Revisiting Quantum Contextuality and Born's rule through Uhlhorn's and Gleason's theorems

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A. Auffèves & P. Grangier, Found. Phys. 46, 121 (2016) http://arxiv.org/abs/1409.2120
A. Auffèves & P. Grangier, Found. Phys. 50,1781 (2020) http://arxiv.org/abs/1910.13738
N. Farouki & P. Grangier, Found. Sci. 26, 97 (2021) http://arxiv.org/abs/1907.11267
P. Grangier, Found. Phys. 51, 76 (2021) http://arxiv.org/abs/2003.03121
A. Auffèves & P. Grangier, Entropy 24 (2), 199 (2022) https://arxiv.org/abs/2111.10758
M. Van Den Bossche & P. Grangier, F. Phys. 53, 45 (2023) https://arxiv.org/abs/2209.01463

Sketch of the talk

1. Einstein, Bohr, Bell, and the experiments : which outcome ?

* From the EPR-Bohr debate (1935) to loophole-free Bell tests (2015)
* Let us forget about Hilbert space and operators and...
  - define a (contextually) objective quantum state then …
  - deduce probabilities from quantization.

  A. Auffèves & P. Grangier, Found. Phys. 46, 121 (2016)
  http://arxiv.org/abs/1409.2120

2. Revisiting contextuality : Gleason rather than Kochen-Specker.

* Reconstructing the quantum formalism

  A. Auffèves & P. Grangier, Found. Phys. 50, 1781 (2020)
  http://arxiv.org/abs/1910.13738
  A. Auffèves & P. Grangier, Entropy 24 (2), 199 (2022)
  https://arxiv.org/abs/2111.10758
* Some other issues... and an algebraic outlook

The Einstein-Bohr debate

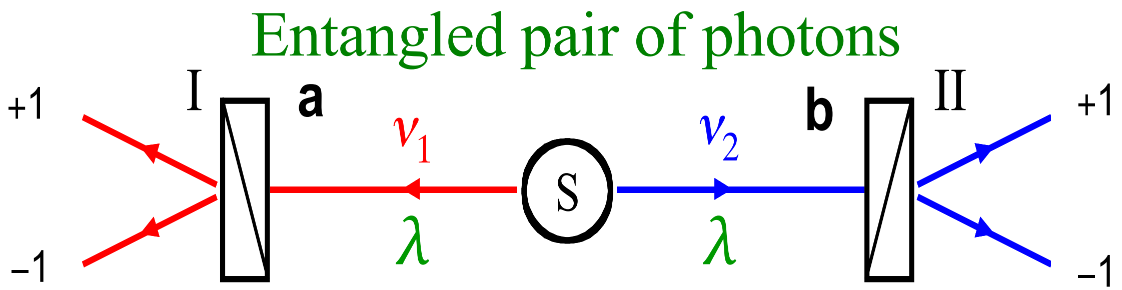
* Einstein, Podolsky, Rosen (EPR) 1935 : quantum mechanics is incomplete ("hidden information")

* Bohr disagrees, intense debate over many years but not much attention from majority of physicists

- Quantum mechanics accumulates success:
  - Understanding nature: structure and properties of matter, quantum theory of light, interactions between light and matter...
  - New concepts, and revolutionary inventions: transistor, laser…
  - No disagreement on the validity of quantum predictions, only on its interpretation: debate considered as "philosophical".

The situation changed radically with Bell' theorem (1964) and the acknowledgement of its importance (1969-82...) : One can make experimental tests of « local realism »

Bell's theorem in a nutshell...



Consider local supplementary parameters theories (in the spirit of Einstein's ideas on EPR correlations):

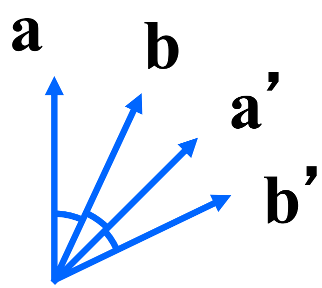
Then the two photons of a same pair have a common property λ

$$A(\lambda,\mathbf{a}) = +1 \text{ or } -1 \qquad B(\lambda,\mathbf{b}) = +1 \text{ or } -1 \qquad \rho(\lambda) \geq 0, \quad \int \rho(\lambda)\, d\lambda = 1$$

Look at the polarization correlation coefficient $E(\mathbf{a},\mathbf{b}) = (\overline{A\,B})_{\text{av.}\lambda}$ between the measurements results, then (Bell-CHSH inequalities) :

$$-2 \leq S \leq 2 \quad \text{with} \quad S = E(\mathbf{a},\mathbf{b}) - E(\mathbf{a},\mathbf{b}') + E(\mathbf{a}',\mathbf{b}) + E(\mathbf{a}',\mathbf{b}')$$

But... $S_{QM} = 2\sqrt{2} = 2.828... > 2$



Conflict !
Experiment ?

Four generations of experiments

Pioneers (1972-76): Berkeley, Harvard, Texas A&M

- Convenient inequalities: CHSH (**Clauser** Horne Shimony Holt)
- First results contradictory (Clauser = QM; Pipkin ≠ QM), but clear trend in favour of Quantum mechanics (Clauser, Fry)
- Significantly different from the ideal scheme

Experiments at Institut d'Optique by **Aspect** et al. (1977-82)

- A source of entangled photons of unprecedented efficiency
- Schemes closer and closer to the ideal GedankenExperiment
- **First test of quantum non locality (relativistic separation)**

Third generation experiments (1984-2014, many places...)

- New sources of entangled pairs (**Zeilinger** et al.)
- Separate closure of loopholes (improved locality, detection..)
- Entanglement at a large distance... towards Q. communications

Fourth generation experiments (2015 - ...)

- Simultaneous closure of all loopholes (nearly ideal expts)

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Four generations of experiments

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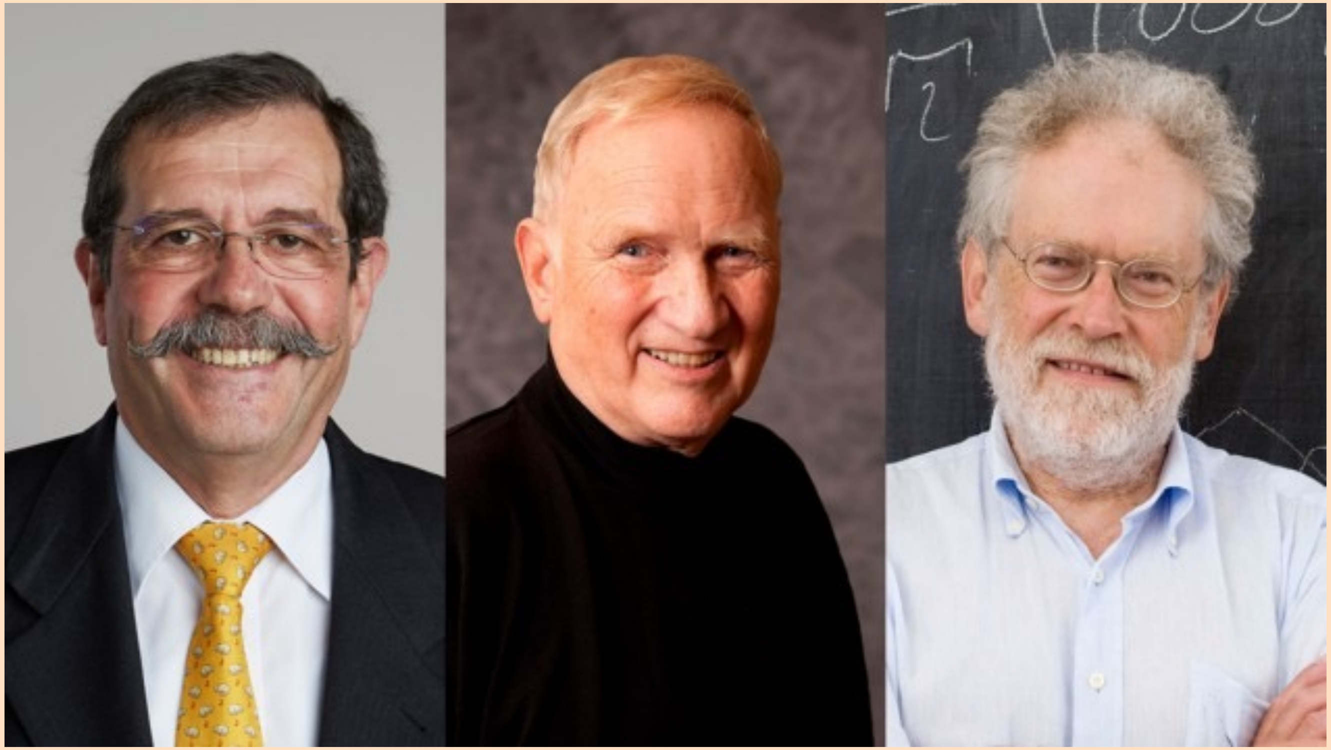


Alain Aspect

John Clauser

Anton Zeilinger

Physics ABOUT BROWSE PRESS

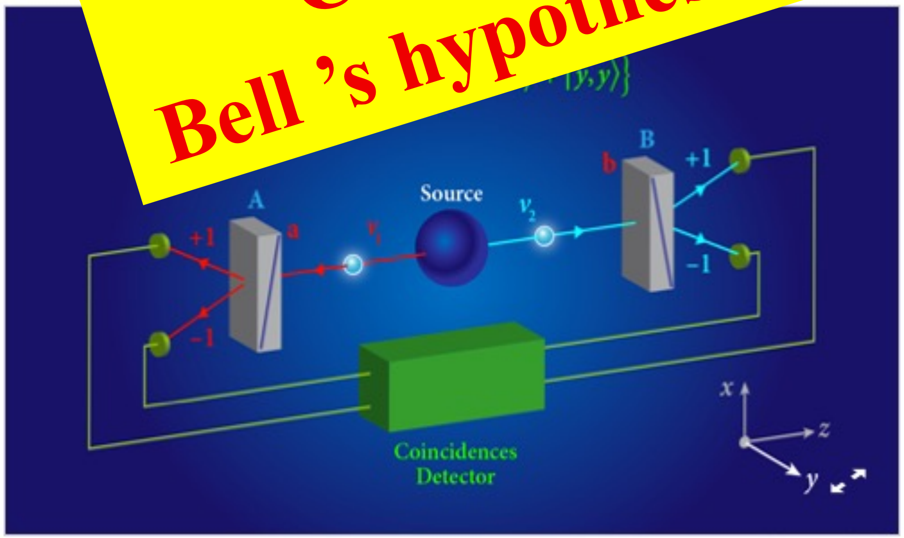
Viewpoint: Closing the Door on Einstein and Bohr's Quantum Debate

Alain Aspect, Laboratoire Charles Fabry, Institut d'Optique Graduate School, ...
Palaiseau, France

December 16, 2015 • *Physics* 8, 123

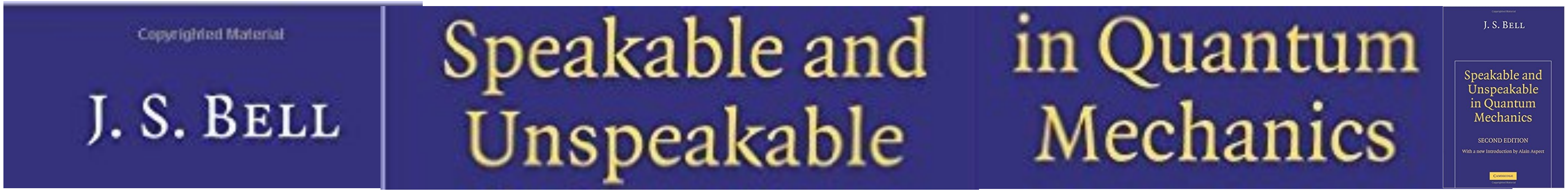
By closing two loopholes at ... s remove the last doubts that we should renoun... uantum information technologies.

Careful but unavoidable conclusion :
Bell 's hypotheses (local realism) are untenable !



APS/Alan Stonebraker

B. Hensen et al., Nature 526, 682 (2015).
"Loophole-free Bell Inequality Violation Using Electron Spins Separated by 1.3 Kilometres"
M. Giustina et al., Phys. Rev. Lett. 115, 250401 (2015).
"Significant-Loophole-Free Test of Bell's Theorem with Entangled Photons"
L. K. Shalm et al., Phys. Rev. Lett. 115, 250402 (2015).
"Strong Loophole-Free Test of Local Realism"
W. Rosenfeld et al, Phys. Rev. Lett. 119, 010402 (2017).
"Event-ready Bell test using entangled atoms simultaneously closing detection and locality loopholes"



Let us anticipate that quantum mechanics works also for Aspect. How do we stand? I will list four of the attitudes that could be adopted.

(1) ~~The inefficiencies of the counter, and so on, are essential. Quantum mechanics will fail in sufficiently critical experiments.~~

(2) There *are* influences going faster than light, even if we cannot control them for practical telegraphy. Einstein local causality fails, and we must live with this. [must be instaneous: N. Gisin et al, Nat. Phys. 8, 868 (2012)]

(3) The quantities *a* and *b* are not independently variable as we supposed. (...). . Then Einstein local causality can survive. But apparently separate parts of the world become deeply entangled, and our apparent free will is entangled with them.

(4) The whole analysis can be ignored. The lesson of quantum mechanics is not to look behind the predictions of the formalism. As for the correlations, well, that's quantum mechanics.

Philosophical standpoint

Many physicists (including me) will support **Physical Realism**, understood as :
The purpose of physics is to study entities of the natural world, existing independently from any particular observer's perception, and obeying universal and intelligible rules.

Many physicists (inc. me) look at **certain and reproducible events as real,** so we like :
If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.
A. Einstein, B. Podolsky and N. Rosen, Phys. Rev. 47, 777 (1935)

but Bell tests show that this view does not work as such... **so don't forget Bohr** :
The **very conditions which define the possible types of predictions regarding the future behavior of the system** constitute an inherent element of the description of any phenomenon to which the term "physical reality" can be properly attached.
N. Bohr, Phys. Rev. 48, 696 (1935)

What are these « very conditions » required by Bohr to speak about the physical reality of quantum phenomena ?

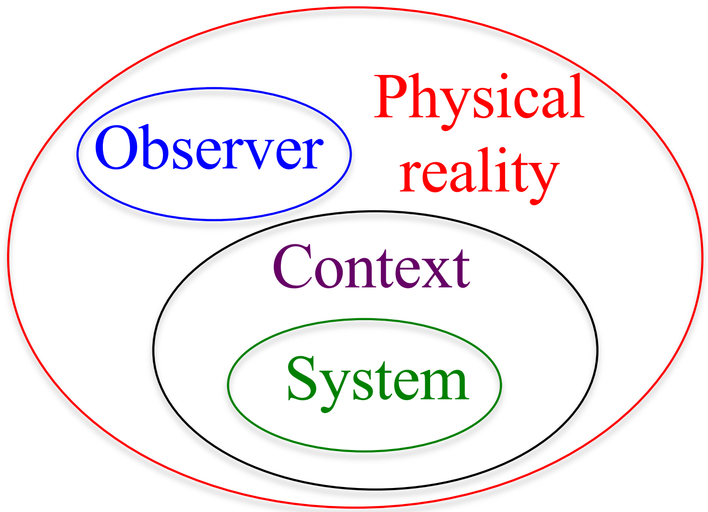
Element of physical reality vs modality

EPR

> If, without in any way disturbing a system we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element physical reality corresponding to this physical quantity.

* This statement agrees with both the **« certainty » required by Einstein and the « very conditions » required by Bohr** to make and to check definite and reproducible predictions (i.e. with objectivity, taken as contextual).

* Therefore **the « object »** carrying the element of physical reality **is a system within a context.**



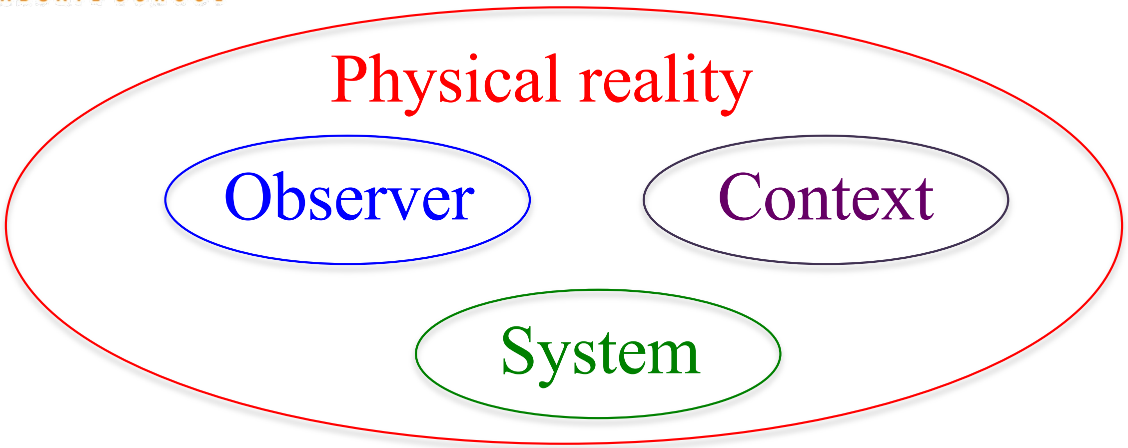
* The « split » between system and context is not a problem for CSM, because a modality is defined in terms of both the system and the context, and the system **cannot include the context.**

« Although it can describe anything, a quantum description cannot include everything »

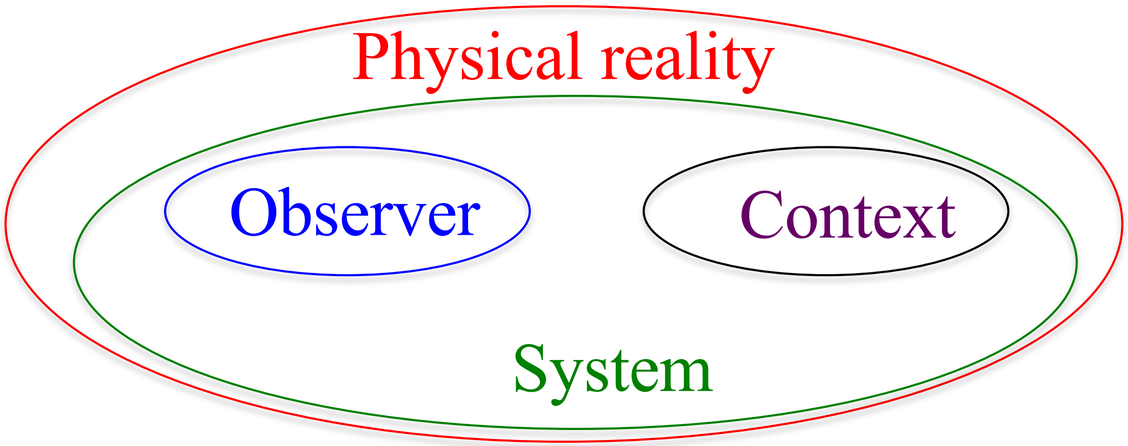
A. Peres and W. H. Zurek, Am. J. Phys. 50, 807 (1982)

Some ontology...

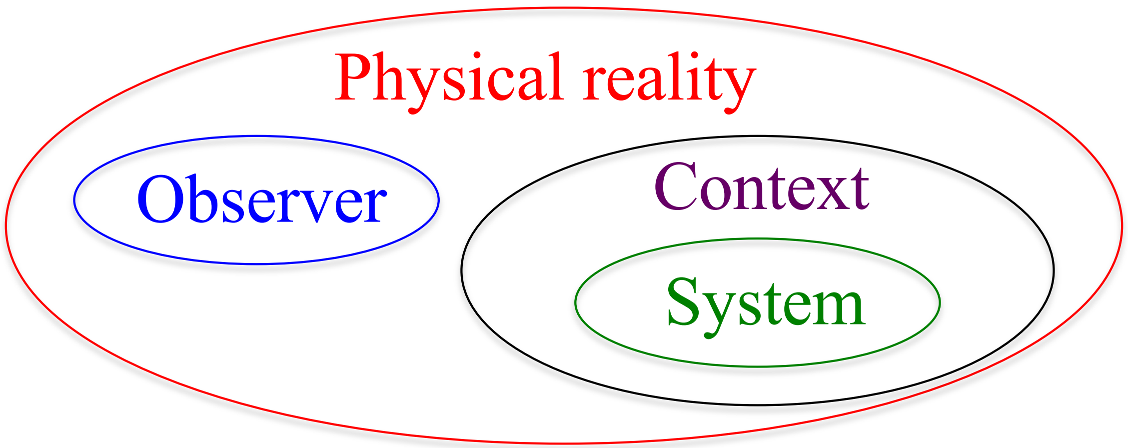
Found. Phys. 46, 121 (2016)
arxiv:1409.2120



Classical ontology :
the observer can know the "real" physical properties of the system, and the context is only used as an auxiliary tool for measurements.



Usual quantum ontology : through successive "entangling" interactions and unitary evolution, the system will include the context, and also (ultimately) the observer.



CSM ontology : the context appears always between the system and the observer, and definite values of the relevant physical properties (modalities) are attributed jointly to the system and the context.

The CSM physical axioms

Contexts, Systems and Modalities

Axiom 1 (modalities)

(i) Given a physical system, a **modality** is defined as the values of a complete set of physical quantities that can be predicted with certainty and measured repeatedly on this system.

(ii) Here "complete" means the largest possible set compatible with certainty and repeatability, for all possible modalities attached to this set. **This complete set of physical quantities is called a context, and a modality is attributed to a system within a context.**

(iii) Modalities in different contexts may be connected with and certainty (**extracontextuality**)

Axiom 2 (contextual quantization)

(i) For a given context, there exist N distinguishable modalities, that are mutually exclusive: if one modality is true, or realized, the others are wrong, or not realized.

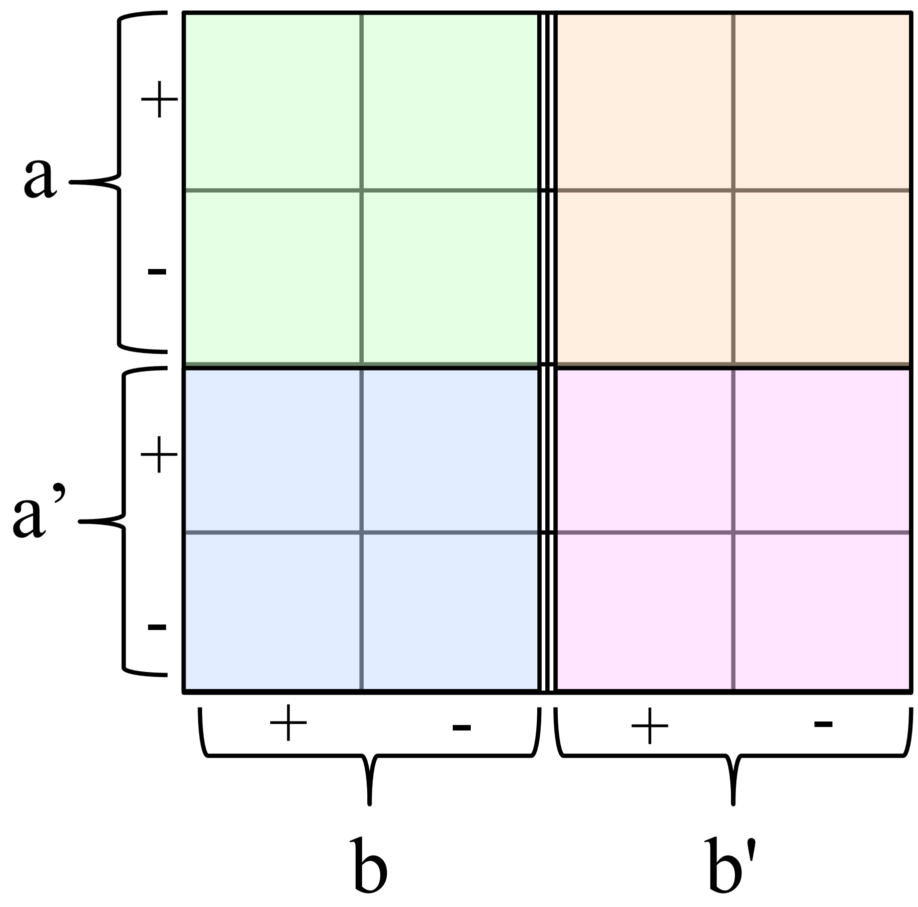
(ii) The value of N, called the dimension, is a characteristic property of a given quantum system, and is the same in all relevant contexts.

Axiom 3 (changing contexts)

Given axioms 1 and 2, the different contexts relative to a given quantum system are related between themselves by continuous transformations which are associative, have a neutral element (no change), and an inverse. Therefore **the set of context transformations has the structure of a continuous group.**

Modalities in a Bell experiment

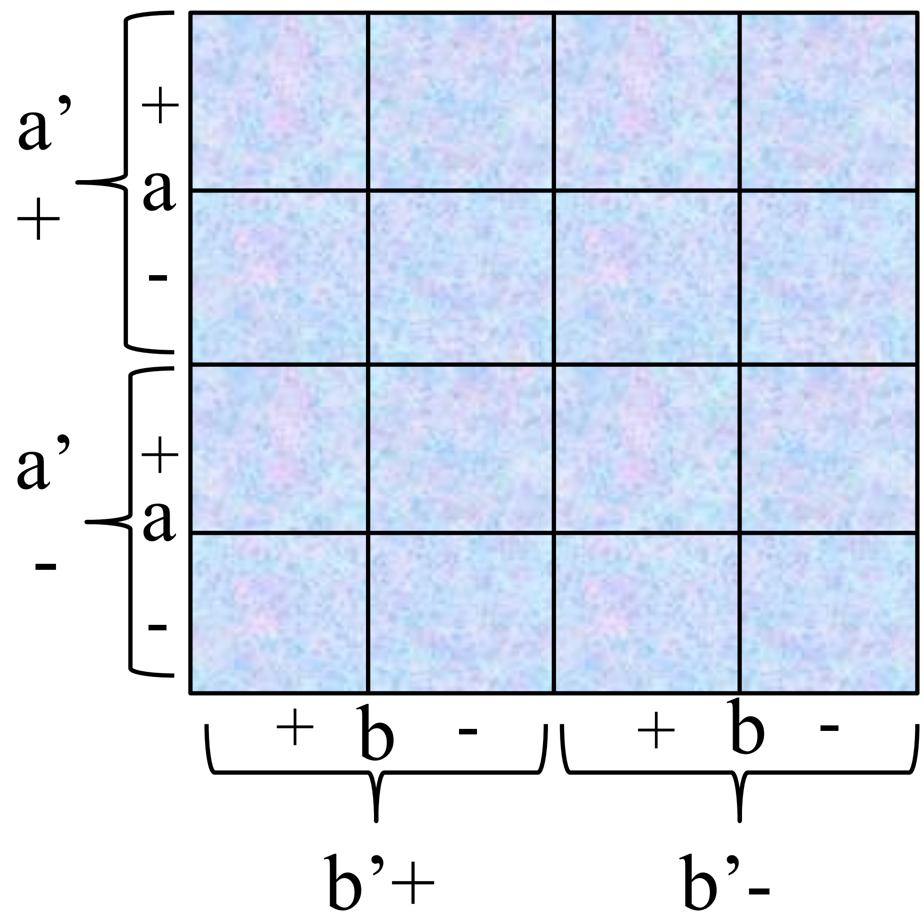
4 different contexts : MQ



N = 4 mutually exclusive modalities in each context

Violation of Bell's ineq. : agreement with expts !

Global context : classical

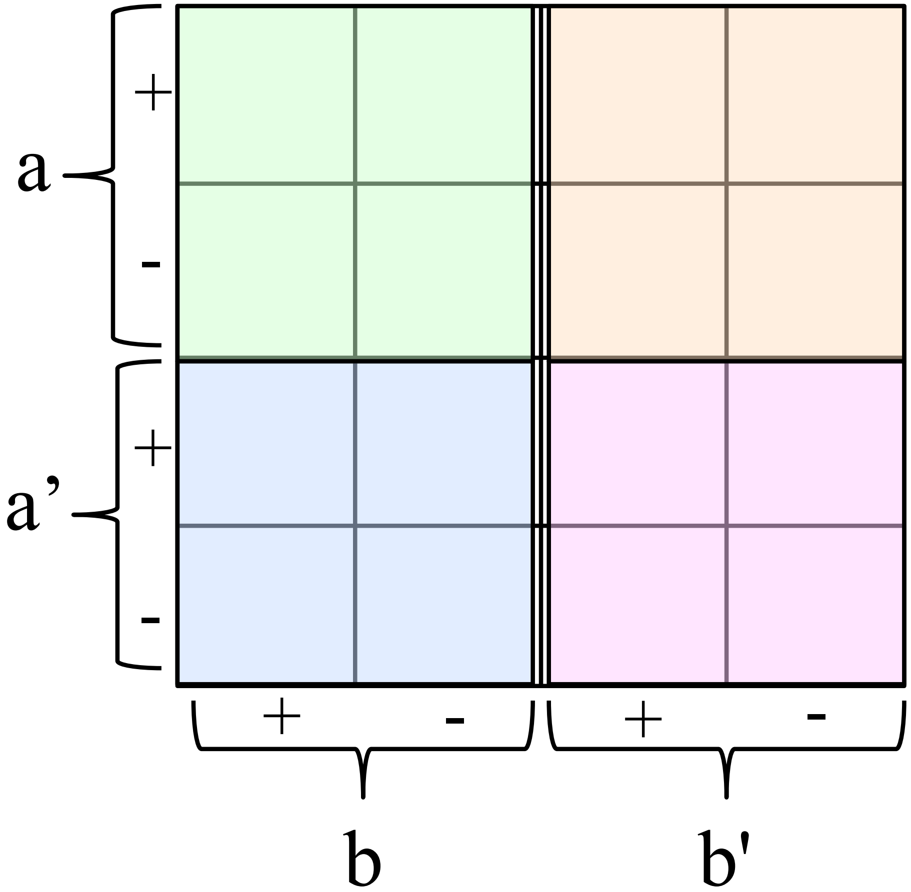


16 mutually exclusive modalities in a global context

Obeys Bell's ineq. : contradiction with expts !

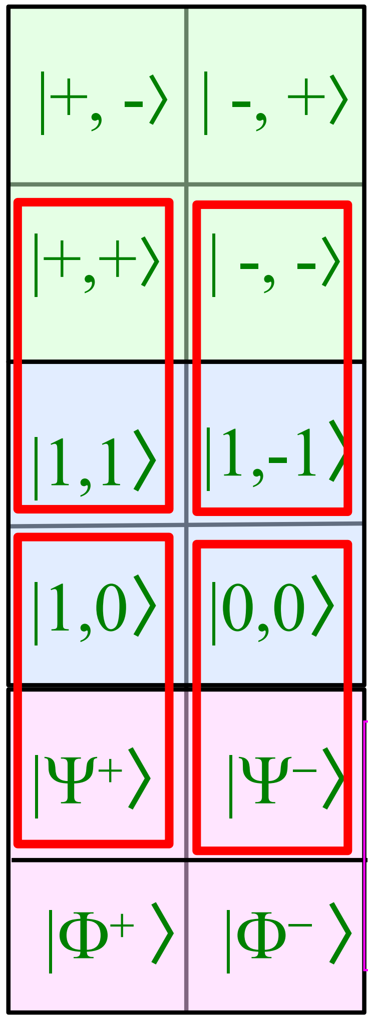
Modalities in a Bell experiment

4 different contexts : MQ



Crucial observation : **The certainty of a modality can be transferred between different contexts : extracontextuality !**

4 other different contexts : MQ



S_{z1}, S_{z2}
$m_1 = \pm 1/2$
$m_2 = \pm 1/2$

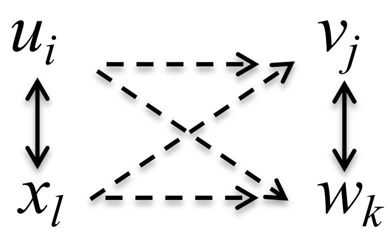
Total spin $\mathbf{S^2}$, S_z
$S = 0,1$
$m = -1,0,1$

Bell states
$|\Phi^{\pm}\rangle = (|+,+\rangle \pm |-,-\rangle)/\sqrt{2}$
$|\Psi^{\pm}\rangle = (|+,-\rangle \pm |-,+\rangle)/\sqrt{2}$

Extra-contextuality and extravalence

Definition : When a system interacts in succession with different contexts, certainty and repeatability can be transferred between their modalities. This is called **extracontextuality**, and this defines an equivalence class between modalities, called **extravalence** (it is reflexive, symmetric, transitive).

Theorem : Given an initial modality and context, the probability to get another modality in another context keeps the same value as long as the initial and final extravalence classes remain the same.

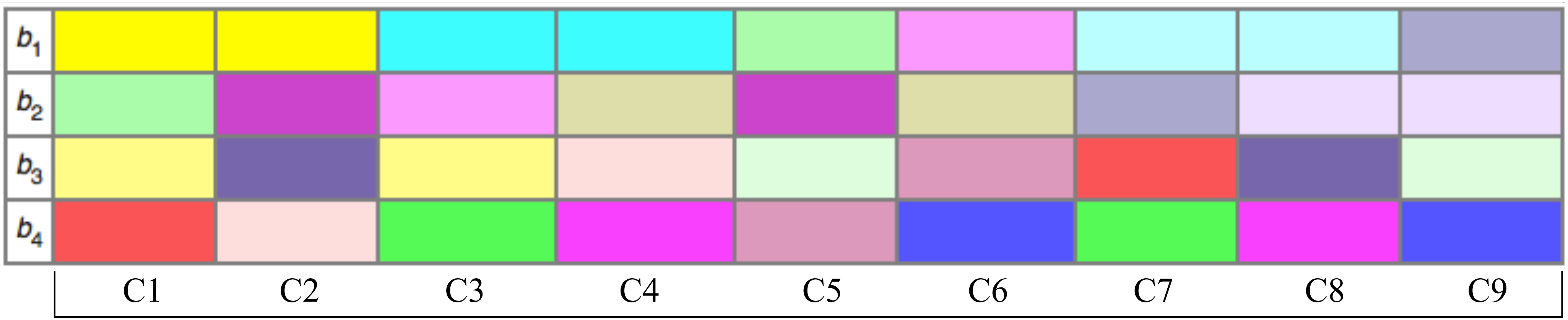


The modalities u_i , v_j , x_l , w_k belong to four different contexts, and u_i is extravalent with x_l , resp. v_j with w_k (full lines). Then all probabilities represented by dashed lines are equal.

$\Rightarrow$ extravalent modalities embed the idea of non-contextuality of probability assignments: the probability belongs to the extravalence class, not to the modality.

Contextuality, non-contextuality and extra-contextuality

Let us consider a version of the Kochen-Specker theorem by A. Cabello



9 contexts (measurements) with 4 modalities (results) $b_{m = 1,2,3,4}$ per context, 36 modalities with 18 pairs of extravalent modalities (same color).

Rules (for a non-contextual hidden variable theory) :

1/ one and only one modality is true for each context

2/ if a modality is true in one context, the extravalent modality is also true

~~**3/ any possible modality must be either true (realized) or wrong (not realized).**~~

From 1/ and 3/, 9 slots must be marked true (one for each context)

From 2/ and 3/, an even number of slots must be marked true

But 9 is not even, so the rules cannot hold together

=> There is no non-contextual hidden variable theory.

Contextuality, non-contextuality and extra-contextuality

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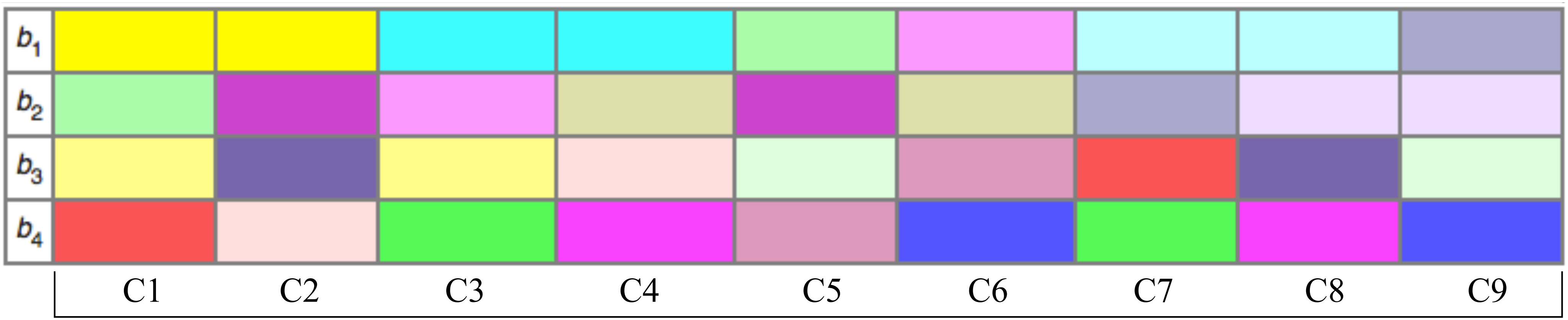

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So let's face it: there is a fundamental randomness in nature !

Then the questions are: how to describe mathematically this non-determinism in a probabilistic way, and how to manage the 'Heisenberg cut' between quantum systems and macroscopic contexts ?

Checking out extra-contextuality

u_1	(0, 0, 0, 1)	(0, 0, 0, 1)	(1, −1, 1, −1)	(1, −1, 1, −1)	(0, 0, 1, 0)	(1, −1, −1, 1)	(1, 1, −1, 1)	(1, 1, −1, 1)	(1, 1, 1, −1)
u_2	(0, 0, 1, 0)	(0, 1, 0, 0)	(1, −1, −1, 1)	(1, 1, 1, 1)	(0, 1, 0, 0)	(1, 1, 1, 1)	(1, 1, 1, −1)	(−1, 1, 1, 1)	(−1, 1, 1, 1)
u_3	(1, 1, 0, 0)	(1, 0, 1, 0)	(1, 1, 0, 0)	(1, 0, −1, 0)	(1, 0, 0, 1)	(1, 0, 0, −1)	(1, −1, 0, 0)	(1, 0, 1, 0)	(1, 0, 0, 1)
u_4	(1, −1, 0, 0)	(1, 0, −1, 0)	(0, 0, 1, 1)	(0, 1, 0, −1)	(1, 0, 0, −1)	(0, 1, −1, 0)	(0, 0, 1, 1)	(0, 1, 0, −1)	(0, 1, −1, 0)

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No hidden variables, modalities are contextually objective and extra contextual.

Born's rule : the CSM way (1)

Now forget QM, and ask : how can we make sure that

- there are only N mutually exclusive modalities in any context

- the certainty of a modality can be transferred between contexts

Inductive reasoning : use projectors !

Let's attribute a N x N projector to an extravalence class, with

- **orthogonal projectors ⇔ mutually exclusive modalities (in a context)**
- **same projector ⇔ mutually certain modalities (in an extravalence class)**

- the probability to find a given result (reproducible with certainty after being found) given an initial 'state' is a function $f(P_n)$, where f depends only on the initial state, and $P_n = |\psi_n\rangle\langle\psi_n|$ is a projector associated with the result.

- the probabilities are additive for mutually orthogonal (commuting) projectors, and $\sum_n f(P_n) = 1$ for any orthogonal set such that $\sum_n P_n = Id$

Born's rule : the CSM way (2)

Deductive part : recovering the usual QM formalism

- Theorem (Uhlhorn) : unitary transformations between contexts.

Consider two contexts Cp (with N mutually orthogonal projectors Pi), Cq (with N mutually orthogonal Qj). Expressing the Pi as a function of the Qj when changing the context **must preserve the orthogonality of the Pi** : then it must be a unitary or antiunitary transformation (Uhlhorn's theorem).

We want also to connect continuously the context change with the identity (no change of context, Cp = Cq) : **unitary transformation only.**

- Theorem (Gleason) : Born's rule.

The previous requirements fit with the hypotheses of Gleason's theorem :

- if the probability 1 is reached when changing contexts then one gets

  Born's rule for pure states, p(j | i) = Trace(Pi Qj).

- otherwise one gets Trace(ρ Qj) where ρ is a density matrix.

Open Access Article

Revisiting Born's Rule through Uhlhorn's and Gleason's Theorems

by Alexia Auffèves [1] and Philippe Grangier [2,*]

Entropy **2022**, *24*(2), 199; https://doi.org/10.3390/e24020199

Theorem 1 (Uhlhorn's theorem [20,21])**.** *Let $\mathcal{H}$ be a complex Hilbert space with dim$(\mathcal{H}) \geq 3$, and let $P_1(\mathcal{H})$ denote the set of all rank-one projections on $\mathcal{H}$. Then, every bijective map Γ: $P_1(\mathcal{H}) \to P_1(\mathcal{H})$, such that $pq = 0$ in $P_1(\mathcal{H})$ if and only if $\Gamma(p)\Gamma(q) = 0$, is induced by a unitary or anti-unitary operator on the underlying Hilbert space.*

Theorem 2 (Gleason's Theorem [23,24])**.** *Let f be a function to the real unit interval from the projection operators on a separable (real or complex) Hilbert space with a dimension at least 3. If one has $\sum_i f(P_i) = 1$ for any set $\{P_i\}$ of mutually orthogonal rank-one projectors summing to the identity, then there exists a positive-semidefinite self-adjoint operator ρ with unit trace (called a density operator), such that $f(P_i) = Trace(\rho P_i)$.*

20. Uhlhorn, U. Representation of symmetry transformations in quantum mechanics. *Arkiv Fysik* **1962**, *23*, 307–340.
21. Chevalier, G. Wigner-Type Theorems for Projections. *Int. J. Theor. Phys.* **2008**, *47*, 69–80.
22. Semrl, P. Wigner symmetries and Gleason's theorem. *J. Phys. A Math. Theor.* **2021**, *54*, 315301.
23. Gleason, A.M. Measures on the Closed Subspaces of a Hilbert Space. *J. Math. Mech.* **1957**, *6*, 885.
24. Cooke, R.; Keanes, M.; Moran, W. An elementary proof of Gleason's theorem. In *Mathematical Proceedings of the Cambridge Philosophical Society*; Cambridge University Press: Cambridge, UK, 1985; Volume 98, pp. 117–128.

Completing the (usual) quantum formalism

1/ A projector $| \psi \rangle\langle \psi |$ **does not define a modality** but an extravalence class, so to make physical sense of the QM formalism one needs

- a state (vector) $| \psi_n \rangle$ or projector $| \psi_n \rangle\langle \psi_n |$

AND

- an observable (operator) $\sum_k a_k | \psi_k \rangle\langle \psi_k |$ with $| \psi_n \rangle \in \{| \psi_k \rangle\}$

Both of them are needed to define a physical modality and to get actual probabilities over a set of mutually exclusive events.

2/ But then the formalism should be able to describe **both** the quantum system and the classical context, i.e. **both sides of the (in)famous « Heisenberg cut »**. **How to do that ?**

It can be said that the usual $| \psi \rangle$ is predictively incomplete ; see
P. Grangier, Entropy 23 (12),1660 (2021) https://arxiv.org/abs/2012.09736
Contextual inferences, nonlocality, and the incompleteness of quantum mechanics

CSM construction : universality and completeness.

Found. Phys. 51, 76 (2021) http://arxiv.org/abs/2003.03121
M. Van Den Bossche & P. Grangier, https://arxiv.org/abs/2209.01463

* Composite systems are described using tensor products as usual.

* **Contexts = infinite tensor product ?** Taking this limit breaks unitarity, and leads to sectorization in type III algebra (see : von Neumann 1939, "On infinite direct products").

Naively, one would expect to get an "infinitely large Hilbert space", still with the same algebraic properties, but this turns out to be completely wrong.

Hint for qubits: $d \equiv 2$, $\dim(H_\infty) = 2^{\aleph_0} = \aleph_1$ i.e. the power of continuum.
=> There is no countable basis dense in H_∞
=> H_∞ is not separable – different from all we use in quantum physics

Quoting von Neumann*: *Infinite (tensor) products differ essentially from the finite ones in this, that they split up into "incomplete tensor products". (...) What happens could be described in the quantum-mechanical terminology as a splitting up of the tensor product into "non-intercombining systems of states", corresponding to the incomplete direct products quoted above."*

* *J. von Neumann, Compositio Mathematica* 6, 1-77 (1939)

CSM construction : universality and completeness.

Found. Phys. 51, 76 (2021) http://arxiv.org/abs/2003.03121

M. Van Den Bossche & P. Grangier, https://arxiv.org/abs/2209.01463

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* **Contexts = infinite tensor product ?** Taking this limit breaks unitarity, and leads to sectorization in type III algebra (see : von Neumann 1939, "On infinite direct products").

* Using a sectorized global algebra : tensor product between two vN algebra, the usual type I non commutative for the system ⊗ type III for the context. Globally all is type III, and this provides a complete description corresponding to the modalities, and not to the usual ψ describing an extravalence class : ok. The algebra is universal, but there is no universal wavefunction.

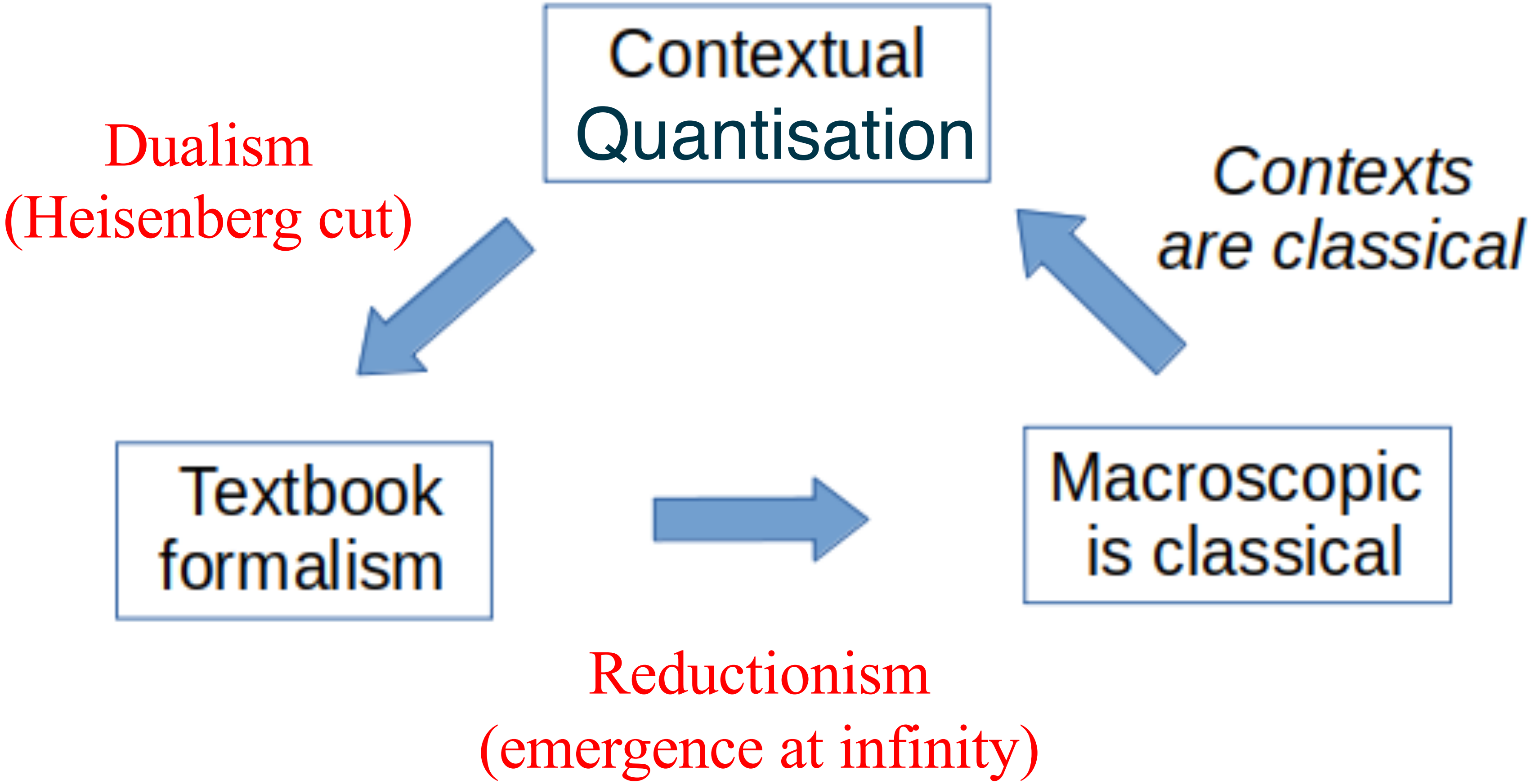
* Major point : there is no need to specify all details for the context (this is not possible : there are « infinitely many » details), and it is enough to label the different sectors by using the commutative 'center' of the type III algebra. This is just what is needed for a classical description of the context.

* This description applies to any (isolated) system within a context, so it is also complete since it fully specifies a modality. It is also universal in the sense that it describes anything, but not everything (it is a ToA, not a ToE).

Closing the loop of CSM

Found. Phys. 51, 76 (2021) https://arxiv.org/abs/2209.01463

Entropy 25, 1600 (2023). https://arxiv.org/abs/2310.06099



Looking more closely at Bell's options

Entropy 23 (12),1660 (2021) https://arxiv.org/abs/2012.09736

(2) There are influences going faster than light => Violation of « elementary locality » without faster than light signalling.

Strong tension with relativistic causality => to be avoided

or not : Bohmian mechanics makes this choice.

(3) The quantities a and b are not independent variable => Violation of « free choice », there are no random independant events.

Full predetermination, no actual randomness => to be avoided

or not : G. t'Hooft makes this choice.

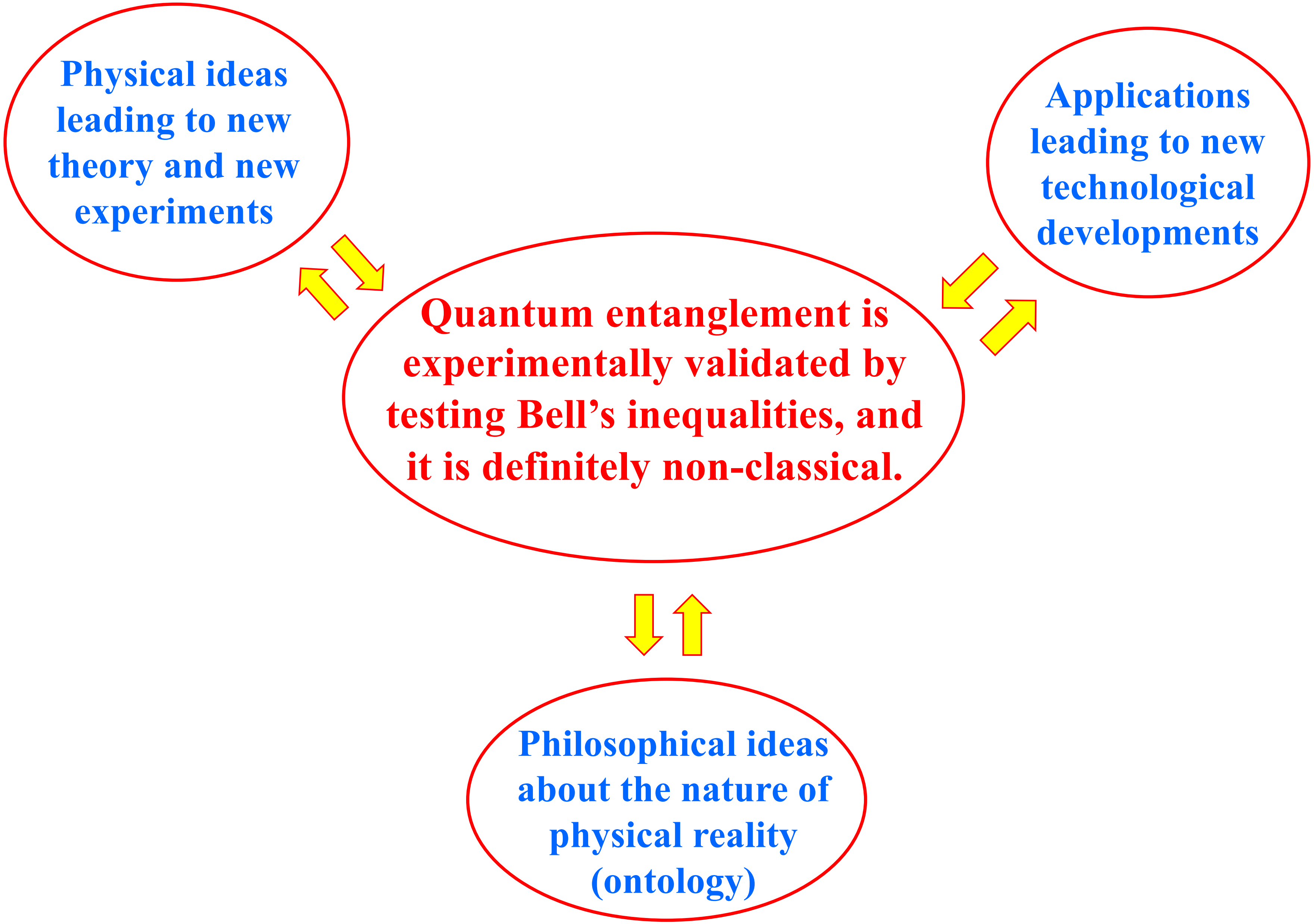
(4) The whole analysis can be ignored, that's quantum mechanics => Violation of « predictive completeness », cannot be formulated in a classical or deterministic framework, but makes sense for QM.

Needs randomness and contextuality, => to be chosen !

then ψ has to be completed by specifying the measurement context.

Completing ψ « from above » by specifying the context is a valid option for physical realism, under the QM empirical constraints.

As a conclusion…



Thank you for your attention !

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Thank you to Franck Laloë, Roger Balian, Anthony Leverrier, Olivier Ezratty...

Some references on Quantum Foundations

* Another view on QM: Contexts, Systems and Modalities (CSM)

A. Auffèves & P. Grangier, Found. Phys. 46, 121 (2016) arxiv:1409.2120

Contexts, Systems and Modalities: a new ontology for quantum mechanics

A. Auffèves & P. Grangier, Found. Phys. 50, 1781 (2020) arxiv:1910.13738

Deriving Born's rule from an Inference to the Best Explanation

A. Auffèves & P. Grangier, Entropy 24 (2), 199 (2022) arxiv:2111.10758

Revisiting Born's rule through Uhlhorn's and Gleason's theorems

* Another view on Bell's theorem: ψ is predictively incomplete

P. Grangier, Entropy 23 (12),1660 (2021) arxiv:2012.09736

Contextual inferences, nonlocality, and the incompleteness of quantum mechanics

* From John von Neumann (1939) to operator algebras

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Contextual unification of classical and quantum physics

M. Van Den Bossche & P. Grangier, Proc. DICE conf (2023) arxiv:2304.07757

Revisiting Quantum Contextuality in an Algebraic Framework

M. Van Den Bossche & P. Grangier, Entropy 25, 1600 (2023) arxiv:2310.06099

Postulating the Unicity of the Macroscopic Physical World

Philosophical options...

Hard subjectivist ("crazy bayesian")

- how certain are you that you are certain ?
- a probability assignment is not a fact (Caves, Fuchs and Schack...)
- **the fact** : it is possible to design a set of measurements so that if you perform it again and again on the same system it will give again and again the same result.

In such a case we tell that the system is in a well defined modality / quantum state.

Hard realist ("deceived lover of hidden variables")

- a state corresponds to a set of elements of physical reality (= the results can be predicted with certainty and measured without changing in any way the system).
- **the fact :** reality is ok, but it must be attributed **jointly** to the context and the system; then a modality is a quite acceptable element of physical reality, and it gives a meaning to « non locality without any spooky action at a distance ».

Philosophical options...

Hard platonist ("mathematical objects do exist")

- a vector in an Hilbert space is not a mathematical tool, but a definition of reality
- unitarity of evolution is basic, the observed classical world must "emerge" from it
- **the fact** : manipulating vectors (projectors) in an Hilbert space is the quantum way to calculate probabilities, it is not a "reality". **The "reality" is the modality, i.e. the set of values of physical properties that you will obtain again and again by performing measurements on the same system in the same context.**

Super-hard platonist ("mathematical objects exist physically")

- nothing else than $|\psi\rangle$ exist (within a universal $|\psi\rangle$)
- the many-world picture must be understood ontologically: there are many « me »
- **the fact** : same as above, but here it is not recognized as a fact, since the only "facts" are about $|\psi\rangle$ itself, so there is no way to agree (physical realism is gone).