Realistic non-singular black holes

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Einstein's equation

- governs how the metric responds to energy and momentum

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{1}$$

- $g_{\mu\nu}$ metric, $T_{\mu\nu}$ energy-momentum tensor
- $R_{\mu\nu}$ Ricci tensor, R Ricci scalar
- G Newton's constant of gravitation, Λ cosmological constant

The Schwarzschild solution

- unique spherically symmetric vacuum solution of the Einstein's equation

$$ds^2 = -\left(1 - \frac{2GM}{r}\right)\mathrm{d}t^2 + \left(1 - \frac{2GM}{r}\right)^{-1}\mathrm{d}r^2 + r^2 d\Omega^2 \quad (2)$$

$$d\Omega^2 = \mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2 \quad (3)$$

- singularities are regions where the curvature (measured by Riemann tensor) becomes infinite
- one scalar quantity for Schwarzschild metric that measures curvature

$$R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} = \frac{48G^2M^2}{r^6} \quad (4)$$

- $r = 0$ is a real singularity, $r = 2GM$ is a coordinate singularity

Energy conditions

- prohibit pathological behaviors of spacetimes
- Null energy condition (NEC): $\tilde{T}_{\mu\nu} k^\mu k^\nu \geq 0$ for any null vector k^μ
- Weak energy condition (WEC): $\tilde{T}_{\mu\nu} v^\mu v^\nu \geq 0$ for any timelike vector v^μ
- Dominant energy condition (DEC): $\tilde{T}_{\mu\nu} v^\mu v^\nu \geq 0$ and $J_\mu J^\mu \leq 0$ hold for any timelike vector v^μ, where $J^\mu := -\tilde{T}^\mu{}_\nu v^\nu$
- Strong energy condition (SEC): $\left(\tilde{T}_{\mu\nu} - \frac{1}{2}\tilde{T} g_{\mu\nu}\right) v^\mu v^\nu \geq 0$ for any timelike vector v^μ

Criteria for physically reasonable non-singular black holes

- C1: Any kind of non-coordinate singularity is absent.
- C2: Closed causal curves are absent.
- C3: $\tilde{T}_{\mu\nu}$ satisfies the standard energy conditions in asymptotically flat regions.
- C4: $\tilde{T}_{\mu\nu}$ satisfies the standard energy conditions on the event horizon of a large black hole.
- C5: The limiting curvature condition (LCC) is respected.
- C6: Realized for a set of non-zero measure in the parameter space of the black-hole solution.
- C7: Dynamically stable.

Spherically symmetric non-singular black holes with a regular center

$$ds^2 = -f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right) \tag{5}$$

$$f(r) := 1 - \frac{2M(r)}{r} \tag{6}$$

- Bardeen:

$$M(r) = \frac{mr^3}{(r^2 + l^2)^{3/2}} \tag{7}$$

- Bardeen black-hole spacetime doesn't satisfy C3, C4 and C5
- Other solutions: Hayward, Dymnikova, Fan-Wang...

References

- S. Carroll: Spacetime and Geometry, An Introduction to General Relativity
- H. Maeda: Quest for realistic non-singular black-hole geometries: regular-center type, JHEP 11 (2022) 108