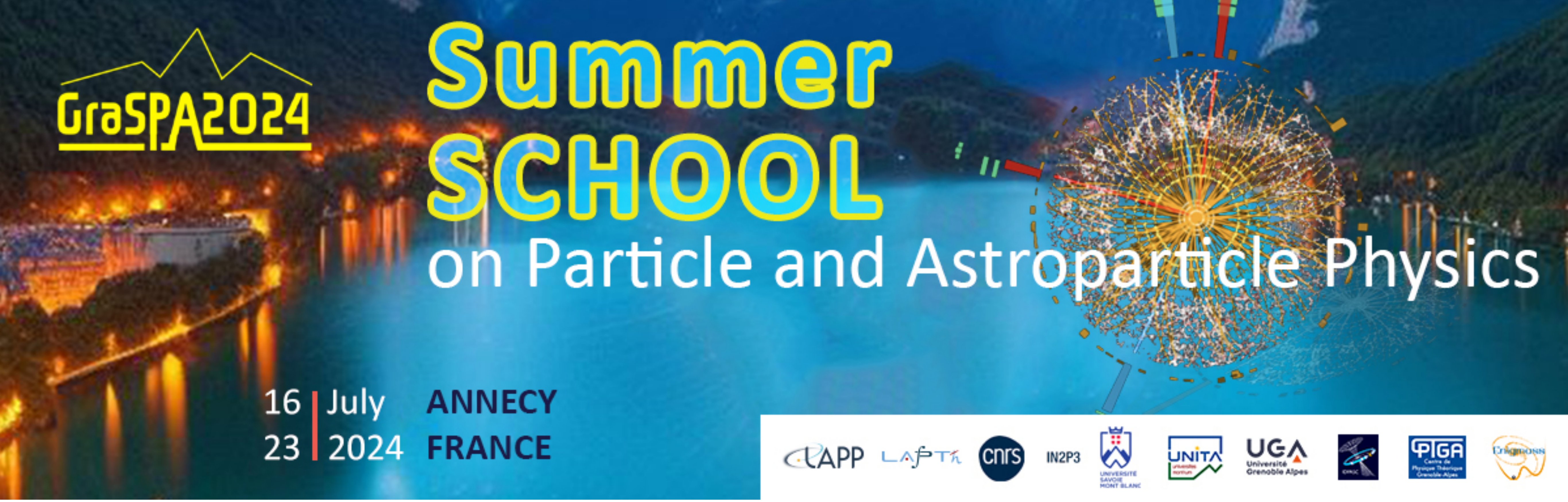

Astroparticle Theory

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What is Astroparticle Physics?

What is Astroparticle Physics?

Particle Astrophysics?
Particle Cosmology?

- Apply **methods** and **tools** from particle physics to astrophysics and cosmological systems
- Use of astrophysical and cosmological **observations** to learn about fundamental physics and BSM
- Provide **quantitative predictions** from/for observational evidence (CMB, BBN)
- Explain origin of **high-energy particles** produced in our current universe (neutrinos, cosmic rays, photons)

Lecture 1

1. A short introduction to cosmology
2. The early Universe thermal history
3. Boltzmann equations for thermal relics

Main reference:

Kolb & Turner, "The Early Universe" (1988)

Chapters 1-3, 5

Observational cosmology

Observational evidence supports the standard model for cosmology

Natural units: $c = k_B = \hbar = 1$ but

$$G_N \equiv M_P^{-2} = (1.22 \cdot 10^{19}\ \text{GeV})^{-2}$$

Ex: Derive Planck fundamental scales (mass, length, time)

Universe expansion

Empirical observation: Mpc-distant galaxies show velocity recession through Doppler red-shift of their spectra

$$z \equiv \frac{\lambda_o - \lambda_e}{\lambda_e} > 0$$ moving away from us

$$\lambda_e \equiv \lambda_{LAB}$$

$$c\,z = H_o d$$

H_o Hubble constant

$$H_o = 100 \cdot h \text{ Km/s/Mpc}$$

$$h \sim 0.7$$

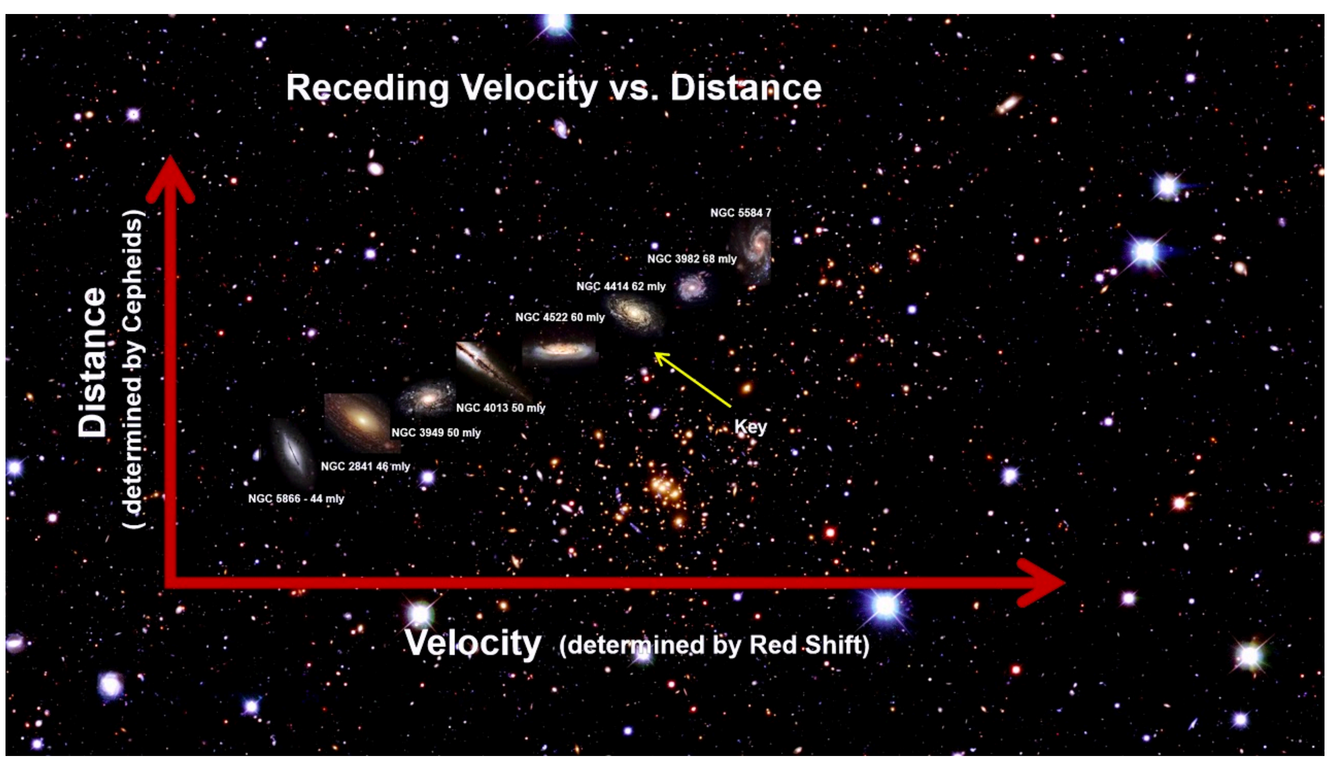
Universe expansion

Interpretation: Red-shift as Doppler effect due to an expanding universe

$$(1+z) = \frac{\lambda_0}{\lambda_e} = \gamma\left(1 + \frac{v_r}{c}\right) \quad ; \quad \gamma \sim 1$$

$$\Rightarrow \quad z \sim \frac{v_r}{c}$$

$$\Rightarrow \quad \boxed{v_r = H_0 d}$$

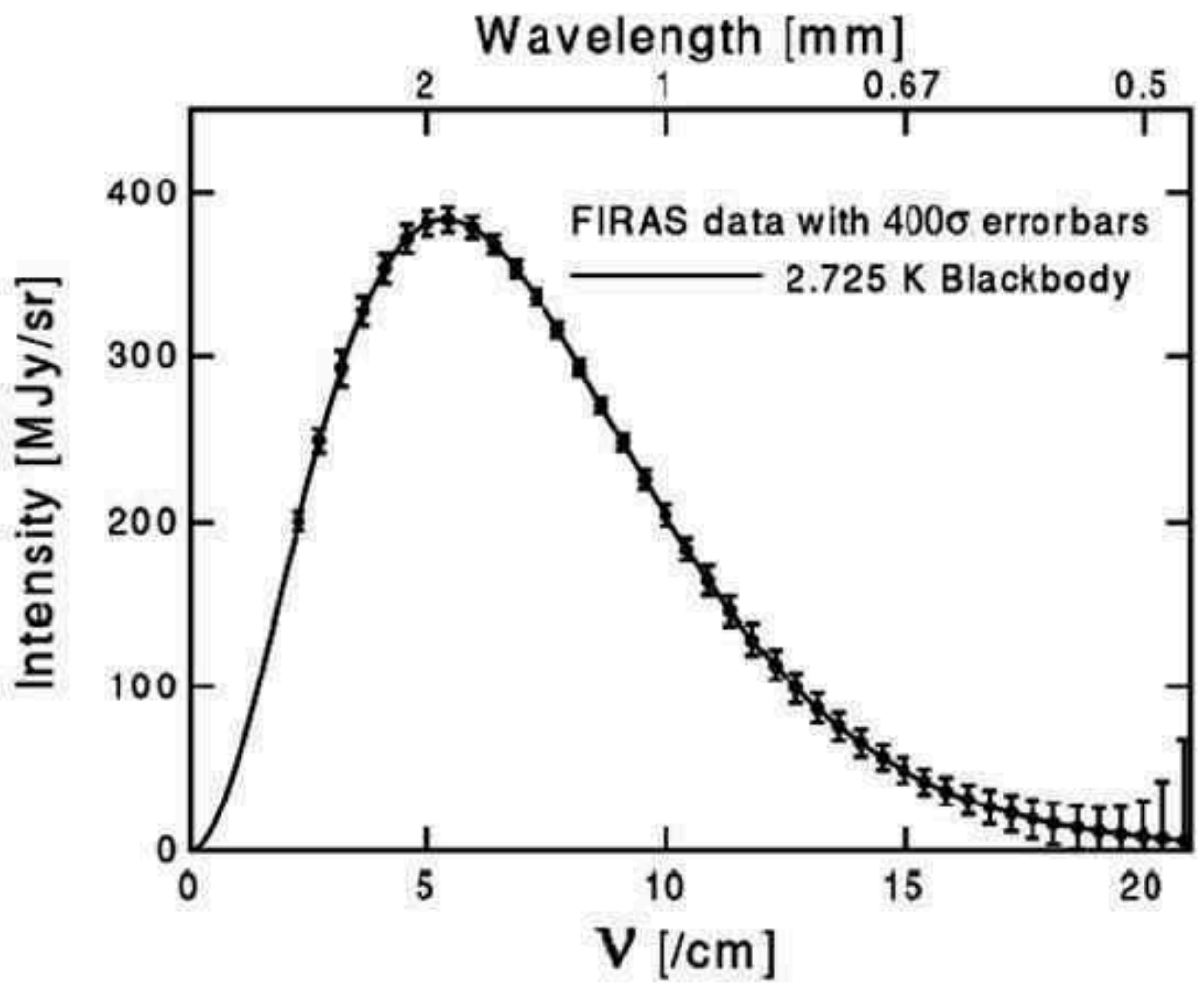


Hubble law (1929)

Validity: (i) Peculiar motion of galaxies can be avg to 0; (ii) Small peculiar velocity ($z < 0.1$)

The cosmic microwave background (CMB)

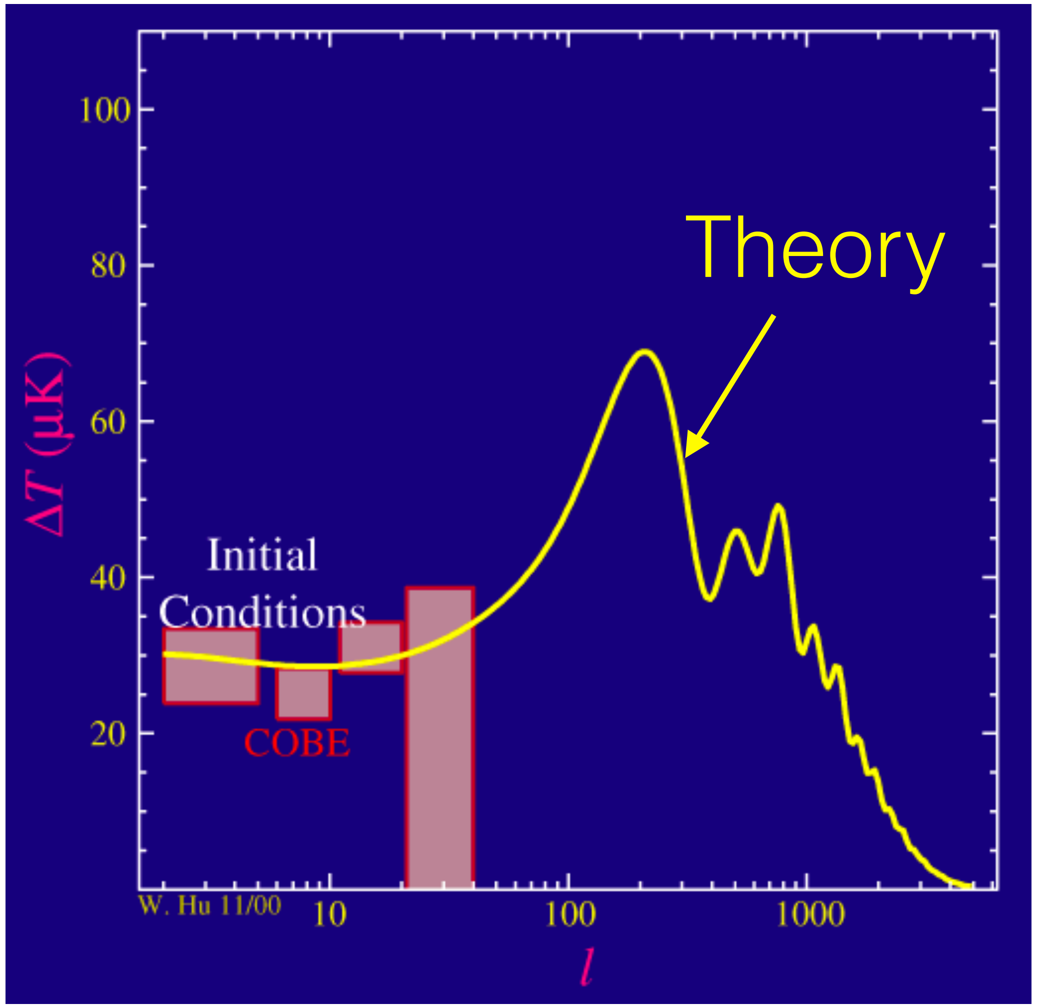
- ’40: Predicted by Gamov, within the theory of Hot Big Bang
- ’64 (Nobel ’73): Observed by Penzias & Wilson as uniform radio white noise in the sky



Best evidence for universe **isotropy** on large scales

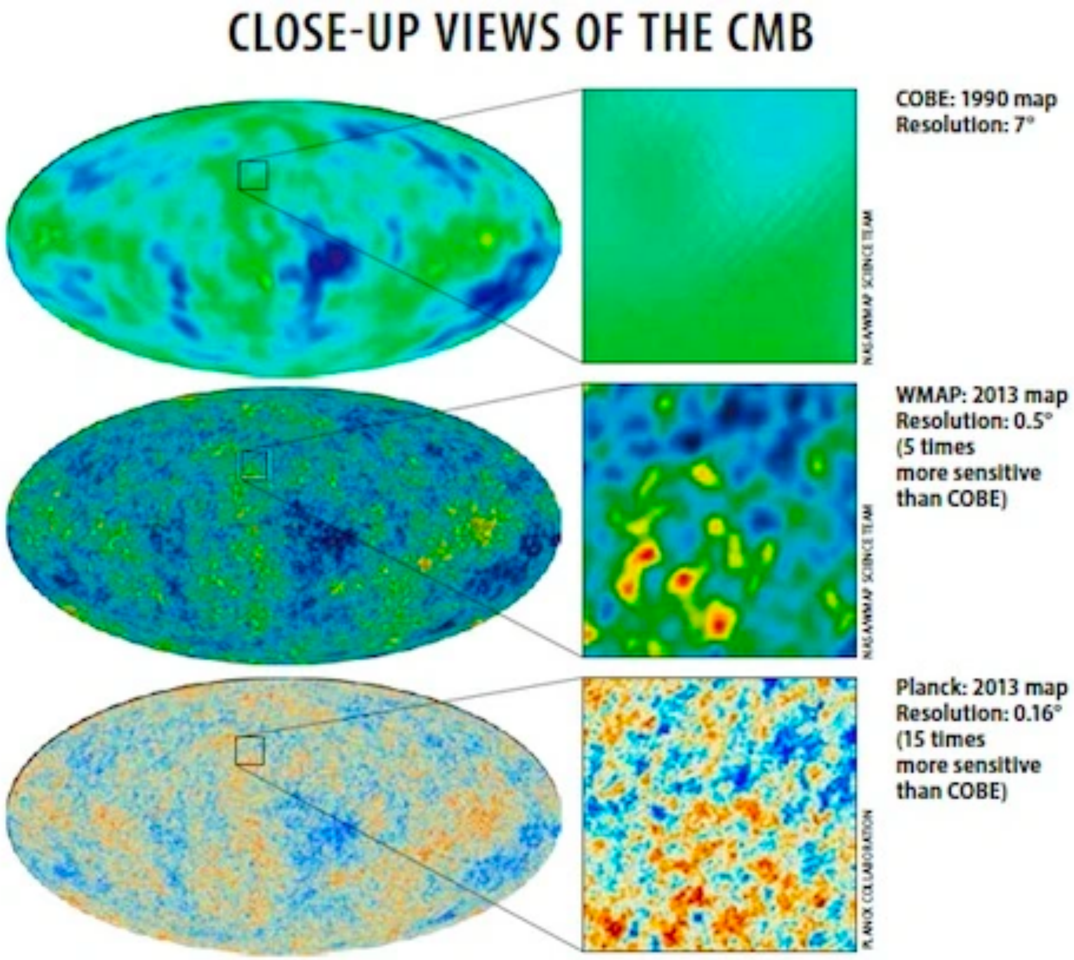
The cosmic microwave background (CMB)

- ’40: Predicted by Gamov, within the theory of Hot Big Bang
- ’64 (Nobel ’73): Observed by Penzias & Wilson as uniform radio white noise in the sky
- ’92: COBE: (i) Measurement of BB spectrum and CMB temperature (2.728 K); (ii) Angular coherence of temperature fluctuations on degrees scale

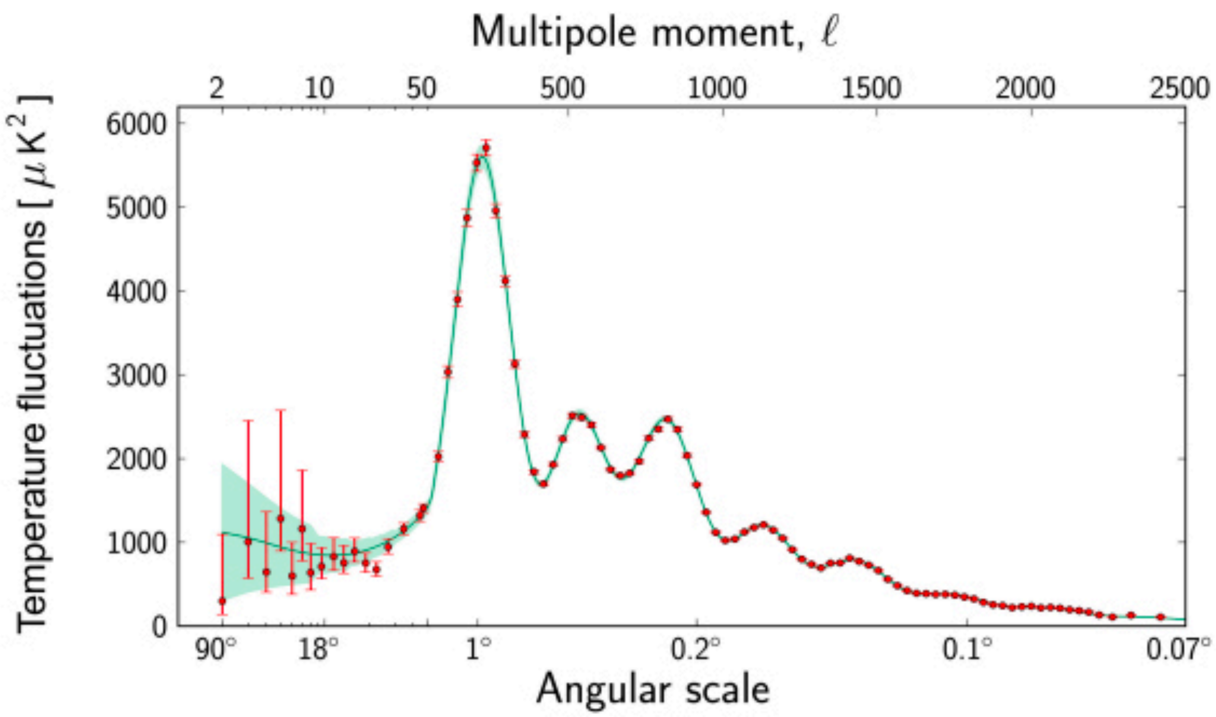


Evidence for **correlations** in T on large angular scales

The cosmic microwave background (CMB)

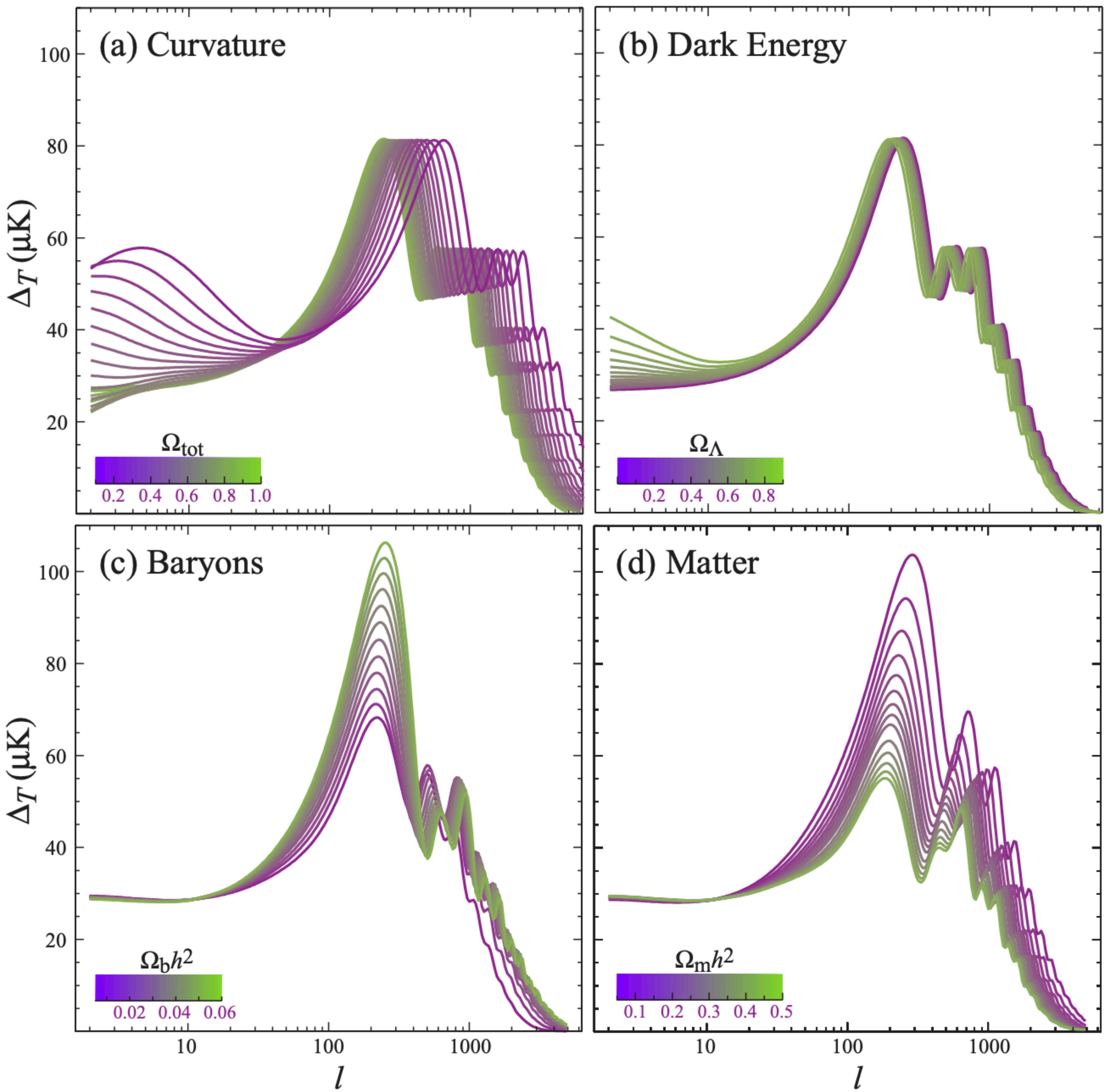


Astronomy: Roen Kelly



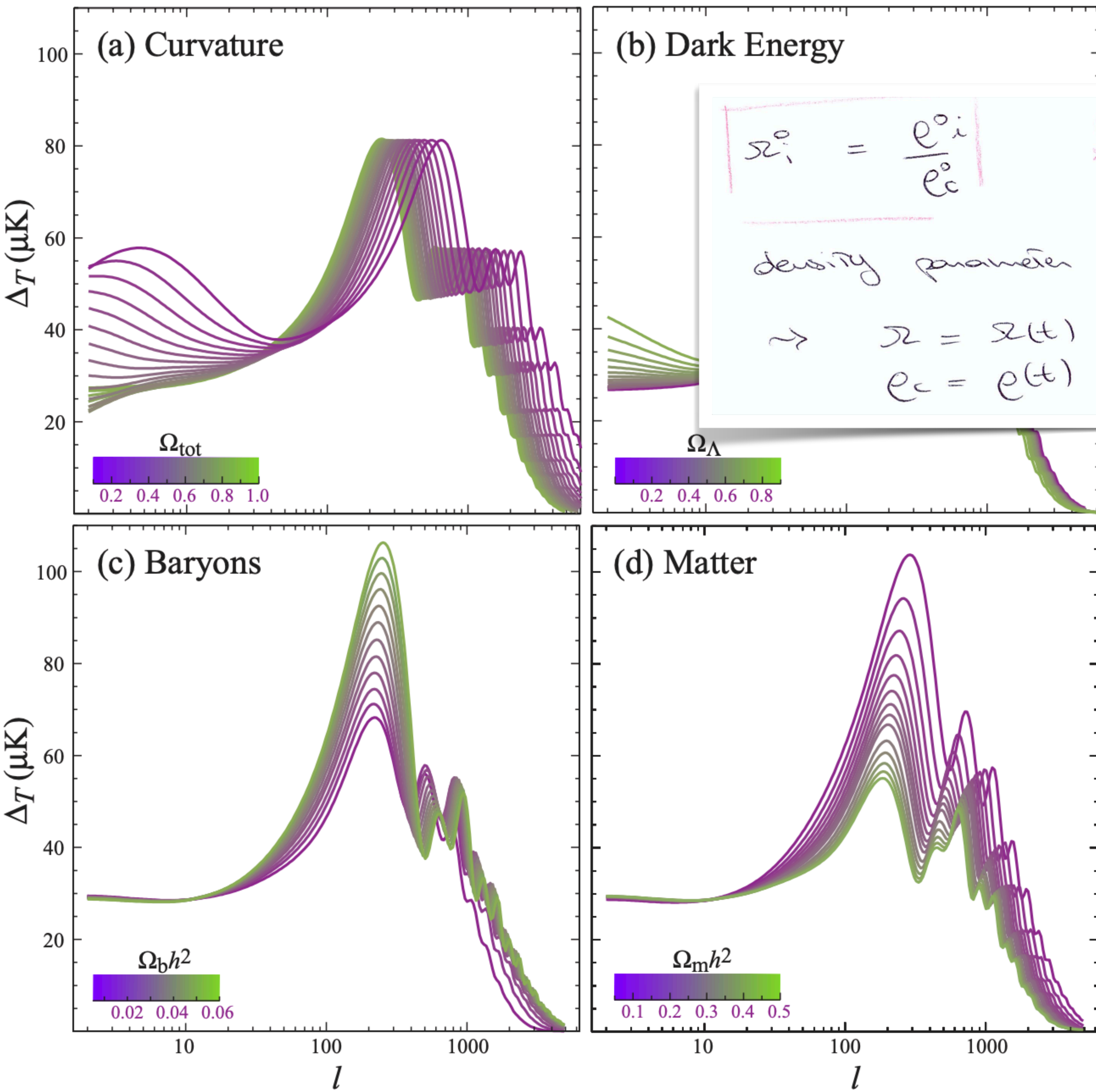
Entering the era of **precision cosmology** of **LCDM**

CMB anisotropies and LCDM



Hu & Dodelson 2002

CMB anisotropies and LCDM



$\Omega_i^0 = \frac{\rho_i^0}{\rho_c^0}$ $\quad \rho_c^0 = \frac{3H_0^2}{8\pi G} \sim 10^{-29}\ h^2 g/cm^3$

density parameter of species i

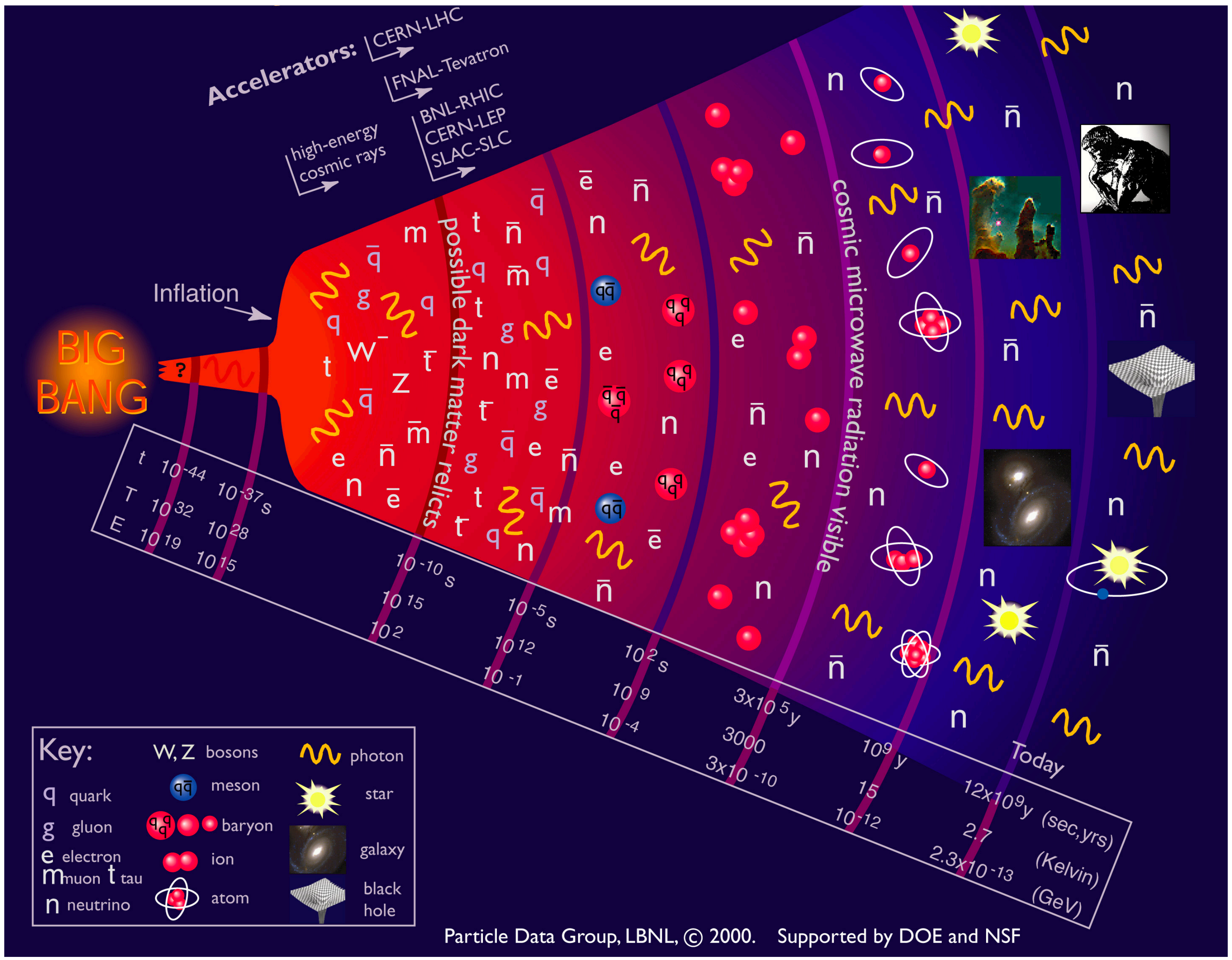
$\rightarrow \quad \Omega = \Omega(t)$
$\rho_c = \rho(t) \longrightarrow H = H(t)$

[see below]

Hu & Dodelson 2002

What can we infer about the universe?

- The universe is a physical, **dynamical** (expansion) system
- The **early** universe (before recombination) is a **thermodynamical** system (plasma, relativistic fluid)



- We can study the *ensemble* properties of the system
- Following one single evolutionary parameter, i.e. the **temperature**
- The relativistic degrees of freedom are set by fundamental particle physics

The standard cosmological model (LCDM)

Cosmological principle: Valid on sufficiently large scales, it postulates the spatial **homogeneity** and **isotropy** of the spatial part of the metric

Universe dynamic evolution described by Einstein eqs.

Ricci Tensor →

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

(R: curvature scalar)

10 eqs coupled
6 indep. eqs

[$\eta_{\alpha\beta} = (+1, -1, -1, -1)$] Minkowski metric

The standard cosmological model (LCDM)

$$d\tau^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

Space-Time 4D

$$x^\mu = (t, x^i)$$

CP ↓

ROBERTSON - WALKER METRIC

$$d\tau^2 = dt^2 - R^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta\, d\varphi^2 \right]$$

The standard cosmological model (LCDM)

$d\tau^2 = g_{\mu\nu}\, dx^\mu dx^\nu$

Space-Time 4D

$x^\mu = (t, x^i)$

CP

ROBERTSON - WALKER METRIC

$$d\tau^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta\, d\varphi^2\right]$$

- $R(t)$ scale factor, describing the stretching of space as a function of time

- $k = 1, 0, -1$ describing the curvature of the 3D space
  - 1 ← spherical (closed)
  - 0 ← flat
  - -1 ← hyperbolic (open)

The standard cosmological model (LCDM)

Cosmological principle => R(t) can only increase, decrease or being constant

R(t) solution determined by the **energy-momentum tensor**

$$\tilde{T}_{\mu\nu} = \underbrace{T_{\mu\nu}}_{\text{energy-matter}} + \underbrace{\frac{\Lambda c^2}{8\pi G} g_{\mu\nu}}_{\text{cosmological constant}}$$

Perfect fluid description applies

The standard cosmological model (LCDM)

$$\tilde{T}_{\mu\nu} = (\tilde{p} + \tilde{\varrho} c^2)\, u_\mu u_\nu - \tilde{p}\, g_{\mu\nu}$$

$$\begin{cases} \tilde{\varrho} \equiv \dfrac{\tilde{T}_{00}}{c^2} = \varrho + \dfrac{\Lambda}{8\pi G} \\[2ex] -\tilde{p}\, g_{ij} \equiv \tilde{T}_{ij} = -p\, g_{ij} + \dfrac{\Lambda c^2}{8\pi G} g_{ij} \end{cases}$$

$$\Rightarrow \quad \tilde{p} = p - \frac{\Lambda c^2}{8\pi G}$$

$(p_\Lambda < 0)$

$p_\Lambda = -\varrho_\Lambda c^2$

The standard cosmological model (LCDM)

$$\tilde{T}_{\mu\nu} = (\tilde{p} + \tilde{\varrho} c^2)\, u_\mu u_\nu - \tilde{p}\, g_{\mu\nu}$$

$$\begin{cases} \tilde{\varrho} \equiv \dfrac{\tilde{T}_{00}}{c^2} = \varrho + \dfrac{\Lambda}{8\pi G} \\ -\tilde{p}\, g_{ij} \equiv \tilde{T}_{ij} = -p\, g_{ij} + \dfrac{\Lambda c^2}{8\pi G} g_{ij} \end{cases}$$

$$\Rightarrow \quad \tilde{p} = p - \frac{\Lambda c^2}{8\pi G} \qquad (p_\Lambda < 0)$$

$$p_\Lambda = -\varrho_\Lambda c^2$$

Equation of state:

e.g.

matter	$p \simeq 0$	
radiation	$p = \varrho/3$	$p = w\varrho$
Λ	$p = -\varrho$	

The standard cosmological model (LCDM)

<u>Friedmann model</u>: How does R(t) evolve with time?

(I) $$\frac{\dot{R}^2}{R^2} + \frac{K}{R^2} = \frac{8\pi G}{3}\rho \qquad \rho \equiv \Sigma_i \rho_i$$

Friedmann eq.

(II) $$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + K/R^2 = -8\pi G p \qquad p = \Sigma_i p_i$$

(III) $\equiv$ (II) $-$ (I) $$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + 3p) \quad ; \quad p_i = w_i \rho_i$$

The standard cosmological model (LCDM)

Friedmann model: How does R(t) evolve with time?

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(III) ≡ (II) − (I) $$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + 3p) \quad ; \quad p_i = w_i \rho_i$$

Hubble parameter:

$$H(t) \equiv \frac{\dot{R}(t)}{R(t)}$$

The standard cosmological model (LCDM)

<u>Friedmann model</u>: How does R(t) evolve with time?

$$(\mathrm{I}) \qquad \frac{\dot{R}^2}{R^2} + \frac{K}{R^2} = \frac{8\pi G}{3}\rho \qquad \rho \equiv \Sigma_i \rho_i$$

Friedmann eq.

$$(\mathrm{II}) \qquad 2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + K/R^2 = -8\pi G p \qquad p = \Sigma_i p_i$$

$$(\mathrm{III}) \equiv (\mathrm{II}) - (\mathrm{I}) \qquad \frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + 3p) \quad ; \quad p_i = w_i \rho_i$$

$$\begin{cases} \ddot{R} > 0 & \text{acceleration if } w < -\frac{1}{3} \\ \ddot{R} = 0 & \text{static if } w = -\frac{1}{3} \\ \ddot{R} < 0 & \text{deceleration if } w > -\frac{1}{3} \end{cases}$$

The standard cosmological model (LCDM)

$a(t) \overset{def}{=} \frac{R(t)}{R_0}$ Scale factor

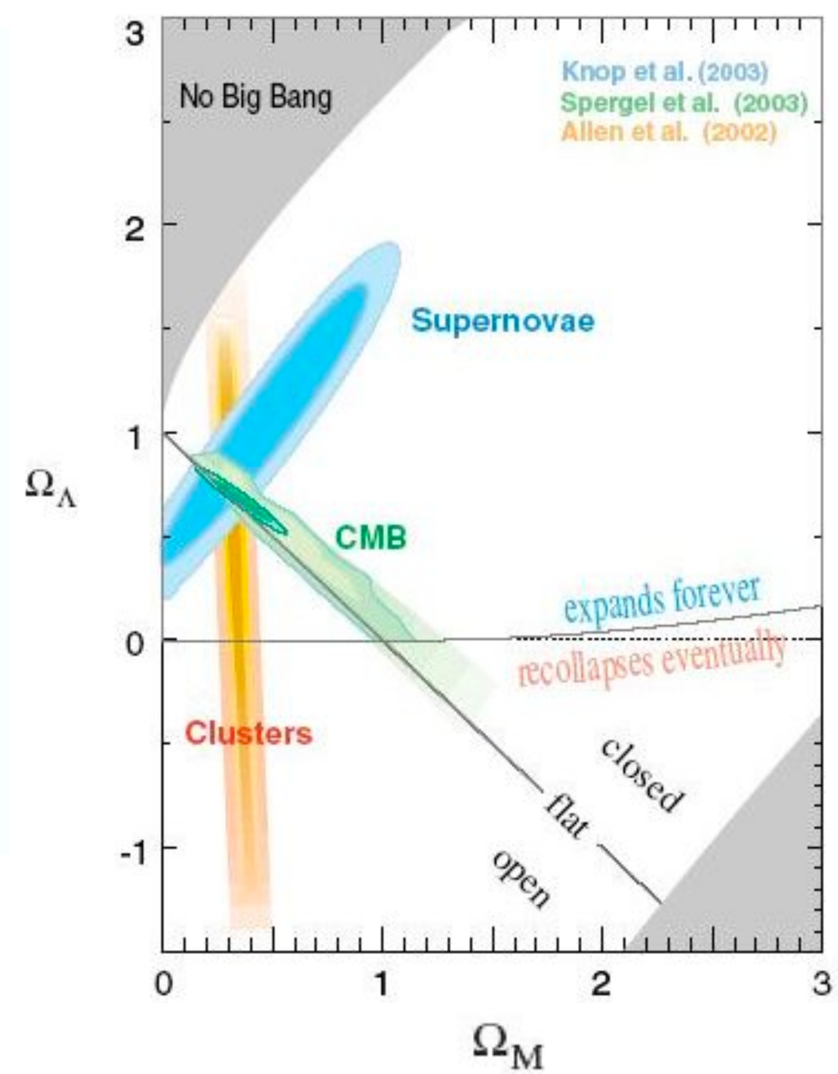
$$\dot{a}^2 = H_0^2 \left\{ \sum_i \Omega_i^0 \, a^{-(1+3w_i)} + (1-\Omega_0) \right\}$$ (F. eqs.)

monocomponent Universe, flat

$\Rightarrow$ i) MD, $w=0$ $\quad a \propto t^{2/3}$

ii) R.D $\quad a \propto t^{1/2}$

iii) Λ $\quad a \propto e^{H_0 t}$



The standard cosmological model (LCDM)

Ex: The universe as thermodynamic system should evolve through equilibrium states. The 1st principle of thermodynamics holds.

$$dU = - p\ dV$$

Demonstrate how the density evolves with R.

The standard cosmological model (LCDM)

Ex: The universe as thermodynamic system should evolve through equilibrium states. The 1st principle of thermodynamics holds.

$$dU = - p\, dV$$

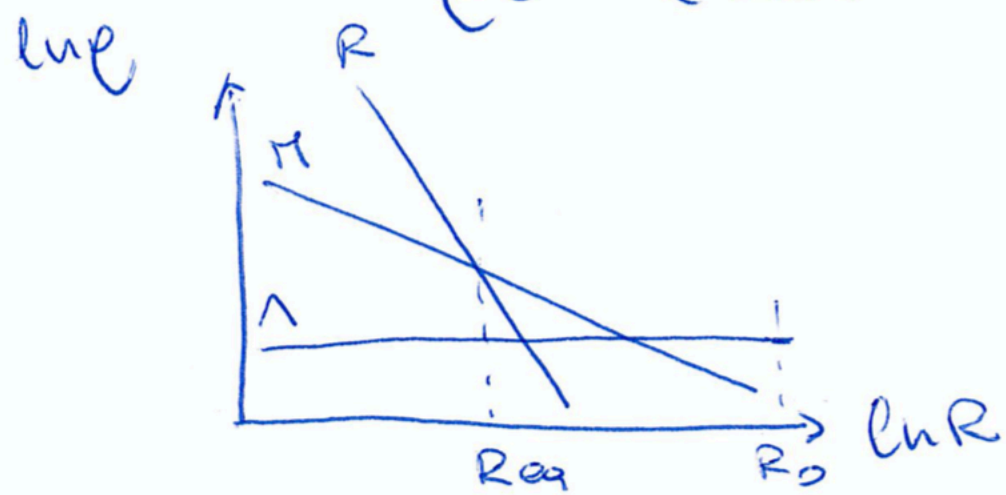
Demonstrate how the density evolves with R.

$$\frac{\rho}{\rho_0} = \left(\frac{R}{R_0}\right)^{-3(1+w)}$$

$$\Rightarrow \quad \rho_M \propto R^{-3}$$

$$\rho_R \propto R^{-4}$$

$$\rho_\Lambda \sim \text{const}$$



Thermal history of the universe

early Universe is radiation dominated

R.D. $R(t) \propto t^{1/2}$

$\rho(t) \propto R^{-4} \propto t^{-2}$

$\Longrightarrow$ ★ high density

★ small causal horizon

$i + j \longleftrightarrow k + l$ interactions sufficiently fast

thermodyn. equilibrium

(kinetic and chemical)

↑ energy exchange processes are fast

↑ particle exchange processes fast

Thermal history of the universe

- thermal distribution of species i

$f(x^i, p^i, t)$ such that $dN = f(\vec{x}, \vec{p}, t)\, d^3\vec{x}\, d^3\vec{p}$

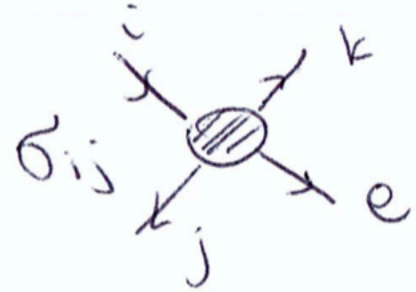
Homogenous and isotropic universe

$f(p, t)$ or equivalently $f(E, t)$

$$\begin{cases} p^2 = |\vec{p}|^2 \\ E^2 = p^2 + m^2 \end{cases}$$

Thermal history of the universe

- equilibrium condition



Def Interaction rate

$$\Gamma_i = \Sigma_j n_j \sim \sigma_{ij}$$

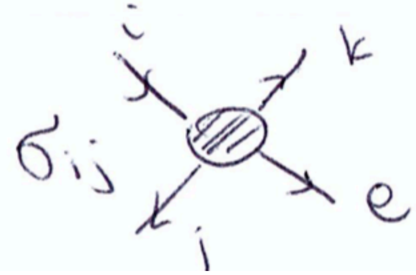
(n_j ↑ Target)

$$[\Gamma_i] = T^{-1}$$

vs Expansion rate $H = \frac{\dot{R}}{R}$

Thermal history of the universe

- equilibrium condition



Def Interaction rate

$$\Gamma_i = \sum_j n_j \, v \, \sigma_{ij}$$

vs Expansion rate $H = \frac{\dot{R}}{R}$

(i) $\lambda^i_I \equiv \Gamma_i^{-1} \ll \lambda_H = H^{-1} \iff \Gamma_i \gg H$

many interactions may reach equilibrium

(ii) $\lambda^i_I \gg \lambda_H \iff \Gamma_i \ll H$

species i cannot interact any longer with thermal bath → out of equilibrium

Thermal history of the universe

- effective descriptive parameter is the temperature T

$$f_i(p) = \frac{1}{e^{\frac{E-\mu_i}{T}} \pm 1} \qquad \begin{cases} +1 & \text{fermions (F.D.)} \\ -1 & \text{bosons (B.E.)} \end{cases}$$

phase-space distribution

μ_i : chemical potential / $\mu_i + \mu_j = \mu_k + \mu_e$

usually $\mu_i \simeq 0$

a) NUMBER DENSITY $\quad n_i \equiv \frac{g_i}{(2\pi)^3} \int d^3p \, f_i(p)$

b) ENERGY DENSITY $\quad \rho_i \equiv \frac{g_i}{(2\pi)^3} \int d^3p \, E \, f_i(p)$

c) PRESSURE DENSITY $\quad p_i \equiv \frac{g_i}{(2\pi)^3} \int d^3p \, \frac{p^2}{3E} f_i(p)$

Thermal history of the universe

i) RELATIVISTIC SPECIES

$$E^2 = p^2 + m^2 \quad ; \quad p^2 \gg m^2 \quad ; \quad E \simeq p$$

$$(T \gg m)$$

★ $$n_i(T) = \frac{\zeta(3)}{\pi^2} g_i T^3 \eta_i' \quad \begin{cases} \eta_i' = 1 & B \\ \eta_i' = 3/4 & F \end{cases}$$

★ $$\rho_i(T) = \frac{\pi^2}{30} g_i \eta_i T^4 \quad \begin{cases} \eta_i = 1 & B \\ \eta_i = 7/8 & F \end{cases}$$

★ $$p_i(T) = \frac{1}{3} \rho_i(T)$$

Thermal history of the universe

ii) NON-RELATIVISTIC SPECIES $(T \ll m)$

$$E = m + \frac{p^2}{2m} \; ; \quad p^2 \ll m^2 \; ; \quad E \sim m$$

$$\star \quad n_i(T) = g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} e^{-m_i/T}$$

$$\star \quad \rho_i(T) = m\, n_i(T) + \frac{3}{2} T\, n_i(T) \sim m\, n_i$$

$$\star \quad p_i(T) = T\, n_i(T) \sim 0 \qquad (T \ll m)$$

Thermal history of the universe

d) TOTAL ENERGY DENSITY (in R.D. epoch)

$$\rho_{tot}(T) \equiv \frac{\pi^2}{30} g_*(T)\, T^4$$

$$g_*(T) \cong g_*^{rel}(T) = \sum_{i=B} g_i \left(\frac{T_i}{T}\right)^4 + \sum_{i=F} \frac{7}{8} g_i \left(\frac{T_i}{T}\right)^4$$

$$\Rightarrow \text{(i)} \quad H(T) \propto \sqrt{g_*(T)} \cdot \frac{T^2}{M_P}$$

e) ENTROPY DENSITY

$$s \equiv \frac{S}{V} = \frac{\rho + p}{T} \qquad dS = 0 \quad \text{adiabatic expansion}$$

dominated by rel. d.o.f.

$$s = \frac{2\pi^2}{45} g_*^s\, T^3$$

$$g_*^s = \sum_{i=B} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{i=F} g_i \left(\frac{T_i}{T}\right)^3$$

Thermal history of the universe

ABUNDANCE PARAMETER $\boxed{Y_i(T) \equiv \frac{n_i}{s} \sim n_i R^3}$ = # density in comoving volume

★ @ Equilibrium

Rel : $Y_i^{eq}(T) = \frac{45\,\zeta(3)}{2\pi^4} \frac{g_i\, n_i'}{g_{*s}(T)} \approx$ const

Non-Rel : $Y_i^{eq}(T) = \frac{45\, g_i}{4\sqrt{2}\,\pi^{7/2}} \frac{(m_i/T)^{3/2}}{g_{*s}(T)} e^{-m_i/T}$

Thermal history of the universe

ABUNDANCE PARAMETER $Y_i(T) \equiv \frac{n_i}{s} \sim n_i R^3$ = # density in comoving volume

* @ Equilibrium

Rel: $$Y_i^{eq}(T) = \frac{45\,\zeta(3)}{2\pi^4} \frac{g_i n_i'}{g_{*s}(T)} \approx \text{const}$$

Non-Rel: $$Y_i^{eq}(T) = \frac{45 g_i}{4\sqrt{2}\,\pi^{7/2}} \frac{(m_i/T)^{3/2}}{g_{*s}(T)} e^{-m_i/T}$$

* $\Gamma(T) \equiv H(T) \to T = T_D$ DECOUPLING

$\Rightarrow$ out of equilibrium

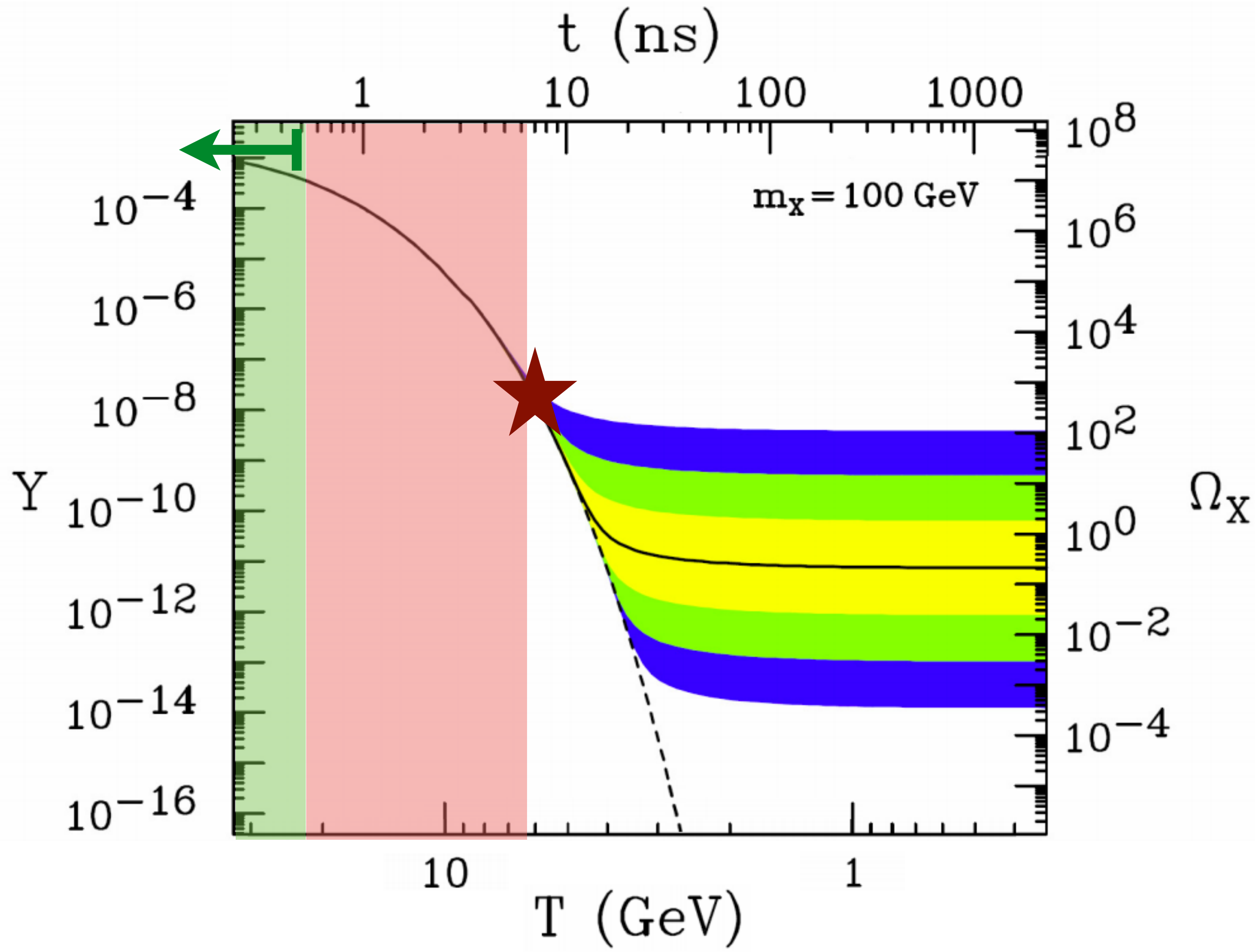
$Y_i(T_D)$ "freezes-out"

$Y_i(T < T_D) = Y_i(T_D) = Y_0$

$\rightsquigarrow$ Predictions:

* CMB
* CνB
* DM relic abundance

Thermal decoupling (freeze-out)



$$Y_i \equiv \frac{n_i}{s} \sim n_i a^3$$ comoving number density

$T \gg m_X \qquad Y_X^{\rm rel} \sim const$

$T \ll m_X \qquad Y_X^{\rm non-rel} \sim e^{-m_X/T}$

★ $Y_X(T_{\rm f.o.}) = Y_0$

Cold relic history very sensitive to details of decoupling because of rapid variation of Y_i ⟶ Sensitivity to **new physics** through:

- Interaction rate, i.e. interaction type
- Number of relativistic d.o.f for the evolution of H(T)

Thermal decoupling (freeze-out)

Formal derivation: phase space density evolution, **collisional Boltzmann equations**

$$L[f] = C[f]$$

$$\dot{n} = -3Hn + \frac{g}{(2\pi)^3}\int \frac{d^3p}{E} C[f]$$

Non-relativistic relic, in equilibrium through processes $X\bar{X} \leftrightarrow a\bar{a}$

$$\langle \sigma \cdot v_{\text{Møl}} \rangle = \frac{\int \sigma(s) \cdot v_{\text{Møl}} f_1(p_1) f_2(p_2) d^3p_1 d^3p_2}{\int f_1(p_1) f_2(p_2) d^3p_1 d^3p_2} \qquad v_{\text{Møl}} \equiv \frac{\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}}{E_1 E_2}$$

$$\dot{n}^{*} = -3Hn - \langle \sigma v_{\text{Møl}} \rangle (n^2 - n_{\text{eq}}^2)$$

$^{*}n = n_X + n_{\bar{X}}$

Thermal decoupling (freeze-out)

<u>Formal derivation</u>: phase space density evolution, **collisional Boltzmann equations**

in R-W metric:

$$L[f(E,t)] = E\frac{\partial f}{\partial t} - \frac{\dot R}{R}|\vec p|^2 \frac{\partial f}{\partial E}$$

from $n(T) = \frac{g}{(2\pi)^3}\int d^3p\, f(p)$

$\downarrow$

$$\frac{dn}{dt} = \dot n = \frac{g}{(2\pi)^3}\int d^3p\, \frac{\partial f}{\partial t}$$

$$\Rightarrow \dot n = \frac{g}{(2\pi)^3}\int d^3p \left\{ \frac{1}{E} L[f] + \frac{\dot R}{R}\frac{|\vec p|^2}{E}\frac{\partial f}{\partial E} \right\}$$

upon integration by parts

$$\dot n + 3\frac{\dot R}{R} n = \frac{g}{(2\pi)^3}\int \frac{d^3p}{E} L[f]$$

B.E for $n(T)$

Thermal decoupling (freeze-out)

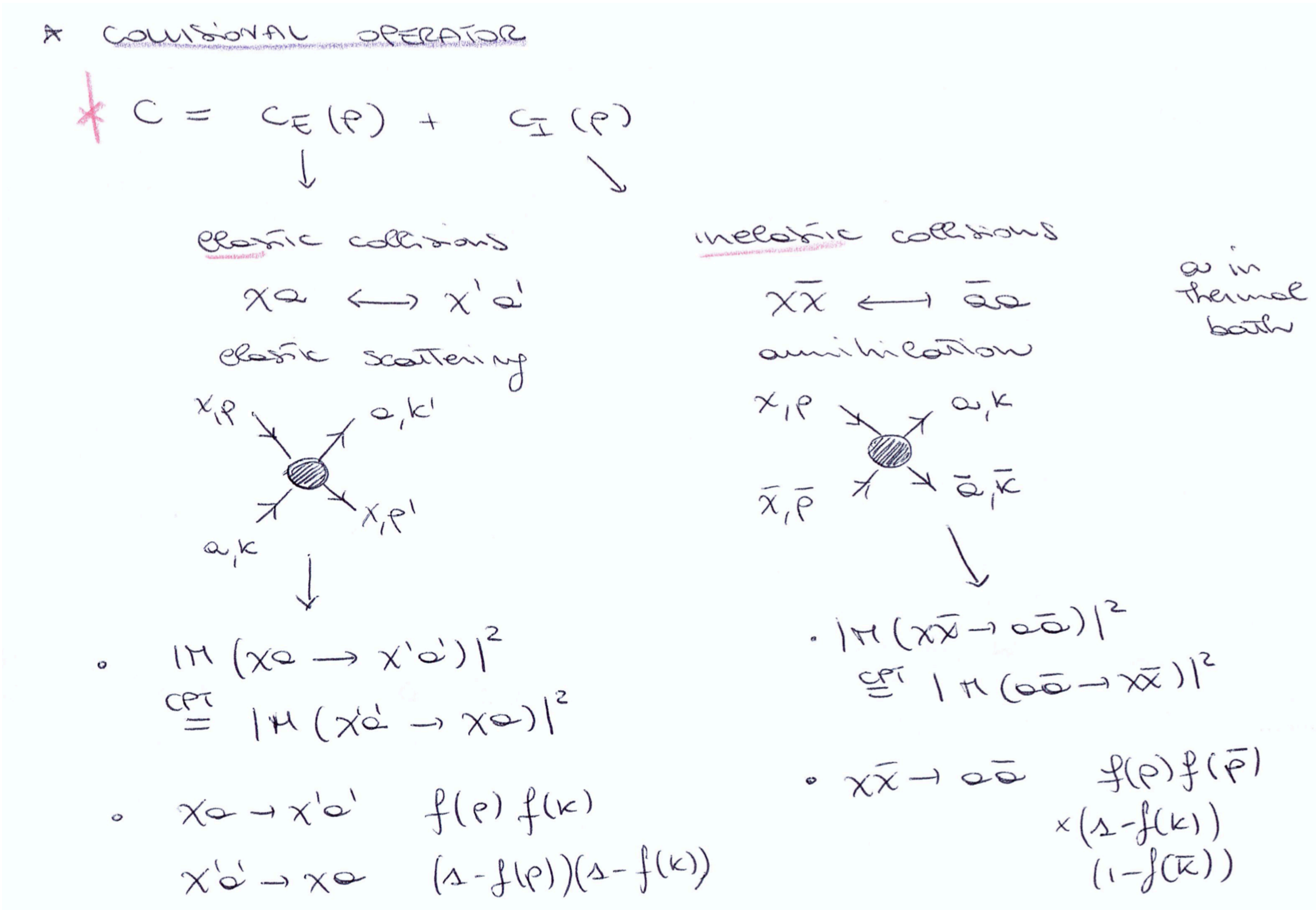
<u>Formal derivation</u>: phase space density evolution, **collisional Boltzmann equations**

$$\Rightarrow \begin{cases} \star \quad \boxed{\dot{n} + 3\frac{\dot{R}}{R} n = 0} \quad \Rightarrow \quad n \propto R^{-3} \\ \circledast \\ \star \quad \boxed{\dot{n} + 3\frac{\dot{R}}{R} n = \frac{g}{(2\pi)^3}\int \frac{d^3p}{E}\, C[f]} \end{cases}$$

$$\star \quad \dot{n} = \underbrace{-3\frac{\dot{R}}{R} n}_{\text{depleted by expansion}} + \underbrace{\frac{g}{(2\pi)^3}\int \frac{d^3p}{E}\, C[f]}_{\text{sustained by collisions}}$$

Thermal decoupling (freeze-out)

<u>Formal derivation</u>: phase space density evolution, **collisional Boltzmann equations**



Thermal decoupling (freeze-out)

Formal derivation: phase space density evolution, **collisional Boltzmann equations**

- $$\frac{g_\chi}{(2\pi)^3}\int \frac{d^3p}{E}\, C_I(p) \neq 0$$

Hp:

1) $[1 - f(q)] \to 1$ corrective phase-space factors

2) classical limit of Fermi-Dirac and Bose-Einstein distributions

$$f(q) \simeq e^{-E_q/T}$$

energy conservation in $\chi\bar{\chi} \to a\bar{a}$

$$\Rightarrow f_a(k)\, f_{\bar{a}}(\bar{k}) \equiv f_{eq}^{\chi}(p)\, f_{eq}^{\bar{\chi}}(\bar{p})$$

3) only one species decouples at the time (no coannihilations)

Thermal decoupling (freeze-out)

Formal derivation: phase space density evolution, **collisional Boltzmann equations**

$$\dot{n}_\chi + 3\frac{\dot R}{R} n_\chi = \frac{g_\chi}{(2\pi)^3}\int \frac{d^3p}{E}\, C_I(p)$$

$$= \frac{g_\chi}{(2\pi)^3}\int \frac{d^3p}{E} \cdot d\Pi_{\bar p}\, d\Pi_{k}\, d\Pi_{\bar k}\, (2\pi)^4\, \delta^4(p-\bar p-k-\bar k) \sum_{\bar\chi o \bar o} |M|^2 \left\{ -f^{\chi}(p) f^{\bar\chi}(\bar p) + f^{\chi}_{eq}(p) f^{\bar\chi}_{eq}(\bar p) \right\}$$

$$\dot{n}_\chi + 3\frac{\dot R}{R} n_\chi = -2\langle \sigma_{ann} v_M \rangle \left(n_\chi n_{\bar\chi} - n_\chi^{eq} n_{\bar\chi}^{eq} \right)$$

Thermal decoupling (freeze-out)

<u>Formal derivation</u>: phase space density evolution, **collisional Boltzmann equations**

for species $\chi = x, \bar{x}$ $\qquad n_x + n_{\bar{x}} = n = 2n_x$

$$\Rightarrow \quad \dot{n} + 3\frac{\dot{R}}{R}n = -\langle \sigma_{ann} v_r \rangle \left(n^2 - n_{eq}^2\right)$$

$$-\langle \sigma_{ann} v_r \rangle n^2 \quad + \quad \langle \sigma_{ann} v_r \rangle n_{eq}^2$$

$\downarrow$ $\dot{n} < 0$ depleted by $x\bar{x} \rightarrow a\bar{a}$

$\downarrow$ $\dot{n} > 0$ sustained by $a\bar{a} \rightarrow \chi\bar{\chi}$

Thermal decoupling (freeze-out)

★ BOLTZMANN EQ FOR $Y(x)$; $x = m/T$

$$\frac{dY}{dx} = -\frac{s\,\langle \sigma_{ann} v_{rel} \rangle}{H(x)\cdot x}\left[Y^2(x) - Y_{eq}^2(x)\right]$$

knowing that (in R.D):

- $H(x) = A\, g_*^{1/2}(x)\, m^2\, x^{-2}$ $\quad A = 1.66\, M_P^{-1}$
- $s = \frac{2\pi^2}{45}\, g_*^{s}(x)\, m^3\, x^{-3}$
- $Y_{ep}(x) = \frac{n_{ep}(x)}{s}$

$$\Rightarrow \frac{dY}{dx} = -(0.264\, M_P)\,\frac{g_*^{s}(x)}{\sqrt{g_*(x)}}\,\frac{m}{x^2}\,\langle \sigma_{ann} v_{rel} \rangle_{(x)}^{x}$$

$$\times\left[Y^2(x) - Y_{ep}^2(x)\right]$$

Relic abundance: Hot relic

★ RELIC ABUNDANCE $Y_0 = Y(x = \infty)$

⊕ HOT RELIC $Y(x_f) = Y_{eq}(x_f) \overset{REL}{=} 0.278 \frac{g_{eff}'}{g_{*s}(x_f)}$

if $T_0^{(\gamma)} < m$ species non-rel today

$$\rho_0 = m \cdot n_0 = m s_0 Y_0 = 2970 \cdot \frac{m}{eV} \cdot Y_0 \, eV \, cm^{-3}$$

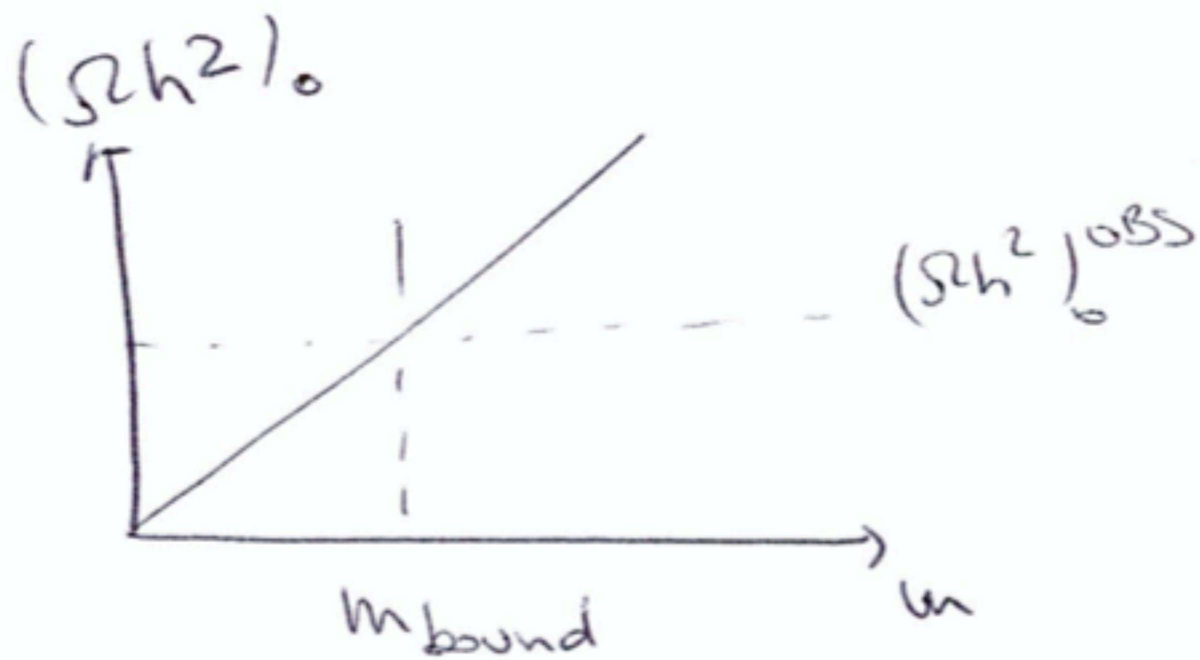
$$= 825 \frac{g_{eff}'}{g_*^s(x_f)} \left(\frac{m}{eV}\right) eV cm^{-3}$$

$$\Rightarrow (\Omega h^2)_0 = 7.83 \cdot 10^{-2} \frac{g_{eff}'}{g_*^s(x_f)} \left(\frac{m}{eV}\right)$$

Relic abundance: Hot relic

Hot relic, non-relativistic today

$$\Rightarrow \boxed{(\Omega h^2)_0 = 7.83 \cdot 10^{-2} \frac{g_{eff}}{g_{*}^{S}(x_f)} \left(\frac{m}{eV}\right)}$$



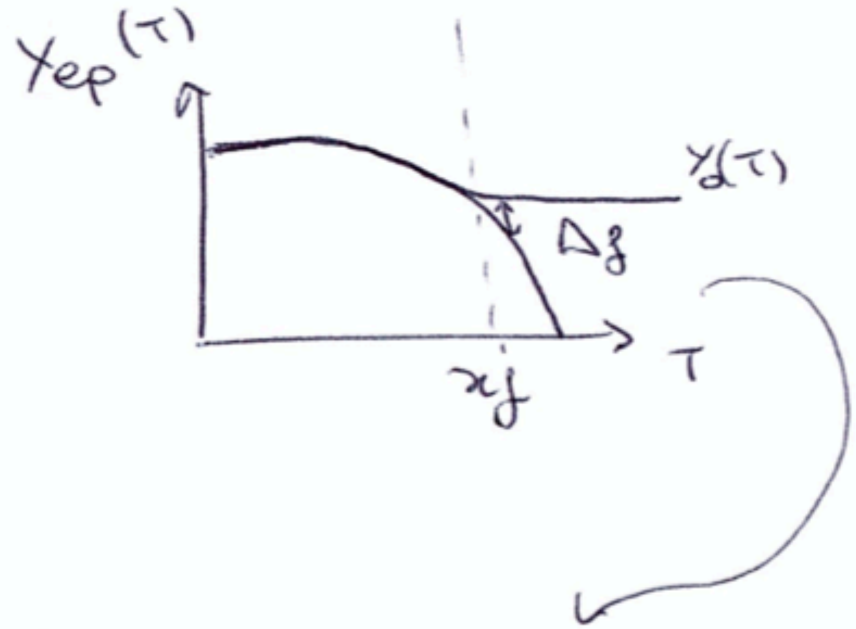
Cowsik-McClelland bound: Cosmological bound on the mass of a stable, light neutrino species

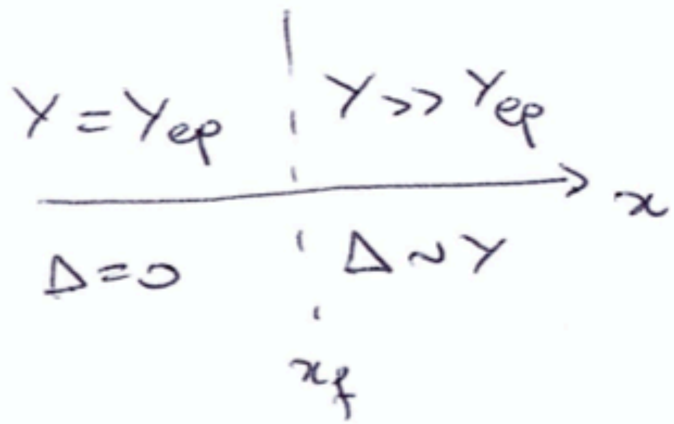
Cowsik, R. and McClelland, J., Phys. Rev. Lett. 29, 669 (1972)

Relic abundance: Cold relic

⊕ COLD RELIC

Details of decoupling are much more relevant to determine Y_0





Approx: 1. instantaneous freeze-out

$$\Delta(x_f) \equiv Y(x_f) - Y_{ep}(x_f) \overset{(*)}{=} c\, Y_{ep}(x_f)$$

$$c \sim O(1)$$

2. $Y(x > x_f) >> Y_{ep}(x)$

Relic abundance: Cold relic

★ RELIC ABUNDANCE Y_0

$x > x_f \qquad Y(x) \gg Y_{eq}(x) \to 0$

$$\Rightarrow \frac{dY}{dx} = -(0.264 M_P) \frac{g_{*s}(x)}{\sqrt{g_*(x)}} \frac{m}{x^2} \langle \sigma_{ann} v_M \rangle|_x \, Y^2(x)$$

Def

$$\langle \sigma_{ann} v_M \rangle_{int} = \frac{1}{m} \int_0^{T_f} \langle \sigma_{ann} v_M \rangle \, dT$$

$$\equiv \int_{x_f}^{\infty} \langle \sigma v \rangle|_x \, x^{-2} \, dx$$

$$Y(x = \infty) \equiv Y_0 = \frac{g_*^{1/2}(T_f)}{g_{*s}(T_f)} \frac{1}{0.264 M_P} \cdot \frac{1}{m \langle \sigma_{ann} v_M \rangle_{int}}$$

Relic abundance: Cold relic

★ RELIC ABUNDANCE Ωh^2

$$(\Omega h^2)_0 \equiv \frac{\rho_0}{10^4 \, eV cm^{-3}} \overset{N.R.}{=} \frac{m s_0 Y_0}{10^{-5} \, GeV cm^{-3}}$$

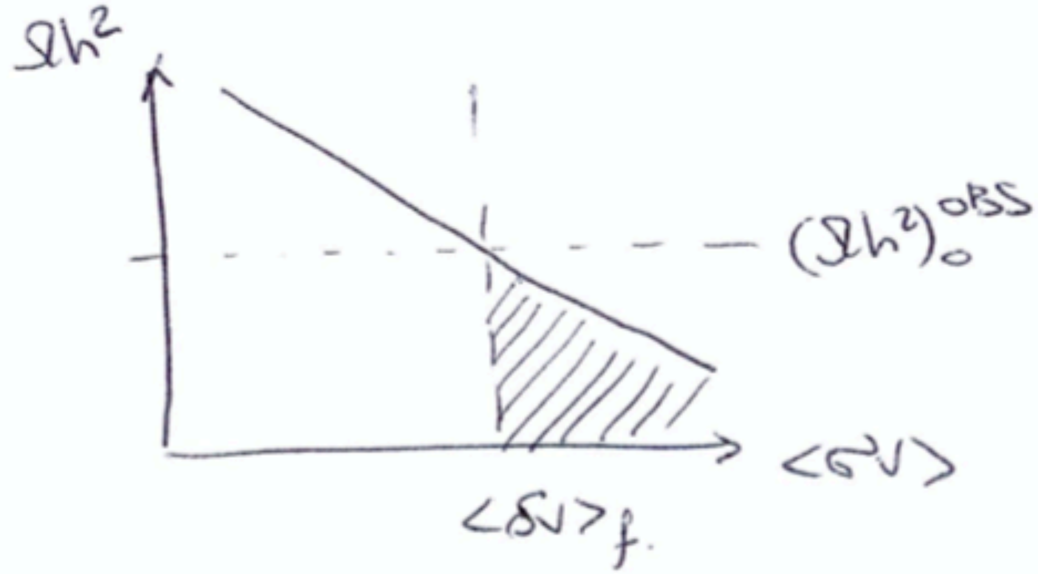
$$\Rightarrow \boxed{(\Omega h^2)_0 = 9.2 \cdot 10^{-11} \, \frac{g_*^{1/2}(T_f)}{g_{*S}(T_f)} \left(\frac{GeV^{-2}}{\langle \sigma_{ann} v_{rel} \rangle_{int}} \right)}$$

Relic abundance: Cold relic

* RELIC ABUNDANCE Ωh^2

$$(\Omega h^2)_0 \equiv \frac{\rho_0}{10^4\ \text{eV cm}^{-3}} \overset{\text{N.R.}}{=} \frac{m\, s_0\, Y_0}{10^{-5}\ \text{GeV cm}^{-3}}$$

$$\Rightarrow \quad (\Omega h^2)_0 = 9.2 \cdot 10^{-11}\ \frac{g_*^{1/2}(T_f)}{g_{*s}(T_f)} \left(\frac{\text{GeV}^{-2}}{\langle \sigma_{ann} v_r \rangle_{int}} \right)$$



~> upper limit on $\langle \sigma v \rangle$ from measurement of $(\Omega h^2)_0$

~> information about type of interactions.

$\langle \sigma v \rangle \uparrow$ later decoupling

$(\Omega h^2) \downarrow$

The WIMP “miracle” or coincidence

Theory prediction

$$(\Omega h^2)_0 = 9.2 \cdot 10^{-11} \, \frac{g_*^{1/2}(T_f)}{g_{*S}(T_f)} \left(\frac{\mathrm{GeV}^{-2}}{\langle \sigma_{ann} v_r \rangle_{int}} \right)$$

Experimental measure

$$(\Omega h^2)_0 \sim 0.1$$

The WIMP "miracle" or coincidence

Theory prediction

$$(\Omega h^2)_0 = 9.2 \cdot 10^{-11} \frac{g_*^{\frac{1}{2}}(T_f)}{g_{*s}(T_f)} \left(\frac{GeV^{-2}}{\langle \sigma_{ann} v_r \rangle_{int}} \right)$$

Experimental measure

$$(\Omega h^2)_0 \sim 0.1$$

$$\Rightarrow \langle \sigma v \rangle_{int} \sim 10^{-10} \; GeV^{-2}$$

$$\sim 0.1\, pb$$

✓ weak cross section

$$1\, barn = 10^{-28}\, m^2$$

$$1\, pb = 2.6 \cdot 10^{-9}\; GeV^{-2}$$

The WIMP "miracle" or coincidence

Theory prediction

$$(\Omega h^2)_0 = 9.2 \cdot 10^{-11} \frac{g_*^{1/2}(T_f)}{g_{*s}(T_f)} \left(\frac{\mathrm{GeV}^{-2}}{\langle \sigma_{ann} v_r \rangle_{int}} \right)$$

Experimental measure

$$(\Omega h^2)_0 \sim 0.1$$

⇒ Thermal history of the Universe naturally explains presence of relic abundance from decoupled particles

If $m \sim 100$ GeV → $\Omega h^2 \sim 0.1$ for $\langle \sigma v \rangle \sim$ weak scale

WEAKLY INTERACTING MASSIVE PARTICLES

can explain cosmological observation of $(\Omega h^2)_{CDM}$

Lecture 2

1. Observational evidence for dark matter
2. Fundamental properties of dark matter
3. The dark matter landscape
4. Searches for WIMPs

References in the slides

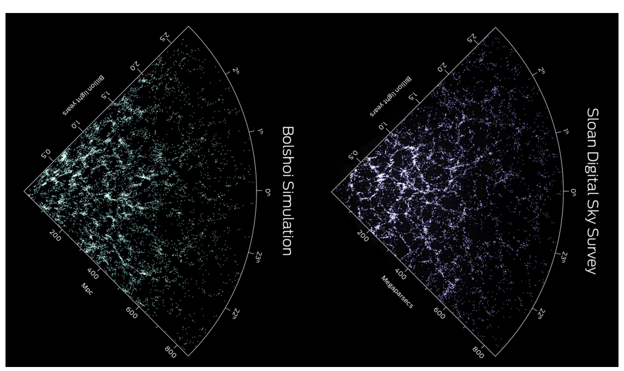
1. Observational evidence for dark matter

Dark matter gravitational evidence

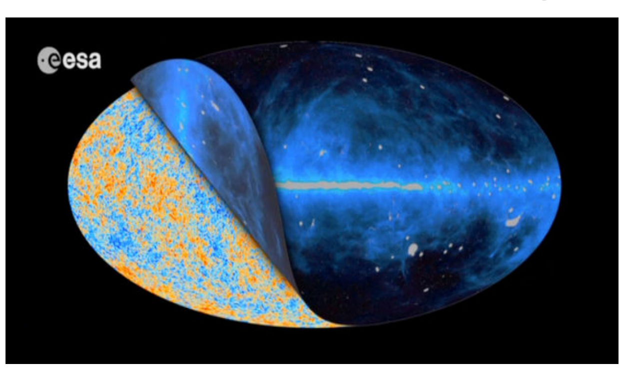
Rotation curves

Galaxy clusters

Large Scale structures



Cosmic microwave background

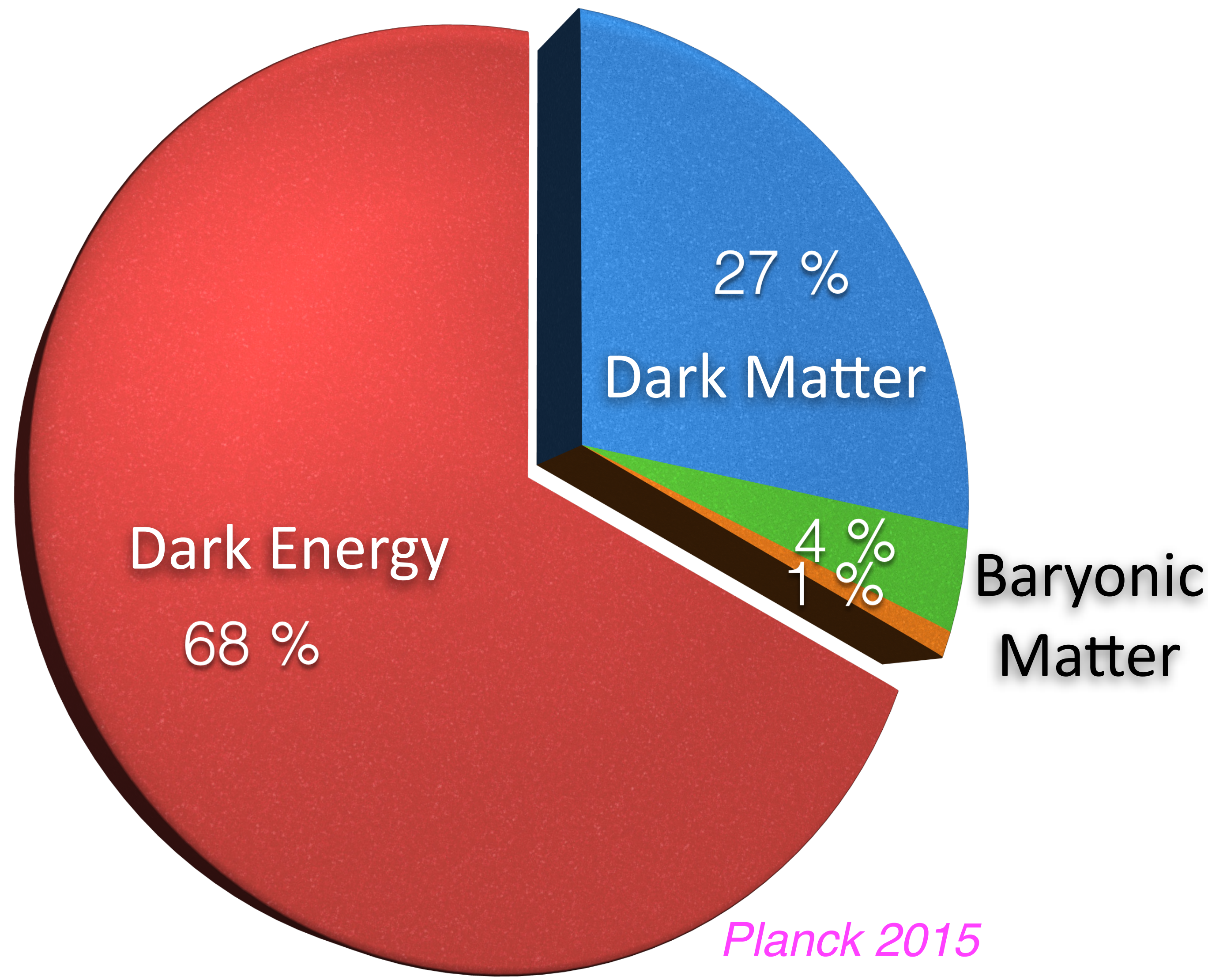


~kpc

~Mpc

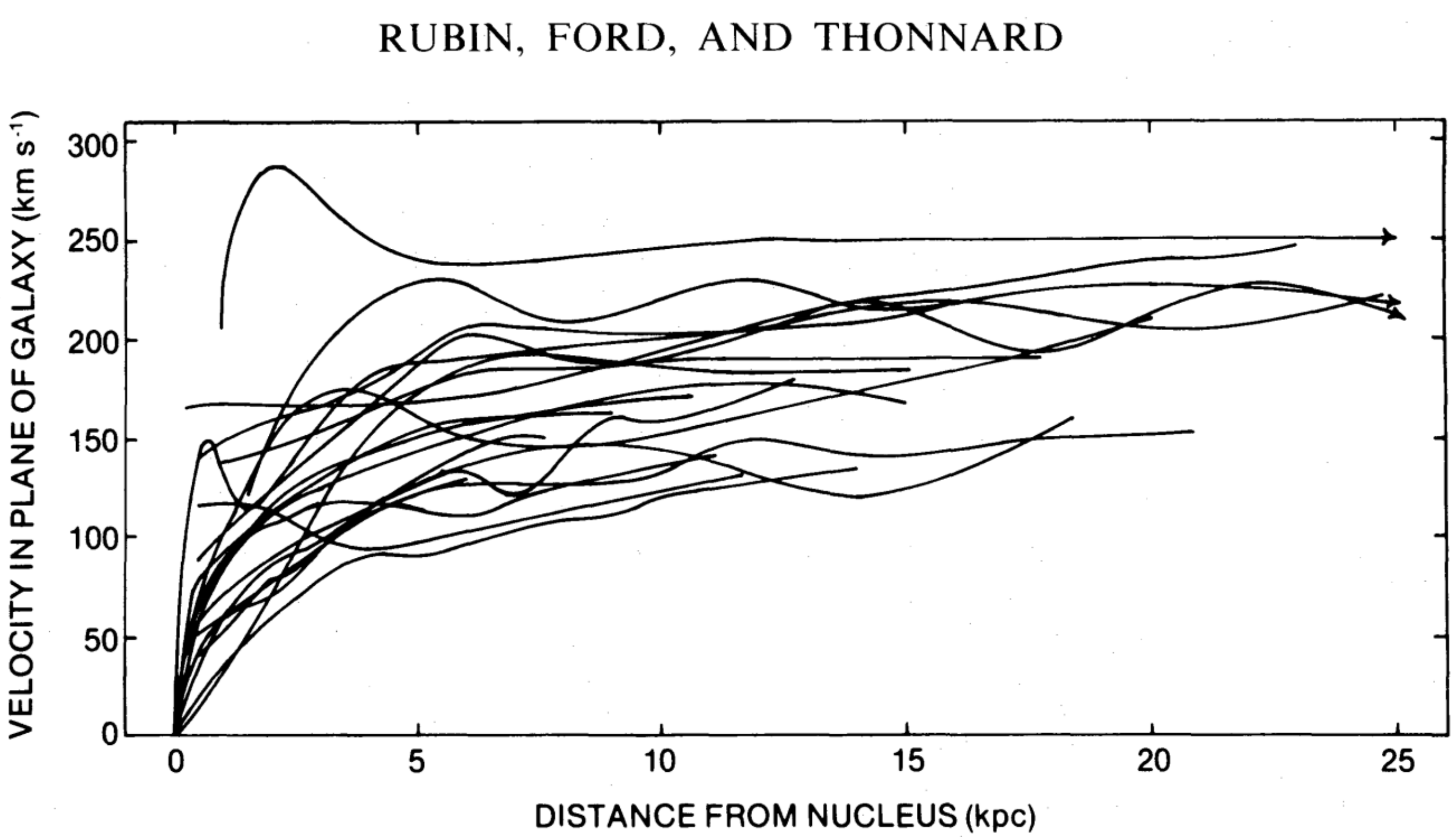
~Gpc

We do not know what most of the Universe is made of!



Dark matter constitutes about 85% of the matter content of the Universe.

Flat galactic rotation curves



'70/'80: observation of spiral galaxies,
rotation supported systems like the Milky Way

V. C. Rubin and W. K. Ford, Jr., ApJ 159, 379 (1970); V. C. Rubin, N. Thonnard and W. K. Ford, Jr., ApJ 238, 471 (1980)

$$v_c^2(<R) = R\frac{d\phi_{\rm tot}}{dR} = \frac{GM(<R)}{R}$$

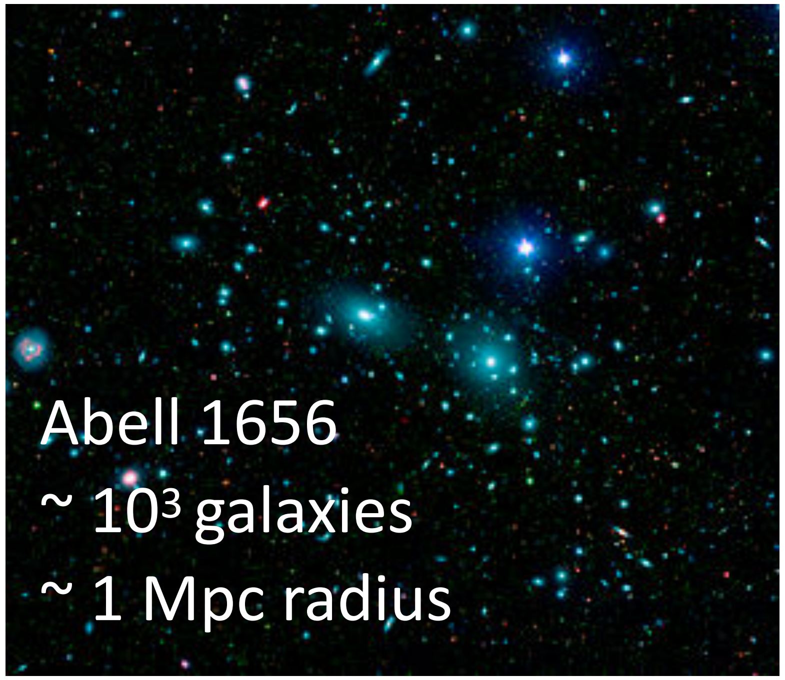
$$M(<R) \equiv 4\pi \int_0^R r^2 \rho(r) dr$$

Predicted from visible light: $v_c^2 \propto \frac{1}{R}$

Observed: $v_c^2 \sim constant$ → $\rho(r) \propto \frac{1}{r^2}$

Data are well described by an additional “matter" component,
but also **MOND** works at these scales

Dark matter in the Coma Cluster



Pioneering application of the **virial theorem** in astronomy

F. Zwicky, Helvetica Physica Acta (1933) 6, 110–127; ApJ (1937) 86, 217

$$2\langle T\rangle + \langle U_{\rm tot}\rangle = 0 \qquad\qquad U(r) \propto r^{-1}$$

$$T = N\frac{m}{2}\langle v^2\rangle$$

$$\langle U_{\rm tot}\rangle \sim -\frac{3}{5}\frac{G_N M^2}{R}$$

gravitational potential of a self-gravitating homogeneous sphere of radius R

$$\longrightarrow \qquad M \sim \mathcal{O}(1)\frac{R\langle v^2\rangle}{G_N} \sim 3 \times M_{\rm visible}$$

X-rays and gravitational lensing

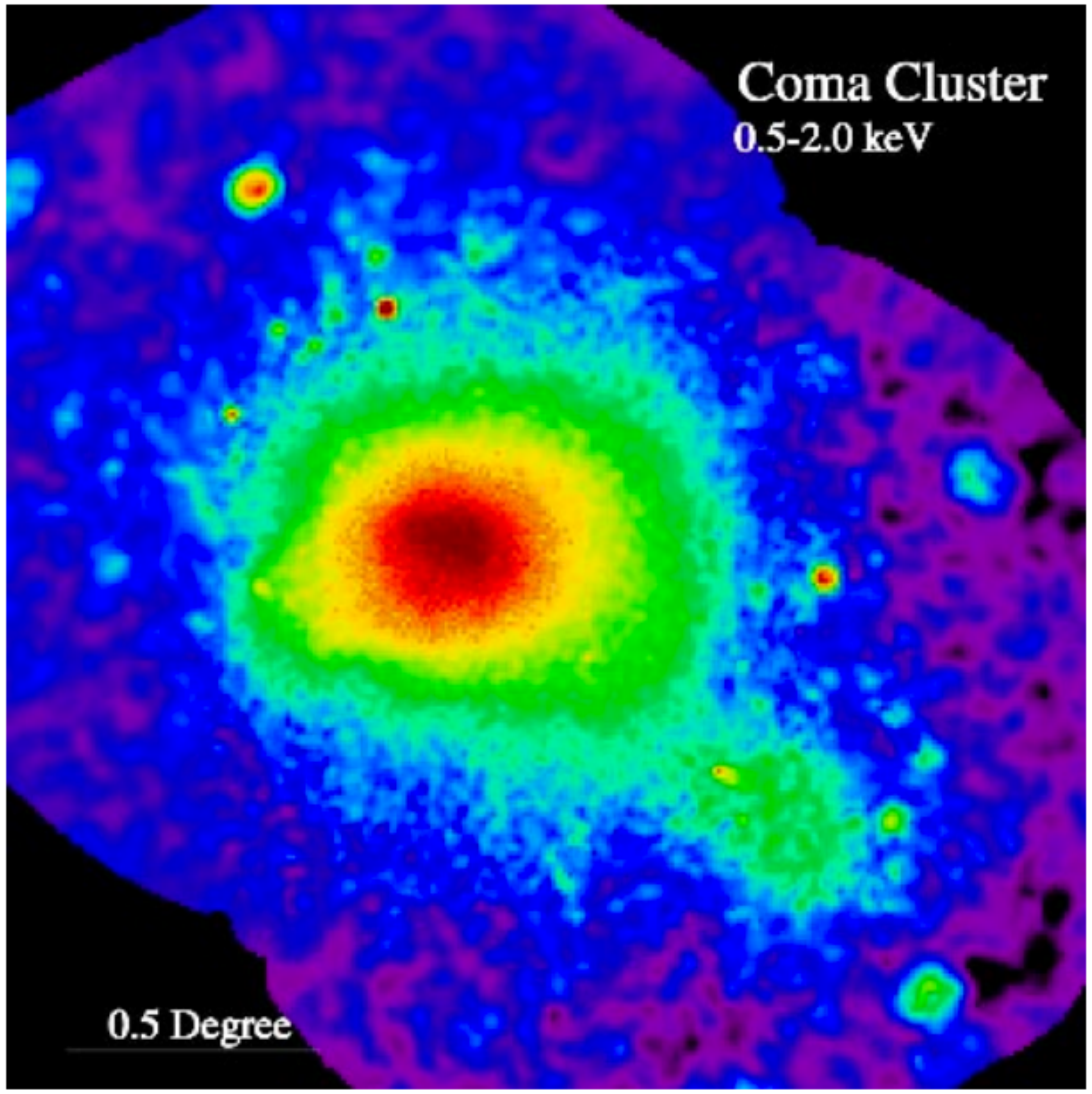
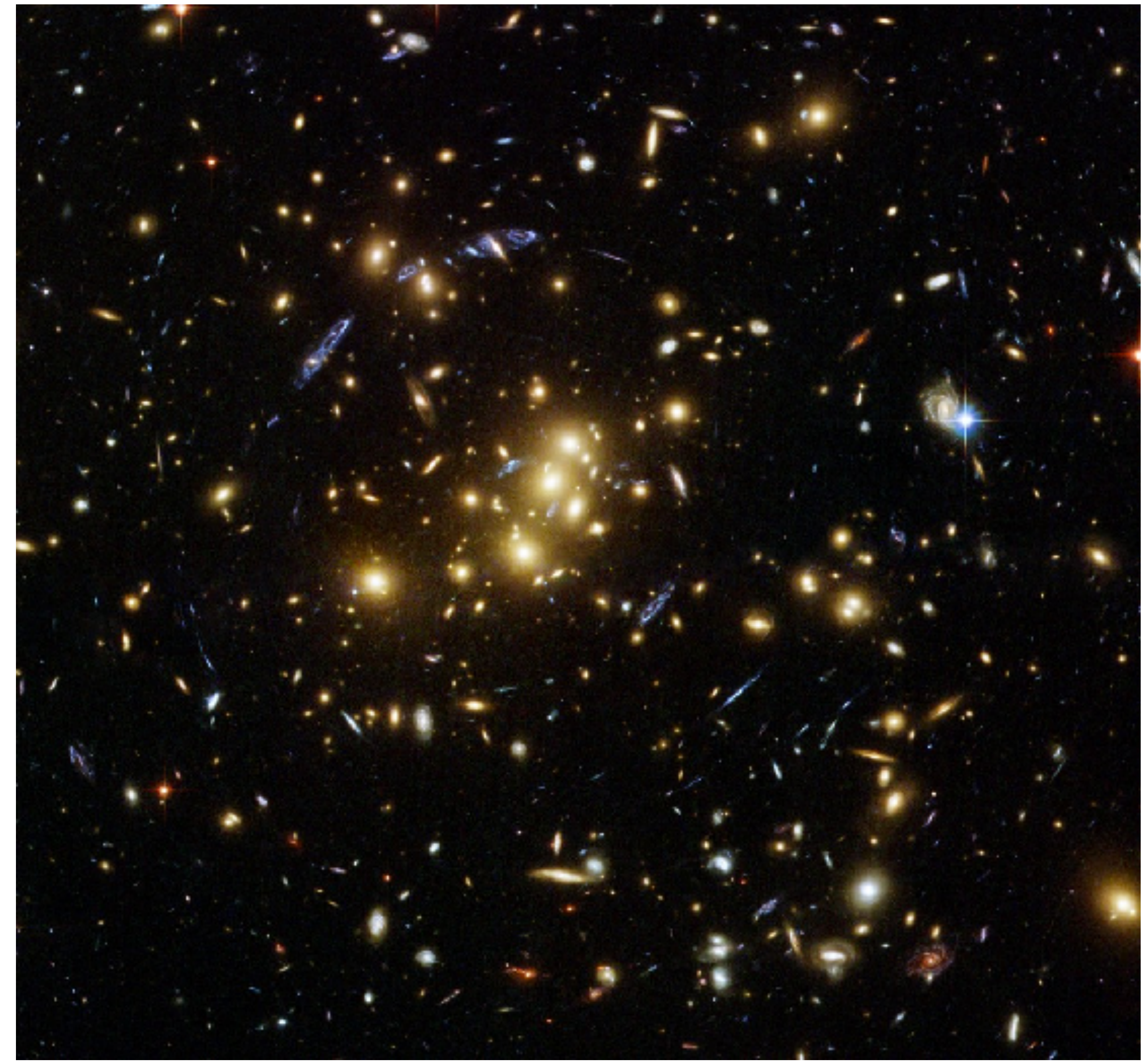


Figure 2. An x-ray image of the Coma cluster obtained with the ROSAT satellite, showing both the main cluster and the NGC4839 group to the south-west. (Credit: S L Snowden, High Energy Astrophysics Science Archive Research Center, NASA.)

Mass in clusters is in the form of hot, intergalactic gas, which can be traced via X rays: X-luminosity and spectrum constrain the mass profile

Lewis, Buote & Stocke, ApJ (2003), 586, 135



Strong gravitational lensing around galaxy cluster CL0024+17, demonstrating at least three layers projected onto a single 2D image.

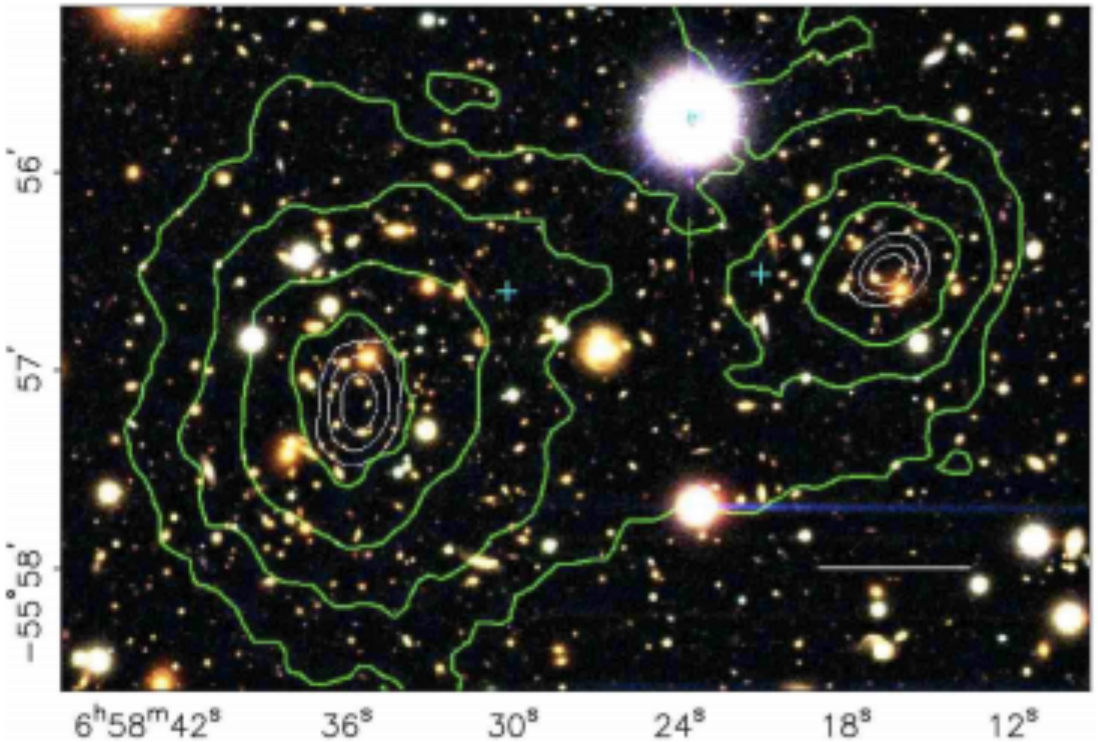
Massey, Kitching & Richard, Rept.Prog.Phys. 73 (2010)

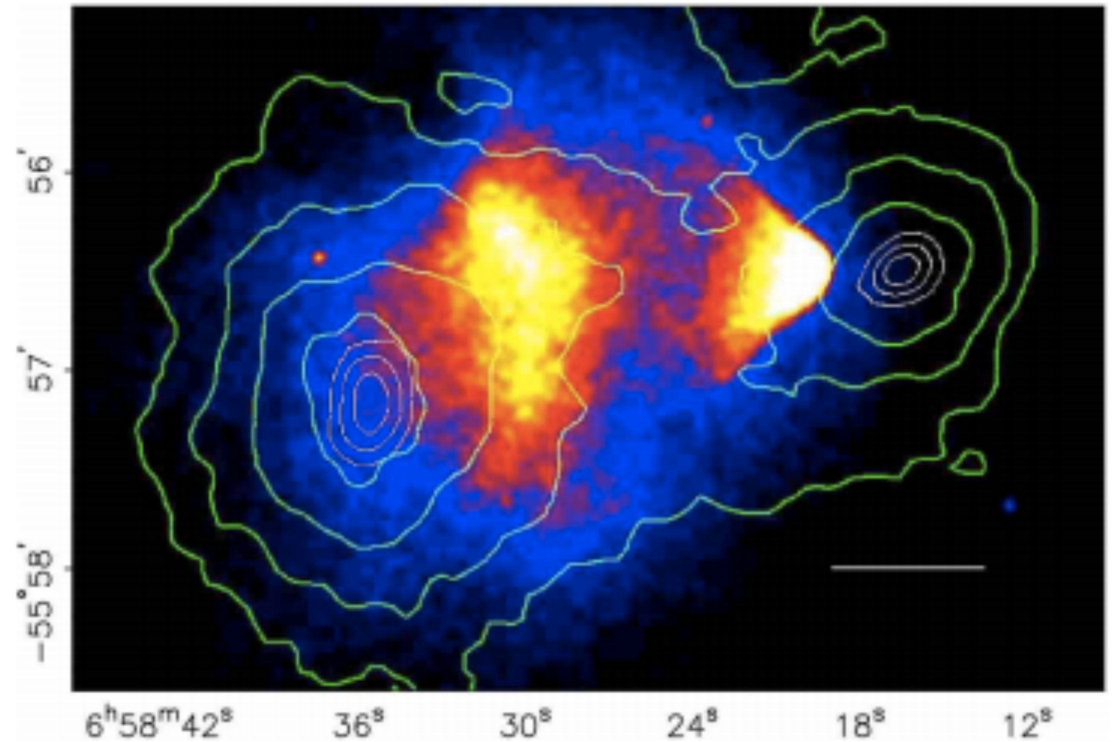

James Webb Space Telescope
JWST
NASA • ESA • CSA

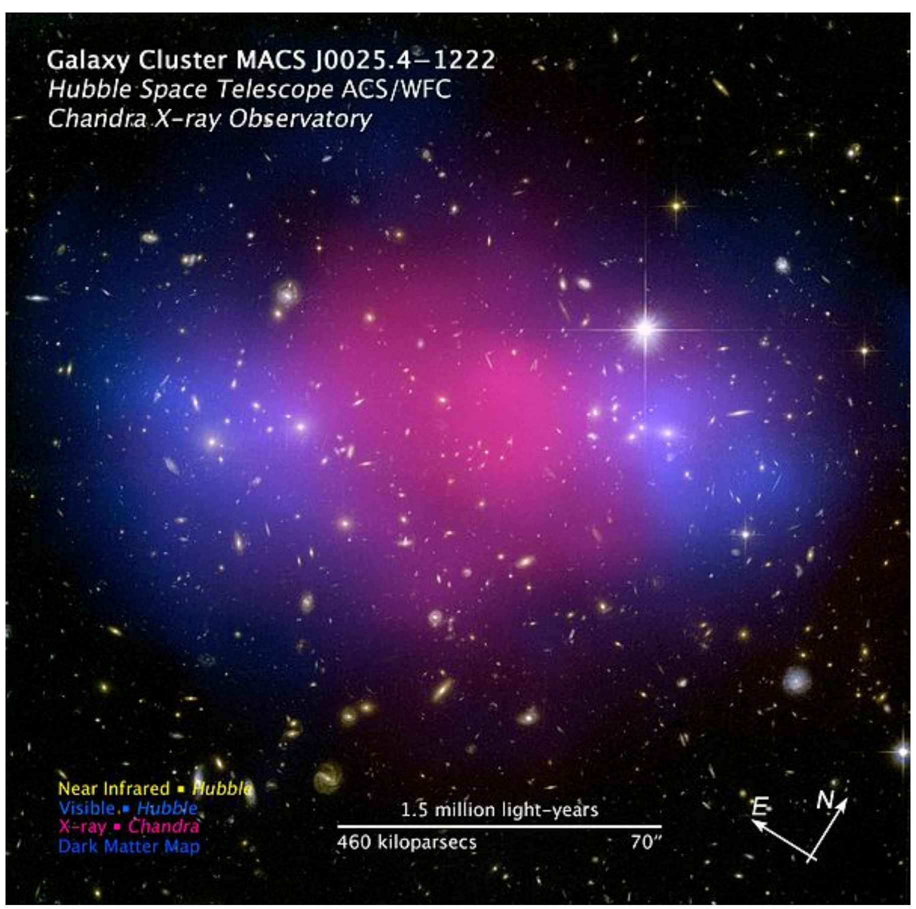
Segregation of matter in clusters

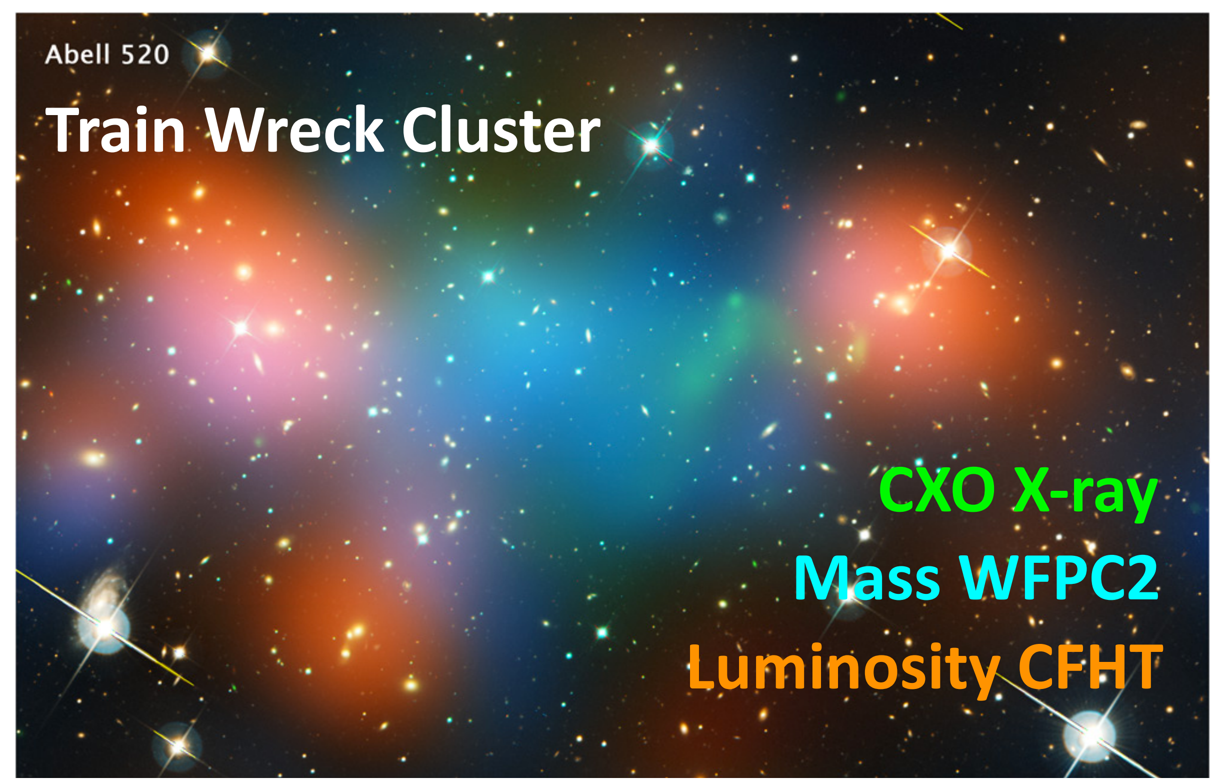
Bullet Cluster (1E 0657-56)

Clowe+, ApJ 604 (2004) 596-603; Clowe+ ApJ, 648 (2006) L109



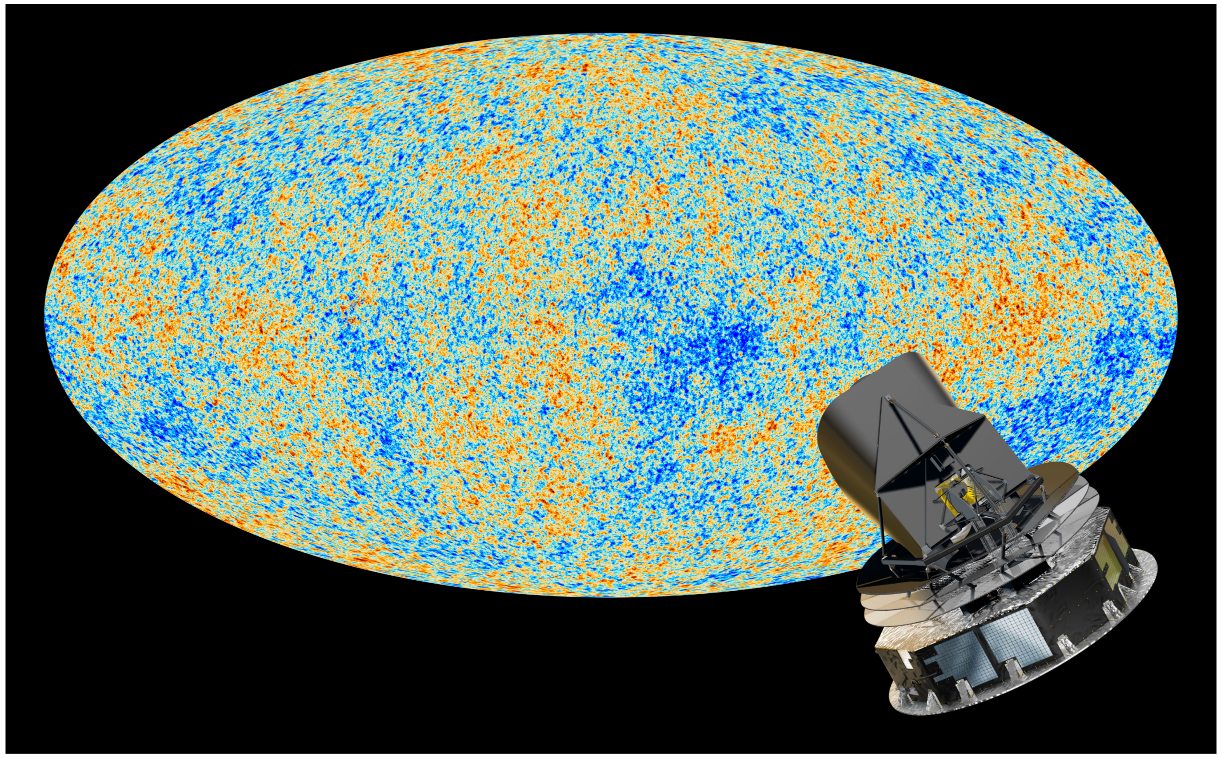






James Jee+, ApJ 783 (2014) 78

Cosmic Microwave Background



$$\Omega_i \equiv \frac{\bar{\rho}_i}{\rho_c}$$ Abundance species *i*

Critical density
(average density of a flat Universe)

$$\rho_{\rm c} \equiv \frac{3H_0^2}{8\pi G_N}$$

10 protons per **cubic meter**
[1 GeV ~ 10^{-24} g]

$$\bar{\rho}_{\rm DM} \simeq 0.3\rho_c \longrightarrow \bar{\rho}_{\rm DM} \sim 10^{10}\,\frac{\rm M_\odot}{\rm Mpc^3} \sim 10^{-6}\,\frac{\rm GeV}{\rm cm^3}$$

Galaxy clusters: 10^5 denser!
Galaxies: 10^6 denser!

$$\frac{\delta\rho}{\rho} \gg 1$$

The Universe today is highly **non-linear**!

Cosmic Microwave Background

T > T_{CMB} tight coupling between photons and baryons
and presence of primordial overdensities $\delta > 0$

Gravitational vs radiation pressure => acoustic oscillations

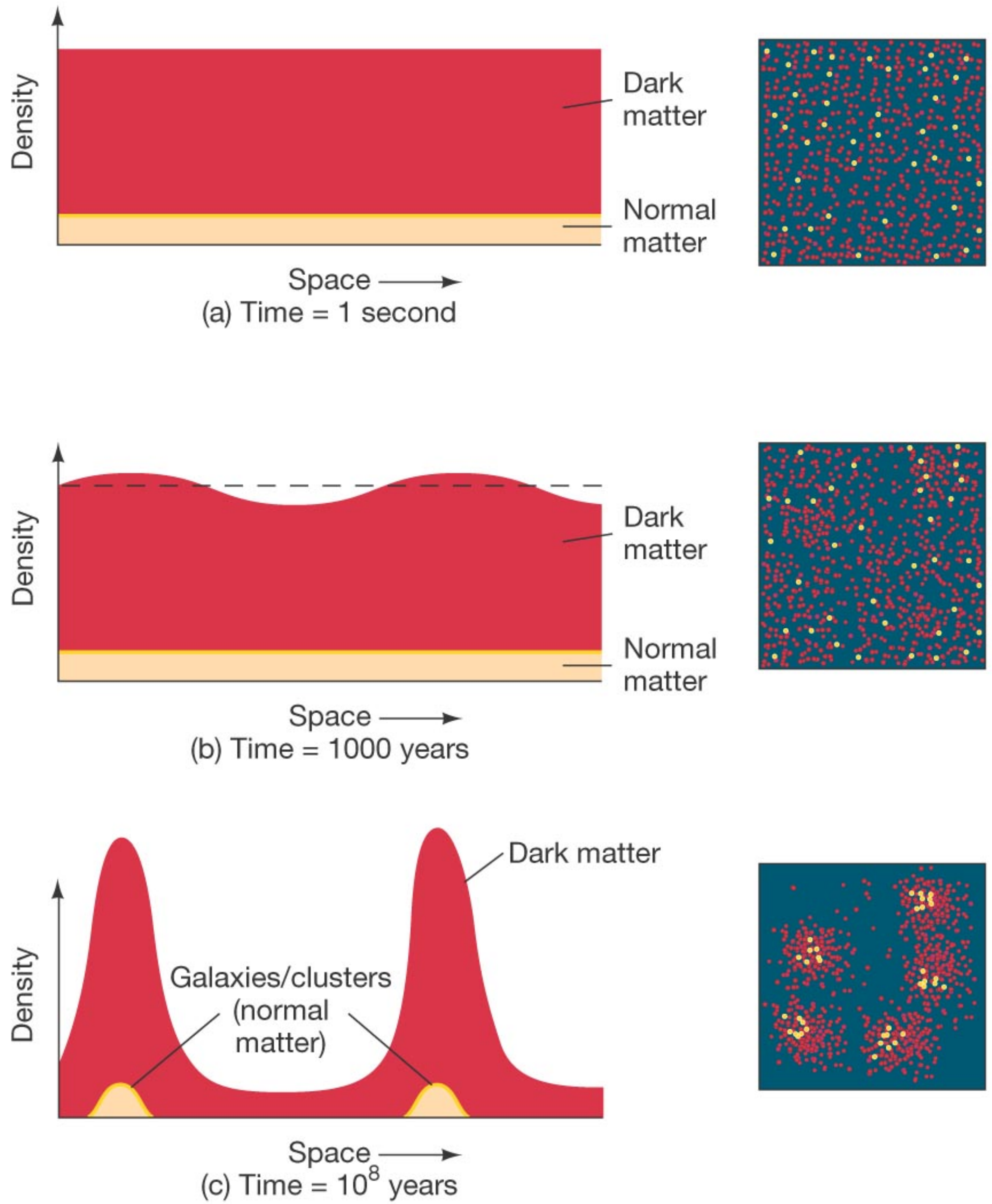
$$\frac{\delta n_\gamma}{n_\gamma} \sim 3\frac{\delta T}{T} \sim \frac{\delta n_b}{n_b} \equiv \delta \qquad n_\gamma \propto T^3$$

$$\frac{\Delta T}{T} \sim 10^{-5}$$ on Mpc scales @ z_{CMB}~ 1100

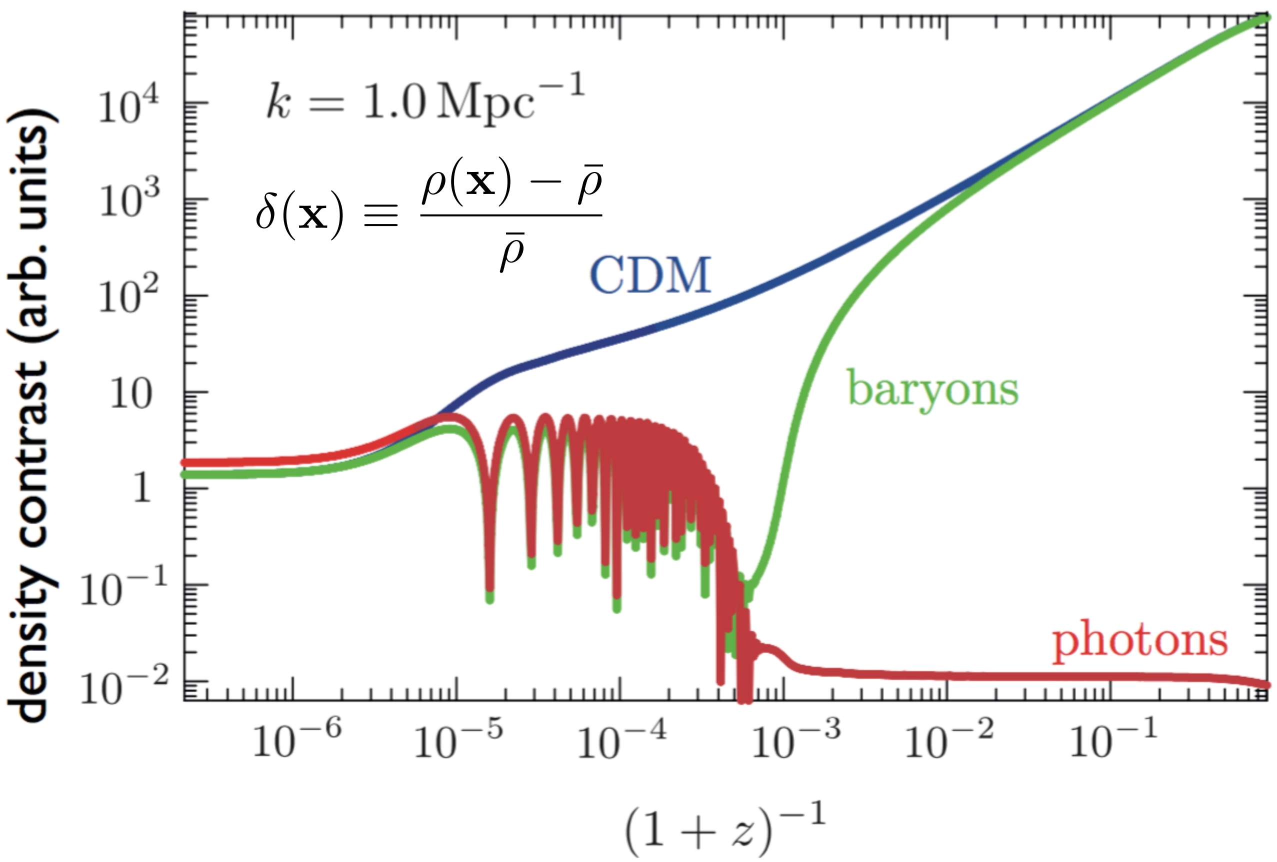
$$\frac{\Delta n_b}{n_b} \sim 10^{-5}(1+z_{\rm CMB})^{-1} \sim 0.01$$ in a **matter** dominated Universe

→ With baryonic matter only, structure formation would be very different! We need a **non-baryonic** component that decouples from photons early enough to create deep potential wells.

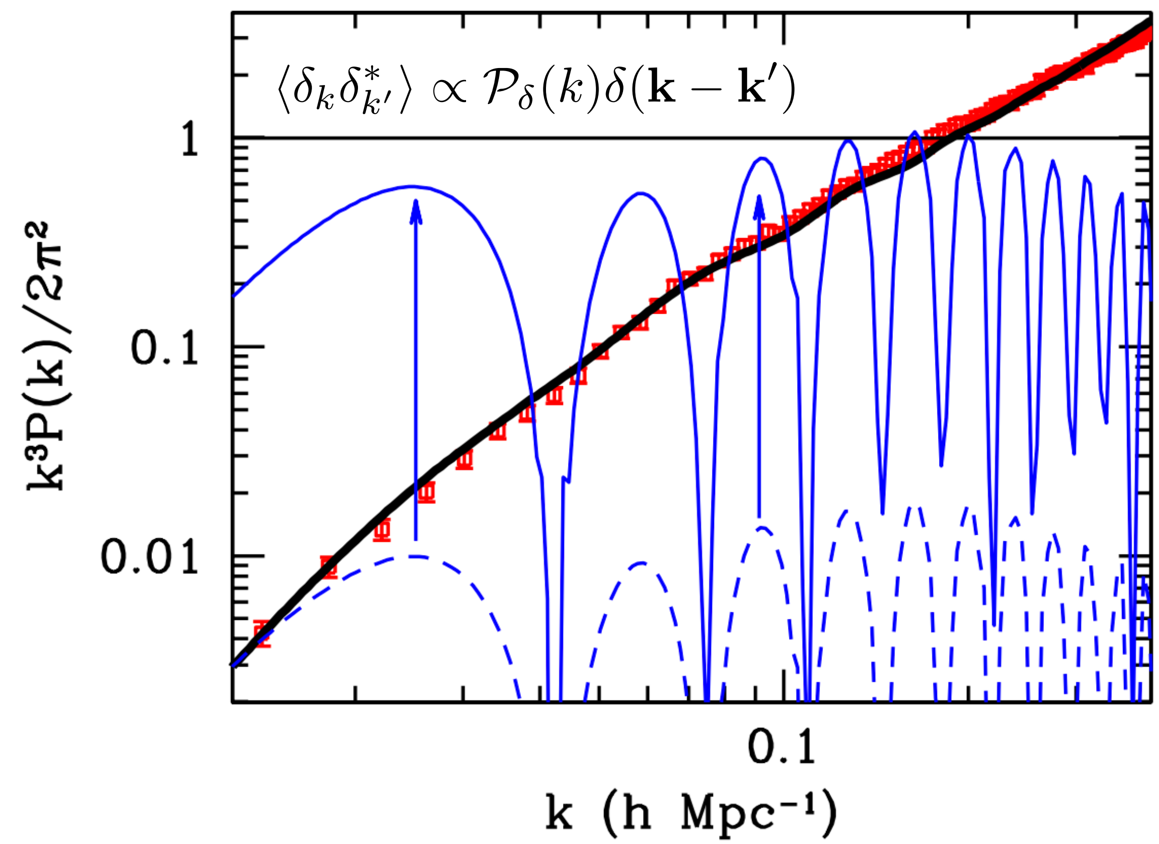
Growth of structures: cartoon



The growth of structures



S. Dodelson, 1112.1320



Baryons coupled to radiation and pressure prevents overdensities from growing, while the (uncoupled, pressureless) CDM mode grows after decoupling.
After recombination, baryons quickly fall in the CDM potential wells.

Even in modified cosmology (TeVeS) structures go non-linear, but the predicted power spectrum of matter density fluctuation is entirely wrong

+ Successful observation of baryonic acoustic oscillations (BAO)!

BOSS Collab., MNRAS 441 (2014), 24-62

2. Fundamental properties of dark matter

Properties of dark matter

What fundamental properties can we infer from this astro/cosmo evidence?

How much dark matter at cosmological scales?

$$\Omega_{\rm CDM} \sim 0.26$$

Planck 2015, 68% CL

The dominant component of dark matter in the Universe should be:

1. Non-relativistic at decoupling, i.e. **cold**
2. **Stable** or long-lived
3. **Sufficiently heavy**, to behave “classically”
4. Smoothly distributed at cosmological scales
5. Dark and **dissipationless**
6. **Collisionless**, i.e. not very collisional

DM evidence requires new physics, beyond current theories
=> **new d.o.f., appealing from a particle physics perspective**

1. Non-relativistic @ decoupling (CDM)

Primordial density fluctuations modified by non-linear effects: gravitation, pressure, dissipation, etc. => N-body simulations are needed to follow the growth in non-linear regime.

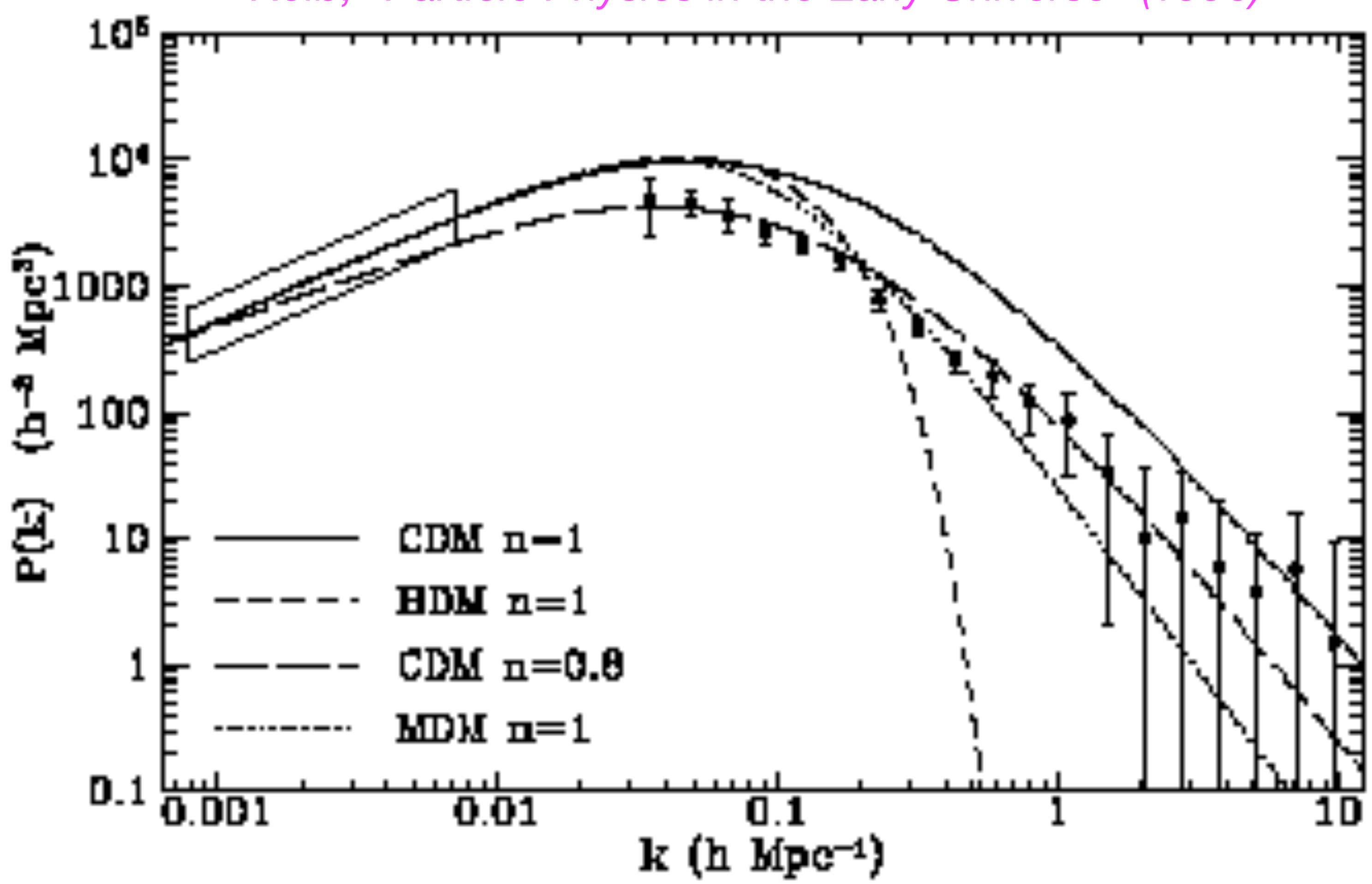
Collisions-less species (neutrinos, DM): free stream from overdense to underdense regions and wash out perturbations => damping of small scale density perturbations

Kolb, "Particle Physics in the Early Universe" (1998)

$$\lambda_{\rm phys} \lesssim \lambda_{\rm fs}$$

$$\lambda_{\rm fs} \sim \frac{\nu(t_{\rm eq})}{H(t_{\rm eq})}$$

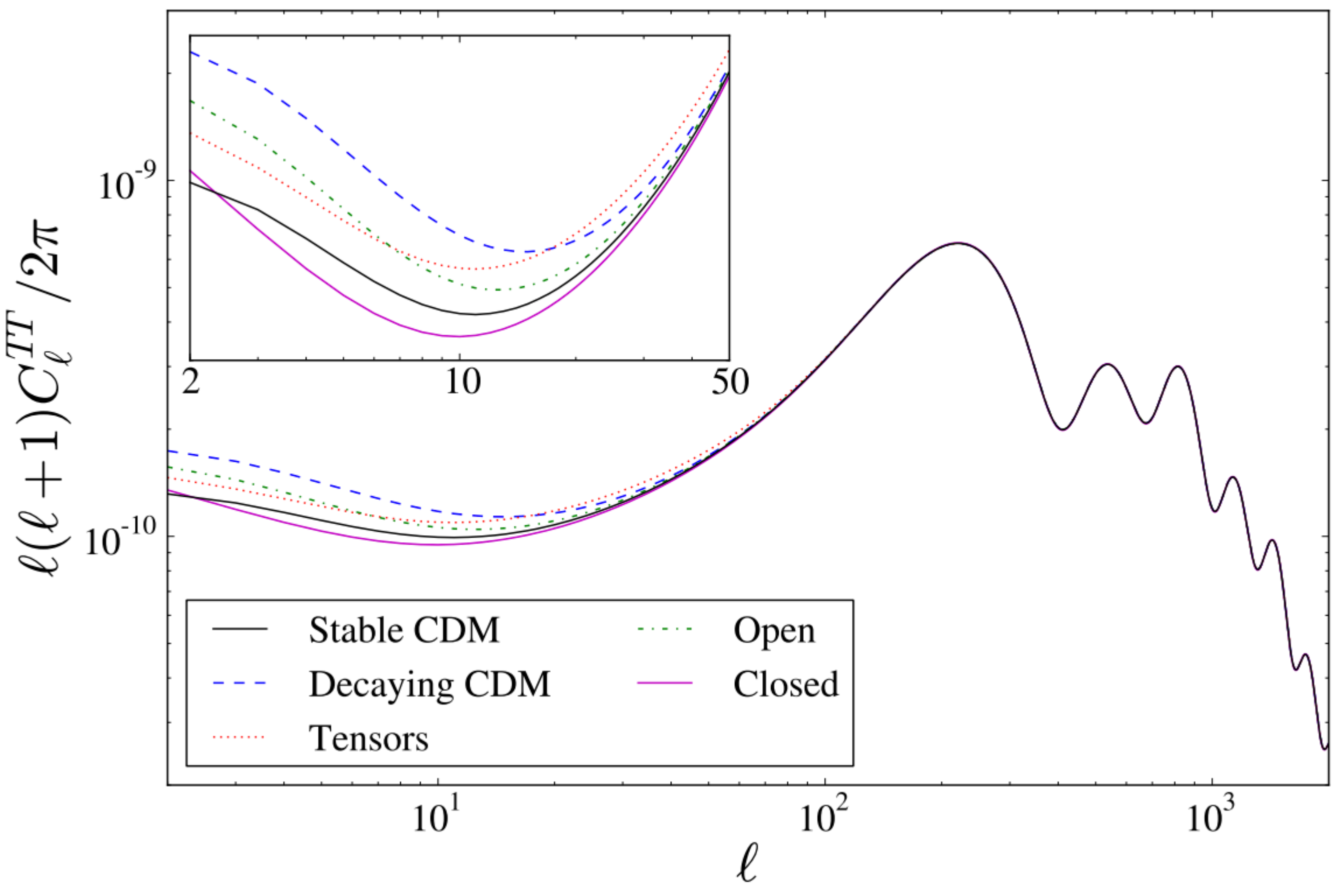
Characteristic imprint in the matter power spectrum and galaxy distribution



2. Stable or long-lived

Model independent bounds exist from CMB & Large Scale structures if decay into invisible species

Audren+, JCAP 12 (2014) 028;
Poulin+, JCAP 1608 (2016) no.08, 036



$\tau \gtrsim 160\,\mathrm{Gyr}$ CMB only

$\tau \gtrsim 170\,\mathrm{Gyr}$ + other consistent data

More stringent bounds from astrophysics and CMB if decaying into "visible" species ($e^\pm$,γ)

$$\tau \gtrsim 10^{26}\,\mathrm{sec}$$

3. Sufficiently massive (LL from localisation)

Evidence of DM at astrophysical scales => Localised therein and behave classically

Dark matter gravitationally bound on scales at least as large as dSph

$$\lambda_{\rm De\,Broglie} = \frac{h}{mv} \lesssim {\rm kpc} \longrightarrow m \gtrsim 10^{-22}\,{\rm eV}\;({\rm v} \sim 100\,{\rm km/s})$$

If DM is a fermion: Pauli exclusion principle holds and the phase space density is further constrained

Tremaine-Gunn bound

$$\bar{f} \lesssim \frac{g}{h^3}$$

$$m > \mathcal{O}(10 - 100\,{\rm eV})$$

Tremaine & Gunn, PRL 42 (1979) 407;
Boyarsky, Ruchayskiy & Iakubovskyi, JCAP 0903 (2009) 005

[From conservation of phase space density of a non-interacting fluid (Liouville equation)]

4. Smoothly distributed (not "granular")

On galaxy scales, we do not detect any "granularity" of dark matter; should have a continuum "fluid" limit

➡ Granular distribution would provide time-dependent gravitational potentials, which might disrupt bound systems of different sizes => heat the galactic disk or disrupt globular clusters

Lacey & Ostriker, ApJ 299 (1985) 633; Moore, ApJ 413 (1993) L93; Rix & Lake, ApJ 417 (1993) L1

➡ Additional Poisson noise into matter power spectrum

Afshordi+, ApJ 594 (2003) L71

$$m \lesssim 10^{3-4}\, M_\odot \sim 10^{70-71}\,\mathrm{eV}$$

5. Optically dark and dissipationless

Very weak e.m. interaction

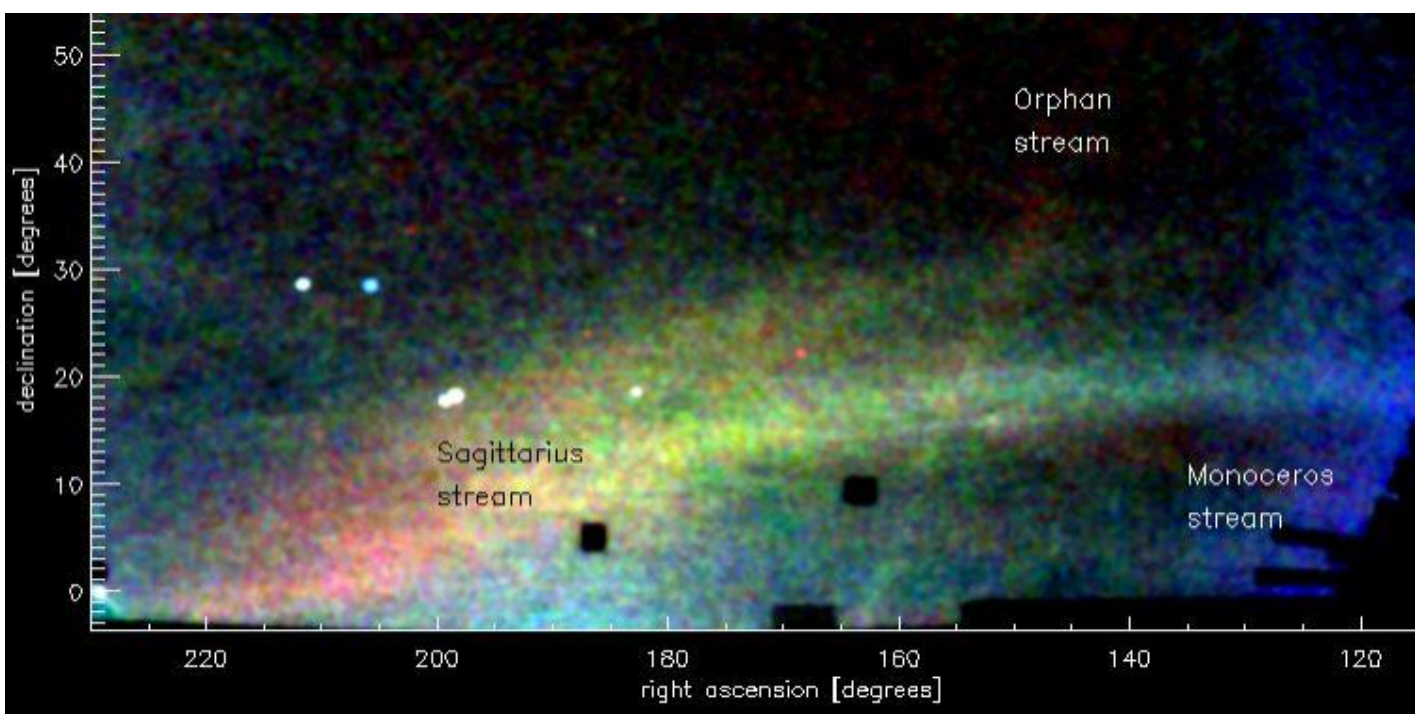
$$\sigma_{DM-\gamma} \leq 8 \times 10^{-31} \, (m_{DM}/\mathrm{GeV}) \; \mathrm{cm}^2$$

Wilkinson+, JCAP 04 (2014) 026

Dark matter can not cool by radiating photons => Strong constraints of the fraction of dissipative dark matter (e.g. through cooling into a rotationally-supported disk)

$$\epsilon_{\mathrm{disk}} \lesssim 0.05$$

J. Fan+, PRL 110, 211302 (2013)



e.g. Law+, ApJ 703 (2009) L67 (2009) & refs. to it

If dissipation and cooling occurs, there may be the consequent formation of a dark disk

NB: Subdominant component

6. Collisionless (or not very collisional)

DM-DM interaction too strong, spherical structures would be obtained rather than triaxial:

$$\sigma \sim \frac{m}{\mathrm{GeV}} \frac{\mathrm{Mpc}}{\lambda}\,\mathrm{barn} \qquad \lambda \sim \frac{1}{\sigma/m\,\rho} > 1\,\mathrm{Mpc}$$

From clusters: $\sigma/m<0.02$ cm^2/g

Miralda-Escudé ApJ 564 60 (2002)

From Bullet cluster: $\sigma/m<0.7$-1.3 cm^2/g

Randall+, ApJ 679,1173(2008); Buckley & Fox, Phys.Rev.D 81,083522(2010)

.....much less than atomic or molecular cross sections!

$$\frac{\mathrm{cm}^2}{\mathrm{g}} = 1.78\,\frac{\mathrm{barn}}{\mathrm{GeV}}$$

DM should **not** have **self-interactions exceeding the barn/GeV** level; **slightly smaller σ could also be beneficial**

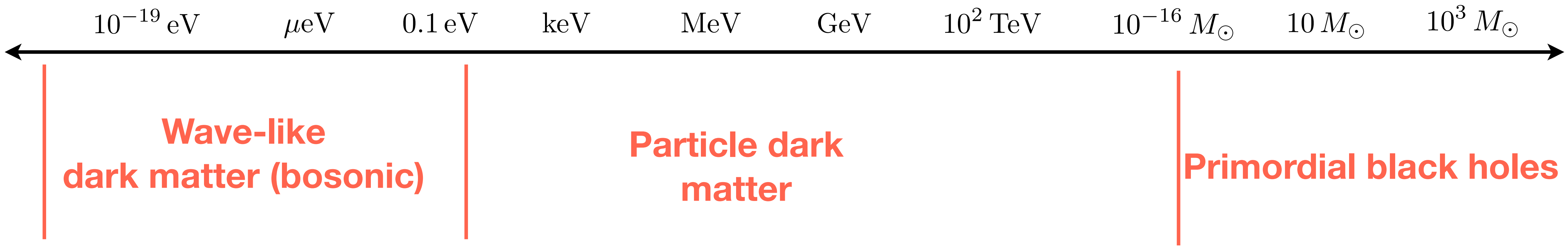
Kaplinghat+ PRL 116, 041302 (2016)

3. The dark matter landscape

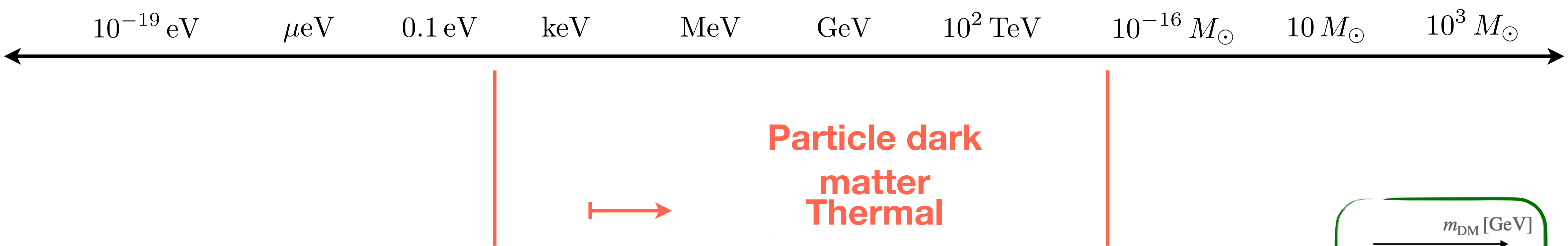
The dark matter landscape

Vast parameter space in mass and interaction strength

The dark matter landscape

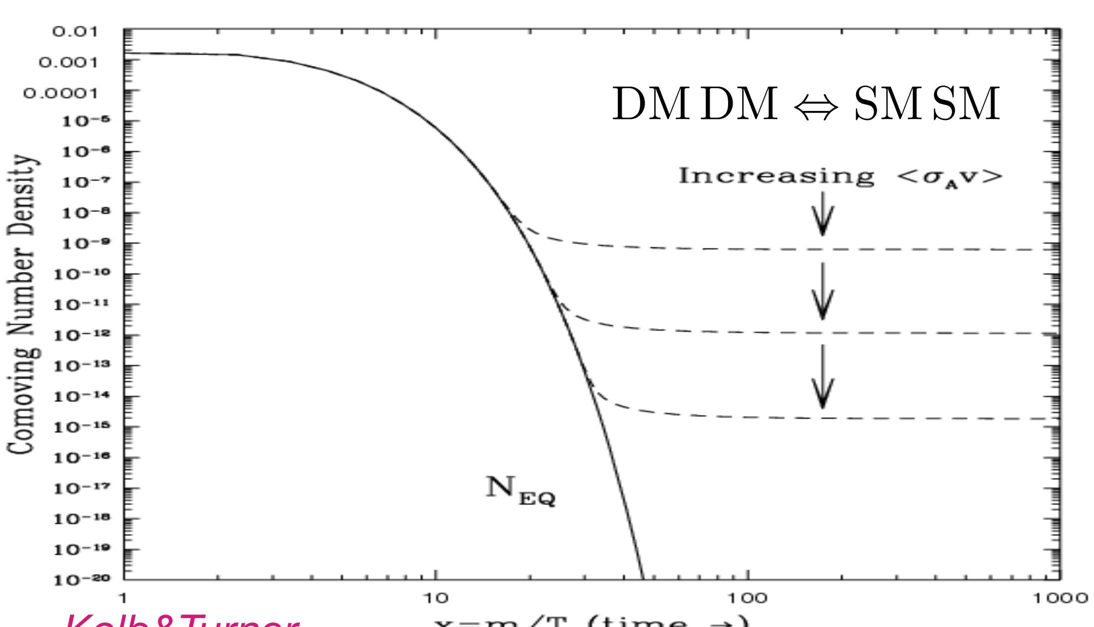


The dark matter landscape



Weakly interacting massive particle (WIMPs)

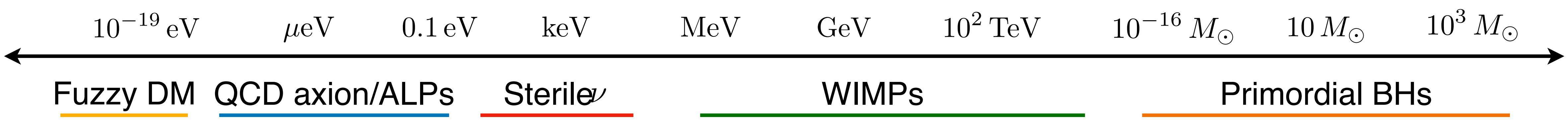
- Freeze-out production mechanism



Kolb&Turner

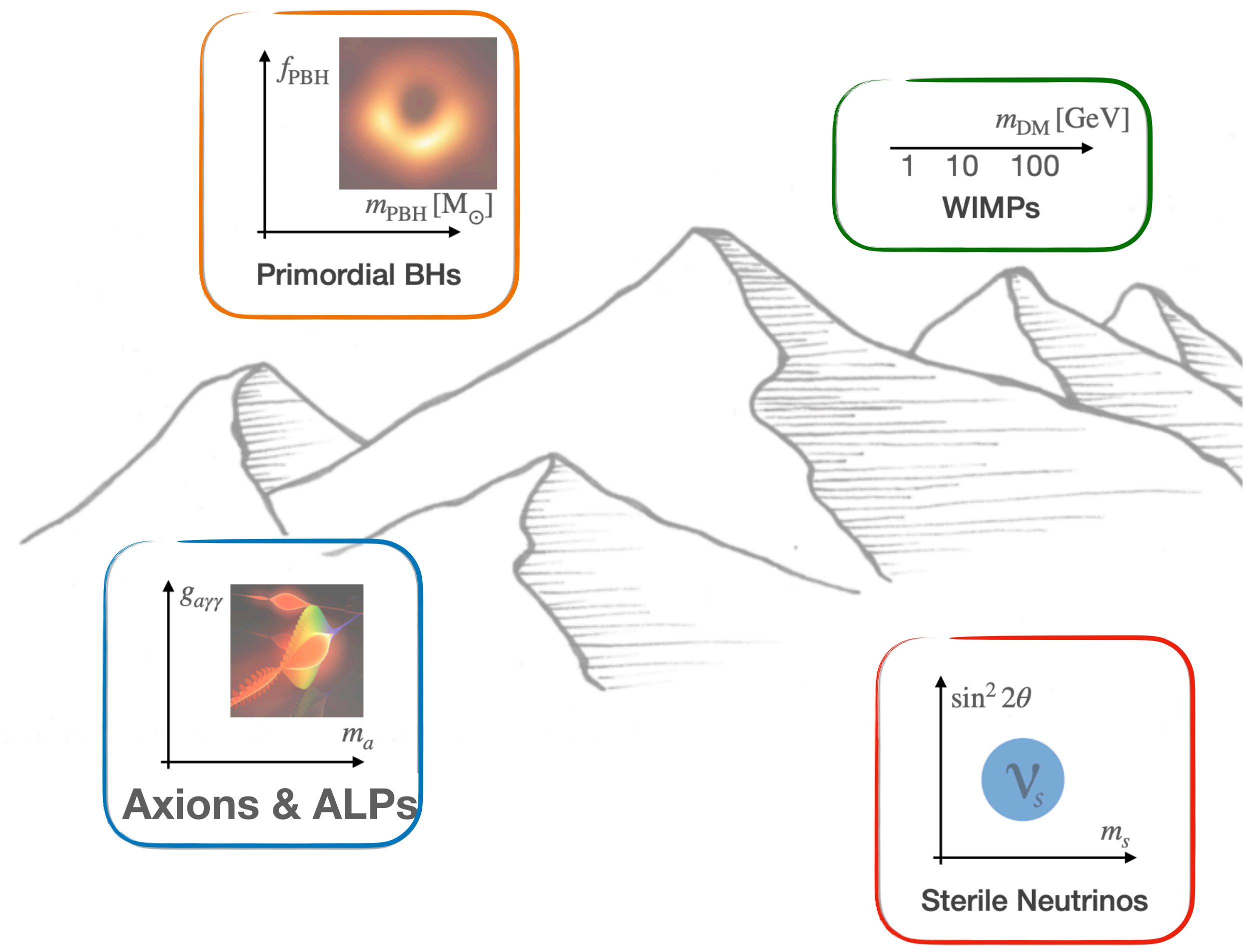
$$\Omega_{DM} h^2 \sim \frac{10^{-27} \mathrm{cm}^3/\mathrm{s}}{\langle \sigma(\mathrm{DM\,DM} \to \mathrm{SM\,SM}) \mathrm{v} \rangle}$$

The dark matter landscape

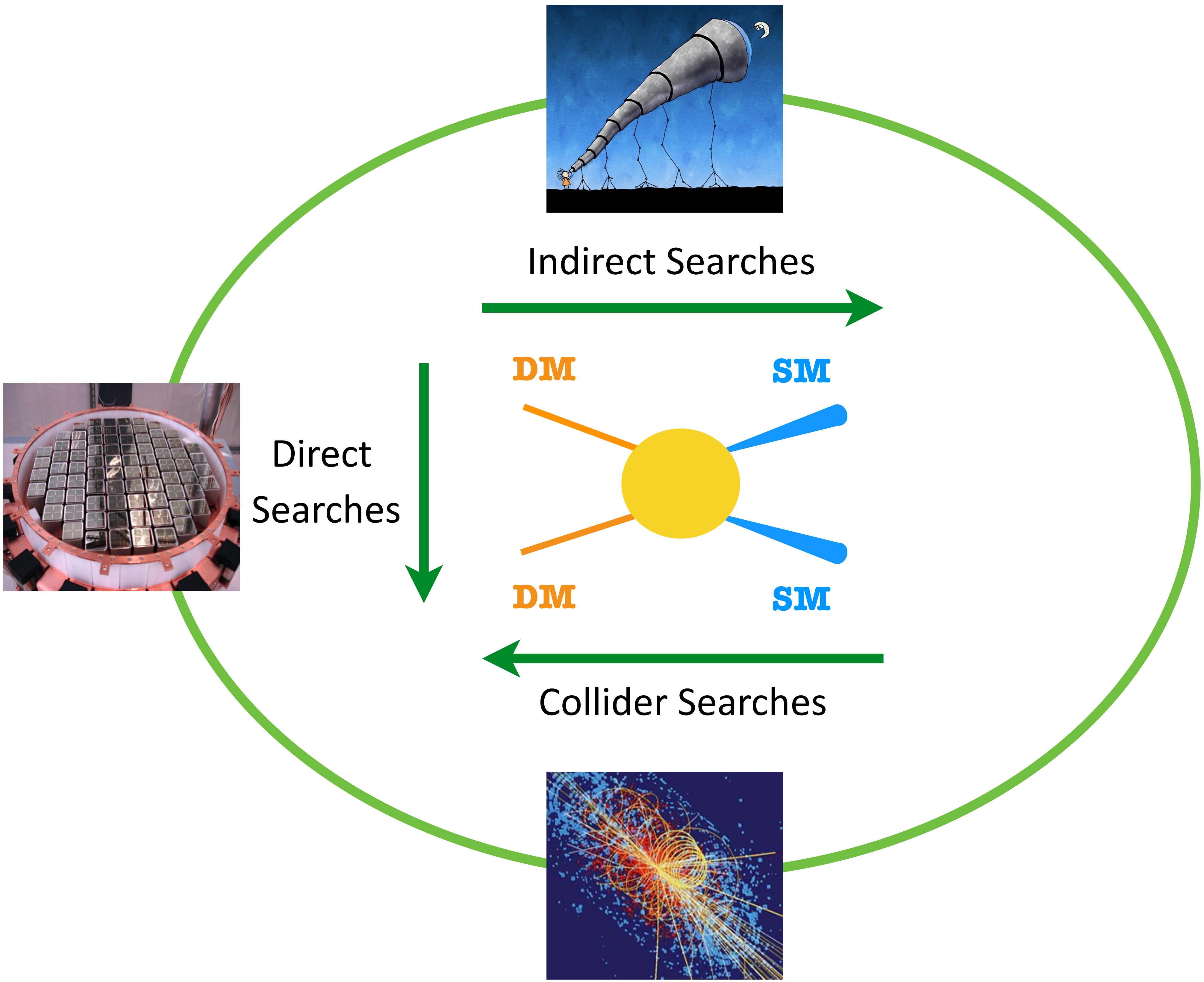


The quest for dark matter is colourful and very broad!!

It leverages on model signatures and available data



WIMP detection strategies



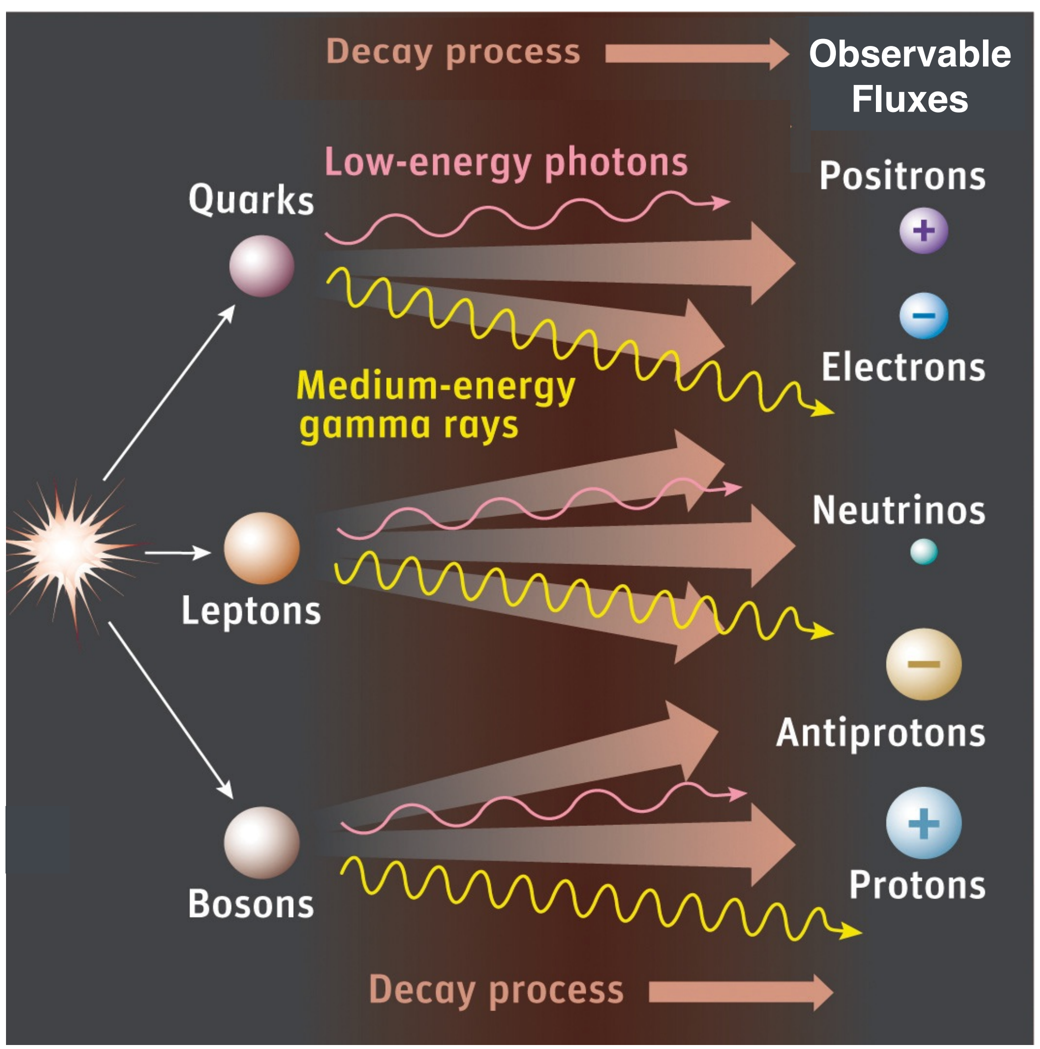
Indirect dark matter detection

Two key-assumptions:

1) **Dark matter exists** and is the main responsible for the gravitational potential inferred in galaxies, clusters and cosmo.

2) **Dark matter is non-gravitationally coupled** to standard matter.

DM annihilation/decay



DM annihilation/decay leads to production of **observable fluxes** of stable particles.

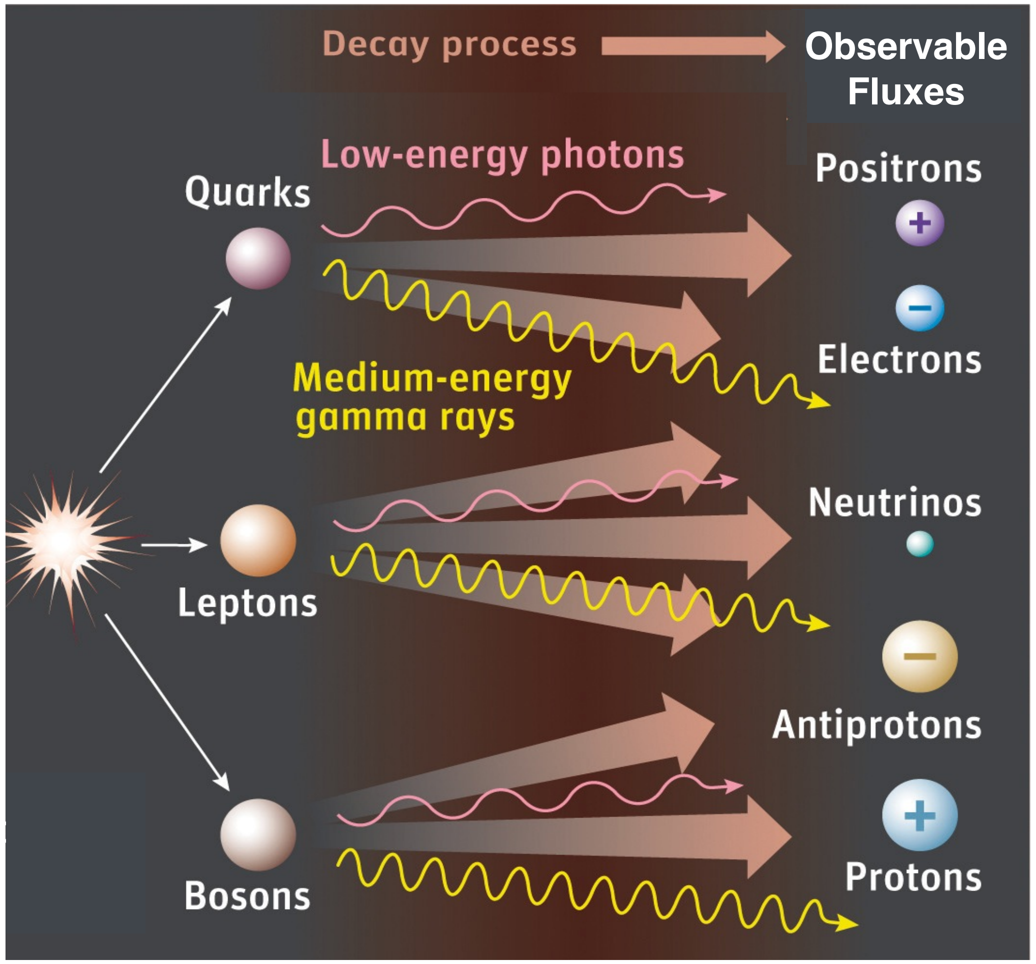
Disclaimer:

1) Not necessarily signatures at the GeV-TeV-scale

2) DM at the electroweak scale is one among possible valuable solutions

Indirect dark matter detection

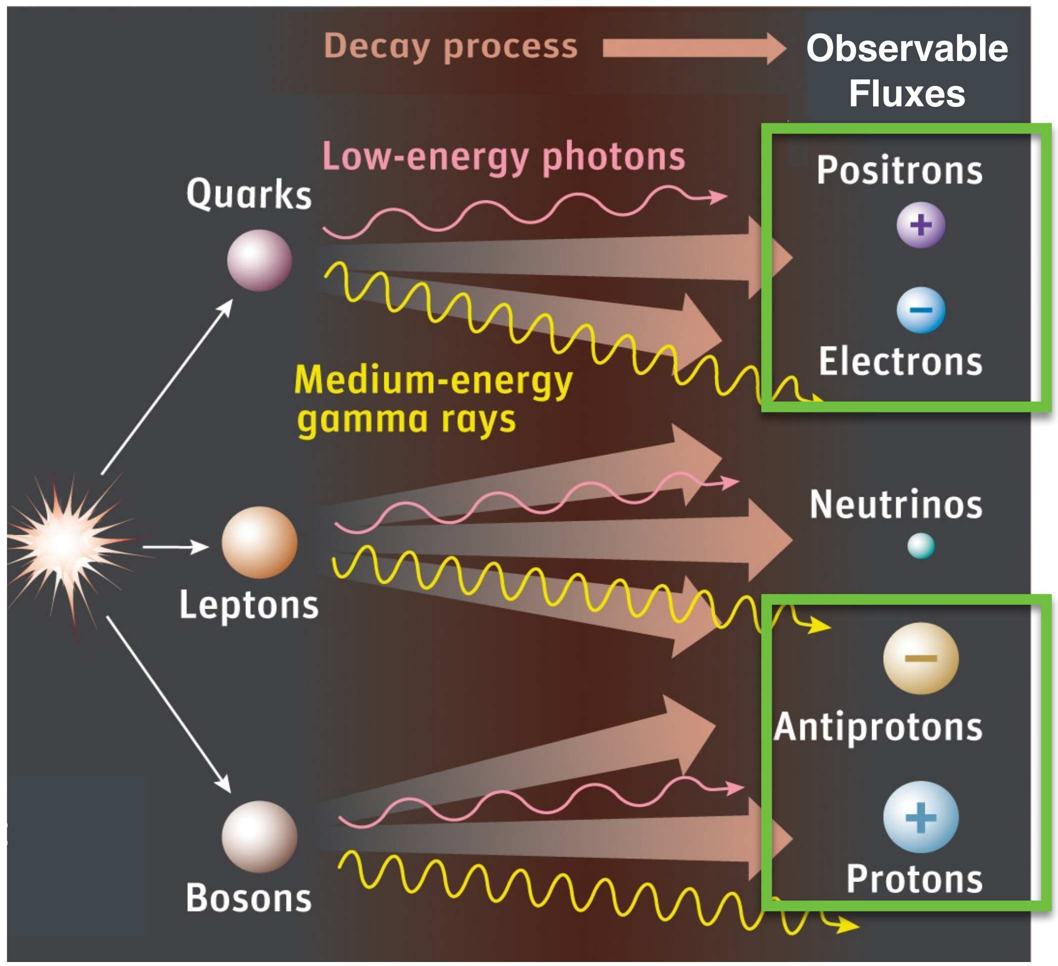
DM annihilation/decay



Indirect searches
for stable dark matter annihilation (or decay) products.

Indirect dark matter detection

DM annihilation/decay



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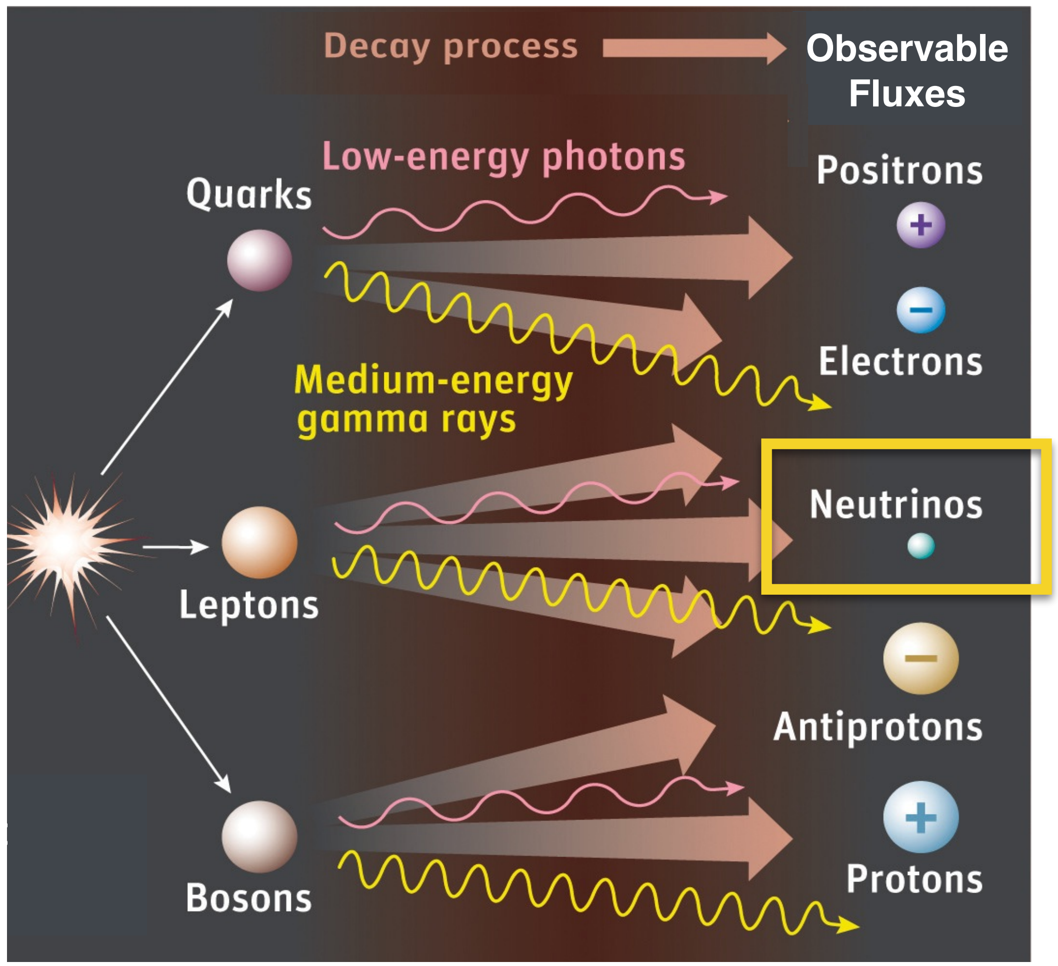

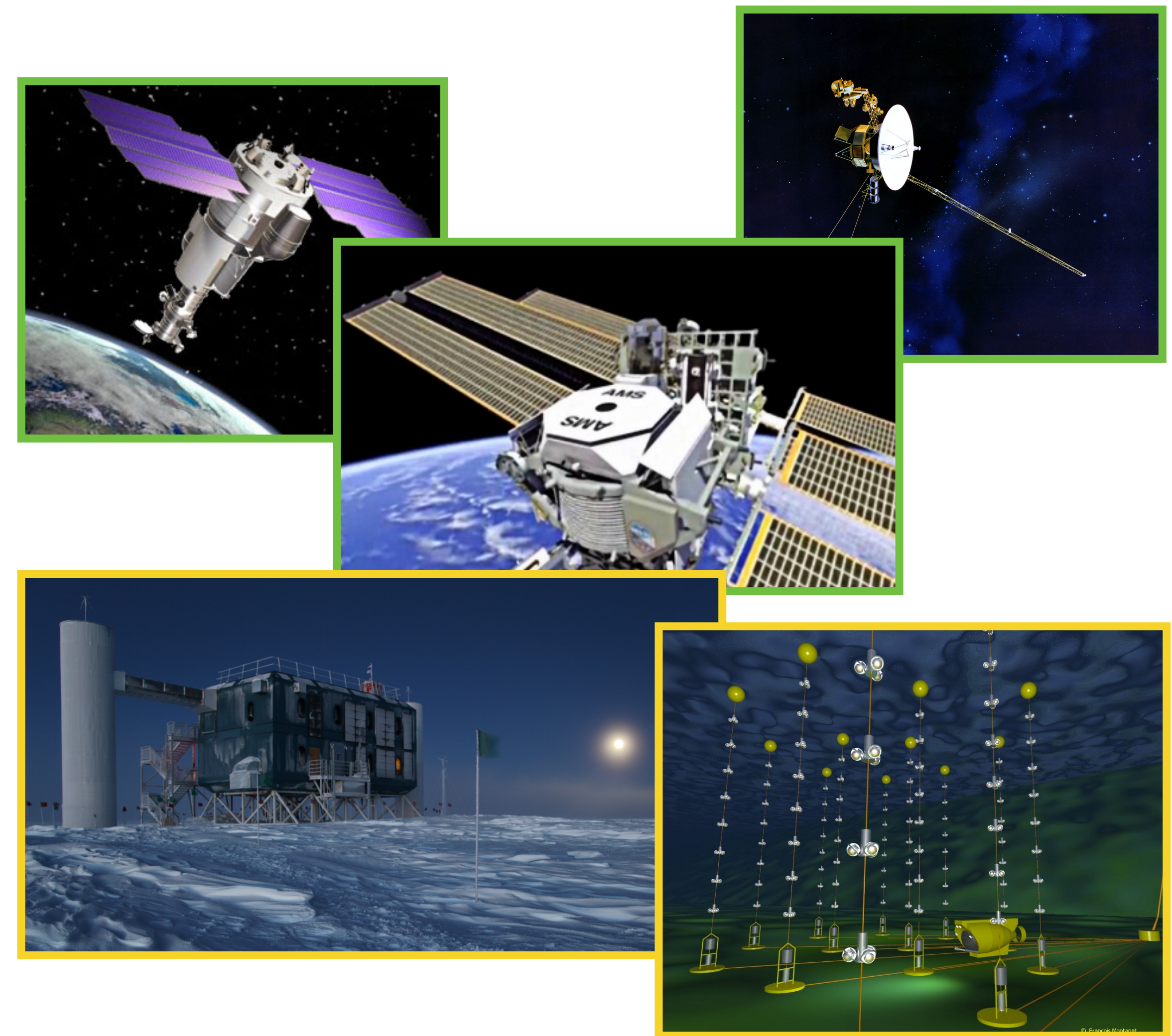

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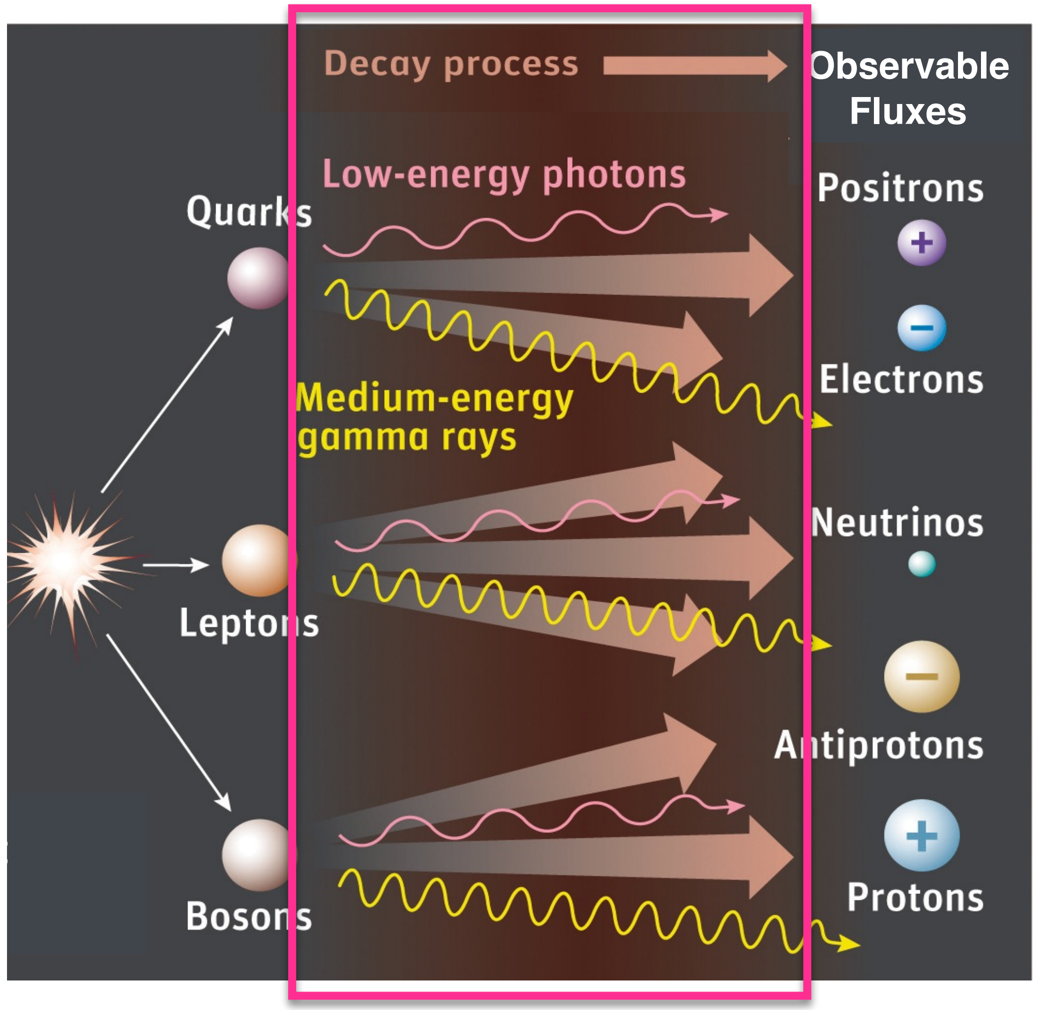

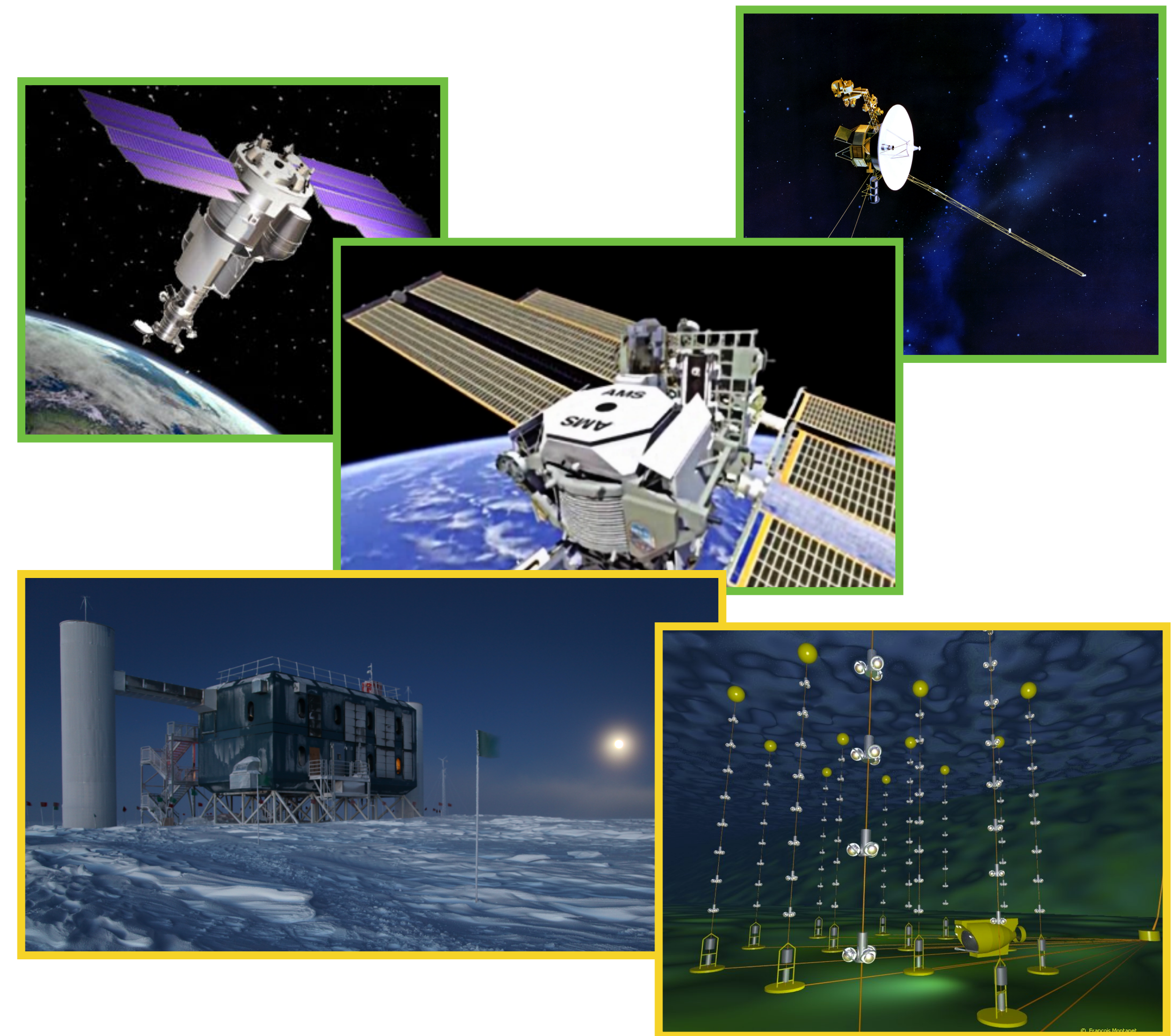

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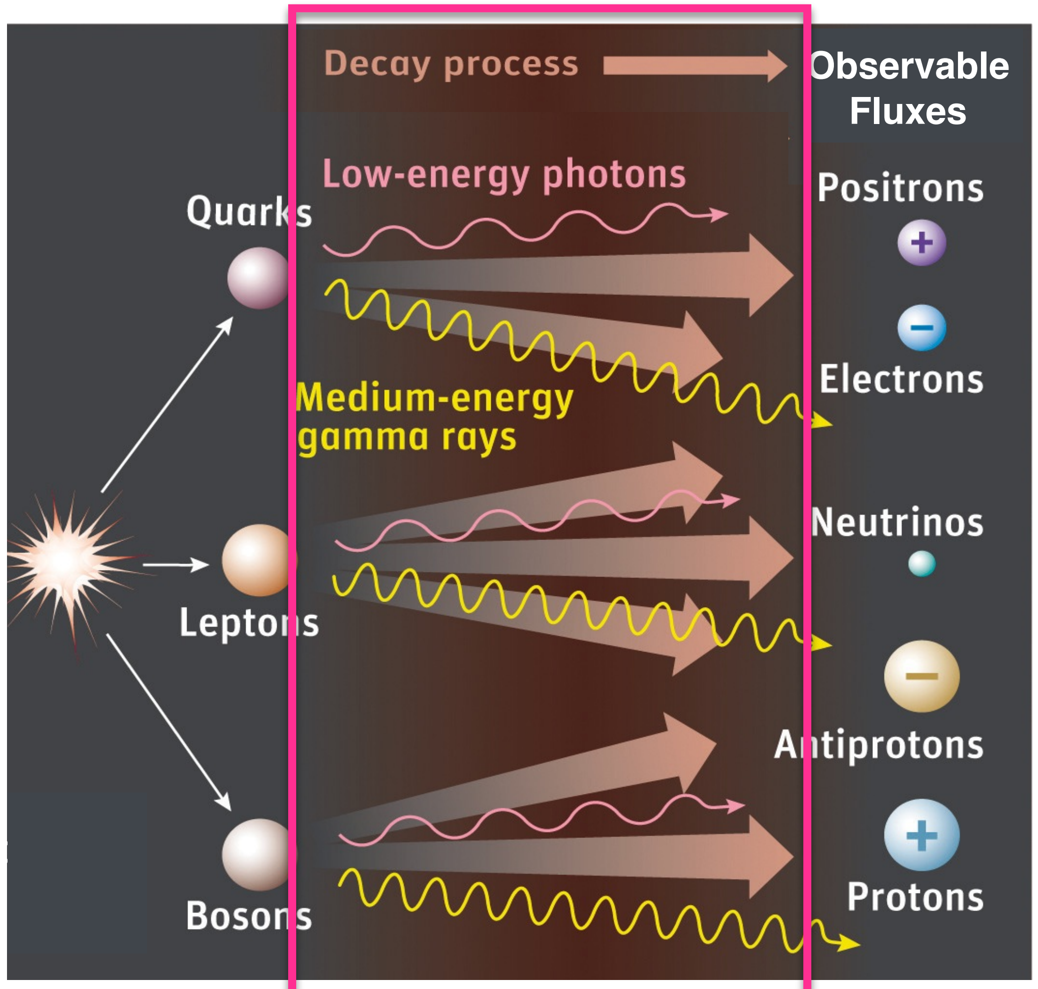
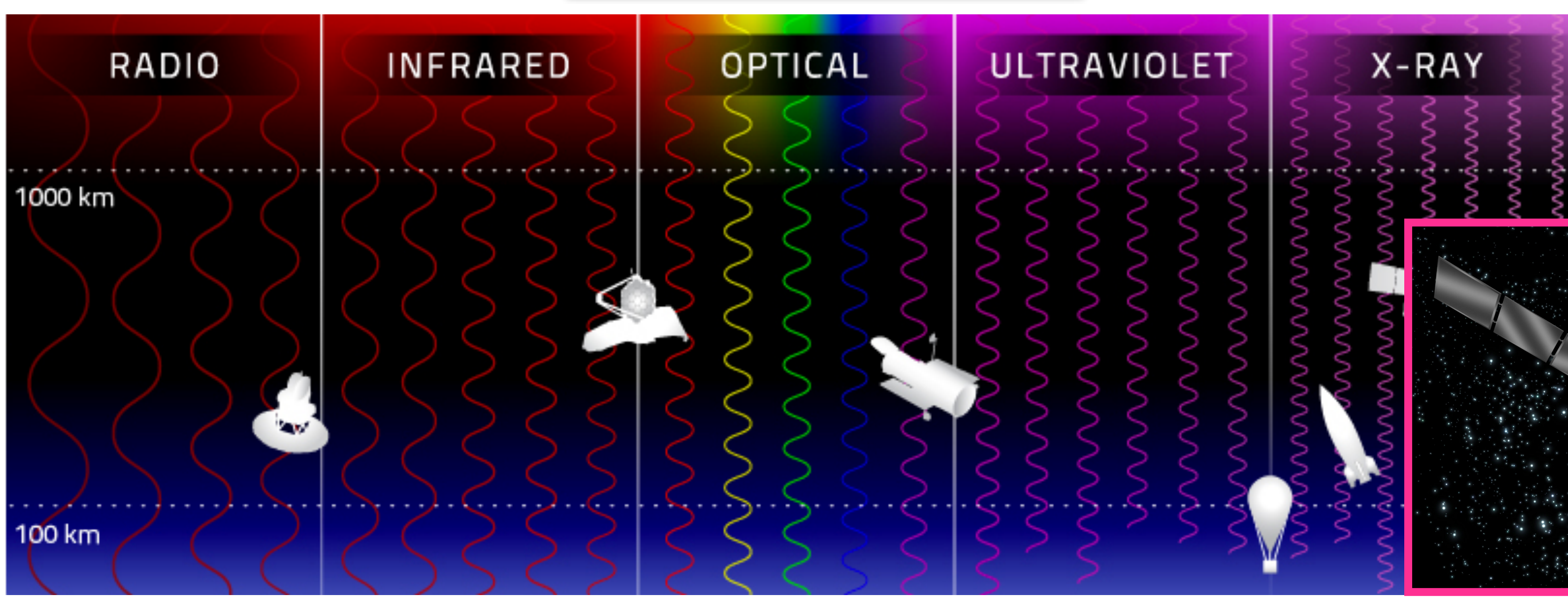


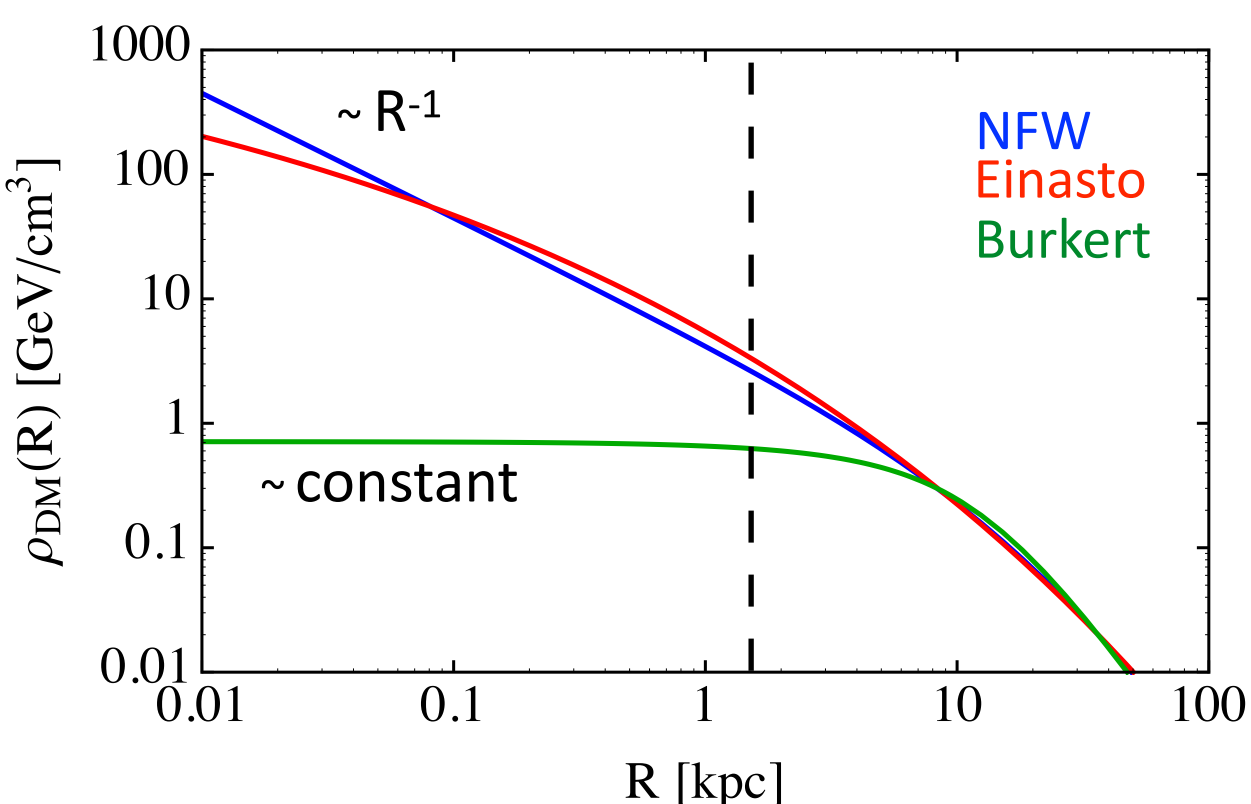


The WIMP gamma-ray flux

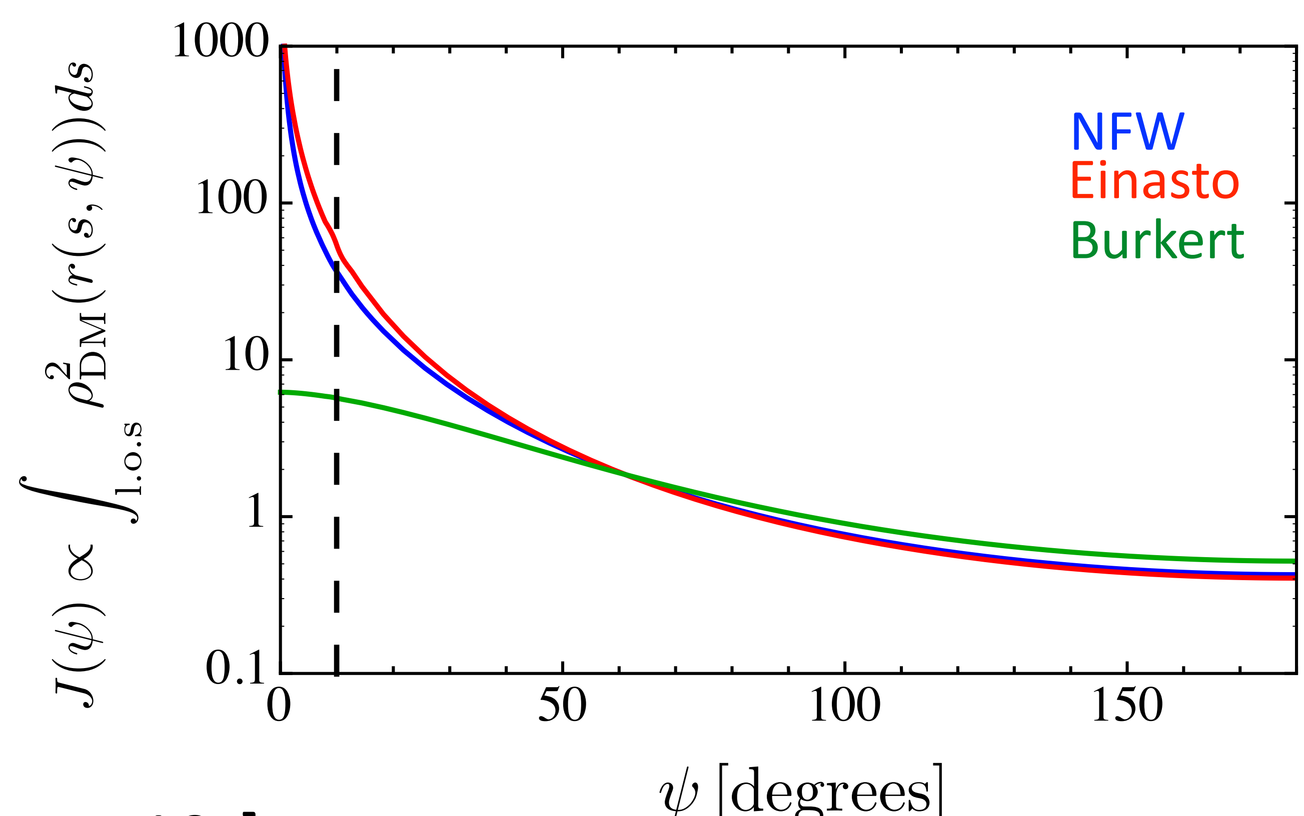
E.g: gamma-ray differential flux from spatial distribution $\rho_{\rm DM}$

$$\frac{d\Phi_\gamma}{dE_\gamma}(E_\gamma, s, \Delta\Omega) \propto \frac{\langle\sigma v\rangle}{2m_{\rm DM}^2} \sum_i B_i \frac{dN_\gamma^i}{dE_\gamma} \frac{1}{4\pi} \int_0^{\Delta\Omega} d\Omega \int_{\rm l.o.s} \rho_{\rm DM}^2(s)ds$$

Dark matter density profiles:



Spatial distribution of the signal:

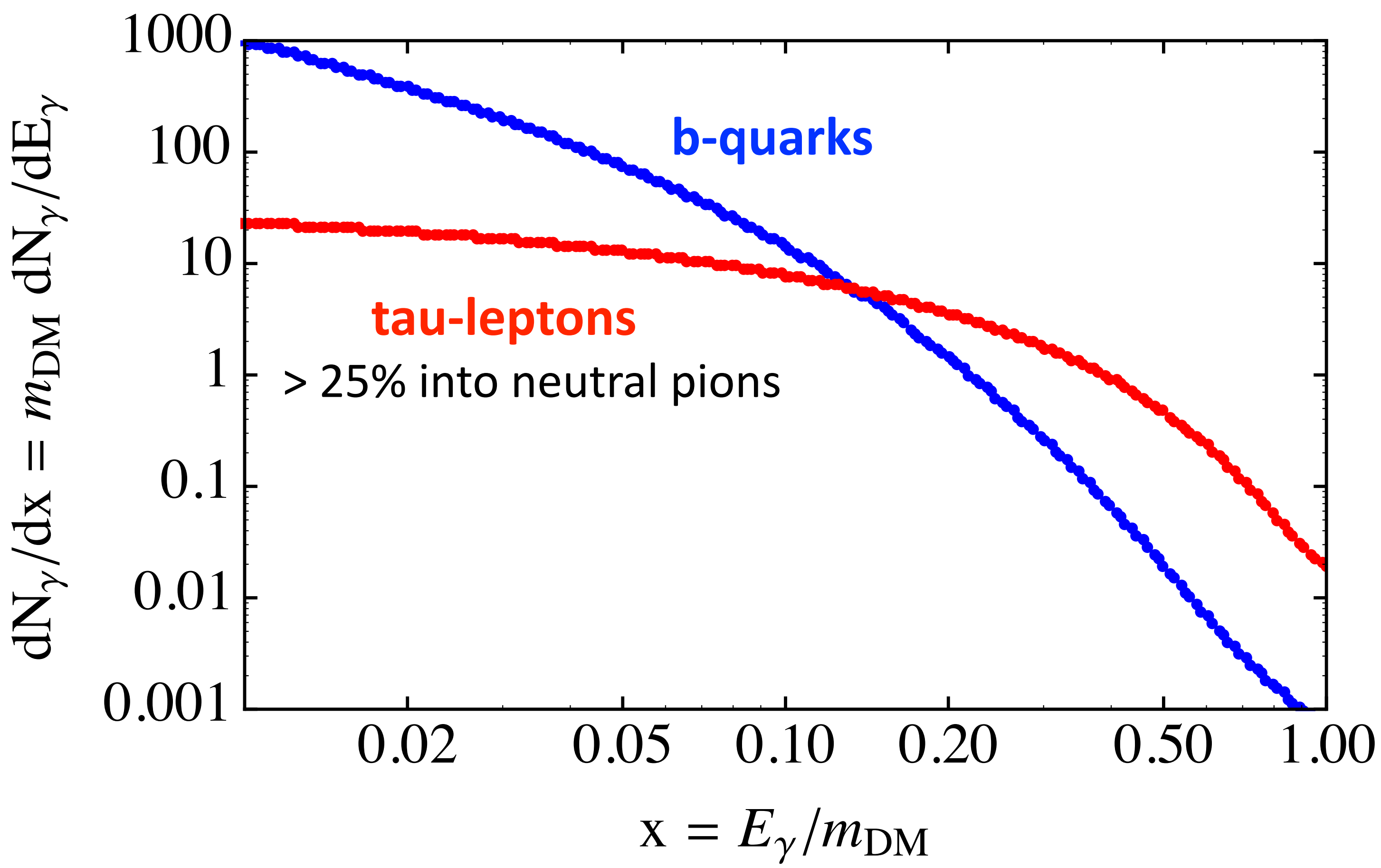


[1.5 kpc ≃ 10°]

Spectra of prompt "secondary" photons

$$\frac{d\Phi_\gamma}{dE_\gamma}(E_\gamma, s, \Delta\Omega) = \frac{\langle\sigma v\rangle}{2m_{\rm DM}^2}\sum_i B_i \frac{dN_\gamma^i}{dE_\gamma}\frac{1}{4\pi}\int_0^{\Delta\Omega} d\Omega \int_{\rm l.o.s} \rho_{\rm DM}^2(s)ds$$

100% Branching ratio (independent on PP model)



$$x \equiv \frac{E_X}{m_\chi}$$

$$\frac{dN_X}{dx} \equiv m_\chi \frac{dN_X}{dE}$$

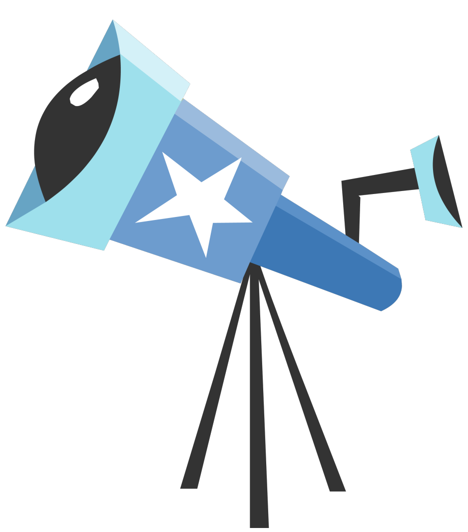
General about DM searches

Observed Flux

$$\Phi_{\text{Obs}}$$

Expected Flux

$$\Phi_{\text{Th}}$$



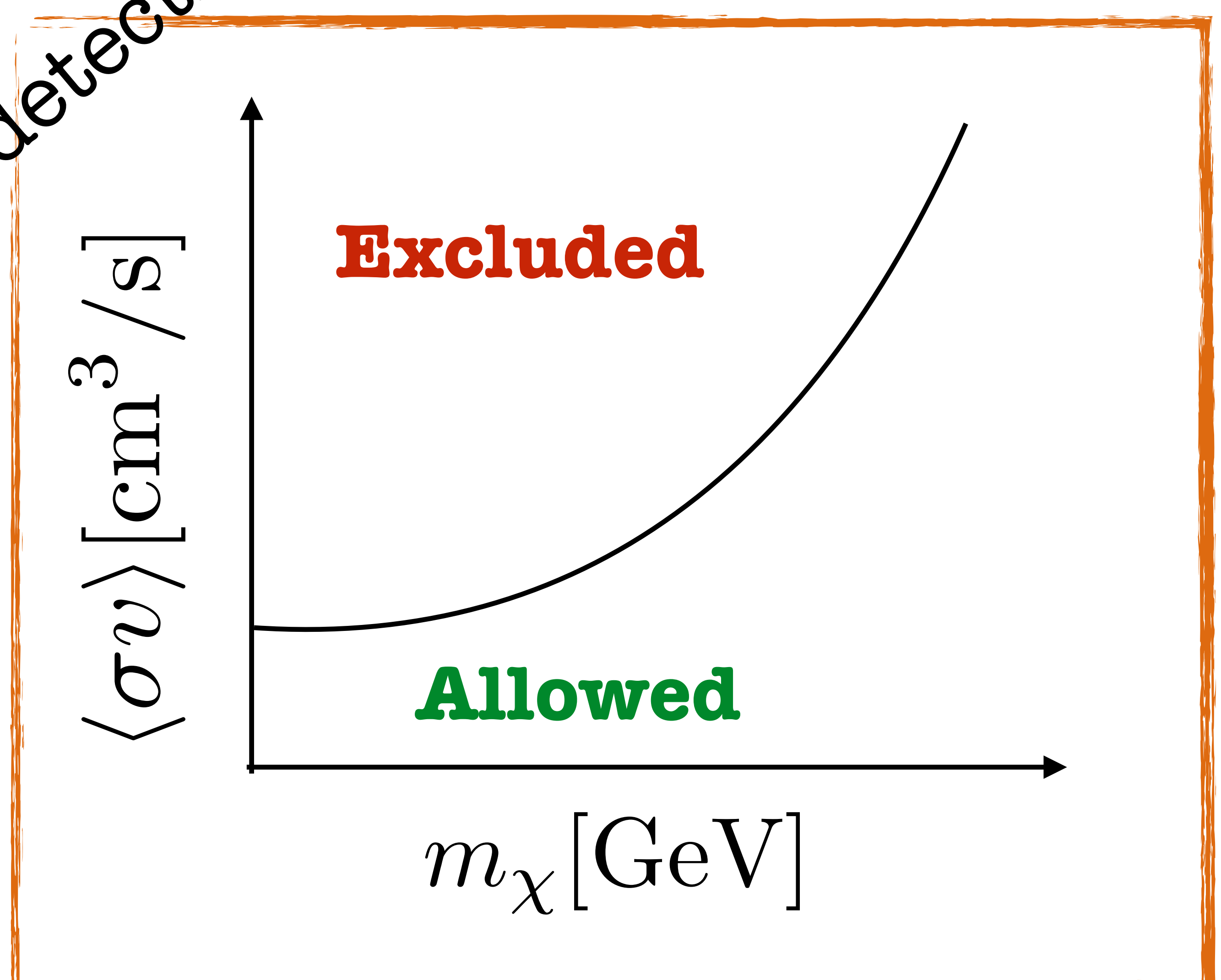
General about DM searches

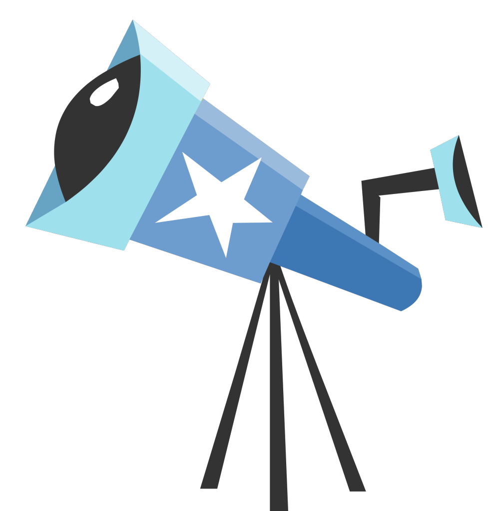
Observed Flux

Expected Flux

$$\Phi_{\mathrm{Obs}} \qquad \Phi_{\mathrm{Th}}$$

No signal detection





General about DM searches

Observed Flux

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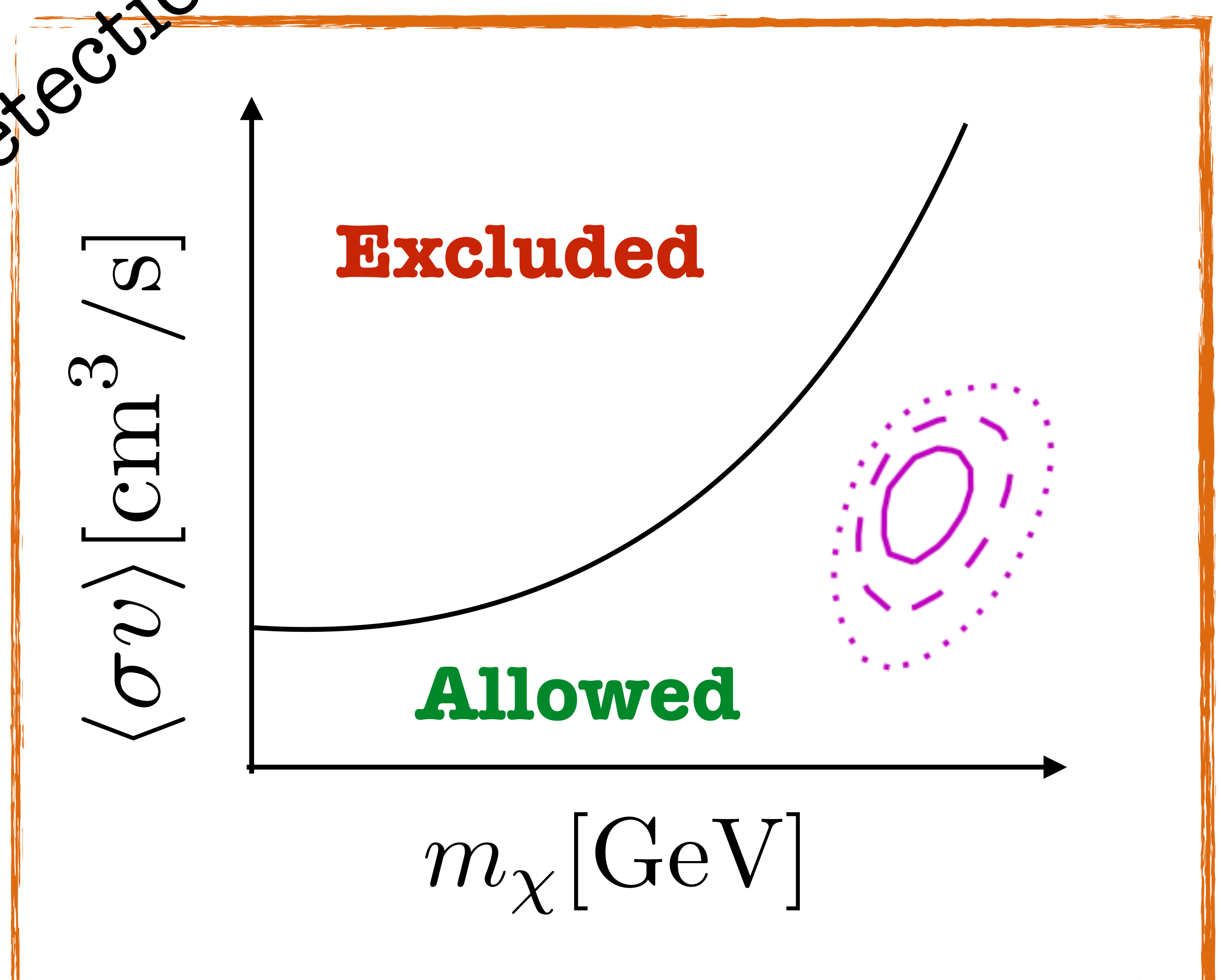
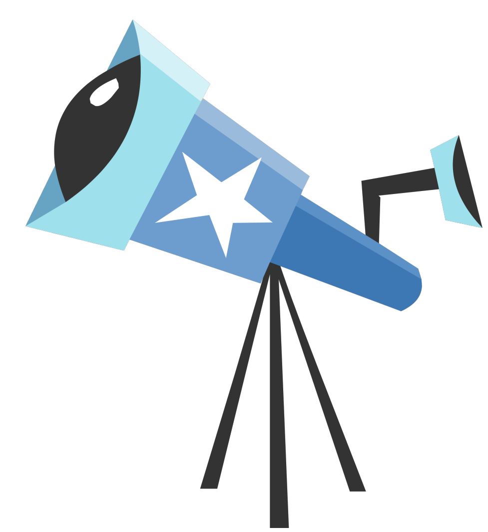
Signal detection





DM phenomenology exercise

Where to look for DM signal? What do we expect? What can we find?