

Black-hole metric and disc physics degeneracy on highly lensed observables in SMBH images

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Journées Théorie PNHE

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General Relativity

- Standard **theory** of **gravity** (1915)
- Gravity = **Curvature** of **spacetime**
- Gravitational **deflection** of **light**
- Eddington experiment (1919)
- Thorough **tests** in the Solar System

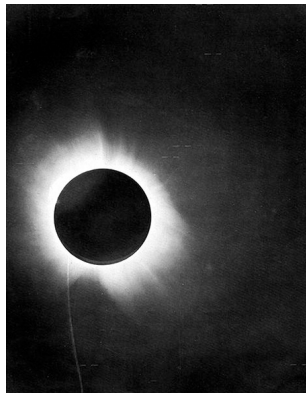


Figure: 1919 solar eclipse.
Credits: Eddington

Black Holes

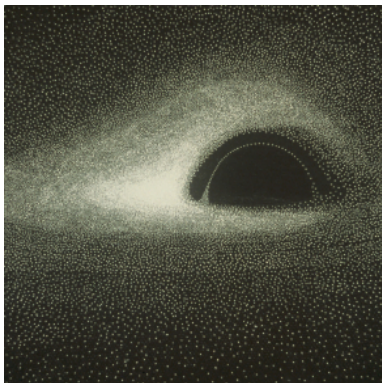


Figure: Simulated photograph of a BH.
Credits: Luminet

- Black hole \leftrightarrow Event horizon
- No-hair theorem \rightarrow Kerr
- Spacetime singularity
- Tests in the **strong field** regime
- **Event Horizon Telescope** (2019)

Black Holes

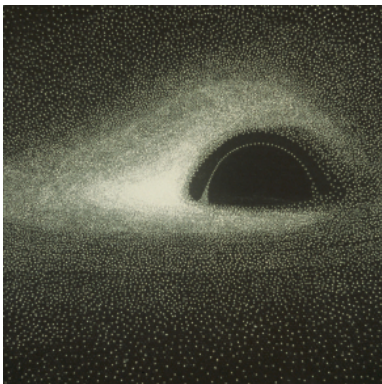


Figure: Simulated photograph of a BH.
Credits: Luminet

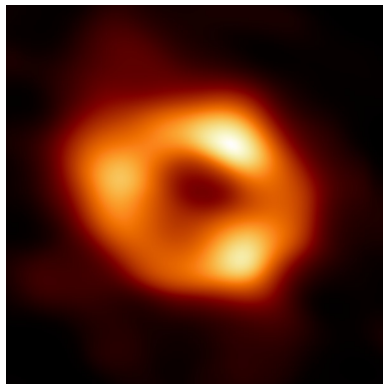


Figure: First image of SgrA*.
Credits: Event Horizon Telescope

Event Horizon Telescope

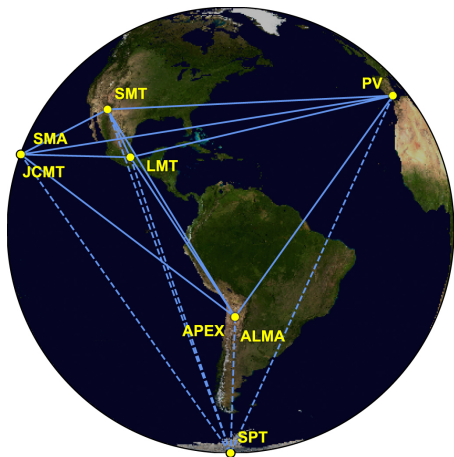


Figure: EHT array of the 2017 campaign.
Credits: Event Horizon Telescope

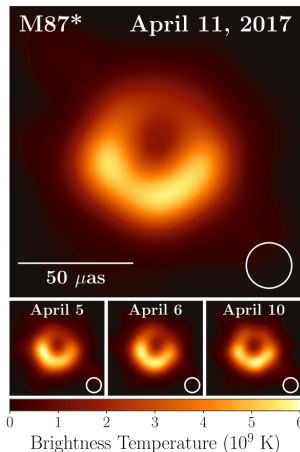


Figure: First image of a black hole.
Credits: Event Horizon Telescope

Project

Scientific question

Can we **detect** a **deviation** from the black-hole **standard model**?

Reasoning steps:

- Find distinctive **image features** induced by the spacetime properties
- Associate reliable electromagnetic **observable signatures**
- **Disentangle** between the **geometry** and the **astrophysics**

Methods:

- **Numerical simulations** via the ray-tracing code **GYOTO**

Trajectories of free photons

- **Photon sphere** = unstable circular orbits of light
- **n = half turns** around the black hole before leaving to infinity
- One point of the disk = **several images**

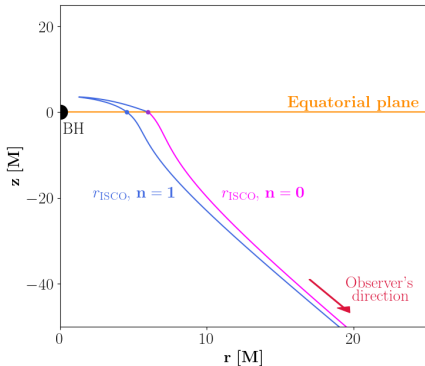
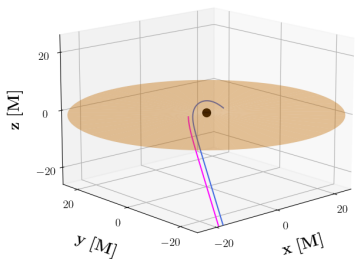


Figure: Geodesics of photons emitted at the innermost stable circular orbit

Image features

- **Critical curve** = projection of the **photon sphere**
- **Inner shadow** = region inside the projected equatorial **event horizon**
- **n -th lensing band** = impact points of light rays of order $n > 0$

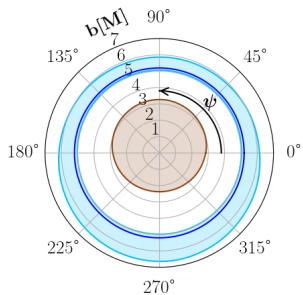


Figure: Horizon, critical curve and $n = 1$ lensing band on the observer's screen.

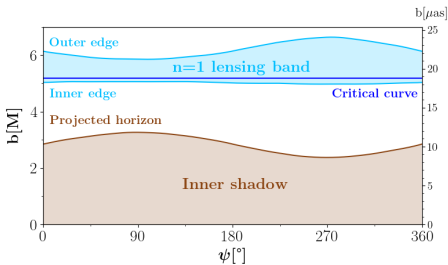


Figure: Impact parameters of the features along the polar angle on the screen.

Photon rings

- Critical curve, horizon, lensing bands = **mathematical regions**
- **Observable** photon rings = radiation of the accretion disk

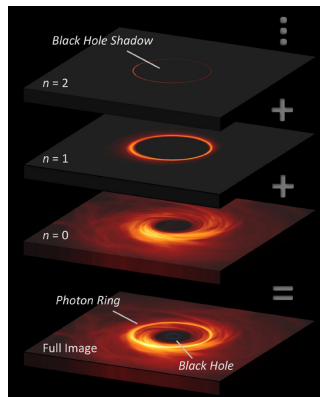
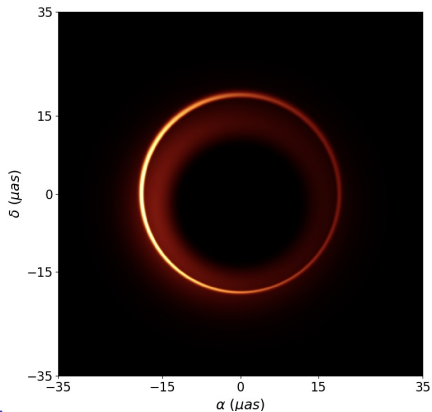
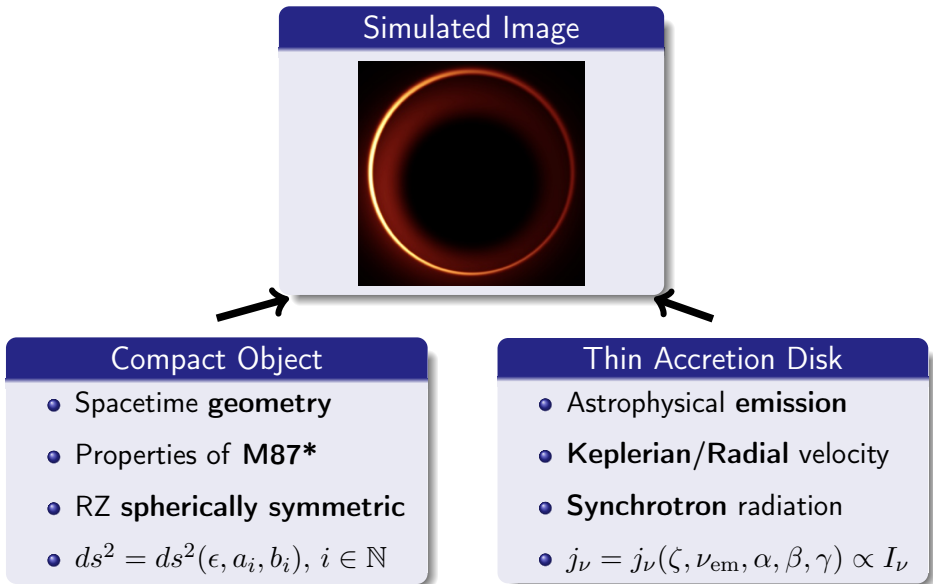


Figure:

Left panel: modeled image of the emission of an accretion disk observed at 230 GHz
 Right panel: embedded rings. Credits: Wong, Johnson

Parametrised framework



Parameters

Accretion disk:

- Emissivity $j_\nu(\zeta, \nu_{\text{em}}, \alpha, \beta, \gamma)$
- $n_e \propto r^{-\alpha}$, $\Theta_e \propto r^{-\beta}$, $B \propto r^{-\gamma}$, $\zeta(B_{\text{inner}}, \Theta_{e;\text{inner}})$

Compact object:

- All **metric parameter** affect the **geodesic motion**
- Only **lower order** parameters affect **near-horizon** phenomena:

| | Event horizon | Photon sphere | ISCO |
|------------|---------------|---------------|------|
| ϵ | ✓ | ✓ | ✓ |
| a_1 | ✗ | ✓ | ✓ |
| b_1 | ✗ | ✗ | ✗ |

- a_0 and b_0 are constrained by observations in the Solar System

1D cross sections

- Separate intensity profiles: $n=0$ image and $n=1$ photon ring
- Measure the **radial position of the intensity peaks**

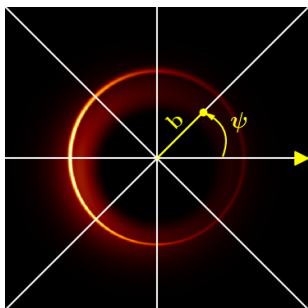


Figure: 1D intensity cuts

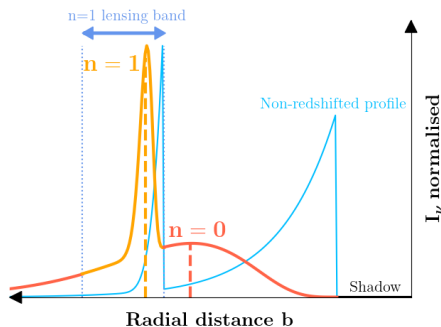


Figure: 1D intensity profile and peaks

Redshift effects

- All I_ν^{em} peak at the radial position of the equatorial event horizon
- Redshifted intensity: $I_\nu^{\text{obs}} = g^3 I_\nu^{\text{em}}$ with $g = \frac{\nu^{\text{obs}}}{\nu^{\text{em}}} = \frac{p^{\text{obs}} \cdot u^{\text{obs}}}{p^{\text{em}} \cdot u^{\text{em}}}$

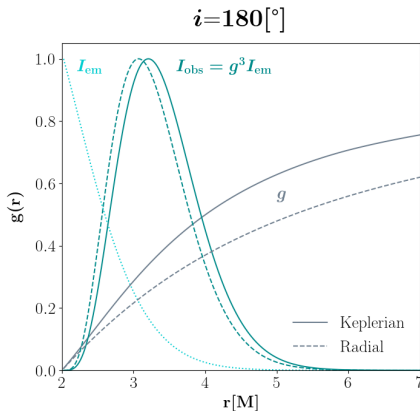


Figure: Redshifted profiles for a Schwarzschild black hole seen face-on

$n = 0$ image

Results

$n = 0$ peaks' position **degenerate** for every disc's dynamics

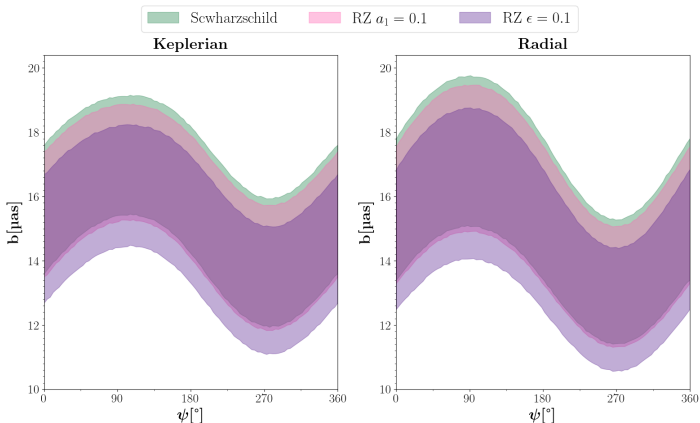


Figure: Impact parameters of the intensity's peaks along the polar angle on the screen

$n = 1$ photon ring

Results

$n = 1$ peaks' position **disentangleable** and always in the Keplerian case

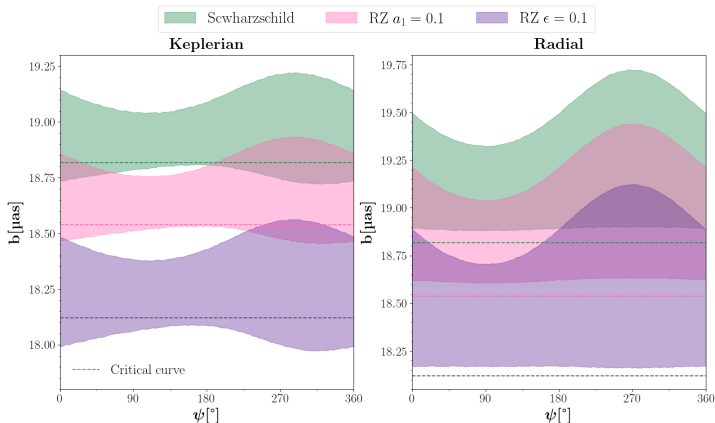


Figure: Impact parameters of the intensity's peaks along the polar angle on the screen

Detectability

- $n = 1$ photon ring not detectable with present instruments
- Angular **resolution** of an interferometer:

$$R \simeq \frac{\lambda}{B},$$

with λ the observed **wavelength** and B the maximum **baseline**

- High-frequency ground array of the **ngEHT** (ongoing)
- **BHEX** space-based array (Small Explorer proposed to NASA)

Avenues

Astrophysics:

- **Geometrically thick disk**
- **Time variability**

Geometry:

- **Rotating** black hole

Methodology:

- **Interferometric signal**

Objects of study:

- **Polarised images**
- **$n=2$ photon ring**

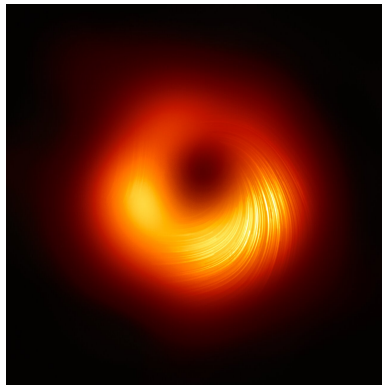


Figure: Polarised image of M87*.
Credits: Event Horizon Telescope

Références

- [1] SE Gralla, A Lupsasca, and DP Marrone. “The Shape of the Black Hole Photon Ring: A Precise Test of Strong-Field General Relativity”. In: *Physical Review D* 102.12 (2020), p. 124004.
- [2] MD Johnson et al. “Universal interferometric signatures of a black hole’s photon ring”. In: *Science advances* 6.12 (2020), eaaz1310.
- [3] L Rezzolla and A Zhidenko. “New parametrization for spherically symmetric black holes in metric theories of gravity”. In: *Physical Review D* 90.8 (2014), p. 084009.
- [4] FH Vincent et al. “Images and photon ring signatures of thick disks around black holes”. In: *Astronomy & Astrophysics* 667 (2022), A170.

Backup slides

Very Long Baseline Interferometry

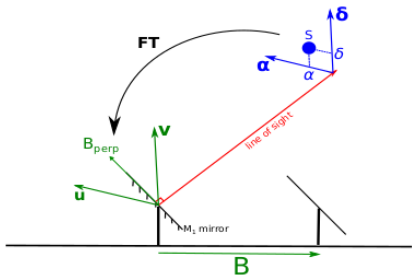


Image: $I(x,y)$
sky plane

FT: $V(u,v)$
 uv plane

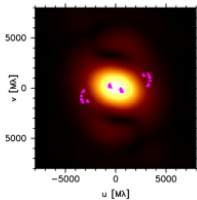
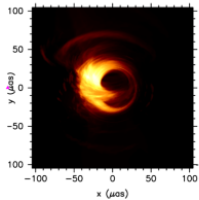


Figure: Relation between the planes of the sky and visibility. Credits: Frédéric Vincent

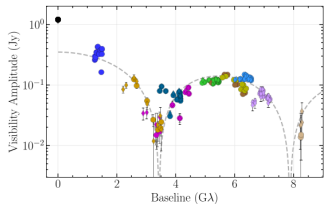
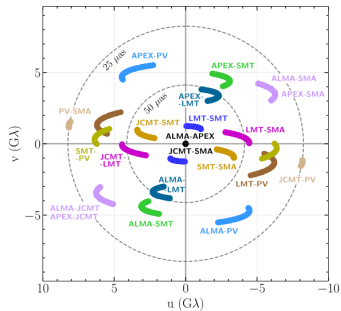


Figure: (u, v) plane coverage and visibility. Credits: EHT

Complex visibility

- **Spatial correlation** function: $\mathcal{V} = \mathcal{V}(\vec{B}_\perp/\lambda)$,
with λ the wavelength and \vec{B}_\perp the projection of the baseline
- **Fourier transform** of the **brightness distribution in the sky** I
- **Van Cittert-Kernike** theorem

$$\frac{\mathcal{V}(u, v)}{\mathcal{V}(0, 0)} = \frac{\iint I(\alpha, \delta) e^{-2i\pi(u\alpha + v\delta)} d\alpha d\delta}{\iint I(\alpha, \delta) d\alpha d\delta},$$

with (α, δ) the usual right ascension and declination
and (u, v) their Fourier conjugate frequencies

Visibility amplitude

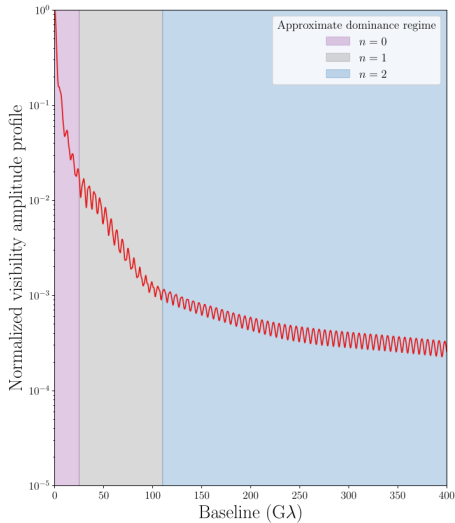


Figure: Visibility amplitude of a Gyoto simulated image. Credits: Pagnat et al. 2022

Photon ring detectability

EHT + space telescope: deviations of **0.1 mJy** detectable at 345 GHz

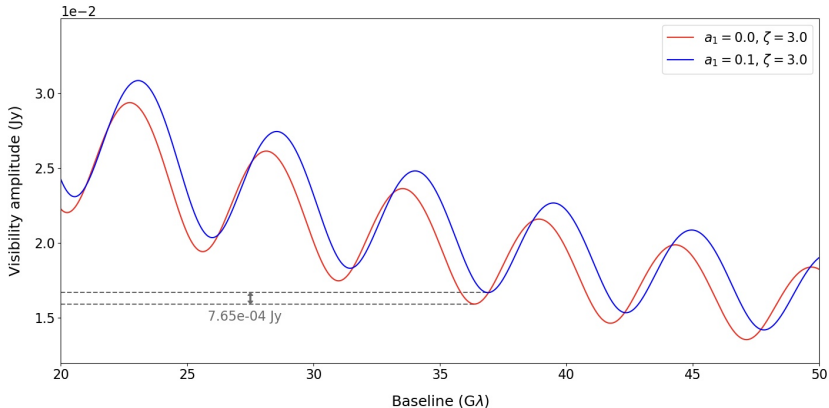


Figure: Visibility amplitude of the $n=1$ photon ring for $\zeta = 3$ and $a_1 = 0$ or $a_1 = 0.1$

Compact object

- Spherically symmetric black hole
- Rezzolla-Zhidenko parametrised and hierarchical metric:

$$ds^2 = -N(r)^2 dt^2 + \frac{B(r)}{N(r)^2} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2),$$

$$\begin{cases} N^2(x) = x[1 - \epsilon(1-x) + (a_0 - \epsilon)(1-x)^2 + \tilde{A}(x)(1-x)^3] \\ B(x) = 1 + b_0(1-x) + \tilde{B}(x)(1-x)^2 \end{cases},$$

$$\tilde{A}(x) = \frac{a_1}{1 + \frac{a_2 x}{1 + \frac{a_3 x}{1 + \dots}}}, \quad \tilde{B}(x) = \frac{b_1}{1 + \frac{b_2 x}{1 + \frac{b_3 x}{1 + \dots}}},$$

where we introduced the variable $x := 1 - r_{\mathcal{H}}/r$ and $1 + \epsilon = 2M/r_{\mathcal{H}}$, with $r_{\mathcal{H}}$ the radius of the horizon and M the mass of the black hole

Metric impact on image features

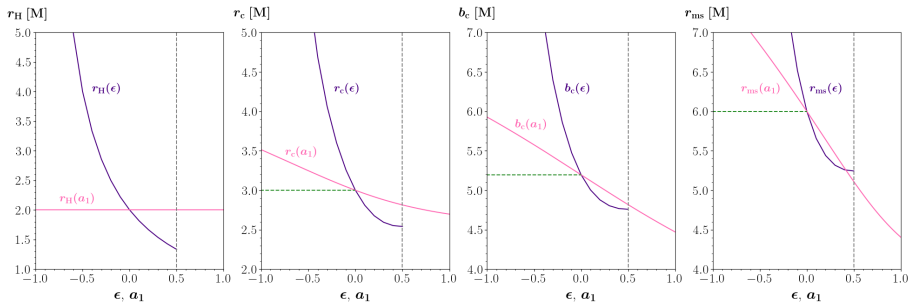


Figure: Dependence of the event horizon, photon shell, critical curve and ISCO on ϵ, a_1

Accretion disk

Set-up:

- **Geometrically thin** disk inclined at 163°

Dynamics:

- **Keplerian velocity** with the **Cunningham hypothesis**
- **Radially infalling velocity**

Emission:

- **Optically thin** disk
- **Thermal synchrotron emission**

Thermal synchrotron emission

- Synchrotron power emitted by an ultrarelativistic electron: P_ν
- Power of a population of electrons in thermal equilibrium:

$$j_\nu = \int_0^\infty P_\nu \frac{dn_e}{d\gamma} d\gamma \propto I_\nu$$

Θ_e, n_e
↓
↑ ↓
 B v_e

with B the magnetic field, Θ_e the dimensionless temperature and n_e the density following a **Maxwell distribution** of speeds v_e

- Hypotheses of power-law fall-offs: $n_e \propto r^{-\alpha}$, $\Theta_e \propto r^{-\beta}$, $B \propto r^{-\gamma}$
- $i_1 = \alpha - 2\beta$ and $i_2 = \gamma + 2\beta$

Emissivity

- $j_\nu \approx \eta \frac{\nu_{\text{em}}[\text{GHz}]}{230} \left(\frac{r}{r_{\text{inner}}}\right)^{-i_1} \exp \left[-\zeta \sqrt[3]{\frac{\nu_{\text{em}}[\text{GHz}]}{230}} \left(\frac{r}{r_{\text{inner}}}\right)^{i_2/3} \right]$

where $\zeta = \left(\frac{3.7 \times 10^5}{B_{\text{inner}} \Theta_{e;\text{inner}}^2 \sin \theta} \right)^{1/3}$

with B_{inner} , $\Theta_{e;\text{inner}}$ the values of B and Θ_e at the inner radius r_{inner} and θ the angle between the emission direction and the magnetic field

- **Various observing frequencies** of the ngEHT: 230, 345 GHz
- $I_\nu^{\text{em}} \propto j_\nu$

Apparent shape of circular rings

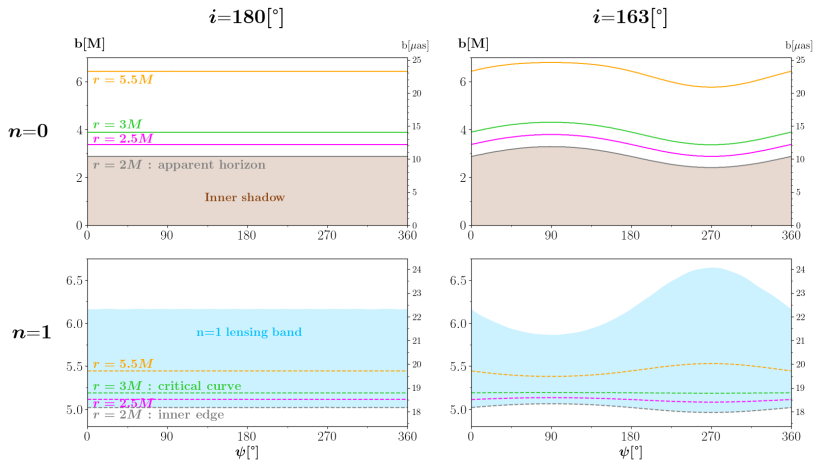


Figure: Direct and first-lensed apparent positions of rays emitted from isoradial distances

Null geodesics

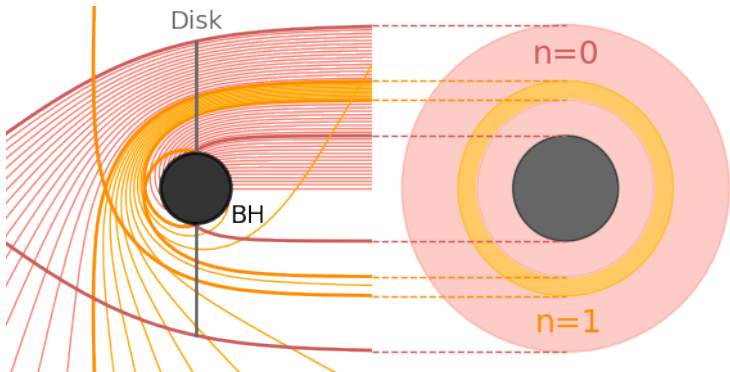


Figure: Geodesics of photons in the Schwarzschild spacetime and corresponding image at infinity of the black-hole seen face-on

GYOTO

Initial condition: $v_{\text{obs}}, x^\mu, \dot{x}^\mu$

Black hole: $g_{\mu\nu} \rightarrow \Gamma_{\alpha\beta}^\mu, r_{\text{inner}}, \tilde{p}_{\text{em}}, \tilde{u}_{\text{em}}$

Accretion disk: ζ

Speed of light in vacuum: $c \approx 2.99 \times 10^8$ m/s

Planck constant: $h \approx 6.63 \times 10^{-34}$ J·s

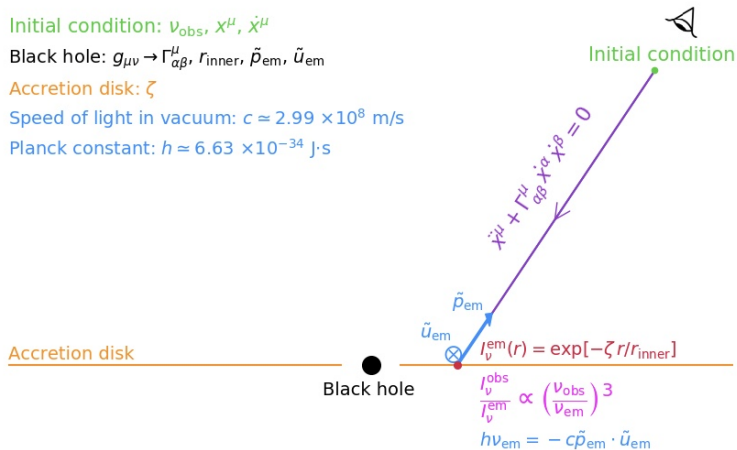


Figure: Scheme of the functioning of GYOTO

Adaptative ray-tracing

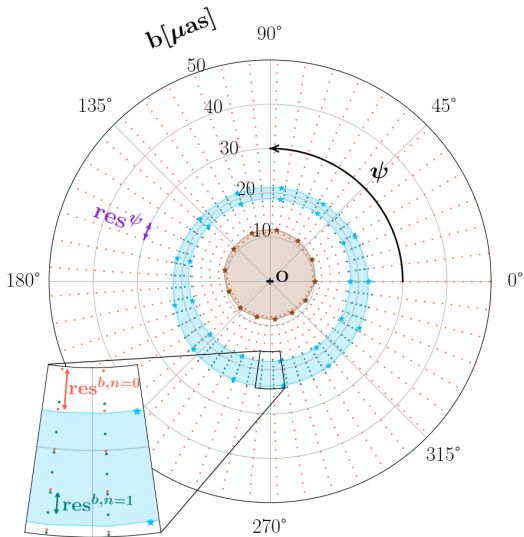


Figure: Adaptively ray-traced points

Polarisation

- **Polarisation:** privileged orientation of the radiation
- Regular magnetic field
- n = number of **half turns**
- Even: same polarisation
- Odd: same polarisation
- Change of polarisation: dependence on the BH **spin**

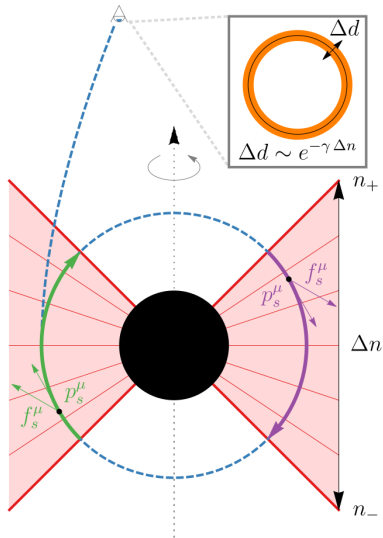


Figure: Ray-tracing in a thick disk.
Credits: Elizabeth Himwich