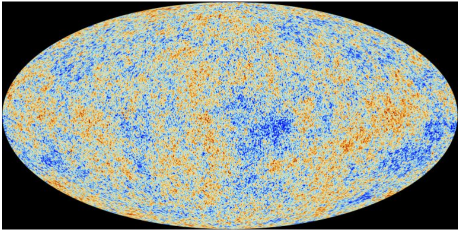
Freeze-out of Cold Dark Matter

Intro

In my thesis I study some aspects about Dark Matter. There are different experimental evidences about the existence of DM:

- Galactic velocities curves
- Velocity distribution inside galaxy clusters
- Anisotropies of the CMB
- Gravitational lensing of both galaxies and galaxy clusters
- Large scale structure of the universe

The anisotropies of CMB testifies that DM has a non-baryonic source and its origin is prior to the CMB. We can then study the relic density of DM studying the number density of particles species in thermal and chemical equilibrium with a thermal bath, in a radiation-dominated universe.



It is interesting to note that the CMB allows us to compute singularly the density matter (baryonic + DM) and baryonic matter. This is because DM and baryonic matter have the same equation of state but, while baryonic matter interacts with the EM (and therefore has a strength that prevents her from clustering, DM does not couple with the EM field).

Freeze-out

If the matter had always remained in thermal and chemical equilibrium it would now have a nearly zero density due to the exponential factor, which we know it is not true.

$$n_\chi = g_\chi \left(\frac{m_\chi T}{2\pi} \right)^{3/2} e^{-m_\chi / T} \tag{1}$$

(number denisty for non-relativistic particles).
To maintain thermal equilibrium with the bath we need to assume a reaction of the form:

$$\chi\chi \leftrightarrow SMSM \tag{2}$$

Istantaneous freeze-out

Then to study the evolution of the number density we can suppose that the decoupling happens exactly when $\Gamma = H$ (instantneous freeze-out). Where Γ is the annihilation decay rate and H is the Hubble constant. Before decoupling $\Gamma >> H$ and then when $\Gamma << H$ the reaction between DM particles are not more effective and so the number density for comoving volume remains constant.
Furthermore we can define Y_χ is the number density for comoving volume and $x = m_\chi / T$.

Boltzmann equation

To study the distribution evolution we can use Boltzmann equation and compute the density of DM. The Boltzmann equation compares the evolution of the phase space equation because due to the geometry of space-time while and because of the presence of decays and creation processes. After some computations one gets a differential equation:

$$\frac{dY_\chi}{dx} = -\langle\sigma_{\rightarrow} v\rangle \frac{s}{Hx}\left[Y_\chi^2 - (Y_\chi^{eq})^2\right] \tag{3}$$

Numerical solutions

We can then compute some numerical solutions. The cross section is assumed of three different magnitudes and it depends one the mass of the DM particles and in our model on a coupling constant α. If someone choose a value of m_χ of about $\simeq 100$ GeV and $\alpha = 0.1$ we get a typical cross section of a weak interaction. This is sometimes regarded as the WIMP miracle.

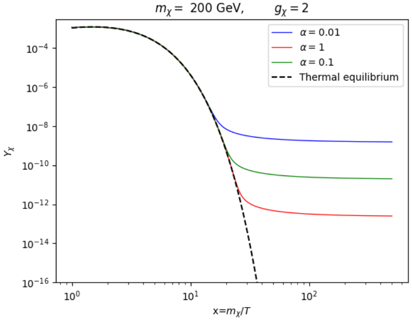


Figure: Boltzmann equation for three different couplings