

The background of the slide is a dark, starry field with a central purple nebula-like glow. The stars are small, bright points of light in various colors, including white, yellow, and orange. The nebula is a soft, glowing purple and blue cloud in the center.

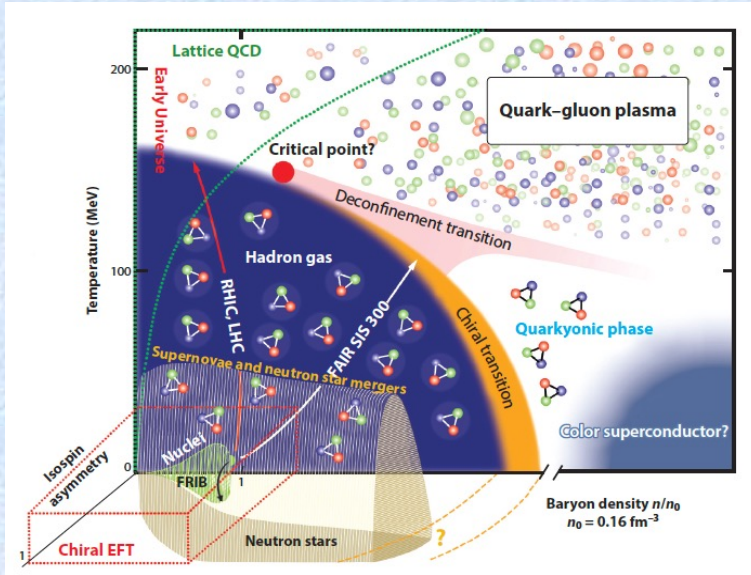
Constraints on the equation of state of dense matter

Anthea F. Fantina

*Journées Théorie de la communauté Hautes Energies
04 – 07 Nov. 2024, Paris (France)*



Probing extreme conditions in NSs



Drischler et al., Ann. Rev. Nucl. Part. Sci. 71, 403 (2021)

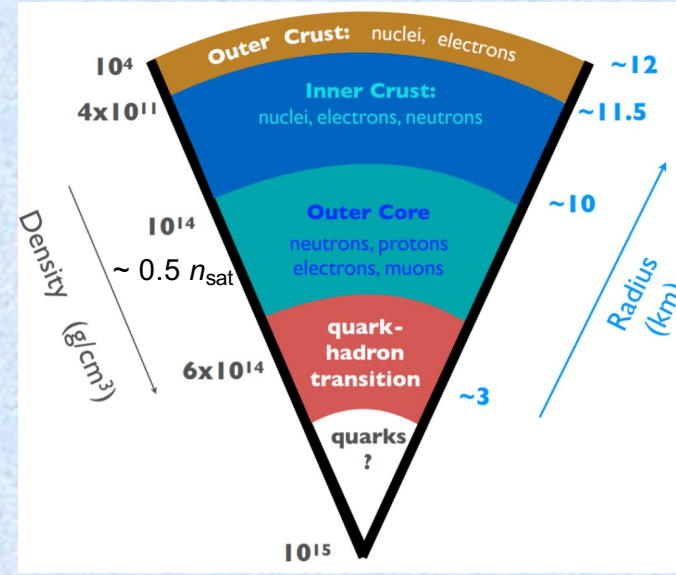


Image Credit: 3G Science White Paper

“cold catalysed matter”
i.e. full equilibrium
(beta equilibrium),
 $T = 0$ ($T < \sim 10^8\text{K}$)
→ EoS $P(n_B)$

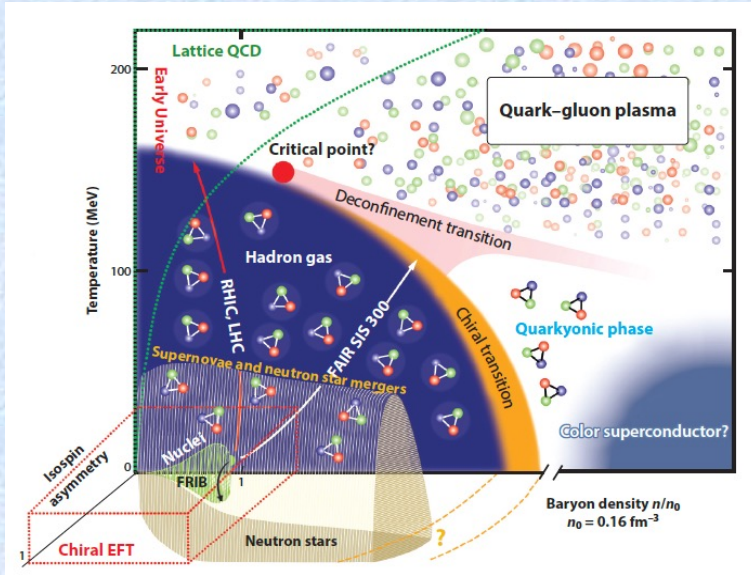
N.B.: In this talk, cold catalysed matter & NS static properties
→ OK for cold NS and pre-merger binary NSs (inspiral phase)

Even then, different states of matter spanned in NSs → inhomogeneous (crust), “pasta” phase, homogeneous (core), “exotic” particles (?) + superfluidity, (strong) \vec{B} , etc.

(see e.g. Haensel et al., “Neutron Star Structure”, Springer 2007; Oertel et al., Rev. Mod. Phys. 2017; Burgio & Fantina, ASSL Springer 2018)



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Drischler et al., Ann. Rev. Nucl. Part. Sci. 71, 403 (2021)

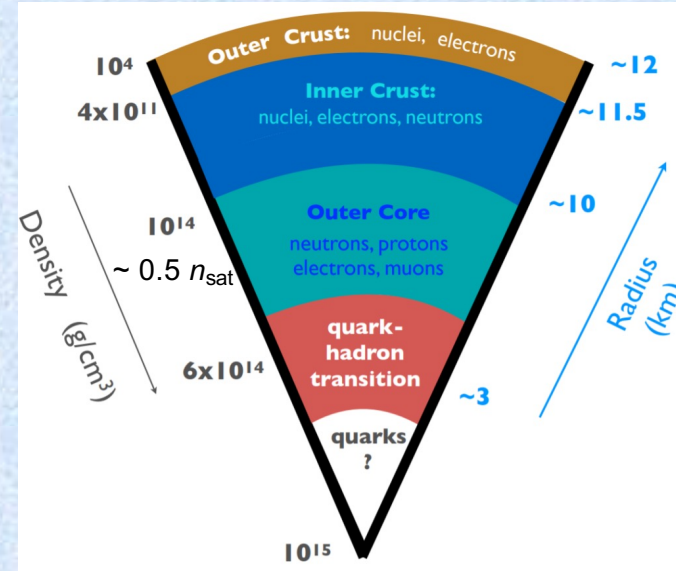


Image Credit: 3G Science White Paper

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Even then, different states of matter spanned in NSs \rightarrow inhomogeneous (crust), “pasta” phase, homogeneous (core), “exotic” particles (?) + superfluidity, (strong) \vec{B} , etc.

\rightarrow Not all conditions can be probed in terrestrial labs \rightarrow theoretical models !
 \rightarrow Consistent description very challenging

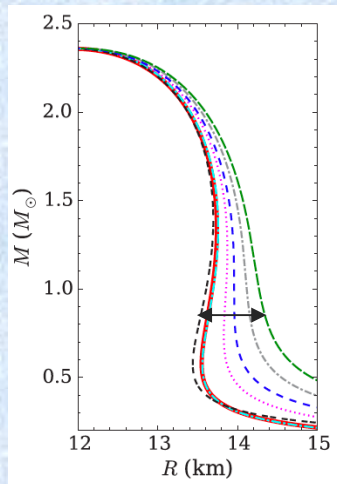
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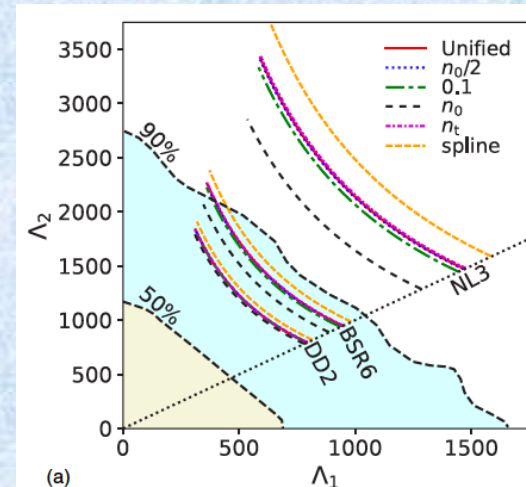
Why a consistent and unified treatment ?

Unified treatment of inhomogeneous & homogeneous matter
→ same nuclear model employed in different regions of star

- Challenging because of wide range of thermodynamic conditions
- Challenging because different states of matter
- But: essential to avoid spurious non-physical effects in numerical modelling



Fortin et al., PRC 94, 035804 (2016)



Suleiman et al., PRC 104, 015801 (2021)

see also Ferreira&Providencia 2020

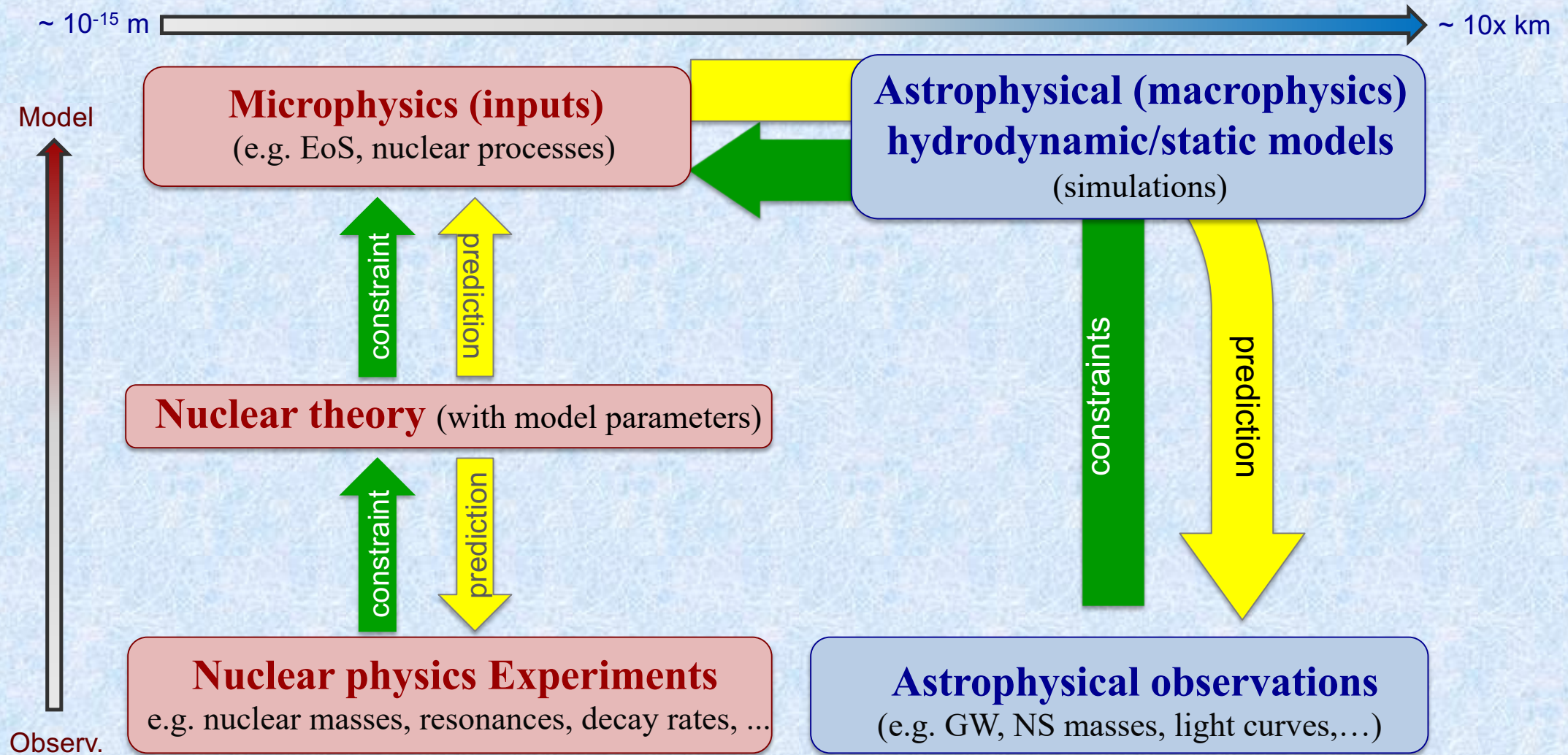


Thermodynamically consistent and unified EoSs for astro modelling & inference analyses

(but not many available, e.g. Douchin&Haensel 2001; Fantina et al. 2013; Raduta&Gulminelli 2015; Viñas et al. 2021; Pearson et al. 2018; Grams et al. 2022; Xia et al. 2022; Scurto et al. 2024; see CompOSE database)

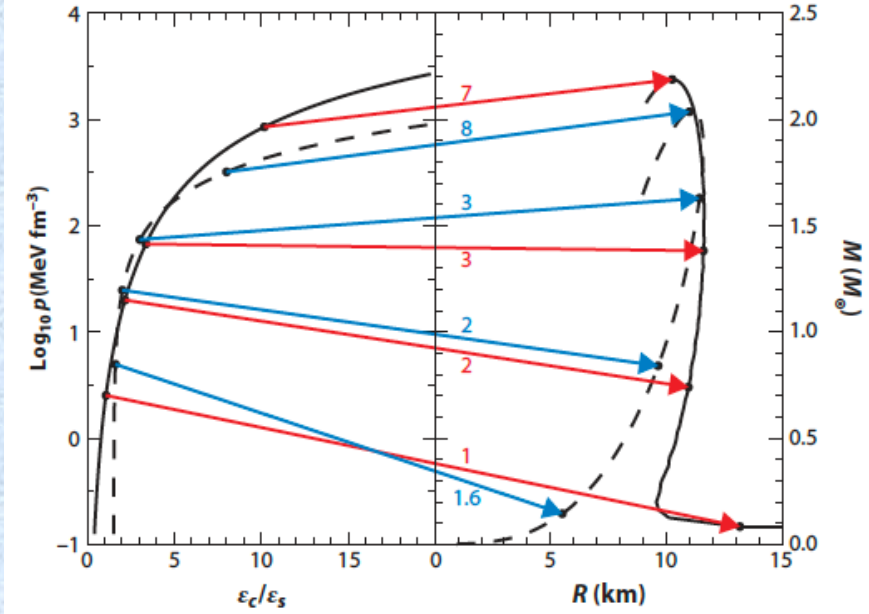


Micro to macro through modelling





EoS \leftrightarrow NS (static) observables (1)



Lattimer, Annu. Rev. Part. Nucl. Sci. 62, 485 (2012)

- **TOV $\rightarrow M(R)$** (Tolmann 1939; Oppenheimer&Volkoff 1939; see also Haensel et al. Springer 2007)

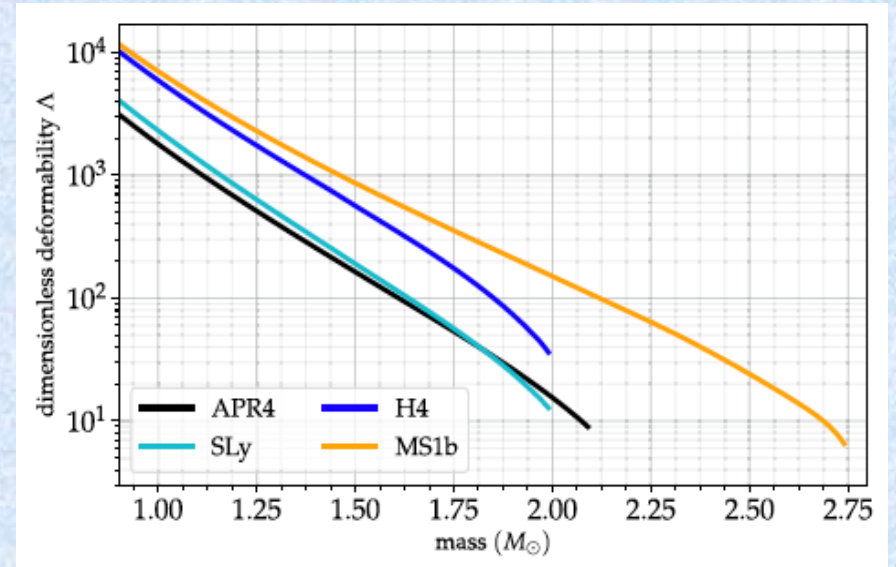
$$\frac{dP(r)}{dr} = -\frac{G\rho(r)\mathcal{M}(r)}{r^2} \left[1 + \frac{P(r)}{c^2\rho(r)} \right] \left[1 + \frac{4\pi P(r)r^3}{c^2\mathcal{M}(r)} \right] \left[1 - \frac{2G\mathcal{M}(r)}{c^2r} \right]^{-1}$$

$$\mathcal{M}(r) = 4\pi \int_0^r \rho(r')r'^2 dr'$$

- **+ eq. for $y(r) \rightarrow \Lambda(M), \Lambda(R)$** $r \frac{dy}{dr} + y(r)^2 + F(r)y(r) + Q(r) = 0$

\rightarrow Love number k_2

$$\rightarrow \Lambda = \frac{2}{3} k_2 \left[\frac{Rc^2}{GM} \right]^5$$



Dietrich et al., Gen. Rel. Gravit. 53, 27 (2021)

\rightarrow GR \rightarrow direct correspondence
EoS \leftrightarrow NS static properties (e.g., M, R, Λ)

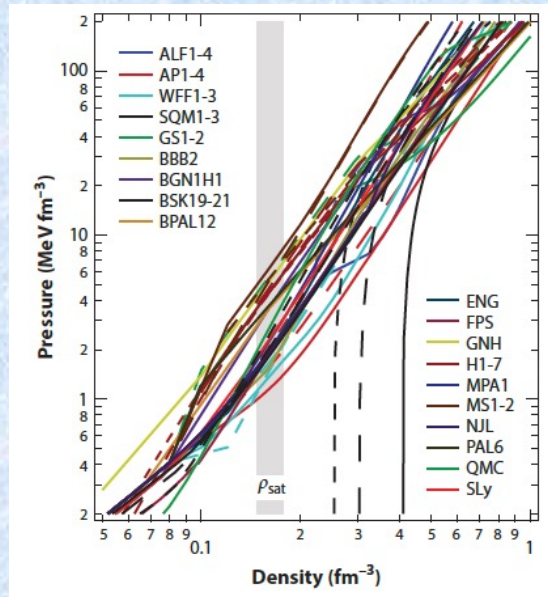
?
 \rightarrow trace back to EoS and composition?



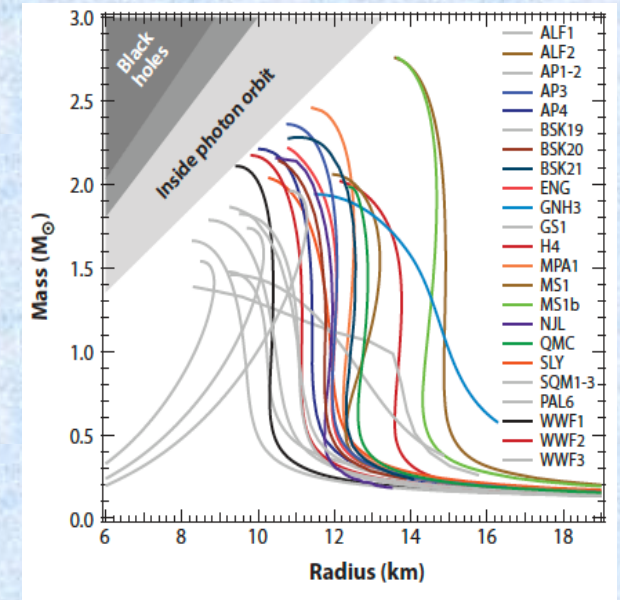
EoS \leftrightarrow NS (static) observables (2)

but:

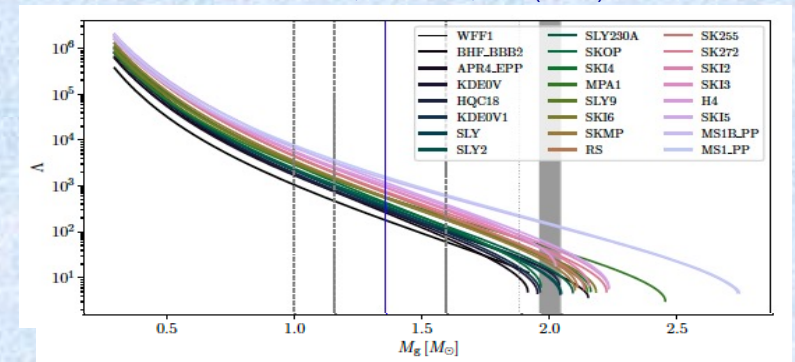
- ✗ EoS model dependent !
- ✗ no ab-initio dense-matter calculations (“microscopic” e.g. chiral EFT, Monte Carlo, Green’s funct., (D)BHF, ...)
- in all regimes (usually applied for infinite matter e.g. the core)
- phenomenological models (e.g. energy density functionals like Skyrme/Gogny, ...)
- ✗ composition \leftrightarrow EoS \rightarrow $M(R)$?



Ozel & Freire, ARAA 54, 401 (2016)



Ozel & Freire, ARAA 54, 401 (2016)



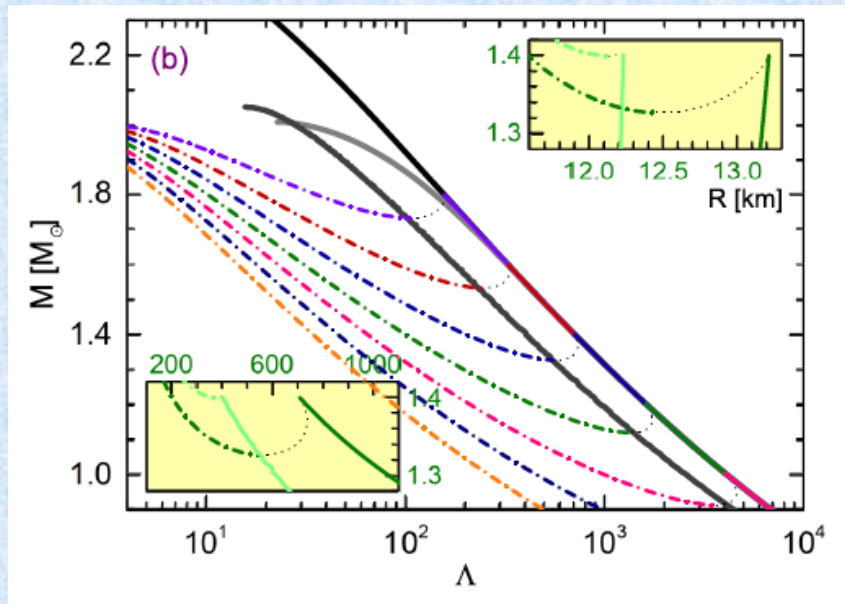
Abbott et al., Class. Quantum Grav, 37, 045006 (2020)



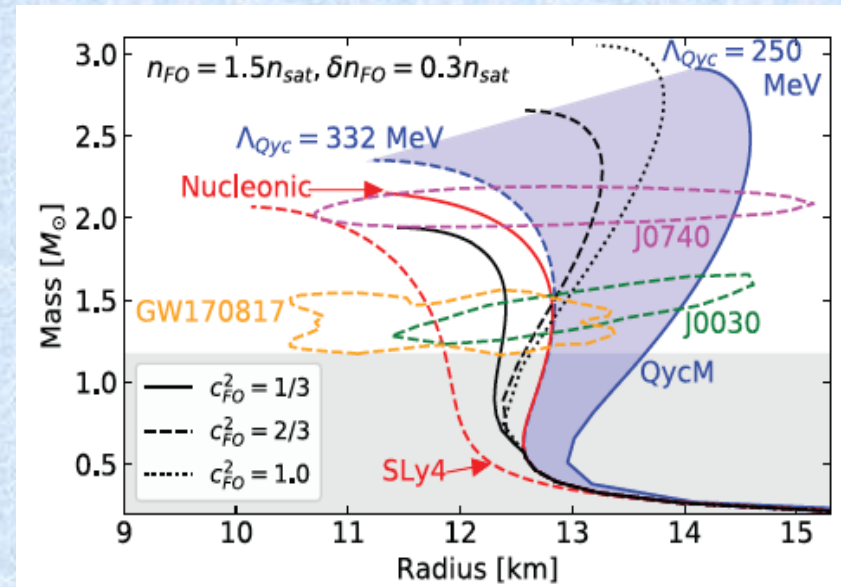
High-density EoS \rightarrow additional d.o.f.?

- Role of “exotic” degrees of freedom?

Hyperons \rightarrow softer EoS \rightarrow lower M_{\max} (+ reduction of R and Λ for intermediate-mass) but large uncertainties on hyperons !
 Quarks \rightarrow not clear



Li et al., PRD 101, 063022 (2020)



Somasundaram & Margueron, EPL 138, 14002 (2022)

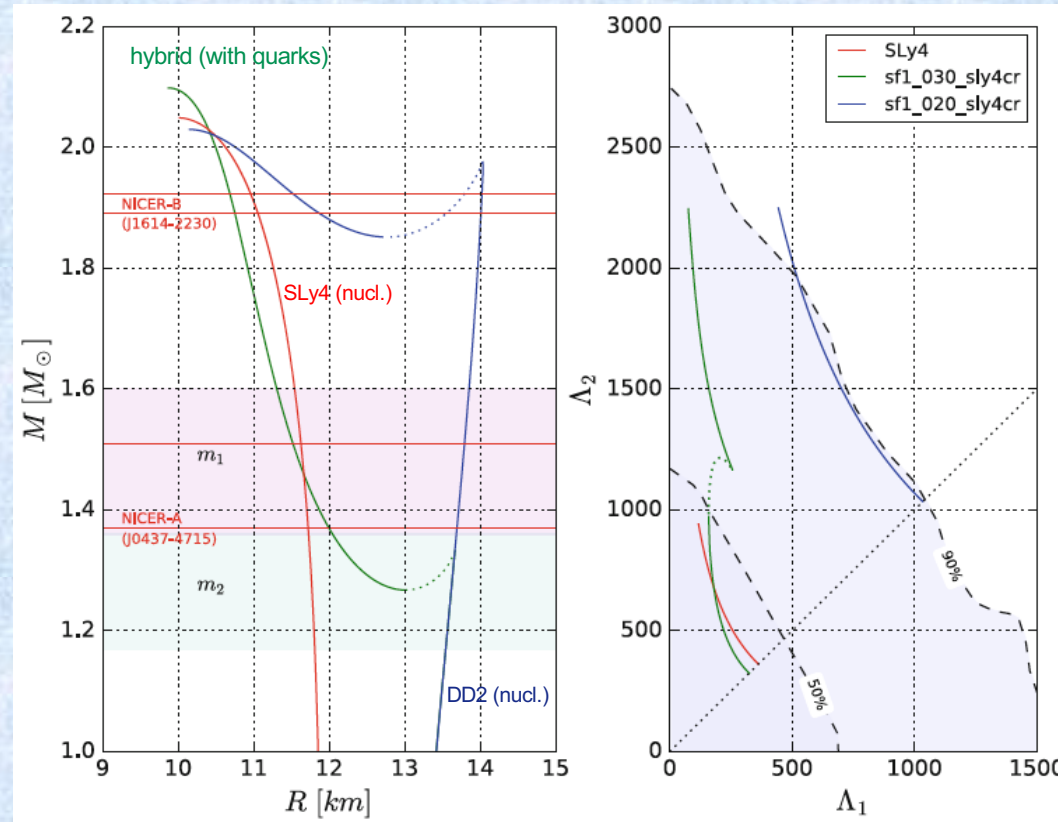


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- “Masquerade” effect

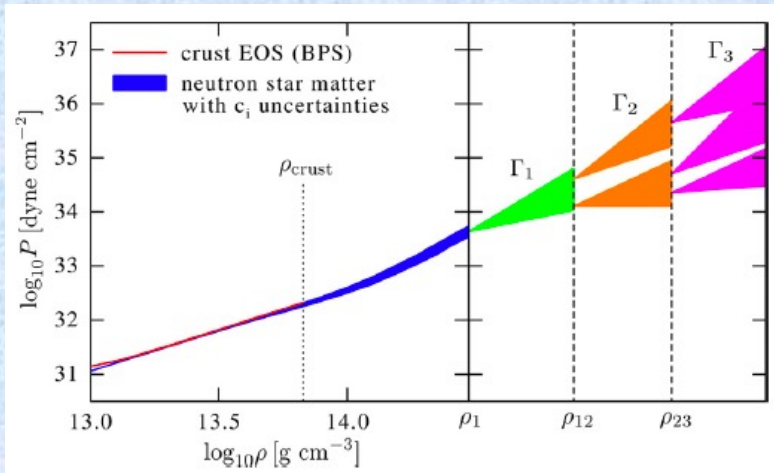


Blaschke & Chamel, ASSL 457, 337 (2018)

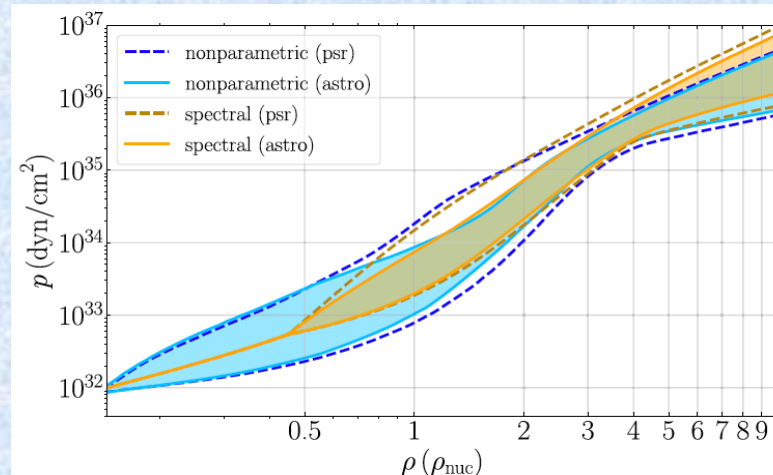


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 - Hyperons \rightarrow softer EoS \rightarrow lower M_{\max} (+ reduction of R and Λ for intermediate-mass) but large uncertainties on Y !
 - Quarks \rightarrow not clear
- “Masquerade” effect
- *Agnostic* (“non-nuclear”) approaches for NS core (e.g. **piecewise polytropes**, c_s models, **Gaussian process**, **spectral funct.**, etc...) (conditioned by astro)



Hebeler et al., ApJ 773, 11 (2013)



Legred et al. PRD 105, 043016 (2022)

- ✓ powerful \rightarrow no underlying hypotheses
- ✗ what about nuclear physics \rightarrow composition ?
- ✗ often unique (non-consistent) low-density EoS \rightarrow uncertainties underestimated



EoS \leftrightarrow nuclear matter parameters

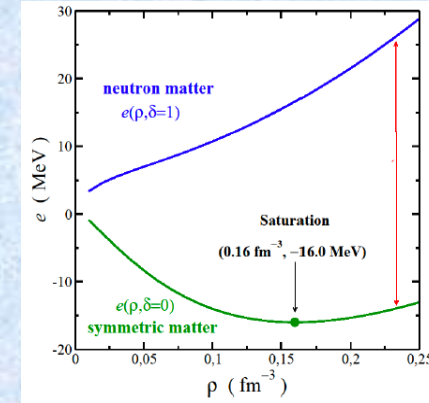
- Expansion in density and asymmetry around n_{sat} and $\delta = 0$

$$x = (n - n_{\text{sat}})/3n_{\text{sat}}$$

$$\delta = (n_n - n_p)/n$$

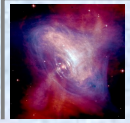
$$e_{\text{is}} = E_{\text{sat}} + \frac{1}{2}K_{\text{sat}}x^2 + \frac{1}{6}Q_{\text{sat}}x^3 + \dots \quad \rightarrow e_{\text{sat}}(n, \delta=0)$$

$$e_{\text{iv}} = E_{\text{sym}} + L_{\text{sym}}x + \frac{1}{2}K_{\text{sym}}x^2 + \frac{1}{6}Q_{\text{sym}}x^3 + \dots \quad \rightarrow e_{\text{sym}}(n) = e(n, \delta=1) - e(n, \delta=0)$$



Centelles, talk@Primosten (2011)

\rightarrow Nuclear empirical parameters (NEP, bulk) $\rightarrow \mathbf{X}_{\text{sat,sym}} = E_{\text{sat}}, K_{\text{sat}}, Q_{\text{sat}}, \dots, E_{\text{sym}}, L_{\text{sym}}, K_{\text{sym}}, Q_{\text{sym}}, \dots$



EoS \leftrightarrow nuclear matter parameters

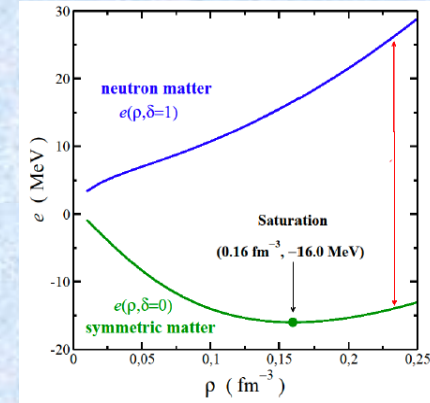
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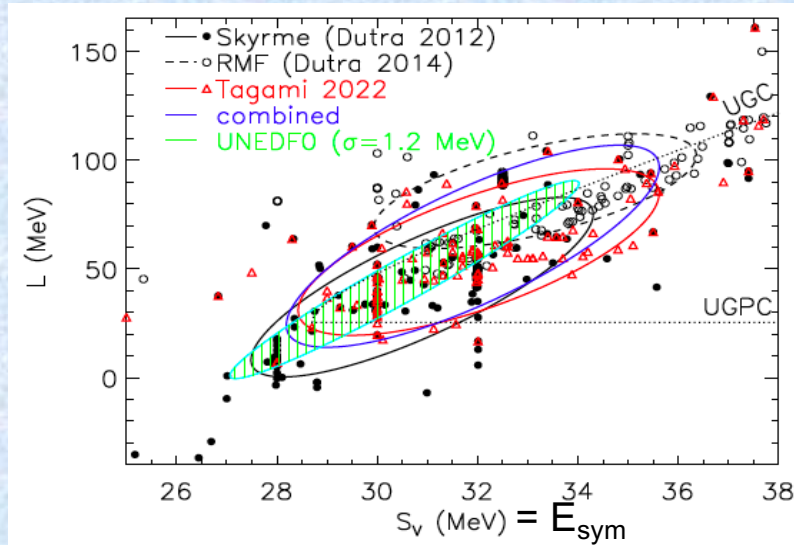
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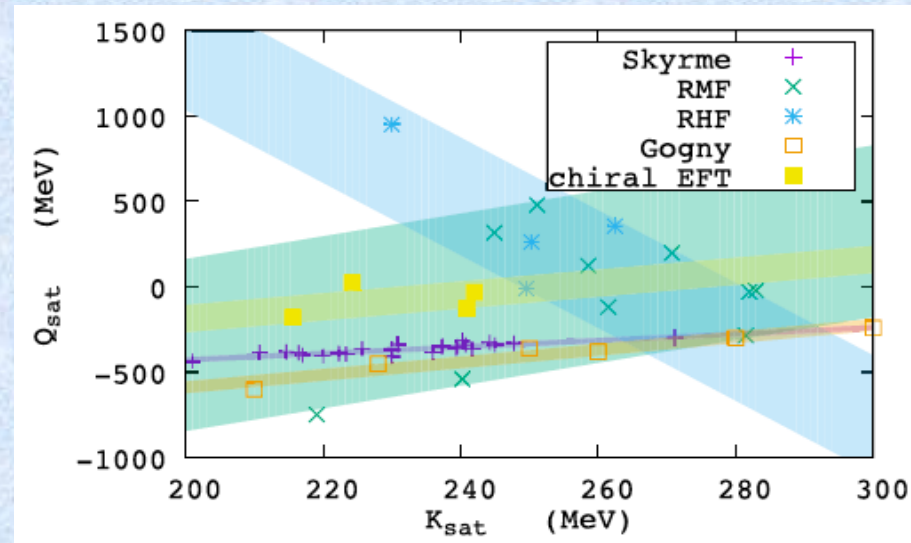


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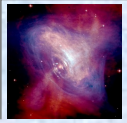


Lattimer, Particles 6, 30 (2023)



Margueron et al., PRC 97, 025805 (2018)

see e.g. Bulgac et al., PRC 97, 044313 (2018), Margueron et al., PRC 97, 025805 (2018), Carreau et al, EPJA 55, 188 (2019), Tews et al., EPJ A 55, 97 (2019), Dinh Thi et al., A&A 654, A114 (2021), Dinh Thi et al., EPJA 57, 296 (2021); Essick et al., PRC 104, 065804 (2021), ...



A semi-agnostic approach: meta-model

- **Nucleonic Meta-model (MM)** (Margueron et al., PRC 97, 025805 (2018); also e.g. Lim&Holt 2019, Tsang et al. 2020; see Char et al., PRD 108 (2023) for a relativistic version)

→ EDF-based but flexible. Based on a Taylor expansion in density and asymmetry.

$$e_{\text{nuc}}(n_B, \delta) = t_{\text{FG}}^*(n_B, \delta) + e_{\text{is}}(n_B) + e_{\text{iv}}(n_B)\delta^2$$

kinetic term with m^*

$$e_{\text{is,iv}}(n_B) = \sum_{k=0}^N \frac{v_k^{\text{is,iv}}}{k!} x^k u_k(x)$$

$N > 2 \rightarrow$ beyond parabolic approx. \rightarrow important for crust

functions of nuclear empirical parameters

zero-density limit

- **For application of MM to NS crust \rightarrow Compressible Liquid Drop Model**

e.g. Carreau et al., EPJA 2019; Dinh Thi et al., A&A 2021; Grams et al., EPJA 2022; Mondal et al., MNRAS 2023; Davis et al., A&A 2024



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- ✓ Possible to reconstruct low-density EoS from any EoS at high density (\rightarrow **CUTER**)
- ✓ Possible to couple MM approach for crust to agnostic EoS (e.g. polytropes, etc.) at high density
- ✓ Vary NEP \rightarrow parameter exploration (without a priori correlations) \rightarrow statistical (Bayesian) analysis

$$p_{\text{post}}(\vec{X}) = \mathcal{N} p_{\text{prior}}(\vec{X}) e^{-\chi^2(\vec{X})/2} w_{\text{LD}}(\vec{X}) w_{\text{HD}}(\vec{X})$$

flat non-informative prior
 \rightarrow large parameter space

nuclear masses
(AME)

Low-Density filters
 \rightarrow ab-initio (EFT)

High-Density filters

\rightarrow causality, stability, $M_{\text{NS,max}}$ (+ NICER, GW)

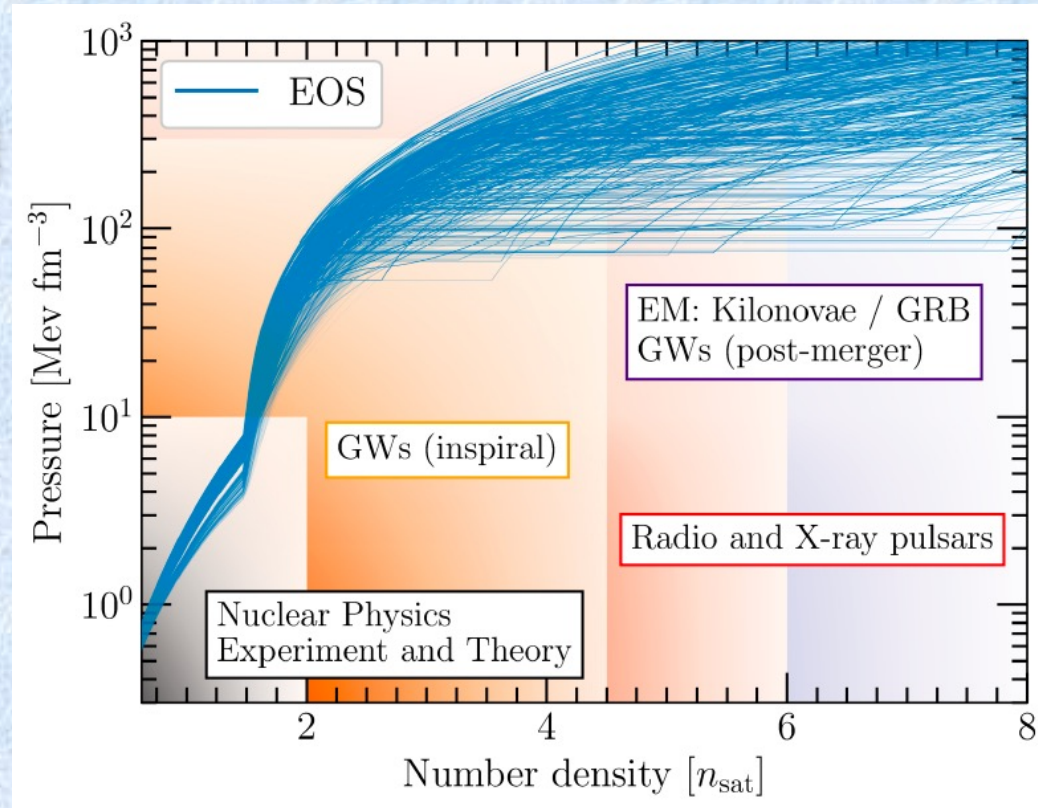


How to get constraints ?

Nuclear physics exp./ theory



“low” density
(better in nucleonic sector)



Astrophysical observations



“high” density

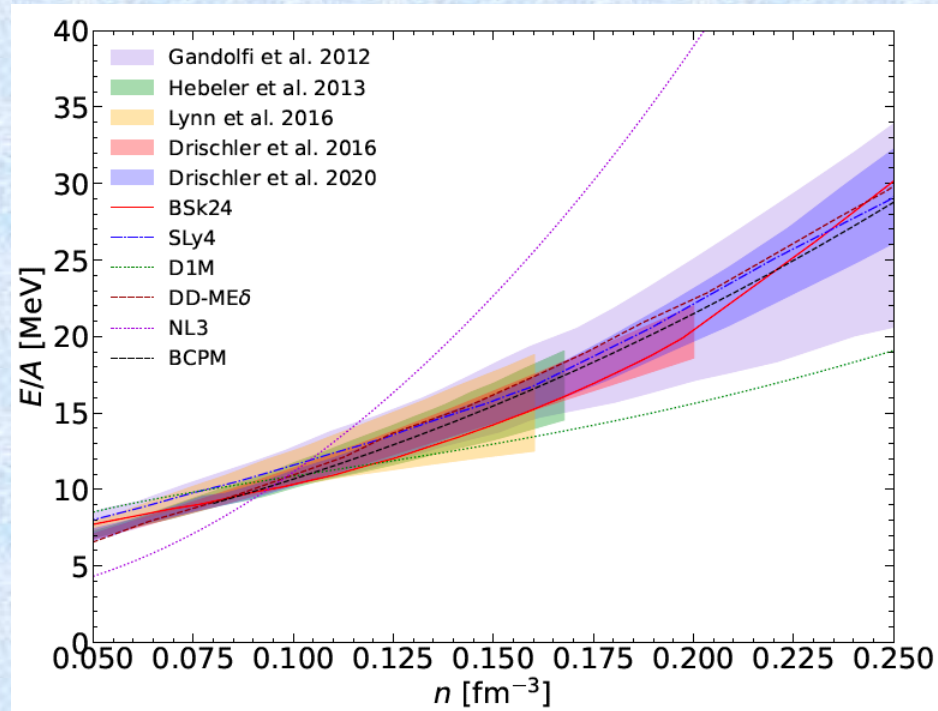
Pang et al., Nat. Comm. 14, 8352 (2023); arXiv:2205.08513

N.B.: rectangles only qualitative → also EoS dependence !



Constraints from nuclear physics: theory

PURE NEUTRON MATTER



Fantina & Gulminelli, J.Phys. Conf. Ser. 2586, 012112 (2023)

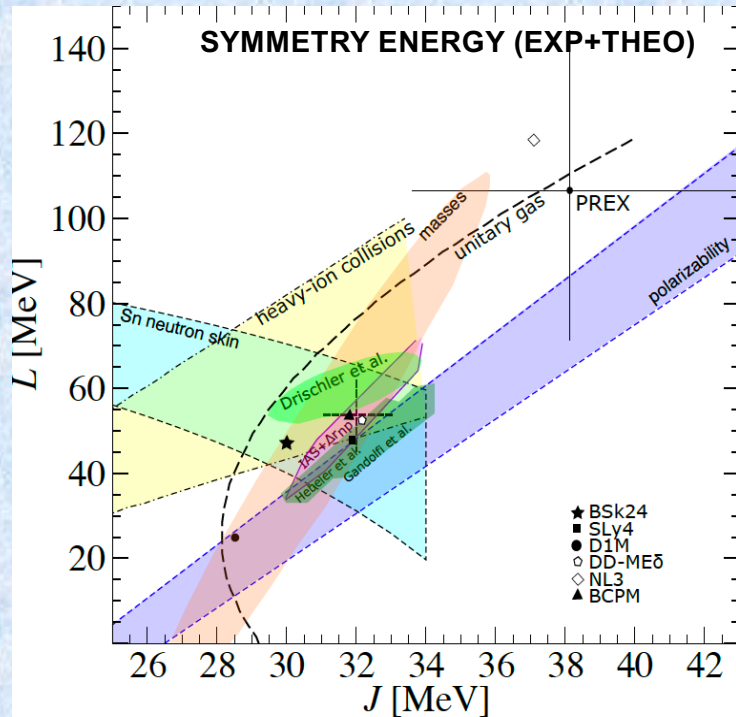
- different calculations (MC, many-body perturbation theory, interaction from chiral-EFT, ...)
- (relatively) low densities

N.B.: ab-initio for symmetric matter much less constraining

- Reasonable agreement of ab-initio (PNM) up to \sim saturation density
- benchmark (for phenomenological models) & constraints
- Not all popular models agree with ab-initio constraints!



Constraints from nuclear physics: experiments

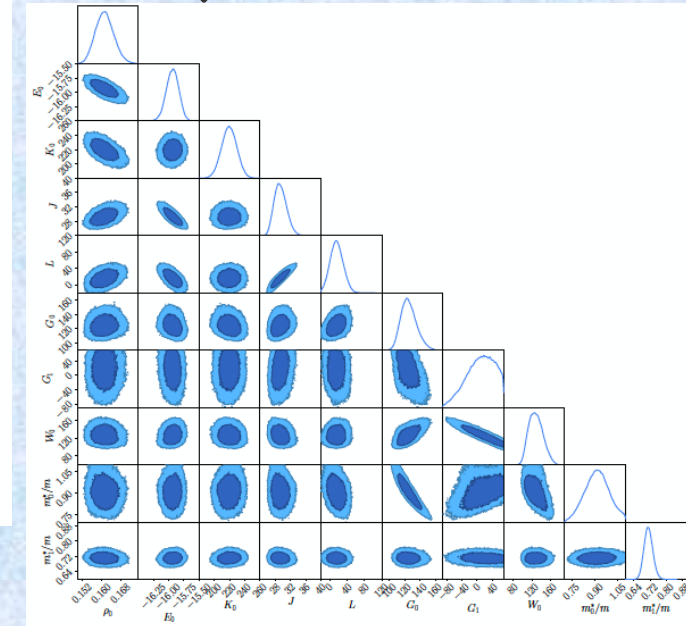


Gulminelli&Fantina, Nucl. Phys. News 31, 9 (2021);
 Fantina&Gulminelli, J.Phys.Conf.Ser. 2586, 012112 (2023)

Ground-state properties			
	$B.E.$ [MeV]	R_{ch} [fm]	ΔE_{SO} [MeV]
^{208}Pb	$1636.4 \pm 2.0^*$	$5.50 \pm 0.05^*$	$2.02 \pm 0.50^*$
^{48}Ca	$416.0 \pm 2.0^*$	$3.48 \pm 0.05^*$	$1.72 \pm 0.50^*$
^{40}Ca	$342.1 \pm 2.0^*$	$3.48 \pm 0.05^*$	-
^{56}Ni	$484.0 \pm 2.0^*$	-	-
^{68}Ni	$590.4 \pm 2.0^*$	-	-
^{100}Sn	$825.2 \pm 2.0^*$	-	-
^{132}Sn	$1102.8 \pm 2.0^*$	$4.71 \pm 0.05^*$	-
^{90}Zr	$783.9 \pm 2.0^*$	$4.27 \pm 0.05^*$	-
Isoscalar resonances			
	E_{GMR}^{IS} [MeV]	E_{GQR}^{IS} [MeV]	
^{208}Pb	$13.5 \pm 0.5^*$	$10.9 \pm 0.5^*$	
^{90}Zr	$17.7 \pm 0.5^*$	-	
Isovector properties			
	α_D [fm 3]	$m(1)$ [MeV fm 2]	A_{PV} (ppb)
^{208}Pb	19.60 ± 0.60	961 ± 22	550 ± 18
^{48}Ca	2.07 ± 0.22	-	2668 ± 113

Klausner et al., arXiv:2410.18598

parameter fit through emulator

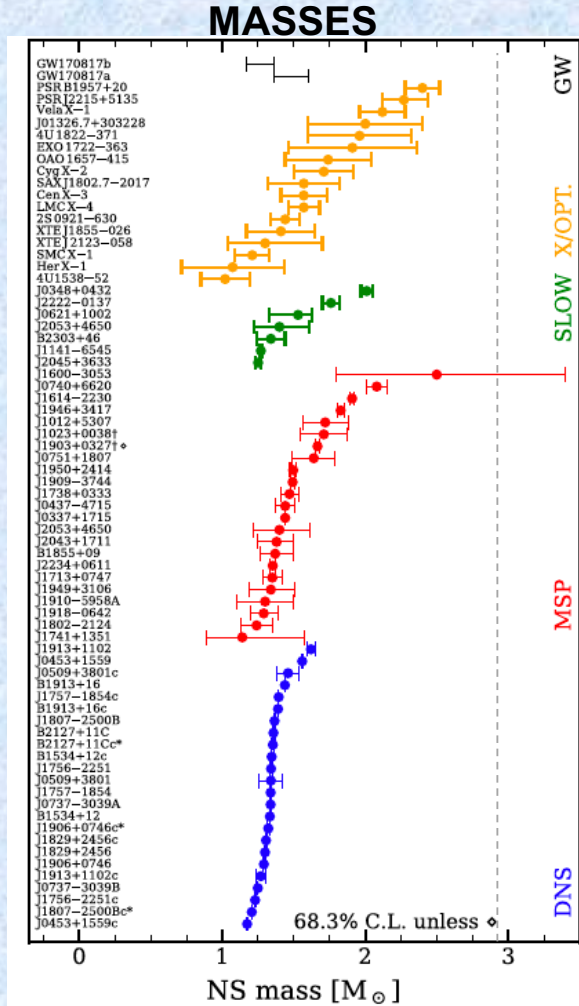


→ Constraints at “low” densities → low-order parameters, and on more on “symmetric” matter
N.B.: deduced constraints from exp. are often *not* raw data, but combined with models
 → model dependence of constraints !

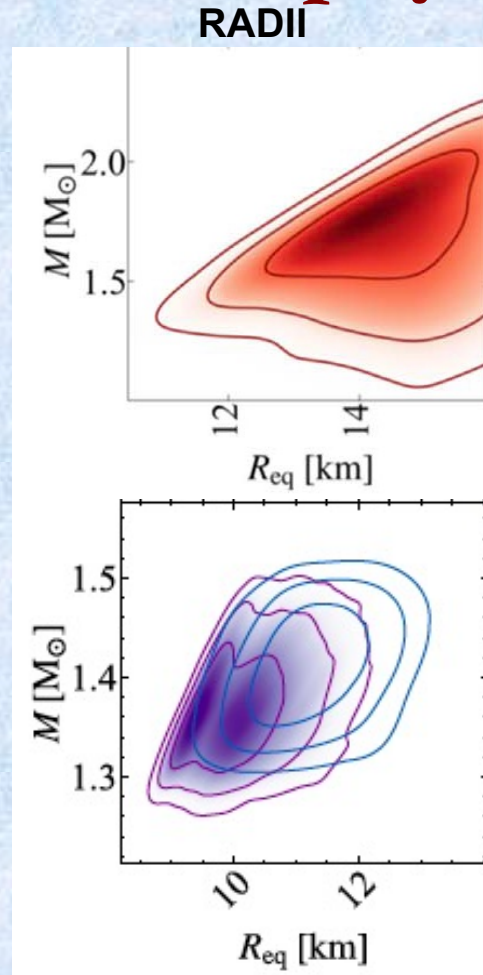
see also Marguerite et al., PRC 97, 025805 (2018) for a compilation



Constraints from astrophysics



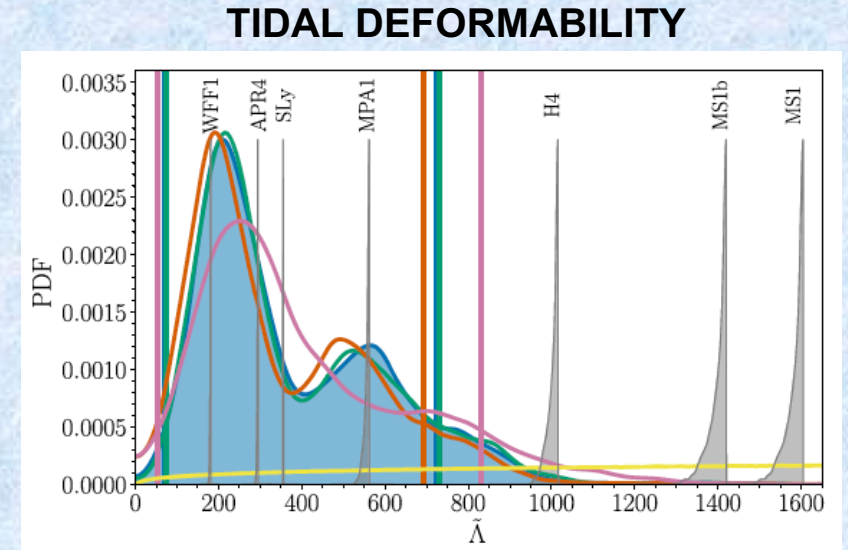
Suleiman et al., PRC 104, 015801 (2021)



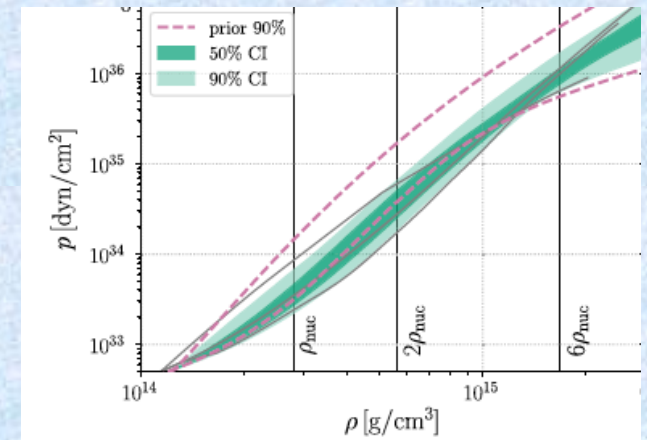
Choudhury et al., ApJL 971, L20 (2024);

Vinciguerra et al., ApJ 961, 62 (2024)

see also Miller et al., Riley et al., ApJL 2019, 2021



Abbott et al., PRX 9, 011001 (2019)



Abbot et al., PRL 121, 161101 (2017)

see also Koehn et al., arXiv:2402.04172 for a compilation



Crustal properties: cc transition

→ important e.g. for NS cooling, rotational evolution of pulsars, pulsar glitches, ...

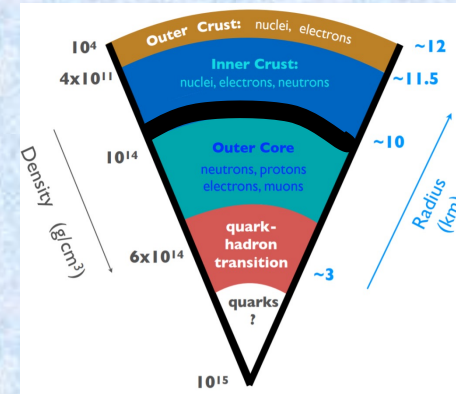
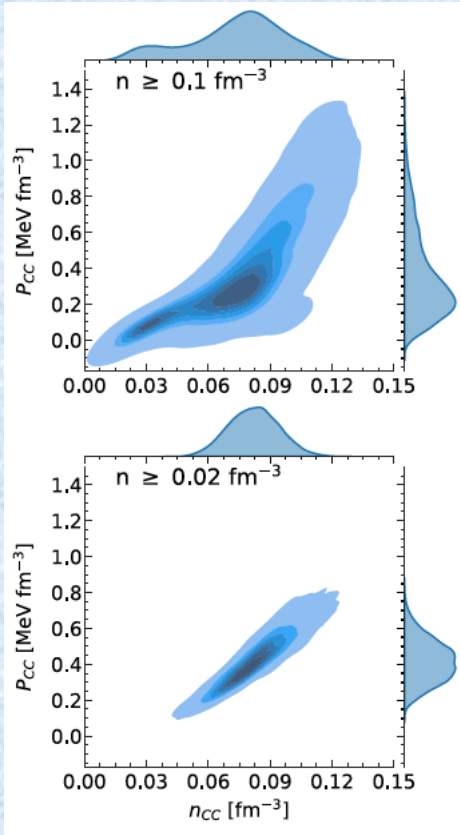


Image Credit: 3G Science White Paper

Dinh Thi et al., A&A 654, A114 (2021);
EPJA 57, 296 (2021) – non rel. MM

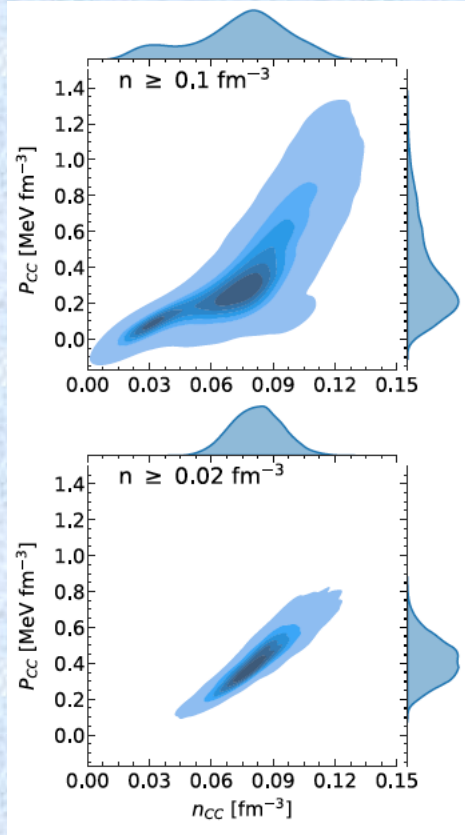
→ importance of low-density EoS

CLDM for crust



Crustal properties: cc transition

→ important e.g. for NS cooling, rotational evolution of pulsars, pulsar glitches, ...



n_{cc}	-0.09	-0.05	0.15	-0.11	0.00	-0.34	-0.69	-0.22	0.55	-0.16	0.10	0.43	-0.17	-0.10
P_{cc}	-0.05	-0.02	0.13	-0.03	-0.03	-0.07	-0.62	-0.38	0.41	-0.04	0.05	0.12	-0.02	-0.05
$R_{crust}^{1.4}$	0.09	-0.18	0.19	0.15	0.09	-0.01	-0.14	0.20	0.64	0.01	-0.09	-0.08	0.04	0.08
	E_{sat}	n_{sat}	K_{sat}	Q_{sat}	Z_{sat}	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}	σ_0	b_s	σ_c	β

Dinh Thi et al., Universe 7, 373 (2021)

bulk

surface

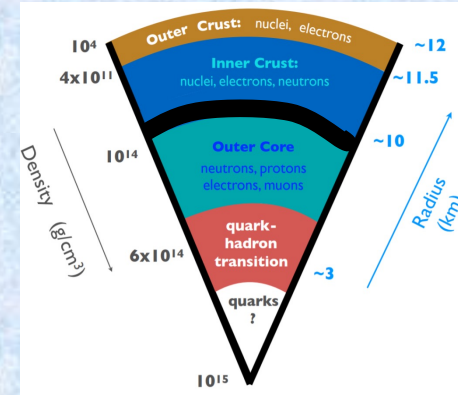


Image Credit: 3G Science White Paper

Dinh Thi et al., A&A 654, A114 (2021);
EPJA 57, 296 (2021) – non rel. MM

→ importance of low-density EoS

→ importance of parameters (*bulk + surface*)

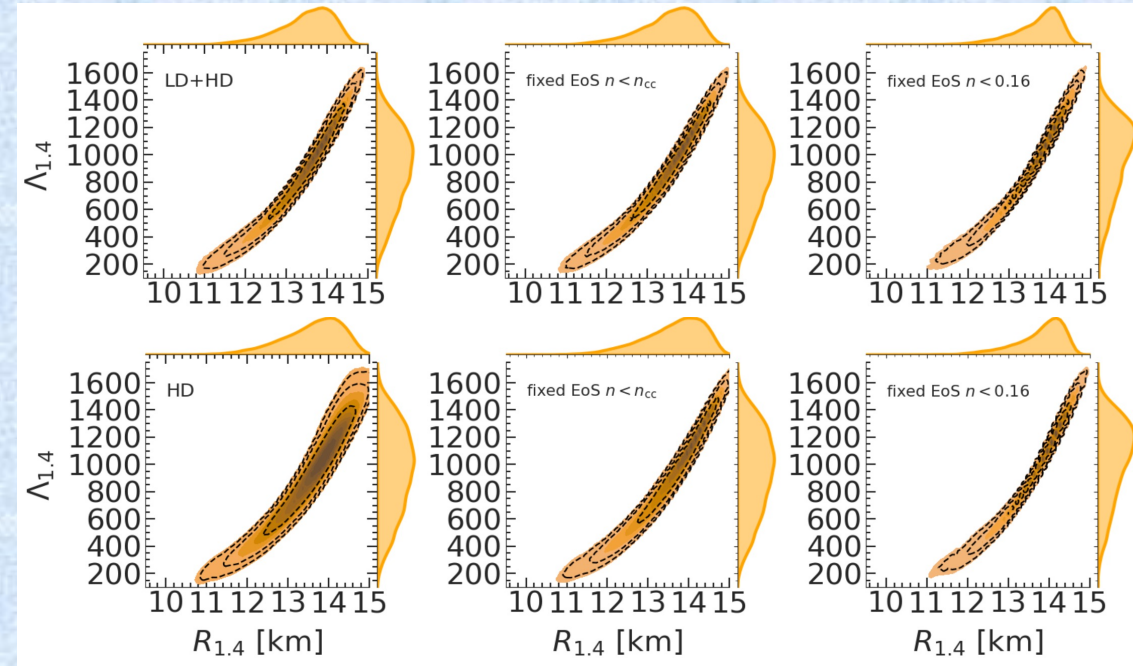
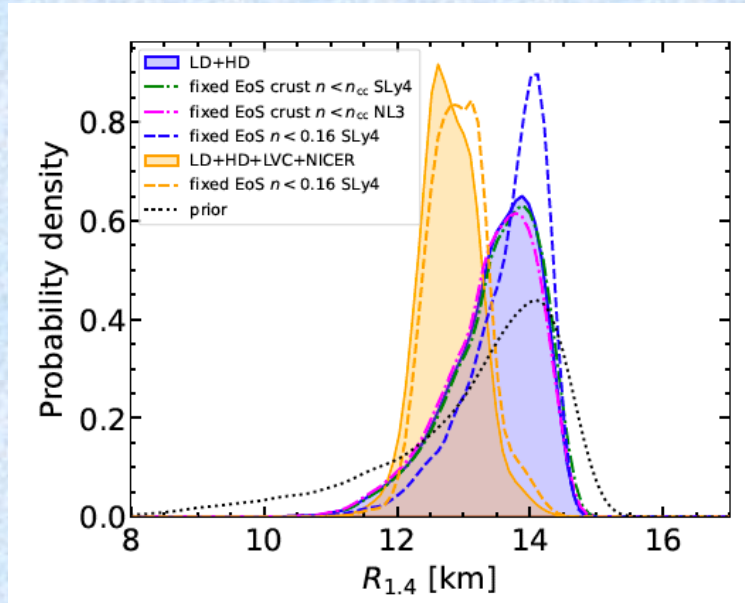
→ importance of higher order ($N > 2$) parameters

CLDM for crust



Effect of the (non-unified) crust (1)

RADIUS & TIDAL DEFORMABILITY



Davis, Dinh Thi, Fantina, Gulminelli, Oertel, Suleiman, A&A 687, A44 (2024)

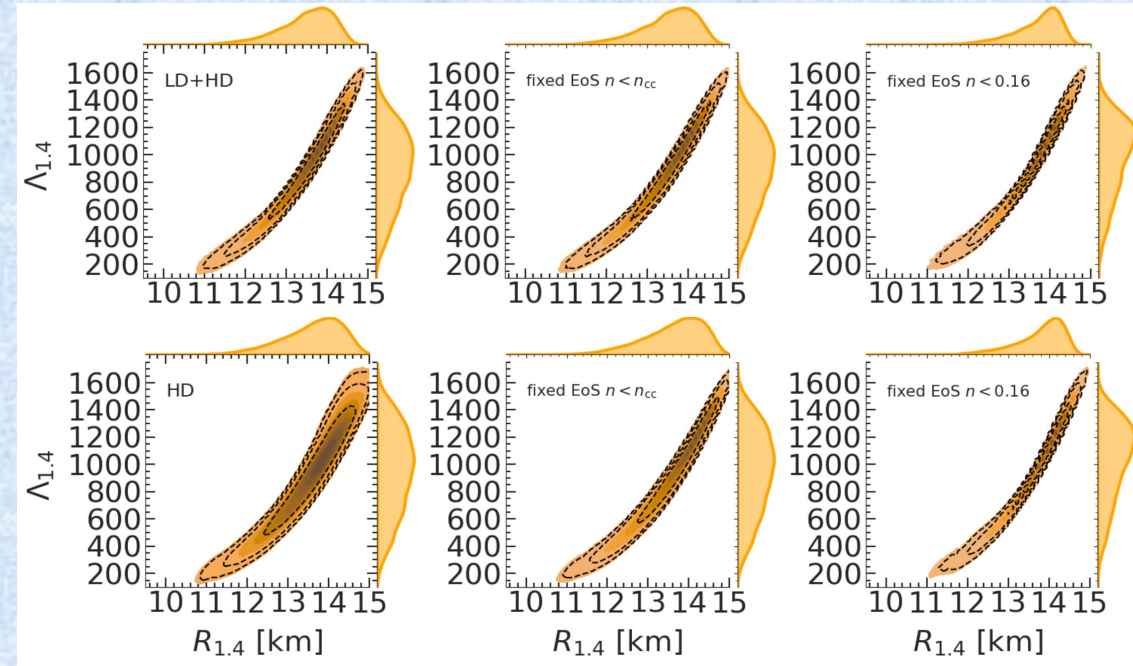
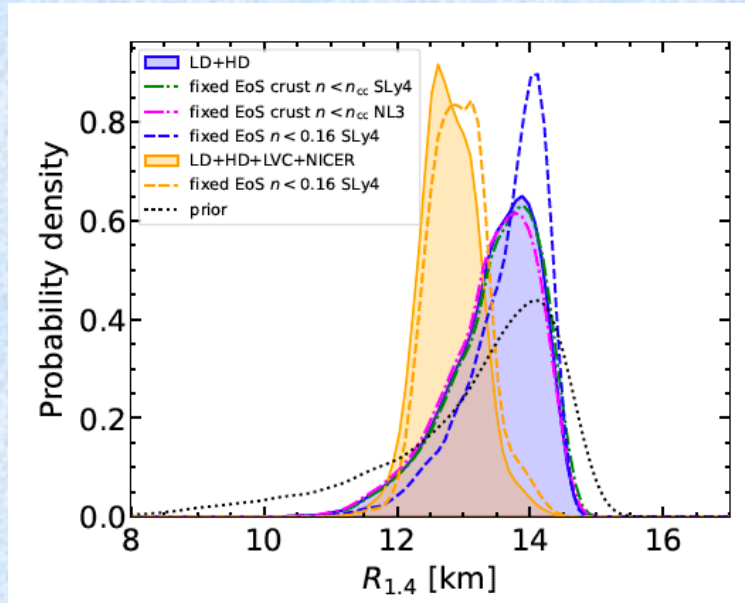
Fantina & Gulminelli, PoS (in prep.)

- use of unique low-density EoS does not change much averages (\sim few %)
 - (see also Gamba et al., *Class. Quant. Grav.* 37, 025008 (2020) \rightarrow \sim 3%)
 - \rightarrow ok if “reasonable” crust applied until cc transition,
 - \rightarrow ok for current GW detectors, but next generation ?
- filters from GW (& NICER) shift distribution (softer EoS) \rightarrow effect of unique crust reduced



Effect of the (non-unified) crust (2)

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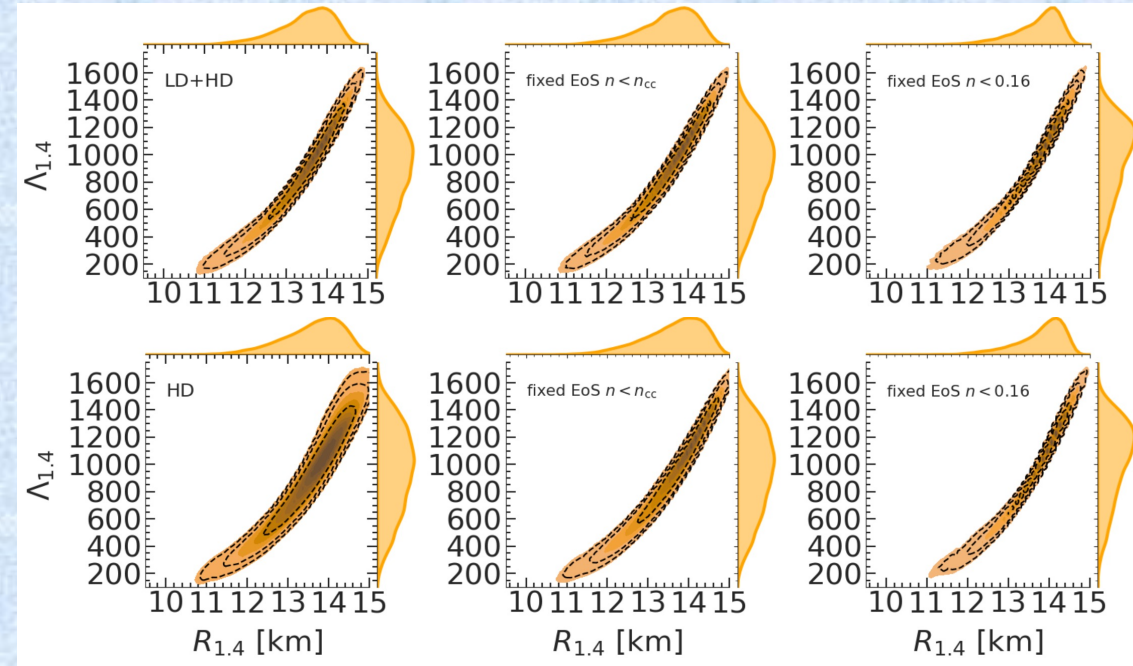
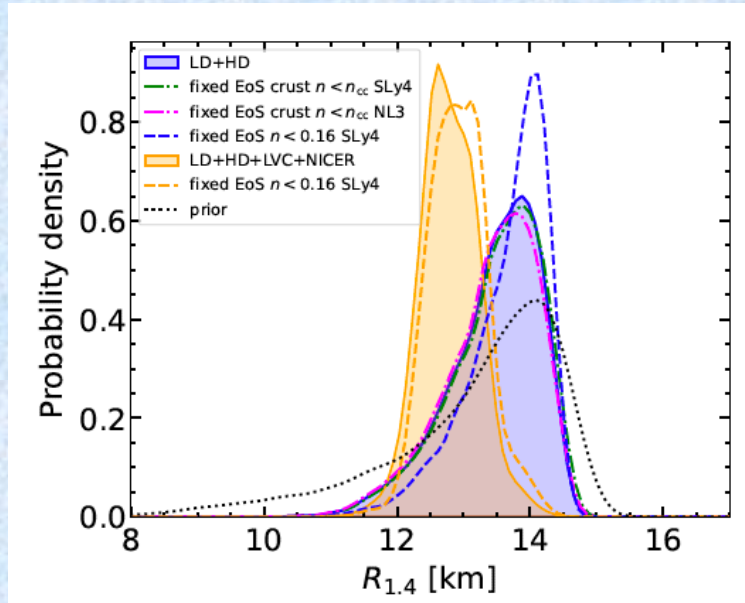
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- filters from GW (& NICER) shift distribution (softer EoS) \rightarrow effect of unique crust reduced
- underestimation of uncertainties in non-consistent approach, especially if no LD filters



Effect of the (non-unified) crust (3)

RADIUS & TIDAL DEFORMABILITY



Davis, Dinh Thi, Fantina, Gulminelli, Oertel, Suleiman, A&A 687, A44 (2024)

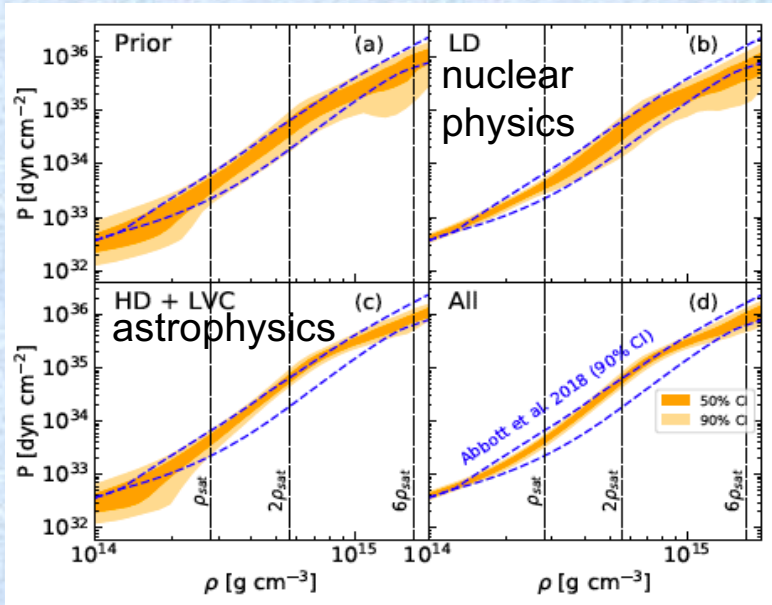
Fantina & Gulminelli, PoS (in prep.)

→ **CUTER code** to reconstruct a *thermodynamically consistent and unified* low-density EoS from a given (high-density) β -equilibrium EoS (non necessarily based on a nuclear model) (available for LVK collaboration, v1 publicly available on Zenodo)

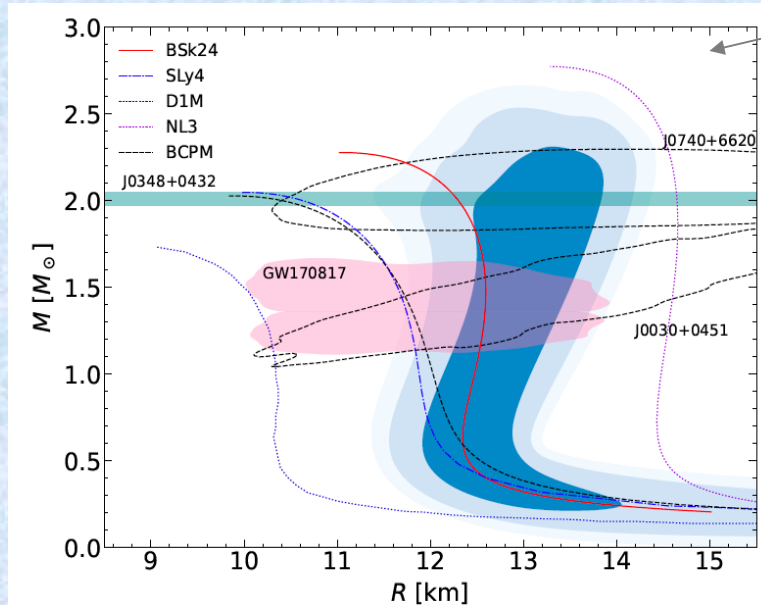
CUTER = Crust Unified Tool for Equation-of-state Reconstruction



EoSs (informed by nucl. phys.) and NS observables

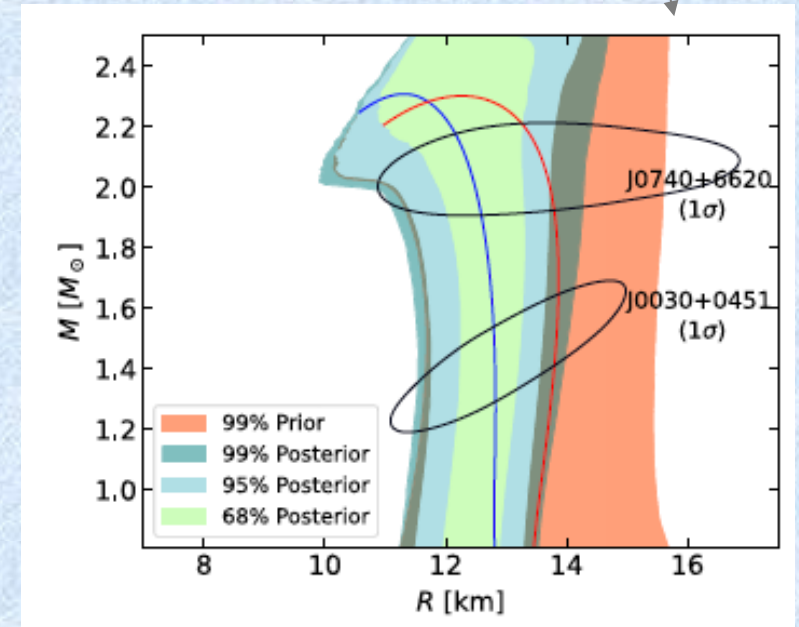


Dinh Thi et al., Universe 7, 373 (2021);
Dinh Thi et al., A&A 654, A114 (2021)



Gulminelli & Fantina, NPN 31, 9 (2021);
Fantina & Gulminelli, J.Phys. Conf.Ser. 2596, 012112 (2023)

Non-relativistic and relativistic MM give compatible predictions !



Char et al., PRD 108, 103045 (2023)

→ posterior compatible with observations, but: some models are not !

→ nucleonic hp compatible with observations → observations not yet constraining enough!

similar conclusions in Lim&Holt, EPJA 2019, Malik et al., ApJ 2022

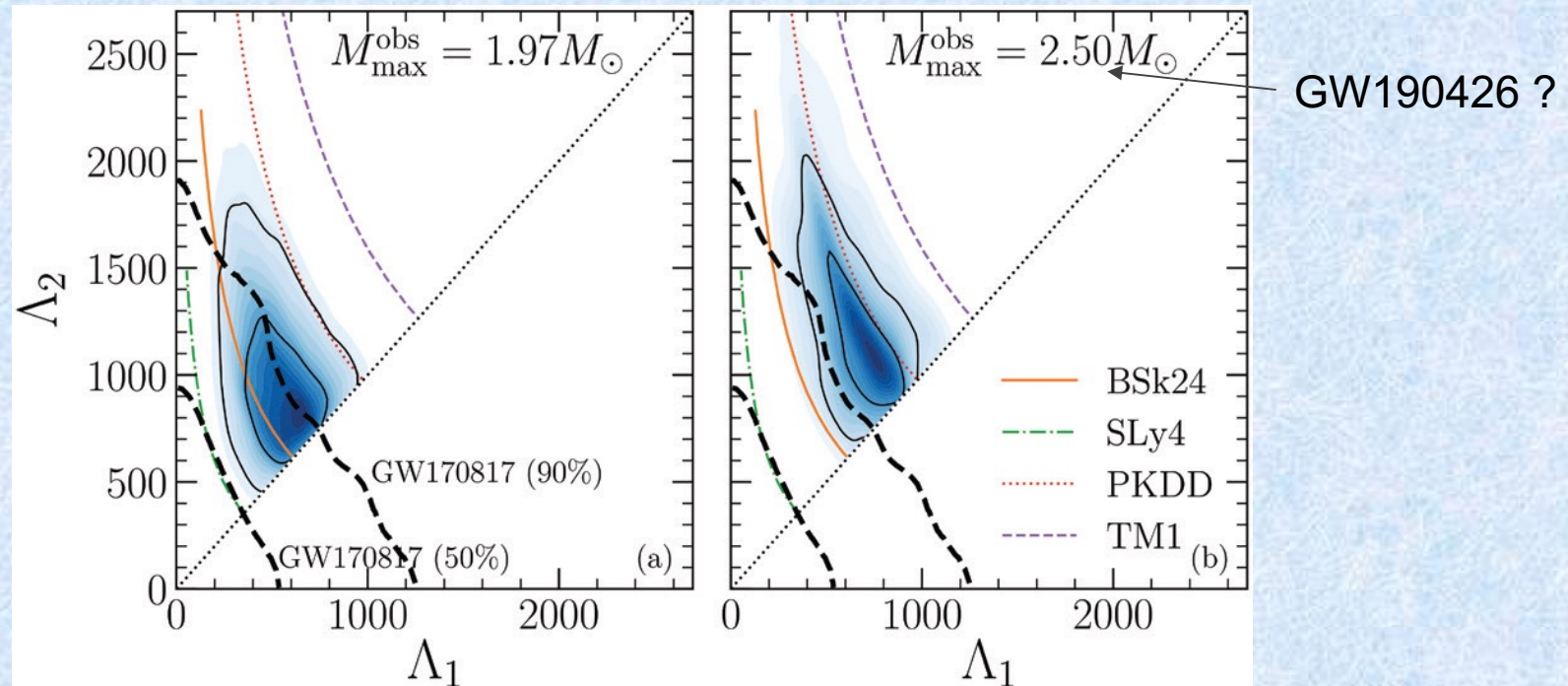
N.B.: Several works within Bayesian analysis trying to constrain NEP / EoS

see also Beznogov & Raduta, PRC (2023); Ghosh et al., EPJA (2022); Char et al., PRD (2023); Imam et al., PRD (2024); Zhu et al., ApJ (2023); Huang et al., arXiv:2303.17518,



How to discriminate models? (astro1)

- more and more precise data (e.g. M , R , Λ , ...) – or even a “smoking gun” observation!



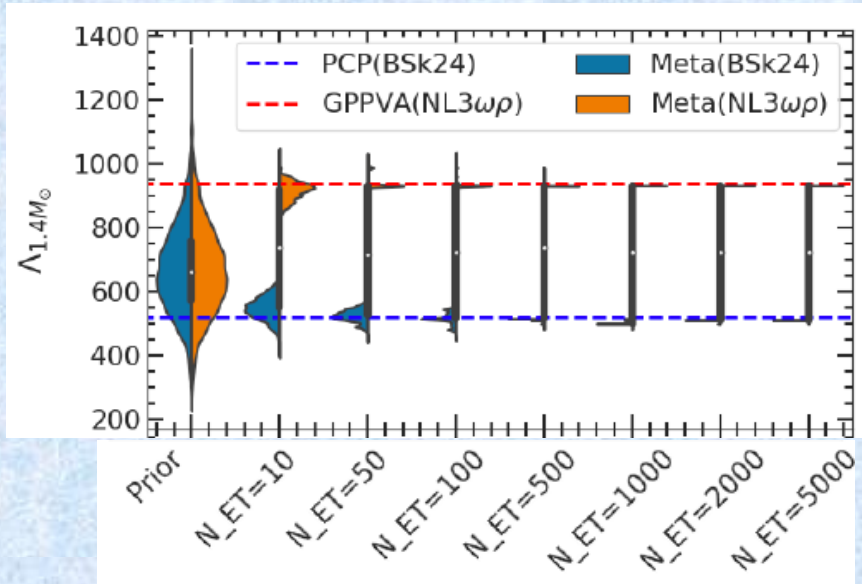
Gulminelli & Fantina, Nucl. Phys. News 31, 2 (2021); T. Carreau, PhD Thesis (2020)

- posterior with nucleonic matter compatible with observations
- but: if $M_{\max} \sim 2.5 M_{\text{sun}}$ → challenge for nucleonic hypothesis ! → “exotica” ?
- Nucleonic hp can be used as null hp!

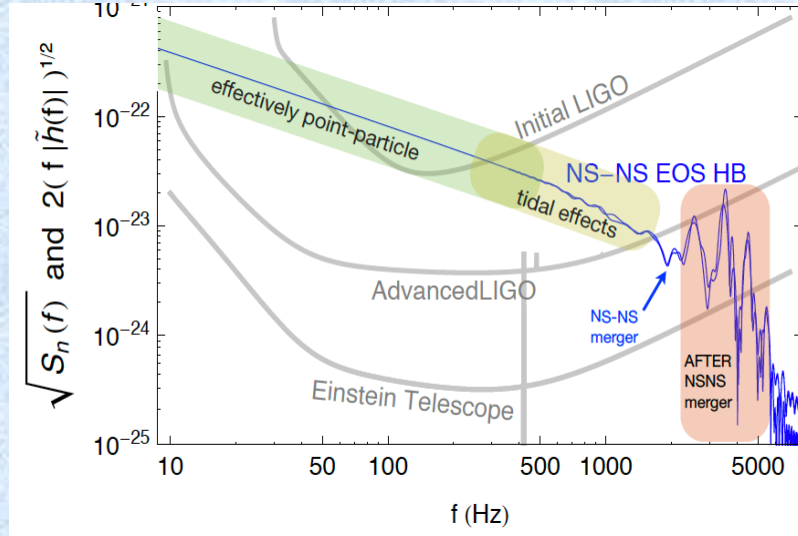


How to discriminate models? (astro2)

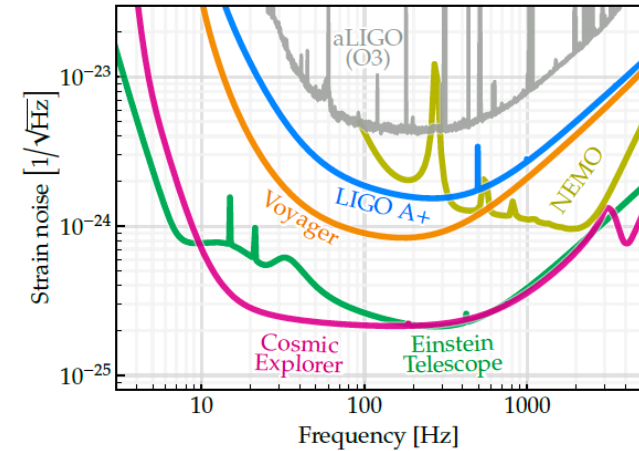
- more and more precise data (e.g. M , R , Λ , ...) – or even a “smoking gun” observation!
- more sensitive detectors → new generation (ET, CE) → post-merger



Iacovelli et al., PRD 2023



Read, CGWAS lecture (2015)



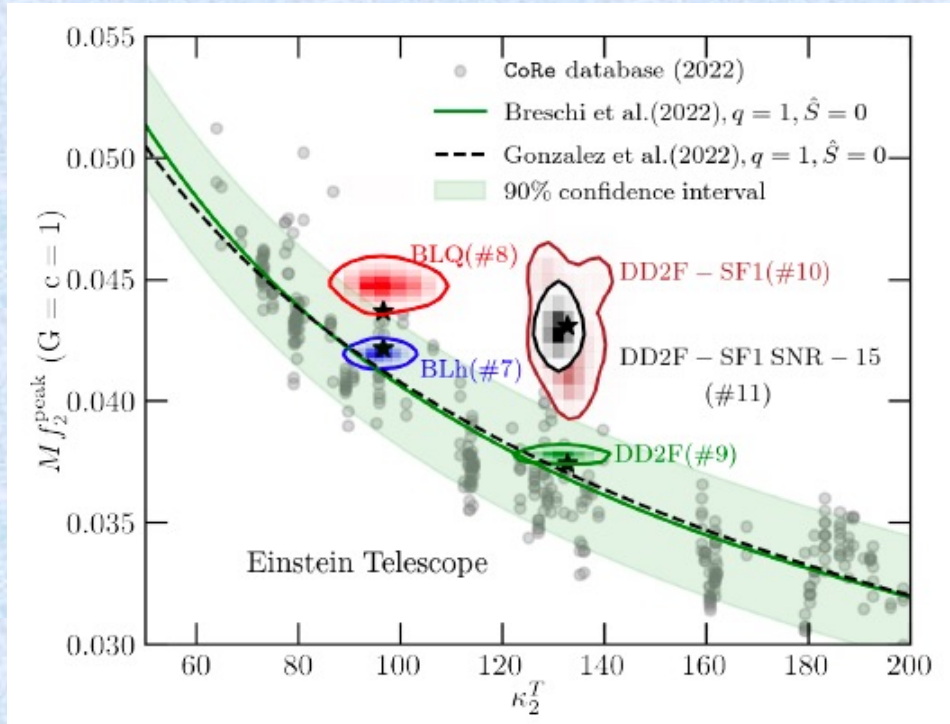
Hall, Galaxies 10, 90 (2022)

- More reliable prediction / interpretation of astrophysical observations
- Better knowledge of dense matter in compact stars → *Phase transition to deconfined matter (quarks, ...)* ?

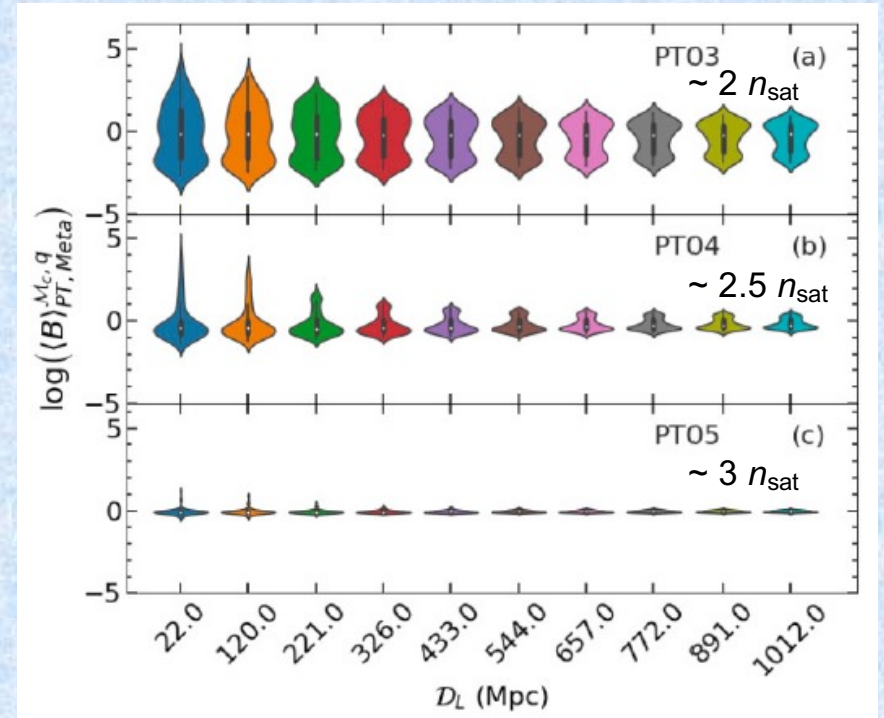
Additional d.o.f. – *Phase Transition to deconfined matter?*

➤ observed discrepancy from (quasi-)universal relations

➤ identification if close detection and early PT



Prakash et al., PRD 109, 103008 (2024)



Mondal et al., MNRAS 524, 3464 (2023)

- if phase transition not strong or “late” → hard to disentangle from other uncertainties
- post-merger signal observations needed → ET / CE!



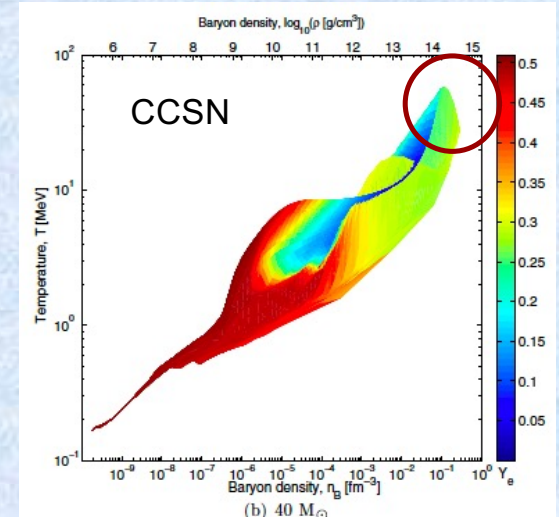
Finite T in BNS mergers / CCSN

- ✓ Finite T , high density met in **CCSN** and **BNS mergers (post-merger)**
→ additional degrees of freedom ? Effect on dynamics ?

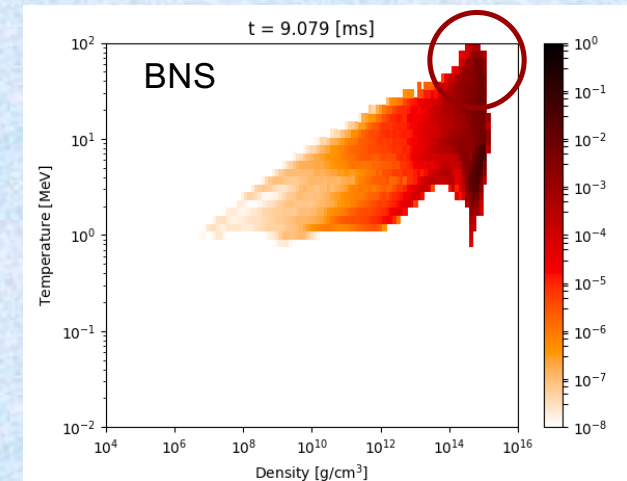
Temperature	$0 \text{ MeV} \leq T < 150 \text{ MeV}$
Baryon number density	$10^{-11} \text{ fm}^{-3} < n_B < 10 \text{ fm}^{-3}$
Electron fraction	$0 < Y_e < 0.6$

- ✗ Consistent treatment of phase transitions challenging
- ✗ Extension of many-body methods and extrapolation of predictions not trivial (e.g. parameters of nuclear phenomenological models usually fitted at $T=0$)
- ✗ Hydro effects (SASI, convection, etc...), B field, ...
→ see e.g. talks by T. Foglizzo, M. Bugli, A. Reoul-Salze

- Need of a *unified* “general purpose” EoS (and composition) in very wide thermodynamic conditions
- Implementation in numerical simulations



Fischer et al., ApJSS 194, 39 (2011)



Perego et al., EPJA 55, 124 (2019)



Conclusions & open questions

- ❖ Nuclear physics + astrophysics → constraints on EoS but still hard to discriminate
→ nucleonic hp compatible with observations, no (dis)proof of exotic matter to now → future observations?
 - ➔ ✓ need of (microscopic) reliable theoretical model when no data & experimental data to calibrate models
 - ✓ need of (more precise / numerous) astrophysical observations
 - ❖ **CUTER** tool for consistent crust EoS reconstruction available
-
- Extrapolation from raw data → model dependence of the constraints
 - Nuclear physics gives constraints up to $\sim 1.5n_{\text{sat}}$ → predicted observable quite robust (with uncertainties)
→ possibility to explore new physics ?
 - Uncertainties in high-density EoS → blurring of different effects ?
 - Astro simulations vs microphysics inputs → uncertainties, consistency of inputs, relative effects of microphysics inputs in astro modelling ?
 - ❖ systematic studies / bayesian analysis → implementation of consistent probability distributions in astro prediction/inference
 - ❖ static properties → if GR, possible “extraction” of EoS (with uncertainties), composition?
dynamical properties → need of (multi-D) hydro simulations
→ computationally challenging, dependence on different inputs



Thank you