THE WORLD IN ELEVEN DIMENSIONS

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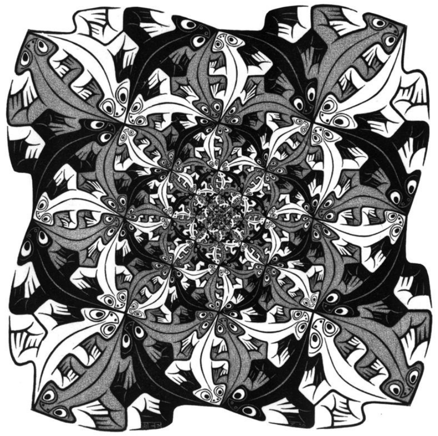
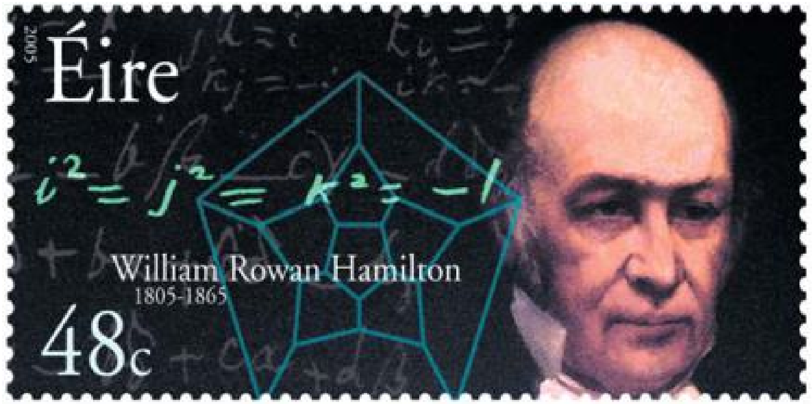
ANNO MCCLXXII

THE WORLD IN ELEVEN DIMENSIONS

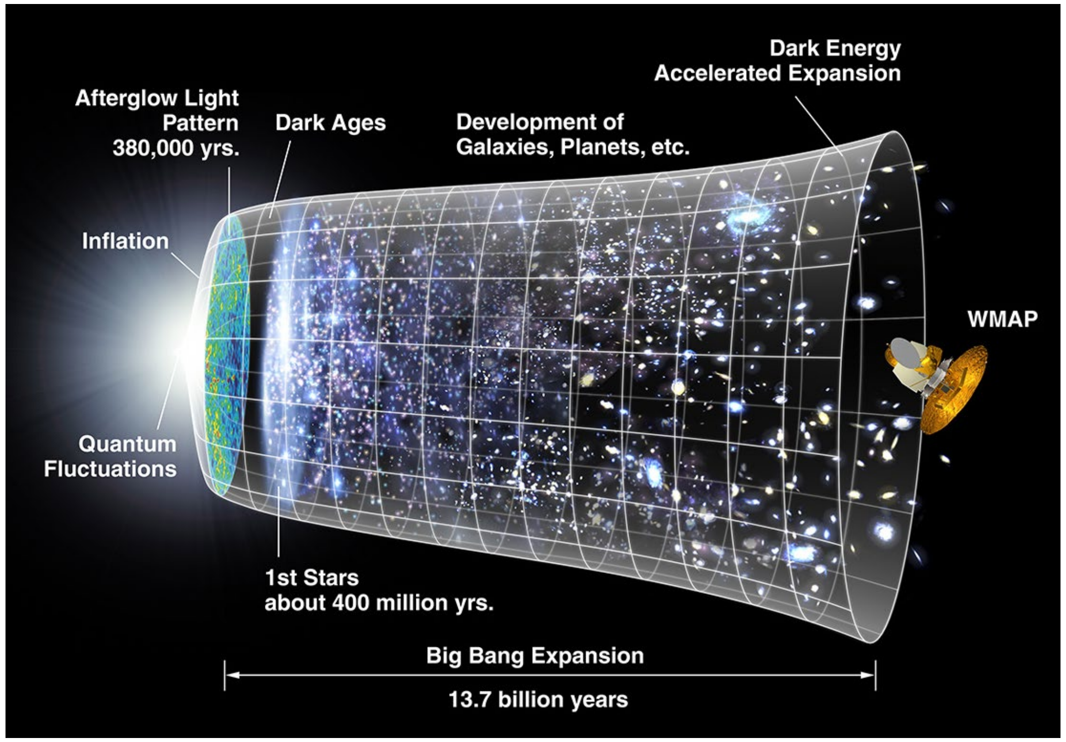
- Part I: Symmetry



- Part II: The division algebras



- Part III: M-theory



WHAT IS A SYMMETRY?

- An operation that leaves something unchanged
- M. C. Esher: illustrator of symmetry

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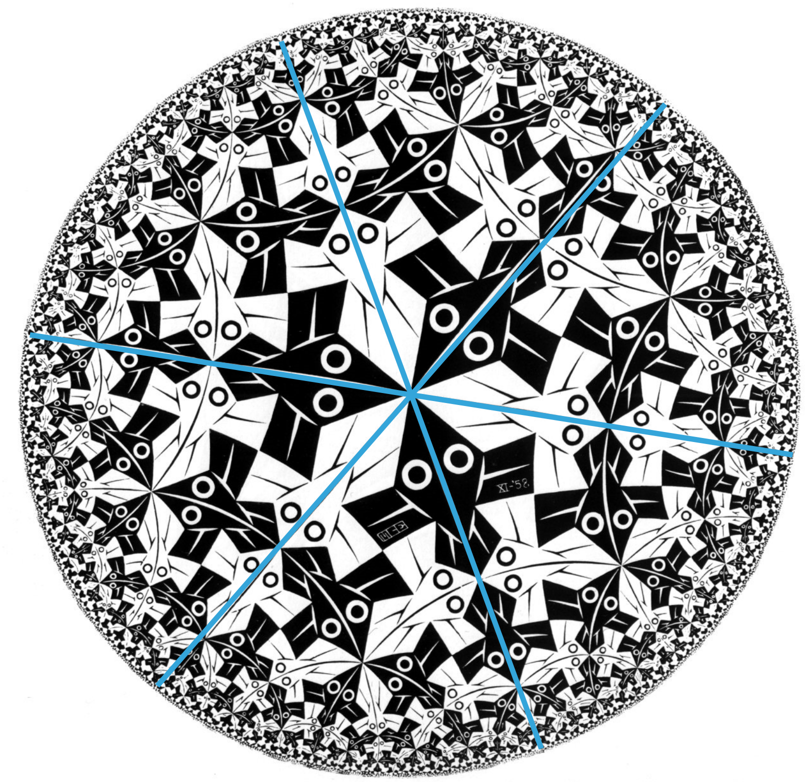
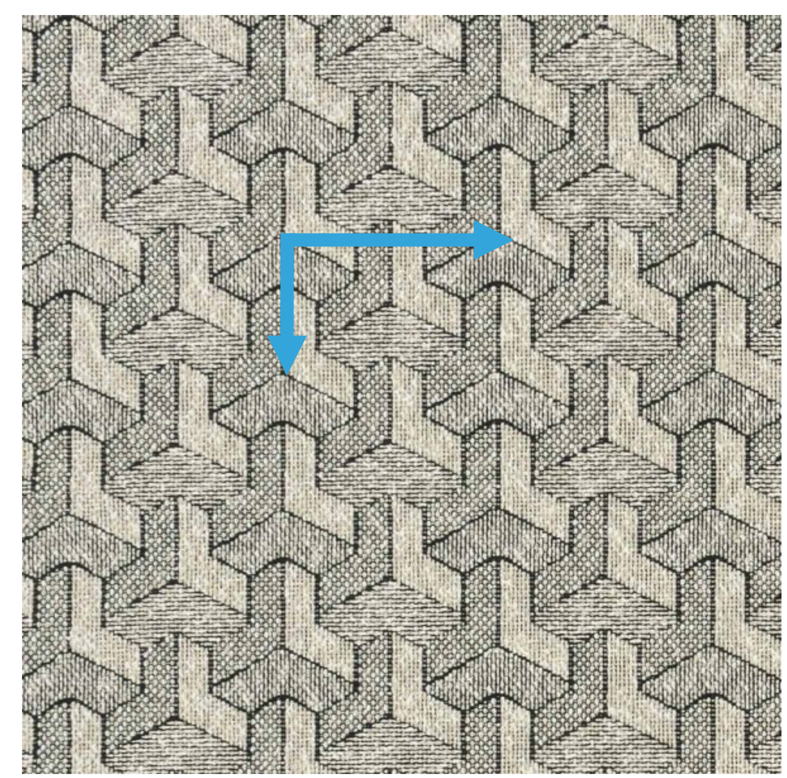
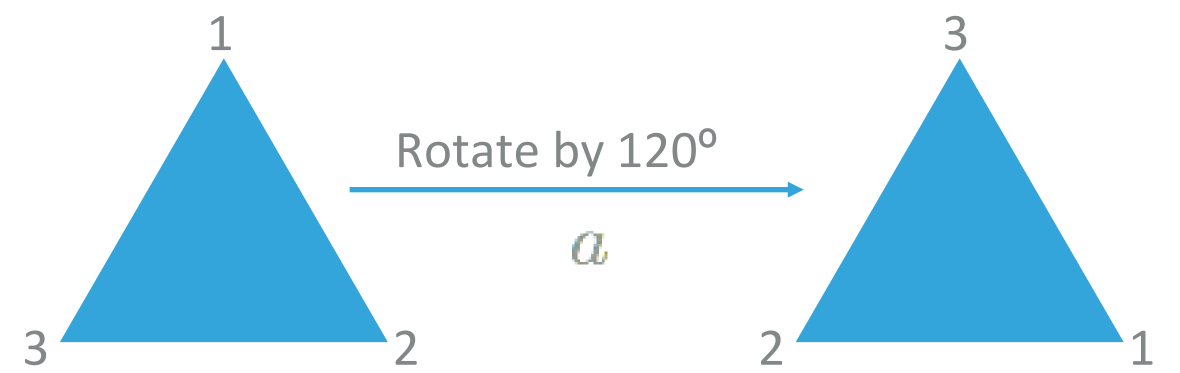
- Reflection
- Translation
- Mathematical articulation: Group Theory





GROUPS:



- $G = \{a, b, c \ldots\}$ s.t. $\forall\, a, b, c \ldots \in G$
- Closure: $a \circ b \in G$
- Identity: $\exists\, e \quad s.t. \quad e \circ a = a = a \circ e$
- Inverse: $\exists\, a^{-1} \quad s.t. \quad a^{-1} \circ a = e$
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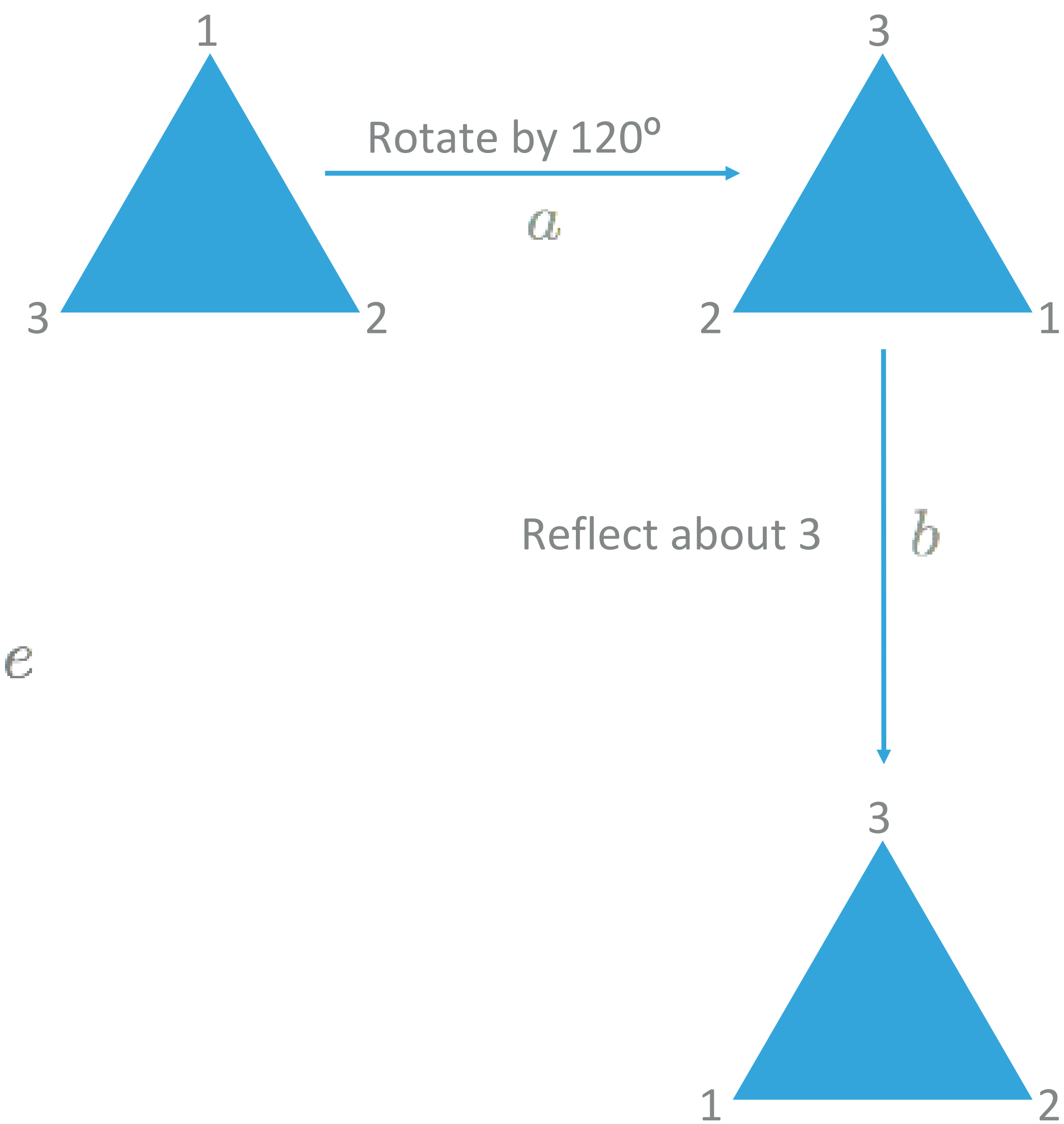

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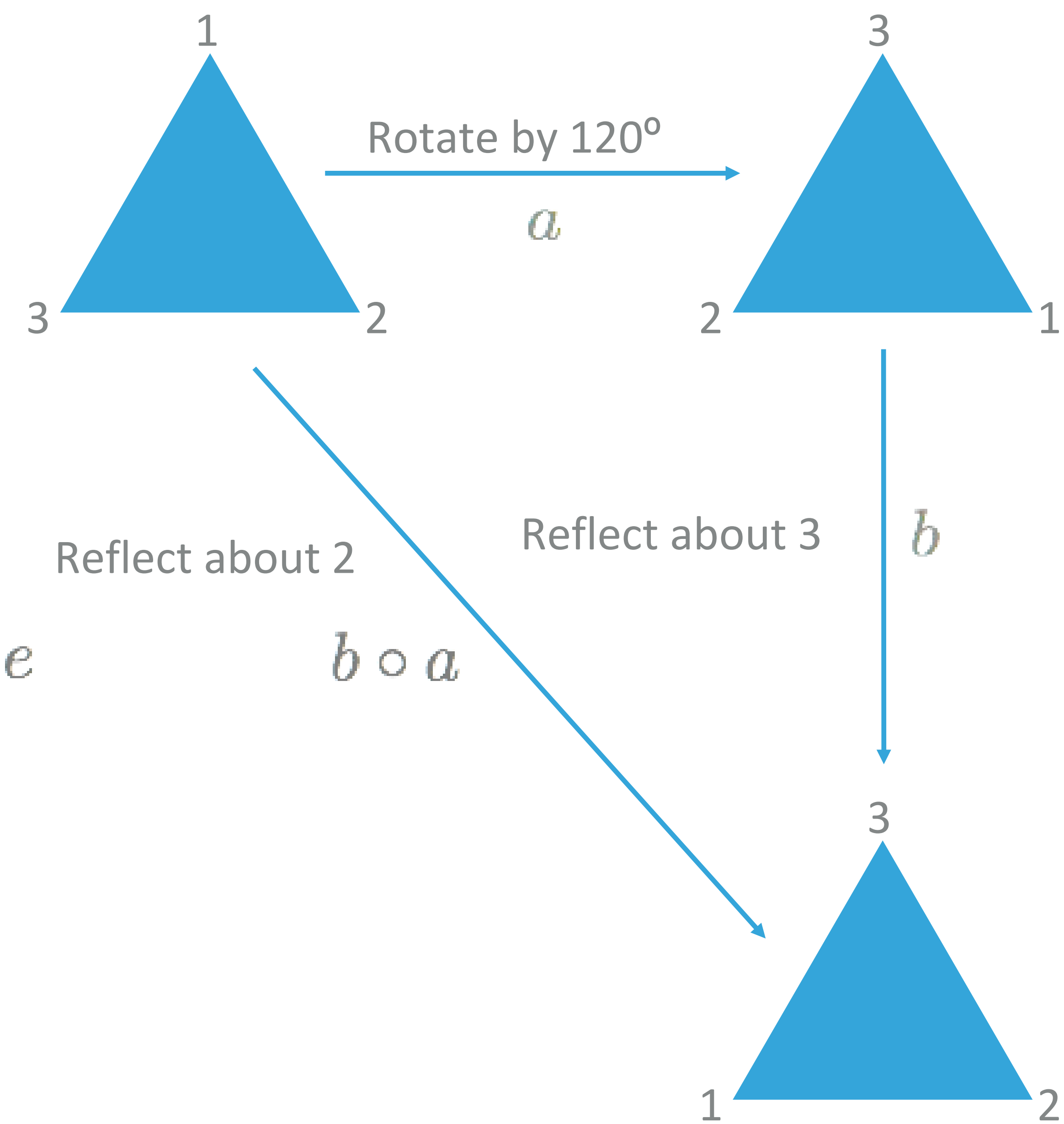

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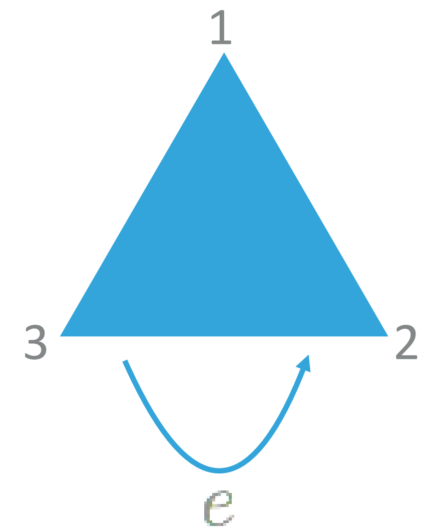

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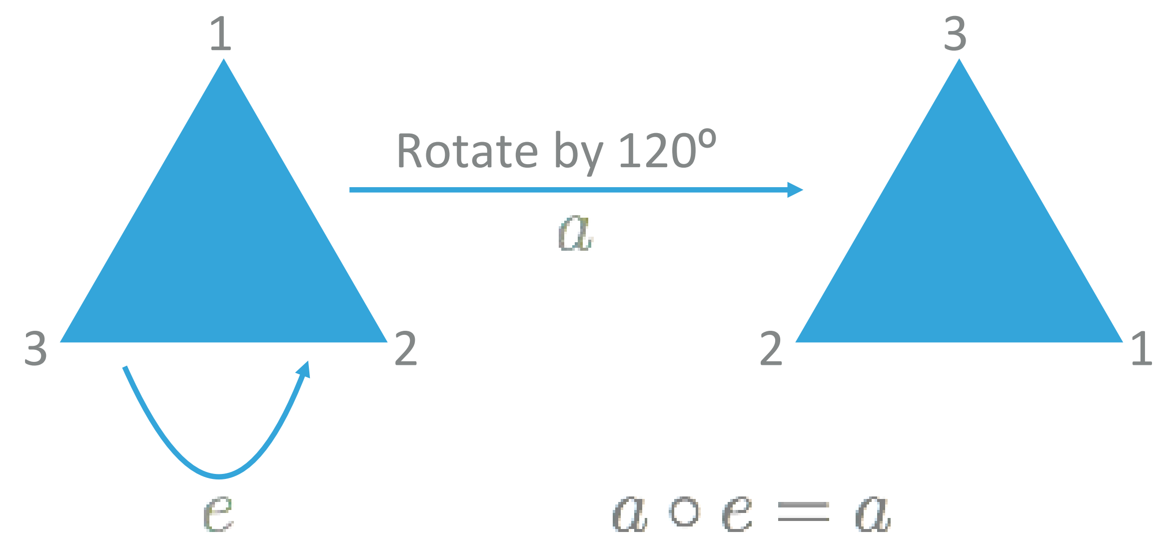

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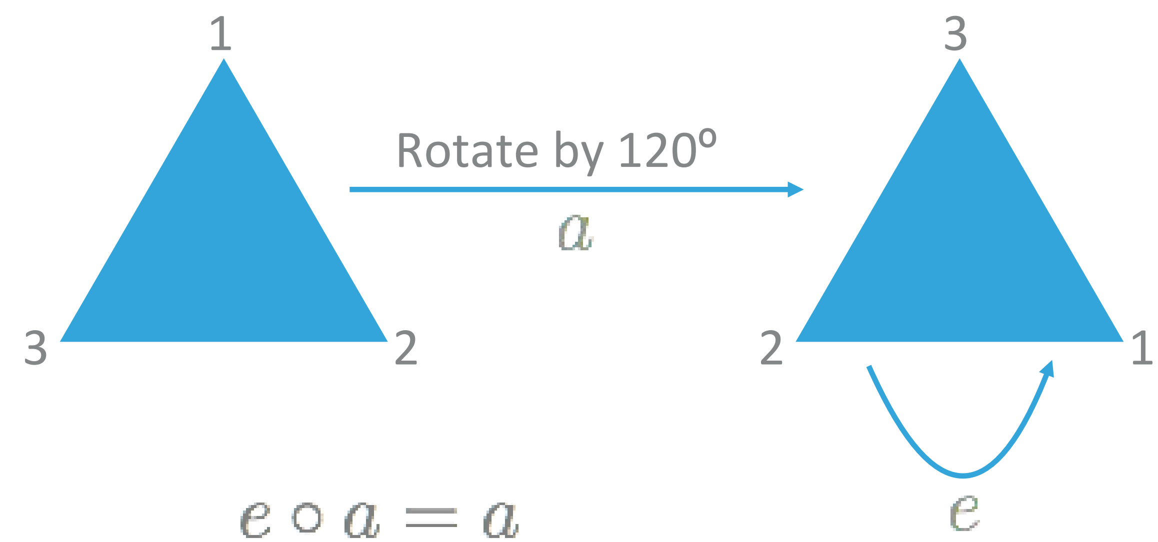

$e \circ a = a$

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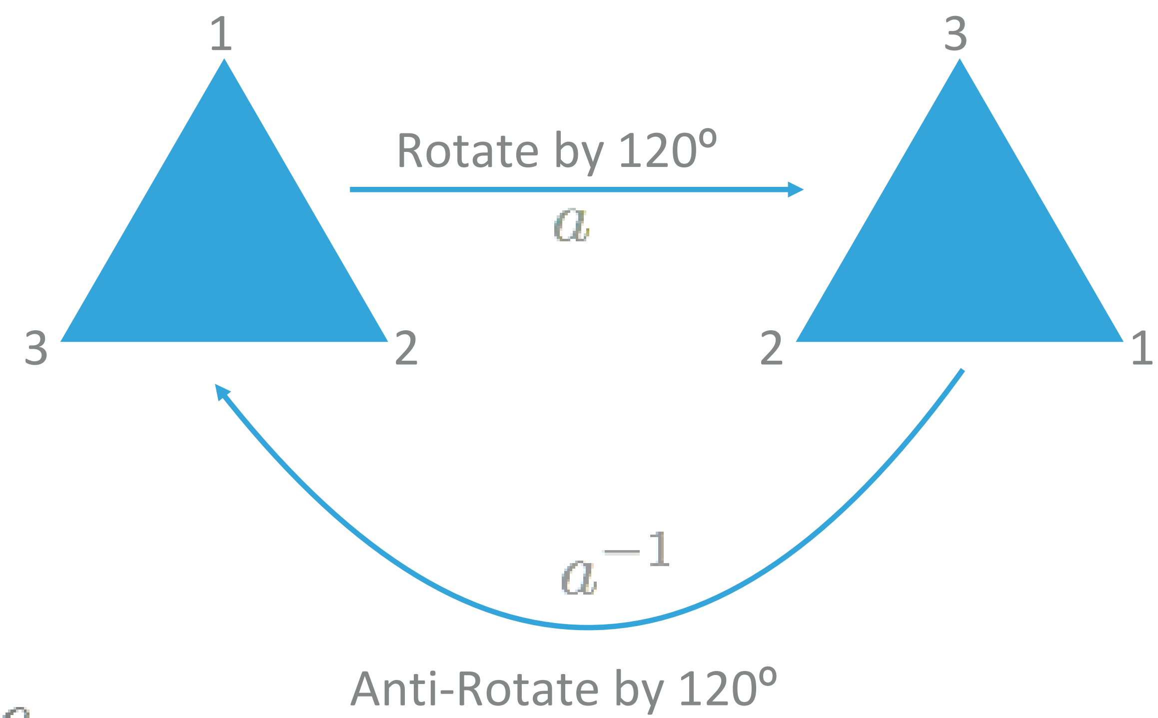

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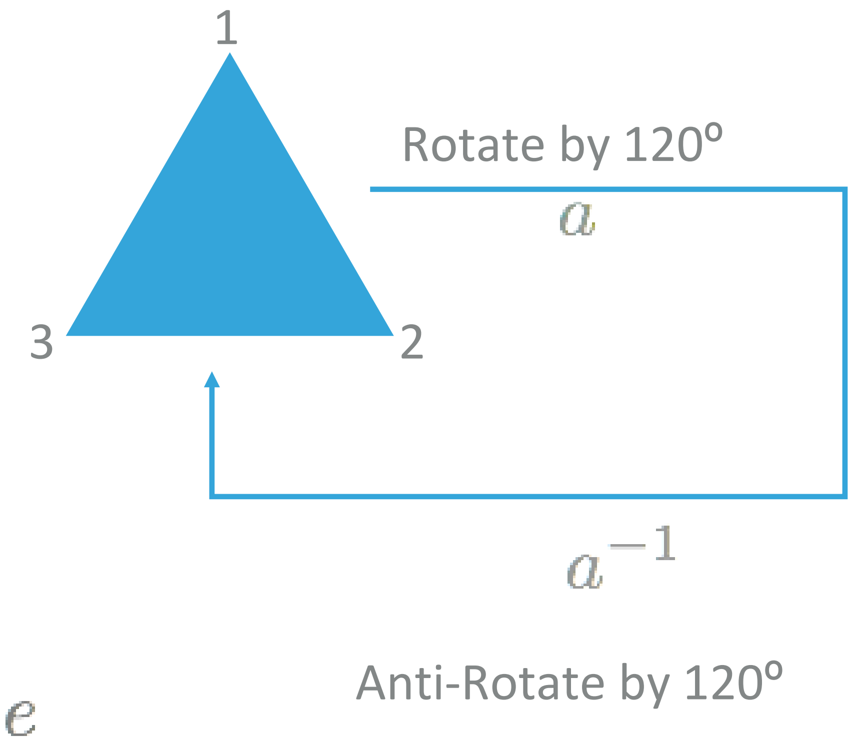

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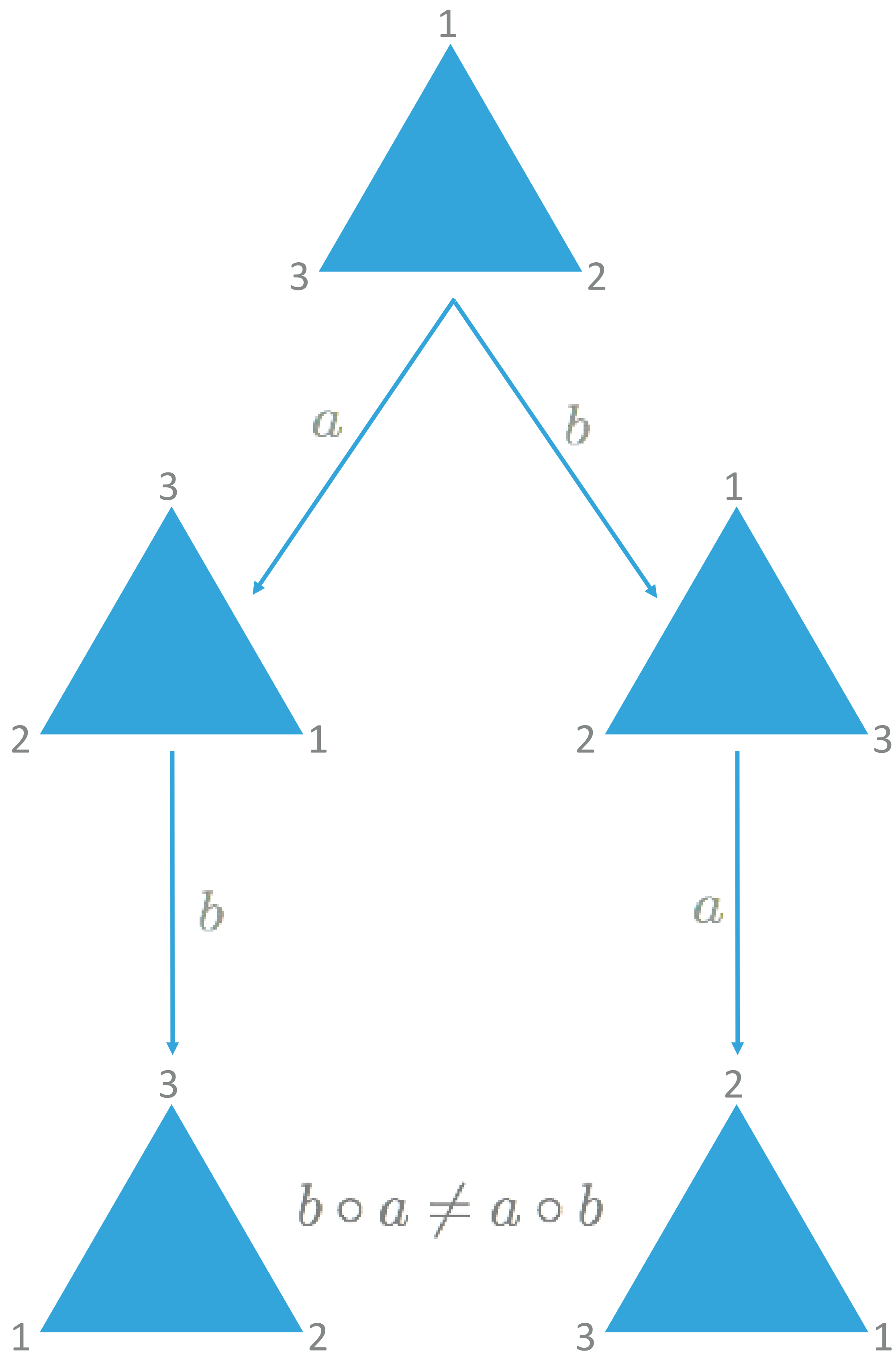
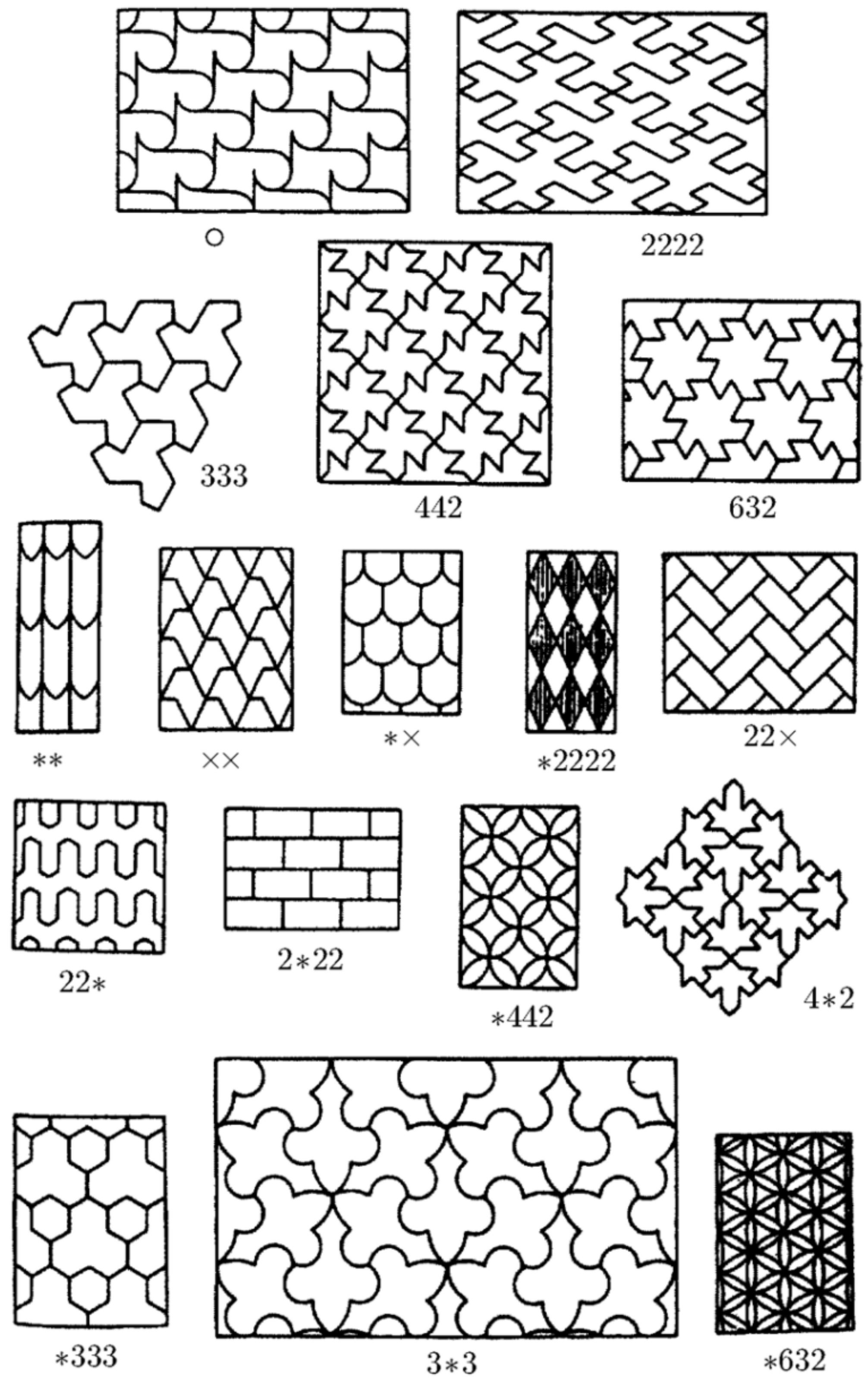


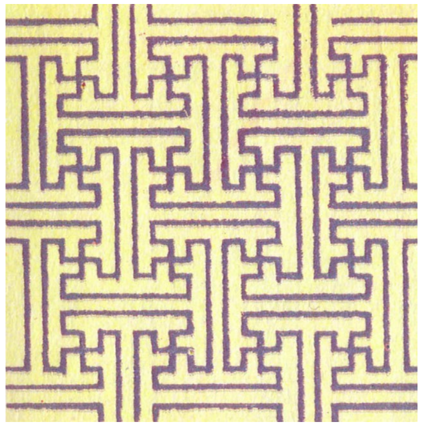
ABSTRACTION AND CLASSIFICATION

- 17 Wallpaper groups, Fedorov 1891
  (Illustration by Pólya 1924)

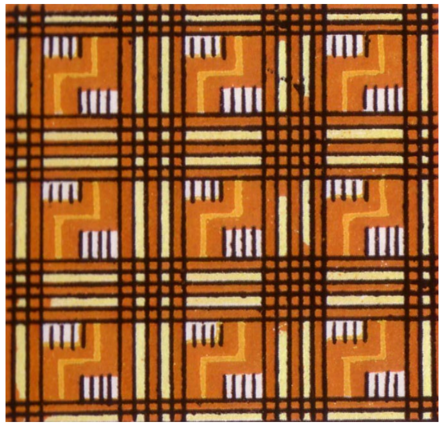


ABSTRACTION AND CLASSIFICATION

- 17 Wallpaper groups, Fedorov 1891 (Illustration by Pólya 1924)
- Aesthetically compelling
- All (or only 13?) found in the Alhambra palace, Granada



Porcelain, China



Cloth, Sandwich Islands

Examples from Alhambra palace

THE ENORMOUS THEOREM

- Finite Simple Groups: “Prime” building blocks
- 3 infinite families and 26 sporadic groups (10,000 pages!)

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- Finite Simple Groups: “Prime” building blocks
- 3 infinite families and 26 sporadic groups (10,000 pages!)
- The Monster (aka Fischer–Griess or Friendly Giant) group:

808,017,424,794,512,875,886,459,904,961,710,757,005,754,368,000,000,000

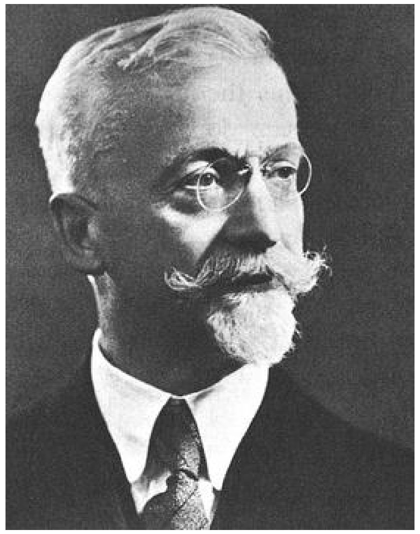
Smallest faithful representation: 196,883 dimensional complex vector space

- McKay 1978: 196,884 ≈ 196,883….Monstrous Moonshine
- Moonshine Beyond the Monster (Terry Gannon)

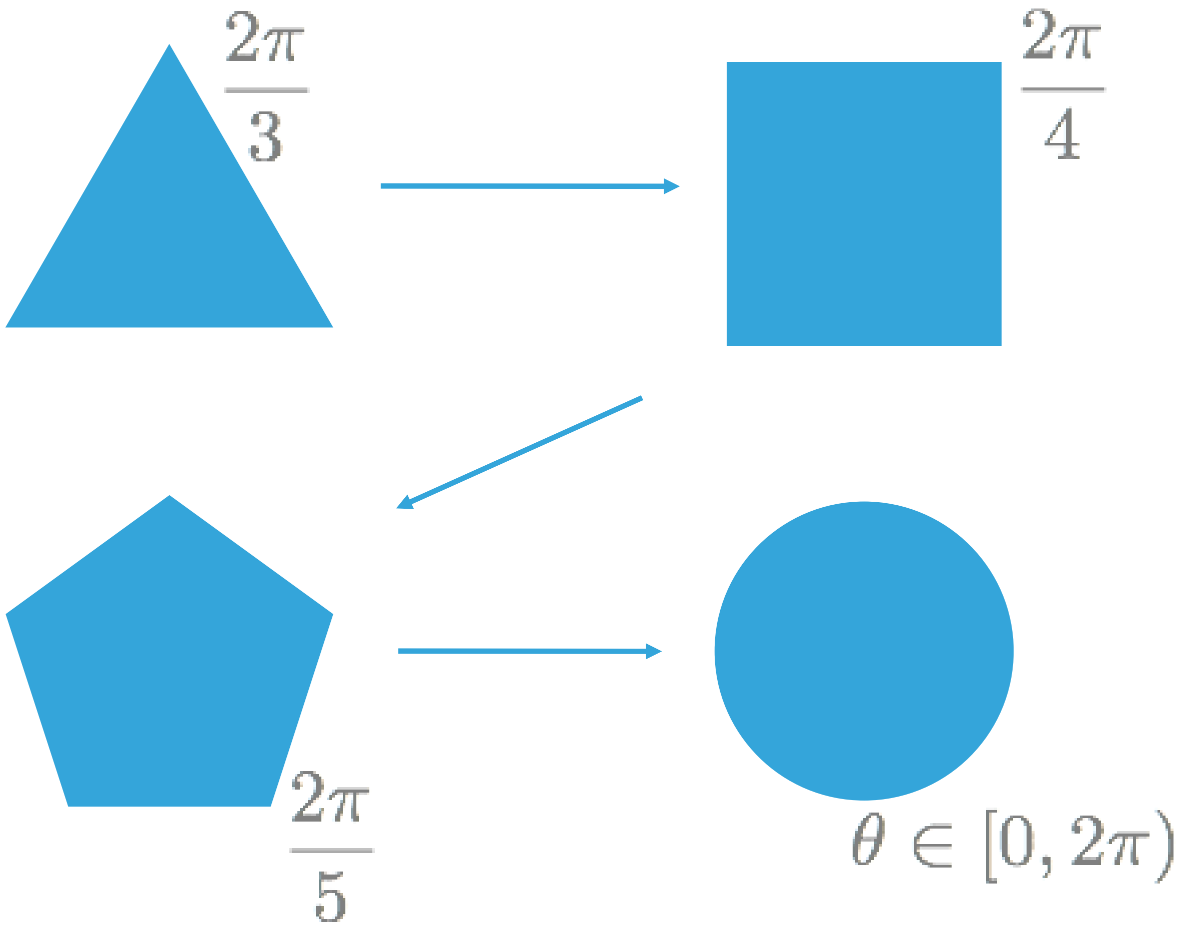
LIE GROUPS

- Continuous symmetry groups

Sophus Lie



Élie Cartan



LIE GROUPS

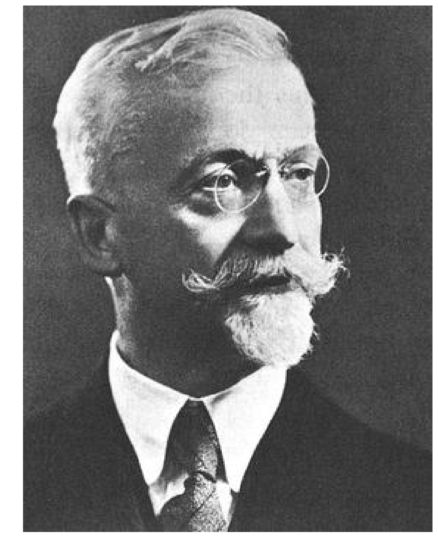
- Continuous symmetry groups
- Classification:

$SO(1,3)$
Lorentz group

Special Orthogonal	$SO(n)$
Special Unitary	$SU(n)$
Symplectic	$Sp(n)$
Exceptional	G_2, F_4, E_6, E_7, E_8

- Special Orthogonal = Rotations
- Exceptional Lie groups are geometrically enigmatic

Sophus Lie



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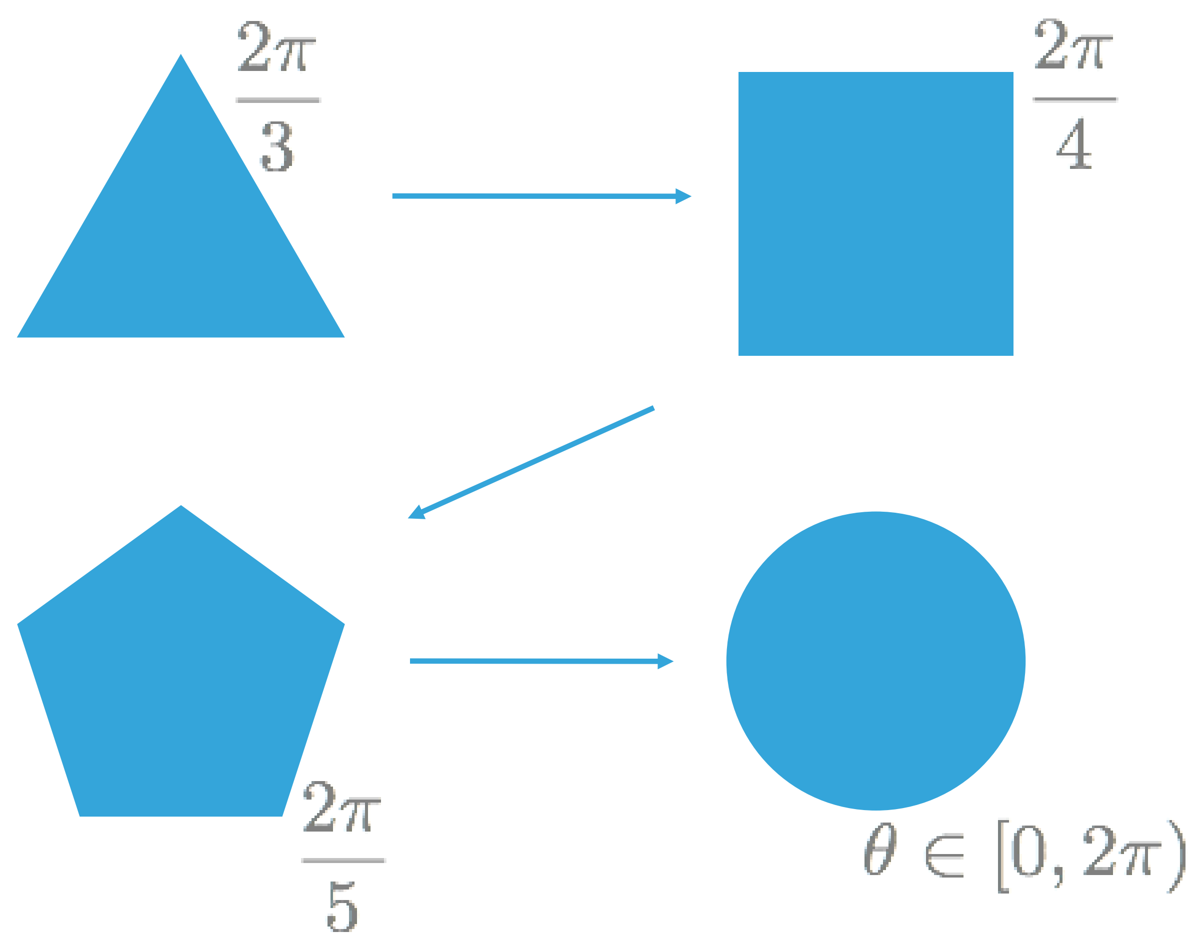
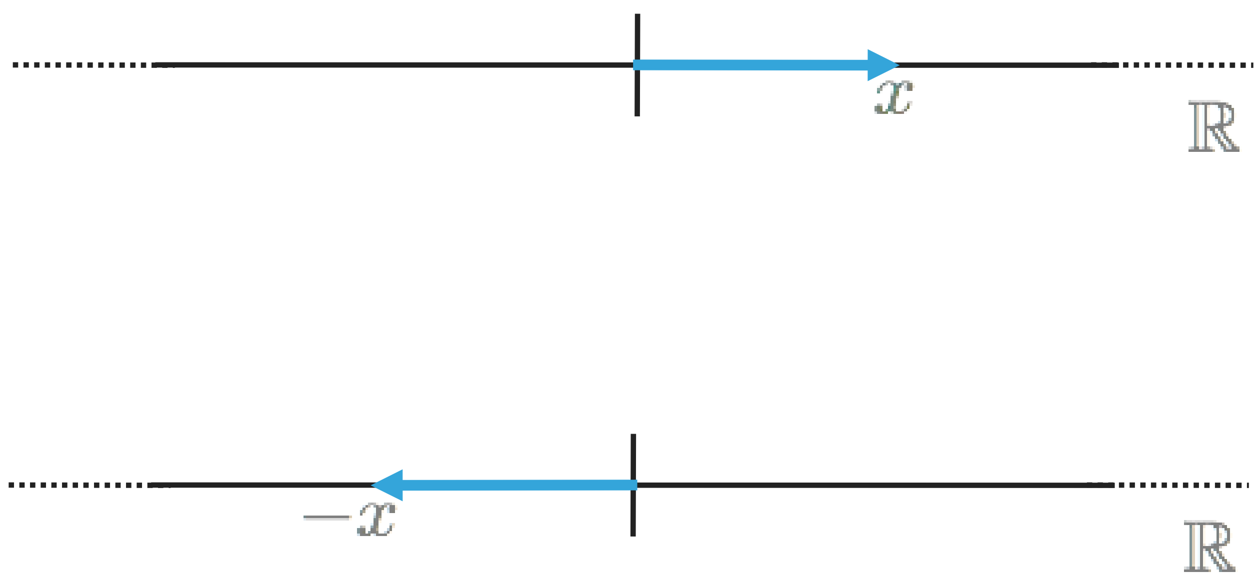


THE ALGEBRA OF SYMMETRY

THE REAL LINE

- Multiplication by ± 1 preserves lengths-squared on the real line $\mathbb{R}$

$$x \mapsto x' = \pm x, \quad |x'|^2 = |x|^2$$



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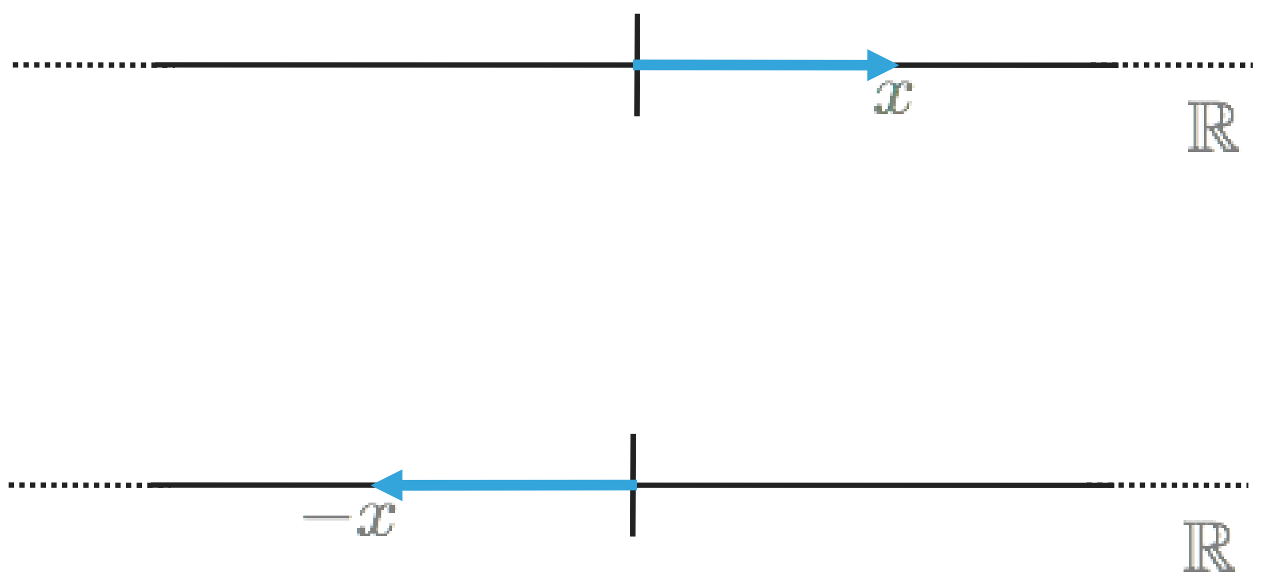
- Finite group

$$O(1) = \{1, -1\}, \quad \circ \to \times$$

- Isomorphic to

$$\mathbb{Z}_2 = \{0, 1\}, \quad \circ \to + \mod 2$$

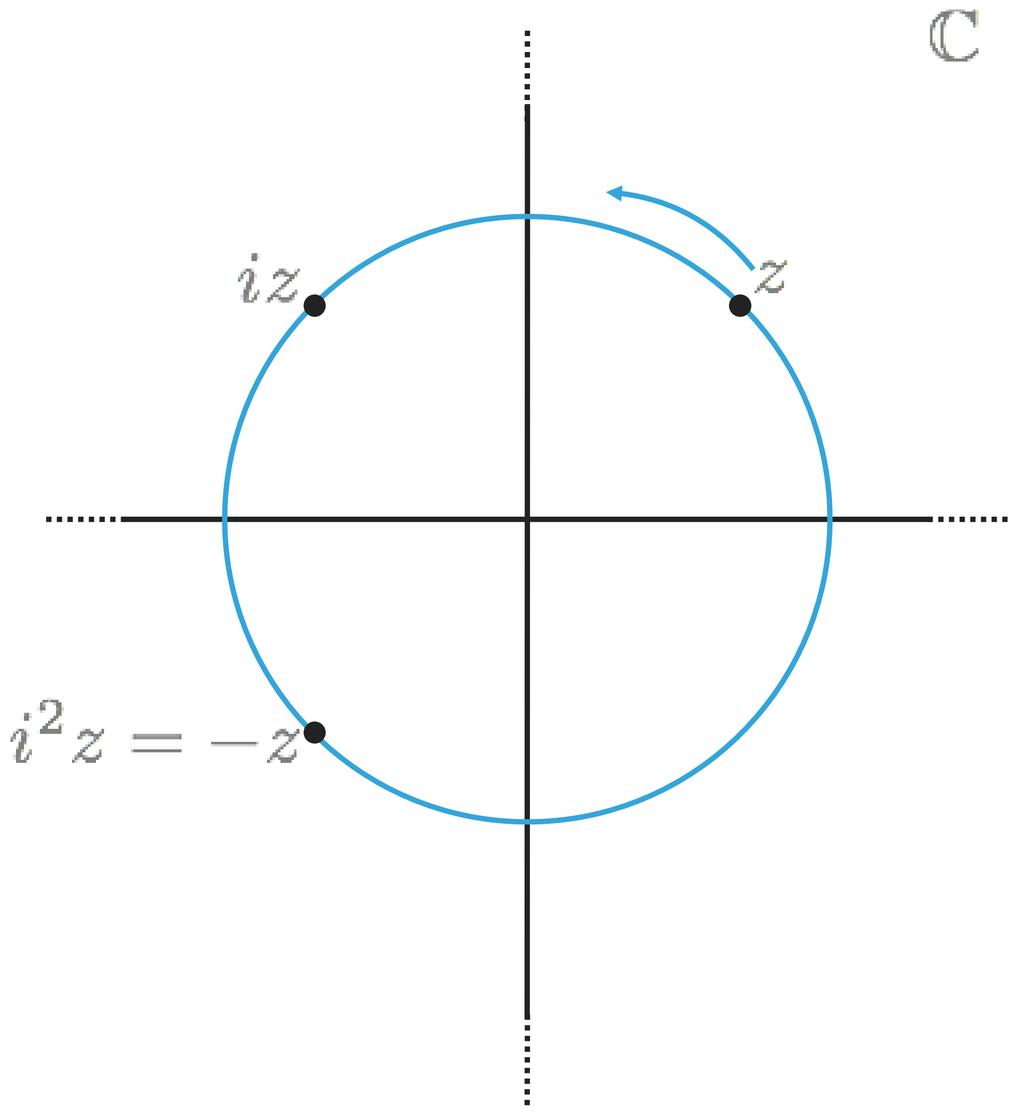


COMPLEX PLANE

- Multiplication by “unit” complexes

$$z \mapsto uz, \quad |u|^2 = u\overline{u} = 1$$

$$|uz|^2 = uz\overline{zu} = u|z|^2\overline{u} = |u|^2|z|^2 = |z|^2$$



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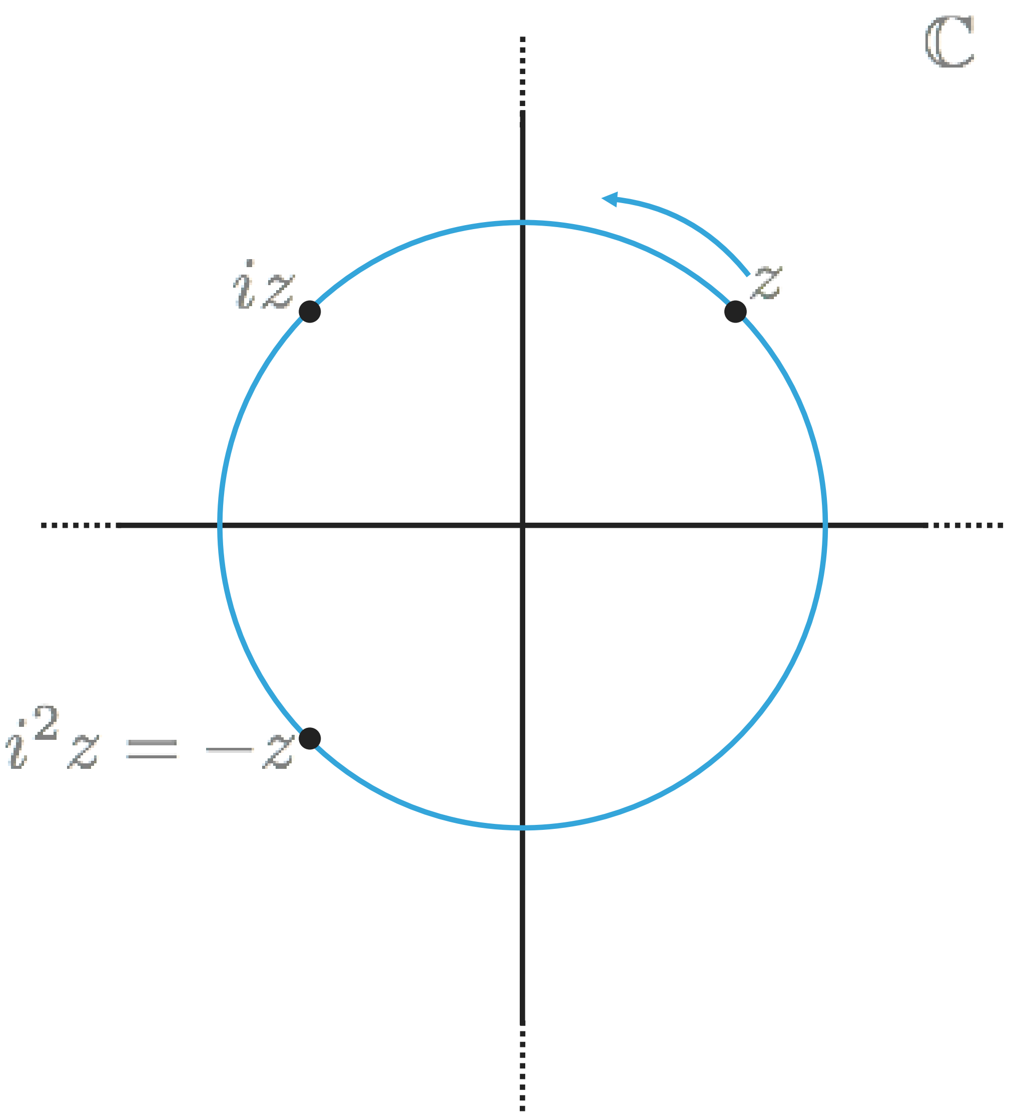
- Angle of rotation $u = e^{i\theta}$

$$e^{i\theta_2}(e^{i\theta_1}z) = (e^{i\theta_2}e^{i\theta_1})z = e^{i(\theta_2+\theta_1)}z$$

- 1-dimensional Lie group

$$U(1) = \{e^{i\theta}\}, \quad \circ \to \times$$

- Isomorphic to rotations in the plane



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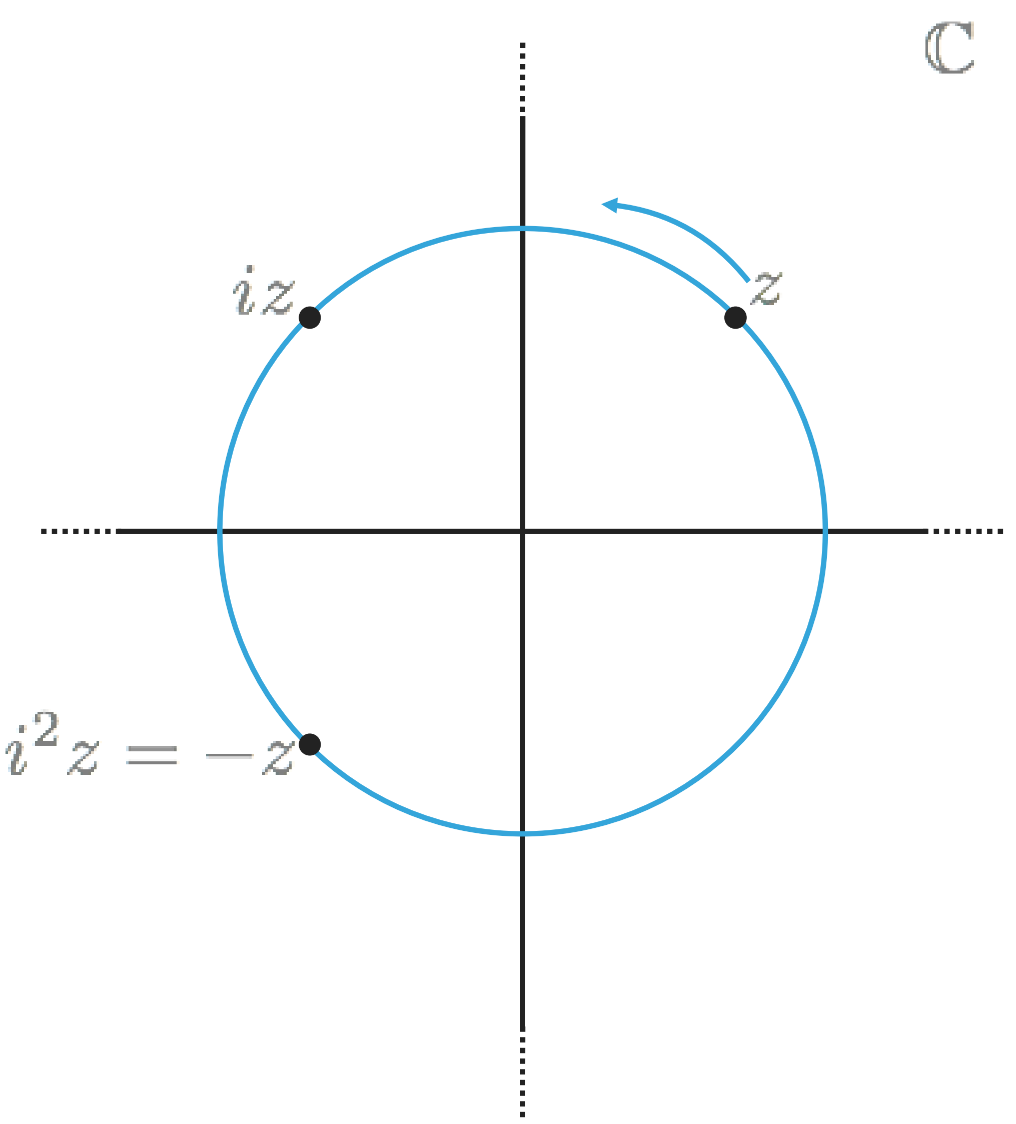
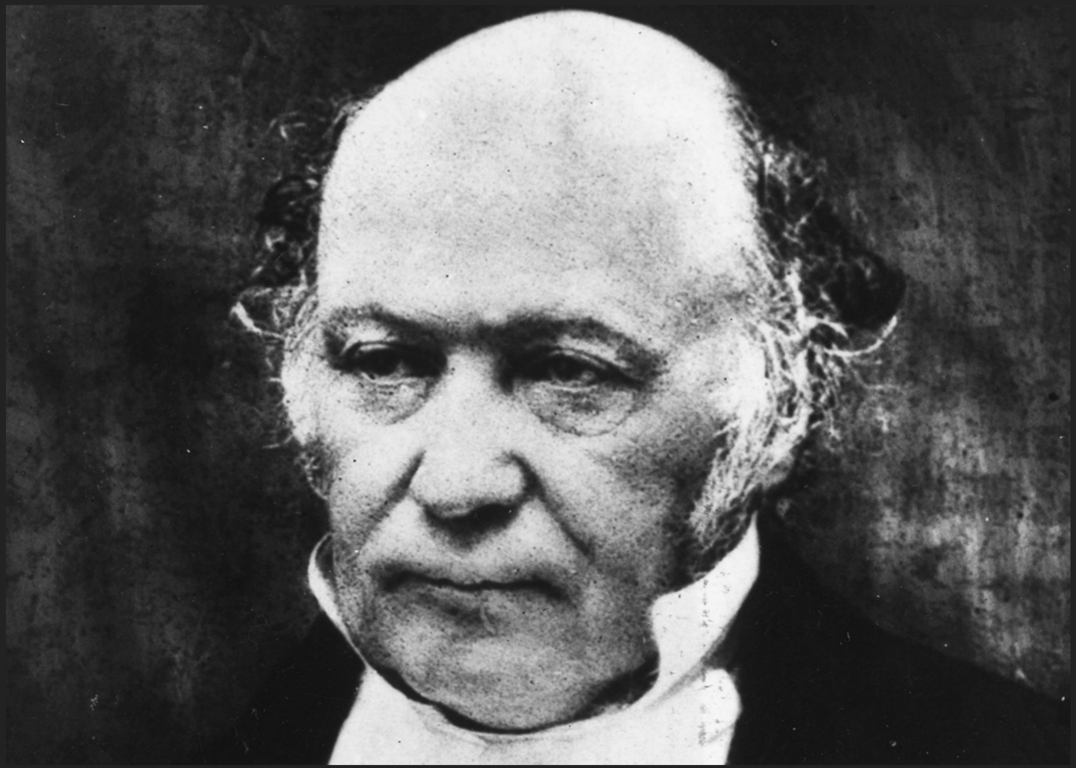
- Isomorphic to rotations in the plane $SO(2)$



HAMILTON’S LAMENT

- Real and complex multiplication: Isometry of 1- and 2-dim space
- Is there a “number” system that does the same for 3-dim space?

“Every morning in the early part of the above-cited month, on my coming down to breakfast, your (then) little brother William Edwin, and yourself, used to ask me: ‘Well, Papa, can you multiply triplets?’ Whereto I was always obliged to reply, with a sad shake of the head: ‘No, I can only add and subtract them’.”



THE ROAD FROM DUNSINK

- Hamilton's epiphany:

On the 16th of October, 1843, walking his wife from Dunsink to a meeting of the Royal Irish Academy on Dawson street:

"That is to say, I then and there felt the galvanic circuit of thought close; and the sparks which fell from it were the fundamental equations between *i, j, k*; exactly such as I have used them ever since."

Hamilton

THE ROAD FROM DUNSINK

- Hamilton's epiphany:

$$i^2 = j^2 = k^2 = ijk = -1$$

16th of October 1843

- The 4-dimensional quaternions, denoted $\mathbb{H}$, where born

- Non-commutative

$$ij = k, \; ji = -k$$

Leron Borsten and me at Dunsink bridge, Dublin, IE

NORMED DIVISION ALGEBRAS

- Generalising the number systems $\mathbb{R}$ and $\mathbb{C}$
- We want to be able to add and multiply - an algebra $\mathbb{A}$ with a unit element 1

$$1a = a1 = a, \quad \forall a \in \mathbb{A}$$

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- A normed division algebra is a normed vector space satisfying the composition property:

$$||ab|| = ||a||||b||, \quad \forall a, b \in \mathbb{A}$$

$$|zw| = |z||w|, \ \forall z, w \in \mathbb{R} \text{ or } \mathbb{C}$$

- Implies division property: $ab = 0 \ \Rightarrow \ a \text{ and/or } b = 0$

WHY STOP AT FOUR?

- We now have

$$\mathbb{R}: \; 1$$
$$\mathbb{C}: \; 1, i$$
$$\mathbb{H}: \; 1, i, j, k$$

- Can we keep going?

- On October 26th John T. Graves, a college friend of Hamilton, wrote with this possibility on his mind

"There is still something in the system which gravels me. I have not yet any clear views as to the extent to which we are at liberty arbitrarily to create imaginaries, and to endow them with supernatural properties…….If with your alchemy you can make three pounds of gold, why should you stop there?"

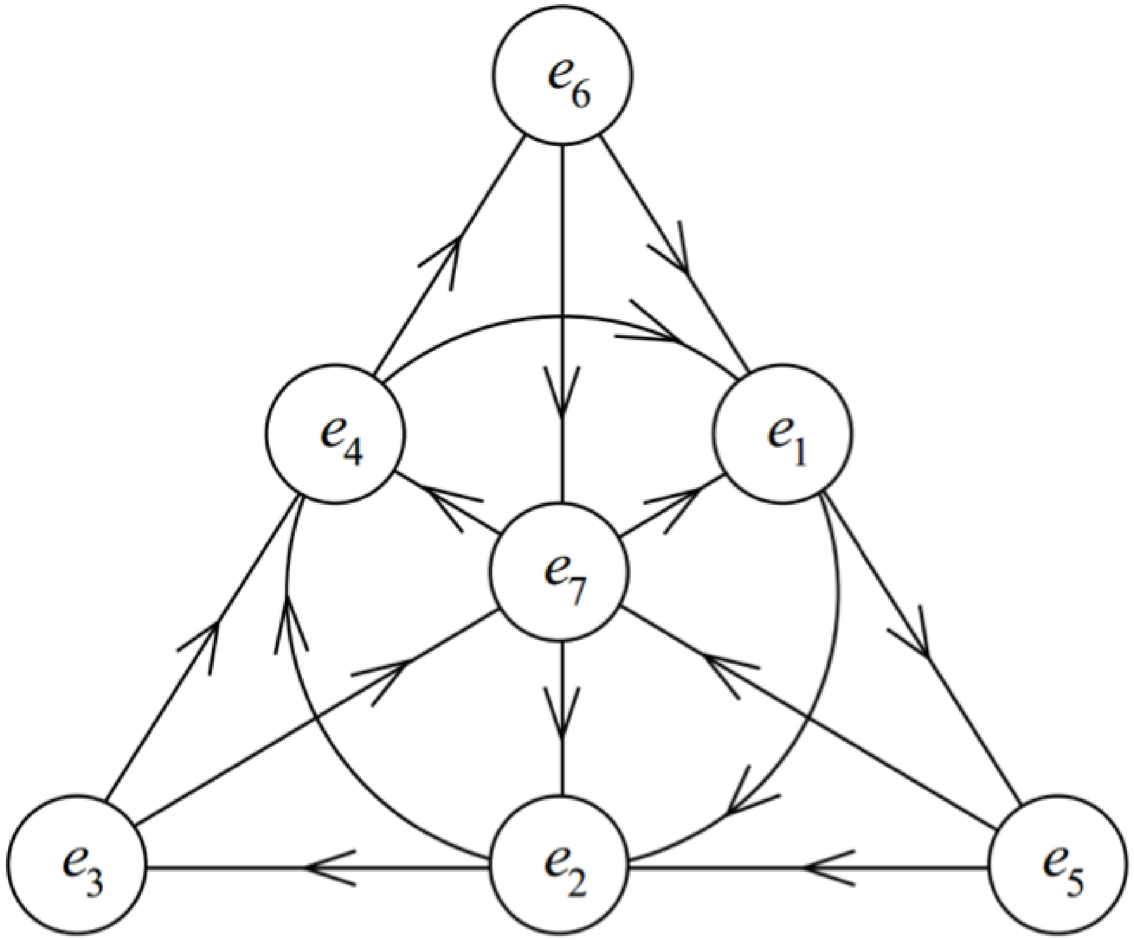
Graves

THE OCTONIONS

- December 26th of that year Graves discovers the octonions

$$\begin{aligned}\mathbb{R} &: \ 1 \\ \mathbb{C} &: \ 1, e_1 \\ \mathbb{H} &: \ 1, e_1, e_2, e_4 \\ \mathbb{O} &: \ 1, e_1, e_2, e_3, e_4, e_5, e_6, e_7\end{aligned}$$

- 8-dimensional normed division algebra



Fano plane

THE OCTONIONS

- Cayley-Dickson doubling

$\mathbb{R}$	ordered, commutative, associative
$\mathbb{C} = \mathbb{R} \oplus e_1\mathbb{R}$	commutative, associative
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"The real numbers are the dependable breadwinner of the family, the complete ordered field we all rely on. The complex numbers are a slightly flashier but still respectable younger brother: not ordered, but algebraically complete. The quaternions, being noncommutative, are the eccentric cousin who is shunned at important family gatherings. But the octonions are the crazy old uncle nobody lets out of the attic: they are nonassociative. "

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- These are the only normed division algebras, Hurwitz 1898

- Can double (sedenions) again but not division

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BACK TO SYMMETRIES

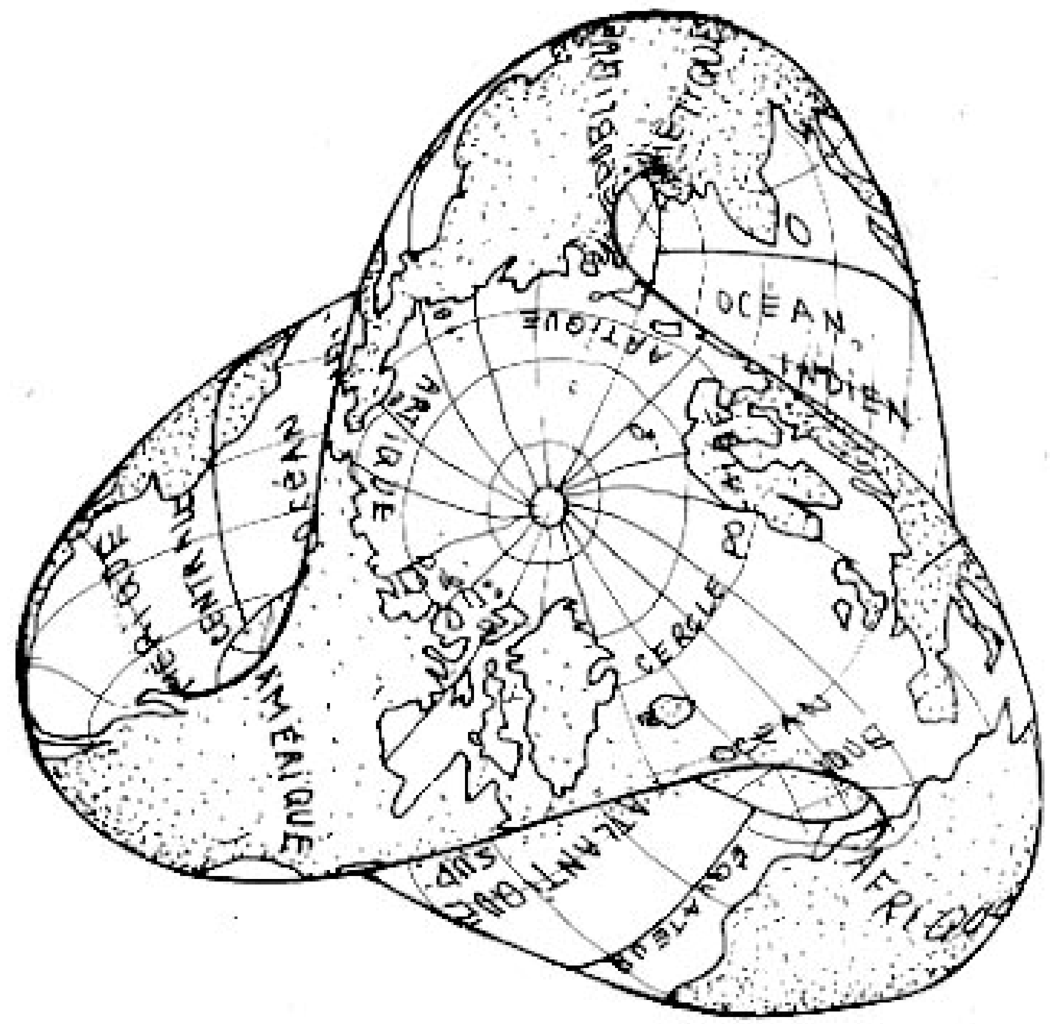
- Recall 3 families of Lie groups

$$SO(n), SU(n), Sp(n)$$

- Rotations in n-dim real, complex and quaternionic spaces

$$\begin{aligned} \mathfrak{iso}(\mathbb{RP}^n) &= \mathfrak{so}(n+1) \\ \mathfrak{iso}(\mathbb{CP}^n) &= \mathfrak{su}(n+1) \\ \mathfrak{iso}(\mathbb{HP}^n) &= \mathfrak{sp}(n+1) \end{aligned}$$

- What about $\mathfrak{iso}(\mathbb{OP}^n) = ??$



THE MAGIC SQUARE

- The Cayley plane

$$\mathrm{Iso}(\mathbb{O}\mathbb{P}^2) \ = \ F_4$$

- Construction breaks down for $n > 2$

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- The Cayley plane

$$\mathrm{Iso}(\mathbb{O}\mathbb{P}^2) = F_4$$

- Construction breaks down for $n > 2$

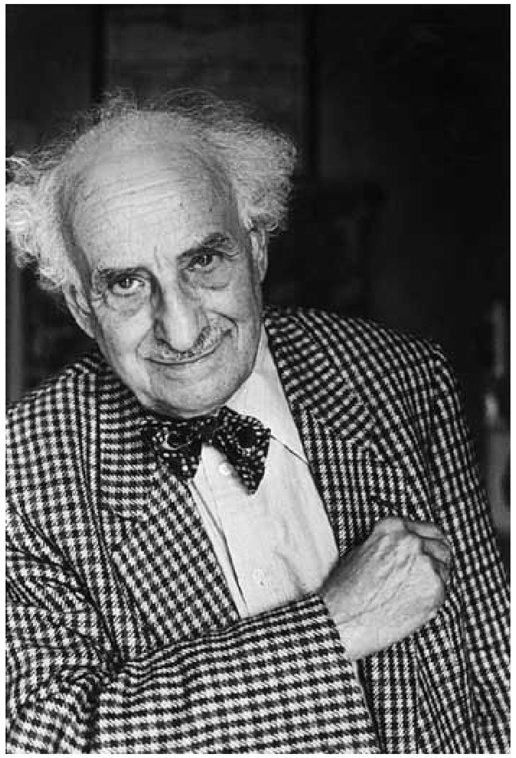
- Noting $\mathbb{R} \otimes \mathbb{O} = \mathbb{O}$, Boris Rosenfeld (1956) proposed:

$$\mathrm{Iso}((\mathbb{R} \otimes \mathbb{O})\mathbb{P}^2) = F_4$$
$$\mathrm{Iso}((\mathbb{C} \otimes \mathbb{O})\mathbb{P}^2) = E_6$$
$$\mathrm{Iso}((\mathbb{H} \otimes \mathbb{O})\mathbb{P}^2) = E_7$$
$$\mathrm{Iso}((\mathbb{O} \otimes \mathbb{O})\mathbb{P}^2) = E_8$$

	$\mathbb{R}$	$\mathbb{C}$	$\mathbb{H}$	$\mathbb{O}$
$\mathbb{R}$	$SO(3)$	$SU(3)$	$Sp(3)$	F_4
$\mathbb{C}$	$SU(3)$	$SU(3)^2$	$SU(6)$	E_6
$\mathbb{H}$	$Sp(3)$	$SU(6)$	$SO(12)$	E_7
$\mathbb{O}$	F_4	E_6	E_7	E_8

Freudenthal-Rosenfeld-Tits Magic Square

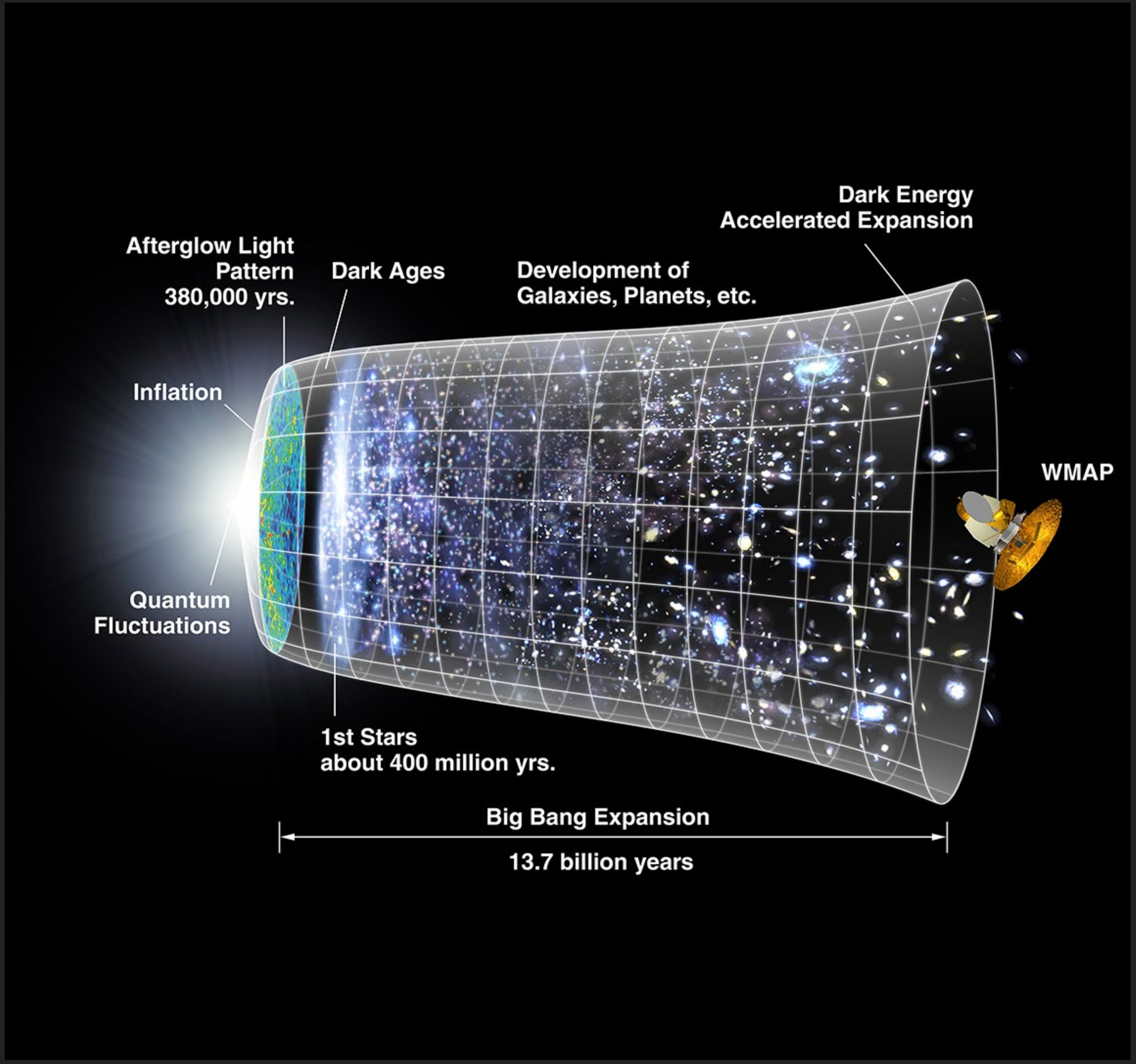
Rosenfeld



Freudenthal

THEORETICAL PHYSICS

- Where have we come from?
- What are we made of?
- Where are we going?



PILLARS OF XX CENTURY PHYSICS

- Quantum Theory

    Non-realist and probabilistic

    Elementary constituents of Nature and their fundamental interactions

    2016 Nobel: “topological phases of matter”

- General Relativity

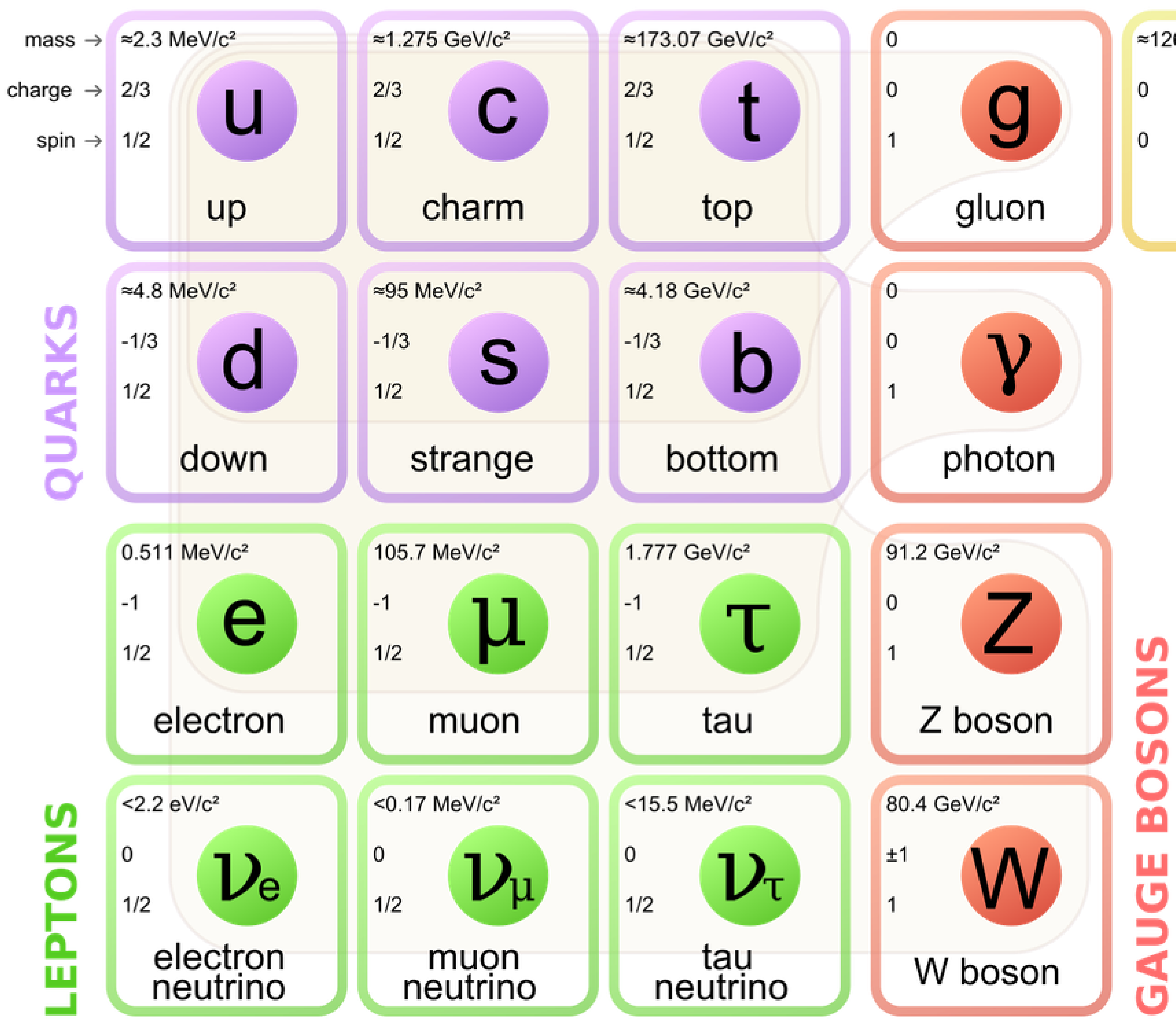
    Classical (realist) and deterministic:

    From planetary orbits to the evolution of the entire universe itself

    2017 Nobel: “for…the observation of gravitational waves".

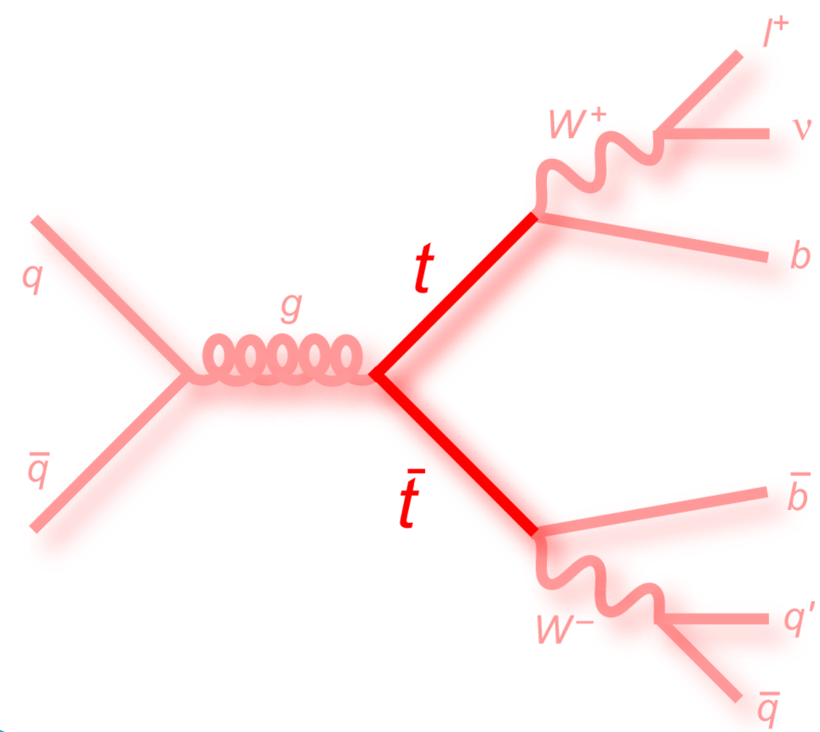
1927 Solvay Conference

THE STANDARD MODEL OF PARTICLE PHYSICS



Yang and Mills

Feynman



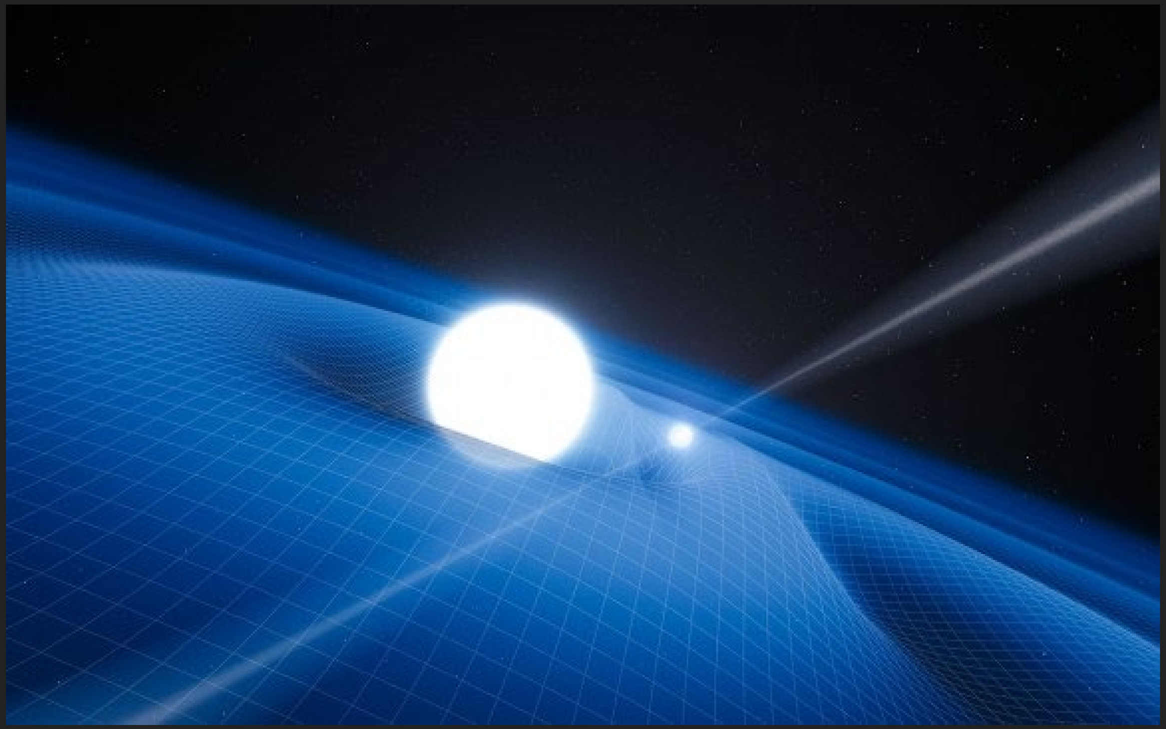
Forces described by Yang-Mills gauge theories

$$SU(3) \times SU(2) \times U(1)$$

SM gauge group

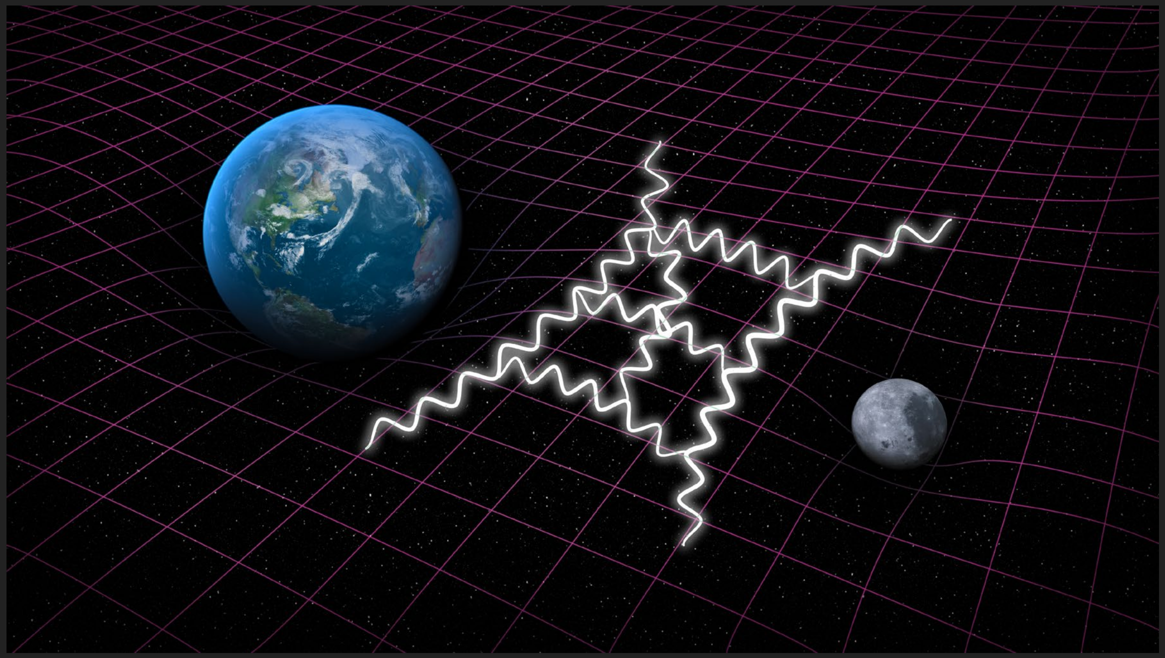
GRAVITY

- The SM plays out of the stage of spacetime
- Gravity = Curvature of spacetime
- Gravity is the stage itself!
- General coordinate invariance
- Ripples in spacetime detected by LIGO and VIRGO



THE DILEMMA OF XXI CENTURY PHYSICS

- GR is naively incompatible with Quantum Theory
- Black holes challenge the very foundations of Quantum Theory

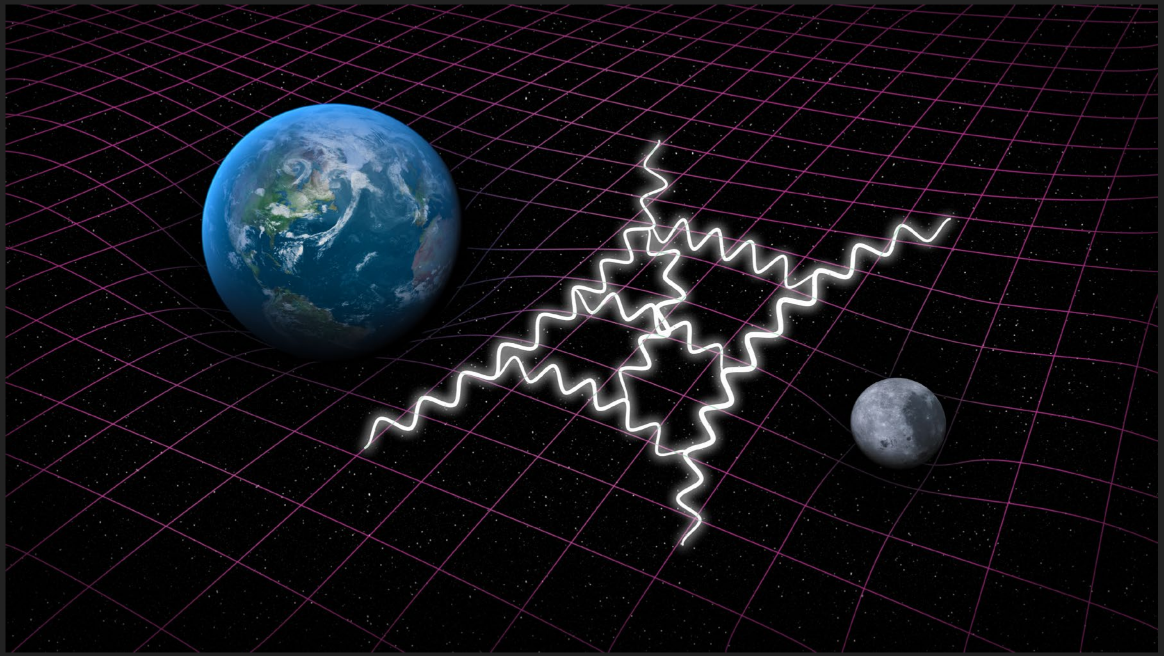


Perturbative quantum gravity diverges at two-loops, Goroff and Sagnotti 1985

Black holes emit Hawking Radiation, Hawking 1974

THE DILEMMA OF XXI CENTURY PHYSICS

- GR is naively incompatible with Quantum Theory
- Black holes challenge the very foundations of Quantum Theory
- The Problem of Quantum Gravity
- Argues the next scientific revolution
- M-theory?



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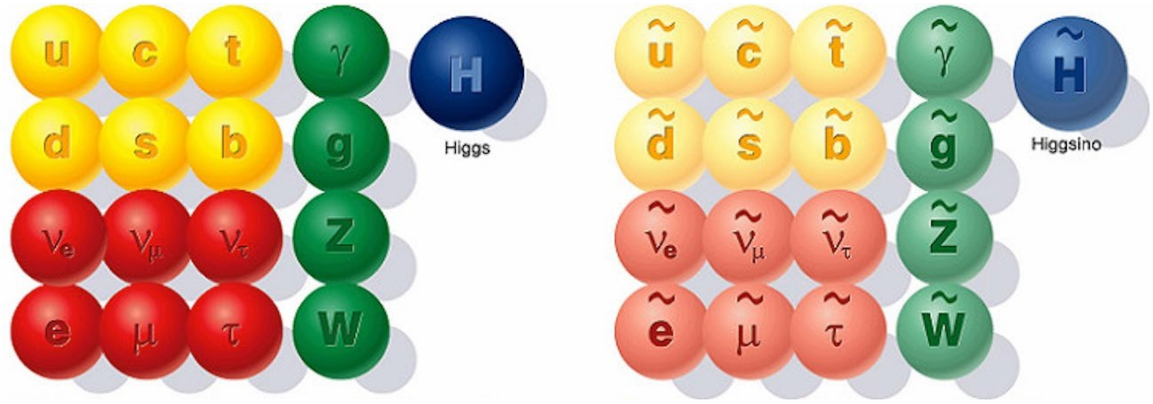
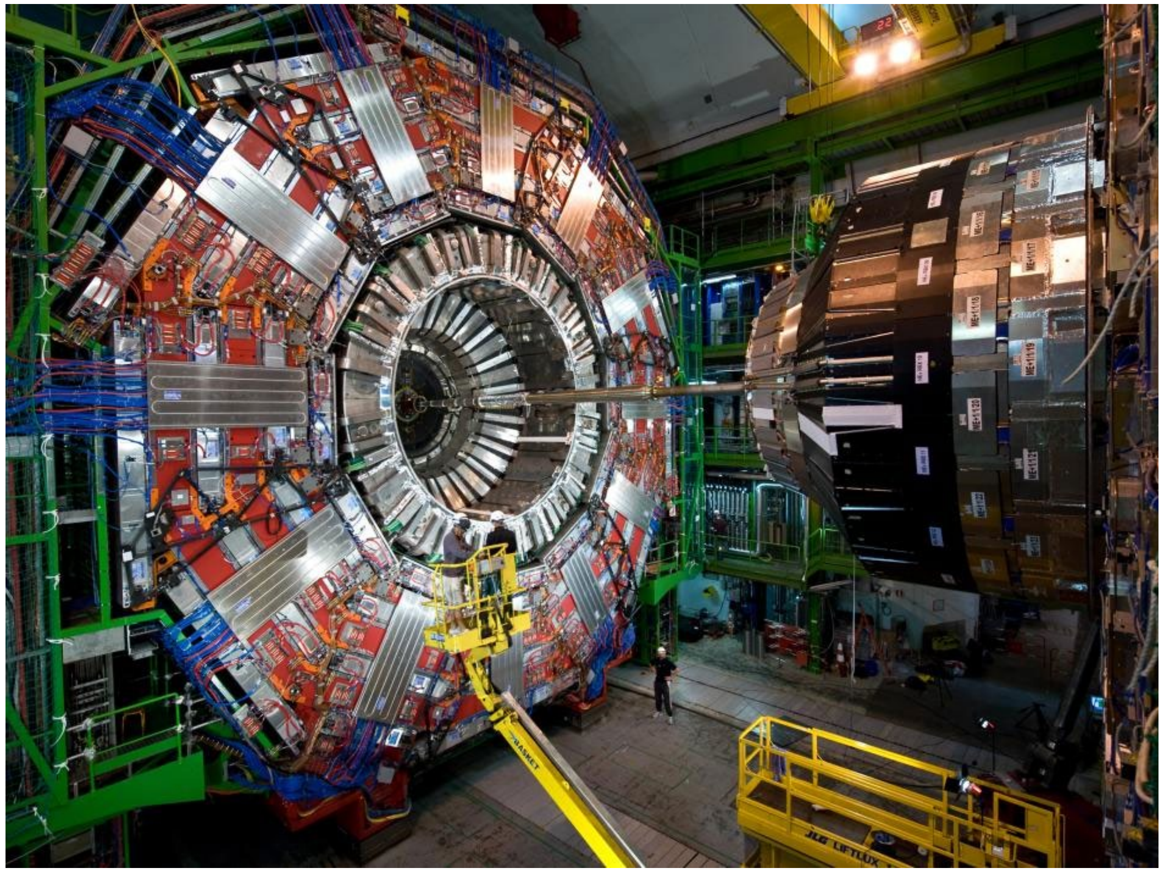
SUPERSYMMETRY

- Supersymmetry unifies bosons and fermions
- Various motivations from particles physics: unification, hierarchy problem, dark matter….
- No SUSY at the LHC (yet)

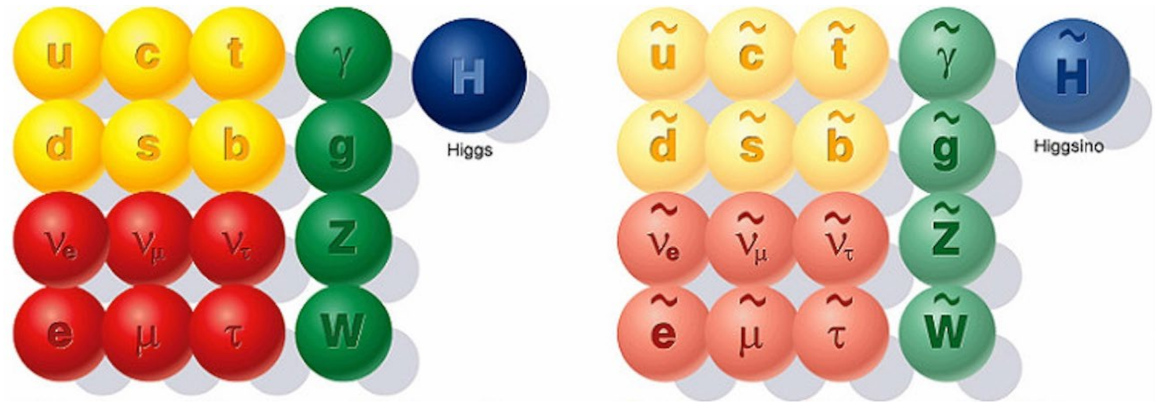


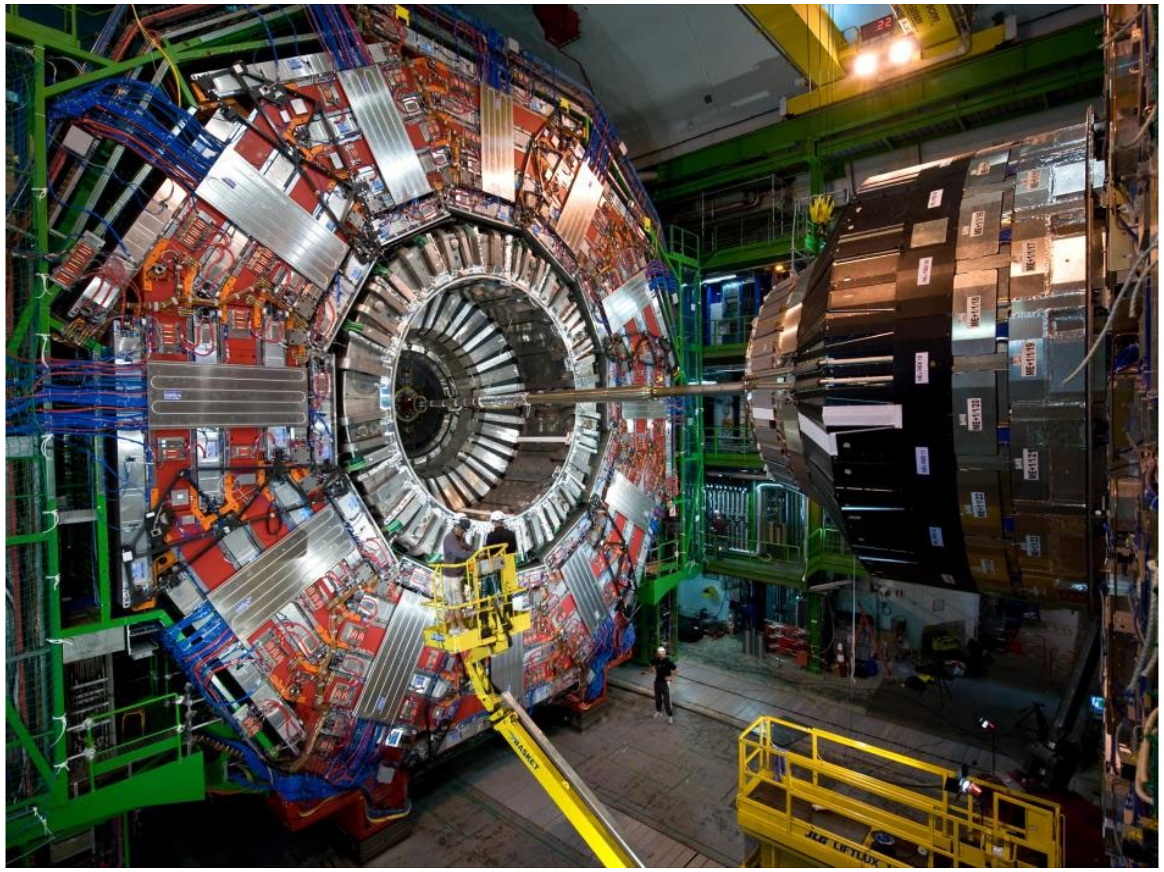


CMS detector used in SUSY searches

SUPERSYMMETRY

- Supersymmetry unifies bosons and fermions
- Various motivations from particles physics: unification, hierarchy problem, dark matter….
- No SUSY at the LHC (yet)
- Local SUSY implies gravity and unifies it with the other forces



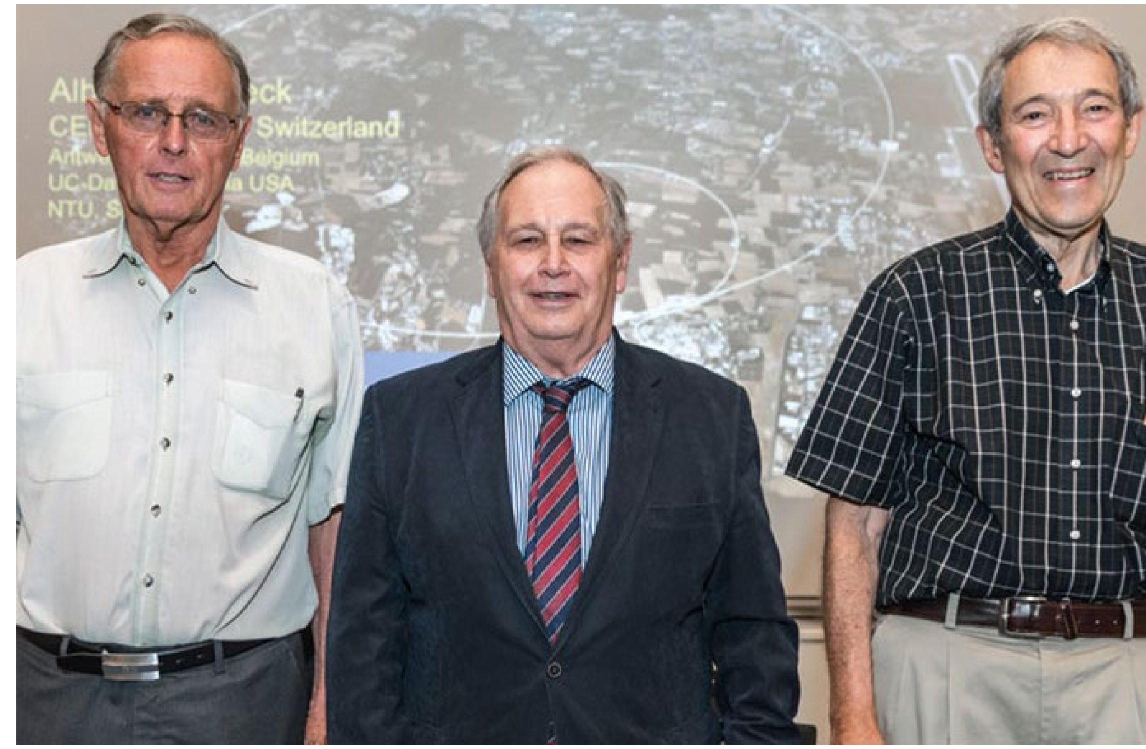


CMS detector used in SUSY searches

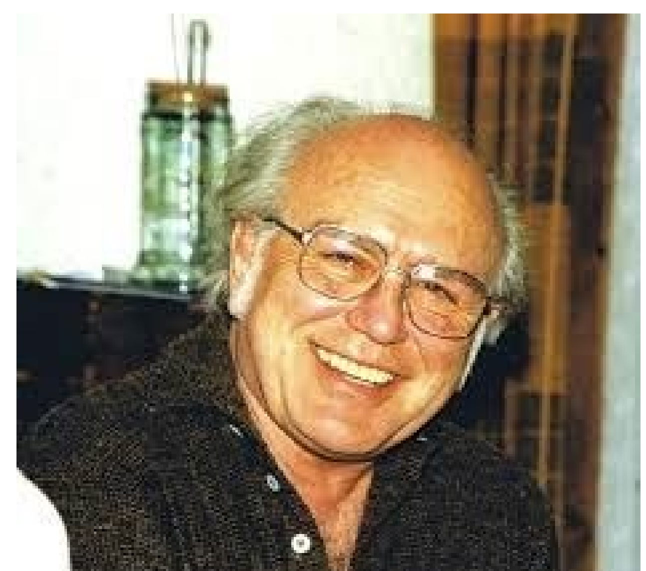
SUPERGRAVITY

- Supergravity - the supersymmetric extension of General Relativity
- Supersymmetry tames infinities
- N=8 supergravity: a theory of everything?

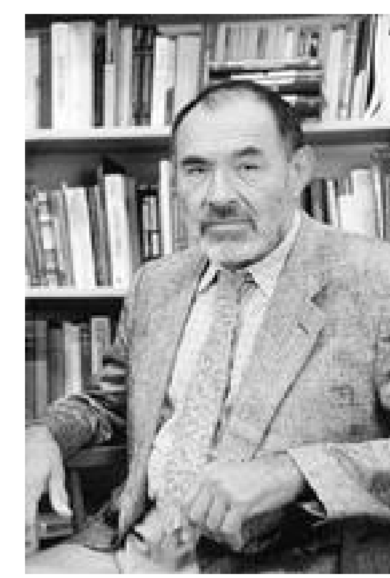
Cremmer and Julia 1978



P. van Nieuwenhuizen, S. Ferrara, D. Freedman



Bruno Zumino



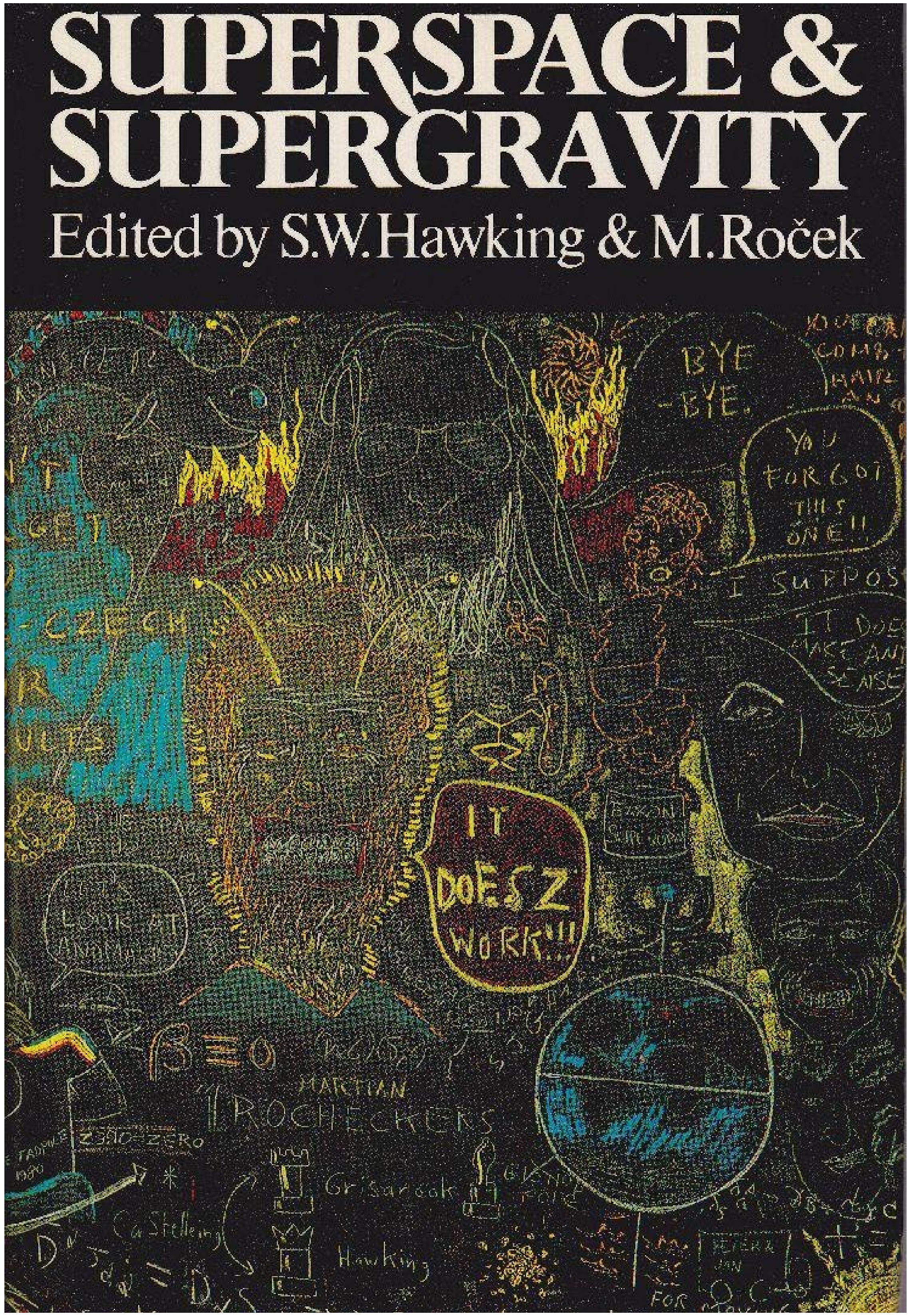
Stanley Deser

SUPERGRAVITY

- Is the End in Sight for Theoretical Physics? : An Inaugural Lecture

“At the moment the N=8 supergravity theory is the only candidate in sight. “

Hawking 1980



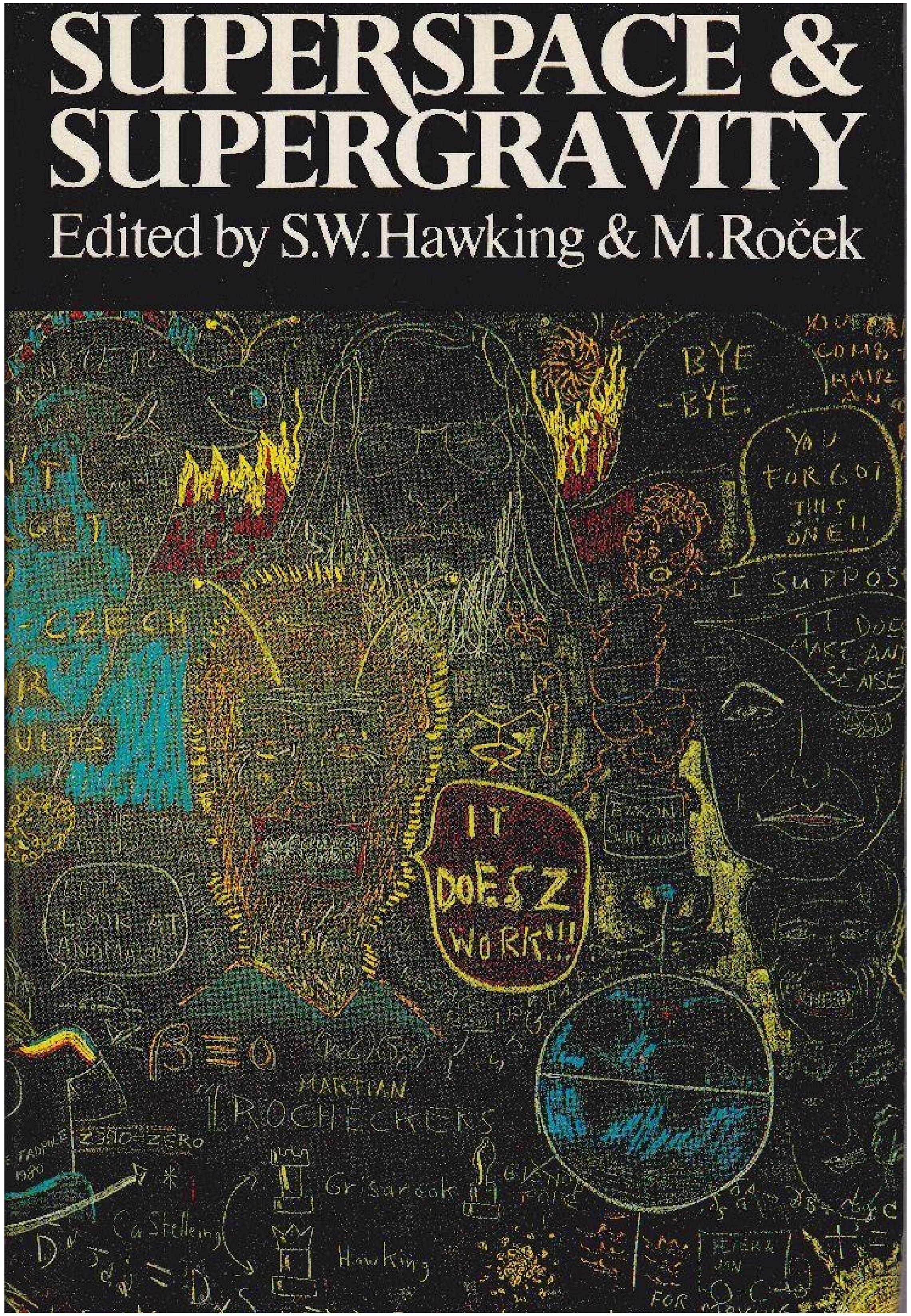
SUPERGRAVITY

- Is the End in Sight for Theoretical Physics? : An Inaugural Lecture

> “At the moment the N=8 supergravity theory is the only candidate in sight. “
>
> Hawking 1980

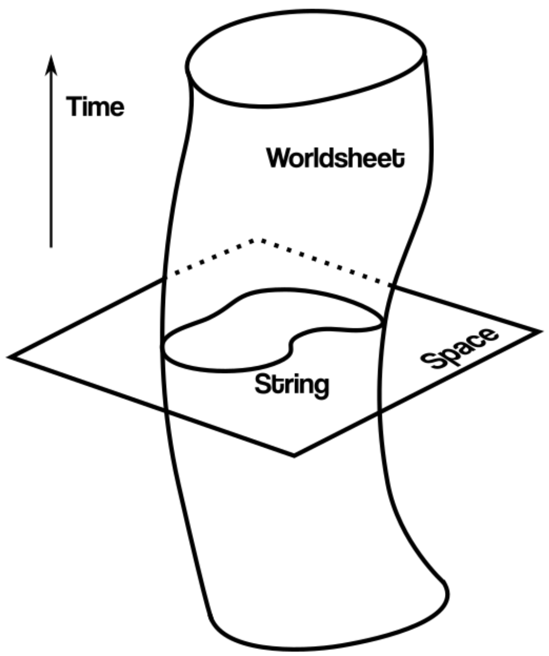
- Supergravity diverges :(

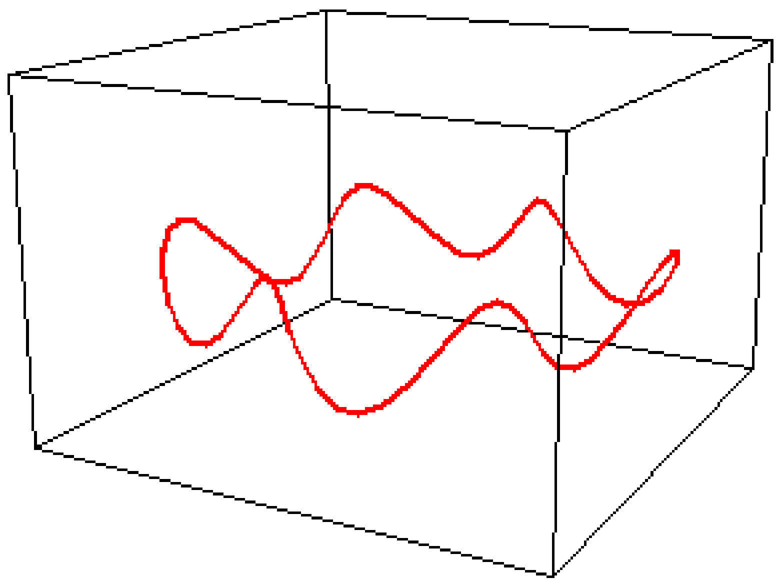
> “There are likely to be a number of crucial calculations within the next few years which have the possibility of showing that the theory is no good.”

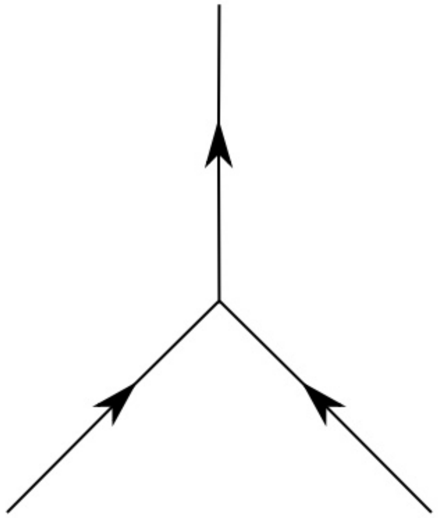


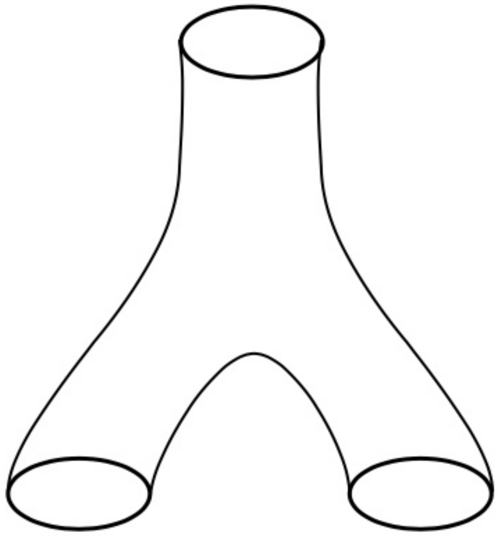
STRING THEORY

- Point particles replaced by strings:









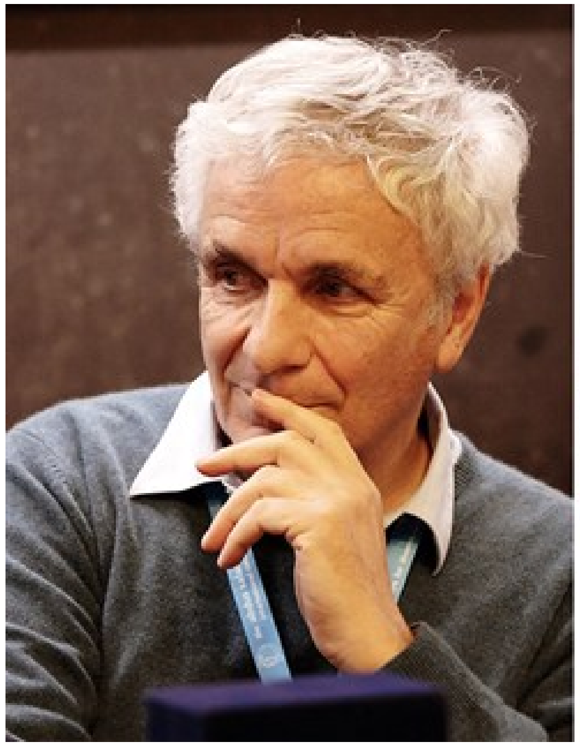
- 1968 - 1973 Veneziano, Nambu, Nielsen, Susskind, Ramond....
- 1973/74 Yoneya, Scherk and Schwarz: Strings Include gravitons!

SUPERSTRING THEORY

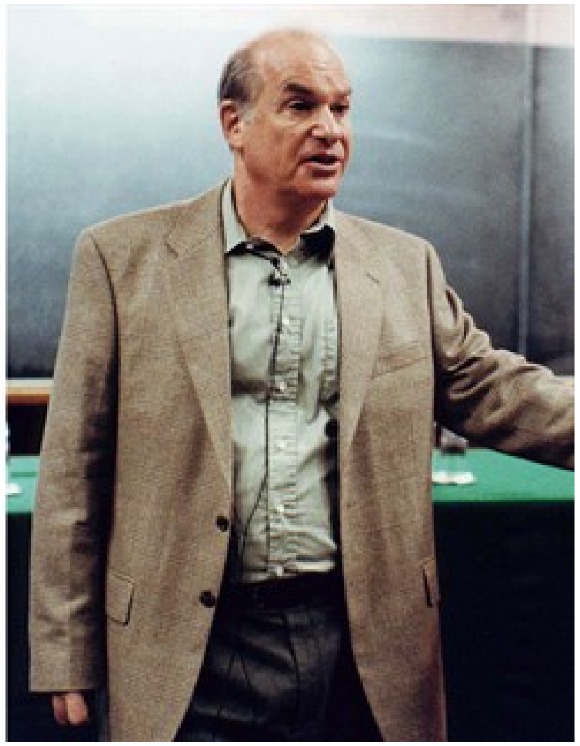
- At low energies:

  closed —> supergravity strings
  —> super Yang-Mills

- Green-Schwarz anomaly cancellation
- The string revolution ~1984



Michael Green



John Schwarz

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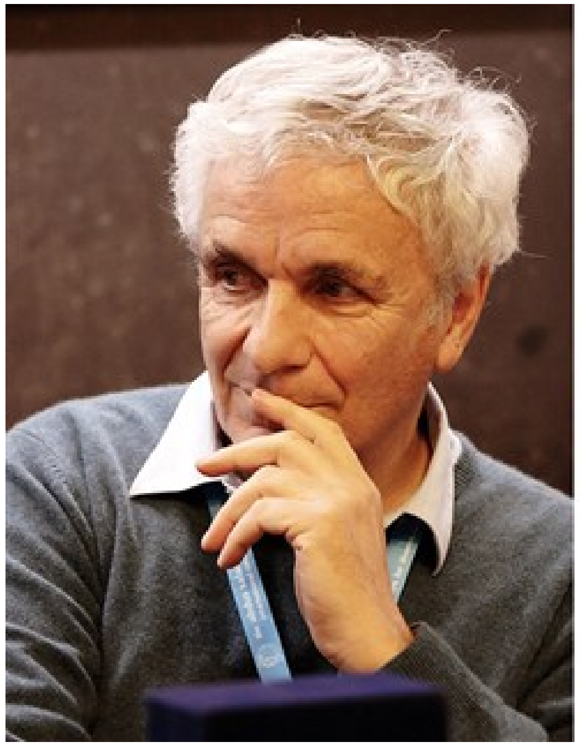

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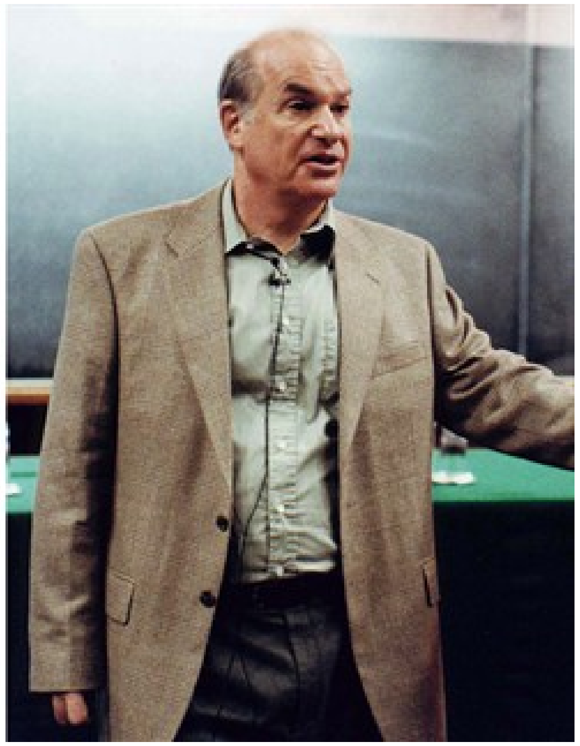
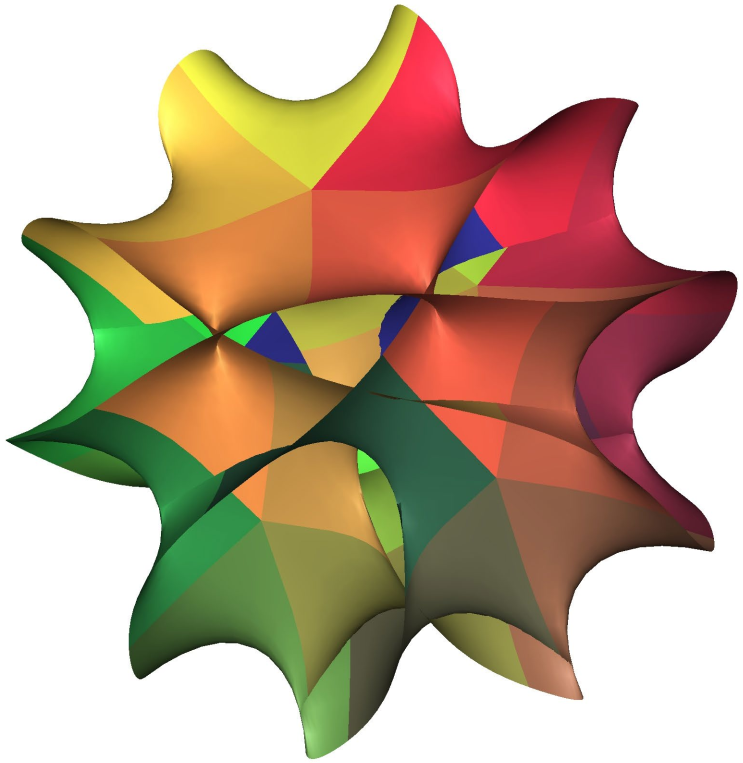
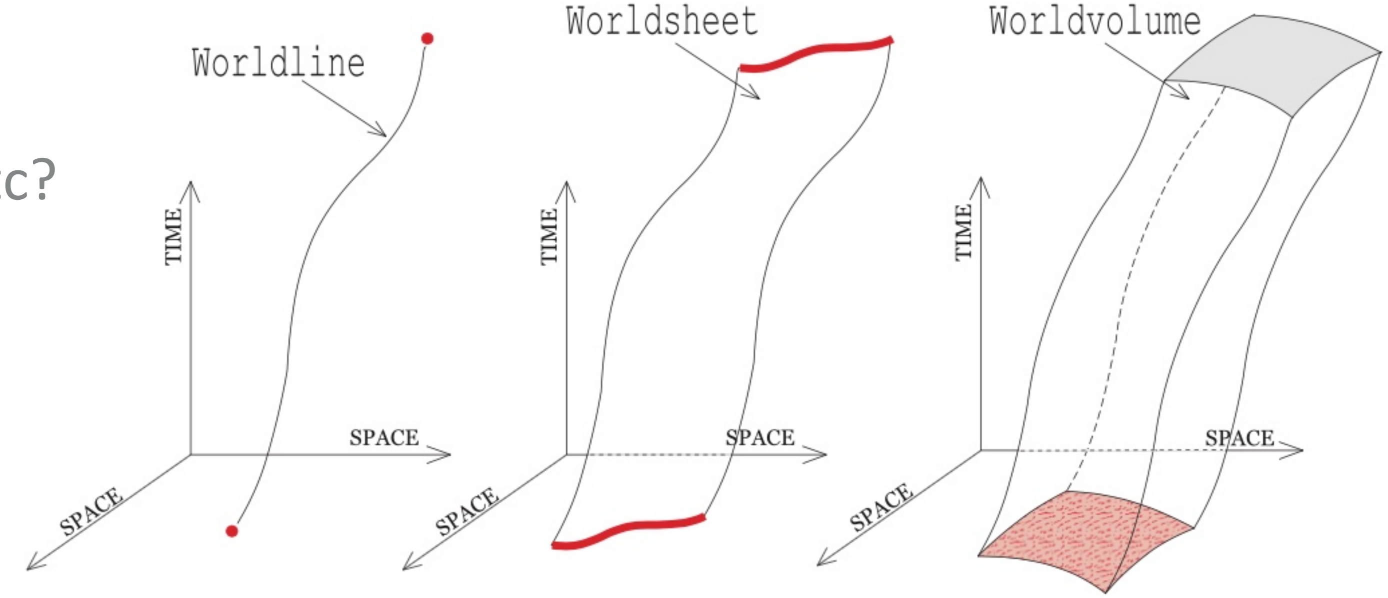


John Schwarz

- Supersymmetric
- 10 spacetime dimensions
- Compactification on a Calabi-Yau 3-fold could include Standard Model?



QUESTIONS, QUESTIONS



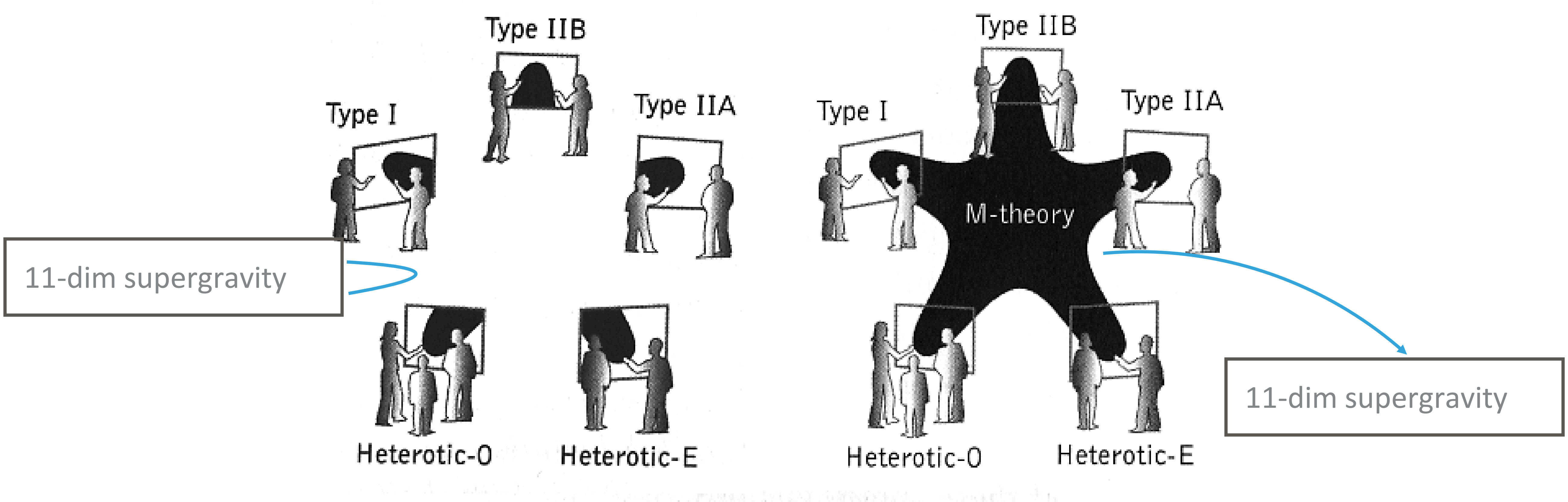
- If strings, why not membranes, 3-branes, etc?
- Five superstring theories: Type I, IIA, IIB, Heterotic-O and Heterotic-E
- Supersymmetry allows up to 11 spacetime dimensions: why do strings stop at 10? Nahm 1977
- Supergravity is unique+simplest in 11-dim

  Cremmer, Julia, and Scherk 1978

Werner Nahm

THE M-THEORY REVOLUTION

- Edward Witten Strings '95: the five consistent 10-dim string theories and 11-dim supergravity are merely corners of a single overarching framework, M-theory

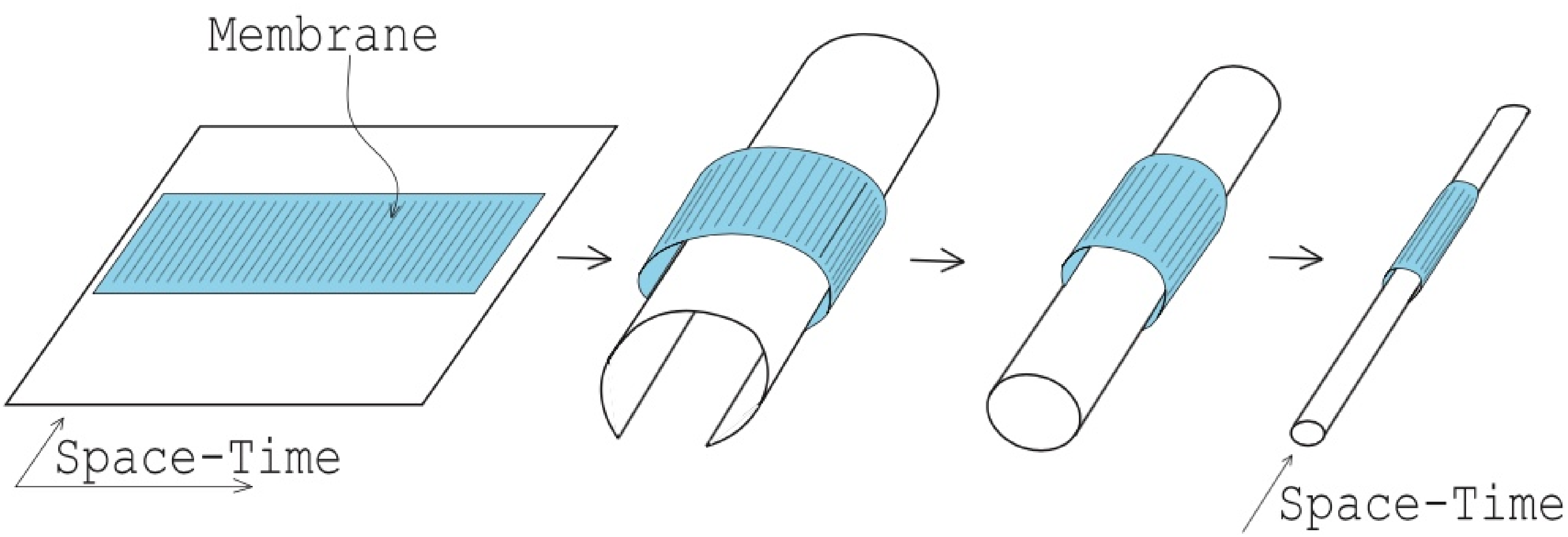


~1986 - 1995: Bergshoeff, Duff, Hull, Khuri, Lu, Pope, Sen, Sezgin, Stelle, Strominger, Townsend…….

M STANDS FOR MAGIC, MYSTERY OR MEMBRANE

- 11-dim supergravity has membrane solution   Bergshoeff, Sezgin and Townsend 1987

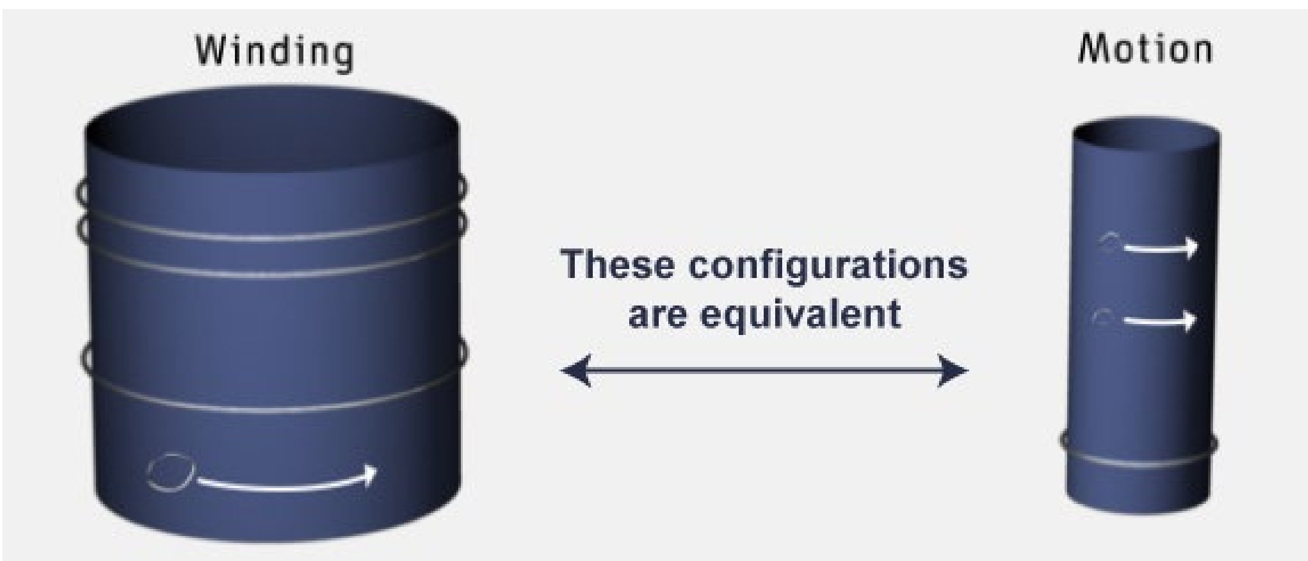
- Wrapping membranes gives type IIA superstrings   Duff, Howe, Inami and Stelle 1987



- M2-branes and M5-branes are the fundamental constituents of M-theory
- D-Branes soon became a required part of string theory (Polchinski)

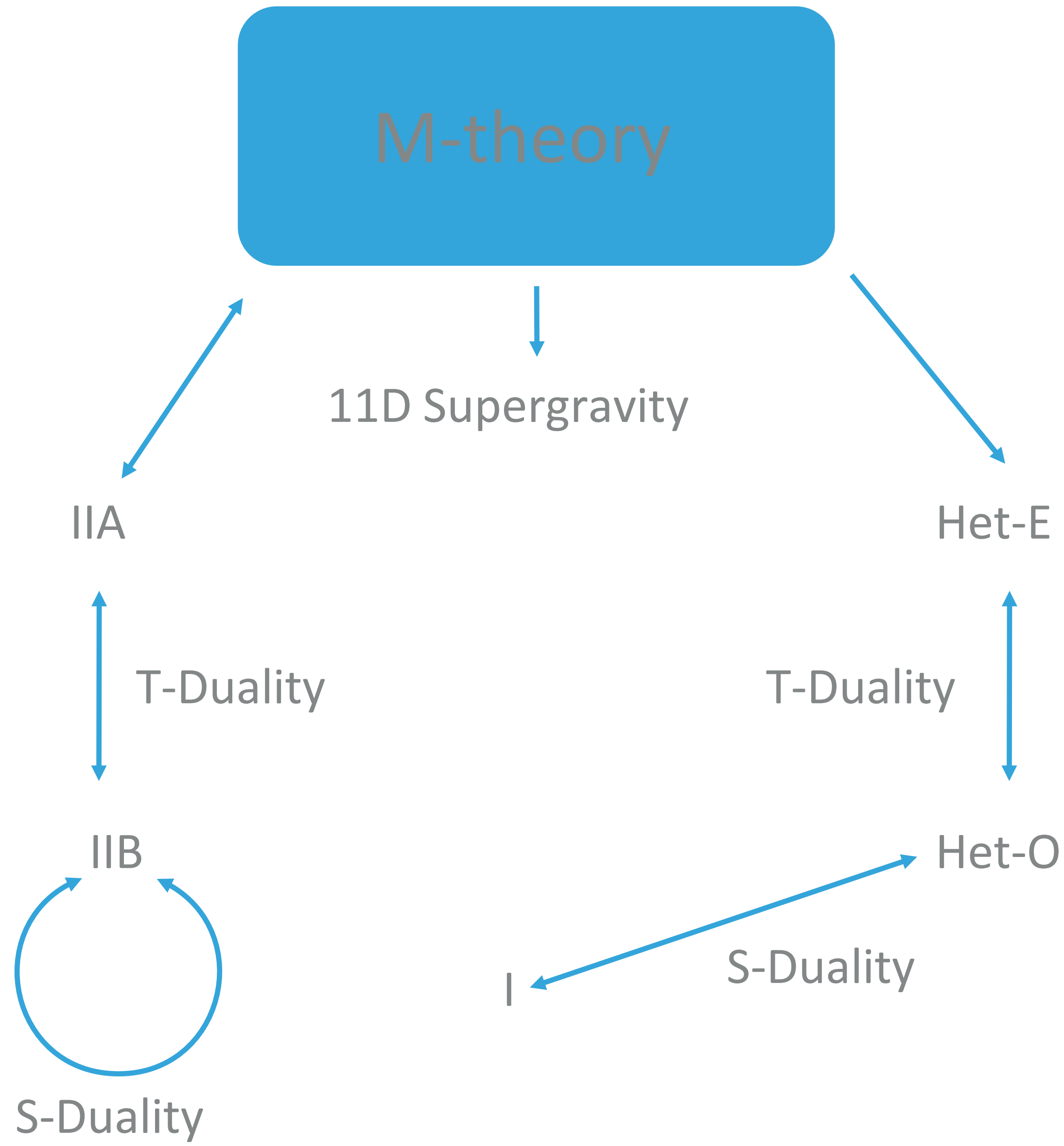
DUALITIES OF M-THEORY

- T-duality



Type IIA

Type IIB



DUALITIES OF M-THEORY

- The M-theory web: U-duality

n	U-duality
1	$E_1(\mathbb{Z}) \cong SO(1,1)$
2	$E_2(\mathbb{Z})$
3	$E_3(\mathbb{Z})$
4	$E_4(\mathbb{Z})$
5	$E_5(\mathbb{Z})$
6	$E_6(\mathbb{Z})$
7	$E_7(\mathbb{Z})$
8	$E_8(\mathbb{Z})$

Hull and Townsend 1995

DIAL M FOR MAGIC

- Is gravity secretly the “square” of Yang-Mills?

[(super) Yang-Mills amplitudes] x [(super) Yang-Mills amplitudes] = (super)gravity amplitudes

Bern, Carrasco, Johansson 2008

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  Bern, Carrasco, Johansson 2008

- In 3 dimensions there are 4 SYM theories with N=1, 2, 4, 8

	$\mathbb{R}$	$\mathbb{C}$	$\mathbb{H}$	$\mathbb{O}$
$\mathbb{R}$	$SO(3)$	$SU(3)$	$Sp(3)$	F_4
$\mathbb{C}$	$SU(3)$	$SU(3)^2$	$SU(6)$	E_6
$\mathbb{H}$	$Sp(3)$	$SU(6)$	$SO(12)$	E_7
$\mathbb{O}$	F_4	E_6	E_7	E_8

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←

D	$\mathbb{A}$	$\text{Spin}(1, 1+\dim\mathbb{A})$
3	$\mathbb{R}$	$\text{Spin}(1,2)$
4	$\mathbb{C}$	$\text{Spin}(1,3)$
6	$\mathbb{H}$	$\text{Spin}(1,5)$
10	$\mathbb{O}$	$\text{Spin}(1,9)$

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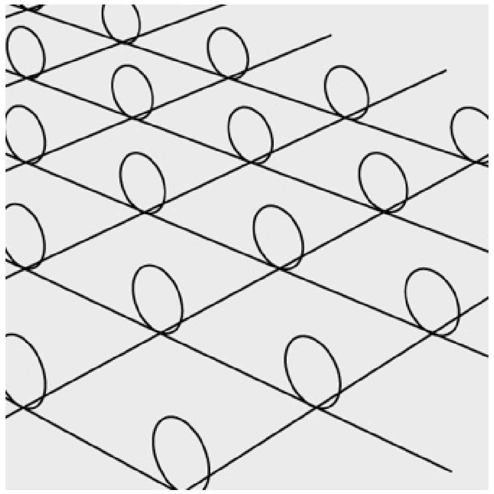
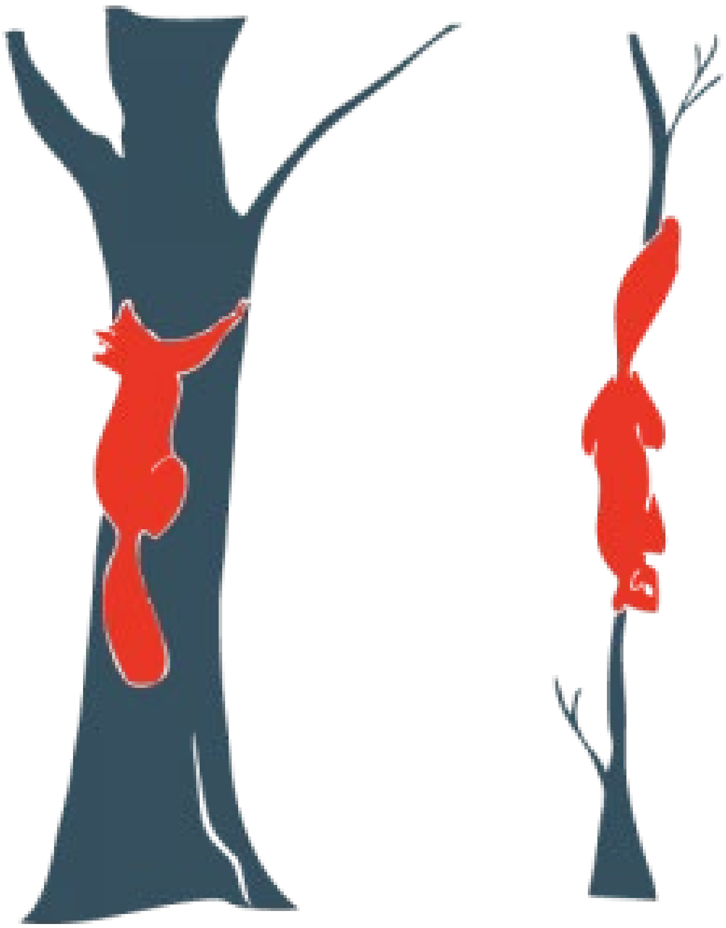
4	$\mathbb{C}$	$\mathrm{Spin}(1, 3)$
6	$\mathbb{H}$	$\mathrm{Spin}(1, 5)$
10	$\mathbb{O}$	$\mathrm{Spin}(1, 9)$

- **M really does stand for Membrane, Magic and Mystery!**

EXTRA DIMENSIONS

- Kaluza-Klein theory ~1926
- 5-dim gravity on a circle





- To a low energy observer it looks just like gravity+electromagnetism (+ a scalar field)

$$h_{MN} \longrightarrow h_{\mu\nu}, \qquad h_{\mu 5} \sim A_\mu, \qquad h_{55} \sim \phi$$

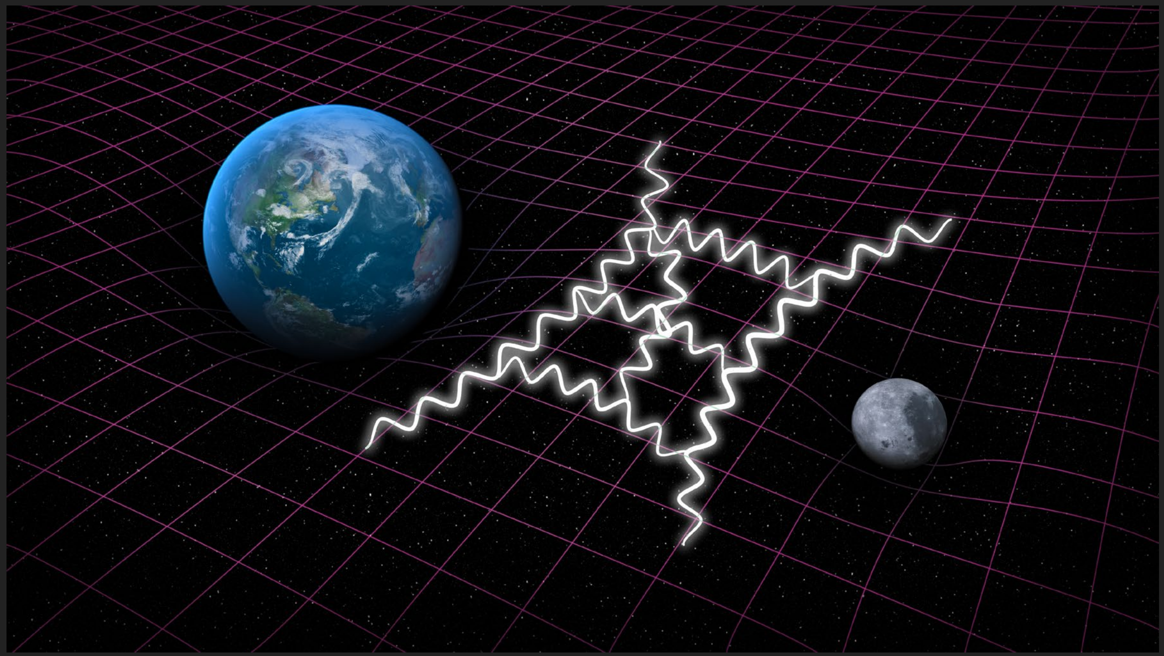
THE DILEMMA OF XXI CENTURY PHYSICS

- GR is naively incompatible with Quantum Theory
- Black holes challenge the very foundations of Quantum Theory
- The Problem of Quantum Gravity

. . . Pauli asked me what I was working on. I said I was trying to quantize the gravitational field. For many seconds he sat silent, alternately shaking and nodding his head. He finally said “That is a very important problem–

but it will take someone really smart!”

Bryce DeWitt



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Thank You!