$b \to s\mu^+\mu^-$ anomalies: New Physics or QCD effects?

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Based on
arXiv: 2011.09813, 2206.03797, 2305.06301, 2312.14146

in collaboration with
T. Feldmann, D. van Dyk, J. Virto, and M. Reboud

Weekly seminars
LAPTh, Annecy

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UNIVERSITY OF CAMBRIDGE

EOS

Talk outline

Introduction

Theoretical framework

- $b \to s\ell^+\ell^-$ effective Hamiltonian
- form factors definition

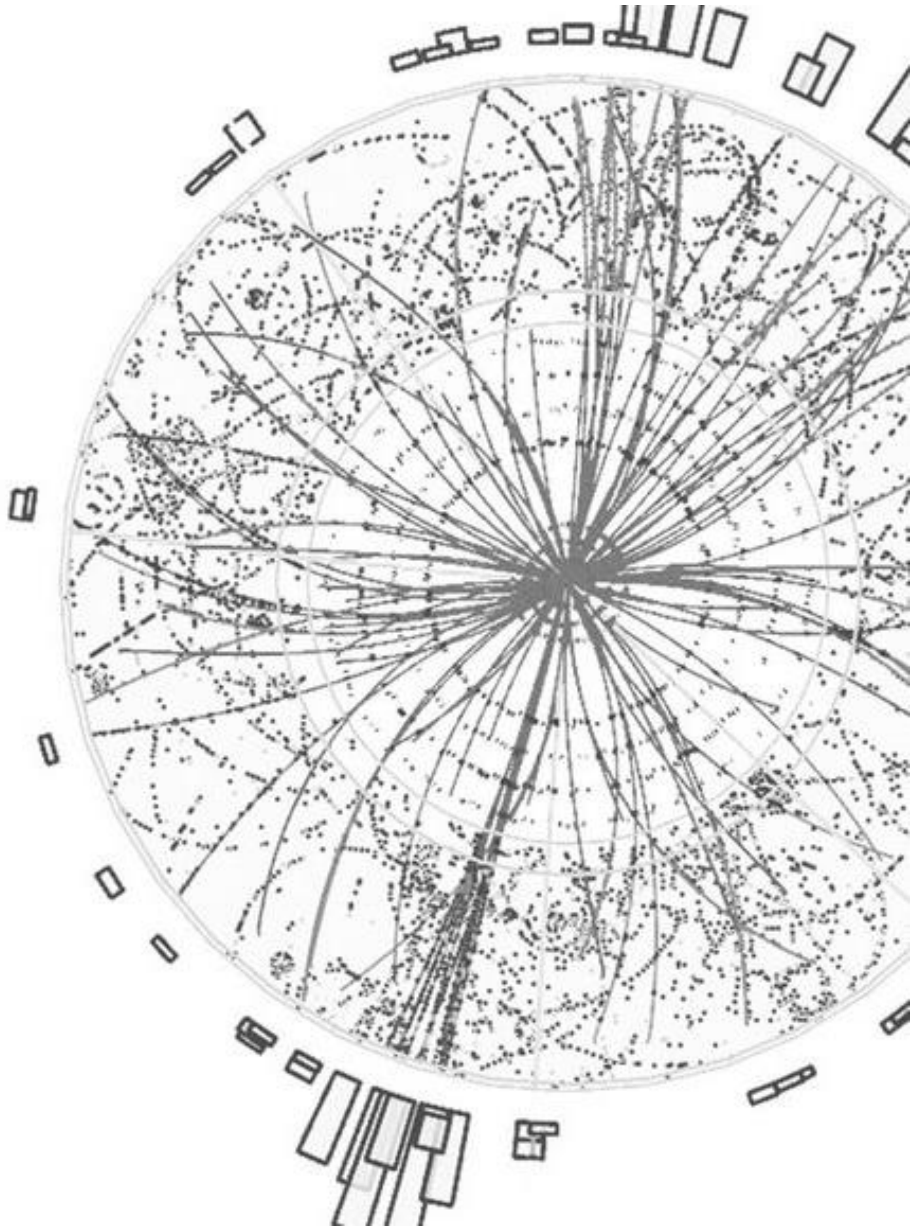
Local and non-local form factors

- parametrization and unitarity bounds
- analysis and results

SM predictions

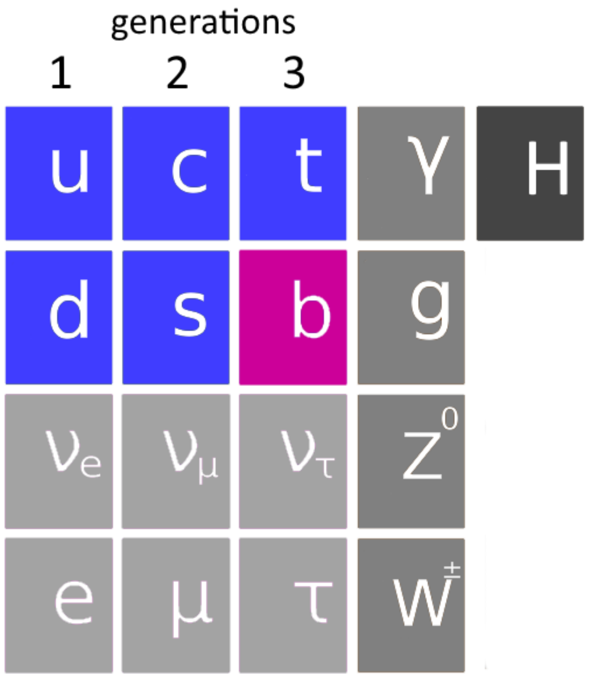
- comparison between SM predictions and data
- global fit to $b \to s\mu^+\mu^-$

Summary and conclusion



Introduction

The beauty of the Standard Model



SM: 6 quark flavours and 6 lepton flavours

flavour physics: investigate the properties, the transitions, and the spectrum of the different quark and lepton flavours

transitions between different (flavours) mediated by $W^{\pm}$

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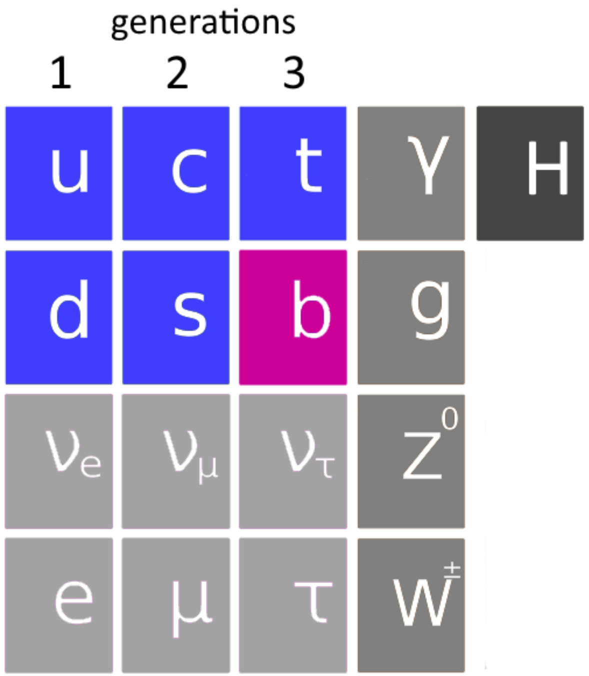

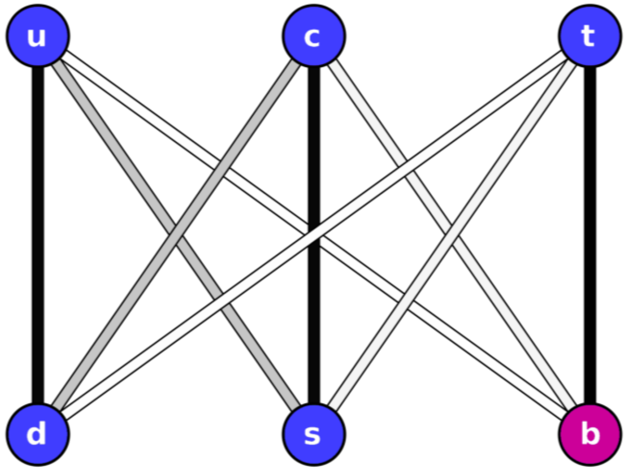

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flavour physics: investigate the properties, the transitions, and the spectrum of the different quark and lepton flavours

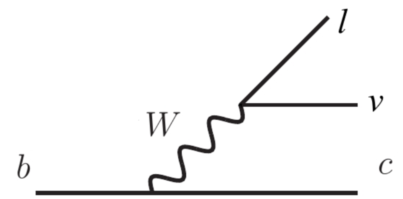
transitions between different (flavours) mediated by $W^\pm$

why is the *b* quark interesting?

- third generation quark
- heaviest fermion that forms bound states ($m_b \gg \Lambda_{\mathrm{QCD}}$)
- lighter than the t quark
  $\Longrightarrow$ decays in quarks of another generation
  $\Longrightarrow$ CKM suppressed decay

Flavour changing currents

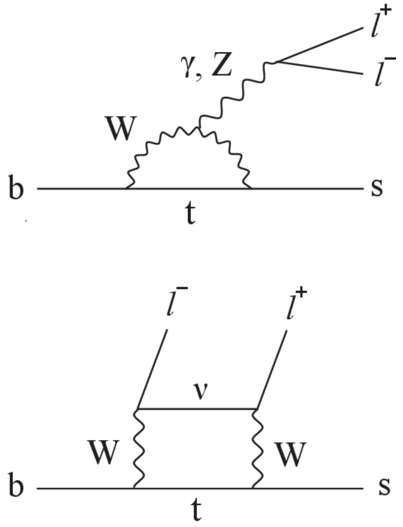
flavour changing charged currents (FCCC) occur at tree level (mediated by $W^{\pm}$) in the SM



FCCC

flavour changing neutral currents (FCNC) absent at tree level in the SM
FCNC are loop, GIM and CKM **suppressed in the SM**

FCNC **sensitive to new physics** contributions
probe the SM through **indirect searches**



FCNC

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probe the SM through **indirect searches**

integrate out DOF heavier than the b
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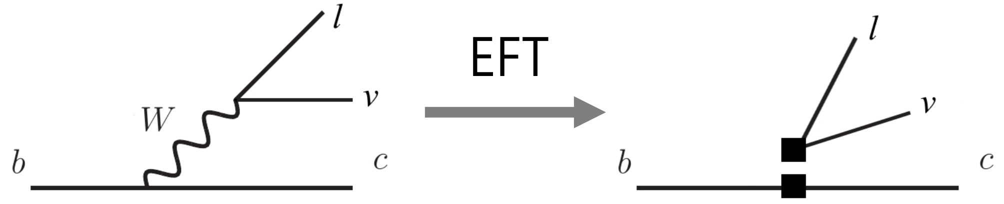
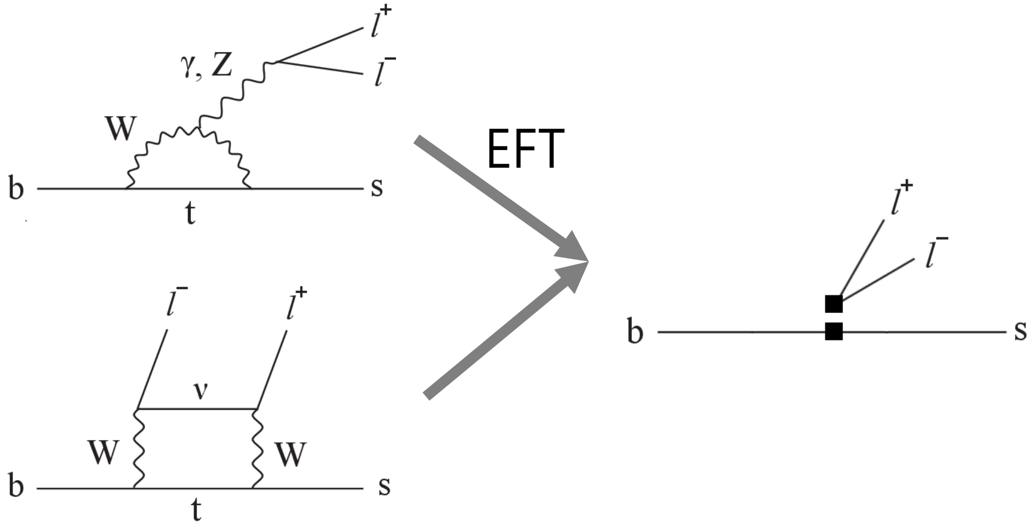
weak effective field theory



FCCC



FCNC

Hadronic matrix elements

study $\boldsymbol{B}$-meson decays to test the SM, focus on to $B \to K^{(*)}\ell^+\ell^-$ and $B_s \to \phi\ell^+\ell^-$
factorise decay amplitude as (neglecting QED corrections)

FCCC: $\langle \bar{D}^{(*)}\ell\nu_\ell|\mathcal{O}_{eff}|B\rangle = \langle \ell\nu_\ell|\mathcal{O}_{lep}|0\rangle\langle D^{(*)}|\mathcal{O}_{had}|B\rangle$

FCNC: $\langle K^{(*)}\ell^+\ell^-|\mathcal{O}_{eff}|B\rangle = \langle \ell\ell|\mathcal{O}_{lep}|0\rangle\langle K^{(*)}|\mathcal{O}_{had}|B\rangle$ + non−fact.

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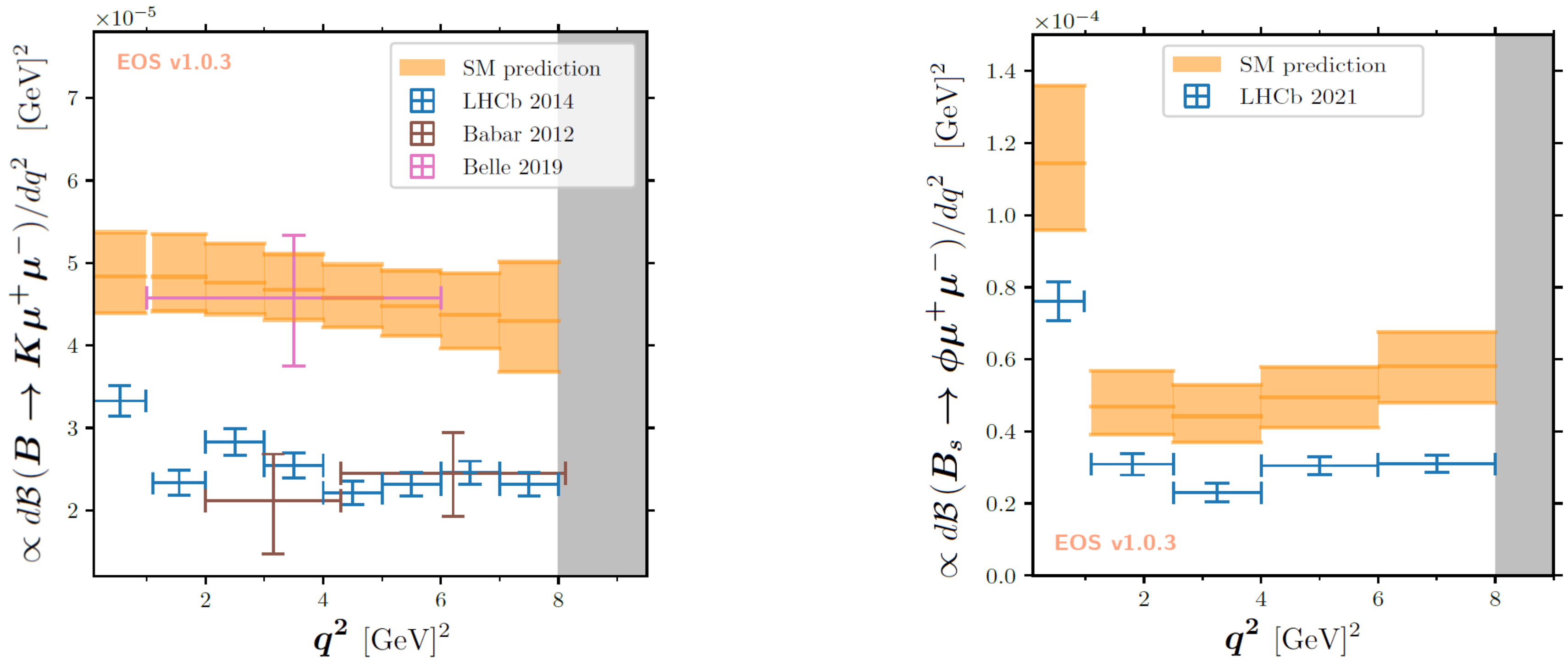
decay amplitudes depend on:

- leptonic matrix elements: perturbative objects, **small uncertainties**
- local hadronic matrix elements (local form factors):
  non-perturbative QCD effects, **moderate uncertainties**
- non-local hadronic matrix elements (charm-loop effects):
  largest source of systematic **uncertainties**

SM predictions for BRs in rare decays

test the SM and constrain New Physics by comparing theory predictions and exp. measurements of, e.g., branching ratios $B \to K^{(*)}\ell^+\ell^-$ and $B_s \to \phi\ell^+\ell^-$



agreement between theory and experiment for LFU ratios R_K and R_{K^*},
but **tension remains for $b \to s\mu^+\mu^-$** observables $\Longrightarrow$ need to understand this tension

$b \to s\mu^+\mu^-$ anomalies: tensions between theory and experiment in $B \to K^{(*)}\mu^+\mu^-$ and $B_s \to \phi\mu^+\mu^-$

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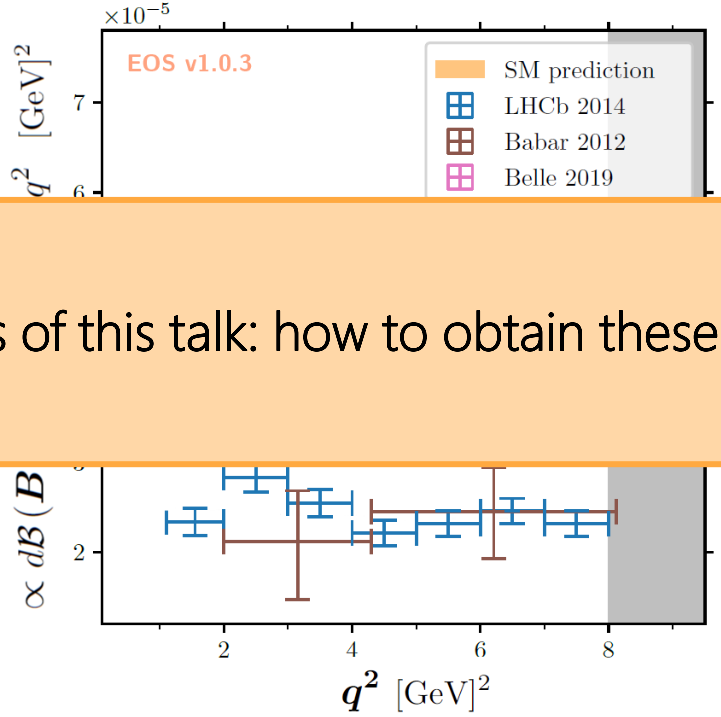

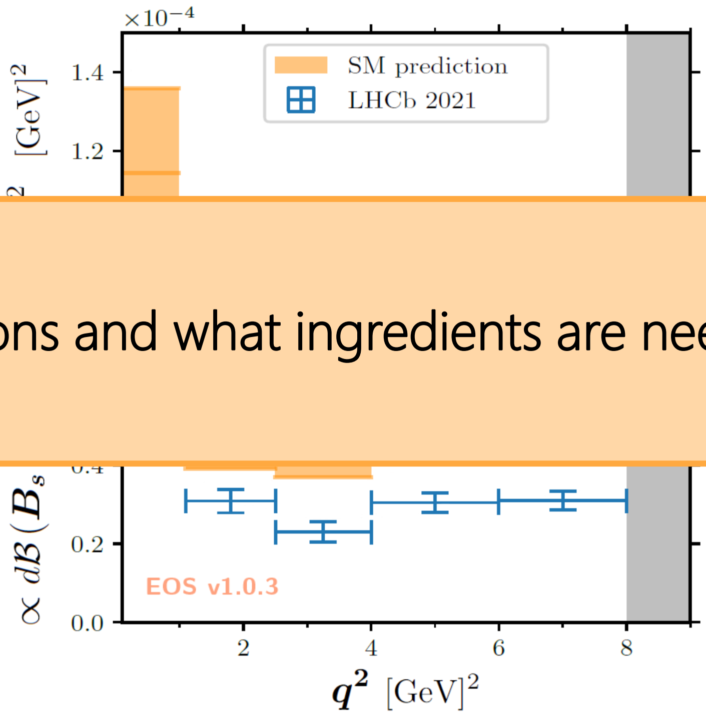

focus of this talk: how to obtain these SM predictions and what ingredients are needed

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$b \to s\mu^+\mu^-$ anomalies: tensions between theory and experiment in $B \to K^{(*)}\mu^+\mu^-$ and $B_s \to \phi\mu^+\mu^-$

Importance of theory predictions

1. understand if $b \to s\ell^+\ell^-$ anomalies are due to
   New Physics or misestimated QCD effects

2. constrain physics beyond the SM
   (SMEFT Wilson coefficients)

very active field of research

$\Updownarrow$

tremendous experimental efforts
(LHCb, Belle (II), CMS, ATLAS)

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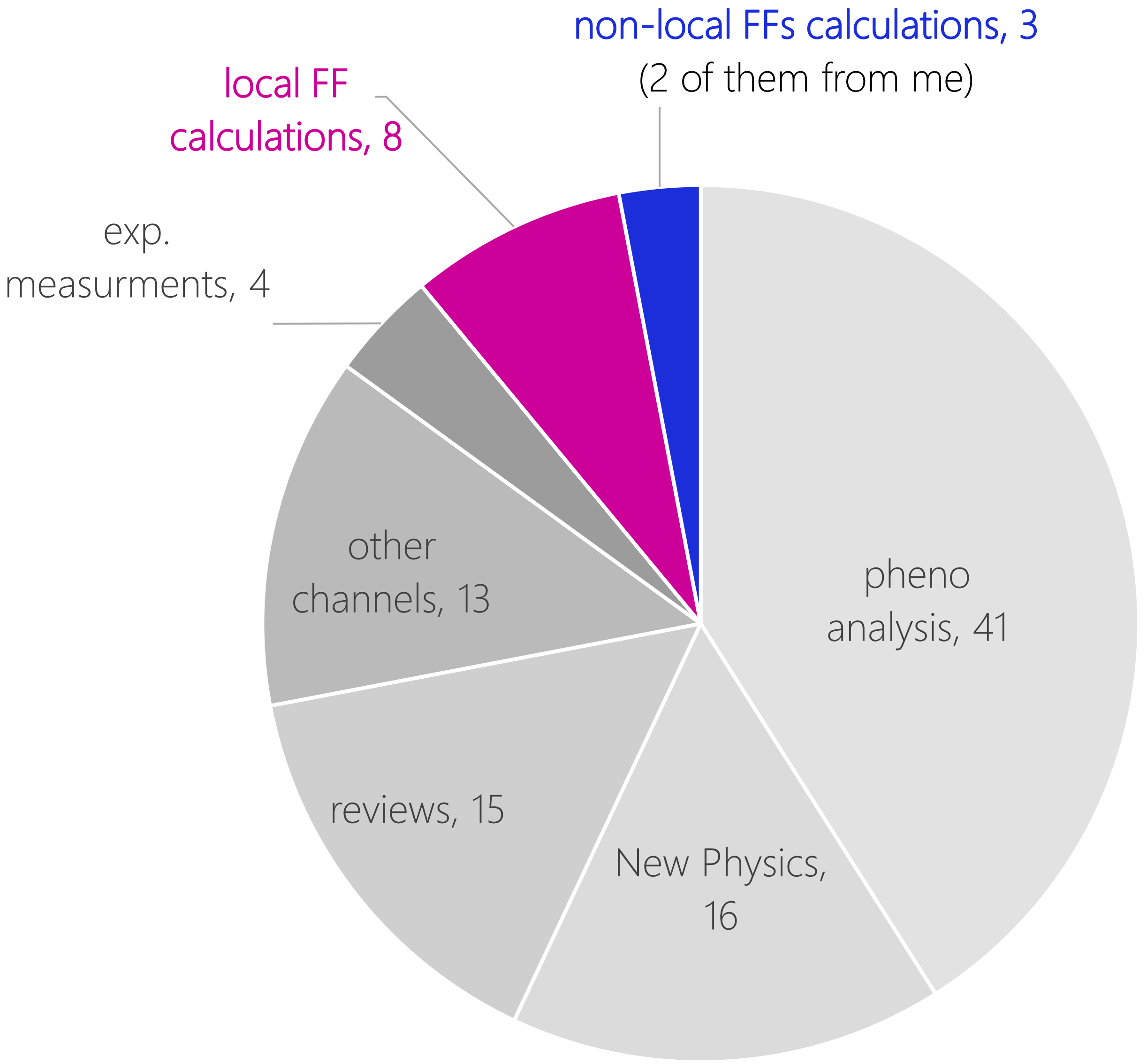
tremendous experimental efforts (LHCb, Belle (II), CMS, ATLAS)

need more theory calculations to fully exploit experimental work

example: distribution of first 100 citations of [NG/van Dyk/Virto 2020]



Theoretical framework

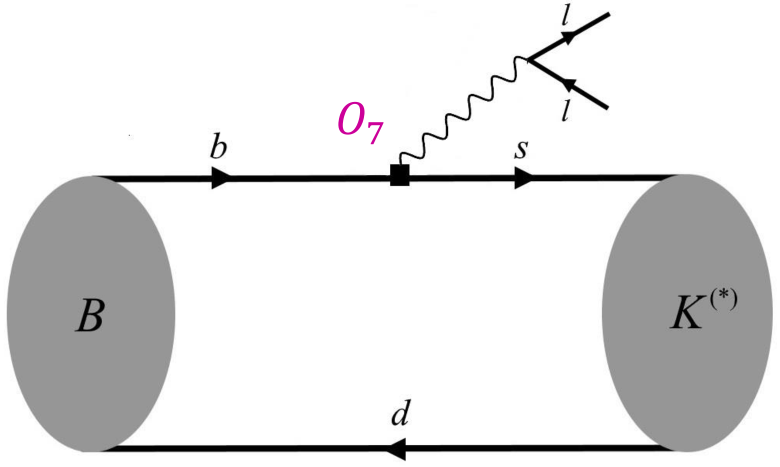
$b \to s\ell^+\ell^-$ effective Hamiltonian

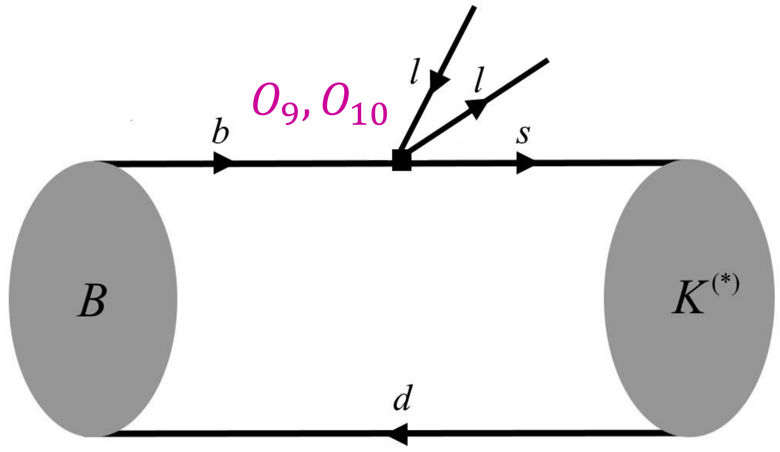
transitions described by the **effective Hamiltonian**

$$\mathcal{H}(b \to s\ell^+\ell^-) = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu) \qquad \mu = m_b$$

main contributions to $B_{(s)} \to \{K^{(*)}, \phi\}\ell^+\ell^-$ in the SM given by local operators O_7, O_9, O_{10}

$O_7 = \frac{e}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}$ $\qquad O_9 = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma^\mu b_L) \sum_\ell (\bar{\ell}\gamma_\mu \ell)$ $\qquad O_{10} = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma^\mu b_L) \sum_\ell (\bar{\ell}\gamma_\mu \gamma_5 \ell)$

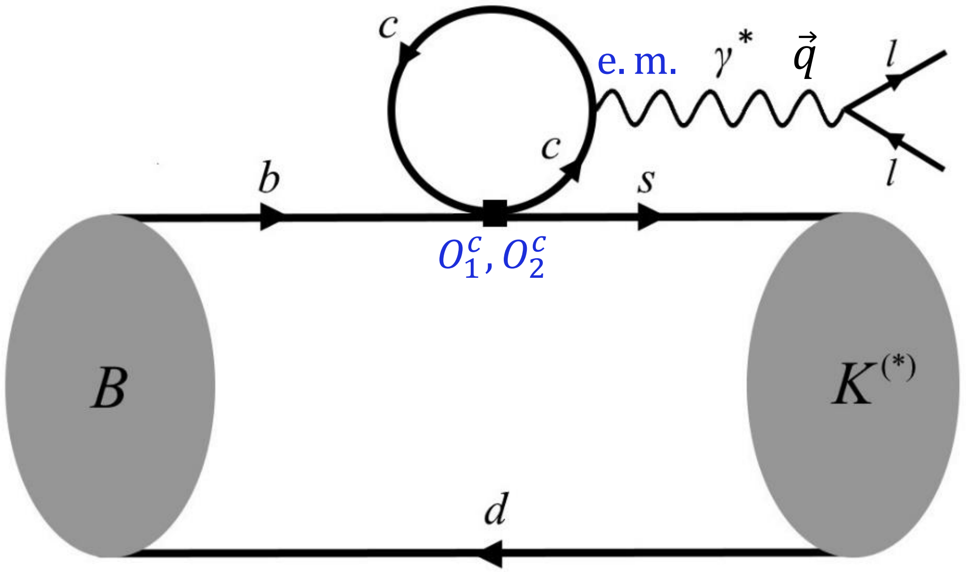




Charm loop in $B \to K^{(*)}\ell^+\ell^-$

additional non-local contributions come from O_1^c and O_2^c combined with the e.m. current (charm-loop contribution)

$$O_1^c = (\bar{s}_L \gamma^\mu c_L)(\bar{c}_L \gamma_\mu b_L) \qquad O_2^c = \left(\bar{s}_L^j \gamma^\mu c_L^i\right)\left(\bar{c}_L^i \gamma_\mu b_L^j\right)$$



Decay amplitude for $B \to K^{(*)}\ell^+\ell^-$ decays

calculate decay amplitudes precisely to probe the SM
$b \to s\mu^+\mu^-$ anomalies: NP or underestimated QCD uncertainties?
(analogous formulas apply to $B_s \to \phi\ell^+\ell^-$ decays)

$$\mathcal{A}(B \to K^{(*)}\ell^+\ell^-) = \mathcal{N}\left[\left(C_9 L_V^\mu + C_{10} L_A^\mu\right) \mathcal{F}_\mu - \frac{L_V^\mu}{q^2}\left(C_7\, \mathcal{F}_{T,\mu} + \mathcal{H}_\mu\right)\right]$$

local hadronic matrix elements

$$\mathcal{F}_\mu = \langle K^{(*)}(k)| O_{7,9,10}^{\mathrm{had}} |B(k+q)\rangle$$

non-local hadronic matrix elements

$$\mathcal{H}_\mu = i \int d^4x\, e^{iq\cdot x} \langle K^{(*)}(k)| T\{j_\mu^{\mathrm{em}}(x), (C_1 O_1^c + C_2 O_2^c)(0)\} |B(k+q)\rangle$$

Form factors definitions

form factors (FFs) **parametrize hadronic matrix elements**

FFs are functions of the momentum transfer squared q^2

local FFs

$$\mathcal{F}_\mu(k,q) = \sum_\lambda \mathcal{S}_\mu^\lambda(k,q)\,\mathcal{F}_\lambda(q^2)$$

computed with **lattice QCD** and light-cone sum rules with good precision $3\% - 20\%$

non-local FFs

$$\mathcal{H}_\mu(k,q) = \sum_\lambda \mathcal{S}_\mu^\lambda(k,q)\mathcal{H}_\lambda(q^2)$$

calculated using an **Operator Product Expansion (OPE)** or QCD factorization or ...

large uncertainties → reduce uncertainties for a better understanding of rare B decays

Local form factors

Methods to compute FFs

non-perturbative techniques are needed to compute FFs

1. Lattice QCD (LQCD)
   numerical evaluation of correlators in a finite and discrete space-time
   more efficient usually at **high q^2**
   reducible systematic uncertainties

2. Light-cone sum rules (LCSRs)
   based on unitarity, analyticity, and quark-hadron duality approximation
   need universal non-perturbative inputs (**light-meson or B-meson** distribution amplitudes)
   only applicable at **low q^2**
   non-reducible systematic uncertainties

complementary approaches to calculate FFs
in the long run LQCD will dominate the theoretical predictions (smaller and reducible syst. unc.)

Local form factors predictions

available theory calculations for local FFs $\mathcal{F}_\lambda$

$B \to K$:

- LQCD calculations at **high** q^2
  [HPQCD 2013/2023] [FNAL/MILC 2015]
  and in the **whole** semileptonic region
  [HPQCD 2023]
- LCSR at **low** q^2
  [Khodjamirian/Rusov 2017] [NG/Kokulu/van Dyk 2018]

$B \to K^*$ and $B_s \to \phi$:

- LQCD calculations at **high** q^2
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  [Bharucha et al. 2015] [NG/Kokulu/van Dyk 2018]

$B \to K$ **FFs excellent** status (need independent calculation at low q^2)

more LQCD results needed for vector states (for high precision K^* width cannot be neglected)

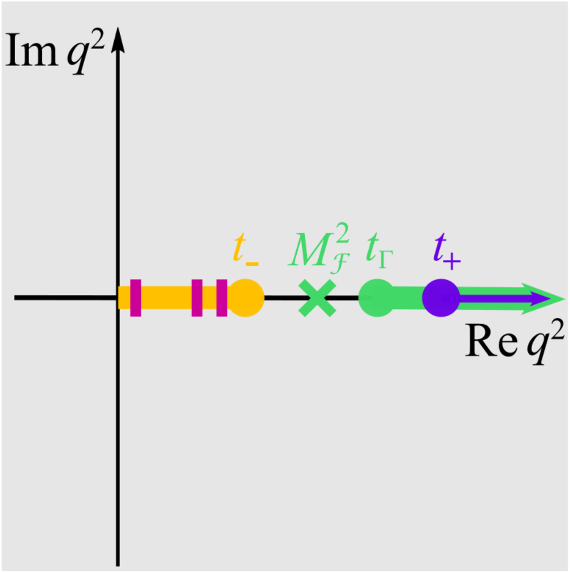
how to **combine** different calculations and obtain result **whole** semileptonic region?

Map for local FFs

obtain local FFs $\mathcal{F}_\lambda$ in the whole semileptonic region by either

- extrapolating LQCD calculations to low q^2
- or combining LQCD and LCSRs

$\mathcal{F}_\lambda$ analytic functions of q^2 except for isolated $s\bar{b}$ poles and a branch cut for $q^2 > t_\Gamma = \left(M_{B_s} + (2)M_\pi\right)^2$



branch cut differs from the pair production threshold:

$t_\Gamma \neq t_+ = \left(M_B + M_{K^{(*)}}\right)^2$ contrary to, e.g., $B \to \pi$

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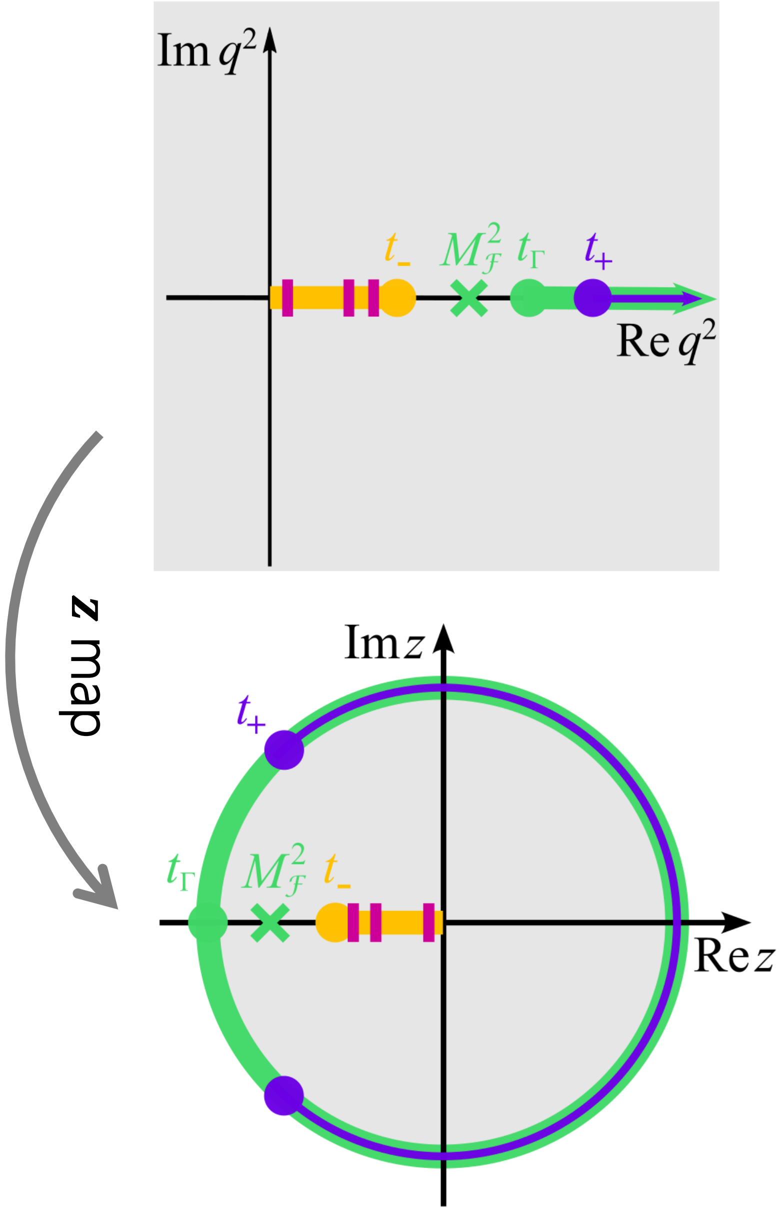
define the map

$$z(q^2) = \frac{\sqrt{t_\Gamma - q^2} - \sqrt{t_\Gamma}}{\sqrt{t_\Gamma - q^2} + \sqrt{t_\Gamma}}$$



Parametrization for $\mathcal{F}_\lambda$

$\mathcal{F}_\lambda$ analytic in the open unit disk $\Longrightarrow$ expand $\mathcal{F}_\lambda$ in a Taylor series in z

we propose a **new parametrization** [NG/van Dyk/Virto 2020]

$$\mathcal{F}_\lambda = \frac{1}{\mathcal{P}(z)\phi(z)} \sum_{k=0}^{\infty} c_k\, p_k(z) \qquad \sum_{k=0}^{\infty} |c_k|^2 < 1$$

$\mathcal{P}(z)\phi(z), p_k(z)$ are known functions

fit c_k coefficients to LQCD (and LCSR) results and use unitarity bounds

first parametrization that is simultaneously:

- valid for $t_\Gamma \neq t_+$
- unitarity bounded

previous works on $B \to K^{(*)}$ local FFs always approximated $t_\Gamma = t_+$
$\Longrightarrow$ non-quantifiable systematic uncertainties

$$p_0^{B\to K}(\hat{z}) = \frac{1}{\sqrt{2\alpha_{BK}}}$$

$$p_1^{B\to K}(\hat{z}) = \left(\hat{z} - \frac{\sin(\alpha_{BK})}{\alpha_{BK}}\right)\sqrt{\frac{\alpha_{BK}}{2\alpha_{BK}^2+\cos(2\alpha_{BK}}}$$

$$p_2^{B\to K}(\hat{z}) = \left(\hat{z}^2 + \frac{\sin(\alpha_{BK})(\sin(2\alpha_{BK})-2\alpha_{BK})}{2\alpha_{BK}^2+\cos(2\alpha_{BK})-1} + \frac{2\sin(\alpha_{BK})(\sin(\alpha_{BK})-2\alpha_{BK})}{2\alpha_{BK}^2+\cos(2\alpha_{BK})-1}\right)$$

$$p_3^{B\to K}(\hat{z}) = \cdots$$

Local form factors predictions

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fit available inputs to

$$\mathcal{F}_\lambda = \frac{1}{\mathcal{P}(z)\phi(z)} \sum_{k=0}^{3} c_k\, p_k(z) \qquad \sum_{k=0}^{3} |c_k|^2 < 1$$

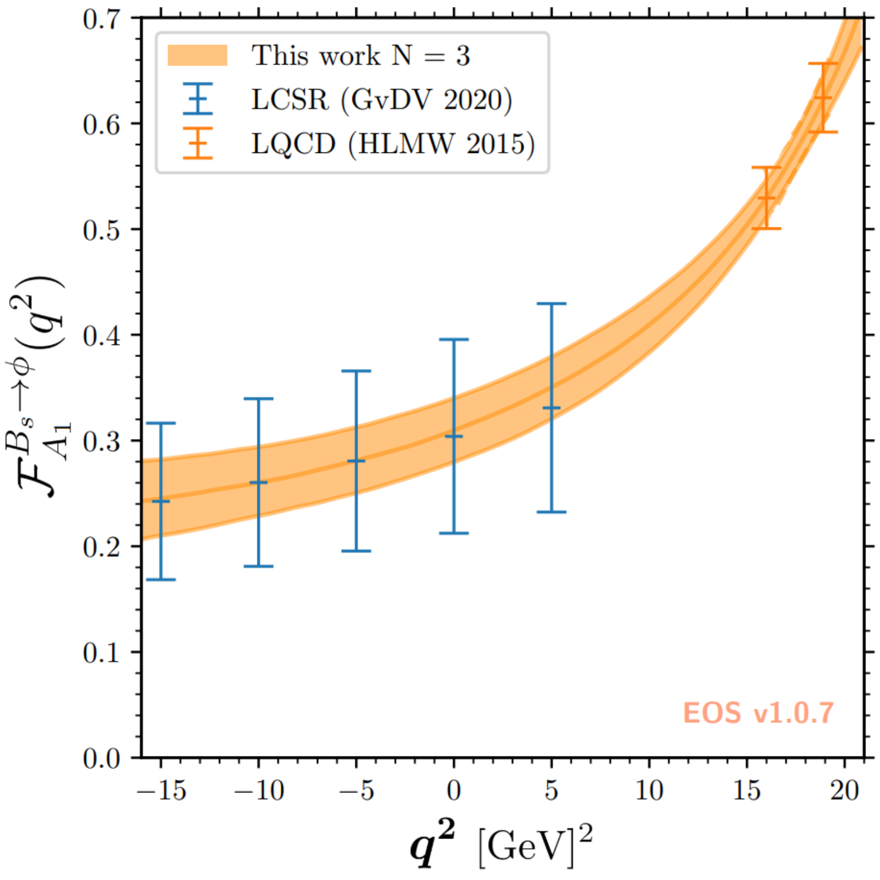
obtain numerical results for $B \to K^{(*)}$ and $B_s \to \phi$ in the whole semileptonic region

[NG/Reboud/van Dyk/Virto 2023]

first simultaneous fit of these FFs

systematic uncertainties under control
large p values

results given in machine readable files



Non-local form factors

Obtaining theoretical predictions for $\mathcal{H}_\lambda$

1. compute the non-local FFs $\mathcal{H}_\lambda$ using a light-cone OPE at negative q^2

$$\mathcal{H}_\lambda(q^2) = C_\lambda(q^2)\mathcal{F}_\lambda(q^2) + \tilde{C}_\lambda(q^2)\mathcal{V}_\lambda(q^2) + \cdots$$

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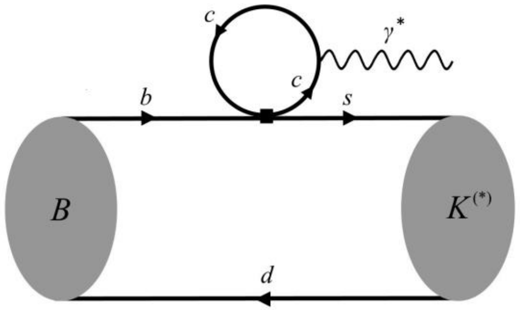
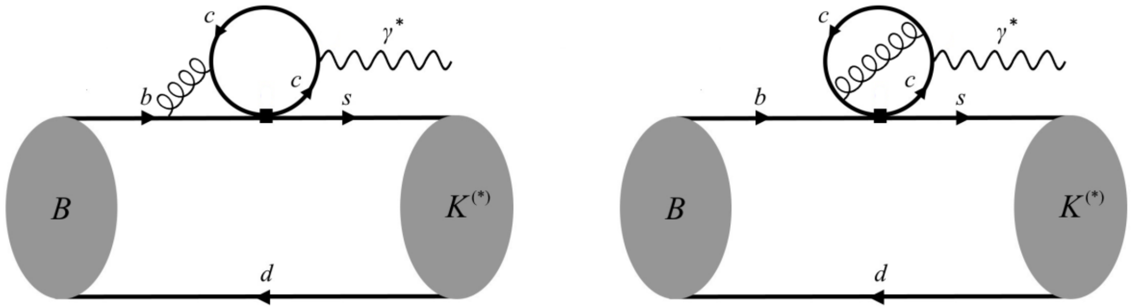
leading power (LO in α_s)



+ hard gluons (α_s) corrections



[Bell/Huber 2014] [Asatrian/Greub/Virto 2019]

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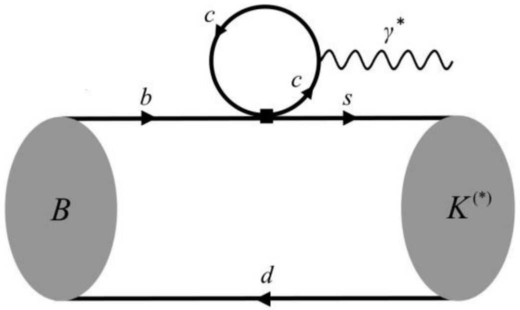

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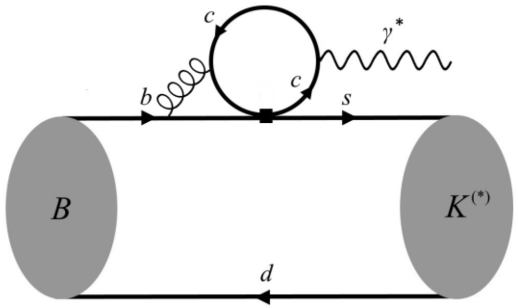
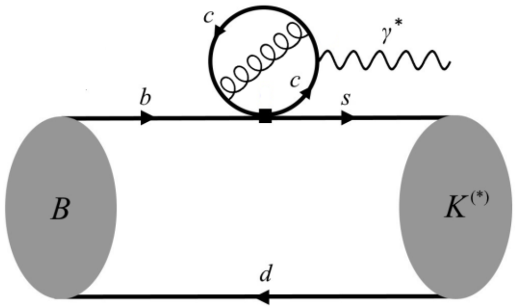


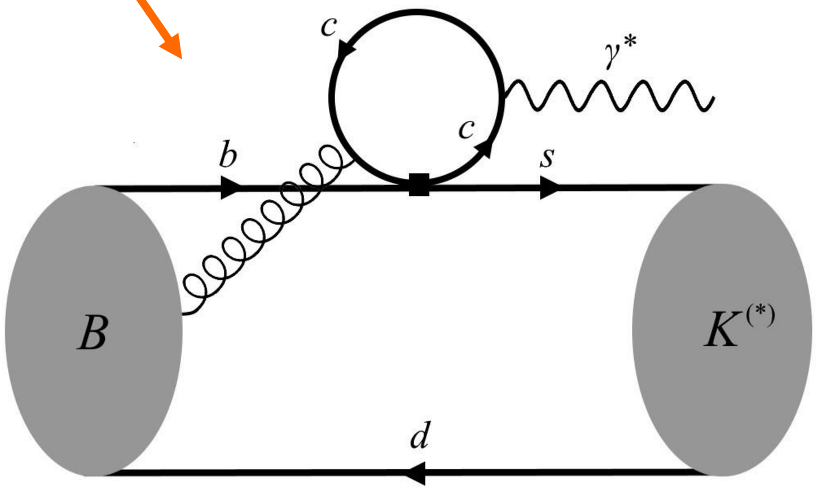


[Bell/Huber 2014] [Asatrian/Greub/Virto 2019]

soft gluon correction

non-perturbative

$\Longrightarrow$ not α_s suppressed



[Khodjamirian et al. 2010]

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2. extract $\mathcal{H}_\lambda$ at $q^2 = m^2_{J/\psi}$ from $B \to K^{(*)} J/\psi$ and $B_s \to \phi\, J/\psi$ measurements (decay amplitudes independent of the local FFs)

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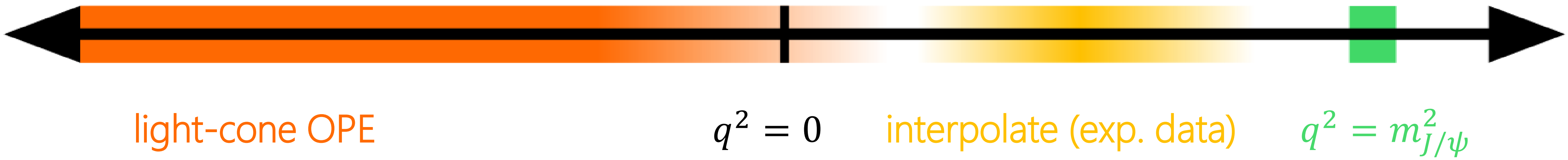
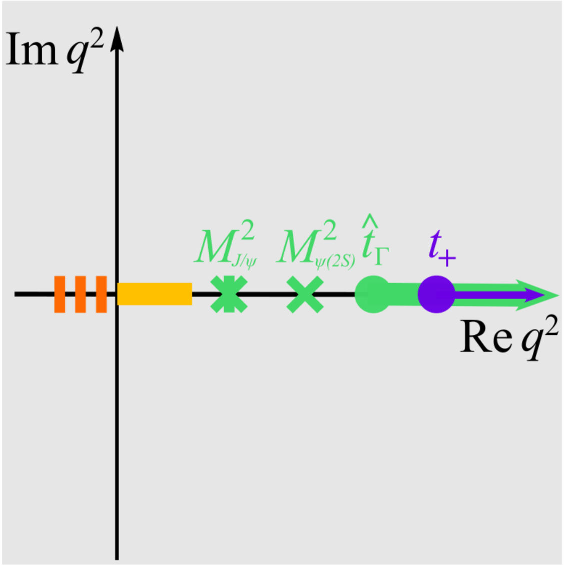
3. **new approach: interpolate** these two results to obtain theoretical predictions in the low q^2 $(0 < q^2 < 8\ \mathrm{GeV}^2)$ region $\Longrightarrow$ compare with experimental data

need a parametrization to interpolate $\mathcal{H}_\lambda$: which is the optimal parametrization?



Map for non-local FFs



similar situation with respect to $\mathcal{F}_\lambda$

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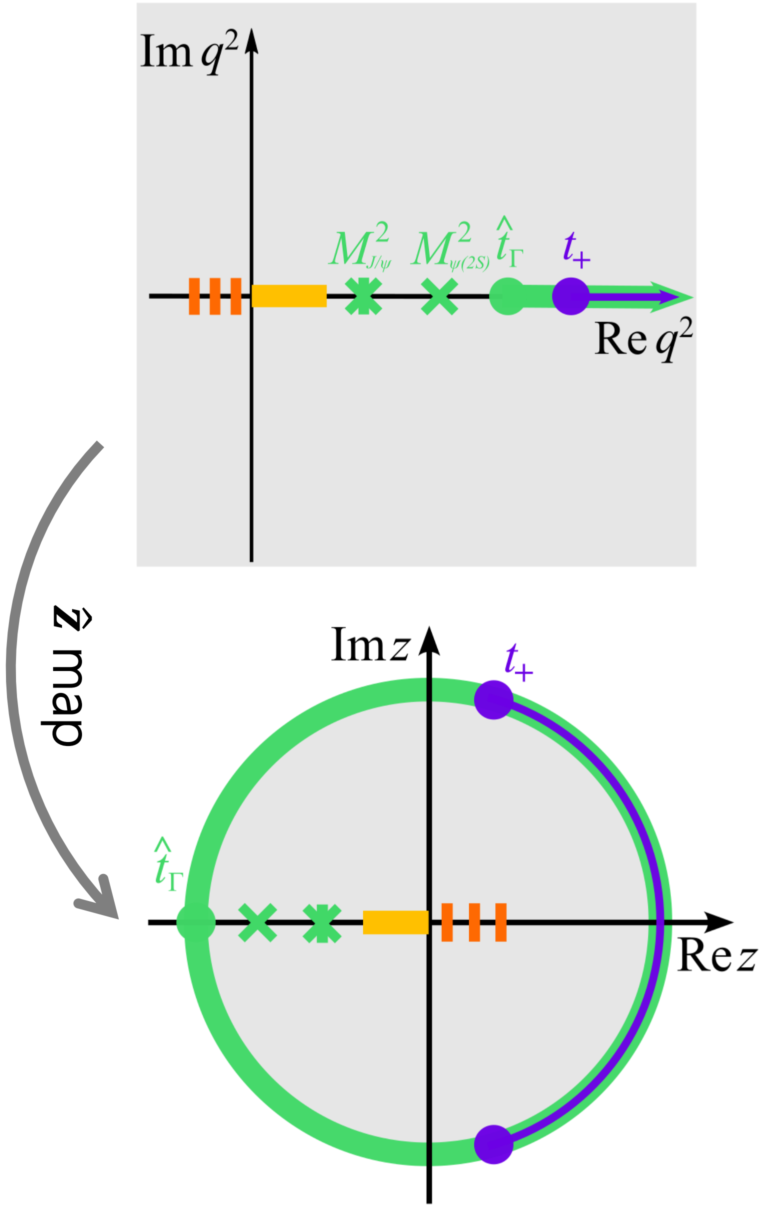
$t_\Gamma \neq t_+ = \left(M_B + M_{K^{(*)}}\right)^2$

define the map

$$\hat{z}(q^2) = \frac{\sqrt{\hat{t}_\Gamma - q^2} - \sqrt{\hat{t}_\Gamma}}{\sqrt{\hat{t}_\Gamma - q^2} + \sqrt{\hat{t}_\Gamma}}$$

only difference between $\mathcal{F}_\lambda$ and $\mathcal{H}_\lambda$ is the threshold $\hat{t}_\Gamma$ and the poles due to more complicate structure of the operators



Parametrizations for $\mathcal{H}_\lambda$

q^2 expansion [Jäger/Camalich 2012, Ciuchini et al. 2015]

$$\mathcal{H}_\lambda(q^2) = \mathcal{H}_\lambda^{\mathrm{QCDF}}(q^2) + \mathcal{H}_\lambda^{\mathrm{rest}}(0) + \frac{q^2}{M_B^2}\mathcal{H}_\lambda^{\mathrm{rest},\prime}(0) + \frac{(q^2)^2}{M_B^4}\mathcal{H}_\lambda^{\mathrm{rest},\prime\prime}(0) + \cdots$$

z Taylor expansion [Bobeth/Chrzaszcz/van Dyk/Virto 2017]

$$\mathcal{H}_\lambda(z) \propto \sum_{n=0}^{\infty} c_n z^n$$

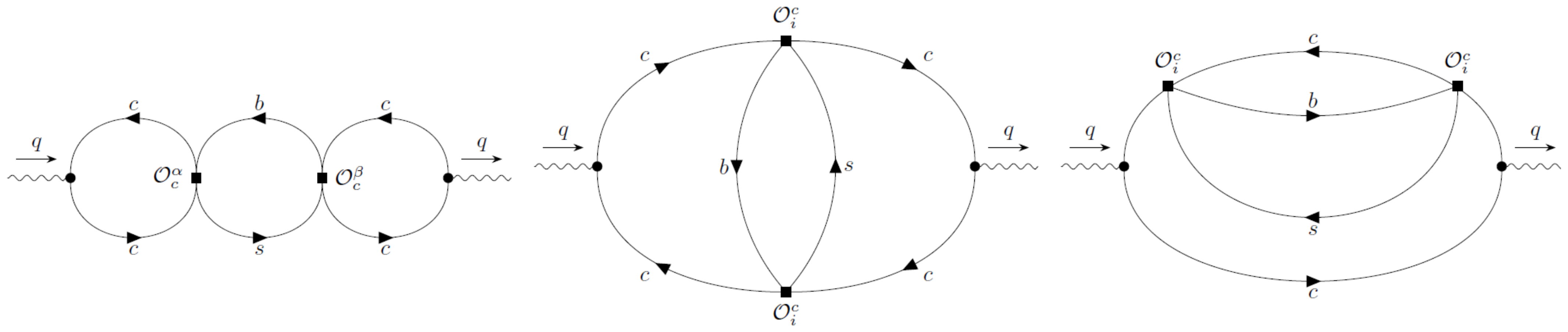
we propose a new parametrization ($\hat{z}$ polynomials) [NG/van Dyk/Virto 2020]

$$\mathcal{H}_\lambda(\hat{z}) = \frac{1}{\mathcal{P}(z)\phi(z)} \sum_{n=0}^{\infty} \beta_n p_n(\hat{z}) \qquad \sum_{n=0}^{\infty} |\beta_n|^2 < 1$$

first unitarity bounds for $\mathcal{H}_\lambda$

Non-local form factors predictions

derivation of the bound requires very **complicated calculations** (normalization of $\phi(z)$ function)



model independent constraint

strengthen the bound by considering different channels simultaneously
($B \rightarrow K^{(*)}\ell^+\ell^-$, $B_s \rightarrow \phi\ell^+\ell^-$, $\Lambda_b \rightarrow \Lambda\ell^+\ell^-$,...)

new approach to constrain syst. unc. of non-local FFs

Non-local form factors predictions

$$\mathcal{A}(B \to K^{(*)}\ell\ell) = \mathcal{N}\left[\left(C_9 L_V^\mu + C_{10} L_A^\mu\right) \mathcal{F}_\mu - \frac{L_V^\mu}{q^2}\left(C_7\, \mathcal{F}_{T,\mu} + \mathcal{H}_\mu\right)\right]$$

obtain numerical results for the non-local FFs $\mathcal{H}_\lambda$

$$\mathcal{H}_\lambda(\hat{z}) \propto \sum_{n=0}^{5} \beta_n p_n(\hat{z})$$

fit the $\hat{z}$ parametrization

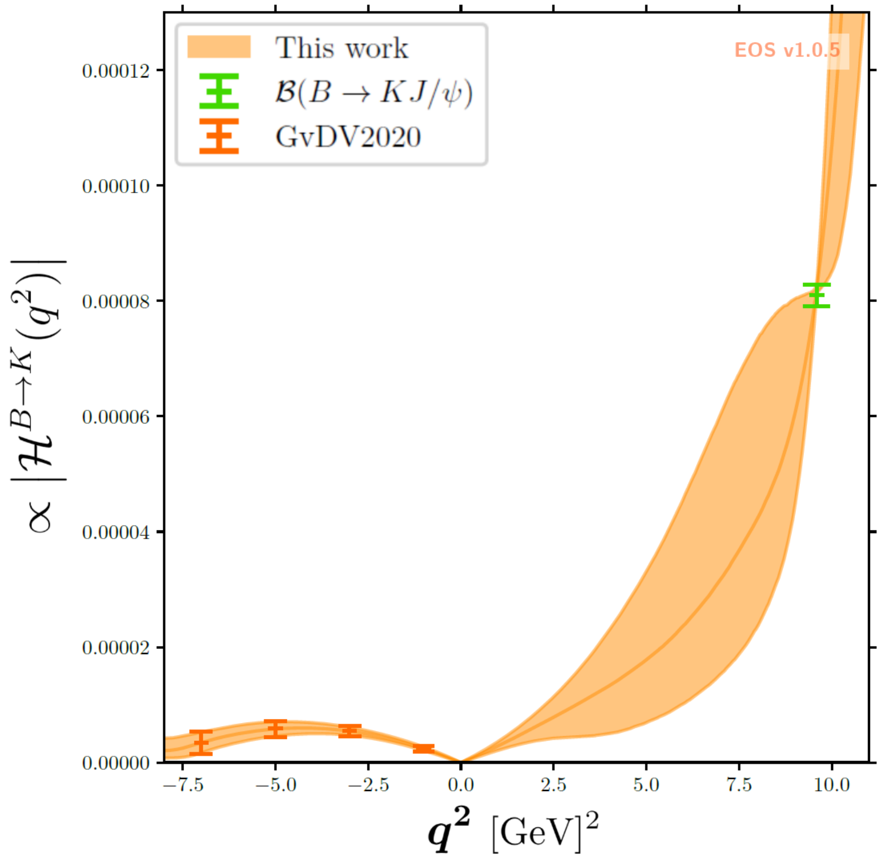
- light-cone OPE calculation at negative q^2

$$\mathcal{H}_\lambda(q^2) = \mathcal{C}_\lambda(q^2)\mathcal{F}_\lambda(q^2) + \tilde{\mathcal{C}}_\lambda(q^2)\mathcal{V}_\lambda(q^2) + \cdots$$

- $B \to K^{(*)} J/\psi$ and $B_s \to \phi\, J/\psi$ measurements at $q^2 = m_{J/\psi}^2$
- unitarity bound

all p values $> 11\%$

results given in machine readable files



[NG/Reboud/van Dyk/Virto 2022]

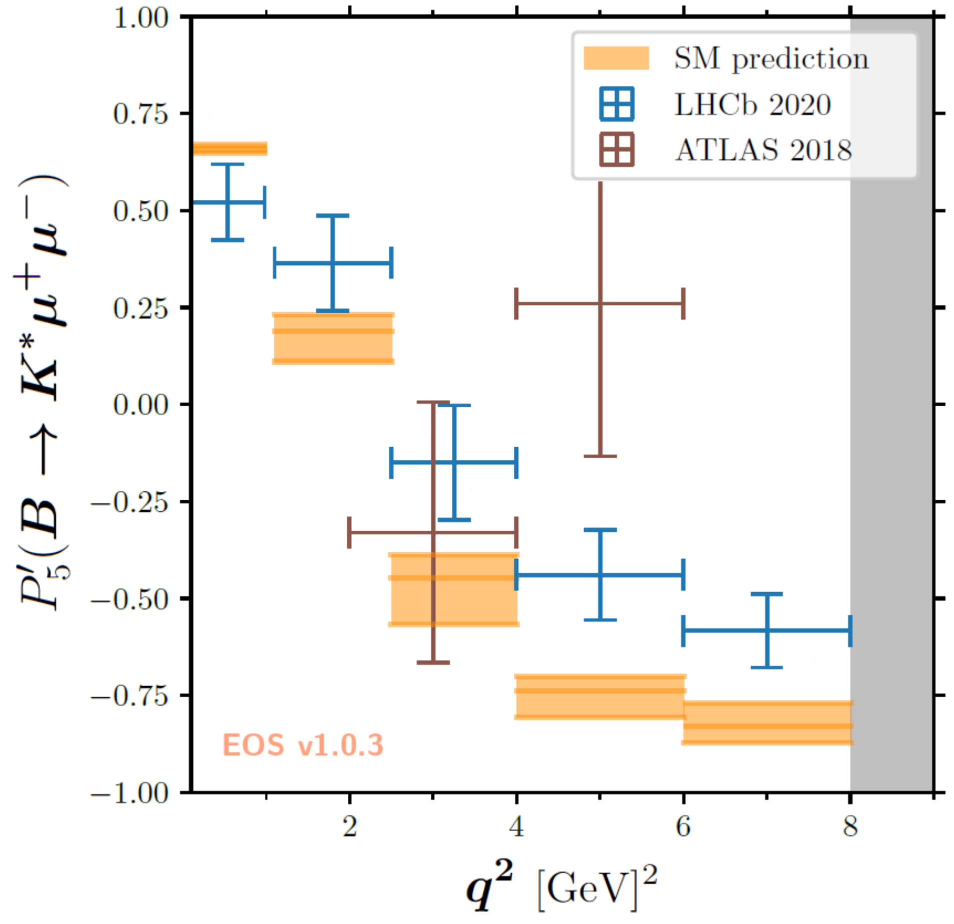
SM predictions and confrontation with data

SM predictions vs. data

predict observables using our $\mathcal{F}_\lambda$ and $\mathcal{H}_\lambda$ results:

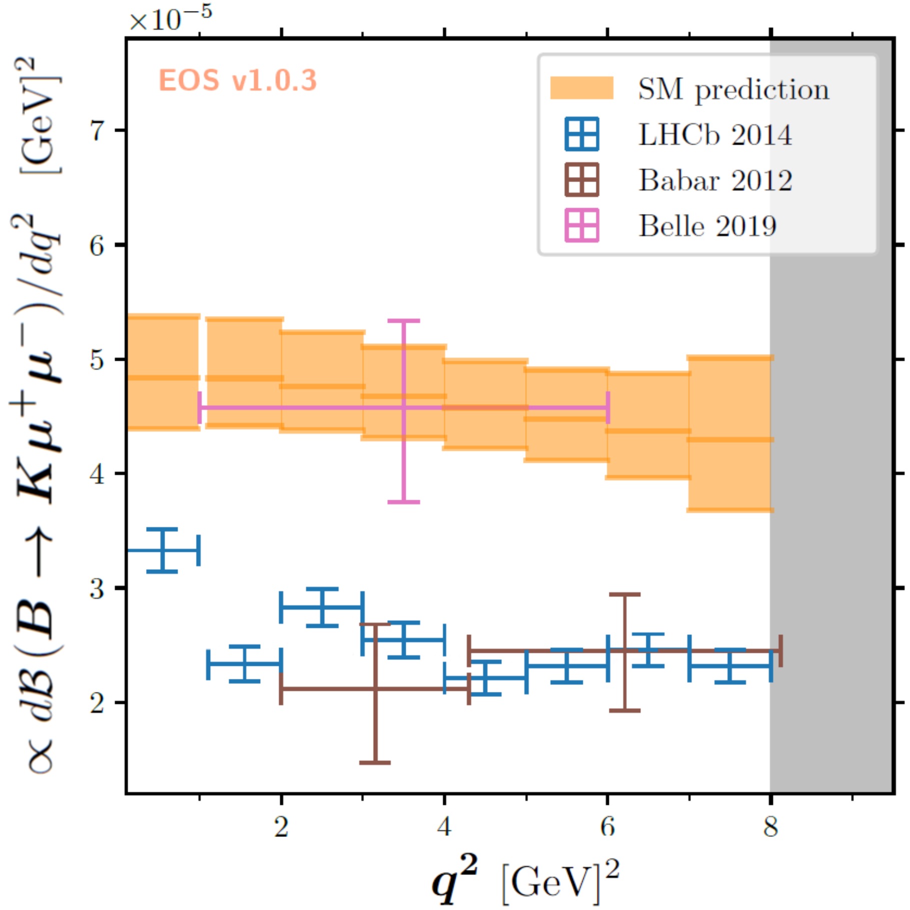
BRs and angular observables
for $B \to K^{(*)}\mu^+\mu^-$, and $B_s \to \phi\mu^+\mu^-$

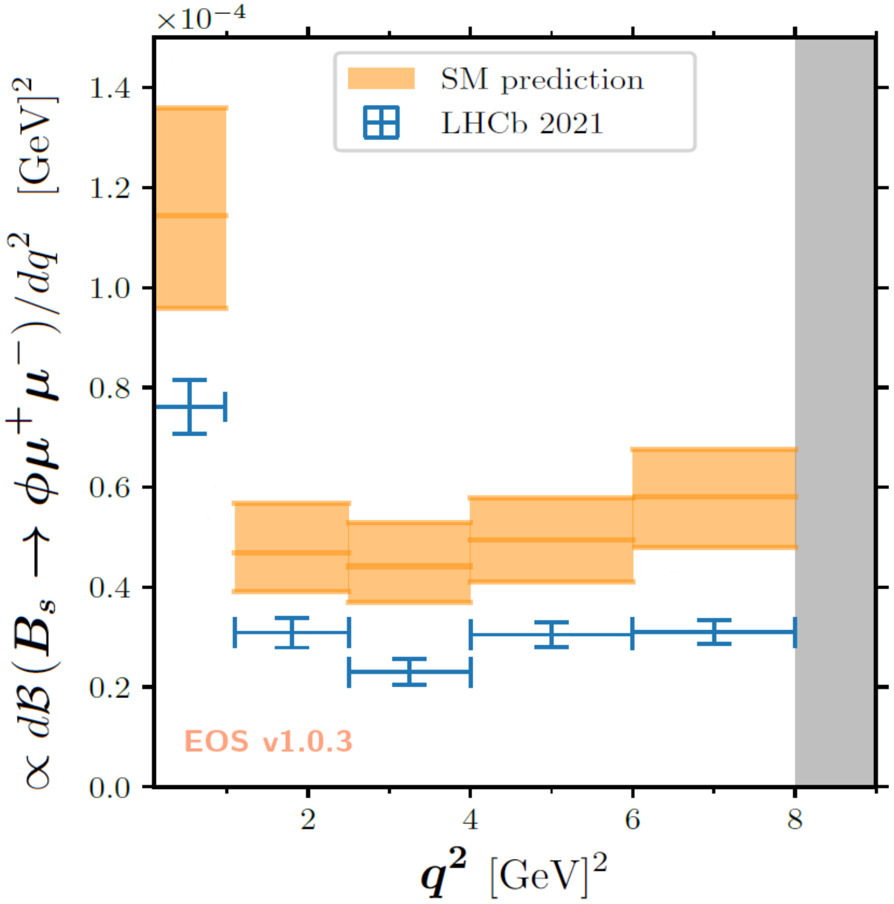
- theory uncertainties mostly due to $\mathcal{F}_\lambda$
- progress in $\mathcal{H}_\lambda$ calculations urgently needed
- more measurements on the way



[NG/Reboud/van Dyk/Virto 2022]

SM predictions vs. data





[NG/Reboud/van Dyk/Virto 2022]

coherent tensions between SM predictions and data

Global fit to $b \to s\mu^+\mu^-$ (setup)

use our predictions for the local and non-local FFs as priors

fit the Wilson coefficients C_9^{NP} and C_{10}^{NP} to the available experimental measurements in $b \to s\mu^+\mu^-$ transitions
$(C_{9,10} = C_{9,10}^{\mathrm{SM}} + C_{9,10}^{\mathrm{NP}})$

we perform three fits, one for each set of the following set of experimental measurements:
(BRs, angular observables, binned and not binned)

- $B \to K\mu^+\mu^- + B_s \to \mu^+\mu^-$
- $B \to K^*\mu^+\mu^-$
- $B_s \to \phi\mu^+\mu^-$

combined fit would be very challenging ⟶ 130 nuisance parameter

Global fit to $b \to s\mu^+\mu^-$ (results)

we **obtain good fits**, agreement between the three fits

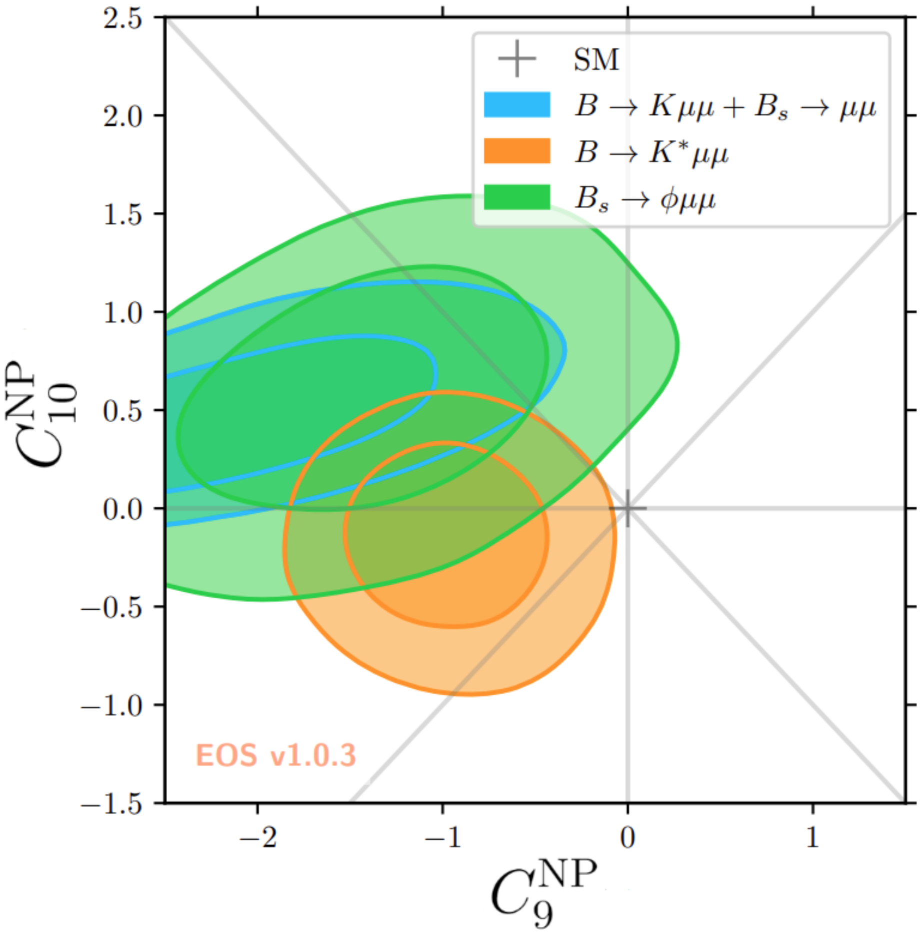
substantial tension w.r.t. SM (in agreement with the literature)

pulls (p value of the SM hypothesis):

- 5.7σ for $B \to K\mu^+\mu^- + B_s \to \mu^+\mu^-$
- 2.7σ for $B \to K^*\mu^+\mu^-$
- 2.6σ for $B_s \to \phi\mu^+\mu^-$

local FFs $\mathcal{F}_\lambda$ main uncertainties

non-local FFs $\mathcal{H}_\lambda$ cannot explain this tension



Open issues

Possible issues on local FFs

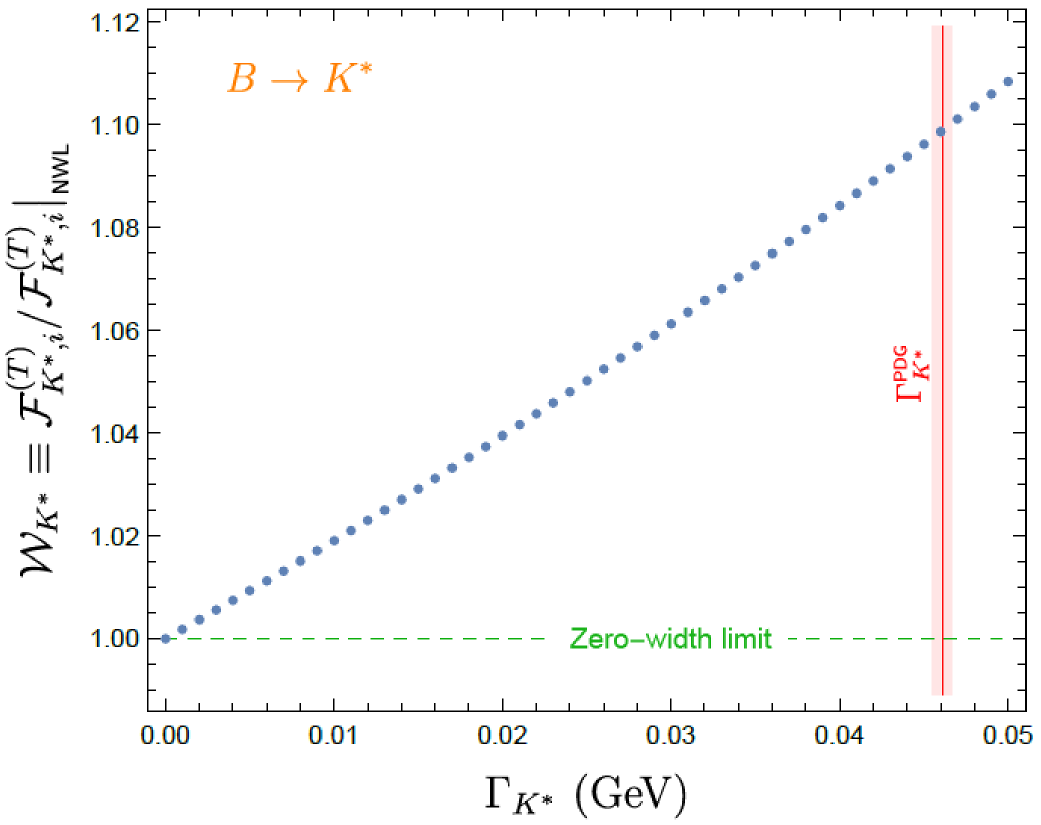
precise LQCD calculations for local $\mathcal{F}_\lambda$ FFs at low q^2 are **essential** to have better theoretical predictions

already available for $B \to K\ell^+\ell^-$ [HPQCD 2022]

w.i.p. for $B \to K^*\ell^+\ell^-$ and $B_s \to \phi\ell^+\ell^-$

K^* has a sizable width
$\Longrightarrow$ $B \to K\pi\ell^+\ell^-$ local FFs calculation
first steps in [Descotes-Genon et al. 2019] using LCSRs

clear path to solve these issues

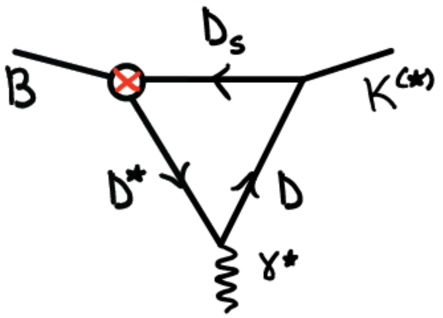


[Descotes-Genon et al. 2019]

Possible issues on non-local FFs

missing contributions?

Ciuchini et al. 2022 (also way before) claim that $B \to \bar{D}D_s \to K^{(*)}\ell^+\ell^-$ rescattering might have a sizable contribution $\Longrightarrow O(20\%)$ at amplitude level



partonic calculation does not yield large contribution (LP OPE and NLO α_s)

$$\mathcal{H}_\lambda(q^2) = C_\lambda(q^2)\mathcal{F}_\lambda(q^2) + \tilde{C}_\lambda(q^2)\mathcal{V}_\lambda(q^2) + \cdots$$

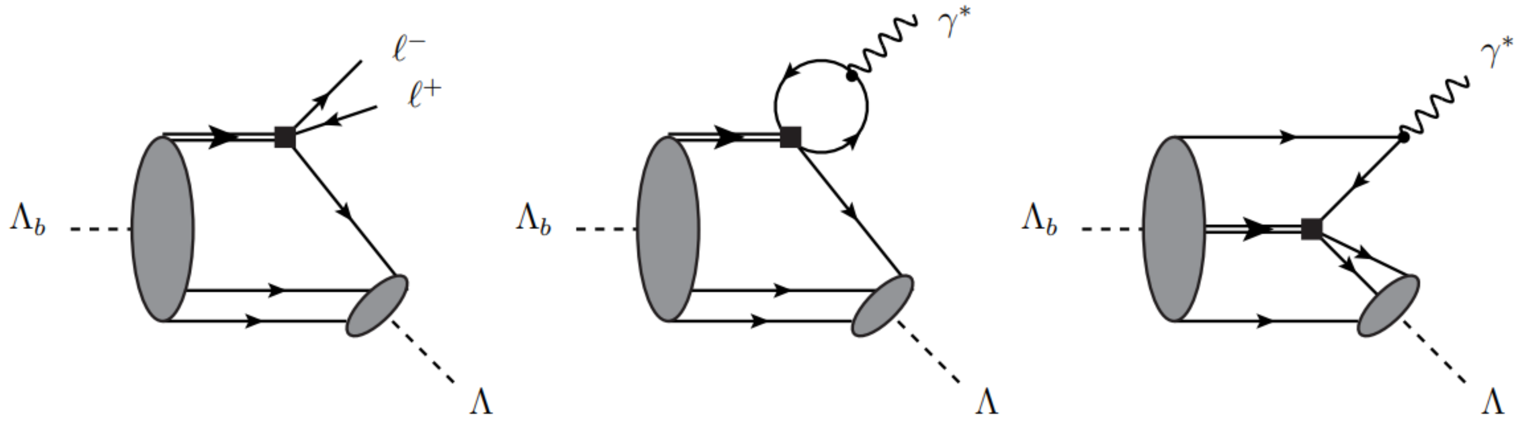
C_λ is complex valued for any q^2 value due to branch cut in $p^2 = M_B^2$ as expected

[Asatrian/Greub/Virto 2019]

multiple ways to solve these issues

1. new theory calculations for $B \to K^{(*)}\ell^+\ell^-$
2. explore different processes ($\Lambda_b \to \Lambda\ell^+\ell^-$, $B \to \pi\ell^+\ell^-$, …)

$\Lambda_b \to \Lambda \ell^+ \ell^-$ decays



[Feldmann/NG 2023]

if $b \to s\mu^+\mu^-$ anomalies are due to New Physics $\Longrightarrow$ **same shift expected in $\Lambda_b \to \Lambda\mu^+\mu^-$**
but rescattering effects are different

already measured by LHCb $\Longrightarrow$ new and more precise measurements on the way

progress needed in theory calculations (no estimate of charm-loop beyond naïve factorization)

first calculation of “annihilation” contributions in [Feldmann/NG 2024]

Summary and conclusion

Summary and conclusion

1. improved parametrization for local FFs $\mathcal{F}_\lambda$ (consider below threshold branch cuts)

   combine LQCD (and LCSRs) inputs to get new results for $\mathcal{F}_\lambda$ in $B \to K^{(*)}\ell^+\ell^-$ and $B_s \to \phi\ell^+\ell^-$

2. new theoretical predictions for $\mathcal{H}_\lambda$ combining our OPE calculation and $B \to K^{(*)}J/\psi$ data

   innovative approach — use unitarity bound to control $\mathcal{H}_\lambda$ uncertainties

3. new and precise SM predictions for observables in $B \to K^{(*)}\ell^+\ell^-$ and $B_s \to \phi\ell^+\ell^-$ decays

   coherent deviations between SM and data in $B \to K^{(*)}\ell^+\ell^-$ and $B_s \to \phi\ell^+\ell^-$ decays

4. progress on the theory side needed more than ever

Thank you!

Backup slides

Parametrizations for $\mathcal{F}_\lambda$

$\mathcal{F}_\lambda$ analytic in the open unit disk $\Longrightarrow$ expand $\mathcal{F}_\lambda$ in a Taylor series in $\boldsymbol{z}$ (up to some known function)

simple (BSZ) z parametrization $\Longrightarrow$ unbounded coefficients [Bharucha/Straub/Zwicky 2015]

$$\mathcal{F}_\lambda = \frac{1}{1-\frac{q^2}{M_{\mathcal{F}}^2}} \sum_{k=0}^{\infty} a_k\, z^k$$

BGL parametrization $\Longrightarrow$ valid only if $t_\Gamma = t_+$, monomials orthonormal on the unit circle [Boyd/Grinstein/Lebed 1997]

$$\mathcal{F}_\lambda = \frac{1}{\mathcal{P}(z)\phi(z)} \sum_{k=0}^{\infty} b_k\, z^k \qquad \sum_{k=0}^{\infty} |b_k|^2 < 1$$

GvDV parametrization $\Longrightarrow$ valid also for $t_\Gamma \neq t_+$, generalization of BGL, polynomials orthonormal on the arc of the unit circle [NG/van Dyk/Virto 2020]

$$\mathcal{F}_\lambda = \frac{1}{\mathcal{P}(z)\phi(z)} \sum_{k=0}^{\infty} c_k\, p_k(z) \qquad \sum_{k=0}^{\infty} |c_k|^2 < 1$$

fit this parametrization to LQCD (and LCSR) results and use new improved bounds

Simultaneous analysis

strengthen the bound by considering different channels simultaneously
($B \to K^{(*)}\ell^+\ell^-$, $B_s \to \phi\ell^+\ell^-$, $\Lambda_b \to \Lambda\ell^+\ell^-$,...)

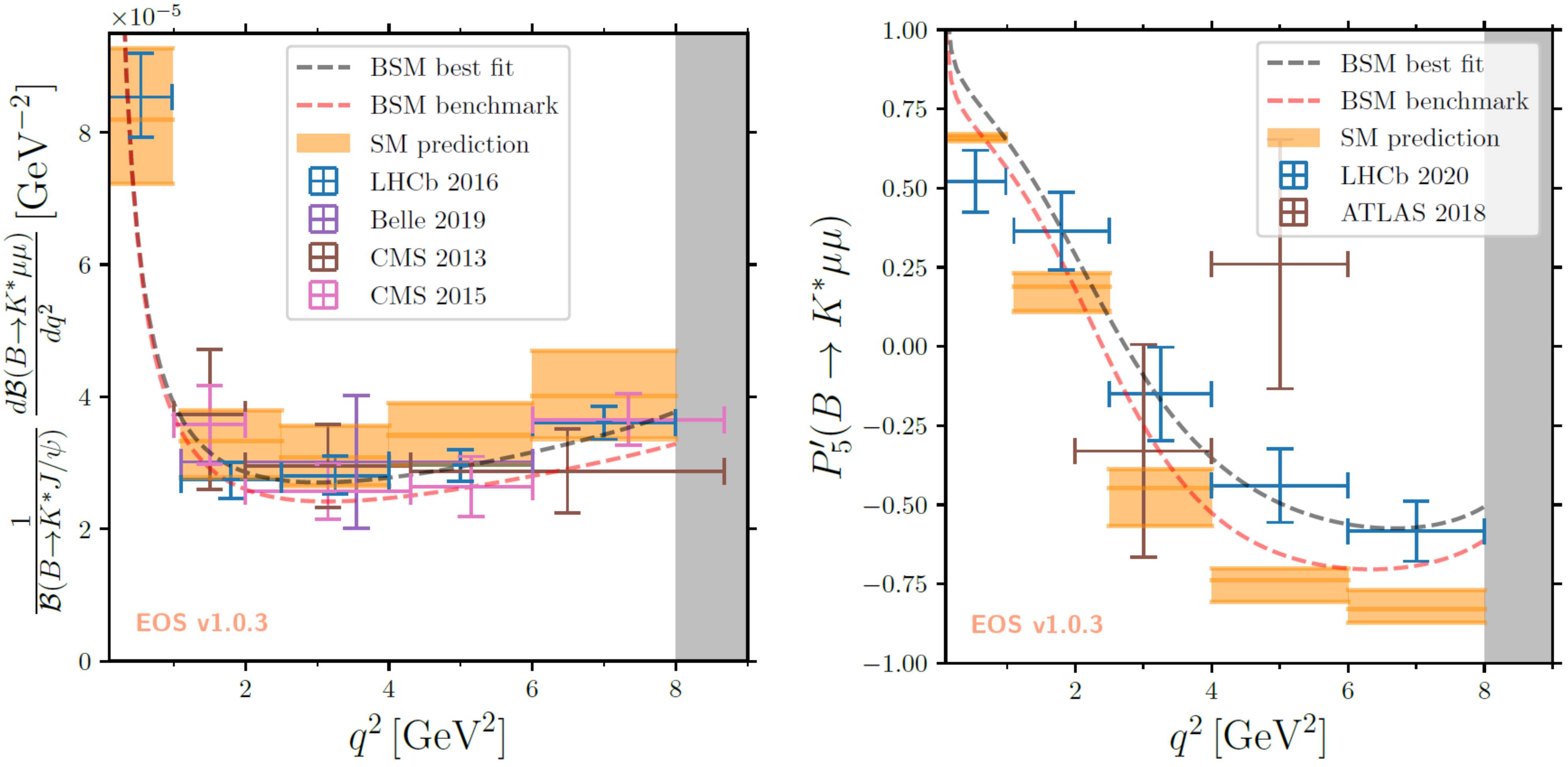
$$\mathcal{H}_\lambda^{B\to K} \cong \sum_\lambda \sum_{n=0}^{\infty} \beta_{n,\lambda}^{B\to K} p_n(\hat{z})$$

$$\sum_\lambda \sum_{k=0}^{\infty} \left|\beta_{n,\lambda}^{B\to K}\right|^2 + \sum_\lambda \sum_{k=0}^{\infty} \left|\beta_{n,\lambda}^{B\to K^*}\right|^2 + \sum_\lambda \sum_{k=0}^{\infty} \left|\beta_{n,\lambda}^{B_s\to\phi}\right|^2 + \sum_\lambda \sum_{k=0}^{\infty} \left|\beta_{n,\lambda}^{\Lambda_b\to\Lambda}\right|^2 + \cdots < 1$$

many nuisance parameters $\Longrightarrow$ technically very difficult but doable

progress in calculations for $\Lambda_b \to \Lambda\ell^+\ell^-$ needed

P_5' angular observable

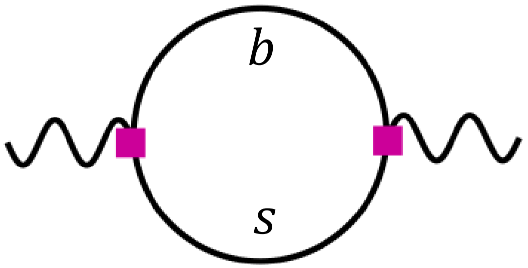


local FFs $\mathcal{F}_\lambda$ inputs crucial (B meson vs. light meson LCSRs)

Unitarity bounds

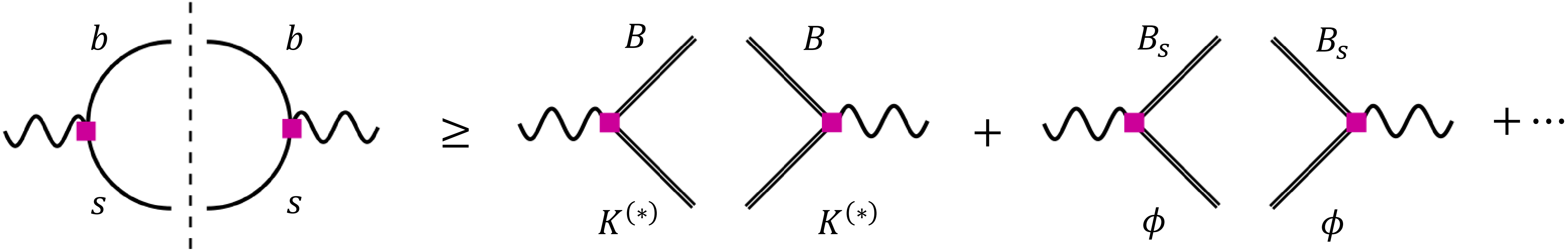
derive unitarity bounds from a correlator

$$\Pi(k,q) = i\int \mathrm{d}^4x\, e^{ikx}\langle 0|T\{O^{\mathrm{had}}_{7,9,10}(x), O^{\mathrm{had},\dagger}_{7,9,10}(y)\}|0\rangle$$



compute $\mathrm{Im}\,\Pi$ perturbatively
inclusive calculation

$\mathrm{Im}\,\Pi$ can also be obtained using **unitarity**
as sum of **exclusive** contributions



$\mathrm{Im}\,\Pi$

$BK^{(*)}$ and $B_s\phi$ contribution to $\mathrm{Im}\,\Pi$ = $B \to K^{(*)}$ and $B_s \to \phi$ local FFs

Derivation of the dispersive bound

define the correlator

$$\Pi(k,q) = i \int \mathrm{d}^4x \, e^{ikx} \langle 0 | T\{\mathcal{O}^\mu(x), \mathcal{O}_\mu(0)\} | 0 \rangle$$

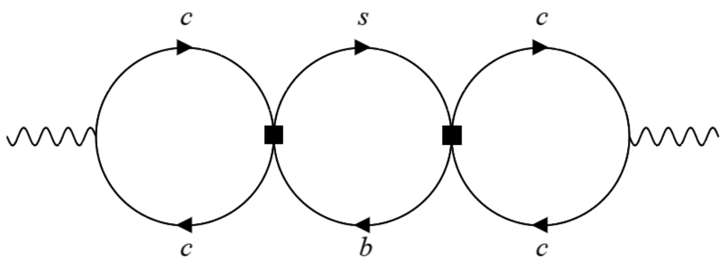
where

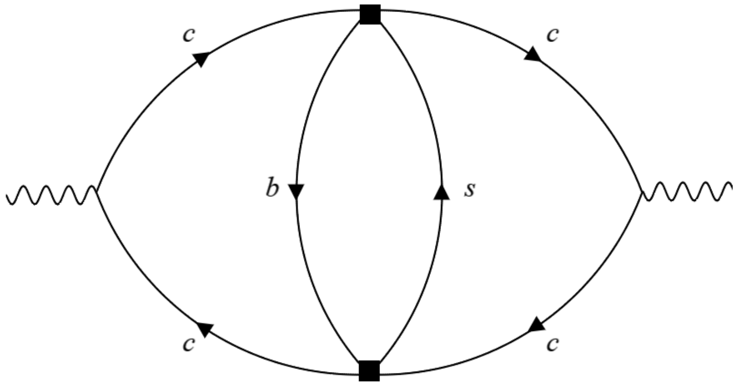
$$\mathcal{O}_\mu \propto \int d^4x \, e^{iq\cdot y} \, T\{ j_\mu^{em}(x+y), (C_1 O_1^c + C_2 O_2^c)(x) \}$$

use a subtracted dispersion relation

$$\chi(q^2) \propto \int_{(M_B+M_K)^2}^{\infty} ds \frac{\mathrm{Disc}_{bs}\Pi(s)}{(q^2-s)^3}$$

calculate $\chi(q^2)$ perturbatively and $\mathrm{Disc}_{bs}\Pi \to \mathcal{H}_\lambda$ using unitarity





Dispersive bound

using unitarity and dispersion relation, we obtain a constraint
on the non-local form factors $\mathcal{H}_\lambda$

dispersive bound

$$1 > \int_{(M_B+M_K)^2}^{\infty} ds\, |\phi^{B\to K}(z)|^2 |\mathcal{H}^{B\to K}(s)|^2 + B \to K^* \text{ and } B_s \to \phi \text{ contr.}$$

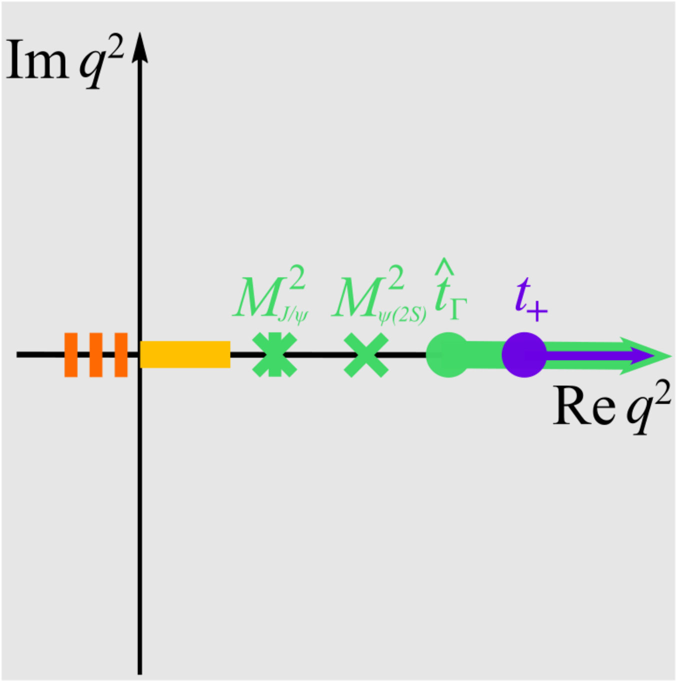
- **first dispersive bound** for $\mathcal{H}^{B\to K}$, $\mathcal{H}_\lambda^{B\to K^*}$, $\mathcal{H}_\lambda^{B_s\to\phi}$
- **model independent** constraint
- strengthen the bound by adding additional contributions (baryons)

Exploit the dispersive bound

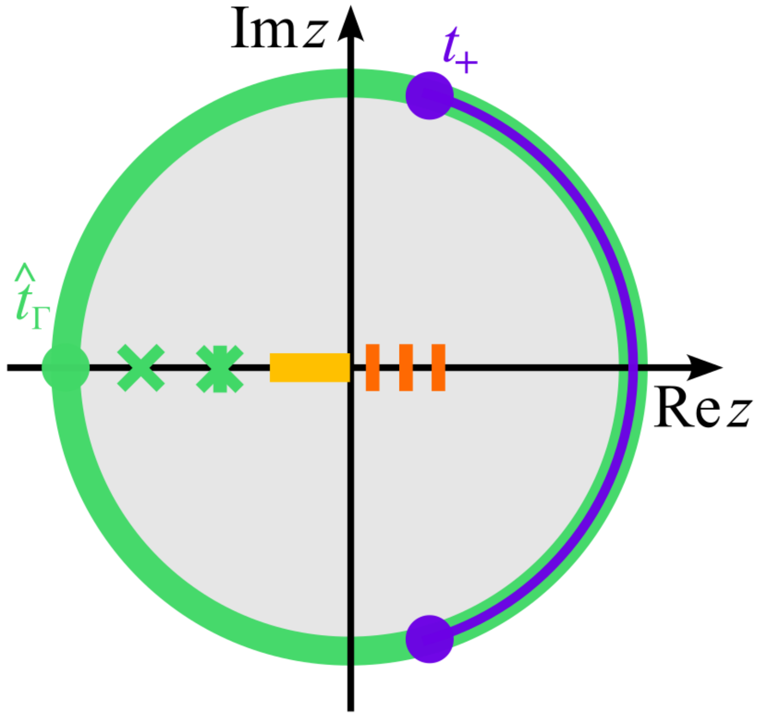
$\mathcal{H}_\lambda$ has a branch cut for $q^2 > \hat{t}_\Gamma = 4M_D^2$ — note that $\hat{t}_\Gamma \neq t_+ \equiv \left(M_B + M_{K^{(*)}}\right)^2$

define the $\hat{z}$ mapping

$$\hat{z}(q^2) = \frac{\sqrt{\hat{t}_\Gamma - q^2} - \sqrt{\hat{t}_\Gamma}}{\sqrt{\hat{t}_\Gamma - q^2} + \sqrt{\hat{t}_\Gamma}}$$



q^2 plane
real axis $q^2 > \hat{t}_+$ $\Longrightarrow$ $\hat{z}$ plane
arc of unit circle



Exploit the dispersive bound

$$1 > \int_{(M_B+M_K)^2}^{\infty} ds\, |\phi^{B\to K}(s)|^2 |\mathcal{H}^{B\to K}(s)|^2 + B \to K^* \text{ and } B_s \to \phi \text{ contr.}$$

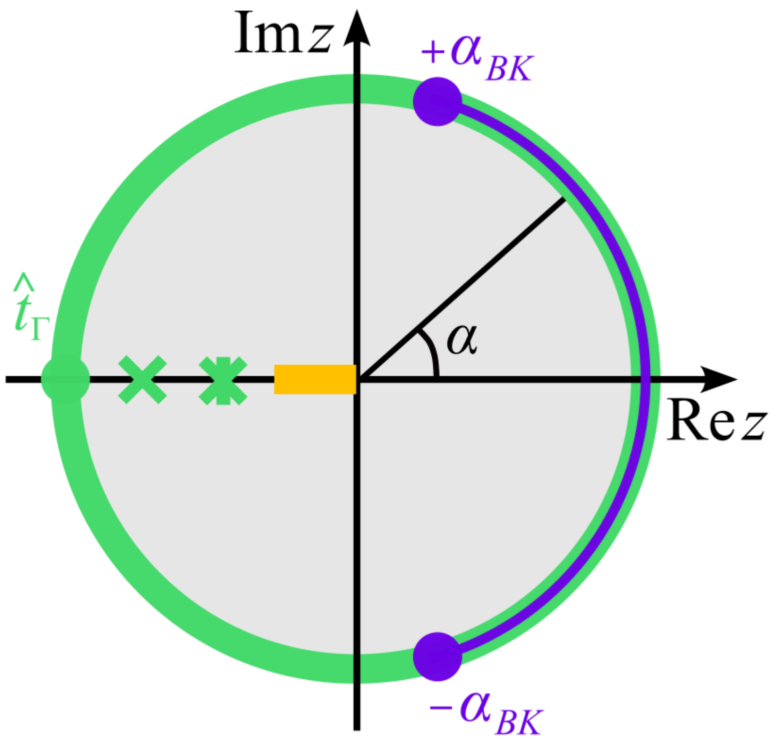
apply the $\hat{z}$ mapping

$$1 > \int_{-\alpha_{BK}}^{+\alpha_{BK}} d\alpha \sum_{\lambda} \left|\widehat{\mathcal{H}}^{B\to K}(\hat{z})\right|^2 + B \to K^* \text{ and } B_s \to \phi \text{ contr.}$$

where $\hat{z} = e^{i\alpha}$ and

$$\widehat{\mathcal{H}}^{B\to K}(\hat{z}) = \mathcal{P}(\hat{z})\, \phi^{B\to K}(\hat{z})\, \mathcal{H}_{\lambda}^{B\to K}(\hat{z})$$

Blaschke factor $\mathcal{P}$, outer function $\phi^{B\to K}$



$\hat{\mathcal{H}}_\lambda$ parametrization

$$1 > \int_{-\alpha_{BK}}^{+\alpha_{BK}} d\alpha \left|\hat{\mathcal{H}}^{B\to K}(\hat{z})\right|^2 + B \to K^* \text{and } B_s \to \phi \text{ contr.}$$

expand $\hat{\mathcal{H}}_\lambda$ in orthogonal polynomials $p_n(\hat{z})$

$$\hat{\mathcal{H}}(\hat{z}) = \sum_{n=0}^{\infty} \beta_n \, p_n(\hat{z})$$

now the dispersive bound reads

$$1 > \sum_{n=0}^{\infty} |\beta_n^{B\to K}|^2 + \sum_\lambda \left(2 \sum_{n=0}^{\infty} \left|\beta_{\lambda,n}^{B\to K^*}\right|^2 + \sum_{n=0}^{\infty} \left|\beta_{\lambda,n}^{B_s\to\phi}\right|^2 \right)$$

no bound for the $\hat{z}$ monomials
(coefficient of the Taylor expansion)

$$p_0^{B\to K}(\hat{z}) = \frac{1}{\sqrt{2\alpha_{BK}}}$$

$$p_1^{B\to K}(\hat{z}) = \left(\hat{z} - \frac{\sin(\alpha_{BK})}{\alpha_{BK}}\right) \sqrt{\frac{\alpha_{BK}}{2\alpha_{BK}^2 + \cos(2\alpha_{BK}) - 1}}$$

$$p_2^{B\to K}(\hat{z}) = \left(\hat{z}^2 + \frac{\sin(\alpha_{BK})(\sin(2\alpha_{BK}) - 2\alpha_{BK})}{2\alpha_{BK}^2 + \cos(2\alpha_{BK}) - 1} \hat{z} + \frac{2 \sin}{}\right.$$

$$p_3^{B\to K}(\hat{z}) = \cdots$$