

# RELATIVISTIC EFFECTS ON THE ORBITS OF THE CLOSEST STARS TO THE BLACK HOLE AT THE CENTER OF THE GALAXY

JCHE: 07/11/2024

Karim ABD EL DAYEM

Supervisors: Frédéric VINCENT, Thibaut PAUMARD, Guy PERRIN

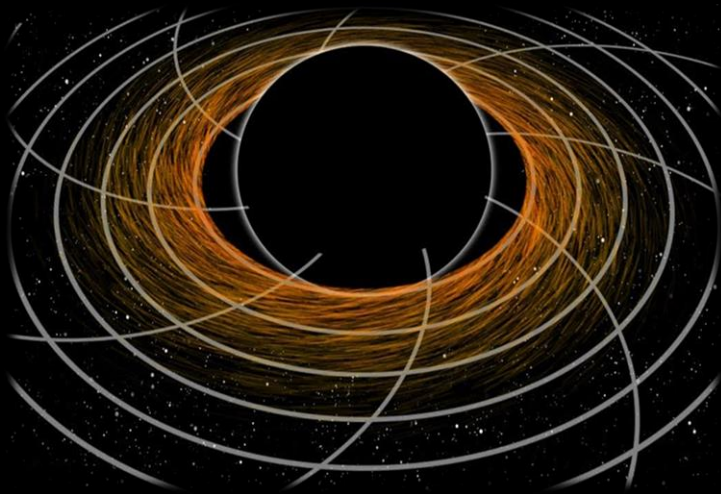


Observatoire  
de Paris

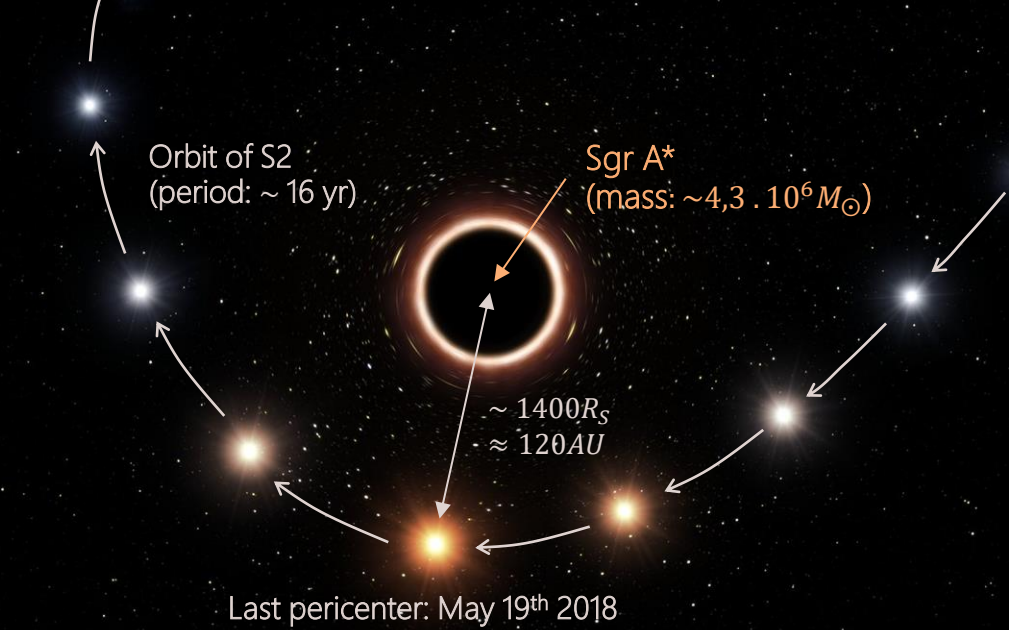
PSL 



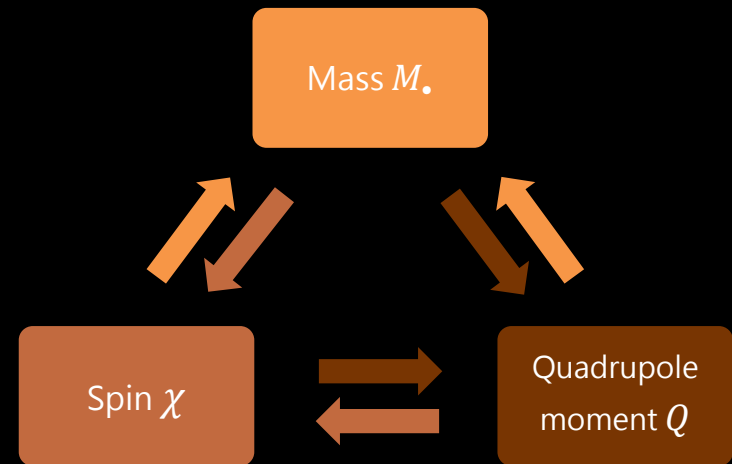
# CONTEXT & AIMS



- Curved space-time ↔ Mass
- Spinning space-time ↔ Spin
- Oblate space-time ↔ Quadrupole moment



No hair-theorem:



# CONTEXT & AIMS

Perturbation to Kepler:

$$a_{2PN} = a_{Sch} + a_{\chi} + a_Q$$

with:

$$a_{\chi} \propto \chi$$

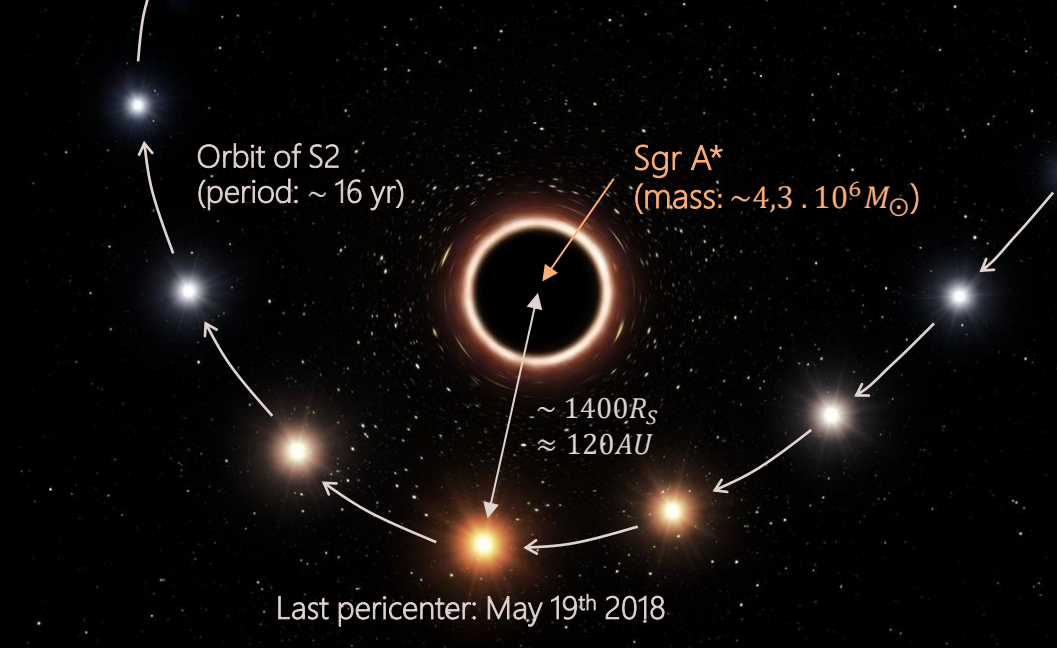
$$a_Q \propto Q$$

Independently fitting  $\chi$  and  $Q$  from the monitoring of S-stars

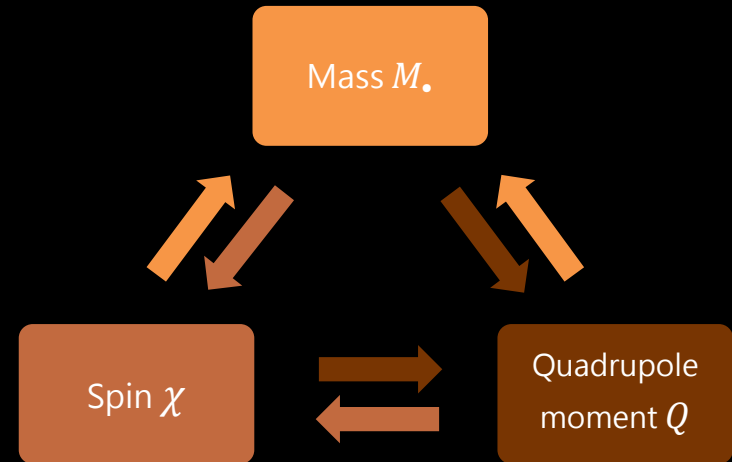


Test the no-hair theorem:

$$Q = -\frac{G^2 M_{\bullet}^3}{c^4} \chi^2$$



No hair-theorem:



# CONTEXT & AIMS

Perturbation to Kepler:

$$a_{2PN} = a_{Sch} + a_{\chi} + a_Q$$

with:

$$a_{\chi} \propto \chi$$

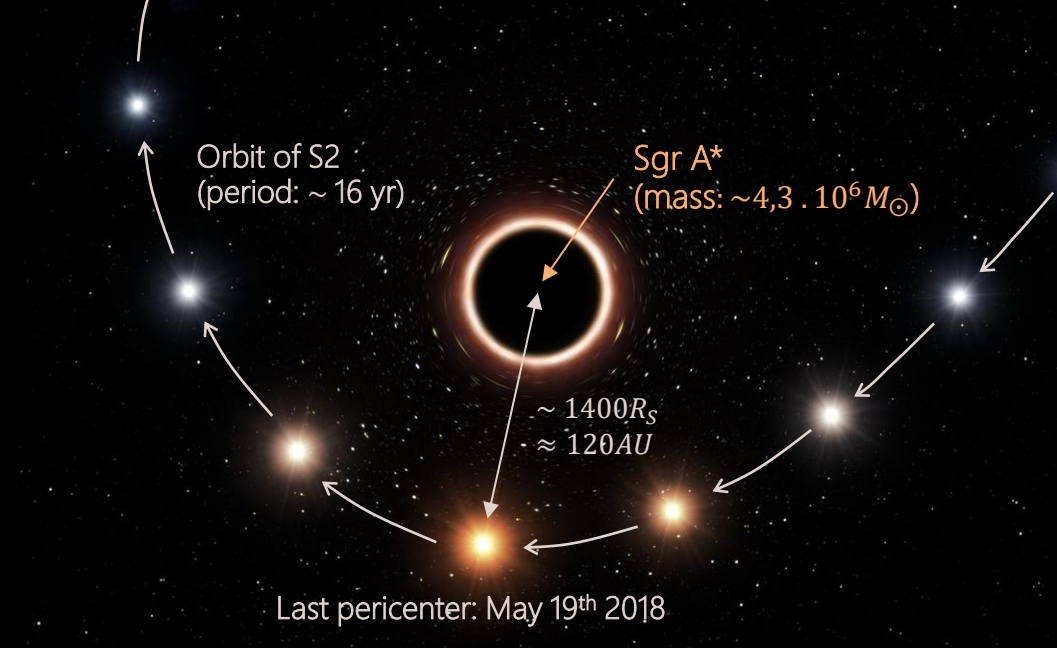
$$a_Q \propto Q$$

Independently fitting  $\chi$  and  $Q$  from the monitoring of S-stars

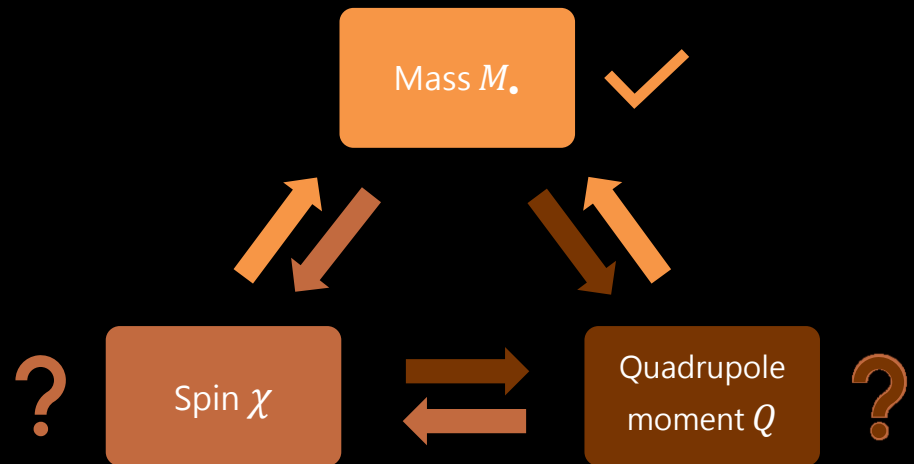


Test the no-hair theorem:

$$Q = -\frac{G^2 M^3}{c^4} \chi^2$$



No hair-theorem:



Maximal accuracy	Astrometric ( $\mu\text{as}$ )	Spectroscopic (km/s)	State
SINFONI (VLT)	–	$\approx 10$	Decommissioned
NIRSPEC (Keck)	–	$\approx 10$	Operational
GRAVITY (VLT)	$\approx 10$	–	Operational
ERIS (VLT)	–	$\approx 10$	Operational
GRAVITY+ (VLT)	$\approx 10$	–	Operational in 2024+
MICADO (E-ELT)	$\approx 50$	$\approx 1$	Operational in 2028+

↳ DEPEND ON THE FLUX REACHING THE INSTRUMENT

# INSTRUMENTATION

VLT (Paranal) :

- GRAVITY (Interferometer)

$$m_K \leq 19$$

- SINFONI (Spectrograph)



Maximal accuracy	Astrometric ( $\mu\text{as}$ )	Spectroscopic (km/s)	State
SINFONI (VLT)	–	$\approx 10$	Decommissioned
NIRSPEC (Keck)	–	$\approx 10$	Operational
GRAVITY (VLT)	$\approx 10$	–	Operational
ERIS (VLT)	–	$\approx 10$	Operational
GRAVITY+ (VLT)	$\approx 10$	–	Operational in 2024+
MICADO (E-ELT)	$\approx 50$	$\approx 1$	Operational in 2028+

GRAVITY+ CAN MULTIPLY  
THE NUMBER OF  
PHOTONS UP TO A  
FACTOR 20!

- Better temporal coverage
- Less systematic errors
- Less photon noise

↳ DEPEND ON THE FLUX REACHING THE INSTRUMENT

|| POSSIBILITY OF DETECTING OTHER STARS



## INSTRUMENTATION

VLT (Paranal) :

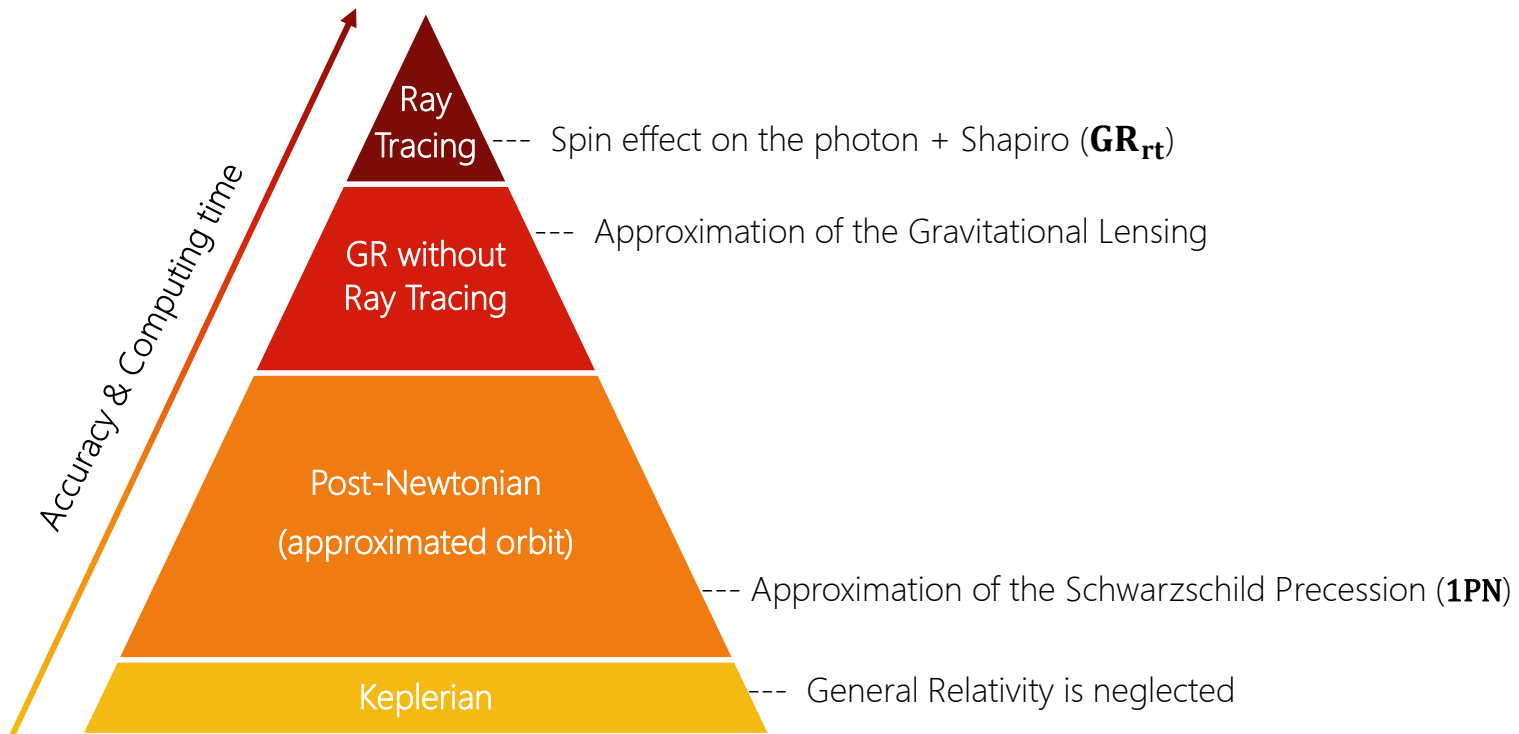
- GRAVITY → GRAVITY + (Interferometer)

$$m_K \leq 19 \rightarrow m_K \leq 22$$

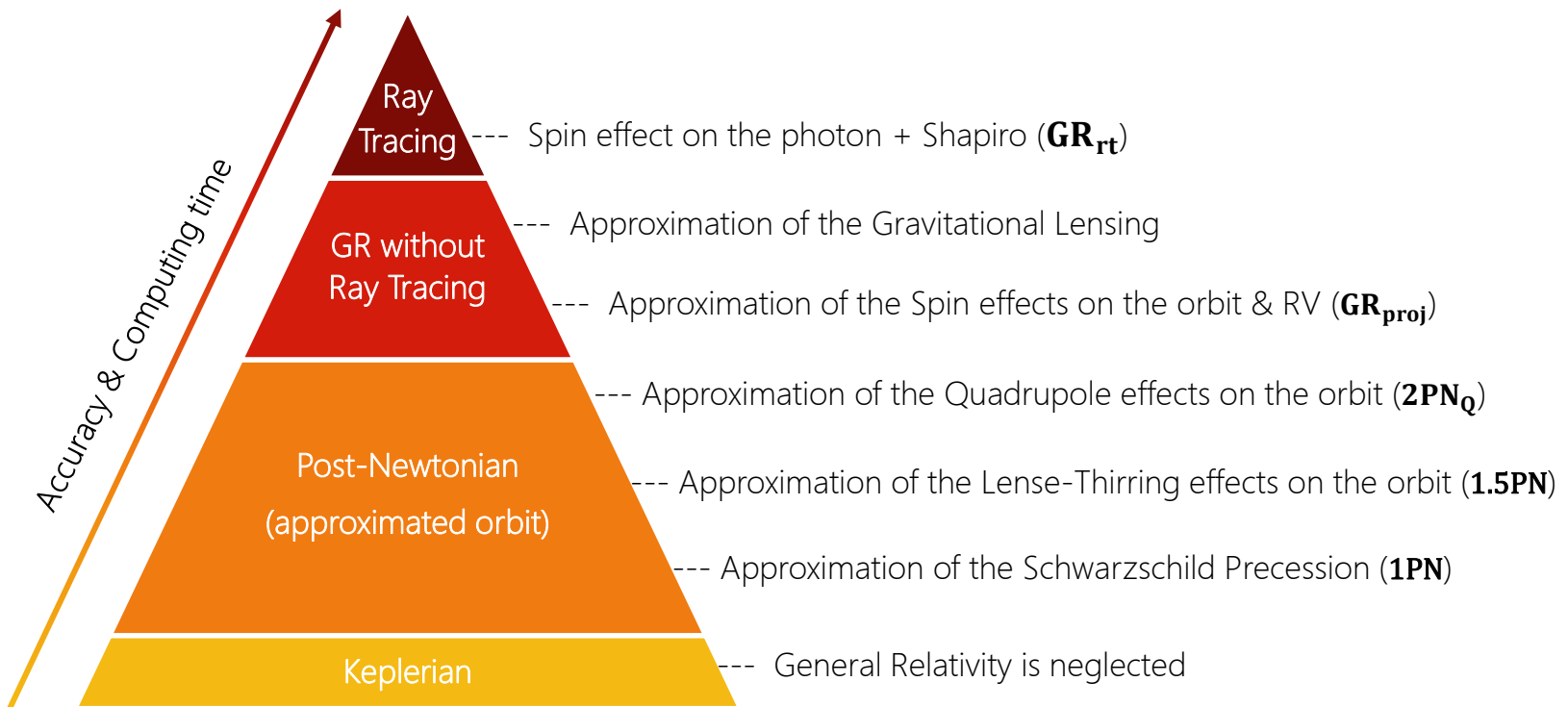
- ERIS (Spectrograph)



# TOOLS: PALETTE OF RELATIVISTIC MODELS



# TOOLS: PALETTE OF RELATIVISTIC MODELS



Development of the **1.5PN**, **GR<sub>proj</sub>** and **2PN<sub>Q</sub>** models





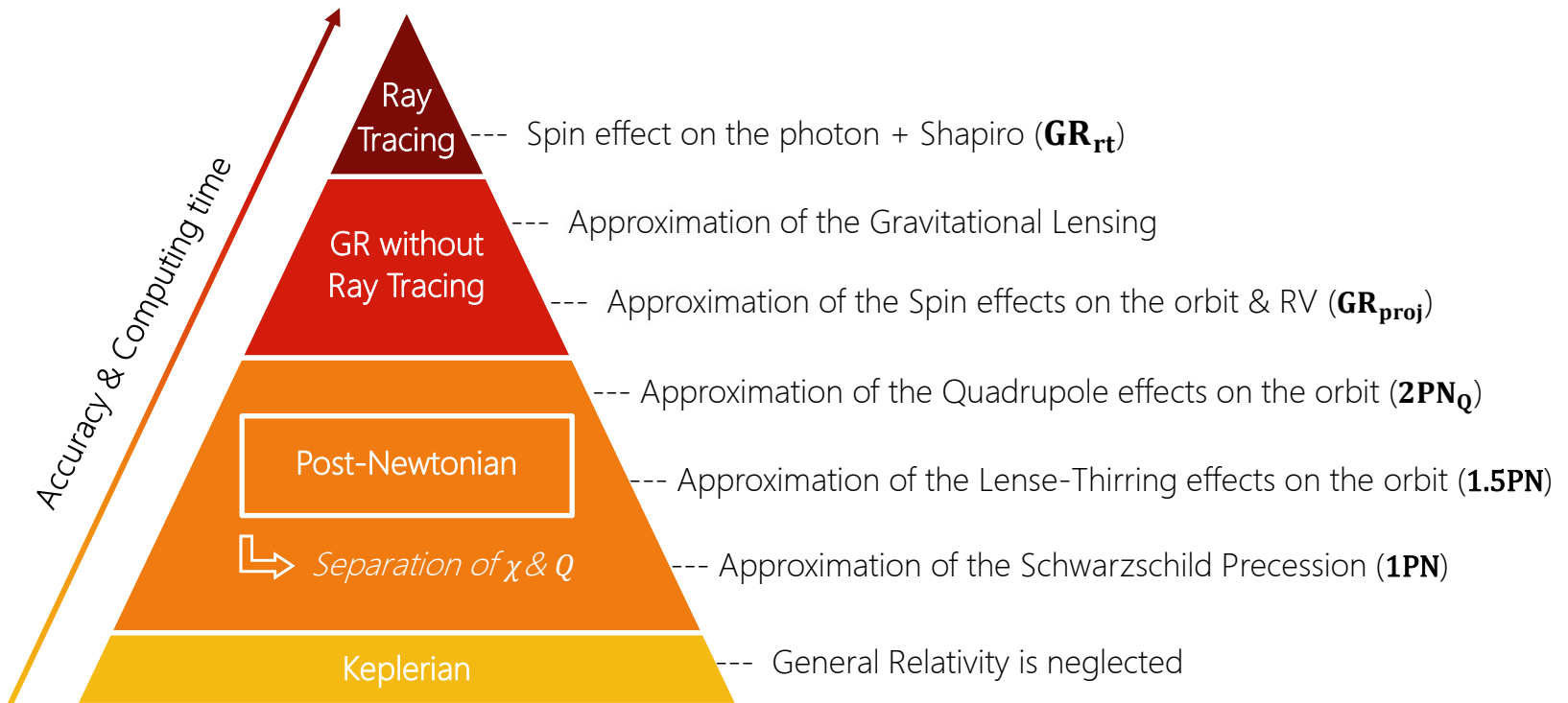
# TOOLS: PALETTE OF RELATIVISTIC MODELS

2PN order for closer-in stars

2PN\_sch : Schwarzschild

2PN\_LT : Schwarzschild +  $\chi$

2PN\_Q : Schwarzschild +  $\chi$  +  $Q$



↳ Development of the **1.5PN**, **GR<sub>proj</sub>** and **2PN<sub>Q</sub>** models

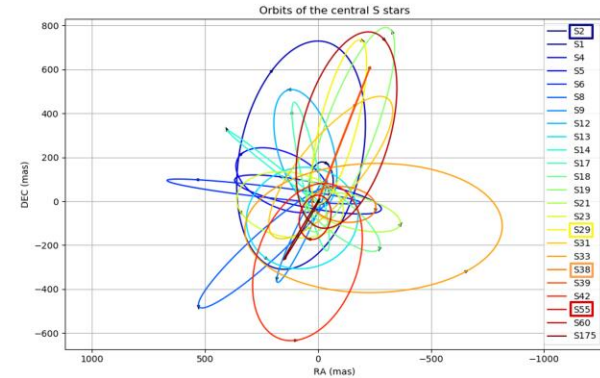
Karim

# 1st Year work:

- Spin effects on other S stars
- Coordinate system effects
- Orbit fitting of spin angles

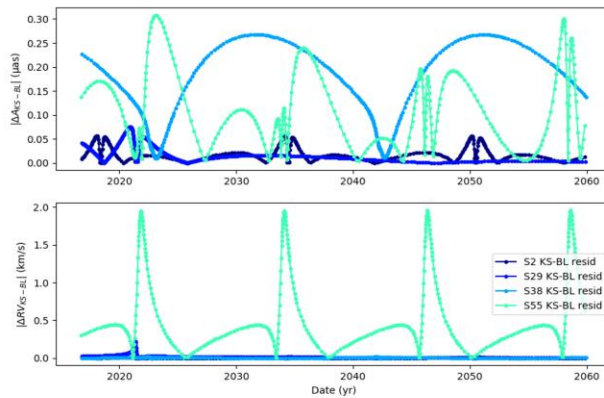
What to do if no closer-in stars were to be found?

➔ Explore the possibility of a spin detection from a collection of eccentric orbits?



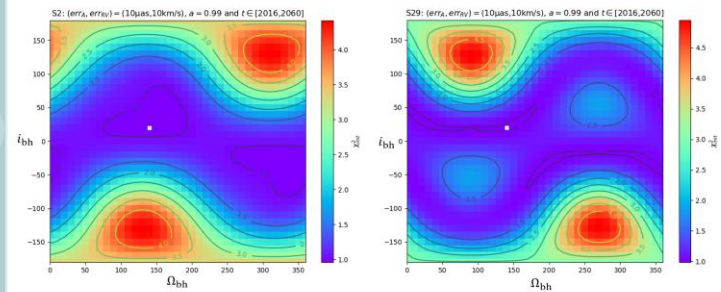
26

A and RV using the GR\_cd model and  $(a, i, \Omega) = (0.0, 45^\circ, 160^\circ)$



37

Studying the detectability of the orientation of the spin

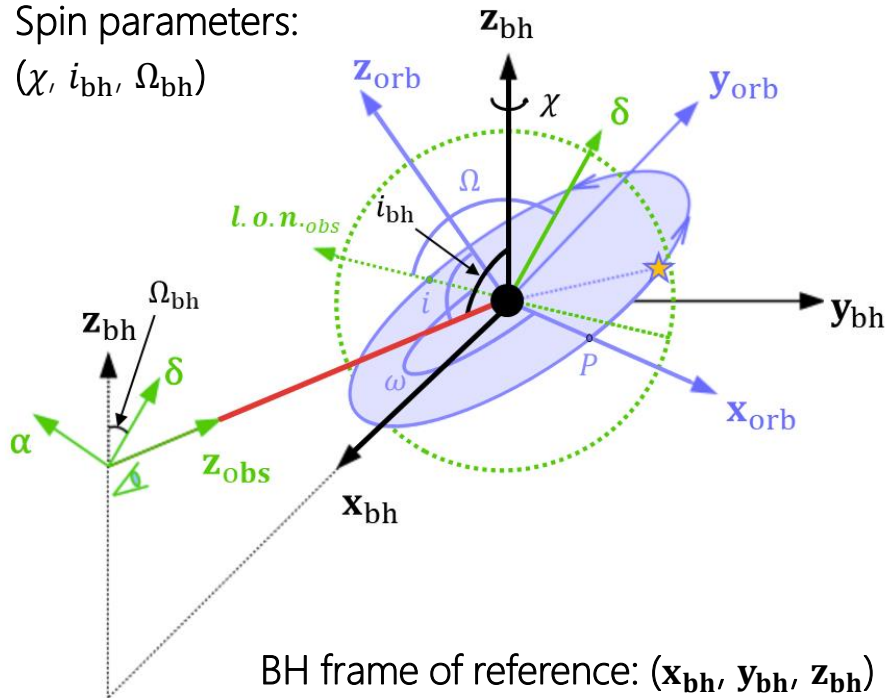


➔ If we fit multiple star orbits at the same time the degeneracy of the spin angles with each other and with the spin magnitude will be reduced ! ➔

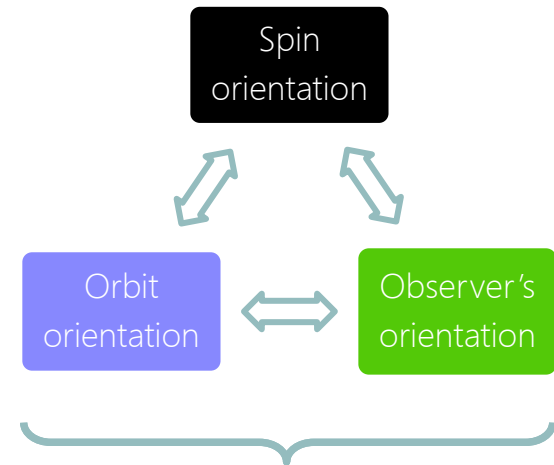
29

## 2<sup>nd</sup> Year work: Impact of the relative orientations

Spin parameters:  
( $\chi$ ,  $i_{bh}$ ,  $\Omega_{bh}$ )



BH frame of reference: ( $x_{bh}$ ,  $y_{bh}$ ,  $z_{bh}$ )  
Orbit frame of reference: ( $x_{orb}$ ,  $y_{orb}$ ,  $z_{orb}$ )  
Observer's frame of reference: ( $\alpha$ ,  $\delta$ ,  $z_{obs}$ )



Impact the secular evolution  
of the pericenter/apocenter

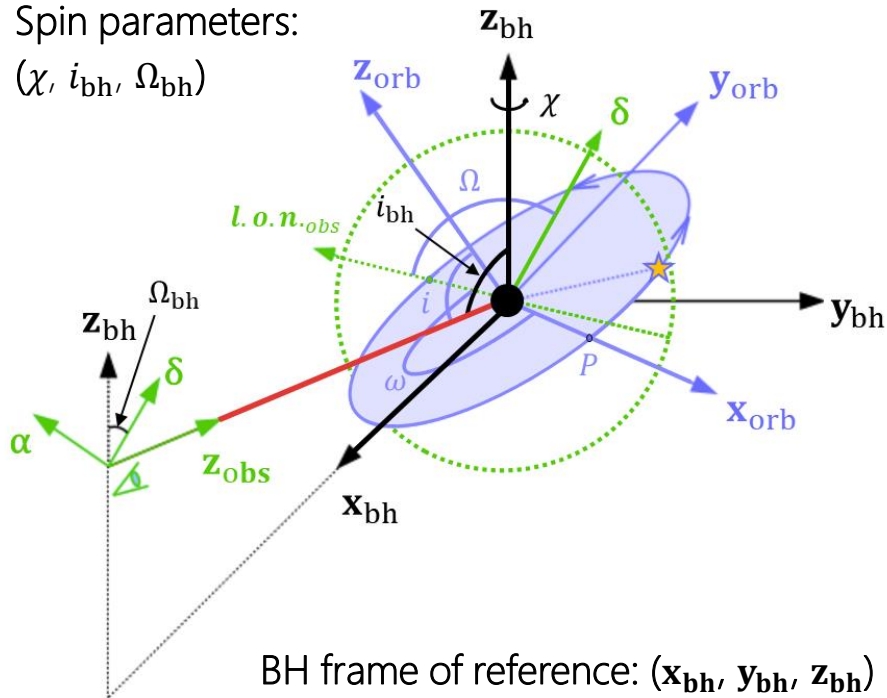
↳ Intuition?



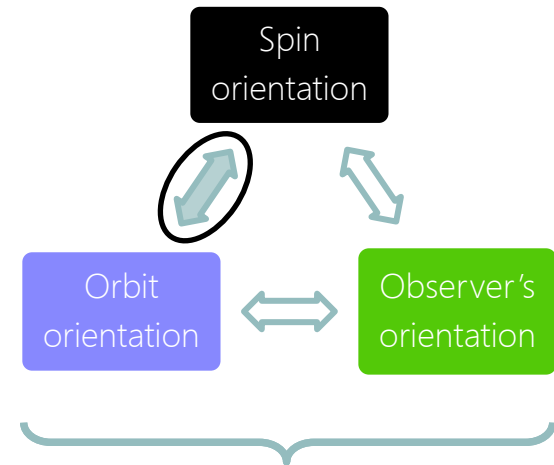
Study the different  
types of precessions

## 2<sup>nd</sup> Year work: Impact of the relative orientations

Spin parameters:  
( $\chi$ ,  $i_{bh}$ ,  $\Omega_{bh}$ )



BH frame of reference: ( $x_{bh}$ ,  $y_{bh}$ ,  $z_{bh}$ )  
Orbit frame of reference: ( $x_{orb}$ ,  $y_{orb}$ ,  $z_{orb}$ )  
Observer's frame of reference: ( $\alpha$ ,  $\delta$ ,  $z_{obs}$ )



Impact the secular evolution  
of the pericenter/apocenter

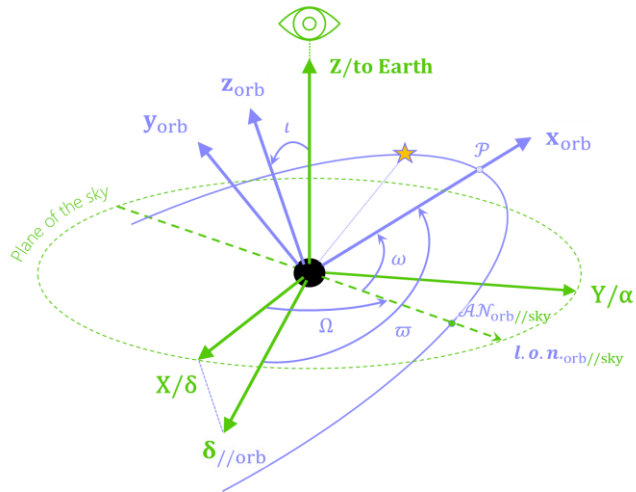
↳ Intuition?



Study the different  
types of precessions

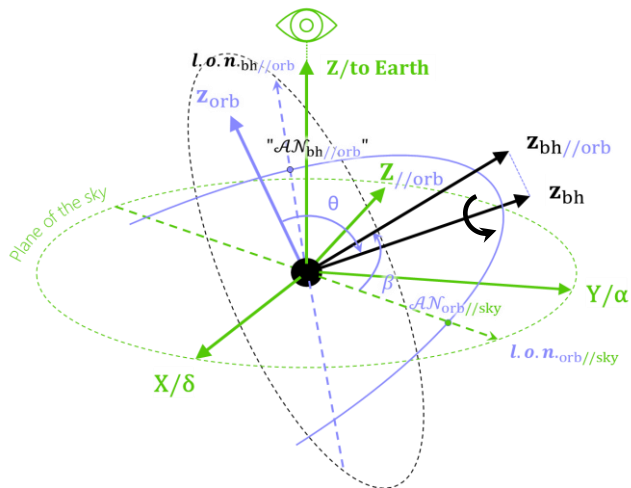
# Studying the secular shift of orbital parameters

## 2 TYPES OF PRESSIONS AT THE 2PN ORDER



### □ IN-PLANE PRESSION:

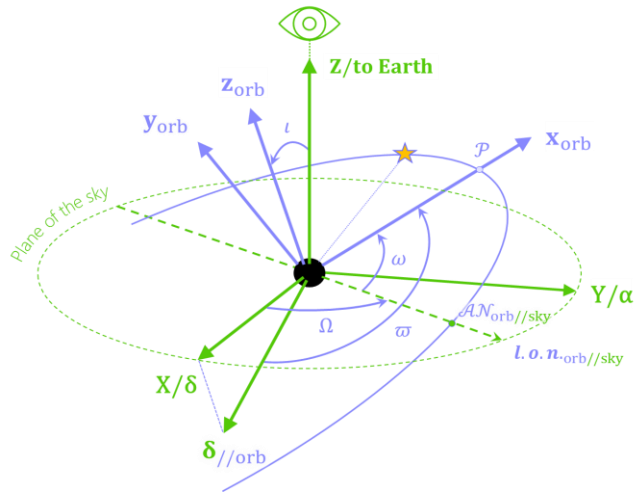
- Around  $\mathbf{z}_{\text{orb}}$
- Due to:
  - Schwarzschild
  - $\chi$
  - $Q$





# Studying the secular shift of orbital parameters

## 2 TYPES OF PRESSIONS AT THE 2PN ORDER



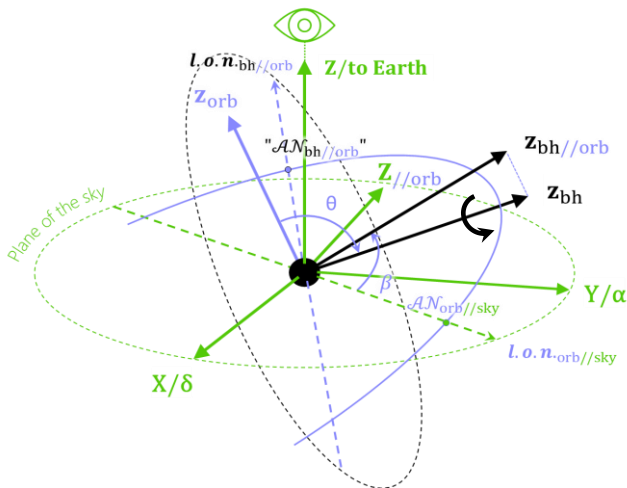
### IN-PLANE PRESSION:

- Around  $\mathbf{z}_{orb}$
- Due to:
  - Schwarzschild
  - $\chi$
  - $Q$
- Characterised by:
 
$$\Delta\varpi = \Delta\omega + \cos i \Delta\Omega$$

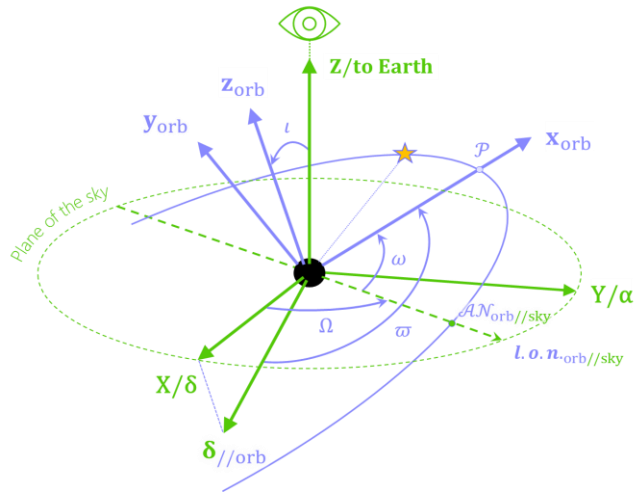
### OUT-OF-PLANE PRESSION:

- Around  $\mathbf{z}_{bh}$  at fixed  $\theta$
- Due to:
  - $\chi$
  - $Q$
- Characterised by:
 
$$\Delta\Theta = \sin\omega\Delta i - \cos\omega\sin i\Delta\Omega$$

Independent of the observer !



# Studying the secular shift of orbital parameters



## IN-PLANE PRECESSION:

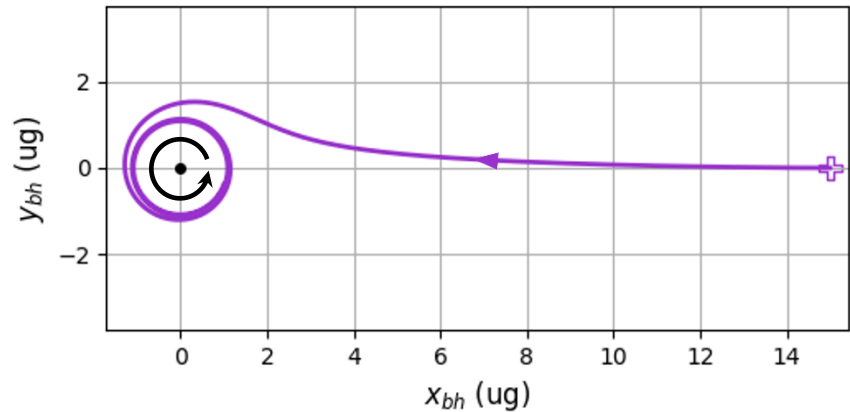
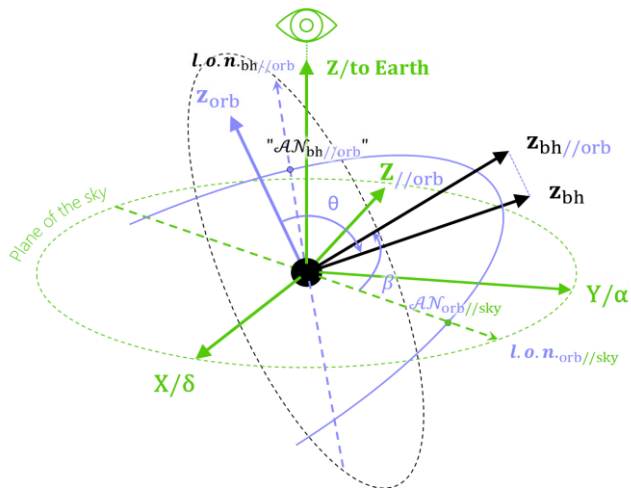
- Around  $\mathbf{z}_{orb}$
- Due to:
  - Schwarzschild
  - $\chi$
  - $Q$
- Characterised by:
 
$$\Delta\varpi = \Delta\omega + \cos i \Delta\Omega$$

## ANALYTICAL EXPRESSION OF THE SECULAR SHIFT OF ORBITAL PARAMETERS:

$$\Delta\varpi_{Sch} = \Delta\omega_{Sch} = \frac{6\pi Gm}{c^2 p} + \frac{\pi G^2 m^2}{2c^4 p^2} (28 - e^2)$$

$$\Delta\varpi_\chi = -\frac{8\pi}{c^3} \left[ \frac{Gm}{p} \right]^{3/2} \cos \theta \chi$$

$$\Delta\varpi_Q = \frac{3\pi}{2c^4} \left[ \frac{Gm}{p} \right]^2 (1 - 3 \cos^2 \theta) \chi^2$$



Timelike geodesic initially at rest  
 ≈ Particle in a whirlpool



## Studying the secular shift of orbital parameters

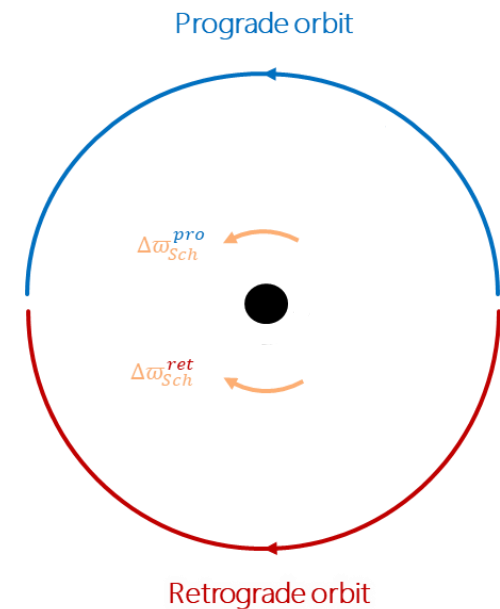
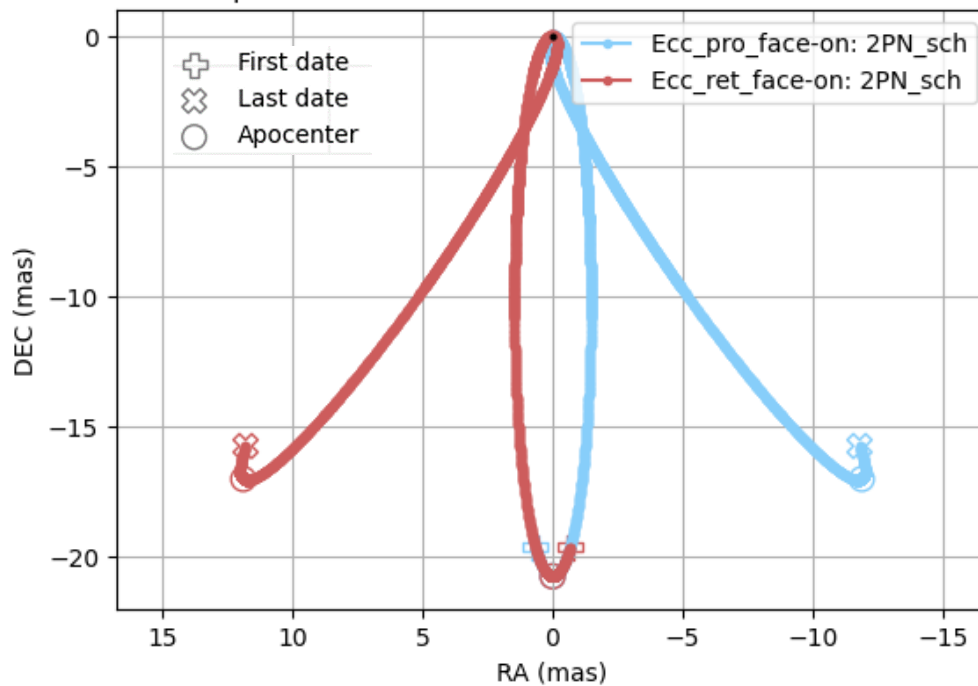
Hypothetical star  
with considerable  
relativistic effects:

Name	"Ecc"
Period	0.38696 yr
Eccentricity	0.99

SIMULATION WITH 3  
CODE MODELS:

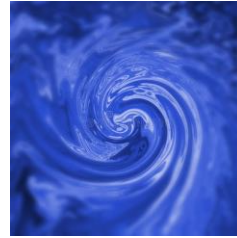
2PN\_sch : Schwarzschild

Equatorial orbit ( $\theta = 0$ ) of Ecc with face-on view



# Studying the secular shift of orbital parameters

WHAT HAPPENS WHEN WE ADD A  
BH SPIN IN THE SAME PLANE ?

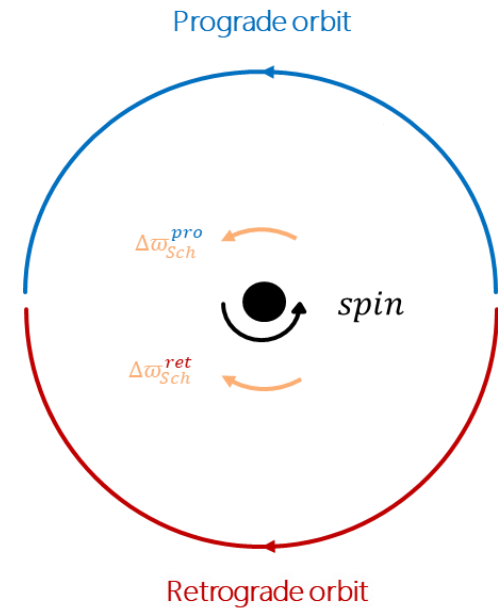
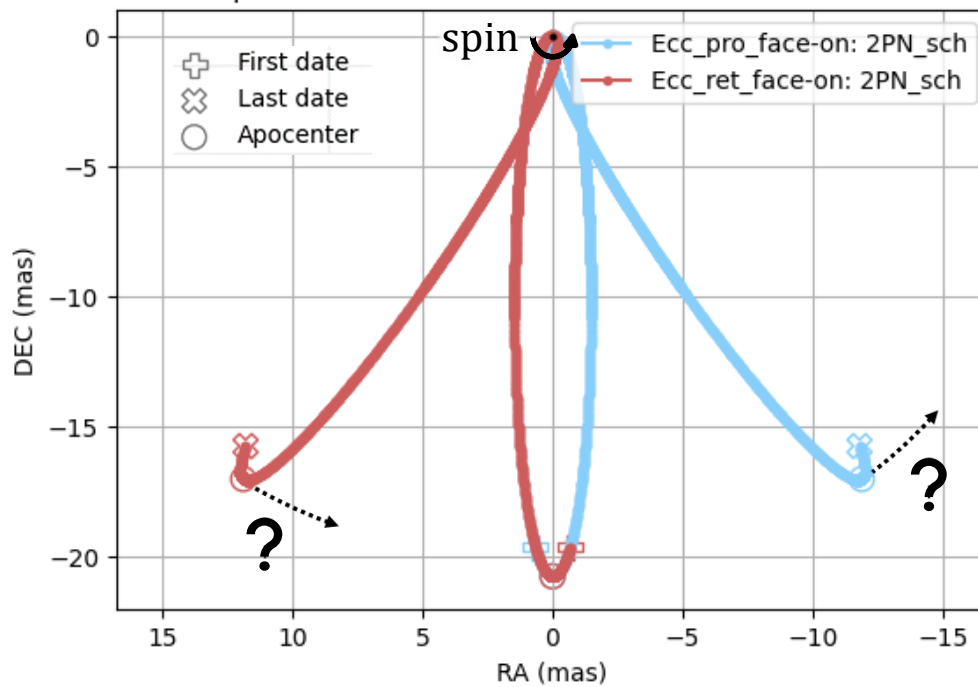


?

SIMULATION WITH 3  
CODE MODELS:

2PN\_sch : Schwarzschild

Equatorial orbit ( $\theta = 0$ ) of Ecc with face-on view



# Studying the secular shift of orbital parameters

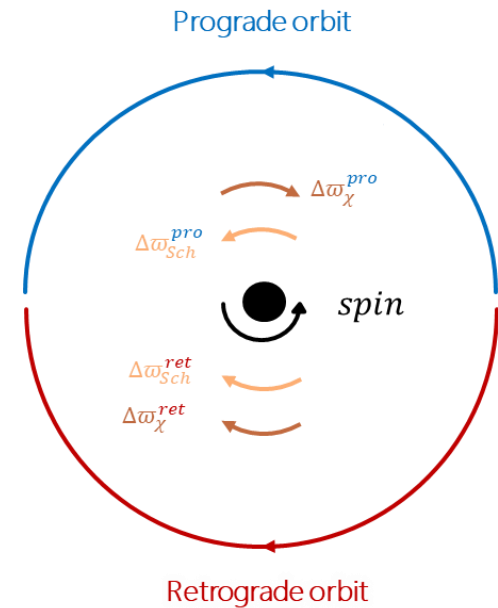
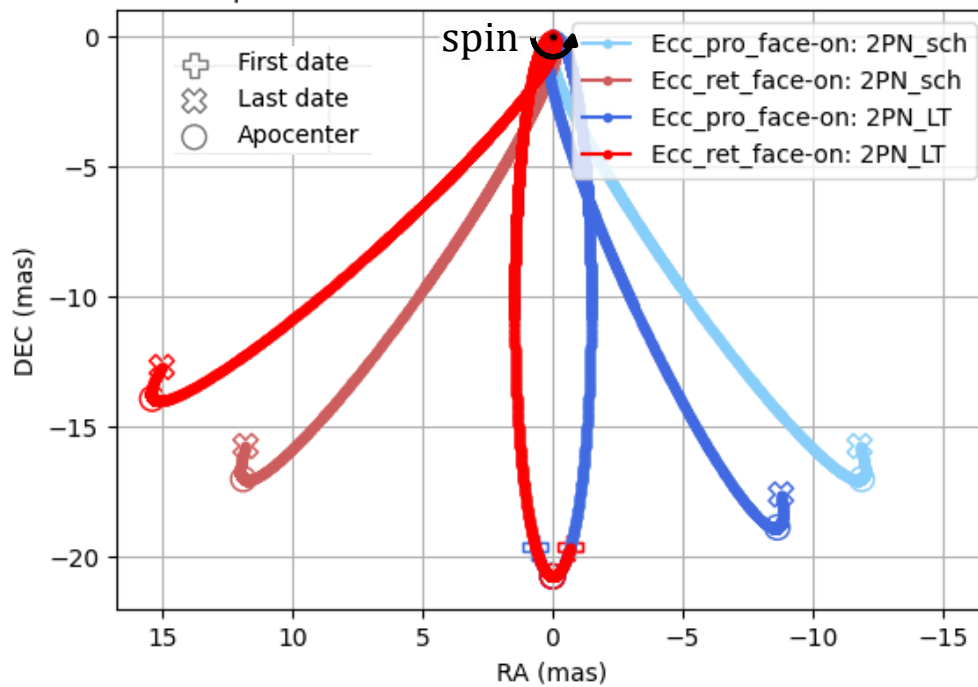
WHAT HAPPENS WHEN WE ADD A BH SPIN IN THE SAME PLANE ?



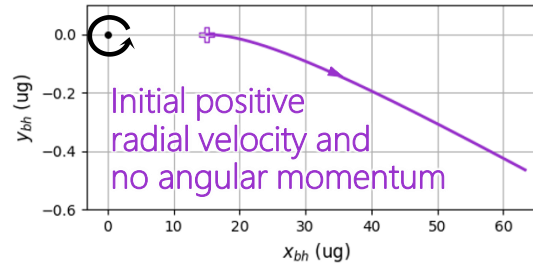
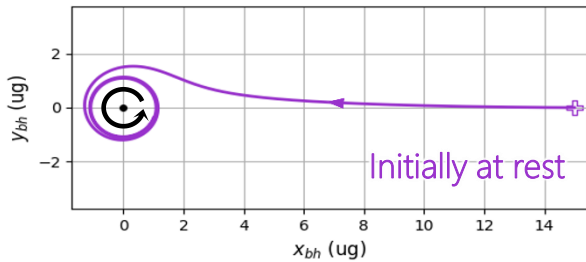
SIMULATION WITH 3 CODE MODELS:

2PN\_sch : Schwarzschild  
 2PN\_LT : Schwarzschild +  $\chi$

Equatorial orbit ( $\theta = 0$ ) of Ecc with face-on view



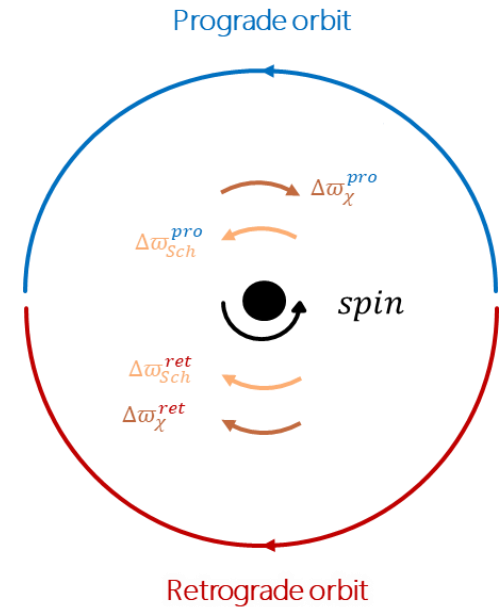
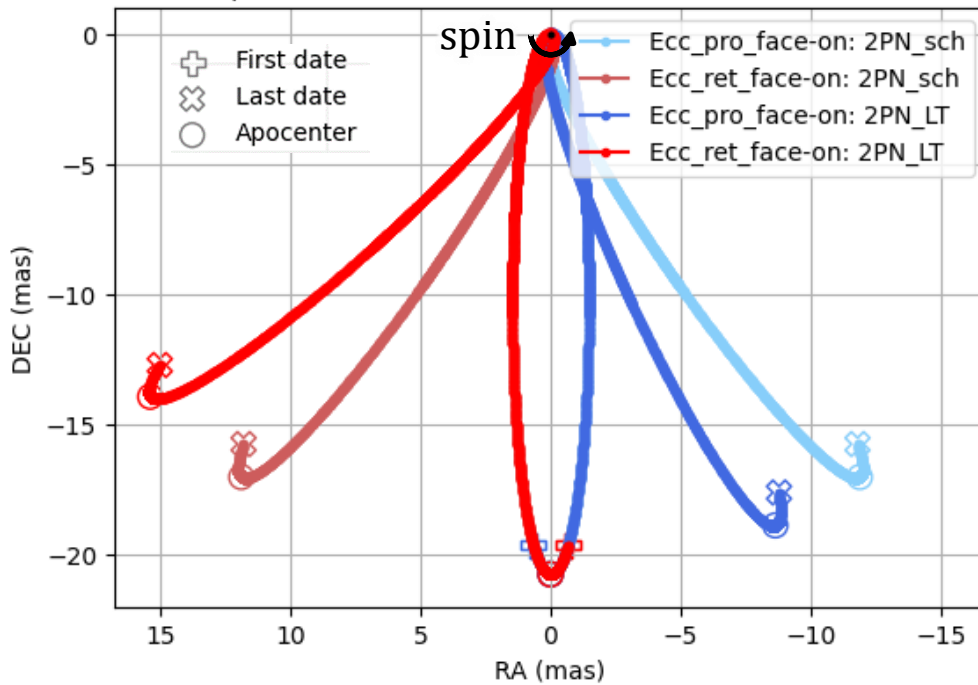
# Studying the secular shift of orbital parameters



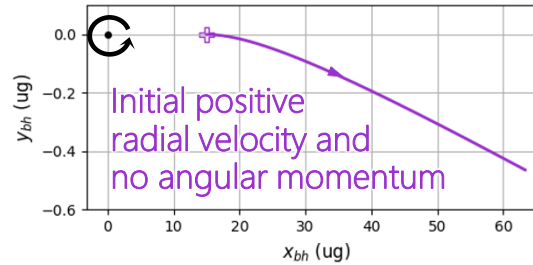
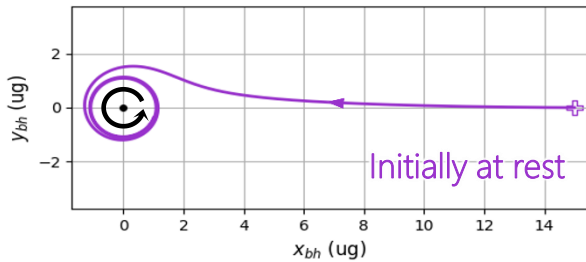
SIMULATION WITH 3 CODE MODELS:

2PN\_sch : Schwarzschild  
 2PN\_LT : Schwarzschild +  $\chi$

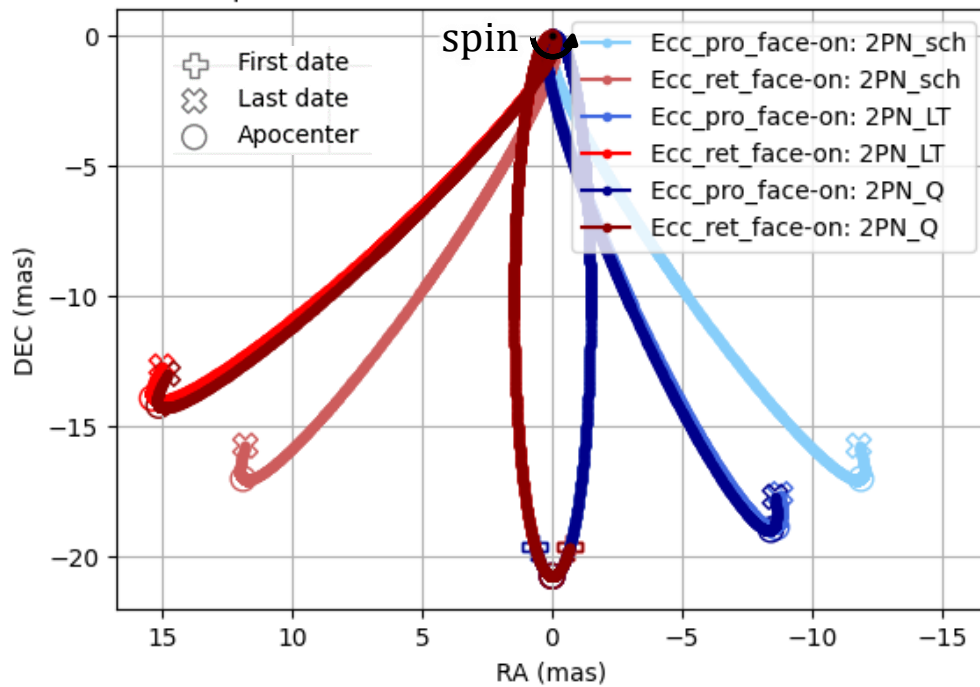
Equatorial orbit ( $\theta = 0$ ) of Ecc with face-on view



# Studying the secular shift of orbital parameters



Equatorial orbit ( $\theta = 0$ ) of Ecc with face-on view

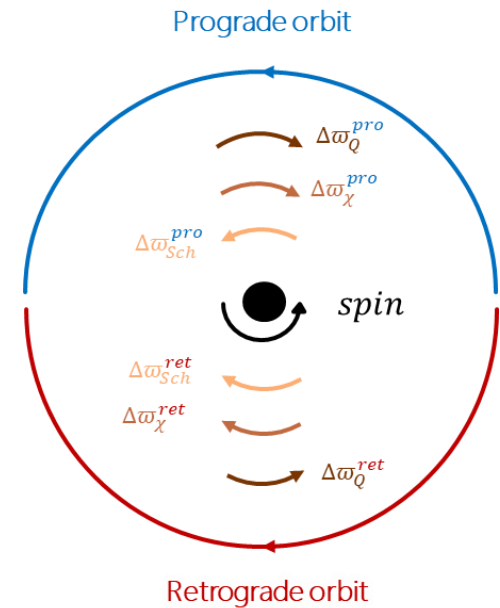


SIMULATION WITH 3 CODE MODELS:

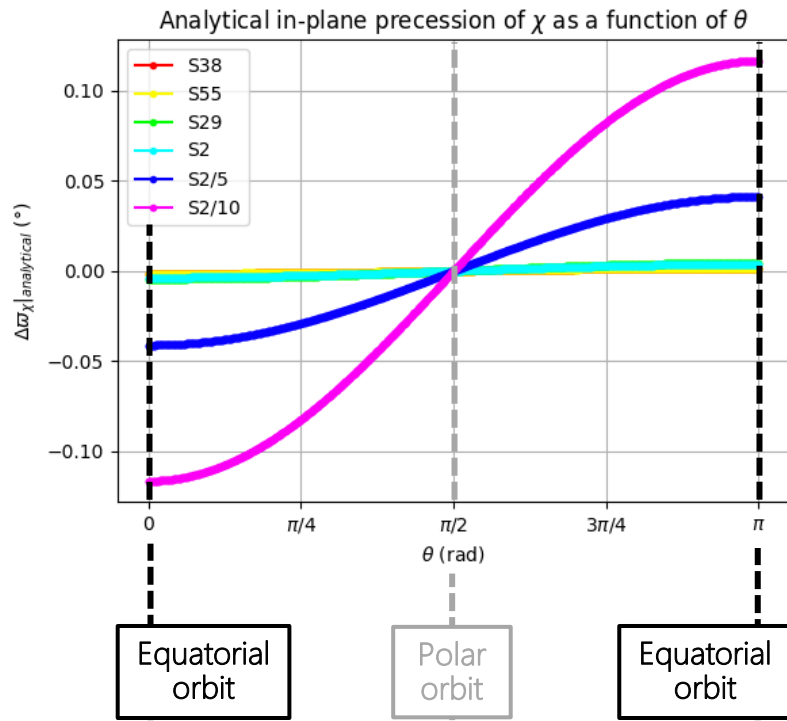
2PN\_sch : Schwarzschild

2PN\_LT : Schwarzschild +  $\chi$

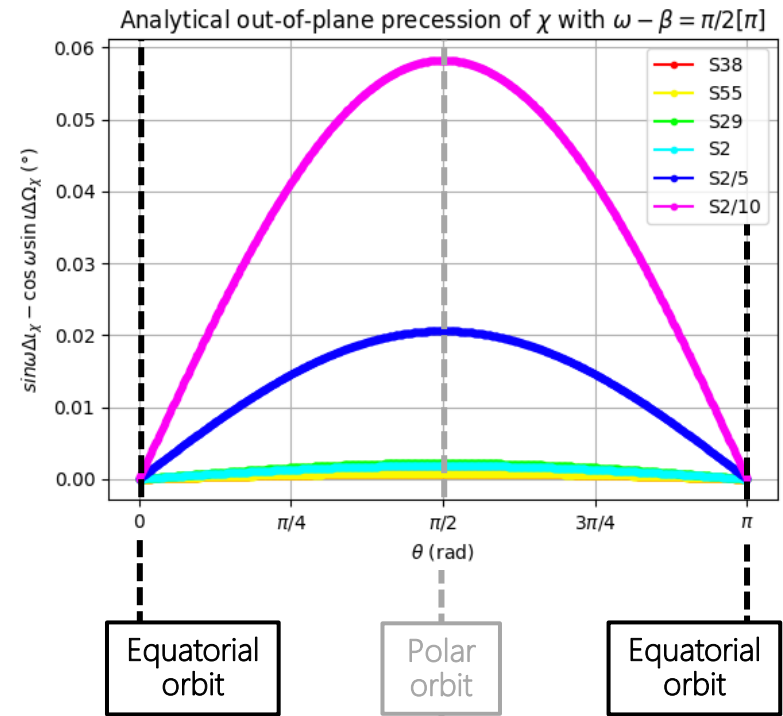
2PN\_Q : Schwarzschild +  $\chi$  +  $Q$



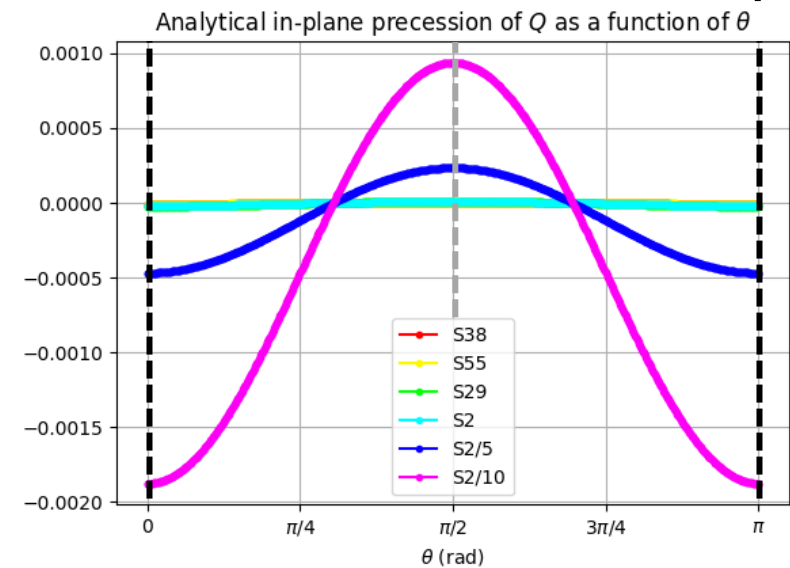
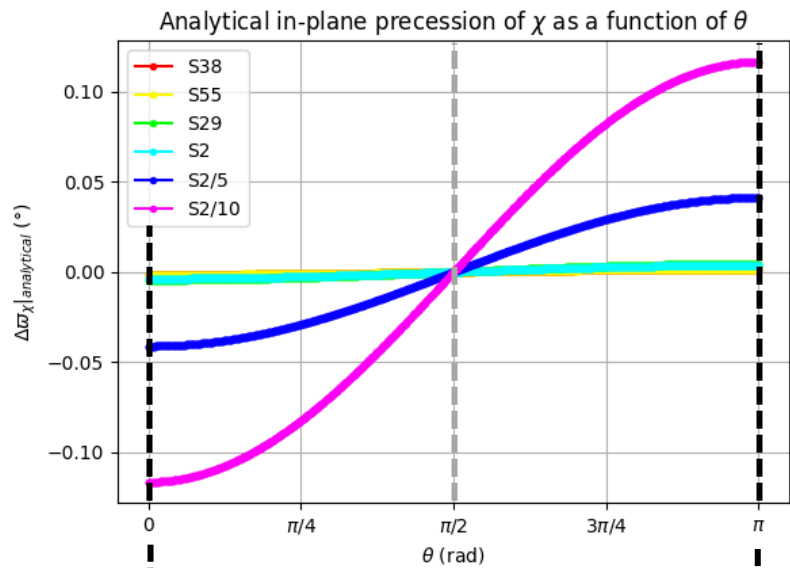
# In-plane precession



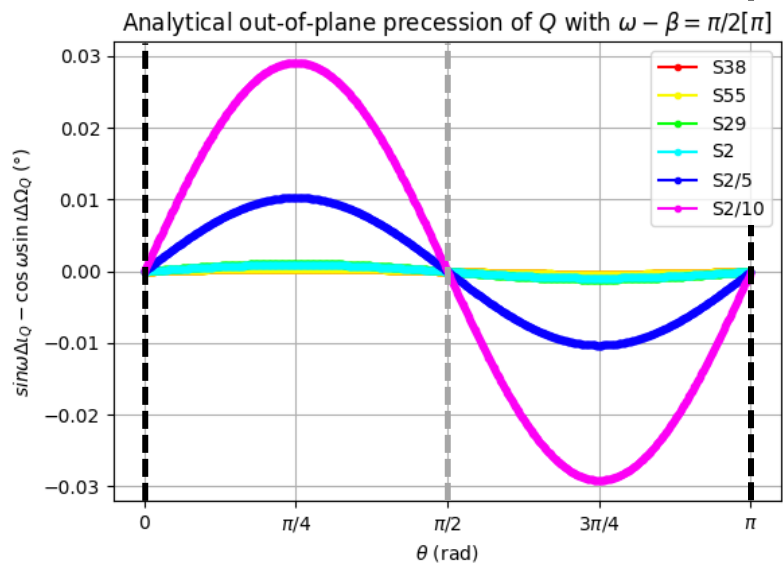
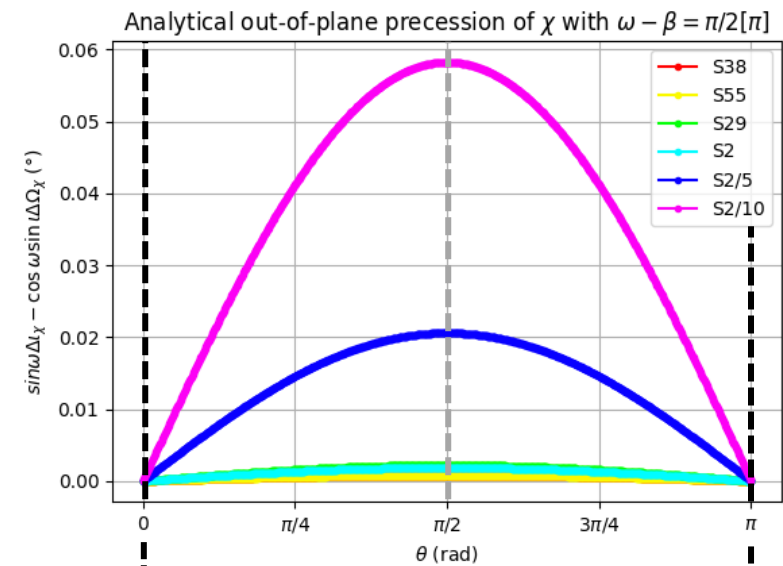
# Out-of-plane precession



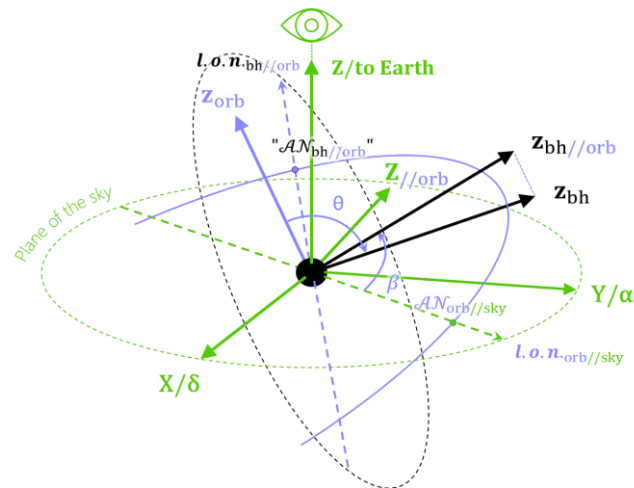
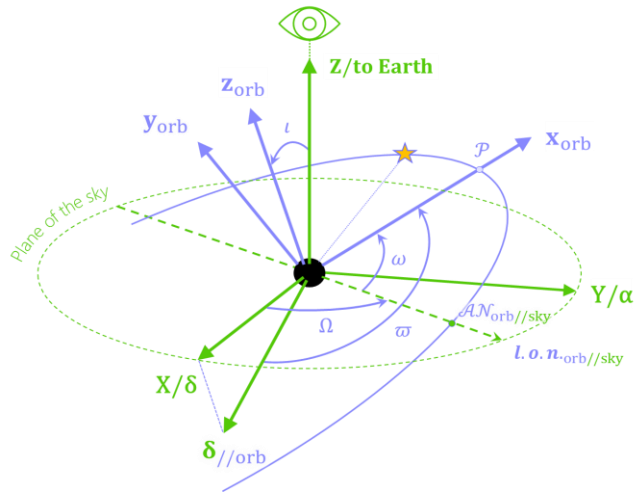
# In-plane precession



# Out-of-plane precession



# Studying the secular shift of orbital parameters



## 2 TYPES OF PRESSIONS AT THE 2PN ORDER

### IN-PLANE PRESSION:

- Around  $\mathbf{z}_{orb}$
- Due to:
  - Schwarzschild
  - $\chi$
  - $Q$
- Characterised by:
 
$$\Delta\varpi = \Delta\omega + \cos i \Delta\Omega$$

### ANALYTICAL EXPRESSION OF THE SECULAR SHIFT OF ORBITAL PARAMETERS:

$$\Delta\varpi_{Sch} = \Delta\omega_{Sch} = \frac{6\pi Gm}{c^2 p} + \frac{\pi G^2 m^2}{2c^4 p^2} (28 - e^2)$$

$$\Delta\varpi_\chi = -\frac{8\pi}{c^3} \left[ \frac{Gm}{p} \right]^{3/2} \cos\theta \chi$$

$$\Delta\varpi_Q = \frac{3\pi}{2c^4} \left[ \frac{Gm}{p} \right]^2 (1 - 3\cos^2\theta) \chi^2$$

↳ Maximal for  $\theta = 0$ : an orbit in the equatorial plane of the BH

### OUT-OF-PLANE PRESSION:

- Around  $\mathbf{z}_{bh}$  at fixed  $\theta$
- Due to:
  - $\chi$
  - $Q$
- Characterised by:
 
$$\Delta\Theta = \sin\omega\Delta i - \cos\omega\sin i\Delta\Omega$$

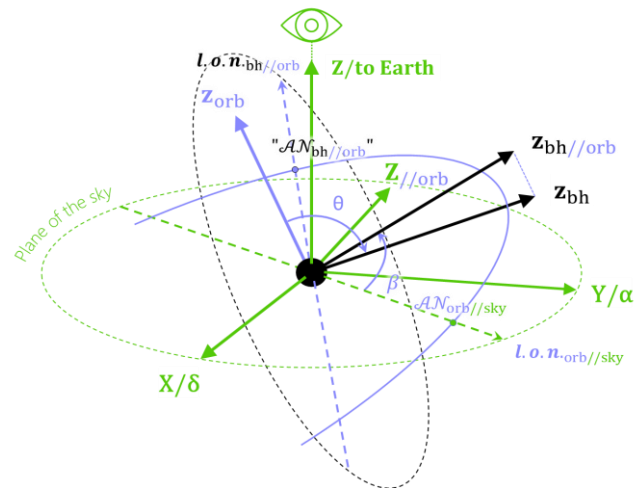
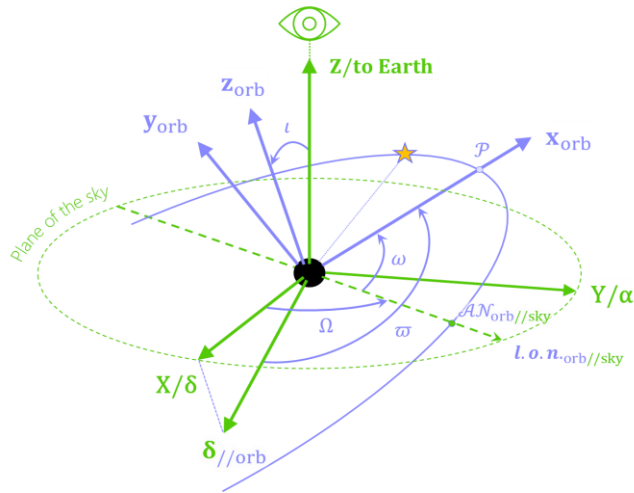
$$\Delta\Theta_\chi = \frac{4\pi}{c^3} \left[ \frac{Gm}{p} \right]^{3/2} \sin\theta \sin(\omega - \beta) \chi$$

$$\Delta\Theta_Q = \frac{3\pi}{c^4} \left[ \frac{Gm}{p} \right]^2 \cos\theta \sin\theta \sin(\omega - \beta) \chi^2$$

↳ Null for  $\theta = 0$ : an orbit in the equatorial plane of the BH



# Studying the secular shift of orbital parameters



## 2 TYPES OF PRESSIONS AT THE 2PN ORDER

### IN-PLANE PRESSION:

- Around  $\mathbf{z}_{orb}$
- Due to:
  - Schwarzschild
  - $\chi$
  - $Q$
- Characterised by:
 
$$\Delta\varpi = \Delta\omega + \cos i \Delta\Omega$$

### ANALYTICAL EXPRESSION OF THE SECULAR SHIFT OF ORBITAL PARAMETERS:

$$\Delta\varpi_{Sch} = \Delta\omega_{Sch} = \frac{6\pi Gm}{c^2 p} + \frac{\pi G^2 m^2}{2c^4 p^2} (28 - e^2)$$

$$\Delta\varpi_\chi = -\frac{8\pi}{c^3} \left[ \frac{Gm}{p} \right]^{3/2} \cos \theta \chi$$

$$\Delta\varpi_Q = \frac{3\pi}{2c^4} \left[ \frac{Gm}{p} \right]^2 (1 - 3 \cos^2 \theta) \chi^2$$

↳ Maximal for  $\theta = 0$ : an orbit in the equatorial plane of the BH

### OUT-OF-PLANE PRESSION:

- Around  $\mathbf{z}_{bh}$  at fixed  $\theta$
- Due to:
  - $\chi$
  - $Q$
- Characterised by:
 
$$\Delta\Theta = \sin \omega \Delta i - \cos \omega \sin i \Delta \Omega$$

$$\Delta\Theta_\chi = \frac{4\pi}{c^3} \left[ \frac{Gm}{p} \right]^{3/2} \sin \theta \sin(\omega - \beta) \chi$$

$$\Delta\Theta_Q = \frac{3\pi}{c^4} \left[ \frac{Gm}{p} \right]^2 \cos \theta \sin \theta \sin(\omega - \beta) \chi^2$$

↳ Null for  $\theta = 0$ : an orbit in the equatorial plane of the BH

# Sgr A\*'s spin and quadrupole moment effects on the orbits of S stars

K. Abd El Dayem<sup>1</sup>, G. Heissel<sup>1,2</sup>, F. Vincent<sup>1</sup>, T. Paumard<sup>1</sup>, and G. Perrin<sup>1</sup>

<sup>1</sup> LESIA, Observatoire de Paris, Université PSL, CNRS, Sorbonne Université, Université Paris Cité, 5 place Jules Janssen, 92195 Meudon, France

e-mail: karim.abdeldayem@obspm.fr

<sup>2</sup> Advanced Concepts Team, European Space Agency, TEC-SF, ES-TEC, Keplerlaan 1, NL-2201, AZ Noordwijk, The Netherlands

Received December ?, 2023; accepted ?, 2024

## ABSTRACT

**Context.** Measuring the astrometric and spectroscopic data of stars orbiting the central black hole in our galaxy (Sgr A\*) offer a promising way to measure relativistic effects. In principle, the “no-hair” theorem can be tested at the Galactic Center by monitoring the orbital precession of S-stars due to the angular momentum (or spin) and quadrupole moment of Sgr A\*.

**Aims.** In this paper we investigate what stellar orbit and spin orientation are needed to measure the spin and quadrupole moment of Sgr A\*. This, allows us to study the detectability of these quantities enabling the testing of the “no-hair” theorem and thus of general relativity (GR).

**Methods.** We consider a collection of S stars as well as a putative stars that orbit closer to Sgr A\*, thus being much more affected by the spin and quadrupole effects. It is possible that either future observations of GRAVITY+ could detect such inner stars that might have been too faint to be detected by GRAVITY. To reach our objectives, we use different relativistic models in order to generate the orbit and radial velocity of the putative stars and analyze how their precession can be affected by the relative orientations of the spin, the orbits and the observer.

**Results.**

**Key words.** black hole physics – gravitation – Galaxy: center – relativistic processes - techniques: interferometric

## 1. Introduction

After years of monitoring S stars in the central parsec of the Milky Way studies have demonstrated the presence of a super-massive black hole (SMBH) called Sgr A\* at the center of the Galaxy (Eckart & Genzel 1996; Ghez et al. 1998, 2003, 2008; Schödel et al. 2002; Gillessen et al. 2009, 2017; GRAVITY Collaboration et al. 2022a). According to the “no-hair” theorem, a black hole can be completely characterized by only three externally observable classical parameters: mass, angular momentum (hereafter referred to as spin), and electric charge which will be set to zero in our study. If we consider in addition the quadrupole moment, it must be linked to the mass and spin according to the no-hair theorem. Thus, the latter can be tested by independently measuring these 3 quantities. Dozens of S star orbits are currently known (Gillessen et al. 2017), including the highly elliptical one of the star S2 with a 16 year period, reaching  $R \approx 1300R_S \approx 120\text{AU}$  from Sgr A\* at its pericenter, where  $R_S = 2\frac{GM_*}{c^2}$  with  $G$  and  $M_*$  being the gravitational constant and the black hole’s mass respectively. By combining the infrared light collected by the 4 Unit Telescopes of the Very Large Telescope (VLT) at Paranal, the interferometric instrument GRAVITY (General Relativity Analysis via Vlt InTerferometrY) was able to estimate the mass of Sgr A\* at  $M_* \approx 4.297 \cdot 10^6 M_\odot$ . Even though the spin and quadrupole moment of the black hole are still unknown, the future monitoring of S stars could, in principle, provide constraints on these parameters (Will 2008; Angéil & Saha 2014; Alush & Stone 2022) which in turn would allow us to test the “no-hair” theorem.

The different relativistic effects that can be observed in the vicinity of a strong gravitational field like the one surrounding Sgr A\* can be divided into 2 categories. The Schwarzschild effects in the case of a non-rotating black hole:

- The Schwarzschild precession which is due to the spacetime curvature on the star trajectory. The orbit precesses because of the gravitational field caused by the central mass (Sgr A\* for instance). Thus, the pericenter and apocenter of the star are shifted from one period to another. This affects both the astrometry and the spectroscopy.
- The Shapiro time delay which is due to the slowdown of the proper time of the photon with respect to the proper time of the observer when the photon crosses a gravitational field. This affects both the astrometry and the spectroscopy.
- The relativistic redshifts which can be decomposed into two components. The transversal Doppler shift appears in special relativity and is due to the relative motion between the emitter and the observer. The gravitational redshift appears in GR and is due to the spacetime curvature. This only affects the spectroscopy.
- The gravitational lensing effect which is due to the curvature of the photon geodesic (including spin effects on the photon trajectory) that changes the apparent position of the star on the sky plane. This affects both the astrometry and the spectroscopy.
- Relativistic aberration is the relativistic version of the aberration of light. This affects both the astrometry and the spectroscopy.

# NEXT

- Wrap up 1<sup>st</sup> paper:  
Abd El Dayem et al. 2024

- Get back to the analysis of the detectability of the spin using multiple stars
- Write 2<sup>nd</sup> paper about multi-star fits

Thank you for your attention

Orbit fitting  
for spin  
detectability

## Acomplished Work

Development of  
the 1.5PN,  
 $GR_{proj}$  & 2PN<sub>Q</sub>  
models

Analytical  
expression of  
secular effects  
to 2PN

Confrontation  
of the results in  
KerrKS and  
KerrBL

Thank you for your attention

Galactic center  
and the effects  
produced by  
Sgr A\*

Astrometry  
and  
spectroscopy

Relativistic  
dynamics and  
use of different  
metrics

## Acquired Knowledge

Intuition on  
the impact of  
the relative  
orientations