim 0.1\text{-}5$ km/s

- Age/Lifetime: 0.3-1 Gyr
  Cluster teenage mortality

- Form in disc of spiral and irregular galaxies

- Binarity ~ 40-100%
  (larger the star mass larger the binarity)





Refs: Salaris+2005, Piskunov+2007, Sana+2012, Netopil+2017,
Moe & Di Stefano 2017, Shukirgaliyev+2018, Cantat-Gaudin+2018

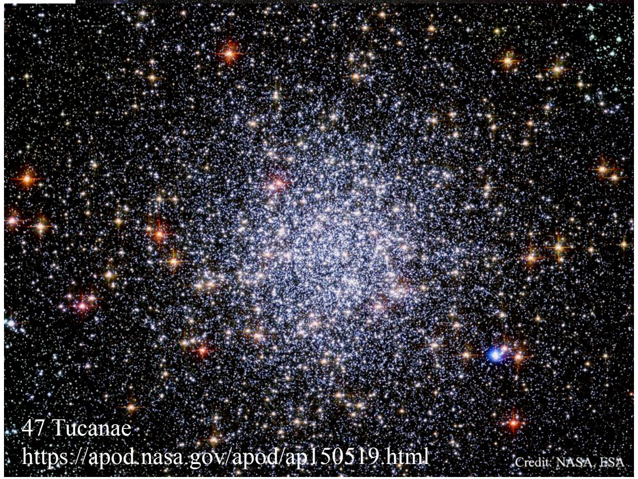
Globular Clusters (GCs):

- $N \sim 180$ known GCs in MW (likely > 200)
- $M_{GCS} \sim 0.001\text{-}0.01\, M_*$; $M_{GCS} \sim 6 \times 10^{-5}\, M_{HALO}$
- In-situ (70%) / accreted clusters (30%)
- Multiple populations (light element abundance variation)
- Cluster properties

  $M_{GC} \sim 10^4\text{-}10^6\, M_{SUN}$

  $R_{HM,GC} \sim 1\text{-}5$ pc

  $\sigma_{GC} \sim 4\text{-}6$ km/s
- Age/Lifetime: 10 Gyr
- Binarity < 20-40% (q>0.5) (.. but they're old!)
- Rotation, spherical/elliptic, IMBHs …



Refs: Harris10, Harris+14, Forbes+10, Milone+15, Lucatello+15, Gratton+19 …

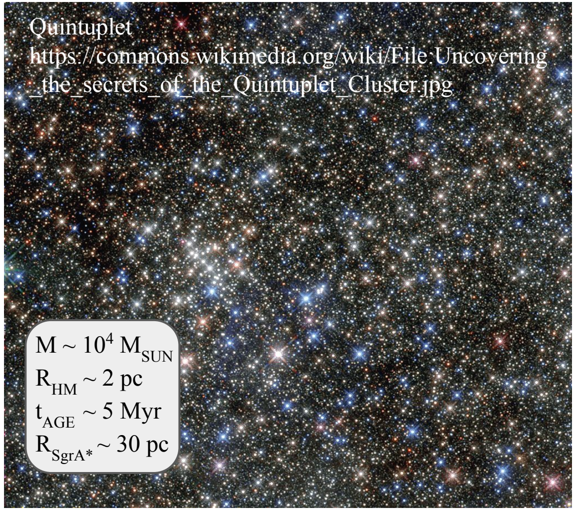
Young Massive Clusters (YCs):

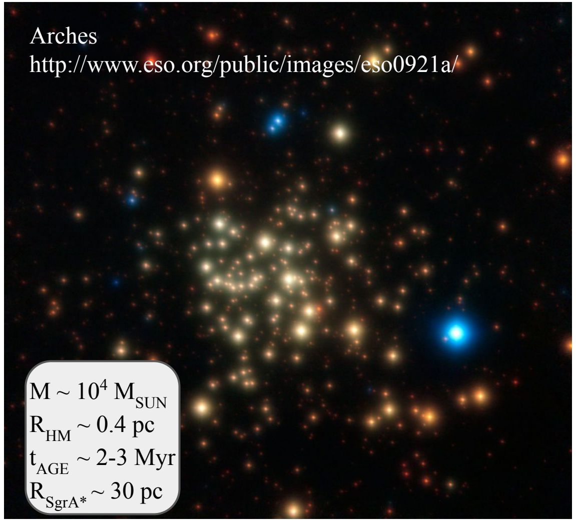
- $N \sim 300$ in MW, LMC, SMC
- GC progenitors?
- gas
- Galactic Centre
- Cluster properties

  $M_{YC} \sim 10^2\text{-}10^5\ M_{SUN}$

  $R_{HM,YC} \sim 1\text{-}4$ pc

  $\sigma_{YC} \sim 0.3\text{-}5$ km/s
- Age/Lifetime: ~ 0.1-2 Gyr
- Binarity ~ 30 - 50%





Refs: Figer+99, Portegies-Zwart+10, Gatto+20, Davies+17 …

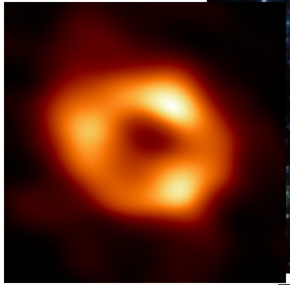
Nuclear star clusters (NC):

- $N \sim 1$ per galaxy
- complex SFH
- Galaxy centre: w/o SMBH (?)
- Scaling relation with host galaxy
- Formation scenarios: in-situ or dry merger
- Cluster properties

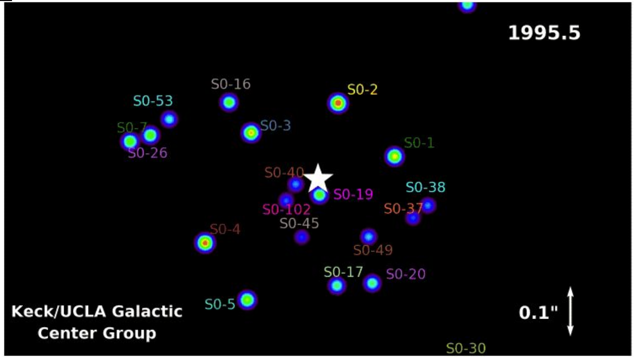
  $M_{NC} \sim 10^5\text{-}10^8\, M_{SUN}$

  $R_{HM,NC} \sim 1\text{-}4$ pc

  $\sigma_{NC} \sim 15\text{-}10^3$ km/s
- Age/Lifetime: ~ 1-10 Gyr
- Binarity ~ 1-5%?

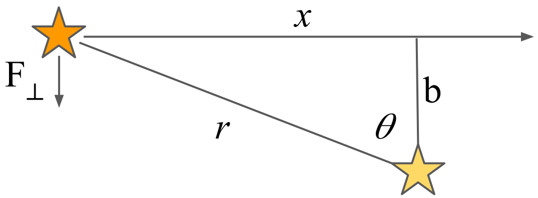


EHT collaboration
https://www.eso.org/public/images/eso2208-eht-mwa/



Refs: Figer+99, Portegies-Zwart+10, Gatto+20, Davies+17 …

Star cluster evolution - #1



A star passes within a distance b from another star, which exerts a force

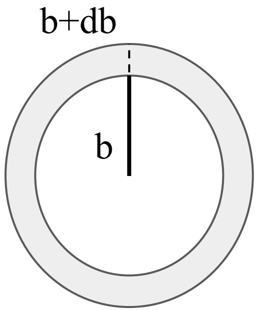
$F_\perp = Gm^2 / (b^2 + x^2) \cos\theta = Gm^2\, b\, (b^2 + x^2)^{-3/2} = Gm^2/b^2\, [1+(vt/b)^2]^{-3/2}$

The "encounter" changes the star velocity vector by $\delta\mathbf{v}$:

$|\delta\mathbf{v}| = \delta v \sim \delta v_\perp = 1/m \int F_\perp(t)\, dt = Gm/b^2 \int[1+(vt/b)^2]^{-3/2}\, dt = 2Gm/bv$

a 90-degrees deflection happens if $\delta v/v \sim 1 \rightarrow b_{90} = 2Gm/v^2$

For a cluster with N stars and size R, the surface density is roughly given by $N/\pi R^2$,
thus our star will undergo a number of encounters with impact parameter between b and b+db



$\delta n \sim N/\pi R^2\, 2\pi b\, db$

Interactions are random, thus $|\delta\mathbf{v}| \sim 0$, but the mean-square velocity variation

$<\delta v>^2 = \Sigma\, \delta v^2 \sim \delta v^2\, \delta n = (2Gm/bv)^2\, (2N/R^2)\, b\, db$

thus the average mean-square velocity variation per crossing will be

$\Delta v^2 = \int \Sigma\, \delta v^2 \sim 8N\, (2Gm/bv)^2 \ln\Lambda$ where $\Lambda = b_{max} / b_{min} \sim R / b_{90}$ is the so-called Couloumb logarithm

Star cluster evolution - #2

The "encounters" cause a diffusion of the star velocity, differently from the steady velocity variation impinged by the overall cluster mass distribution
The typical velocity of a star moving in a cluster with N stars and size R is roughly

$v^2 \sim GNm/R$

thus the number of crossing needed for the star to vary its velocity by a factor $\Delta v^2 / v^2$ will be

$n_{relax} \sim v^2 / \Delta v^2 = N / (8 \ln \Lambda)$

The crossing time, i.e. the time needed for the star to cross the cluster, is roughly $t_{cross} = R/v$, thus the timescale over which encounters will change the initial velocity of our target star is the **relaxation time**

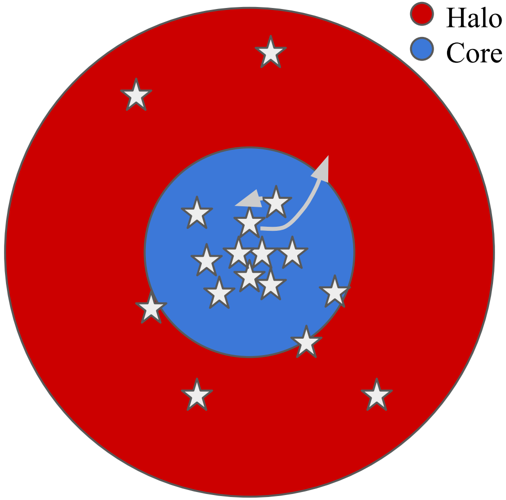
$t_{relax} = t_{cross}\, n_{relax} \sim 0.1\, N / (\ln N)\, t_{cross} \sim 0.1N/(\ln N)\, R^{3/2} (GNm)^{-1/2}$,

note that $\Lambda = b_{max} / b_{min} \sim R / b_{90} = R / (2Gm/v^2) = N$

Clusters with $t_{relax} < t_{age}$ are "collisional"

cluster type	**N**	**R (pc)**	**t_{cross}(Myr)**	**t_{relax}/t_{cross}**	**t_{age} (Gyr)**
open clusters	10^2	2	4.2	2	0.01-1
young clusters	10^4	2	0.4	10^2	0.1-1
globular clusters	10^5	2	0.1	$8x10^2$	10
nuclear clusters	10^7	2	0.01	$6x10^4$	10

Star cluster evolution - #3



The cluster can be divided into

halo - low density, long relaxation time
core - high density, short relaxation time

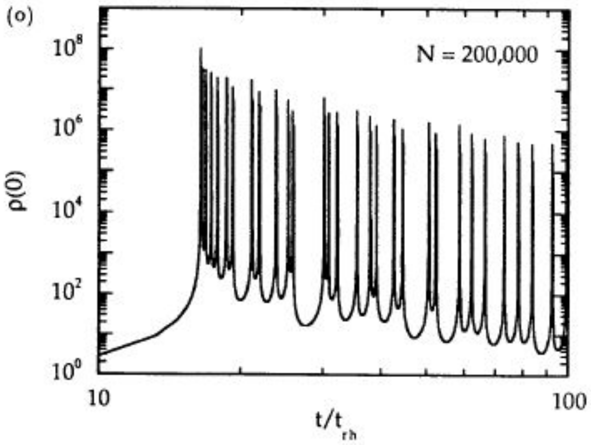
Stars diffuse from the core outward:
some stars gain energy → mass loss (evaporation)
some stars lose energy → cluster contracts

We can treat the core as a gaseous medium with negative heat capacity

The situation proceeds "indefinitely" up to the core-collapse = zero radius, infinite density (see also gravothermal catastrophe)

$t_{CC} \sim 15 t_{relax}(r = 0)$ (e.g. Cohn 1980)

Note: for a GC $t_{relax} \sim 1$ Gyr → t_{CC} > Hubble time (?)



What happens afterwards?
For N > 8,000 stars the core develope *gravothermal oscillations*

Star cluster evolution - #4

All the things we discussed so far rely upon an idealised single mass star cluster, but real clusters are characterised by a broad mass spectrum… so?
Let's consider a still simplified case

a) two populations of stars with masses either $m_2 >> m_1$
b) $\sum m_2 << \sum m_1$

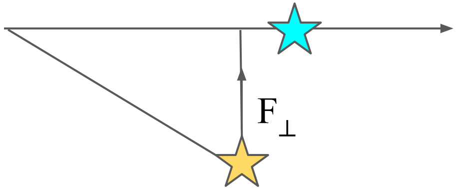
Thus, the mean kinetic energy of the population is roughly $3m_1\sigma^2$

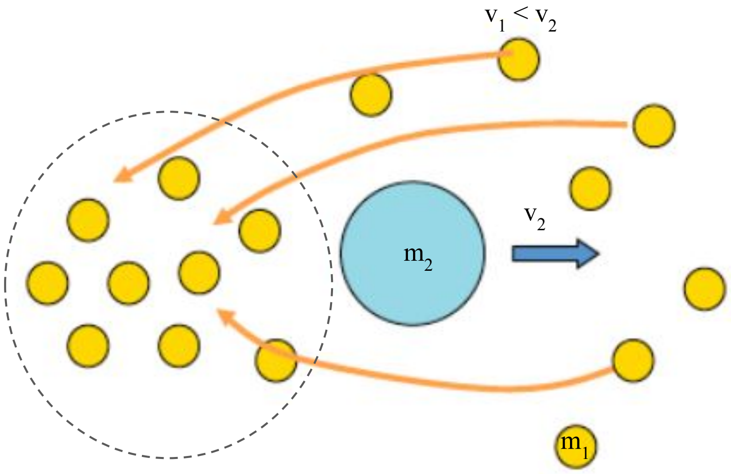
Since $m_2 >> m_1$, diffusion is dominated by one main term,
called ***dynamical friction*** (Chandrasekhar 1943):

$$d\mathbf{v}/dt = -16\pi G^2 m_2 m_1 \ln\Lambda\, \mathbf{v}_2/v_2^3 \int_0^{v2} dv_1\, v_1^2 f(v_1)$$

The heavy star transfers energy to the lighter stars that move slower, thus:

lighter stars are pushed on wider orbits, whilst heavier stars sink toward the cluster centre ***(mass-segregation)***





Star cluster evolution - #5

If the cluster is represented by an isothermal sphere, a massive star would drift into the cluster centre at a rate

$r\,dr/dt \sim -0.302\,Gm/\sigma \ln\Lambda$ (Binney and Tremaine 2008)

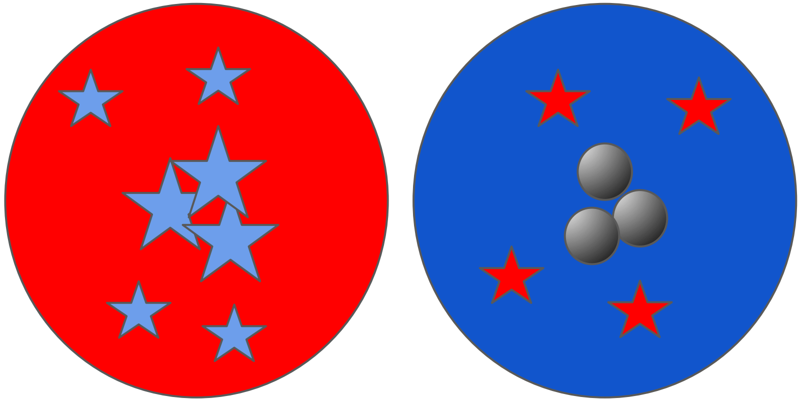
which can be integrated over time from 0 to t_{DF} to derive the mass-segregation timescale

$t_{DF} \sim 1.17/\ln\Lambda\, M_c(r)/m\, t_{cross}$

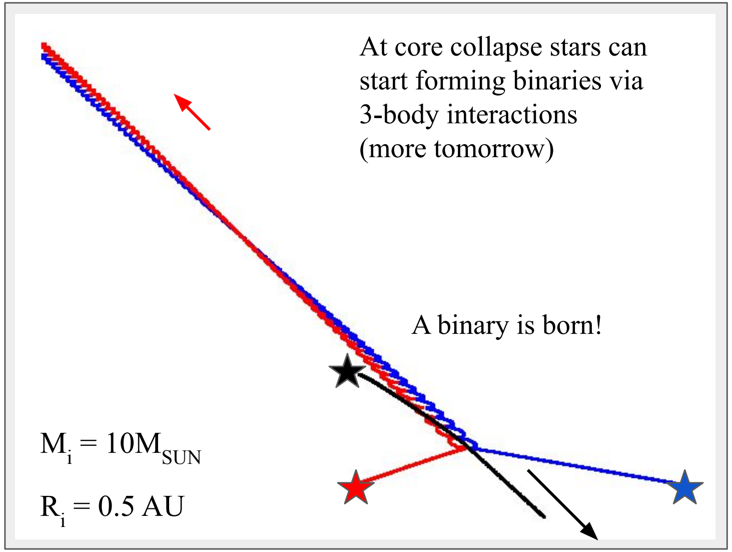
The process leads to core-collapse on a shorter time compared to the single-mass case
$t_{CC} \sim 0.2\, t_{relax} \sim 0.02\, N / (\ln \lambda N)\, t_{cross}$ (Portegies-Zwart and McMillan 2002)

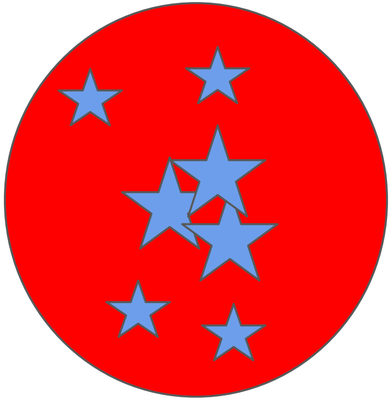
Roughly, a star with 10(100) M_{SUN} ends its life over a time ~ 15(4) Myr, thus we have two teams!

cluster type	N	R (pc)	t_{cross}(Myr)	t_{DF}(Myr)	t_{CC} (Myr)
open clusters	10^2	2	4.2	4.4	2.3
young clusters	10^4	2	0.3	14.1	7.2
globular clusters	10^5	2	0.1	36.8	19
nuclear clusters	10^7	2	0.01	257	131

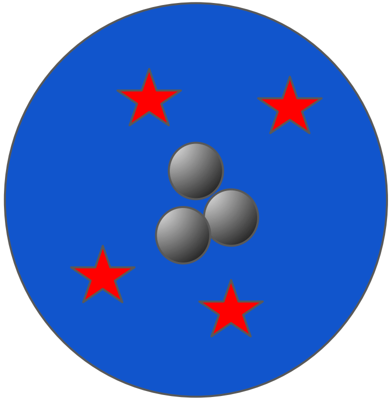


Star cluster evolution - #6





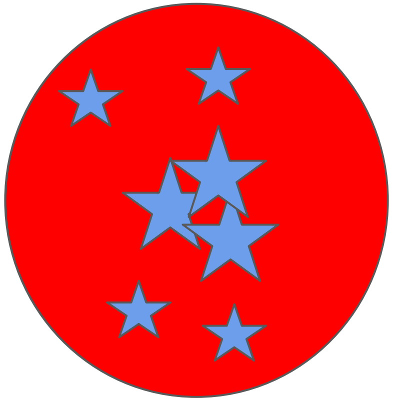
Interaction time-scale < stellar evolution
- stellar collisions and mergers
- possible formation of a very massive star
- possible formation of an IMBH

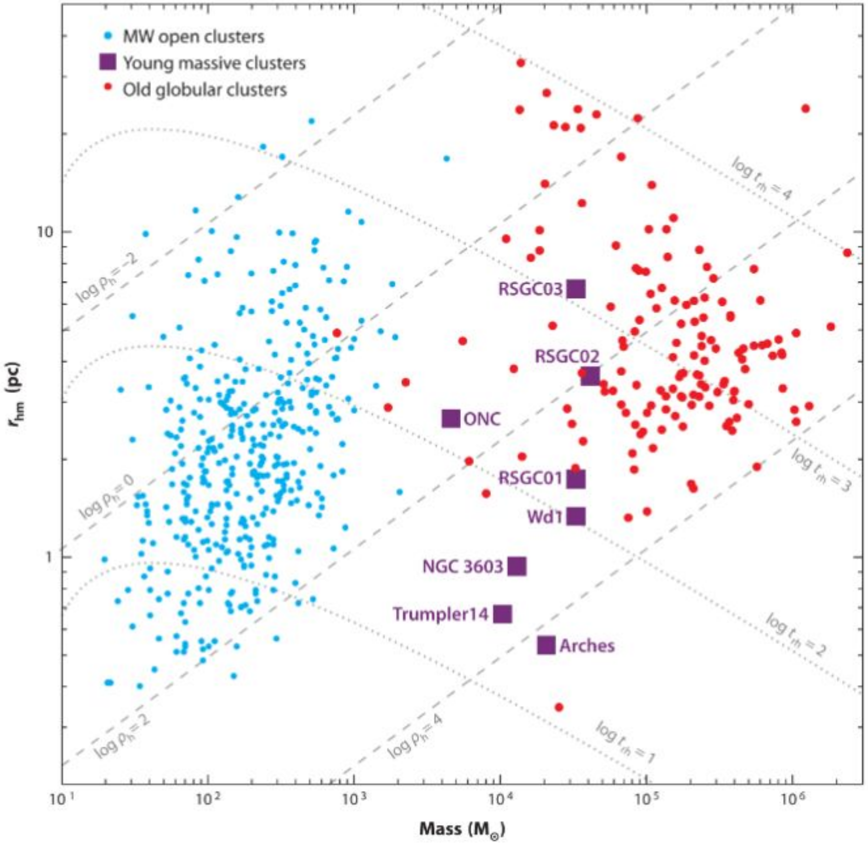


Interaction time-scale > stellar evolution
- BHs form a sub-system and dominate dynamics
- possible BHBH mergers
- possible onset of hierarchical BHBH mergers

Star cluster evolution - #7

Clusters with short collapse times

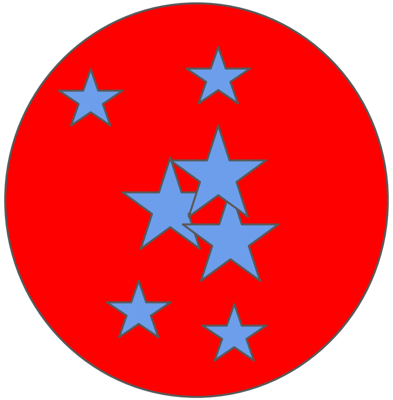


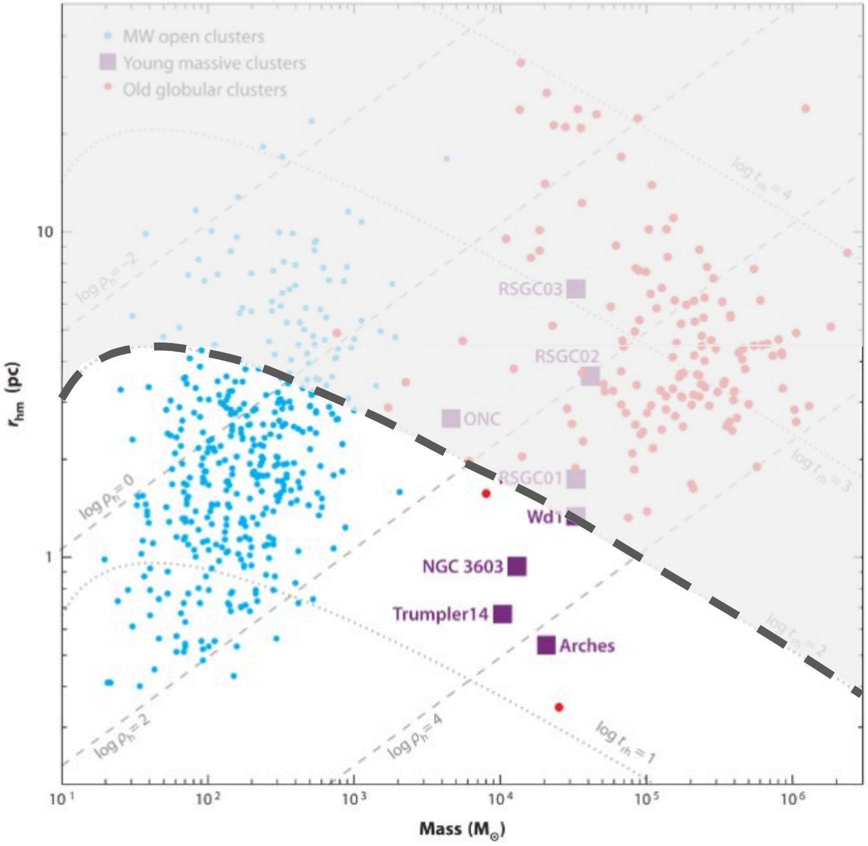


Portegies-Zwart+(2010)

Star cluster evolution - #7

Clusters with short collapse times



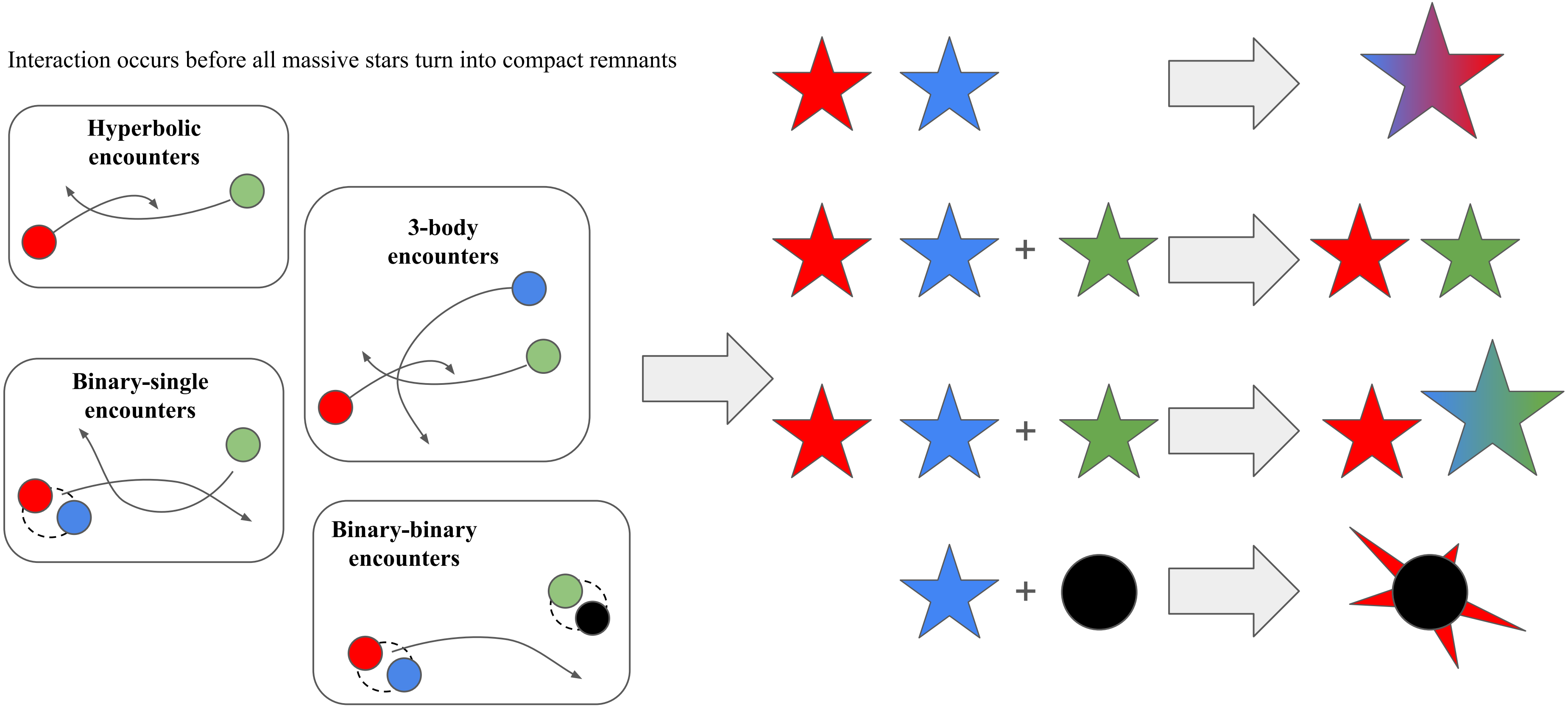


Portegies-Zwart+(2010)

Star cluster evolution - #7

Clusters with short collapse times

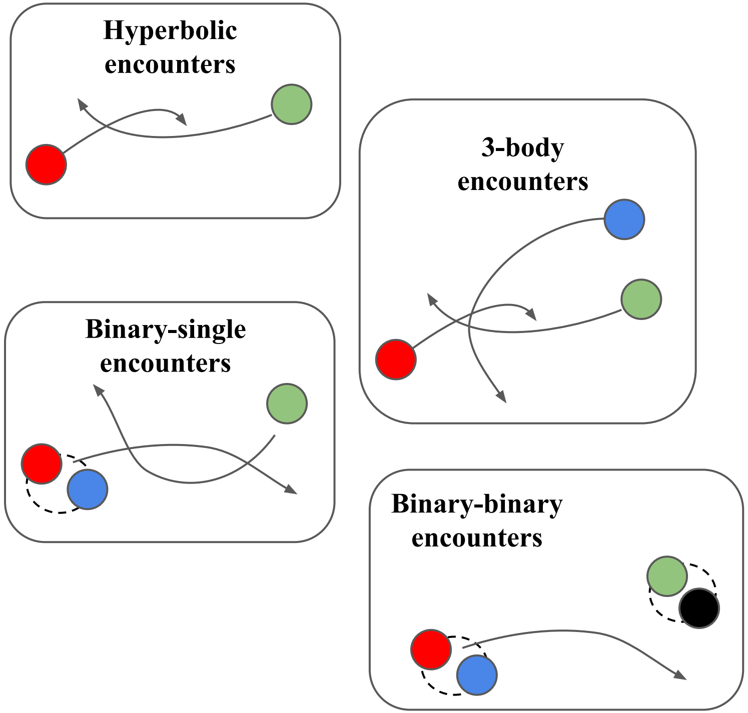
Interaction occurs before all massive stars turn into compact remnants

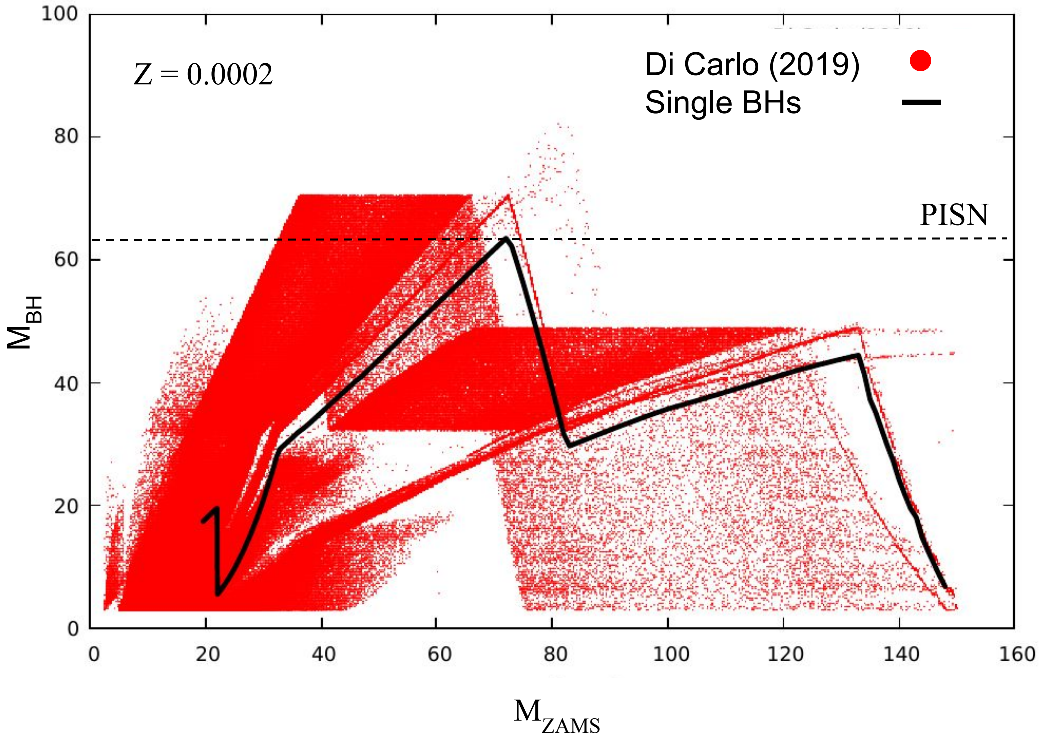


Star cluster evolution - #7

Clusters with short collapse times

Interaction occurs before massive stars turn into compact remnants





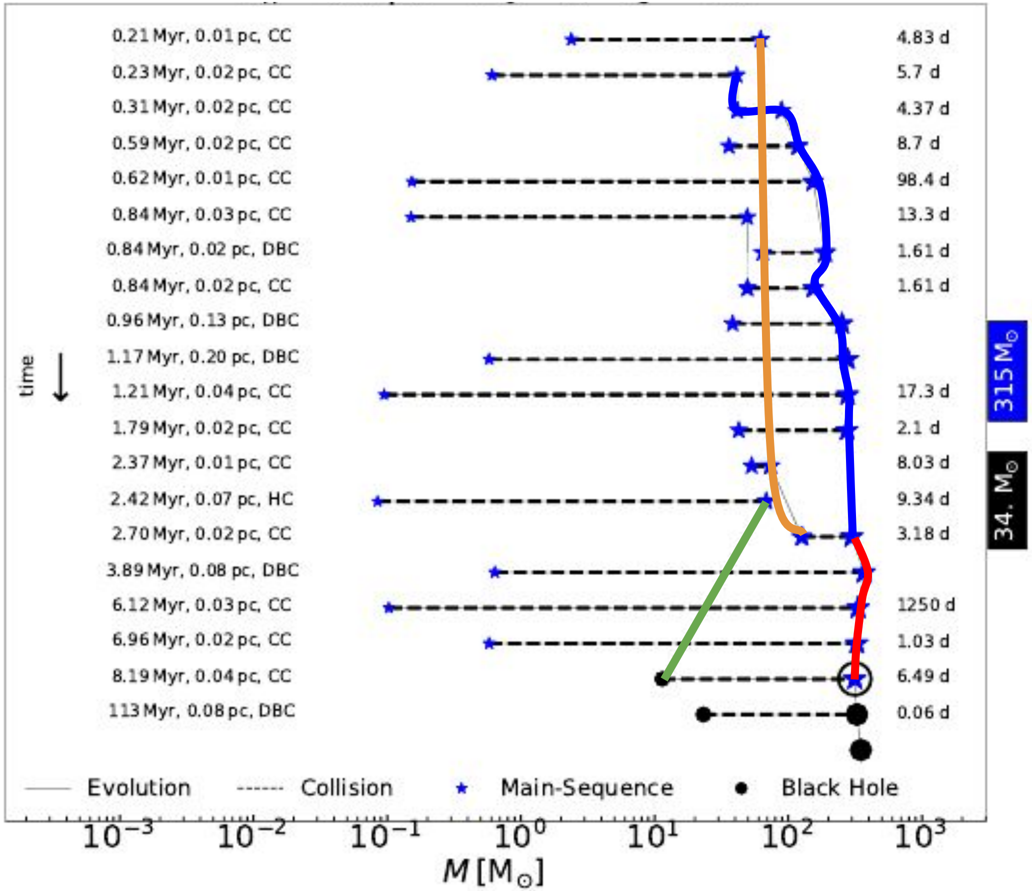
Star cluster evolution - #7

Clusters with short collapse times

N-body simulations by Rizzuto et al (2021,2022)

Progenitor #1
Progenitor #2
Progenitor #12
Progenitor #3
IMBH

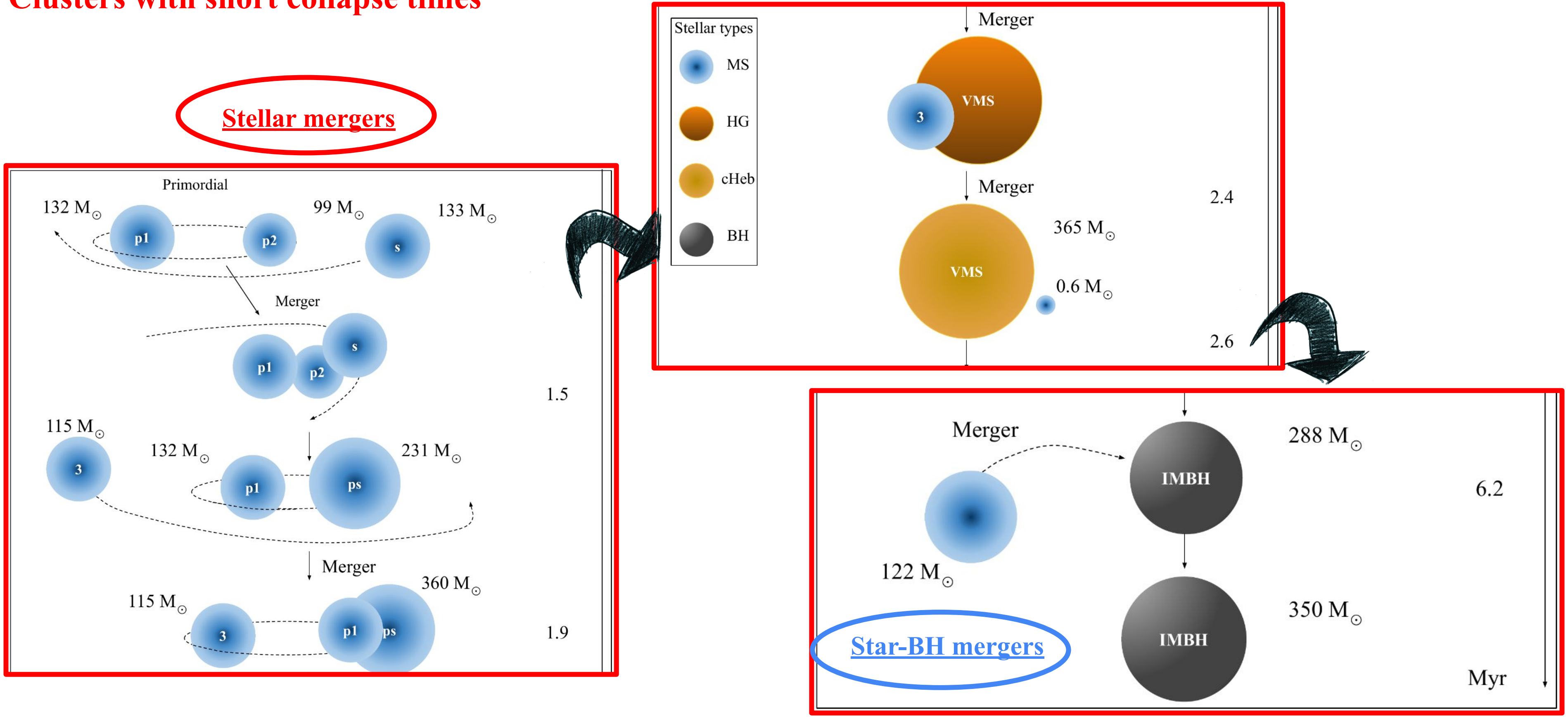
2.7 Myr
8.2 Myr



Star cluster evolution - #8

Clusters with short collapse times

Arca Sedda et al (2023)



Star cluster evolution - #9

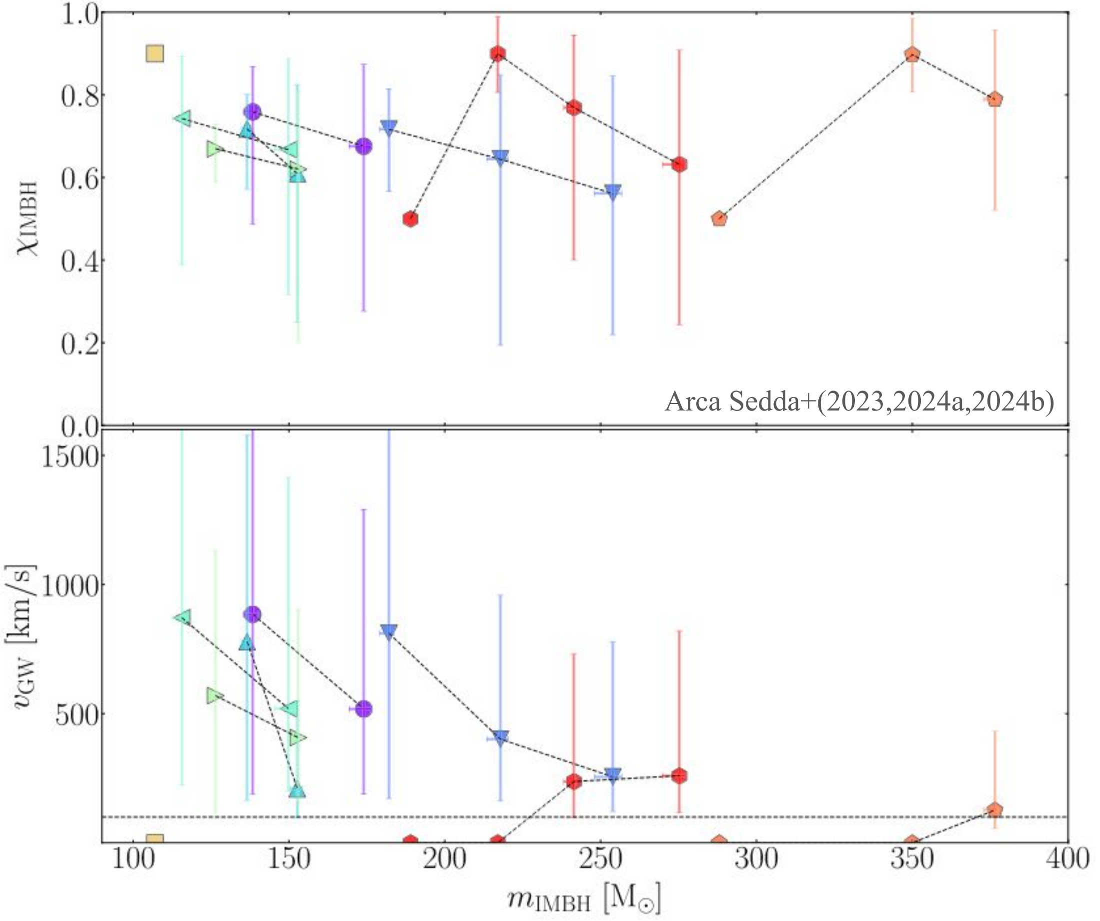
Clusters with short collapse times

The central escape velocity is roughly

$$v_{\rm esc} = (34 \pm 3){\rm km/s}\left(\frac{M}{10^5\ {\rm M}_\odot}\right)^{1/2}\left(\frac{R_{\rm HM}}{1{\rm pc}}\right)^{-1/2}$$

While the typical recoil kick received during a strong dynamical encounter is given by (Heggie75)

$$v_{\rm rec} = \left[\frac{G m_1 m_2}{a_{\rm fin}(m_1+m_2)}\frac{m_p}{m_1+m_2+m_p}\left(1-\frac{a_{\rm fin}}{a}\right)\right]^{1/2}$$



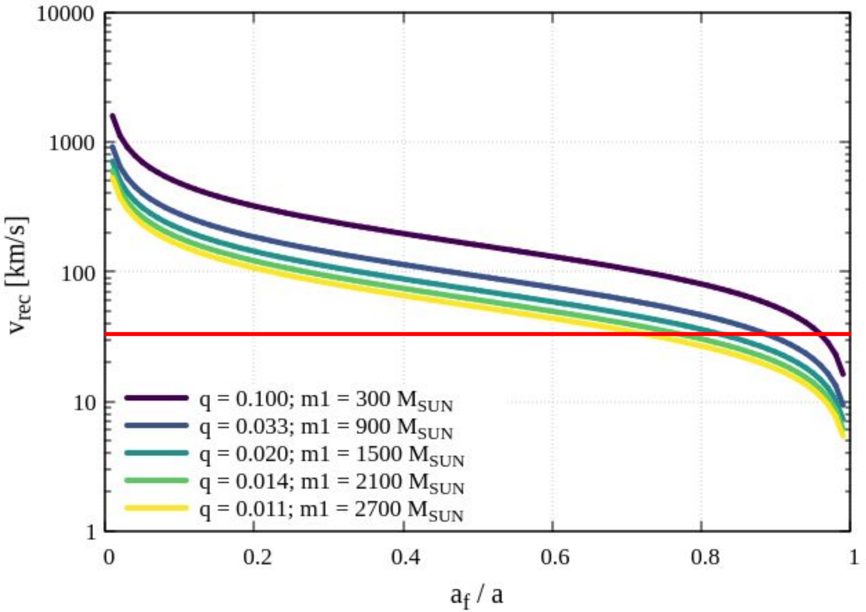
Star cluster evolution - #9

Clusters with short collapse times

For $m_1 >> m_{2,p}$ the recoil reduces to

$$v_{\rm rec} \sim 160{\rm km/s} \times$$
$$\times\, q_{12}^{1/2} \left(\frac{m_p}{30{\rm M}_\odot}\right)^{1/2} \left(\frac{a_f}{1\,{\rm AU}}\right)^{-1/2} \left(1-\frac{a_f}{a}\right)^{1/2}$$

The more efficient the hardening, the more powerful the kick.



Star cluster evolution - #9

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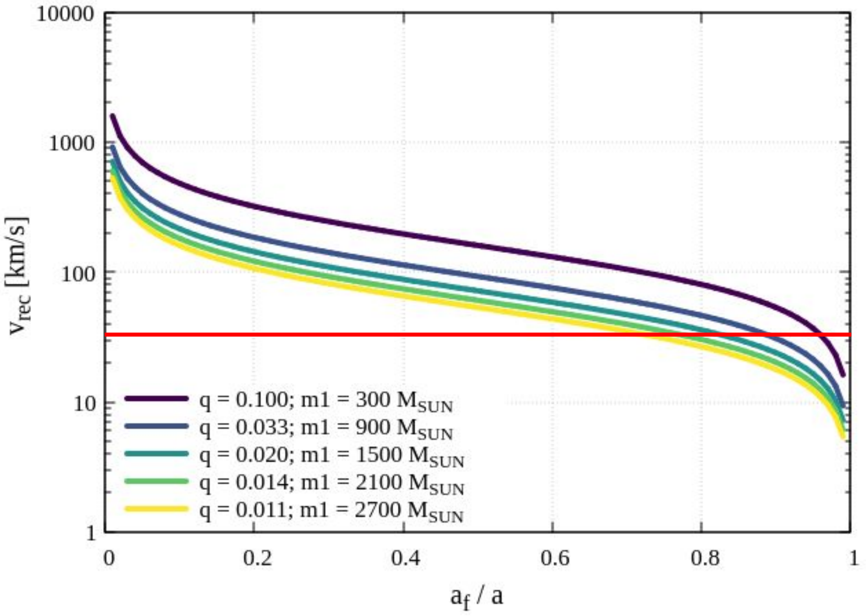
The more efficient the hardening, the more powerful the kick.

The IMBH can escape with its companion and possibly merge for the last time outside the cluster



Star cluster evolution - #9

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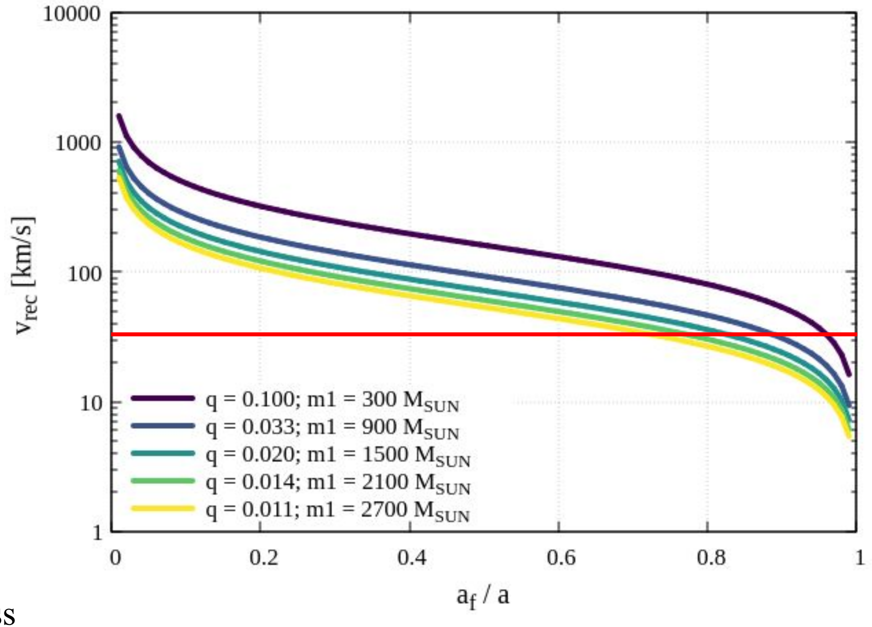


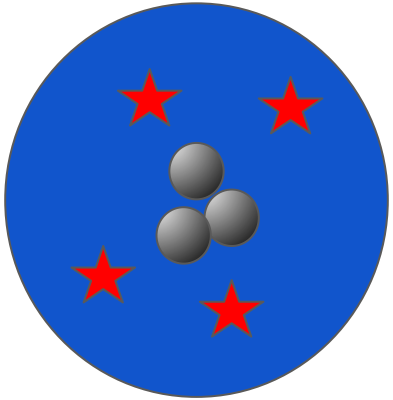
The more efficient the hardening, the more powerful the kick.

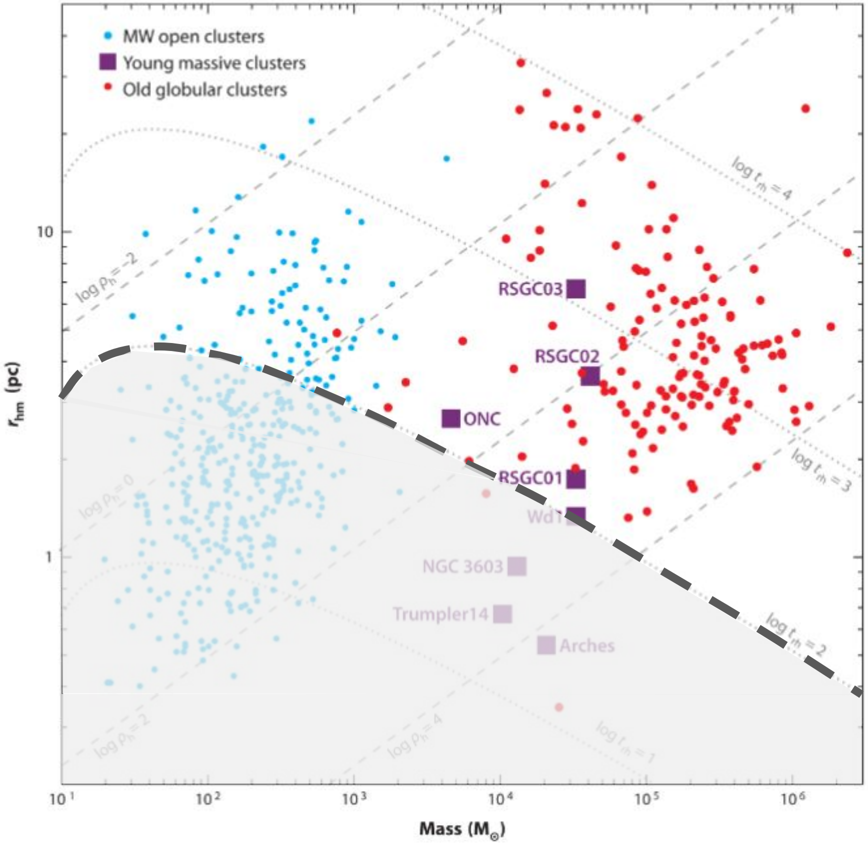
The IMBH can escape with its companion and possibly merge for the last time outside the cluster

The maximum IMBH mass is generally dictated by the collision process

Star cluster evolution - #10

Clusters with long collapse times

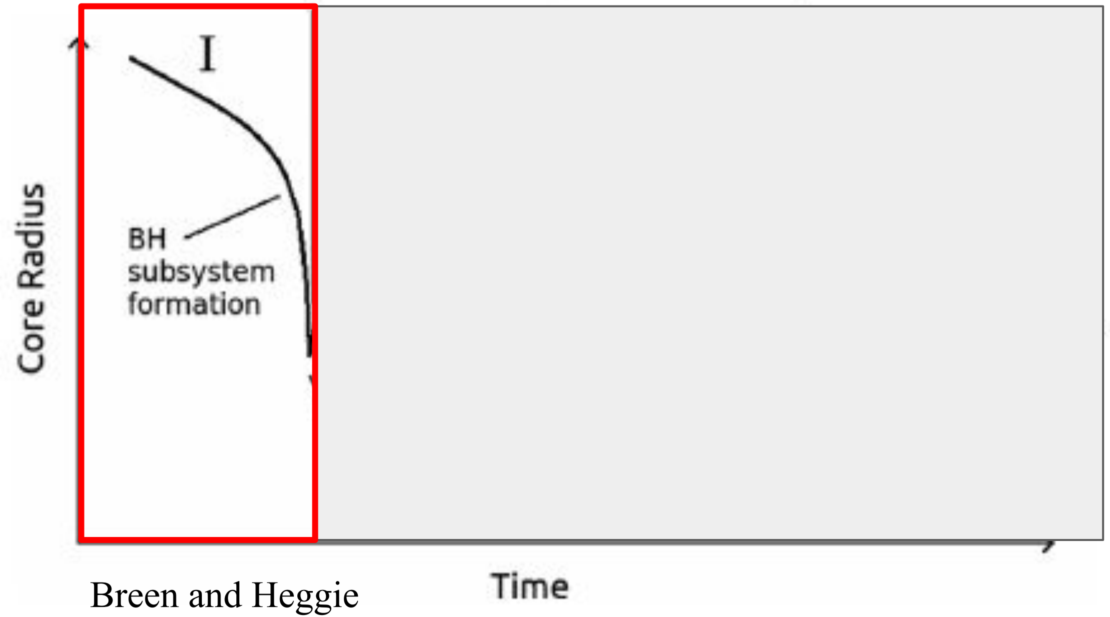




Portegies-Zwart+(2010)

Star cluster evolution - #10

Clusters with long collapse times



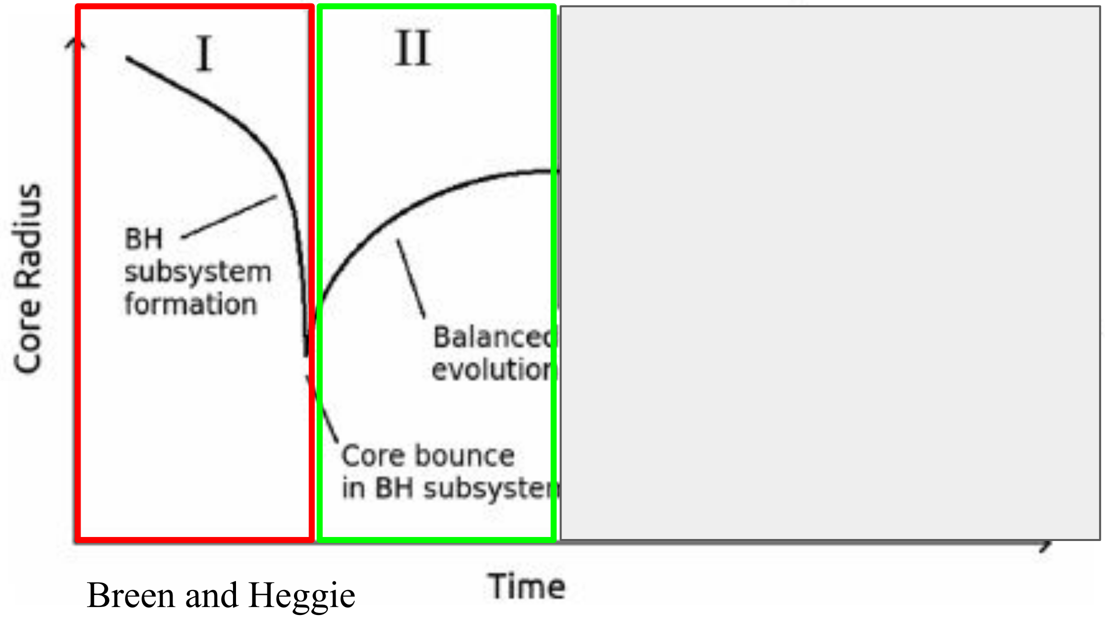
Breen and Heggie (2013a,b)

Phase I.

The cluster undergoes core-collapse, BHs form a self-interacting subsystem (BHS)

Star cluster evolution - #10

Clusters with long collapse times



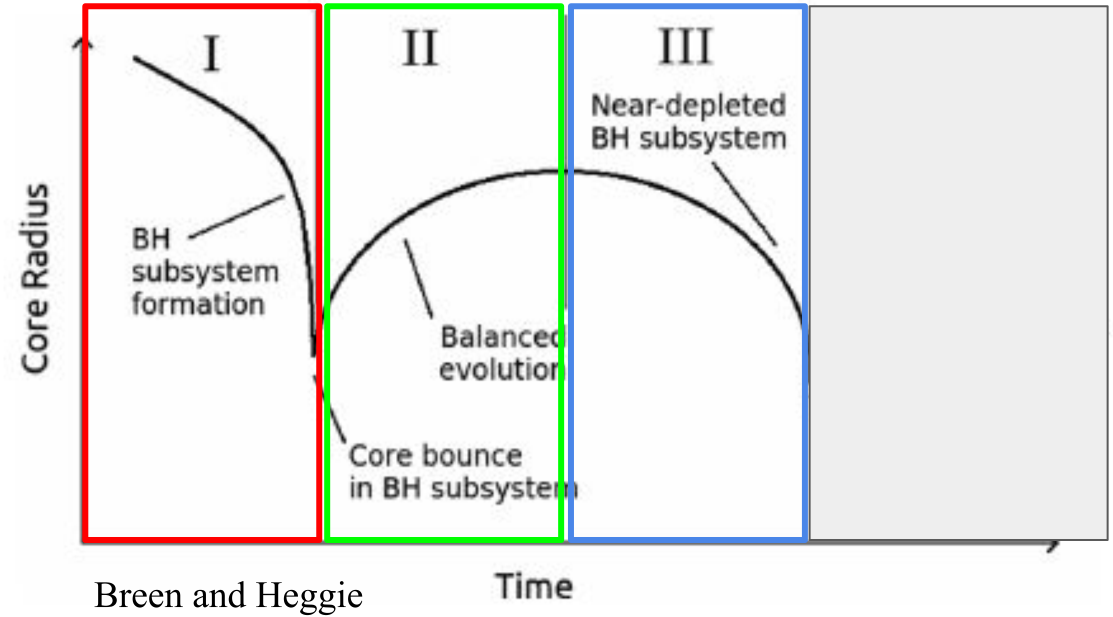
Breen and Heggie (2013a,b)

Phase I.
The cluster undergoes core-collapse, BHs form a self-interacting subsystem (BHS)

Phase II.
Strong scatterings eject the most massive BHs from the cluster

Star cluster evolution - #10

Clusters with long collapse times



Breen and Heggie (2013a,b)

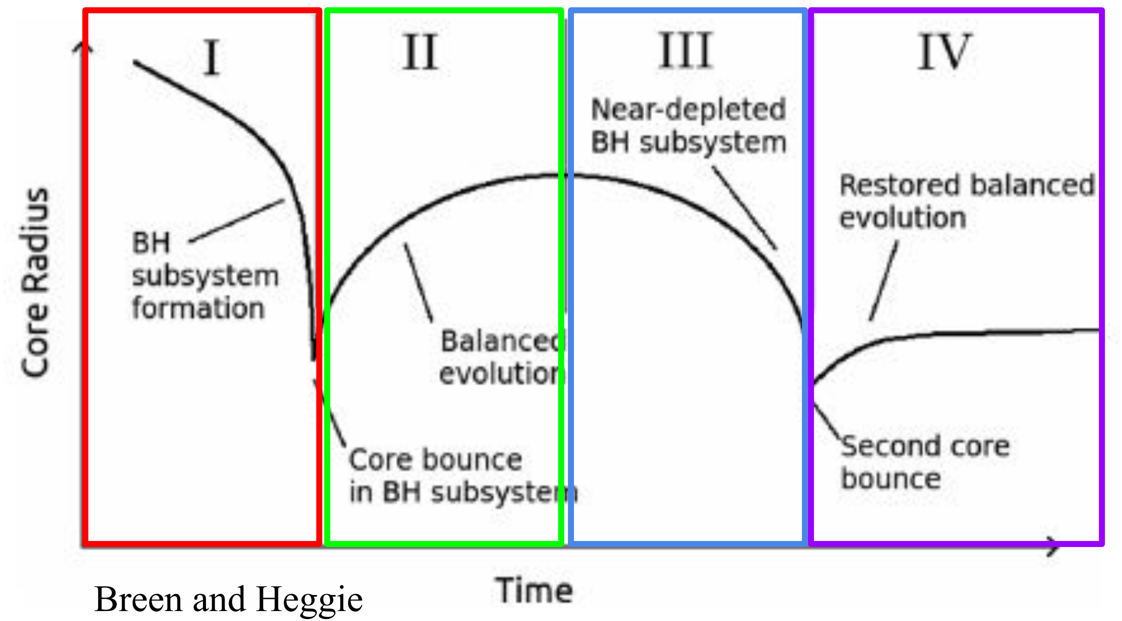
Phase I.
The cluster undergoes core-collapse, BHs form a self-interacting subsystem (BHS)

Phase II.
Strong scatterings eject the most massive BHs from the cluster

Phase III.
The energy support from hard binary is missing, the core re-contracts

Star cluster evolution - #10

Clusters with long collapse times



Breen and Heggie (2013a,b)

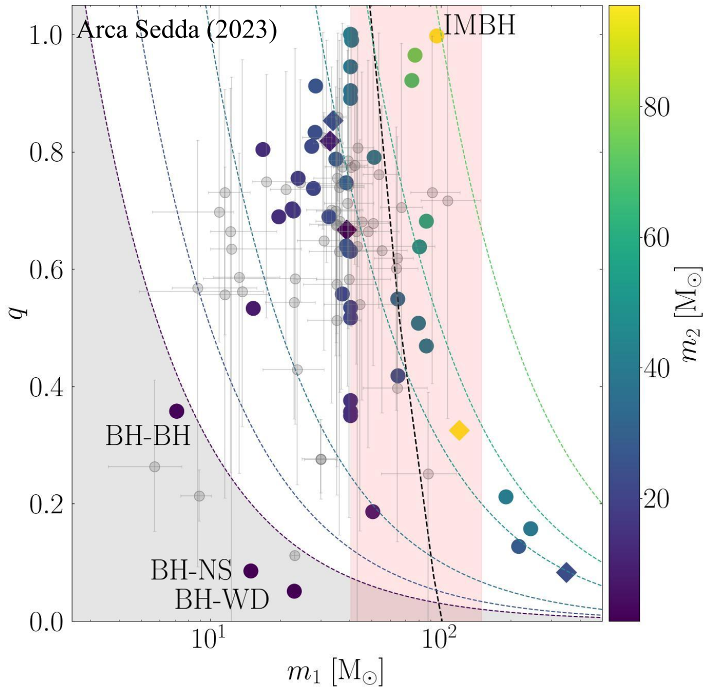
Phase I.
The cluster undergoes core-collapse, BHs form a self-interacting subsystem (BHS)

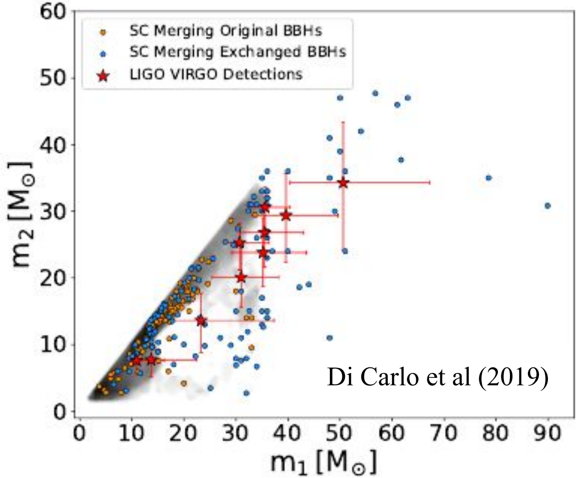
Phase II.
Strong scatterings eject the most massive BHs from the cluster

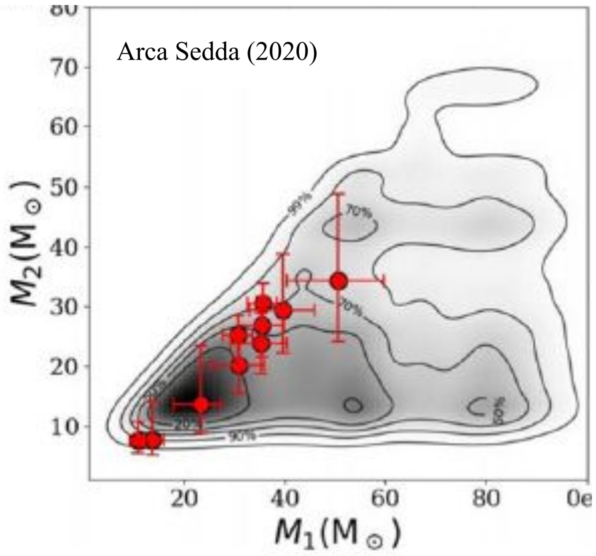
Phase III.
The energy support from hard binary is missing, the core re-contracts

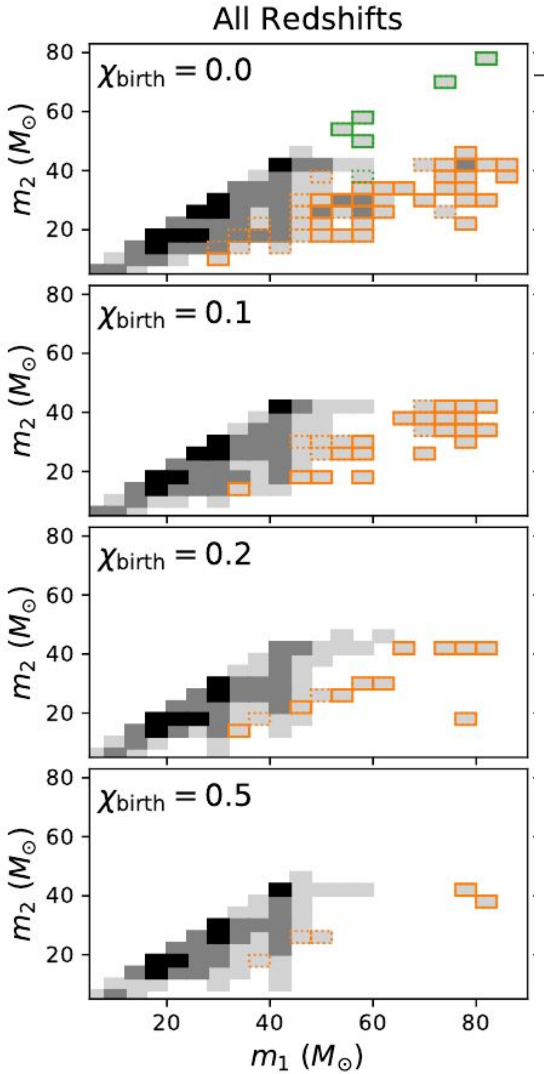
Phase IV.
New energetic binaries are formed, core expands and reach a ~ equilibrium (slow expansion)

Dynamical formation

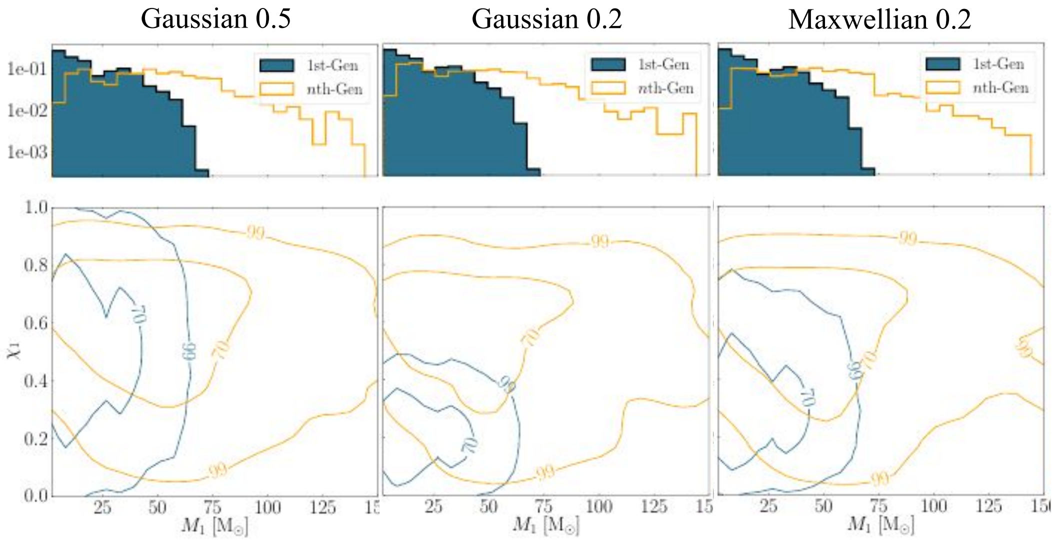


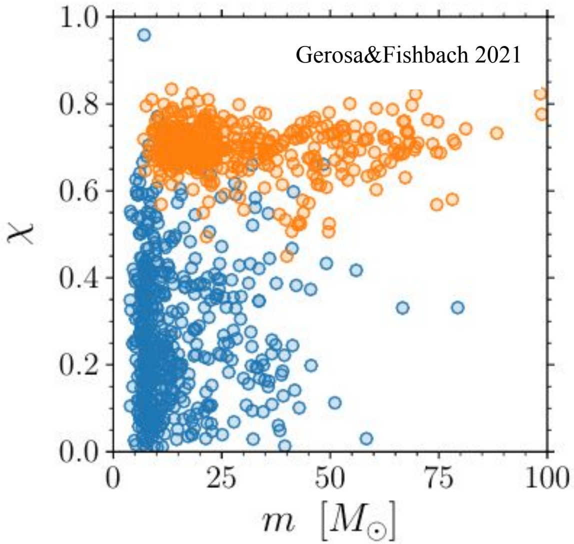






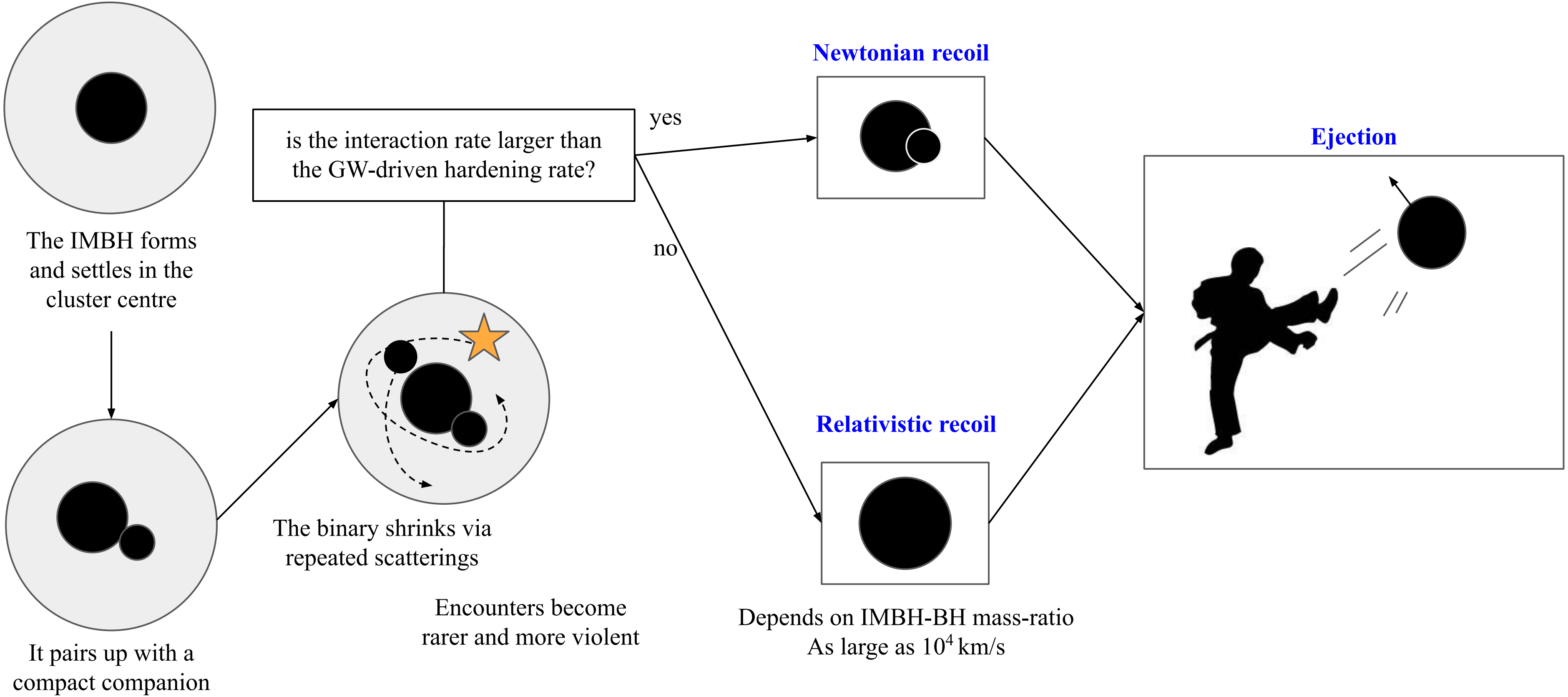
Dynamical formation





Dynamical formation

It’s hard to grow up: the impact of Newtonian and relativistic dynamics on IMBH retention



Dynamical formation

- GWs carry away linear momentum from the binary
- In circular equal-mass binaries, the momentum stolen over one orbit is ~0 for symmetry reason
- This IS NOT the case for non equal-mass binaries and/or spinning BHs.

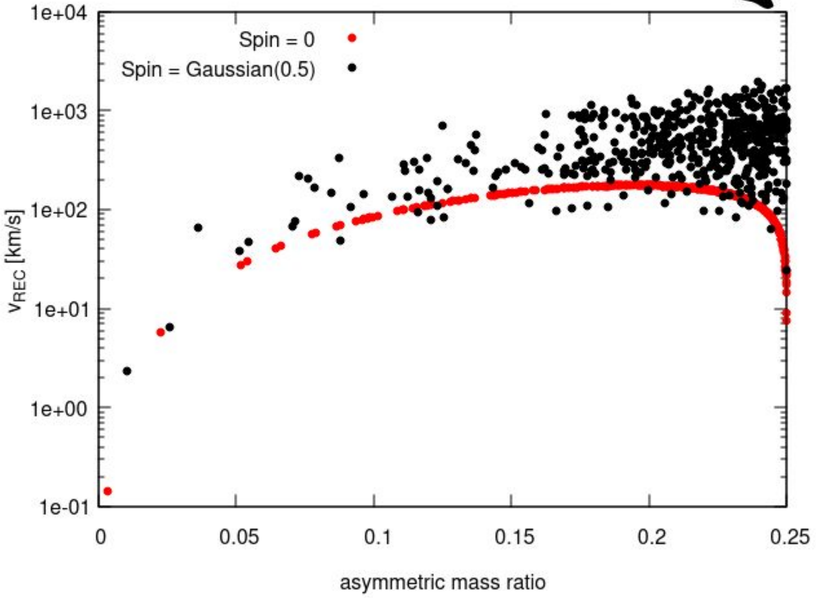
Numerical relativity simulations show that (Campanelli+07, Gonzalez+07, Lousto&Zlochower08, Lousto+12)

$$\boldsymbol{v}_k = v_m \hat{e}_{\perp,1} + v_\perp(\cos\xi \hat{e}_{\perp,1} + \sin\xi \hat{e}_{\perp,2}) + v_\parallel \hat{e}_\parallel,$$

$$v_m = A\eta^2\sqrt{1-4\eta}(1+B\eta)$$

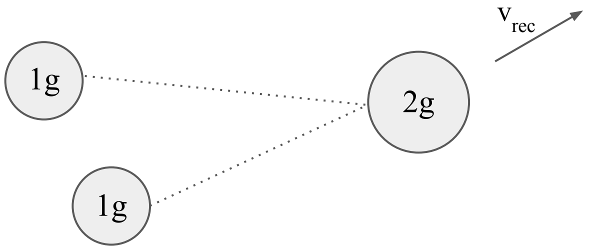
$$v_\perp = \frac{H\eta^2}{1+q_{\rm BBH}}(a_{2,\parallel} - q_{\rm BBH} a_{1,\parallel})$$

$$v_\parallel = \frac{16\eta^2}{1+q_{\rm BBH}}[V_{11} + V_A \Xi_\parallel + V_B \Xi_\parallel^2 + V_C \Xi_\parallel^3] \times |\boldsymbol{a}_{2,\perp} - q_{\rm BBH}\boldsymbol{a}_{1,\perp}| \cos(\phi_\Delta - \phi_1).$$



Dynamical formation

Hierarchical mergers

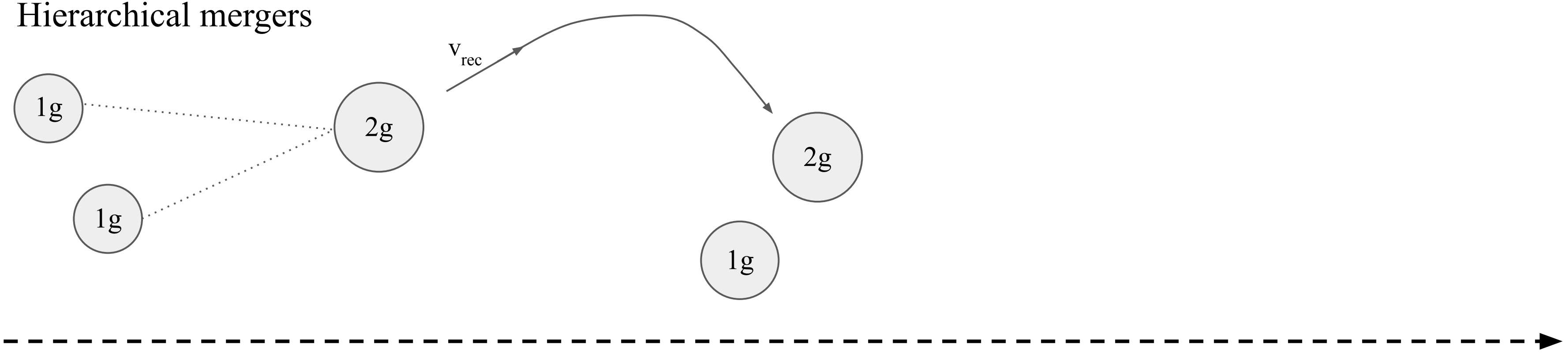


$t = t_{form} + t_{SEV} + t_{DF} + t_{3bb} + t_{bs} + t_{mer}$

Dynamical formation

Hierarchical mergers

$v_{rec} < v_{esc} = 2\Phi \sim (GM_c/R_c)^{1/2}$

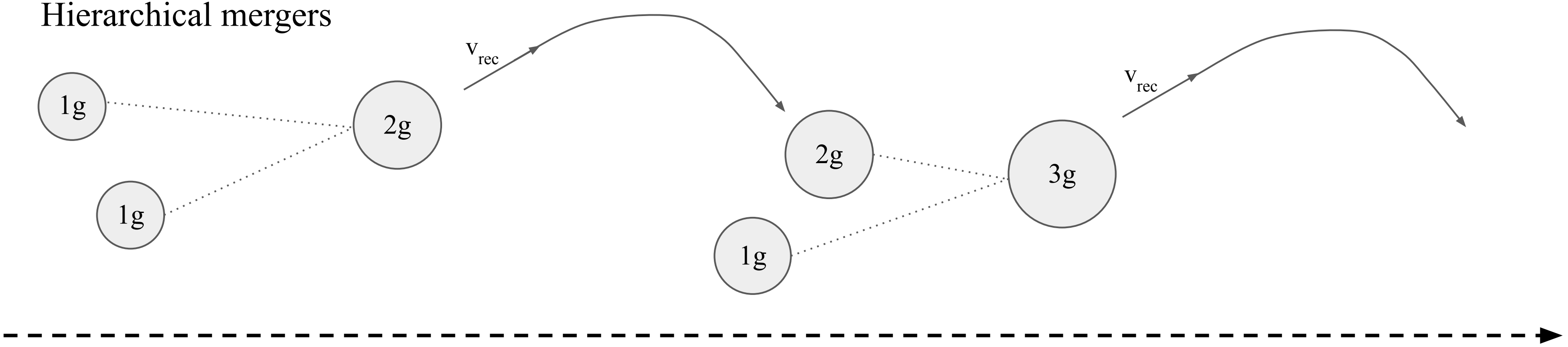


$t = t_{form} + t_{SEV} + t_{DF} + t_{3bb} + t_{bs} + t_{mer}$ t_{trav} t_{DF}

Dynamical formation

Hierarchical mergers

$v_{rec} < v_{esc} = 2\Phi \sim (GM_c/R_c)^{1/2}$



$t = t_{form} + t_{SEV} + t_{DF} + t_{3bb} + t_{bs} + t_{mer}$ t_{trav} t_{DF} $\min(t_{3bb}, t_{bs})$

Dynamical formation

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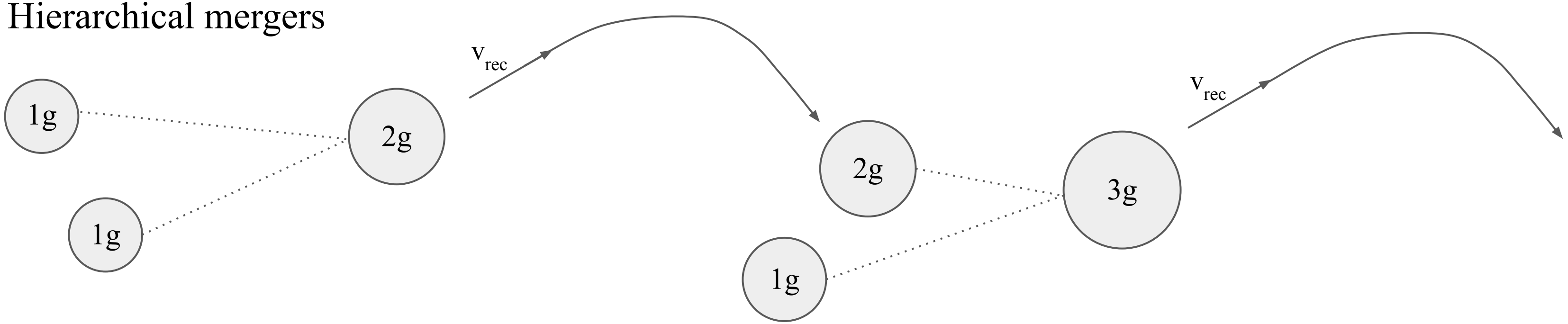
Hierarchical mergers



$t = t_{form} + t_{SEV} + t_{DF} + t_{3bb} + t_{bs} + t_{mer}$ t_{trav} t_{DF} $\min(t_{3bb}, t_{bs})$

Dynamical Mergers:

Natal spins?
Mass ratios?
Remnant spin?

BH mass function?
BH evolution?

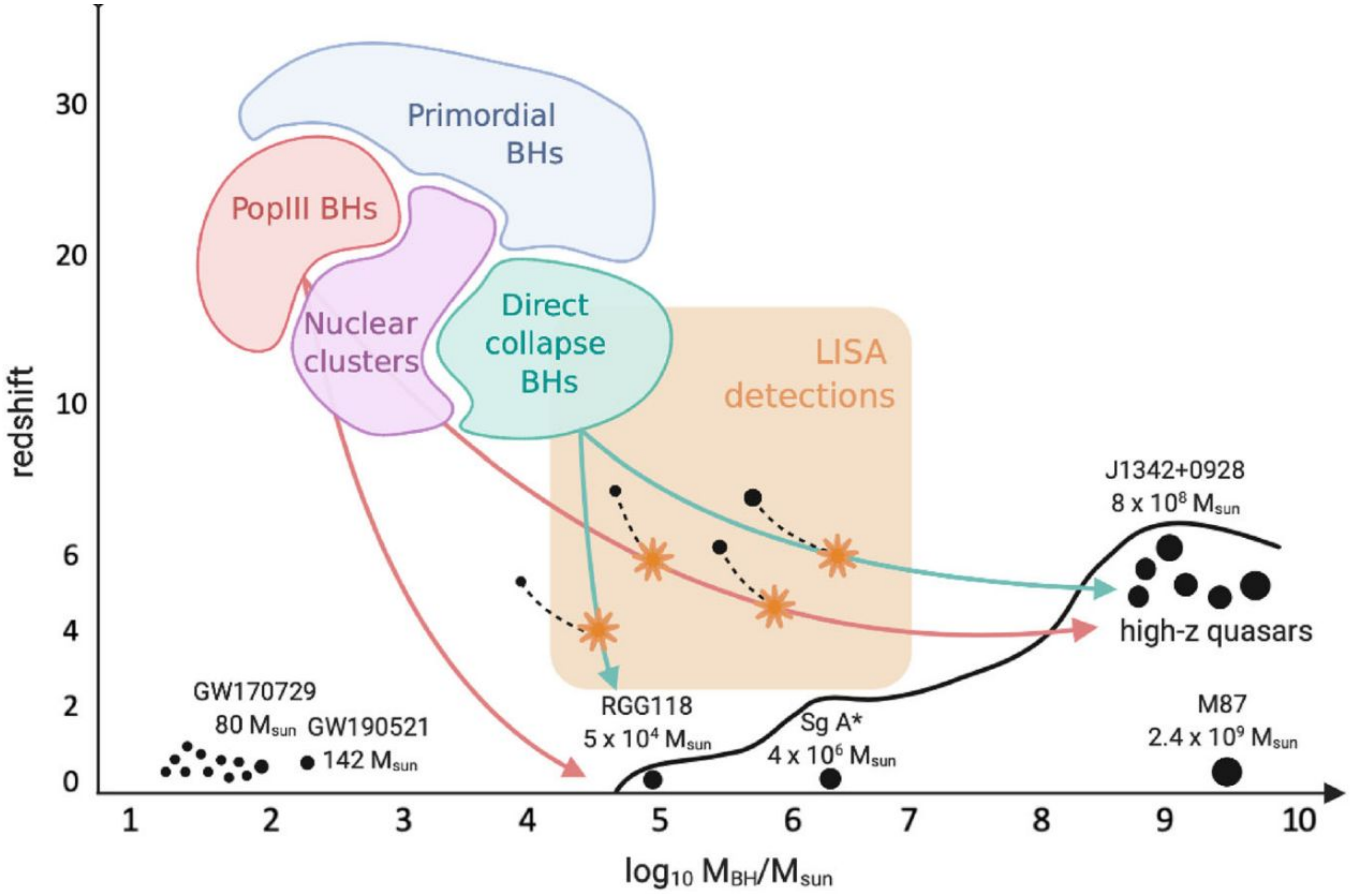
Cluster lifetime?
Cluster evolution?

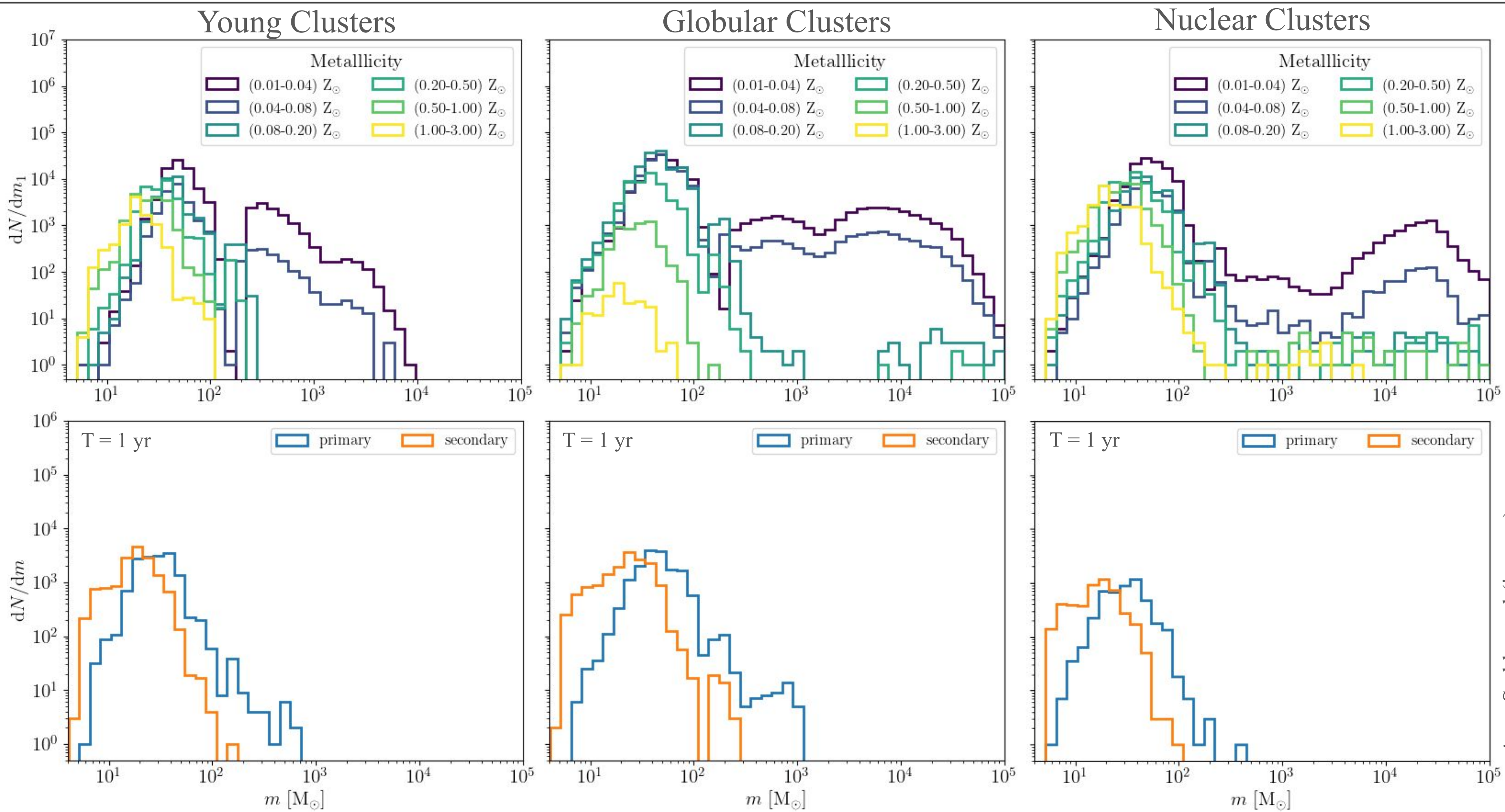
Isolated Mergers:

Galaxy velocity curve

2g + 1g unlikely

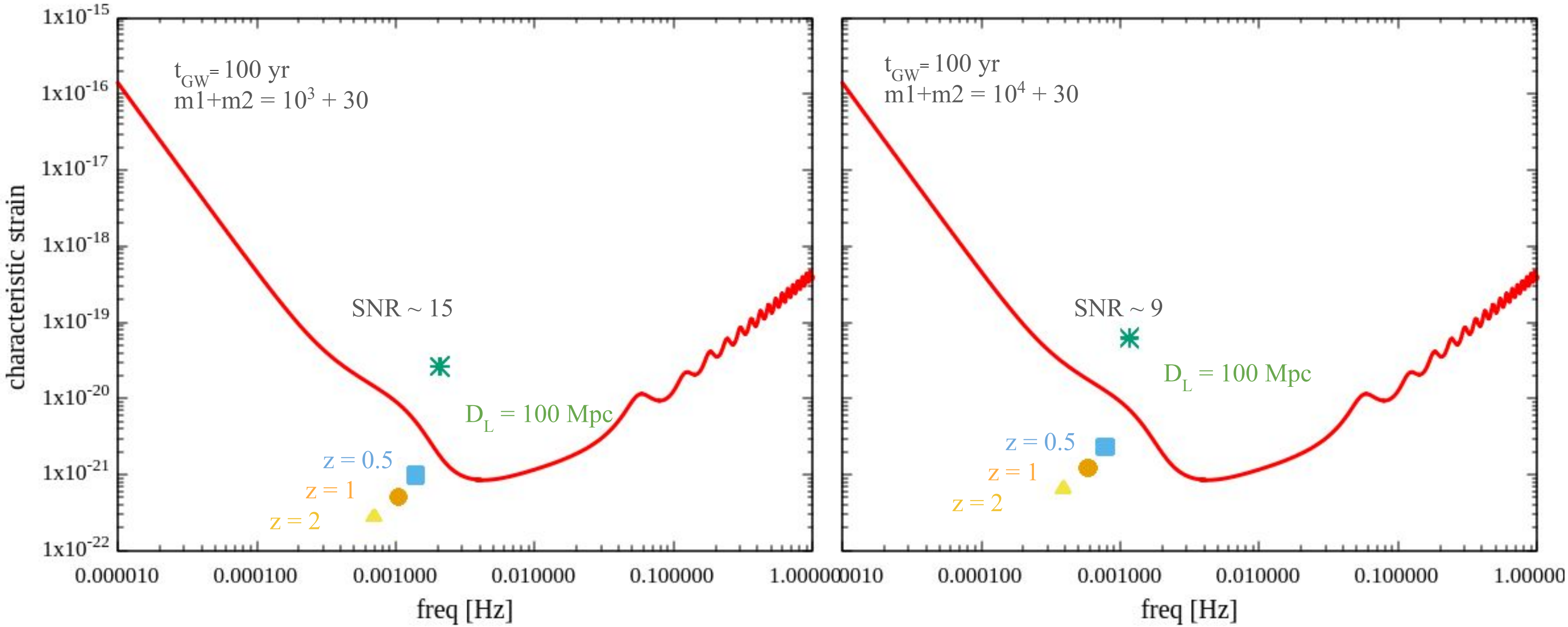
Dynamical formation





Arca Sedda et al (in prep)

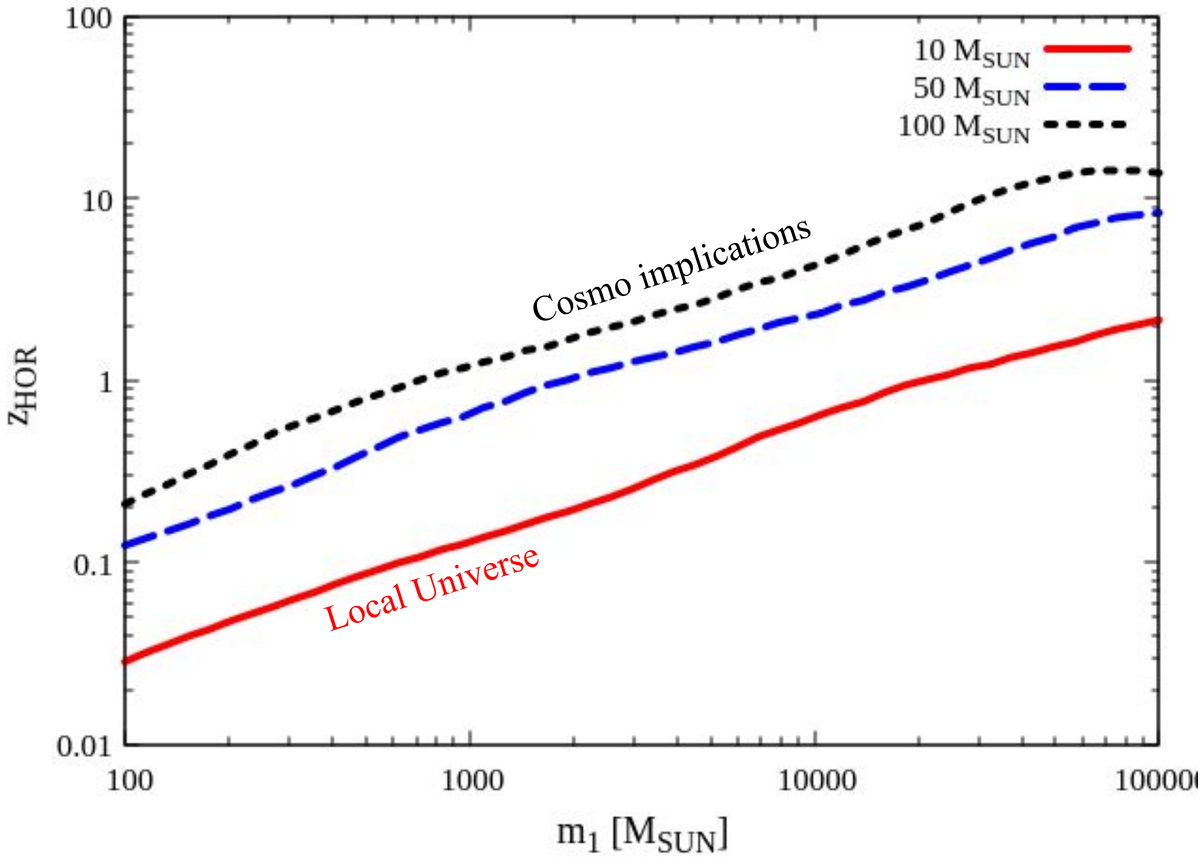
Dynamical formation



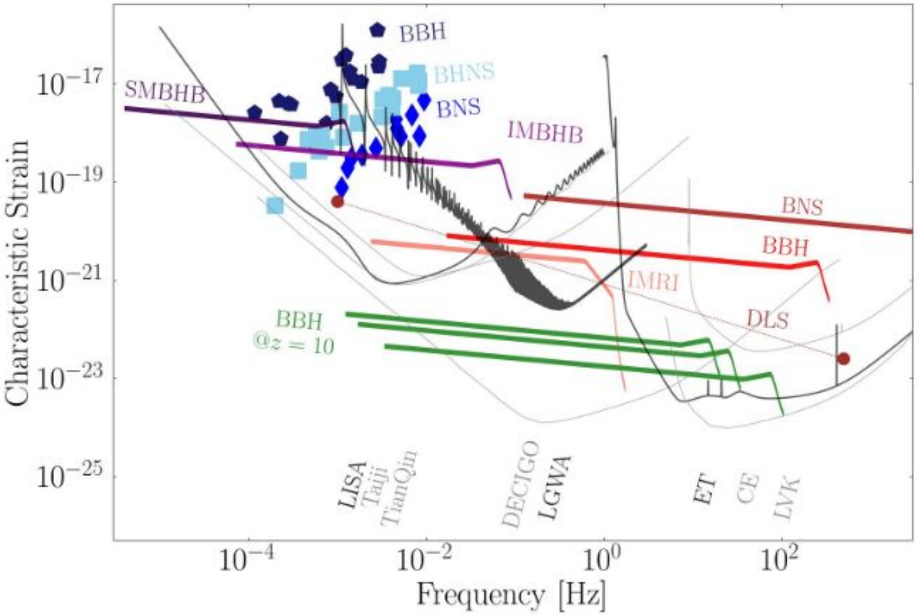
Dynamical formation
Horizon

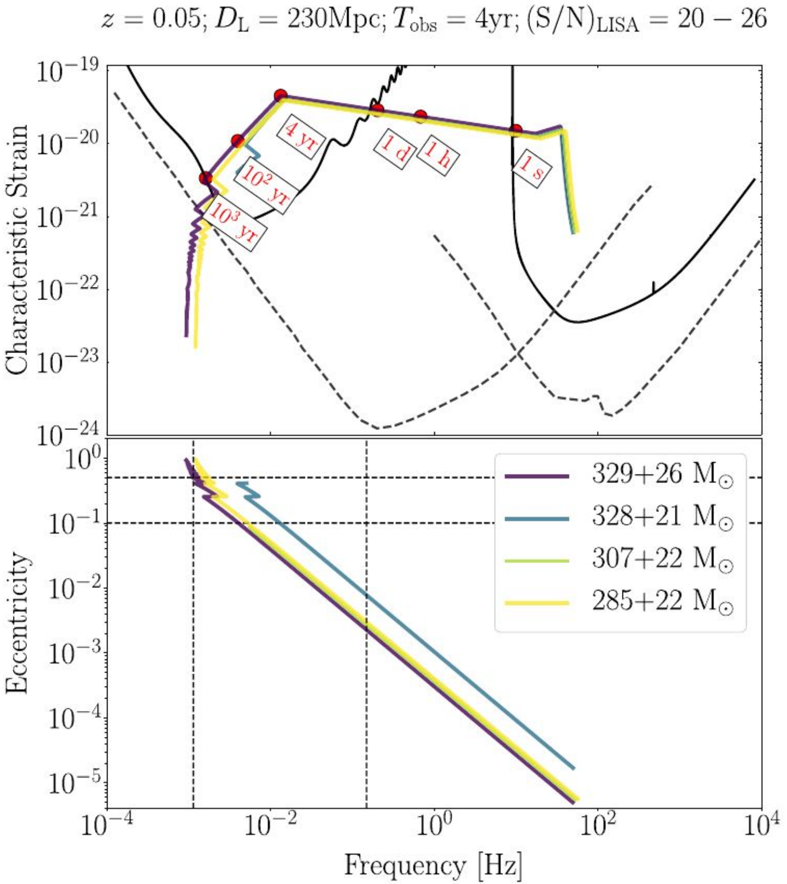
- max redshift at which signal is detected with SNR

- general threshold ~8



Dynamical formation
Horizon



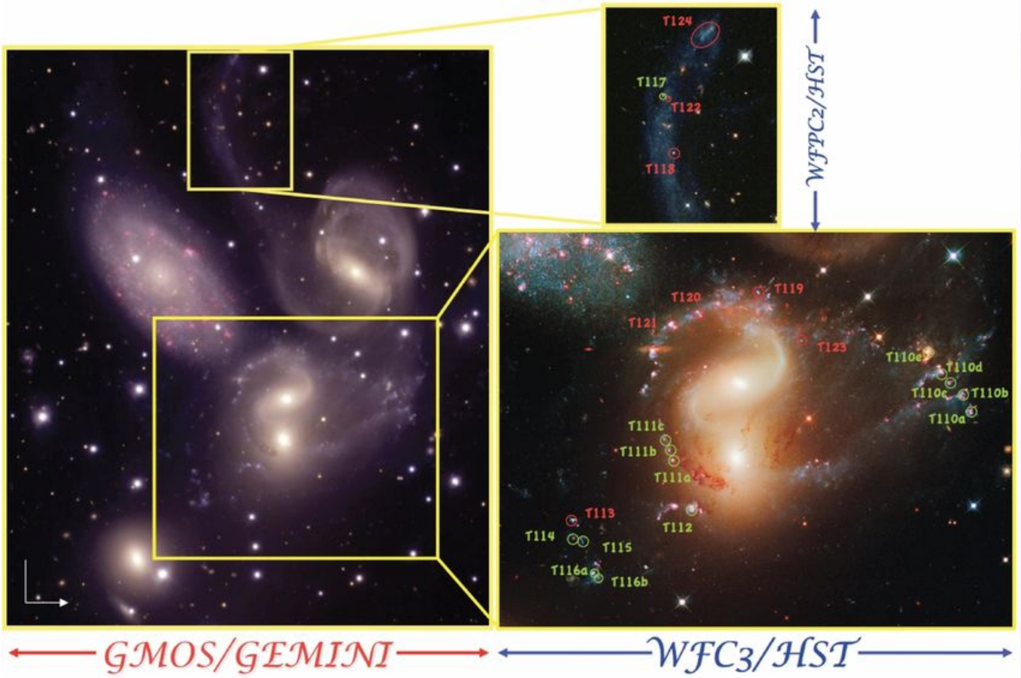


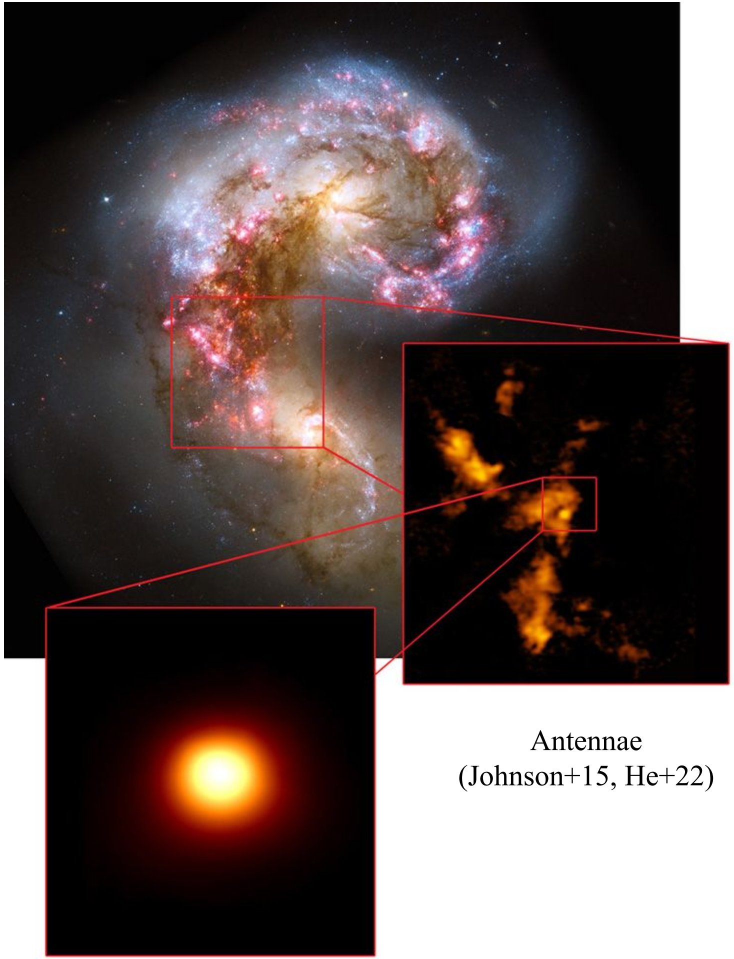
IMBH binaries in merging clusters

- Interacting regions in merging galaxies are site of intense bursts of star formation (triggered by compression of gas and increase in the gas density, e.g. Maji+17)

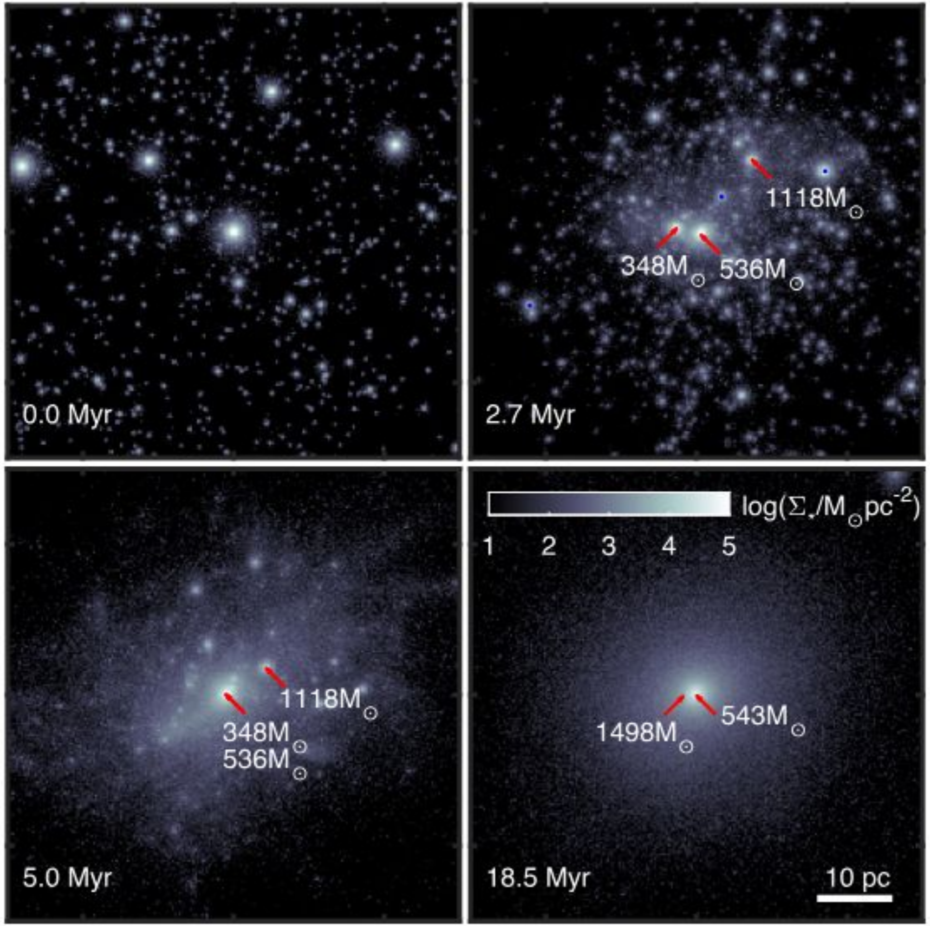
- SC complexes form in relatively small boxes (10^2 pc) → interactions and mergers (e.g. Fellauer&Kroupa02)

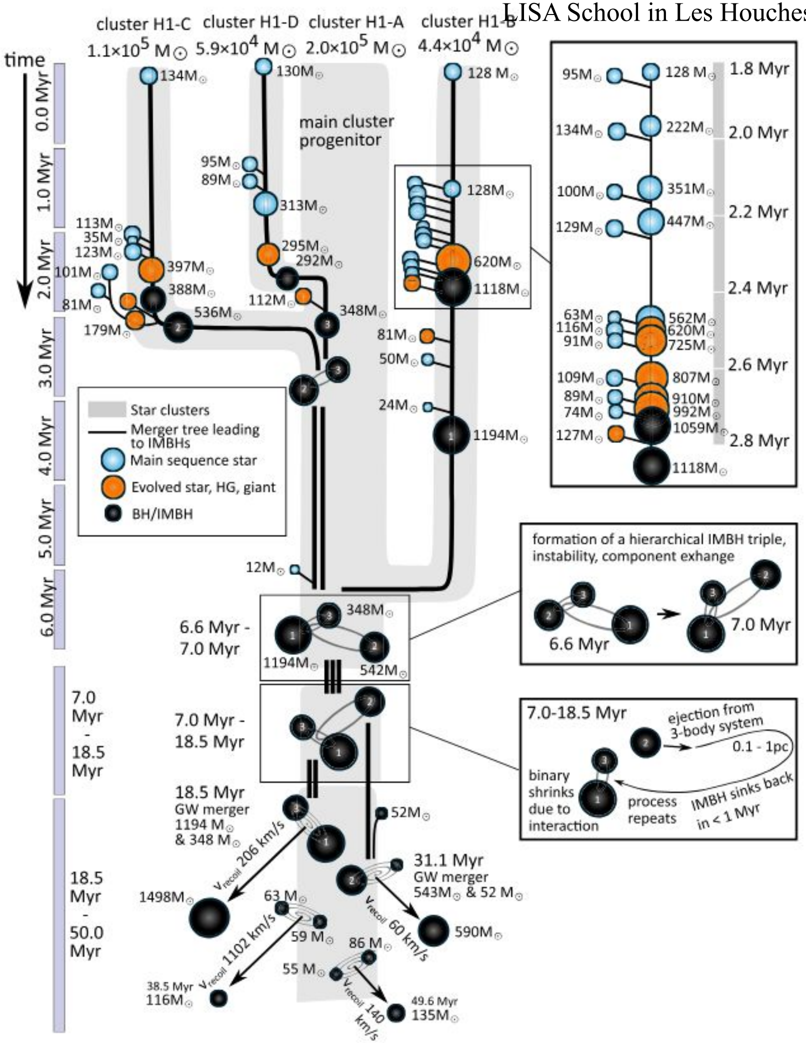
Stephan's Quintet (Gallagher+01, Trancho+12)





Antennae
(Johnson+15, He+22)





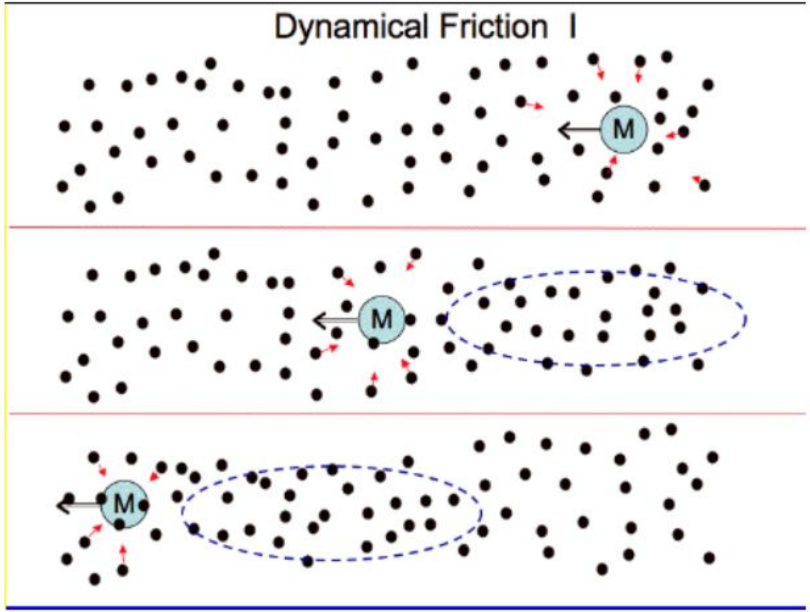
Rantala et al 2024

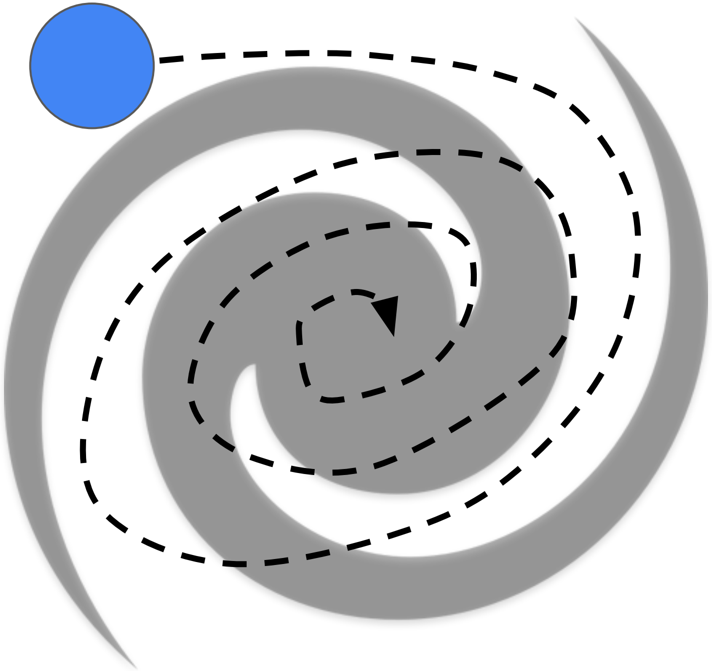
IMBHs as gravitational-wave sources: heavy - IMRIs

- Merging star cluster in galactic centres

Main processes: dynamical friction (Chandrasekhar 1943, Binney&Tremaine08):

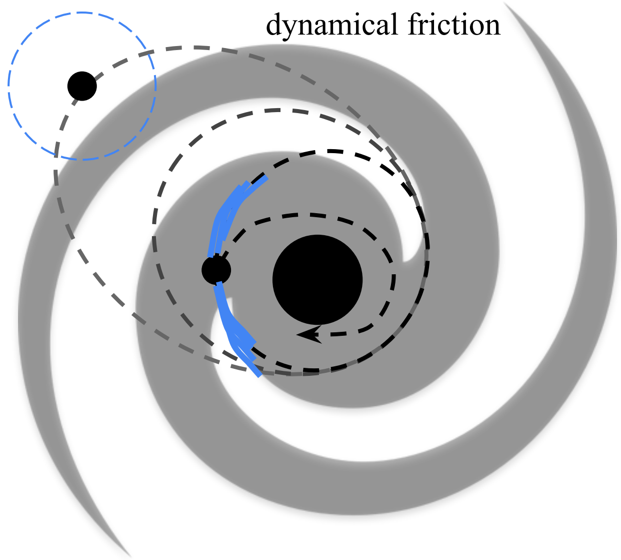
$$\left.\frac{d\mathbf{v_M}}{dt}\right|_{\mathbf{v_m}} = -2\pi \ln(1+\Lambda^2) G^2 m(M+m) \int f(\mathbf{v_m}) d^3\mathbf{v_m} \frac{(\mathbf{v_m}-\mathbf{v_M})}{|\mathbf{v_m}-\mathbf{v_M}|^3},$$

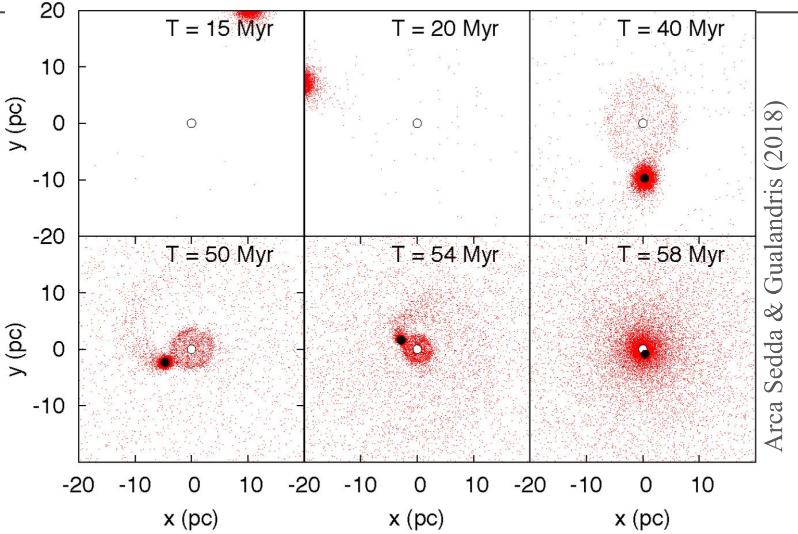


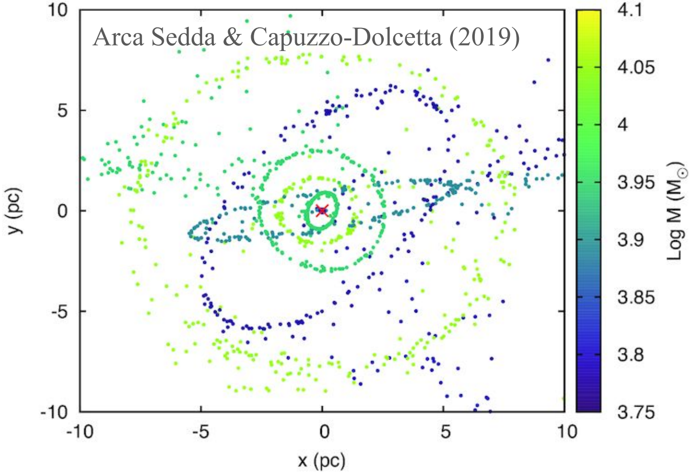


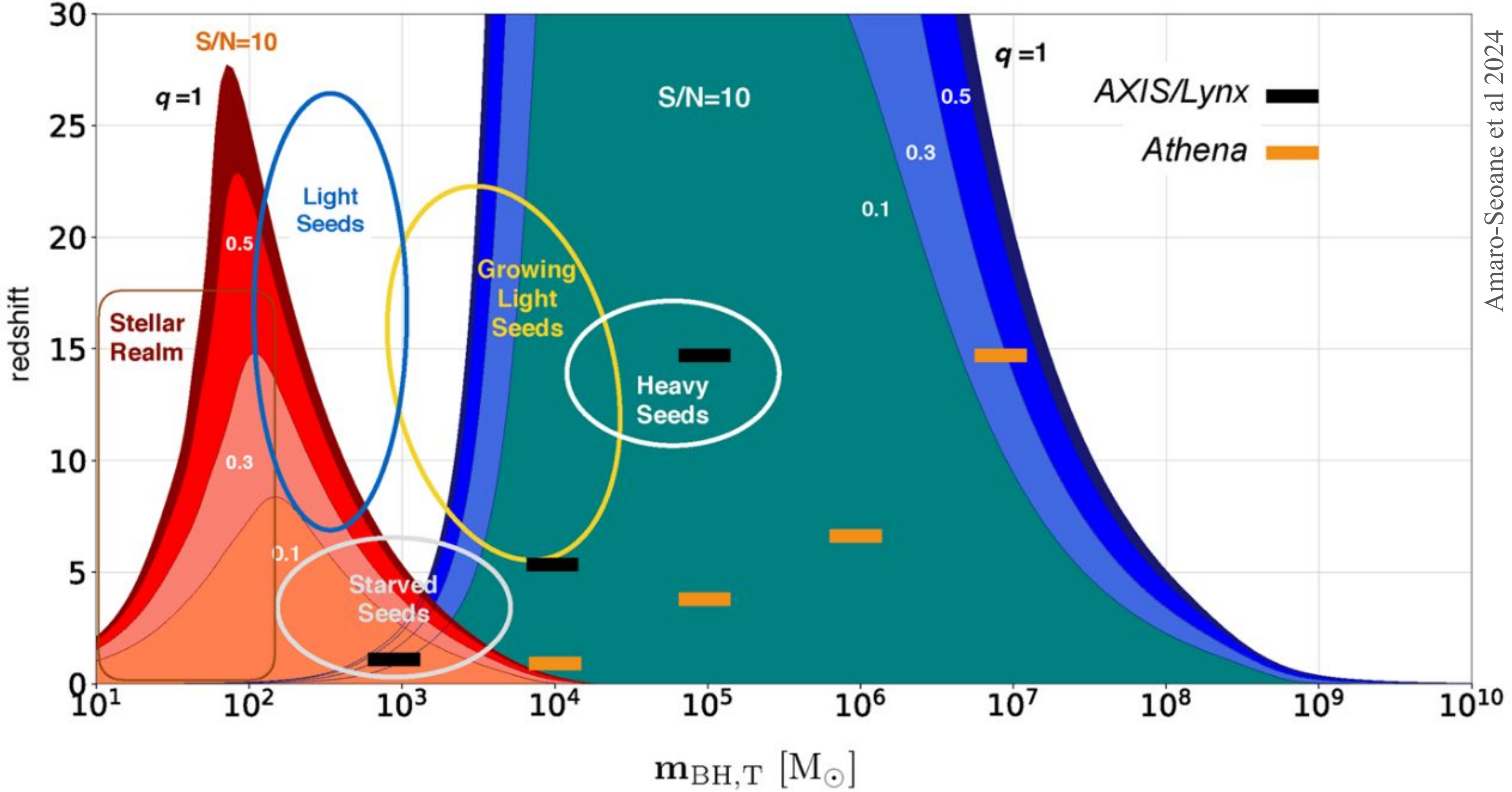
IMBHs as gravitational-wave sources: heavy - IMRIs

Heavy IMRIs: IMBH + SMBH
- Minor galaxy mergers & Nuclear clusters
(Ebisuzaki+01, Portegies-Zwart+06, Arca Sedda&Gualandris18, Arca Sedda&Capuzzo-Dolcetta19, Fragione22)

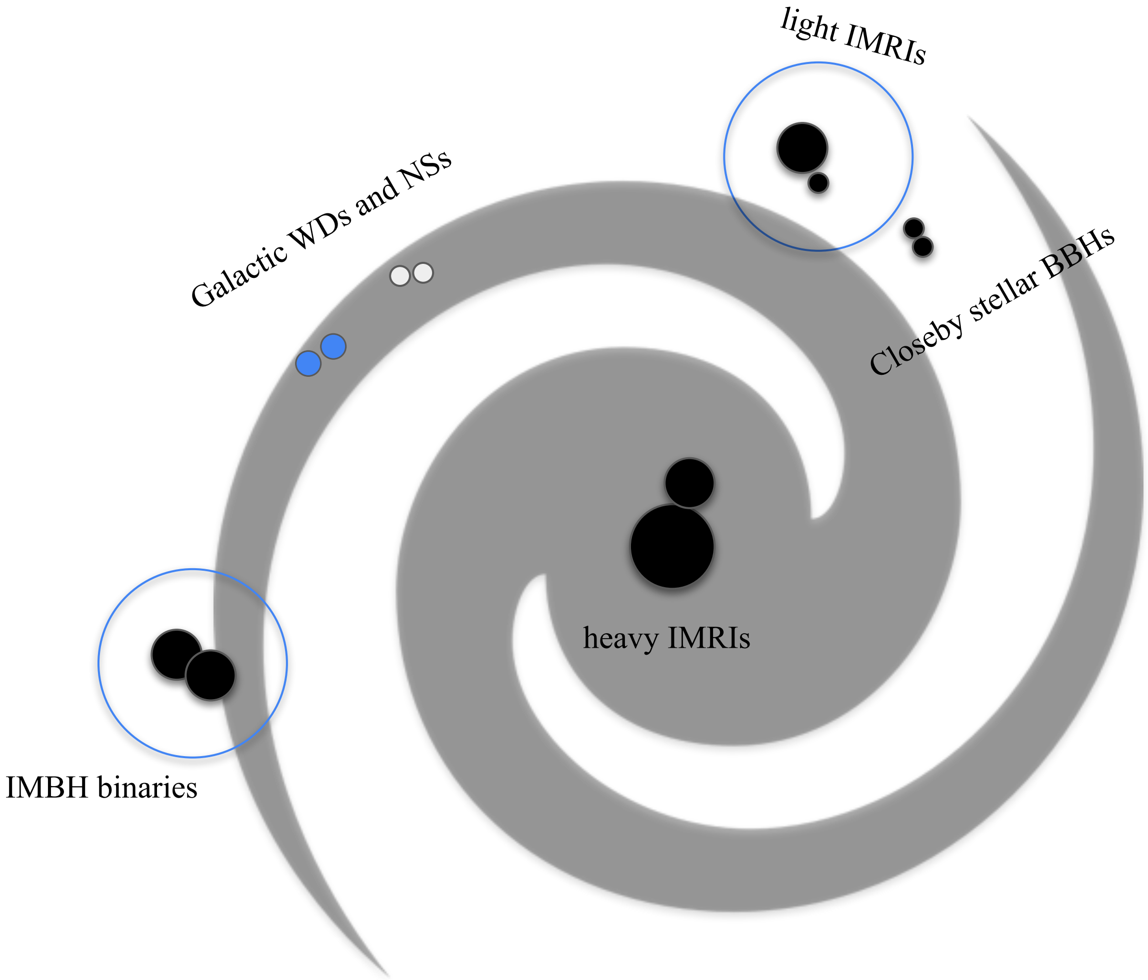






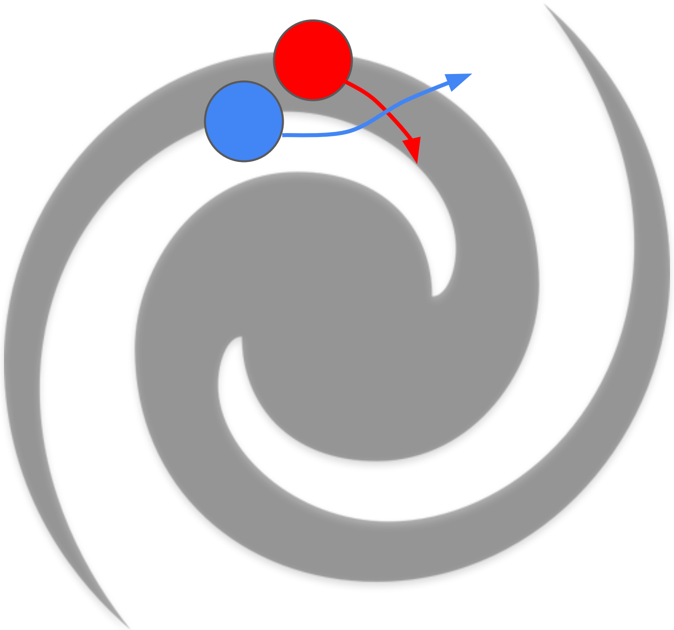


Amaro-Seoane et al 2024


light IMRIs
Galactic WDs and NSs
Closeby stellar BBHs
heavy IMRIs
IMBH binaries

IMBH binaries in merging clusters

- Merging star cluster in galactic discs



- In a MW-sized galaxy we expect 1 SC merger per Gyr
(e.g. Khopersov+18)

- If $P_{IMBH} \sim 20\%$ →1-2 IMBHB per galactic disc in 10 Gyr
(e.g. Arca Sedda & Mastrobuono-Battisti19)

- At redshift < 1 the number of MW-like galaxies is (e.g. Abadie+10)

$$N_G = \frac{4\pi}{3}\left(\frac{D_L}{1\mathrm{Mpc}}\right)^3 (2.26)^{-3} 0.0116$$

- Naively, the number of mergers per yr

$z < 1 \rightarrow R = 0.1\ \mathrm{yr}^{-1}$
$z < 2 \rightarrow R = 1.7\ \mathrm{yr}^{-1}$
$z < 3 \rightarrow R = 7.4\ \mathrm{yr}^{-1}$

IMBH binaries in merging clusters

MBH binary evolution

Phase #1: host systems merge

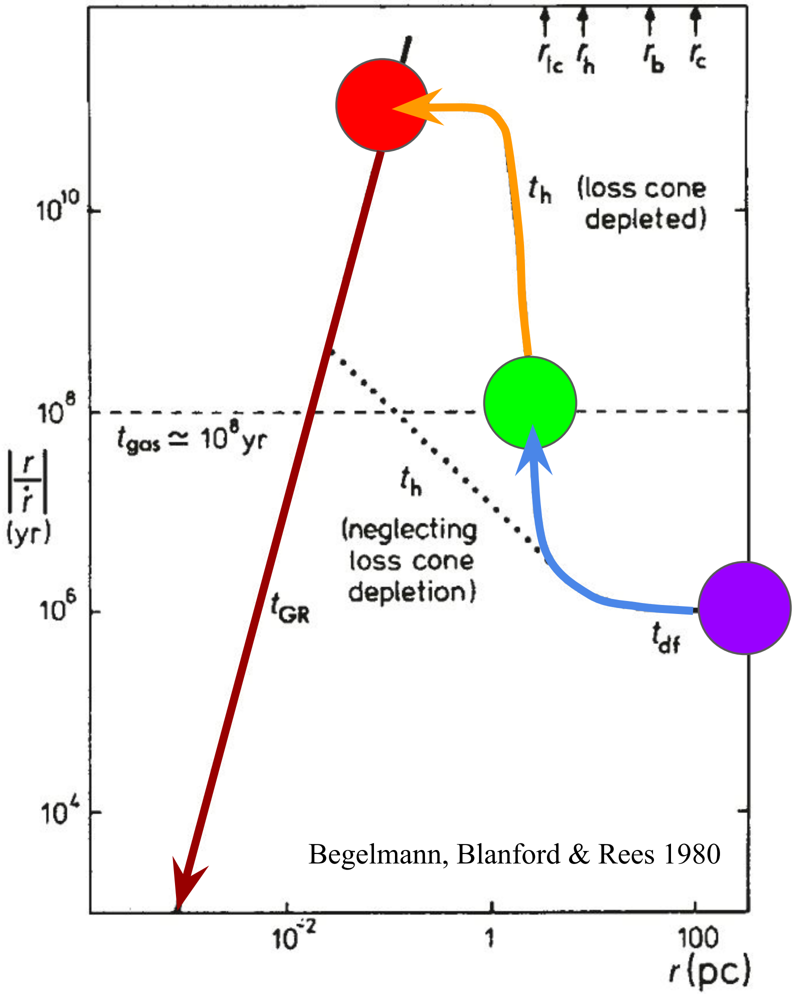
Phase #2: the MBHs inspiral in the merger remnant owing to dynamical friction

Phase #3: the MBHs find each other and form a bound pair

Phase #4: close passages of stars with angular momentum < than a threshold (loss cone) pass through the MBH binary orbit and steal energy, causing the hardening

Phase #5: the binary shrunk so much that GW emission becomes the dominant hardening process

Phase #6: the MBH binary merge and the remnant receives a kick as large as 10^4 km/s



Begelmann, Blanford & Rees 1980

IMBH binaries in merging clusters

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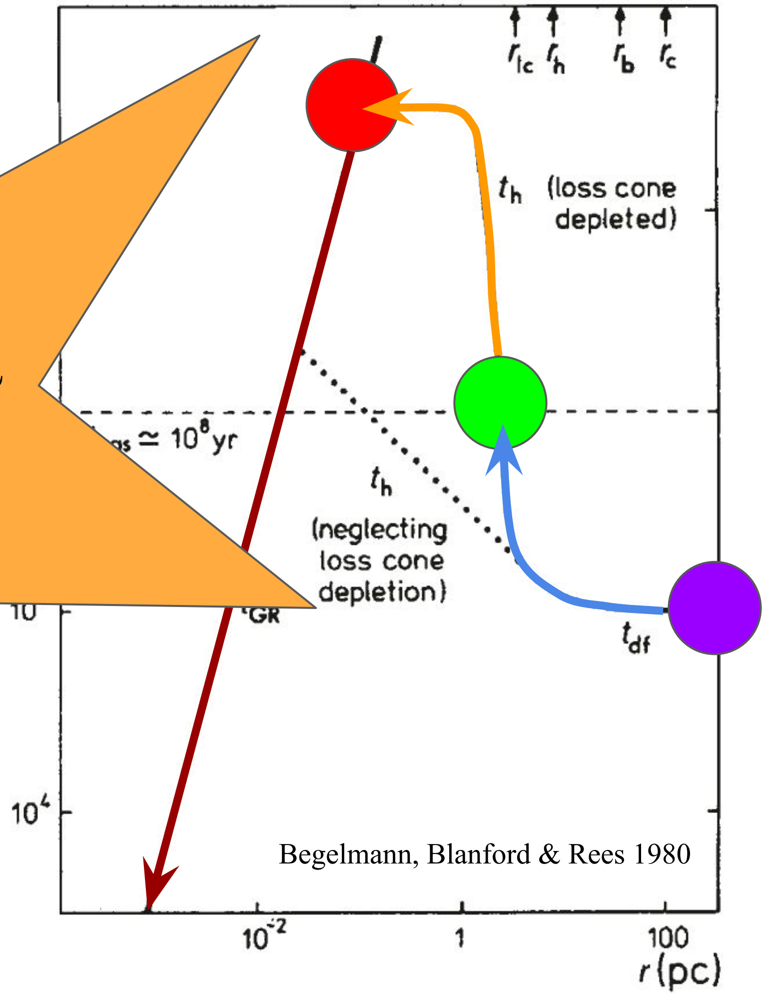
See Elisa Bortolas' lecture tomorrow!!



Begelmann, Blanford & Rees 1980