A parametric study of neutron star electrospheres

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Outline

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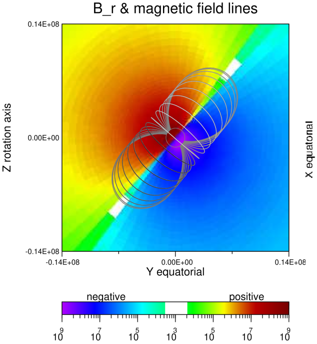
Electrospheres, input physics and definition

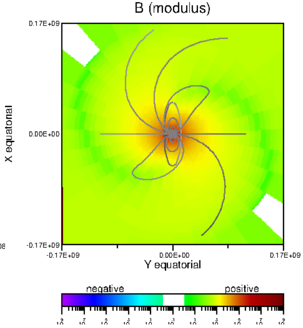
The fundamental structure of an electrosphere through past and present simulations

Influence of various parameters

Data for a neutron star (NS) magnetosphere simulation

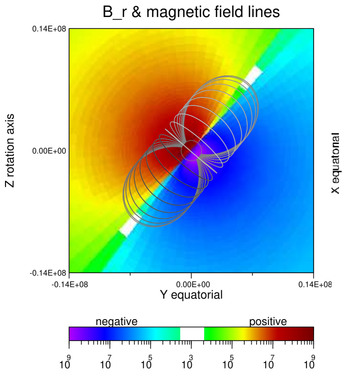
- Neutron star $\leftarrow$ a rotating sphere, with a strong magnetic field, and a surface with a large electrical conductivity.
- The surface magnetic field $B_r(R_*, \theta, \phi)$ is dipolar, with an inclination i to the axis of rotation.
- A surface corotation electric field $\vec{E} = -\vec{v}_\phi \times \vec{B}$.
- The electromagnetic field is non-trivially expanded into vacuum (analytical solution) [Deutch, Petri, Bonazzola+]
- A central electric charge, and the charges surrounding the NS are the other causes of electromagnetic fields.

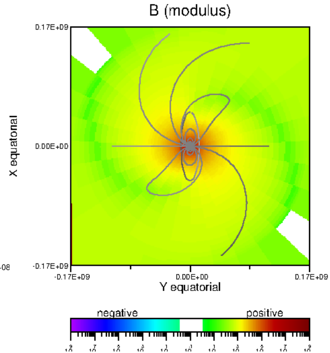




At the initial stage of the simulation, the NS surface magnetic field is expanded into vacuum. Left: $i = 45^o$, mid distance. Right: $i = 75^o$ inside and beyond the light cylinder.

Particles and radiation in a nutshell



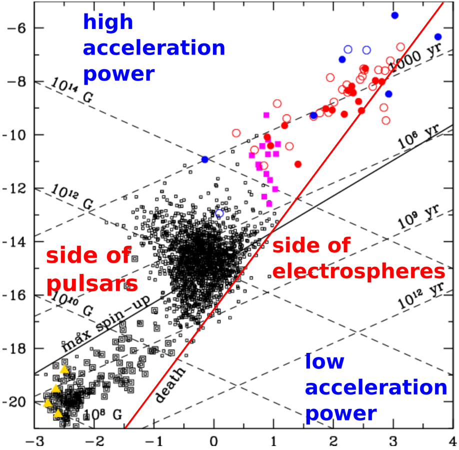


At the initial stage of the simulation, the NS surface magnetic field is expanded into vacuum. Left: $i = 45^o$, mid distance. Right: $i = 75^o$ inside and beyond the light cylinder.

- The surface is hot, $T \sim 10^6$ K, possibly covered by a thin atmosphere of light gas. Electrons and ion can leave the NS surface freely.

- The strong electric fields feed the NS environment with highly energetic particles.
- Particle can reach high Lorentz factors and radiate gamma rays.
- A significant proportion of the particle energy is dissipated by radiation.

Electrospheres: a few commonplace conjectures



Observed pulsars in the period P and period derivative $\dot{P}$ diagram. Top : stronger magnetic fields. Left : higher spin rates.

- Pulsar = solution with electron-positron pair cascades.
- Electrosphere = solution without pair cascades.
- Electrospheres have tenuous plasma, weak electric currents, they do not radiate strongly.
- Electrospheres are not observed.
- Electrospheres occur with neutron stars with low magnetic field and low spin rate ($\sim$ weak energy).
- The pulsar graveyard contains electrospheres, and the huge majority of NS environments.

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Electrospheres, input physics and definition

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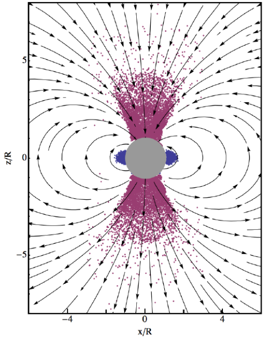
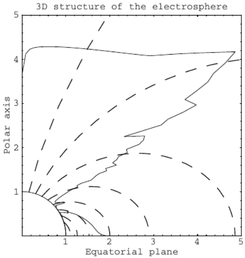
Previous simulations of electrospheres



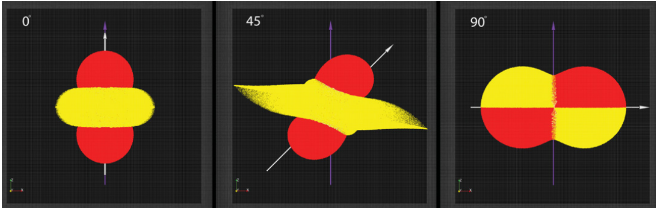


Aligned electrospheres
PIC simulation of ultrafast NS with "indeterminate" magnetic field [Spitkowski+ 2014]

Right: force-free semi analytic, more realistic and detailed study [Petri 2002]

- Dome-torus structures of opposite charges.

Previous simulations of electrospheres



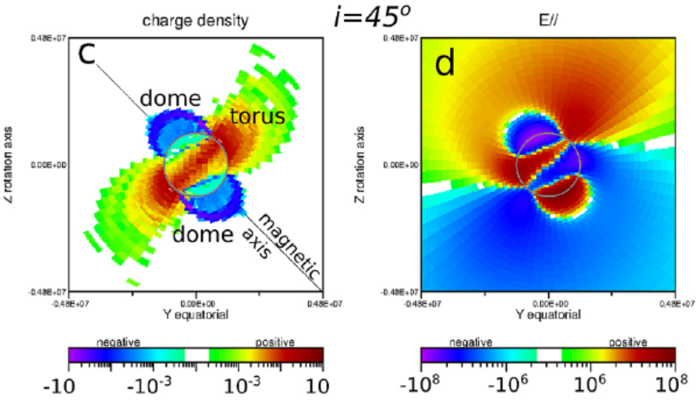
3D PIC simulations, of ultrafast NS with "indeterminate" magnetic field, and force-free inner boundary conditions [McDonald+Shearer 2009] "initial simulations", with no follow-up. Red: electron density. Yellow: positron density.

The *Pulsar ARoMa* code

[Mottez, 2024]

- Stationary solution through iteration process.
- Each iteration include long particle trajectories ($\sim$ 10000 time steps).
- 3D spatial grid in spherical coordinates.
- $\sim$ 10 interlocking spherical shells of variable sizes.
- Inner boundary: $\Delta r \sim 1$ cm. Outer boundary: $\Delta r \sim 100$ m.
- Solve Maxwell's equations with spectral methods (very efficient if soft gradients) [Novak, Petri].
- Particles have a statistical weight depending on their initial energy.
- Trajectories: solve diff. eq. of motion with variable Δt.
- Particles finite inertia $\rightarrow$ parallel motion + guiding center drifts.
- Energy loss by curvature radiation [Vigano 2015] for radiation reaction.
- No force-free hypothesis (boundaries, motion, EM field).

Une electrosphère trop choupi



Inclination $i = 45$ deg, $R = 12$ km, and $Q_b = Q_c$ and $B_1 = 10^9$ G, $P = 10$ ms.

- Two electron domes aligned with the magnetic axis.
- A proton belt.
- The solution is not force-free.

Non exhaustive table of simulations 1/2

The plots and data sheets can be found in the repository XXX. Options: A: no radiation braking, M: multipolar magnetic field, G: two simulations with different grids, *: unrealistic parameter set

P s	B_1 G	R km	Q/Q_c	i deg	
0.01	10^9	12	1	1.	
0.01	10^9	12	1	0.	
0.01	10^9	12	1	0.1, 0.2, 0.4	
0.01	10^8	12	0.8	0	
1.	10^{10}	12	0.8	0	
0.01	10^9	12	0.	90	
1.	10^{11}	12	0.	90	
0.010	10^7	12	0.8	45	
1.	10^8	12	0.5, 0.8	45	
1.	10^{10}	12	0.8	45	
0.010	10^9	1.2	1.5	-	M modes 32 & 10
0.100	10^{10}	1.2	1.5	-	M modes 32 & 10
0.010	10^8	1.2	1.5	-	M modes 32 & 10
1.000	10^{10}	1.2	1.5	-	M modes 32 & 10
1.	10^{10}, 10^{11}	120	0.	90	*
0.1	10^9	12	0., -0.2, -0.5	90	
0.0001	10^5	0.001	0.	0	*
0.001	10^3	1200	1.3	0	*
0.100	10^3	1200	1.3	0	*
0.010	10^9	1.2	0.0	90.	G

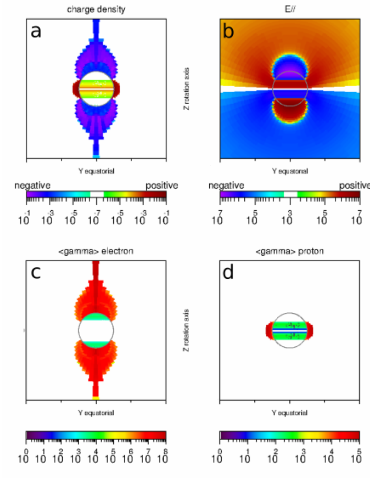
Non exhaustive table of simulations 2/2

Options: A: no radiation braking, M: multipolar magnetic field, G: two simulations with different grids, *: unrealistic parameter set

0.010	10^8	1.2	0	-	M mode 33
1.	10^{10}	1.2	0	-	M mode 33
0.100	10^8	1.2	0	-	M mode 33
1.000	10^8	1.2	0	-	M mode 33
0.010	10^9	1.2	0.8	0.	
1.	10^{10}	12	0.8, 0.2	0	
1.	10^8	12	0.8, 0.5	45	
5.	10^{11}	12	0.5	75	
0.01	10^6	12	0.8	45	
0.01	10^8	12	0	90	A
0.01	10^8	12	0	90	A
0.01	5.10^7	12	0.5	45	
0.01	5.10^7, 10^8	12	0.8	45	
0.1	10^9	12	0.8	0	
0.1	10^9	12	0, 0.5, 1	90	
0.0001	2.10^9	12	0.5	0	*
1.	10^9	12	0.5	0	
1.	10^{10}	24	0.8,0.5	45	
0.01	5.10^7	24	0.8	0	
0.01	5.10^7	24	0.8	45	
1.	10^{11}	120	0.	90	G*
0.01	10^7	120	0.8	0	*
0.01	10^8	120	0.	90	*, A*
0.01	10^9	120	0.2	0	*

Outline

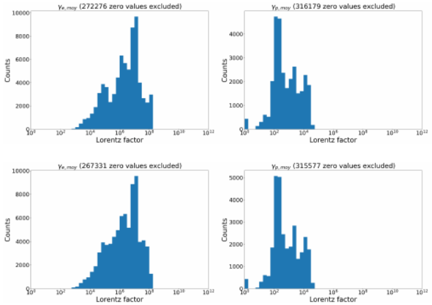
Most decisive parameters



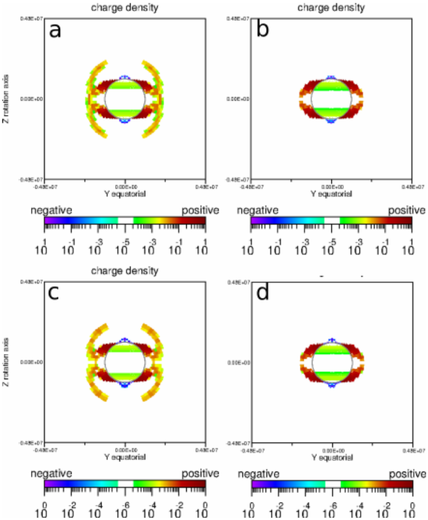
$R = 12$ km, and $Q_b = 0.8Q_c$ and $B_1 = 10^8$ G, $P = 10$ ms, or $B_* = 10^{10}$ G, $P = 1$ s. Same product ΩB, same electrospheric solution.

Parameters by range of importance:

- The magnetic field configuration: dipole inclination i, multipoles...
- The total electric charge Q/Q_c inside the NS (imposed + self consistent evolution).
- The product ΩB_1 rotation rate times surface magnetic field amplitude.
- NS radius R and last... B_1 or Ω.



Multipole magnetic field: ΩB_* is also a primary parameter

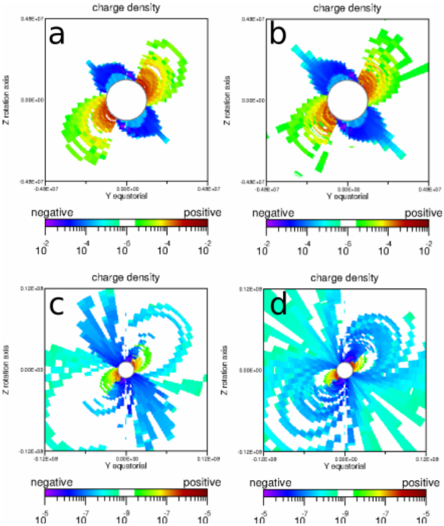


$B_r/B_* = Y_1^0(\theta,\phi) - Y_3^2(\theta,\phi)$. (a) $B_* = 10^9$ G, $P = 10$ ms, (b) $B = 10^{10}$ G, $P = 100$ ms, (c) $B_* = 10^8$ G, $P = 10$ ms, (d) $B = 10^{10}$ G, $P = 1$ s.

Parameters by range of importance:

- The magnetic field configuration: dipole inclination i, multipoles...
- The total electric charge Q/Q_c inside the NS (imposed + self consistent evolution). Q_c is corotation charge.
- The product ΩB_1 rotation rate times surface magnetic field amplitude.
- NS radius R and last... B_1 or Ω.

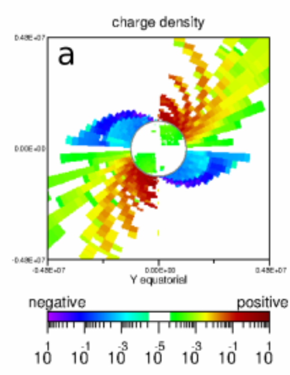
Influence of ΩB_1 and of Q/Q_c

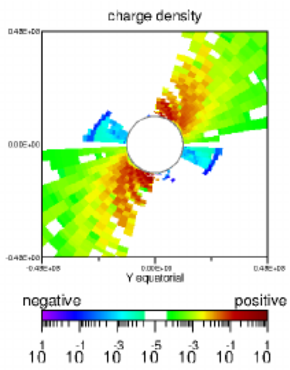


From (a) to (b) and (b) to (c), $OmegaB_1$ is divided by 10.
(a,b,c) $Q/Q_c = 0.8$, (d) $Q/Q_c = 0.5$.

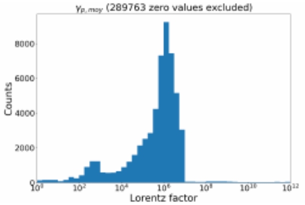
- Lower values of ΩB_1 imply larger electron domes.
- Lower value of Q/Q_c imply larger electron domes, or even less confined electrons (as suggested intuitively).

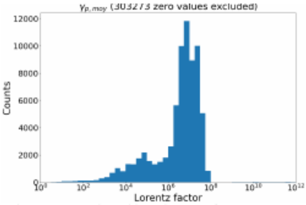
NS radius is "mainly" a scaling factor



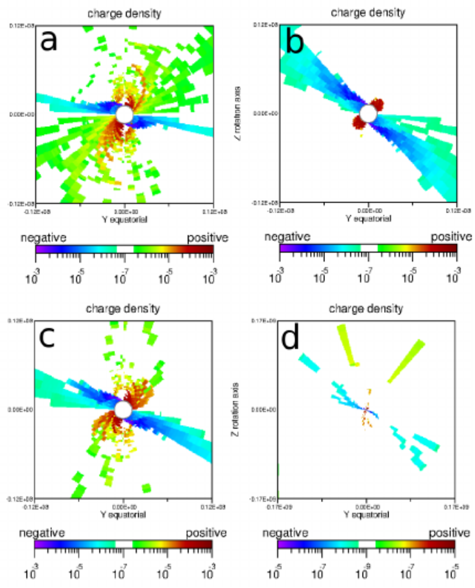


$i = 90$ deg, $B = 10^1 1$ G, $P = 1$ s, $Q/Q_c = 0$. Left: $R = 12$ km. Right: $R = 120$ km.





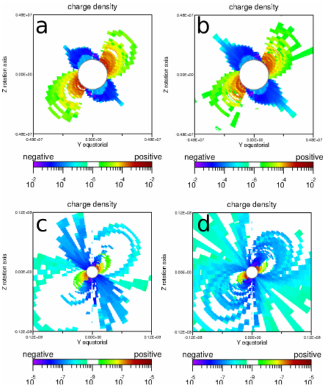
Electrospheres with particle outflows



- (a) $Q/Q_c = 0$. No outflow
- (b) $Q/Q_c = -0.5$. Electron outflow. Total charge must increase.
- (c,d) $Q/Q_c = -0.2$, two scales. Ion and electron outflow. Can be a "long term" solution with a constant total charge.

$R = 12$ km, $i = 90$ deg, $B = 10^{9}1$ G, $P = 100$ ms.

Conclusion

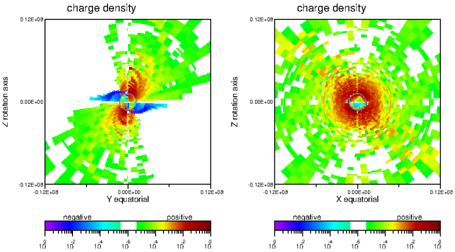


- *Pulsar ARoMa* is designed for self-consistent simulation of NS environments with realistic parameters.
- The first parametric study of electrospheres.
- The solution that we found are not force-free.
- The most influential parameters are the magnetic field configuration, the total electric charge, and ΩB_*.
- The radius is mostly a scaling factor, in the range of realistic diameters (8 to 20 km).
- We could simulate outflowing electrospheres.

Pulsar ARoMa features, pros and cons

- Stationary solution through iteration process. ☹ No guarantee of convergence.
- Each iteration include long particle trajectories ($\sim$ 10000 time steps).
- 3D spatial grid in spherical coordinates. ☺ Smooth NS surface.
- $\sim$ 10 interlocking spherical shells of variable sizes. ☺ Technical but robust
- Inner boundary: $\Delta r \sim 1$ cm. Outer boundary: $\Delta r \sim 100$ m. ☺
- Solve Maxwell's equations with spectral methods (very efficient if soft gradients) [Novak, Petri].☺ Technical but robust
- Particles have a statistical weight depending on their initial energy. ☺ Keep info about the tail of the distribution.
- Trajectories: solve diff. eq. of motion with variable Δt ☺ Tricky and robust.
- Particles finite inertia $\rightarrow$ parallel motion + guiding center drifts.☺ Tricky and robust.
- Energy loss by curvature radiation [Vigano 2015]. ☹ Risk of numerical instability.
- No force-free hypothesis (boundaries, motion, EM field). ☺ More realistic account of particle kinetic energies.

A realistic perpendicular electrosphere ($i = 90^o$)



- Neutron star radius: 12 km;
- Surface magnetic field: $B_* = 10^9$ G;
- NS rotation period: $P_* = 10$ ms;
- Magnetic inclination angle $i = 90^o$.

- **Electrons+protons : still a proton torus.**
- No radiation reaction force: a "Quad-lobe" plasma distribution. Electrons behave like protons.
- Electrons-positrons (unrealistic) : "Quad-lobe" plasma distribution as in Mc Donald [2009].

My way $\rightarrow$ ARoMa
Take benefit from the approximation of large magnetic field

There is only one component of impulsion, here Px stands for $P_{||}$ parallel to the local magnetic field.

- Very large magnetic field in NS environment.
- Synchrotron radiation kills perpendicular momentum of any particle in $\sim 10^{-17}$ s (near the NS).
- Momentum has only one component parallel to the magnetic field (classical mechanics, no QED).
- So momentum space is 1D.
- Phase space is 3D (space)+1D (momentum) = 4D. Data can be handled with present generation small computers.

```
FFF(N E MI:N E MAX,N R MIN:N R MAX,N THETA MIN:N THETA MAX,N PHI MIN:N PHI MAX,2,N ESPECES)
```

ARoMa
Take benefit from the approximation of a time invariant solution -1-

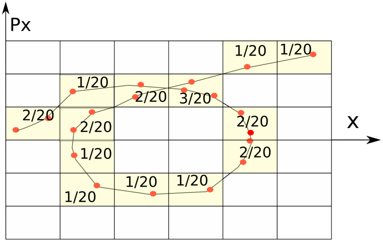
Trajectory starting from a given cell of the phase space, calculated for 20 time steps.

- An iteration corresponds to the computation of full particle trajectories.
- A particle trajectory has a maximum number `PmaxToutesParticules` of elements/time steps.
- Number of trajectory elements: $0 \leq$ `PmaxCetteParticule` $\leq$ `PmaxToutesParticules`.
- Same interpolation process (simplified actually) as in Vlasov codes.
- Simultaneous contribution of all trajectory elements.
- This is done for all the phase space cells with a value $\neq 0$.

```
TRAJECTOIRE X(0:LONGUEUR MAX TRAJECTOIRE) et LONGUEUR MAX TRAJECTOIRE= P MAX TOUTES PARTICULES

subroutine OU VAS TU TRAJ EQ UNIFIEE TOUS DEPARTS DT VARIABLE
```

ARoMa
Take benefit from the approximation of a time invariant solution -2-



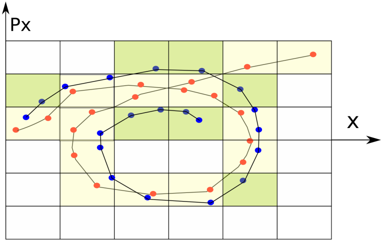
Trajectory starting from a given cell of the phase space, calculated for 20 time steps.

- The particles are not "re-centered" at the middle of a cell at each time-step: less diffusion, more precision.
- Same interpolation process (simplified actually) as in Vlasov codes.
- Simultaneous contribution of all trajectory elements.
- Trajectories start at every phase space cells containing a value $\neq 0$.

```
subroutine DISTRIBUTION EVOLUTION MILLE PATTES PLUSIEURS ECHANTILLONS
         call OU VAS TU TRAJ EQ UNIFIEE TOUS DEPARTS DT VARIABLE
                  call POUSSEUR...
         TRAJECTOIRE X...E → C2 POUR RHO JJJ, C2 POUR FFA... → FFF, RHO, JJJ
```

ARoMa

More trajectories for a better evaluation of ρ and $\vec{J}$

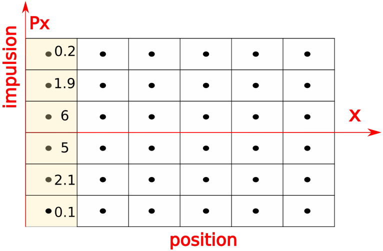


2 trajectories starting from the same phase space cell, calculated for 20 time steps.

- One point in the middle of each phase space cell may cause a lack of precision.
- For each new trajectory, choose `Npcell` $\sim$ 3-10, particles with random position inside each phase space cell. (In the spirit of PIC codes.)
- This is more costly than a standard Vlasov method, but it reduces again the noise level and diffusion.
- In 4D phase space (not 6D), it is worth the cost.
- Use the central symmetry of the physical configuration when evaluating ρ and $\vec{J}$. It is like multiplying `Npcell` by two.

`subroutine DISTRIBUTION EVOLUTION MILLE PATTES PLUSIEURS ECHANTILLONS`

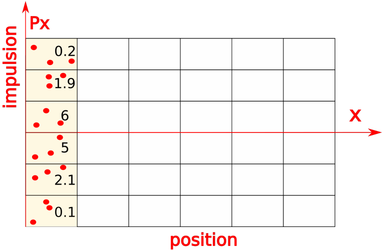
ARoMa
Take benefit from boundary conditions



The cells in yellow represent the NS surface. Other cells contain a null value.

- In electrospheres, particles come only from the star'surface (left hand side of the grid, cells colored in yellow).
- We compute only the trajectories starting from these cells.
- If `PmaxToutesParticules` is large enough, trajectories will fill the rest of the phase space.
-

ARoMa
Combine boundary conditions and Npcell particles / cell

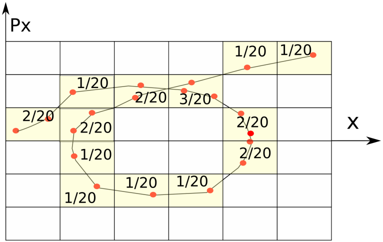


The cells in yellow represent the NS surface. Other cells contain a null value. The red dots are the starting points of the trajectories that need to be computed.

- In electrospheres, particles come only from the star'surface (left hand side of the grid, cells colored in yellow).
- We compute only the trajectories starting from these cells.
- If `PmaxToutesParticules` is large enough, trajectories will fill the rest of the phase space.
- `Npcell` trajectories start from each cell corresponding to the star'surface.

```
subroutine PREPARE CONDITIONS BORD INTERNE
        → EEE, BBB, RHO(...RAYON ASTRE...), ...FFF(...RAYON ASTRE...)
```

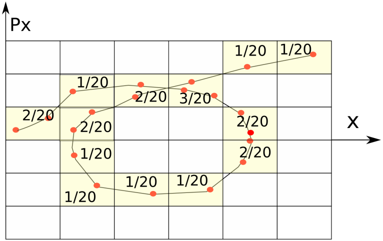
ARoMa
Mitigate the approximation of parallel particle velocities
And we see that the code is both PIC and Vlasov



Each particle trajectory has 6 components, it lies in a 6D phase space. We use it to compute ρ and $\vec{J}$ in 3D space, and $f(\vec{r}, p_{||})$ in the 4D "reduced" phase space, and then erase it from memory.

- When a trajectory is computed, 6 time series $x(t), y(t), z(t), v_x(t), v_y(t), v_z(t)$ are stored momentarily.
- Actually, there is no need to suppose $v_\perp \neq 0$.
- The sources $\rho(r, \theta, \phi)$ and $\vec{J}(r, \theta, \phi)$ are deduced from the 6 series of components ($p_\perp$ can be $\neq 0$).
- The distribution $f(x, y, z, p_{||})$ is computed, the information on $v_\perp \neq 0$ is lost for f.
- The trajectory is erased, another one can be computed.
- f will be used for particle-photon interactions; $\rho(r, \theta, \phi)$ and $\vec{J}(r, \theta, \phi)$ are used in Maxwell eqs.

ARoMa
The great advantage of this code over a standard PIC code



Each particle trajectory has 6 components, it lies in a 6D phase space. We use it to compute ρ and $\vec{J}$ in 3D space, and $f(\vec{r}, p_{||})$ in the 4D "reduced" phase space, and then erase it from memory.

- In a PIC code, all the particles have the same statistical weight.
- In ARoMa, each particle have its own statistical weight that is related to its place in the "reduced" 4D distribution function.
- Then, I keep information about the "tail" of the particle distribution = the rare particles with the highest energies.
- The tail of the distribution function is the source of gamma rays and of electron-positron pairs.