First neutrino interaction recorded in hydrogen bubble chamber

NEUTRINO PHYSICS: EXPERIMENTAL ASPECTS

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GRASPA School
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Overview

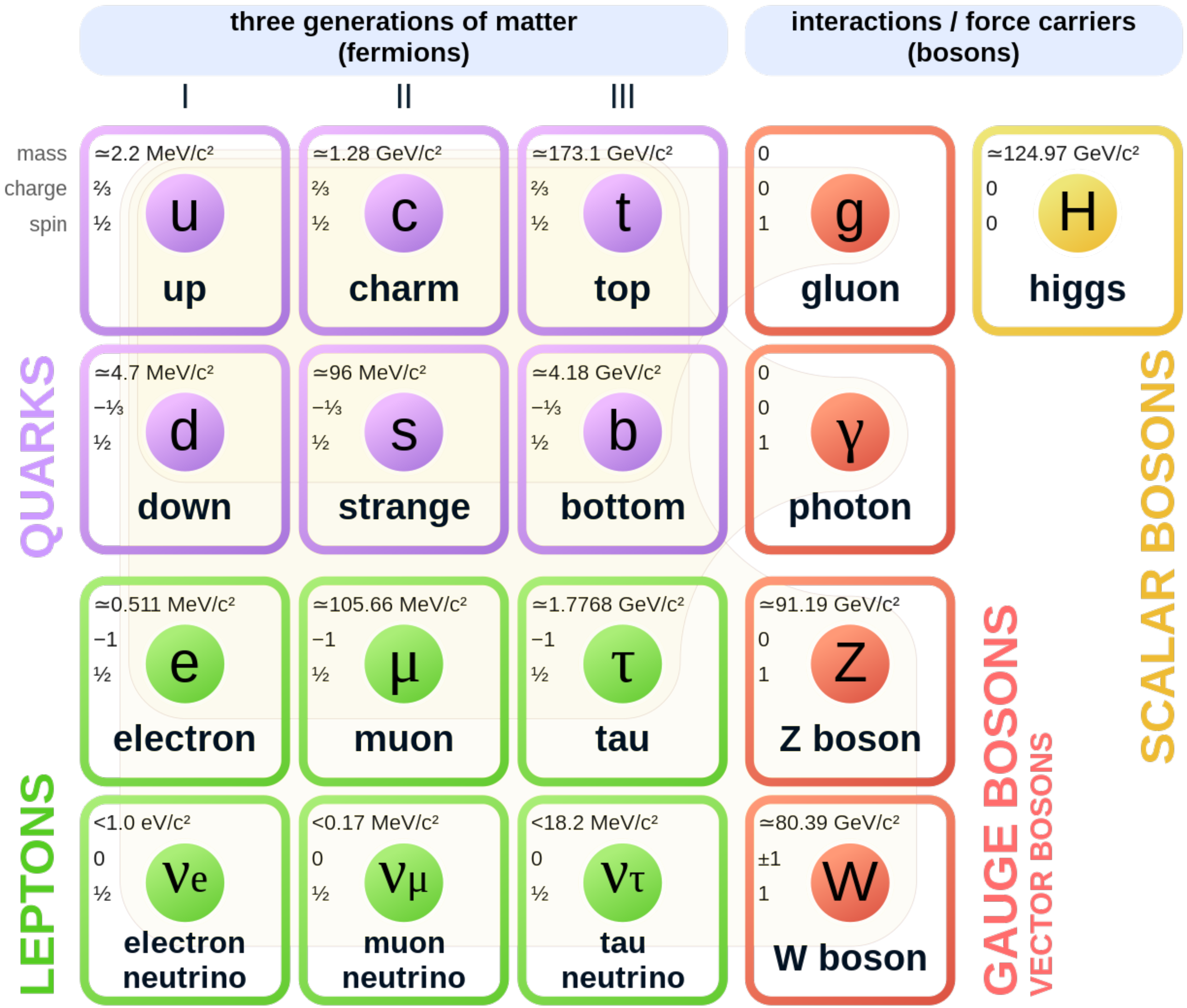
○ Neutrino properties, sources, interactions

○ Neutrino Oscillation with a « historical » approach

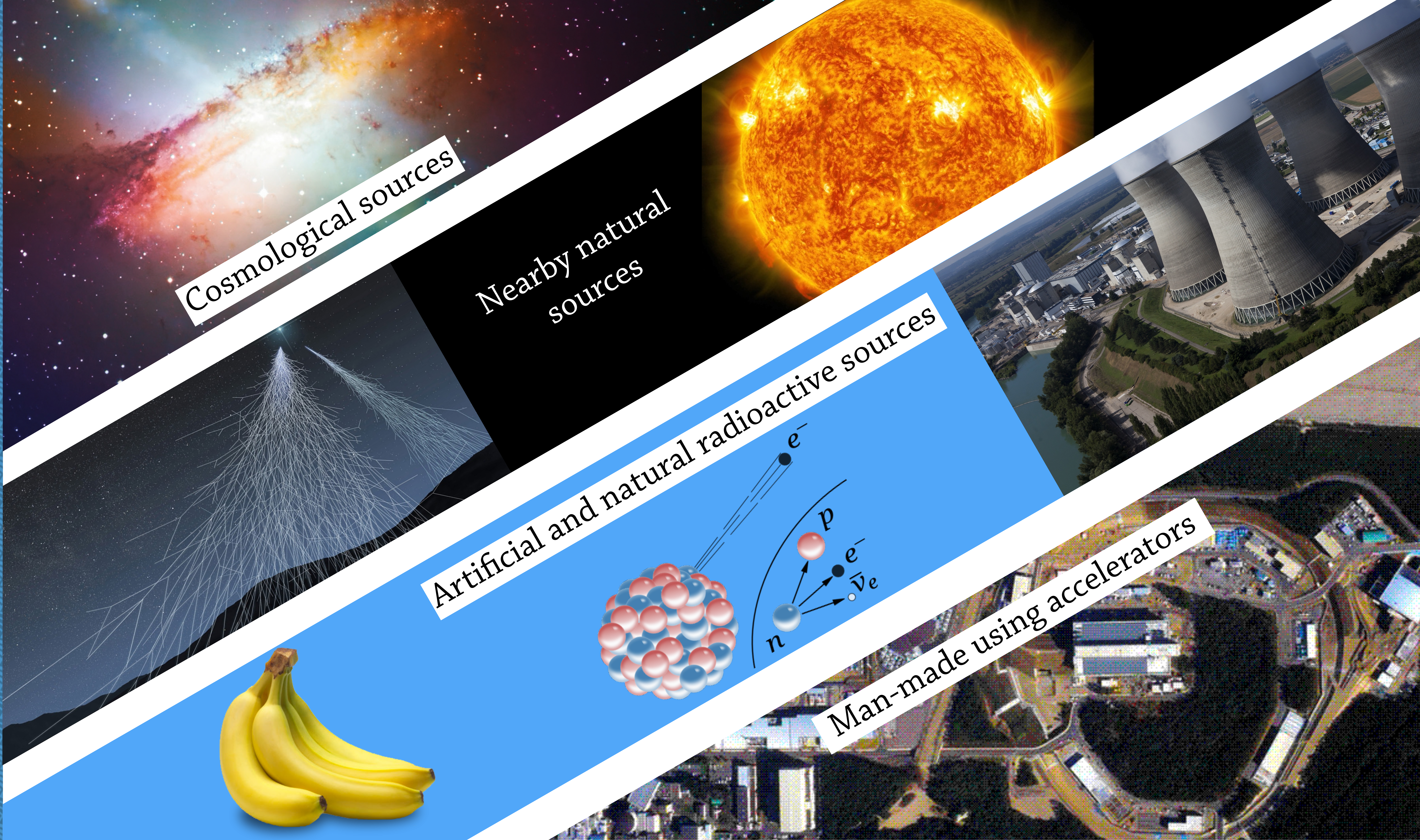
○ Produce, Detect and neutrino oscillations today & tomorrow

In the Standard Model

- Neutrinos are leptons
- 3 flavors linked to their corresponding charged counterpart
- Can only interact through weak force (through W± and Z⁰ bosons)



Neutrino Sources



Cosmological sources

Nearby natural sources

Artificial and natural radioactive sources

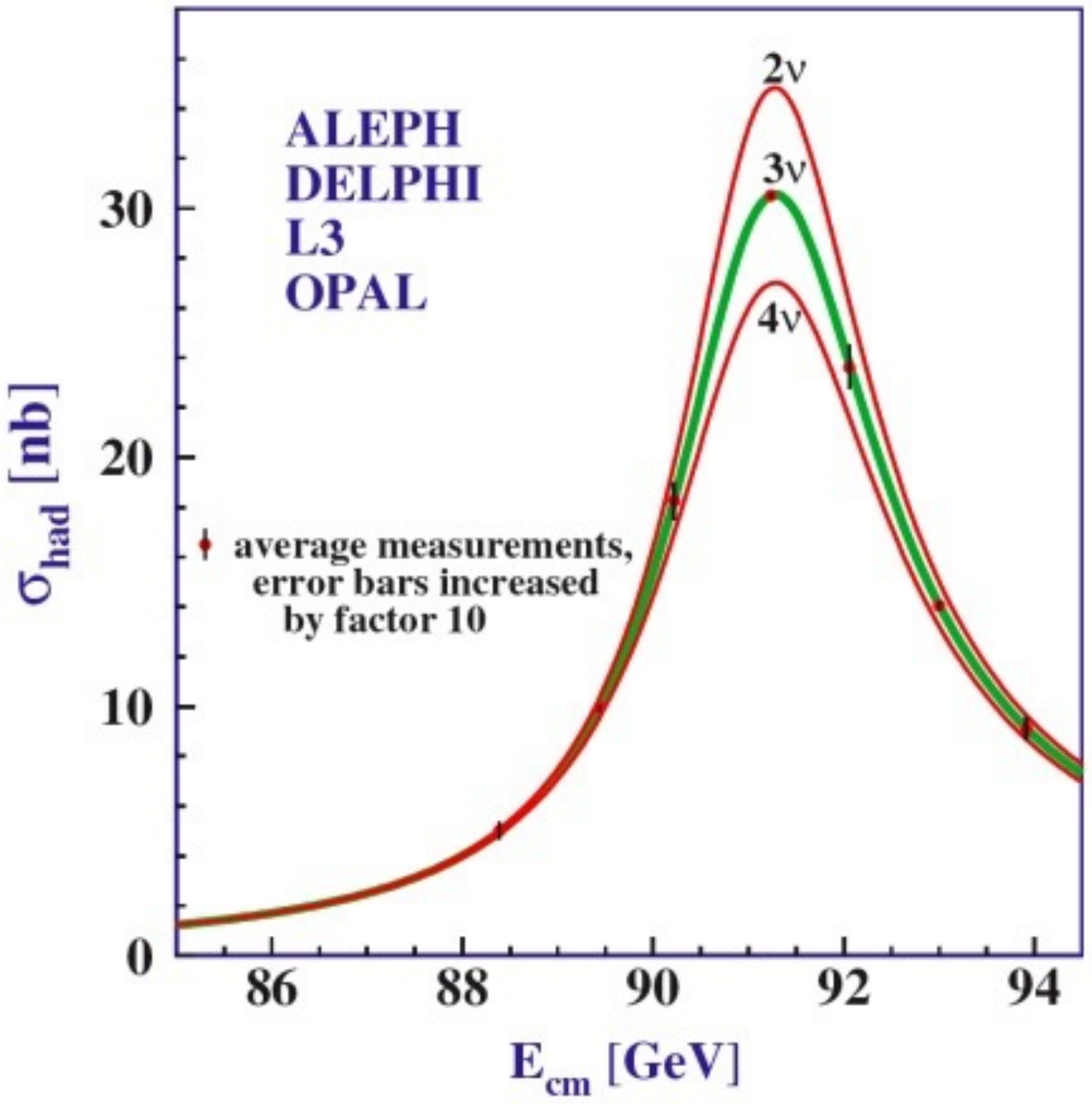
Man-made using accelerators

Key facts

○ Three flavors of light and active neutrinos named ν_e, ν_μ, ν_τ

→ In 1989, LEP mesures the Z invisible decay width :

$$N_\nu = 2.984 \pm 0.008$$



○ Neutrinos are only left-handed

→ Cannot couple to the Higgs field, therefore the neutrinos are considered massless in the Standard Model

→ But they in fact do have a mass:

$$m_\nu < 1 \text{ eV} \; ; \; \sum m_\nu > 0.06 \text{ eV}$$

○ Most abundant massive particule

$\Phi_{sun} = 65\times10^9\ \nu_e/cm^2/s$ on earth

$\Phi_{reactor} = 2\times10^{20}\ \bar{\nu}_e/s/GW_{th}$

$\Phi_{atmo} = 4\times10^2\ \nu_{e+\mu}/m^2/s/sr$

$\Phi_{accelerator} \sim 1\times10^{12}\ \nu_\mu/m^2$

○ Only interact through weak interaction

→ Small cross section :

$\sigma \sim 10^{-42}\ cm^2$ for IBD

$\sigma \sim 10^{-38}\ cm^2$ at 1 GeV

→ 50% chance a ν_e from the sun interact in you in your lifetime

HOW TO DETECT NEUTRINOS

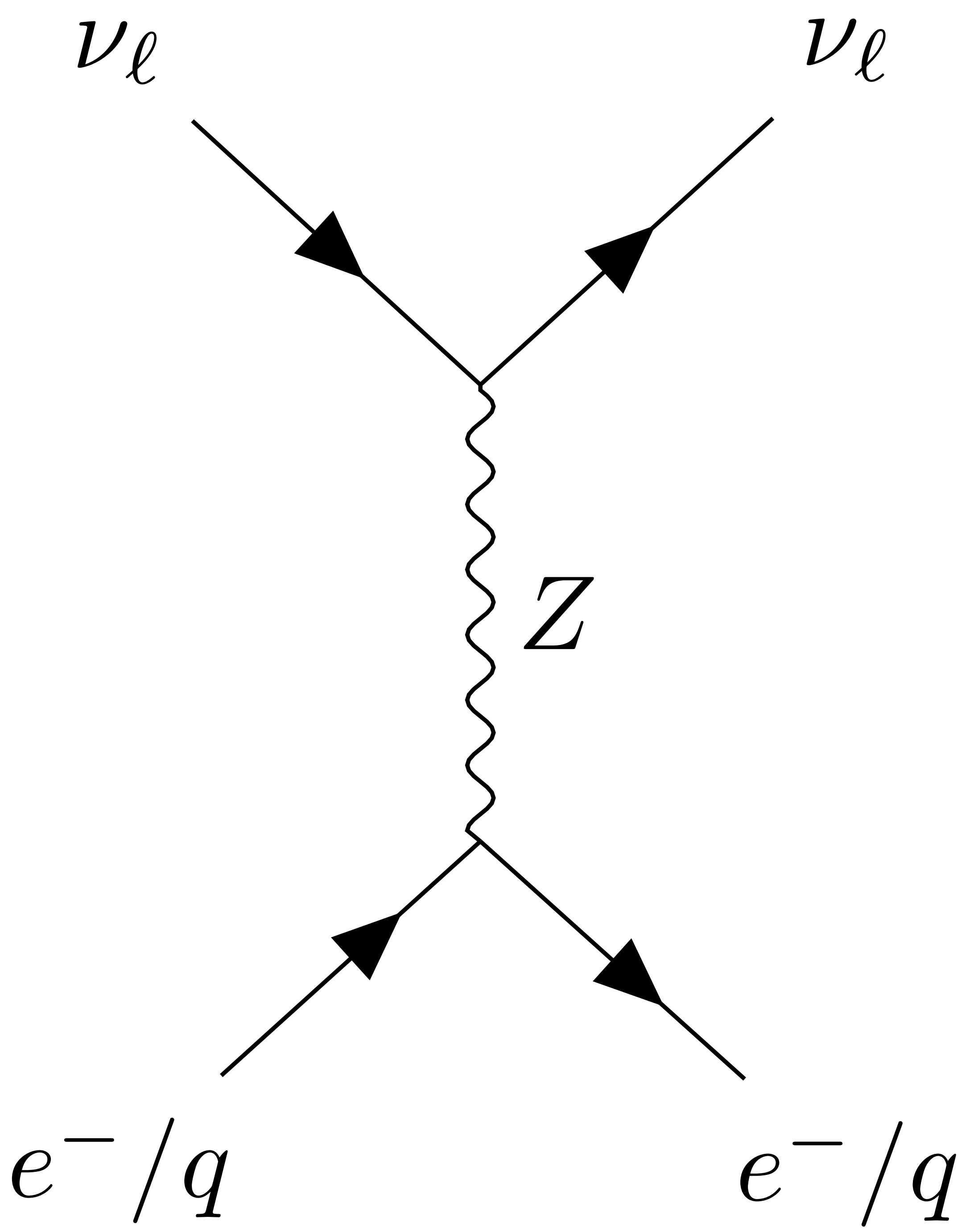
Charged and neutral currents

Neutrino have no electric charge -> We cannot detect them directly

We have to :
- Wait for a neutrino to interact
- Detect the products of the interaction
- Retrieve the original neutrino flavor/direction/energy/sign

Neutrinos can only interact by weak interaction
-> Through Z^0 exchange = Neutral Currents
-> Through $W^\pm$ exchange = Charged Currents

Neutral currents interactions



Elastic scattering

○ Cannot identify the incoming neutrino flavor

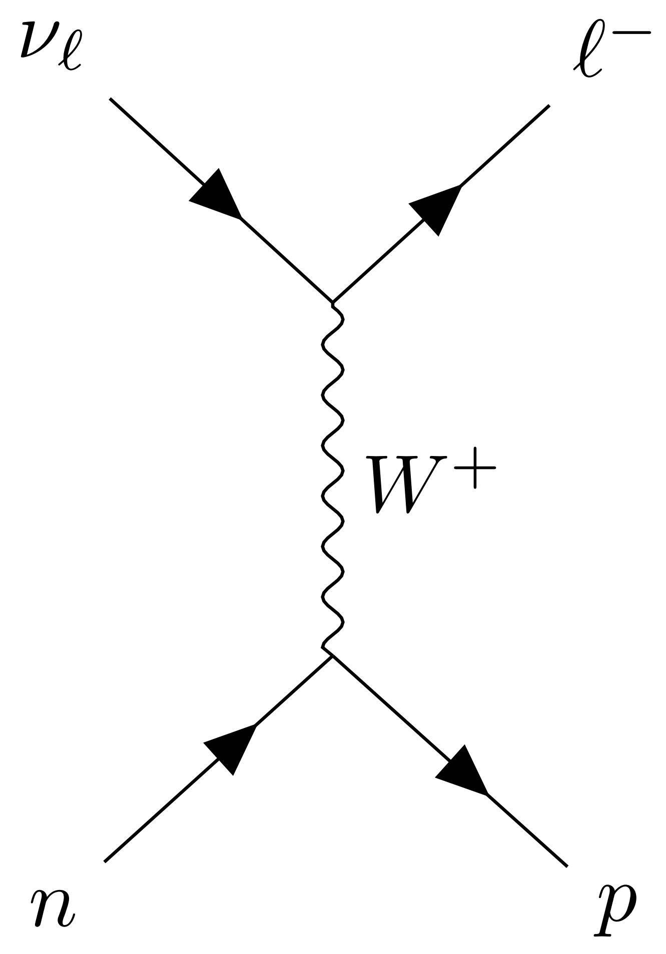
○ All neutrino interact with same potential

Charged currents interactions

Charged currents interactions

The Quasi-Elastic interaction

The Golden Channel



- ν flavor & sign tagged by the lepton
- E_ν reconstructed with the lepton kinematics :

$$E_\nu = \frac{m_f^2 - (m_i - E_b)^2 - m_\mu^2 + 2(m_i - E_b)E_\mu}{2(m_i - E_b - E_\mu + p_\mu \cos\theta_\mu)}$$

m_i, m_f : initial, final nucleon masses;
E_b : nucleon binding energy in the nucleus

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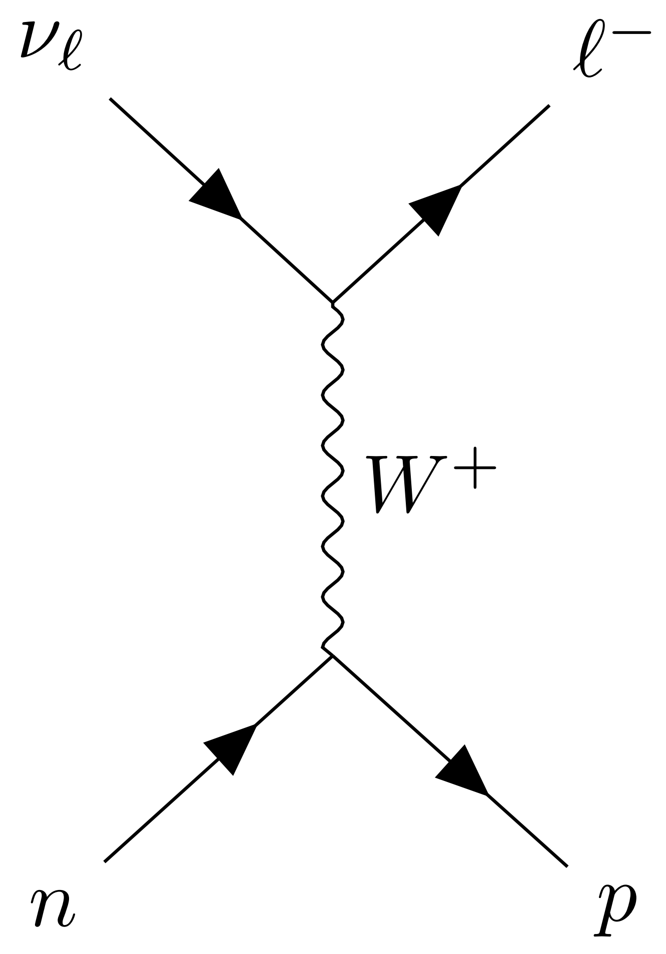

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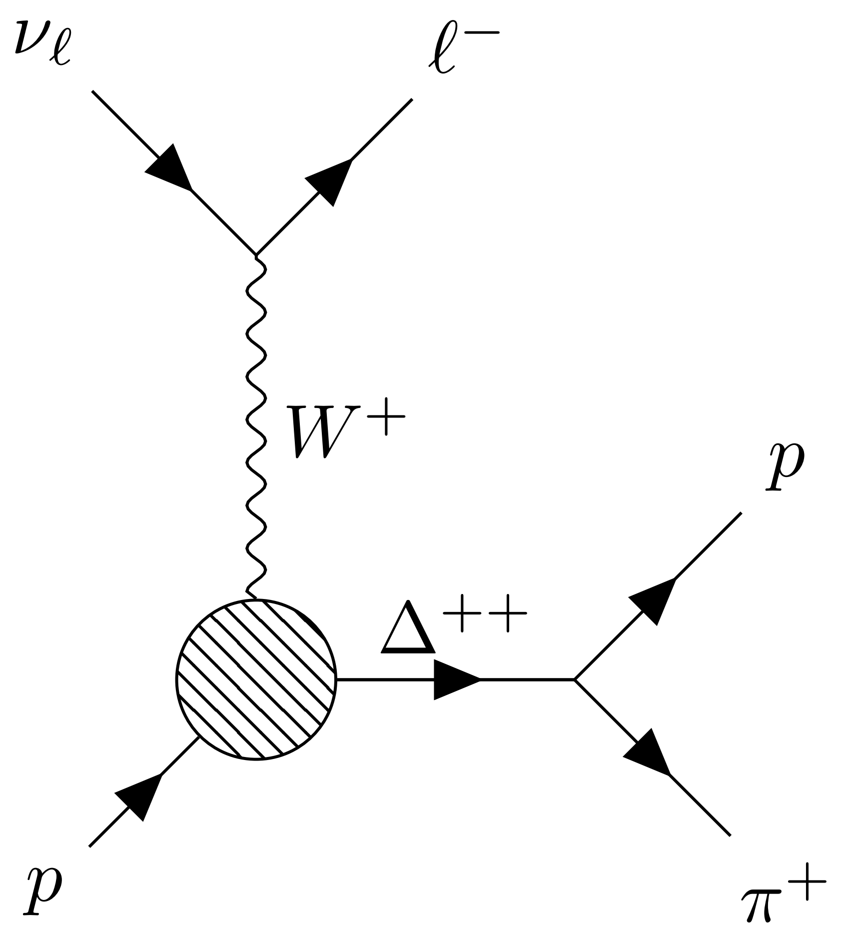
At higher energies, more complex interaction topologies:

The Resonant interaction



Nucleon is excited
-> Many final states

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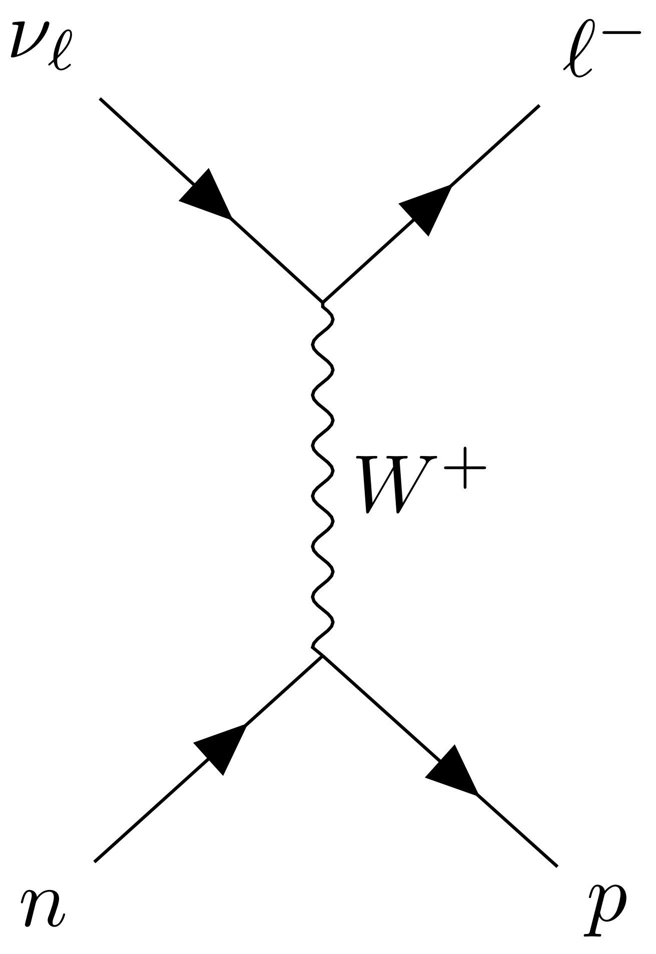

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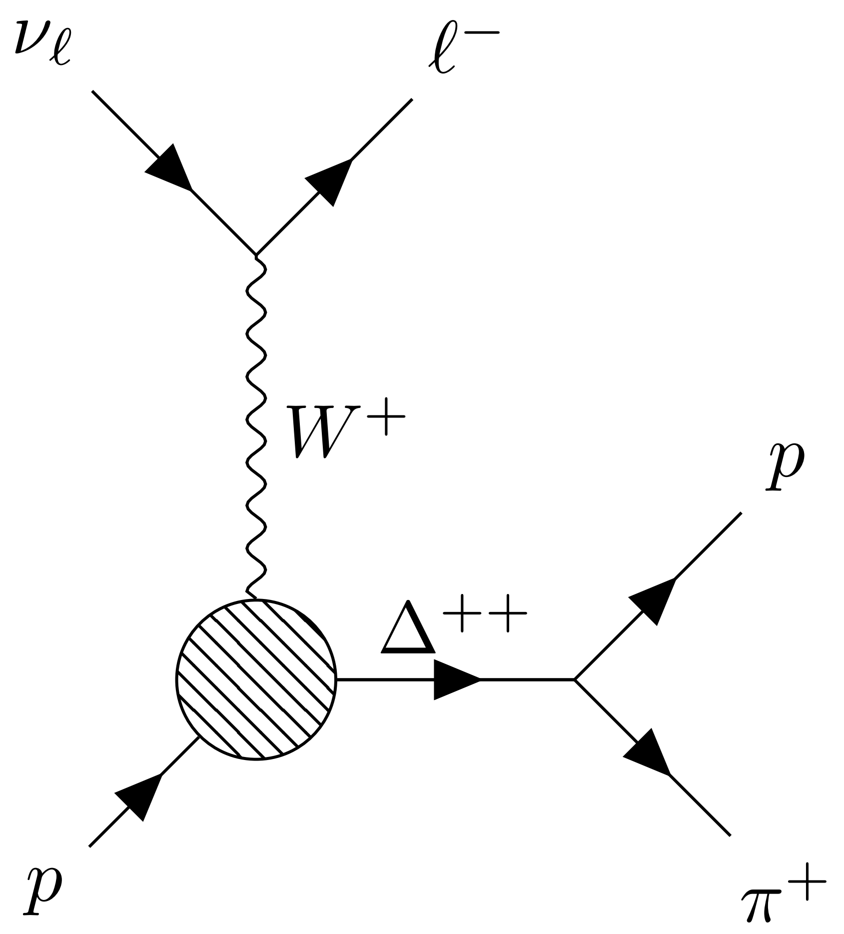

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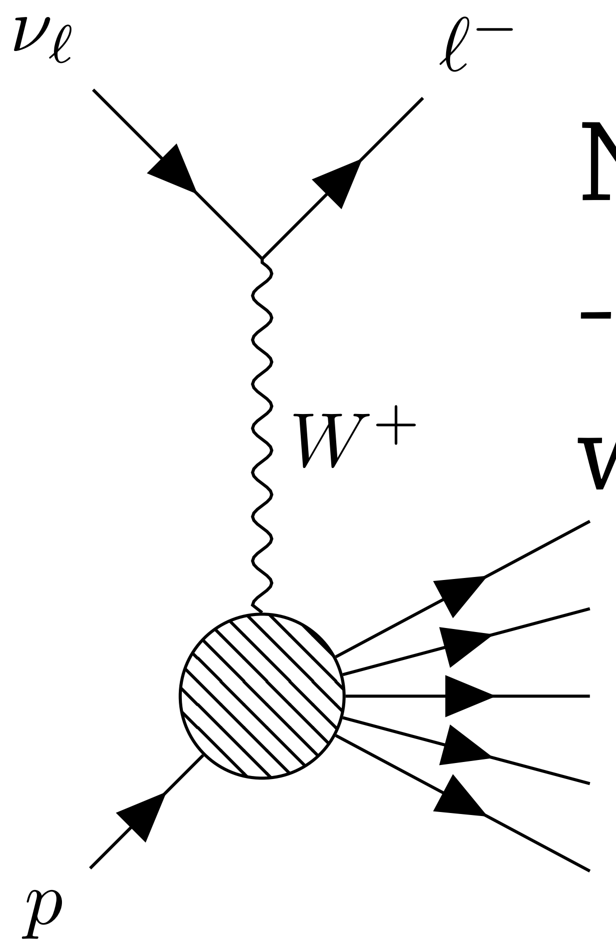
The Deep-Inelastic interaction



Nucleon breaks
-> Interactions with the quarks

Charged currents interactions

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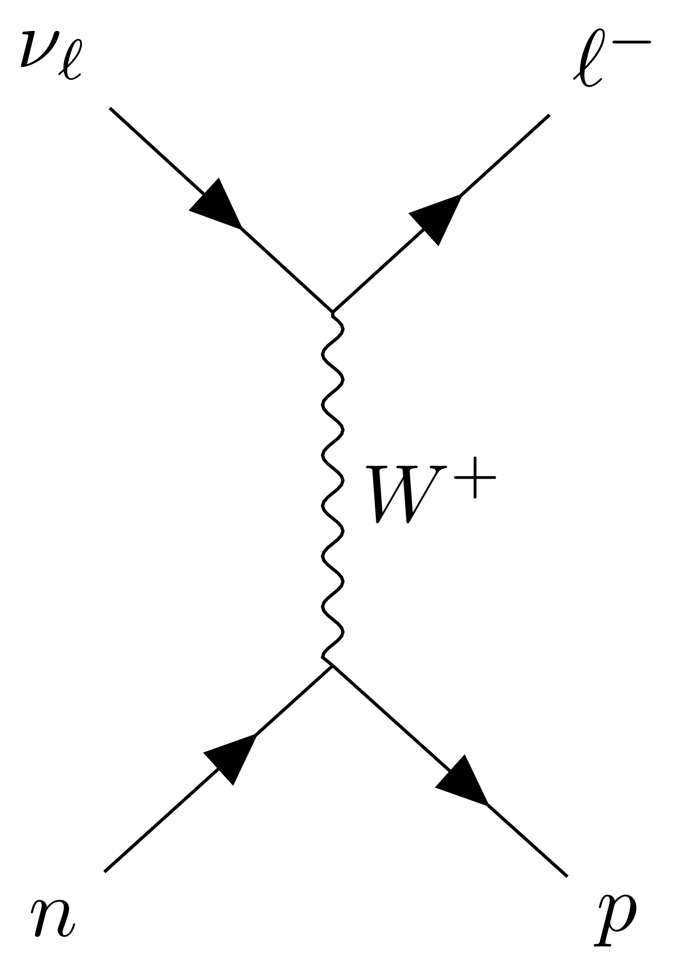

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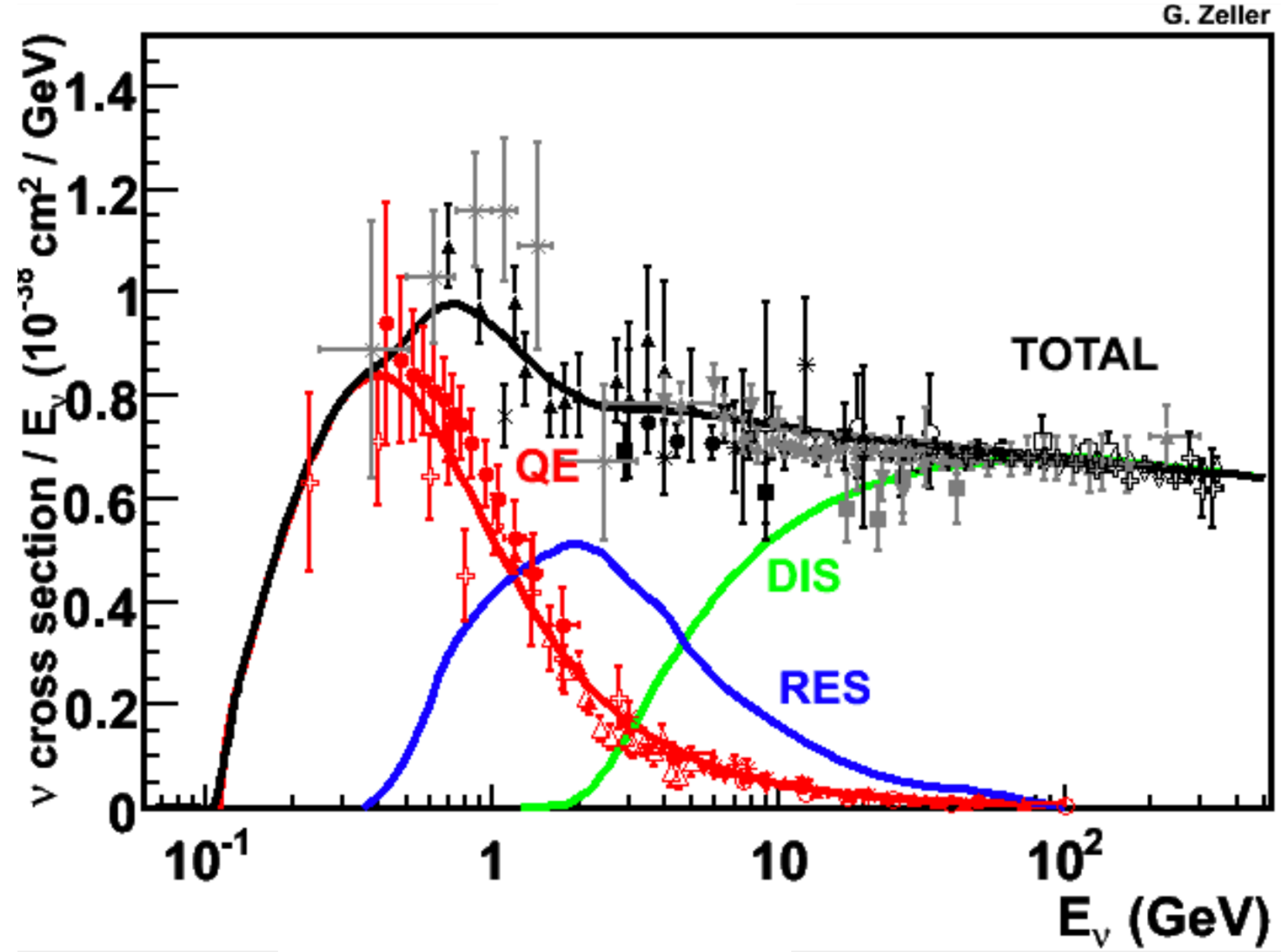
Neutrino cross section increase with energy but the final states are more complex



Charged currents interactions

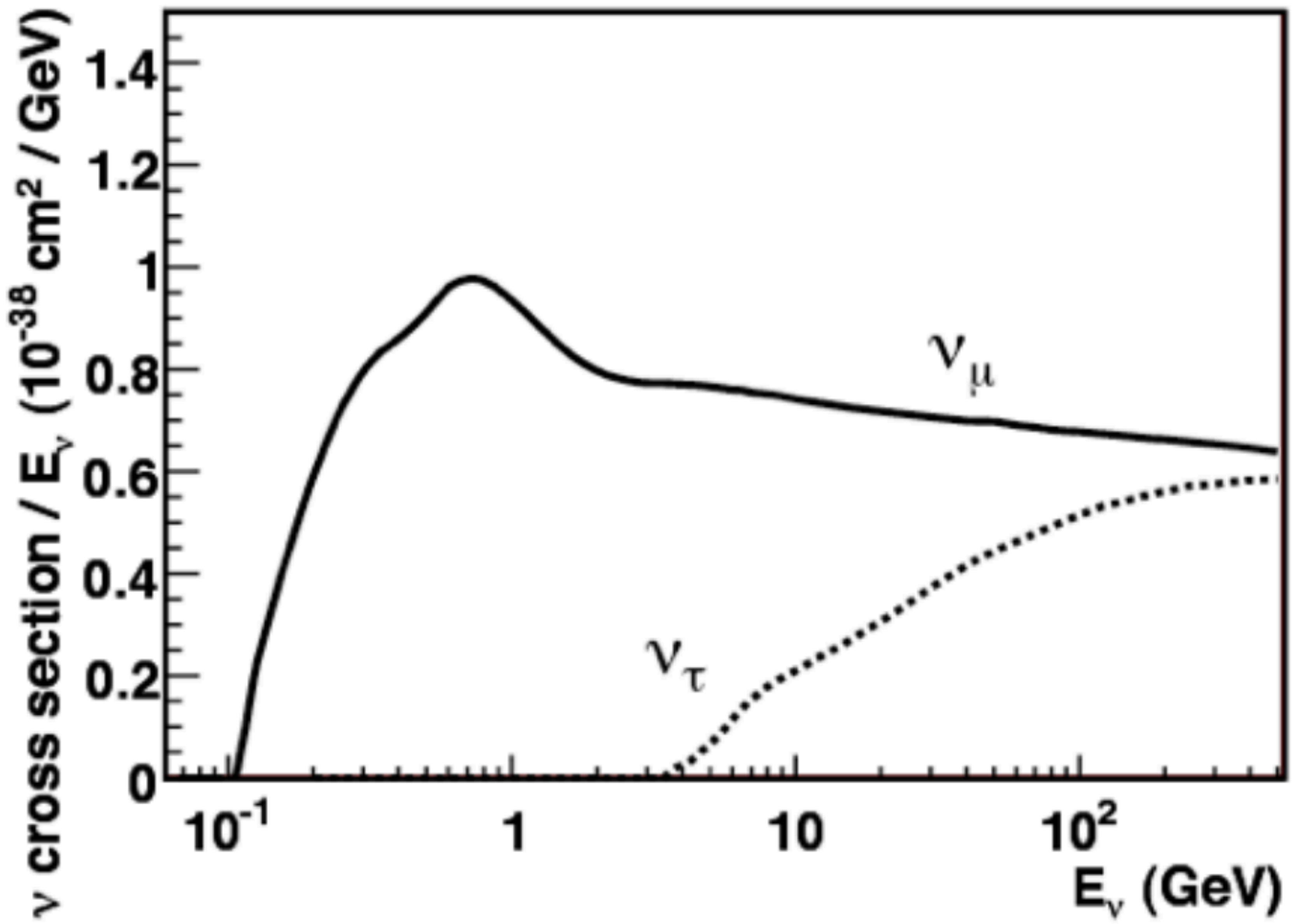
Interaction threshold

For CC interactions : $\nu_\ell + n \to \ell^- + p$

$$E_\nu \geq \frac{(m_\ell + m_p)^2 - m_n}{2m_n}$$

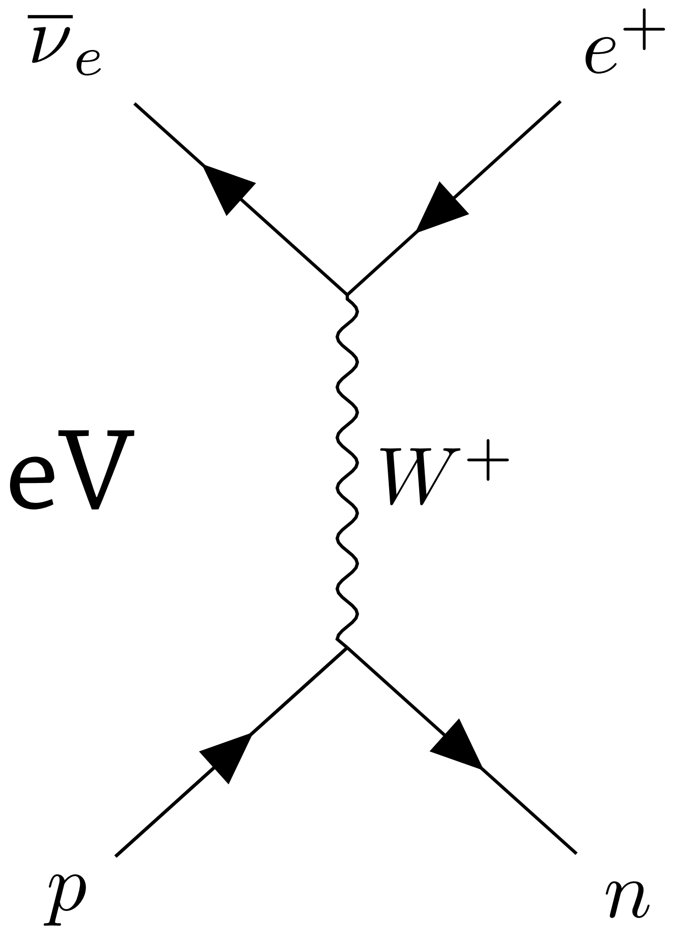
$E_{thr}(\nu_\mu)$ = 110 MeV

$E_{thr}(\nu_\tau)$ = 3.45 GeV

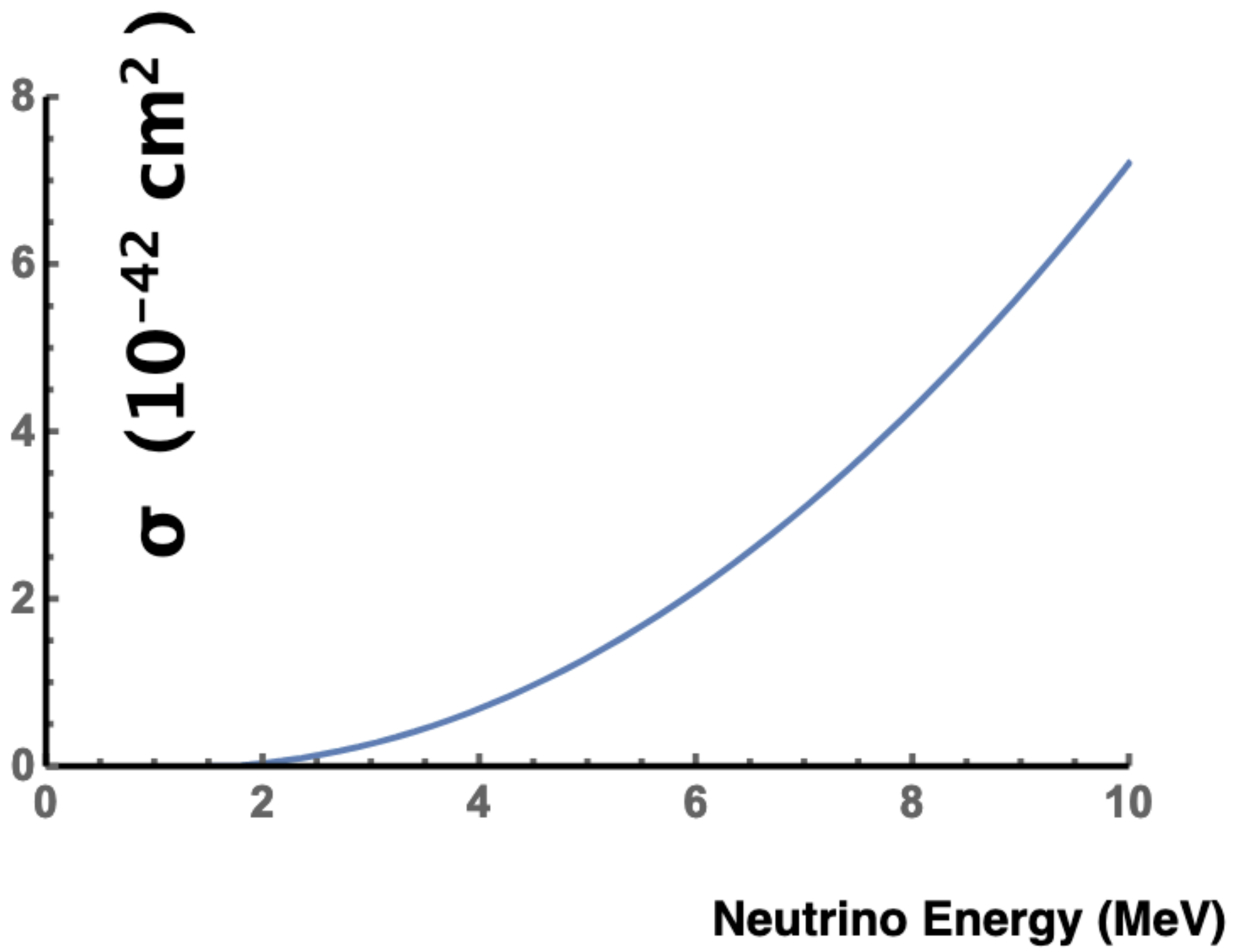


$\bar{\nu}_e$ low energy interaction

inverse β-decay :



$E_{thr}(\bar{\nu}_e)$ = 1.806 MeV

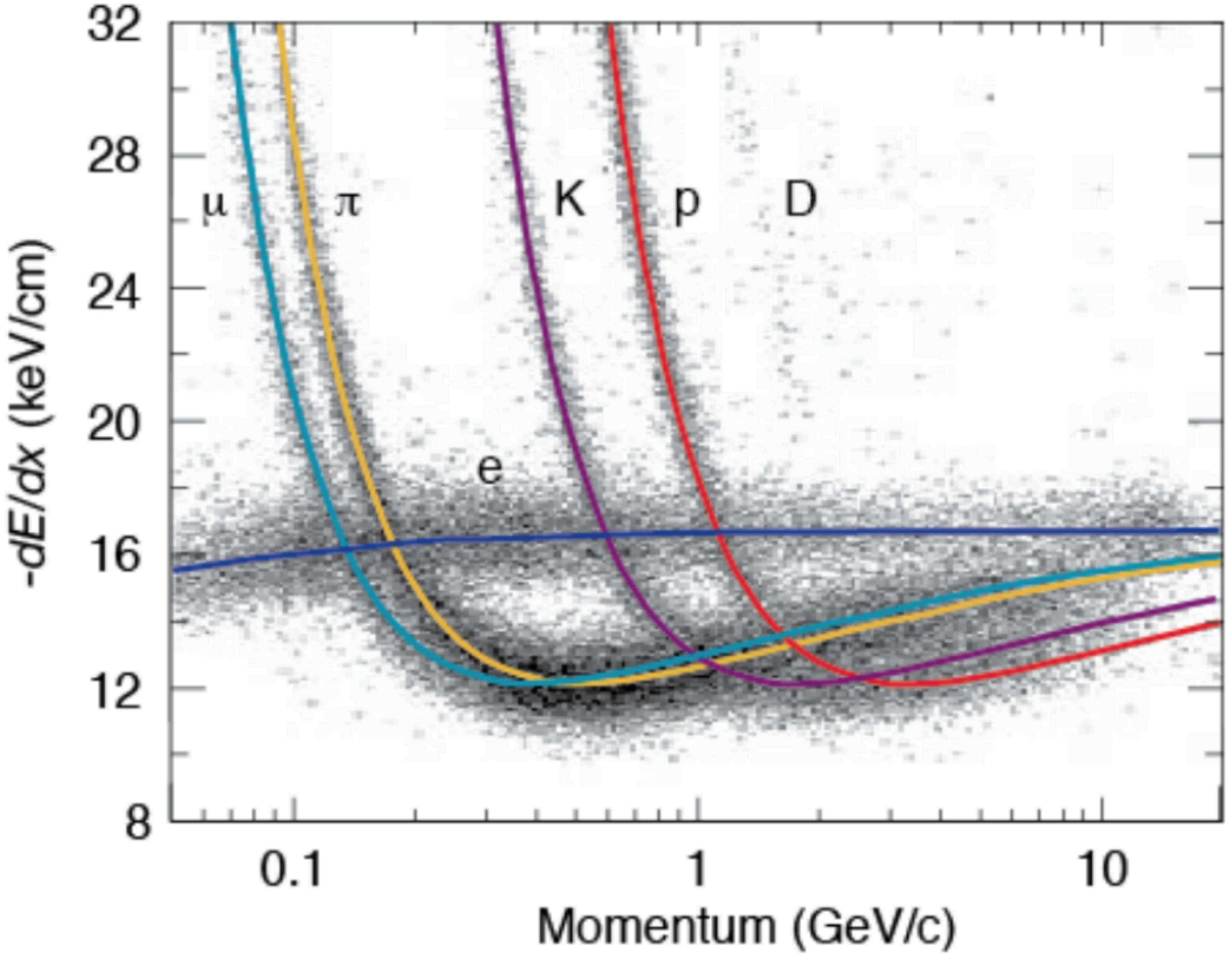


Using the Ionization potential

Principle : When a charged particle crosses a medium, it looses energy through ionization.
The mean amount of energy lost per cm through ionization is parametrized by the Bethe Bloch formula and depends on the particle energy (βγ)

$$\left\langle -\frac{dE}{dx} \right\rangle = Kz^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 W_{max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

PEP-4 detector, gas mixture 80:20 of Ar:CH_4



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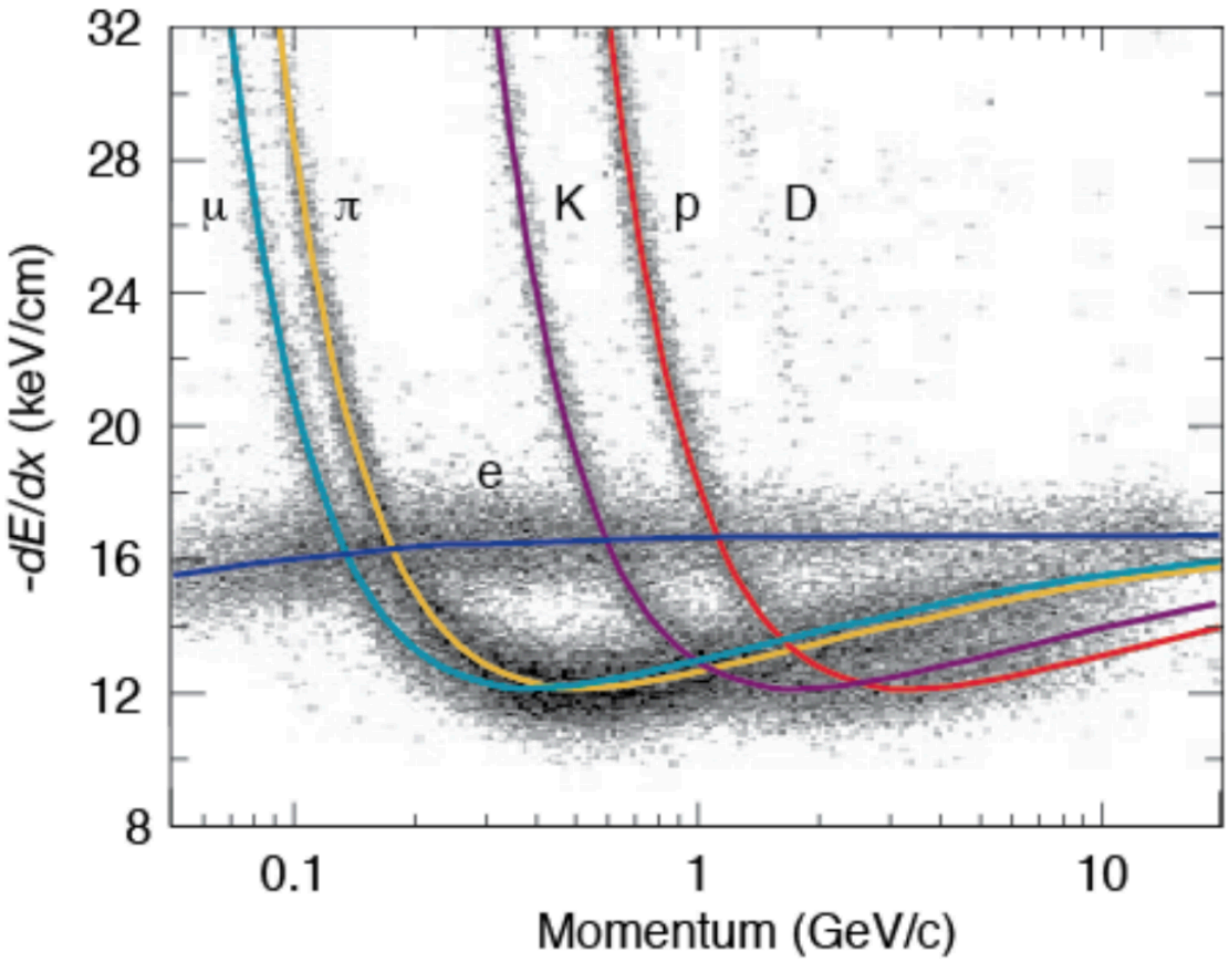

PEP-4 detector, gas mixture 80:20 of Ar:CH_4

If this energy lost can be seen, one can have a 2D (or even 3D) image of the interaction. Through track topology, one can know the daughters identity.

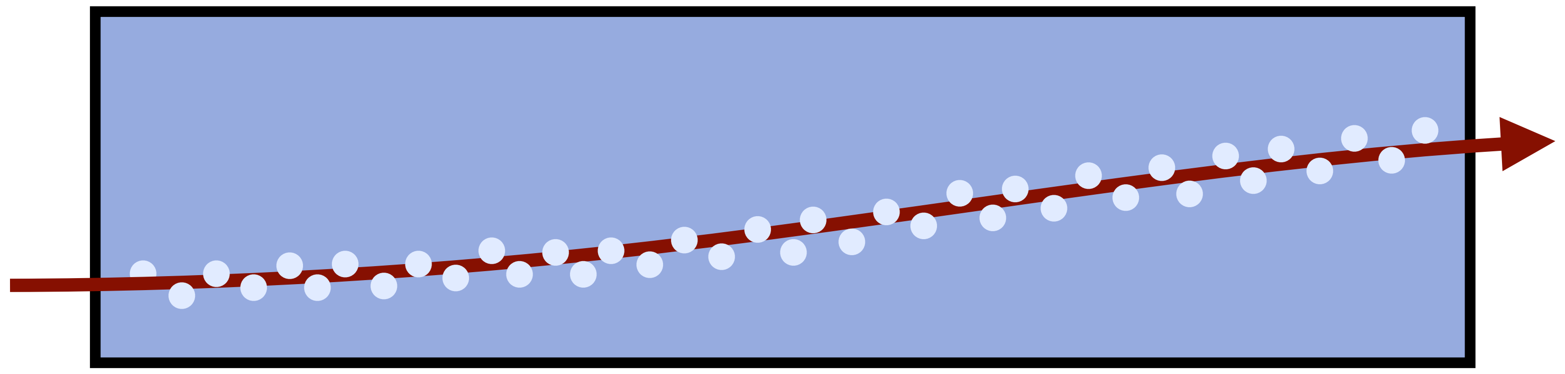
Moreover, if this energy can be collected, one can reconstruct the energy of the daughters, and hence fully reconstruct the interacting neutrino kinematics.

Using the Ionization potential

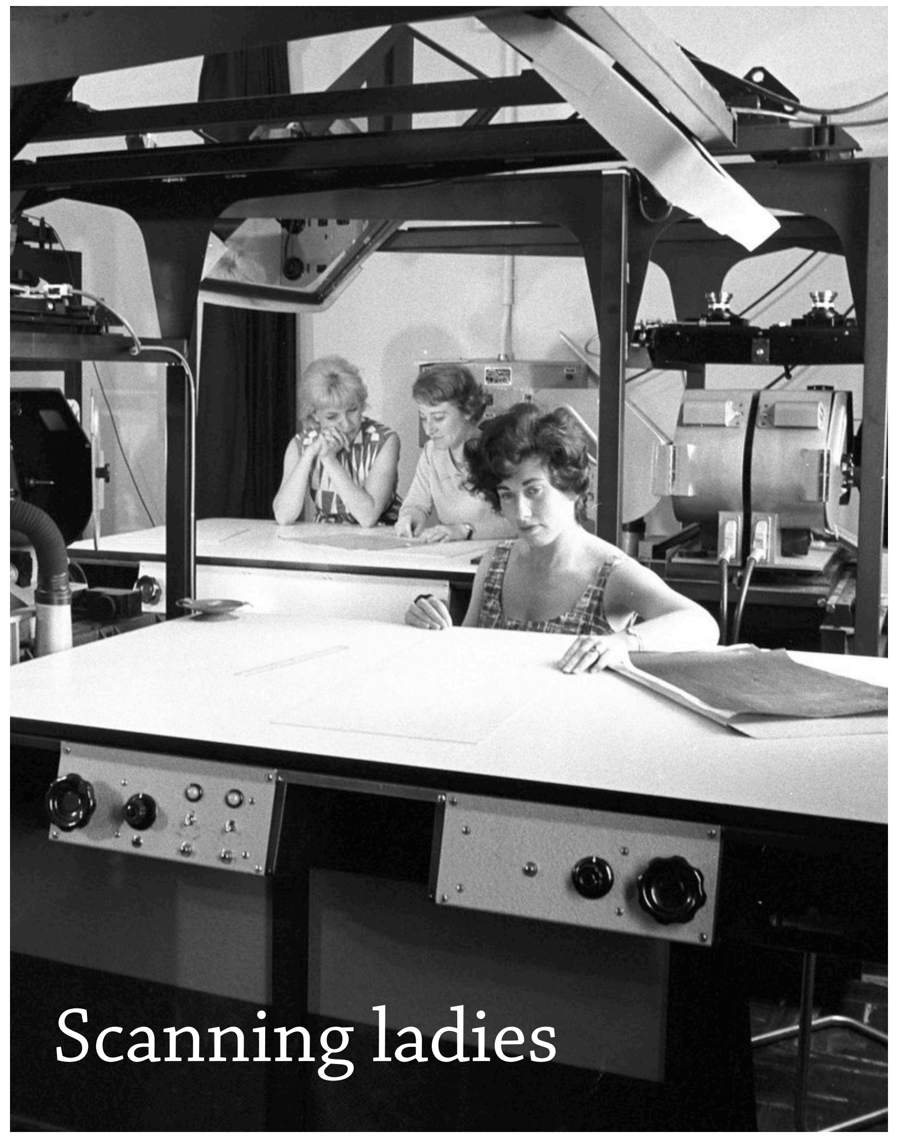

BEBC at CERN

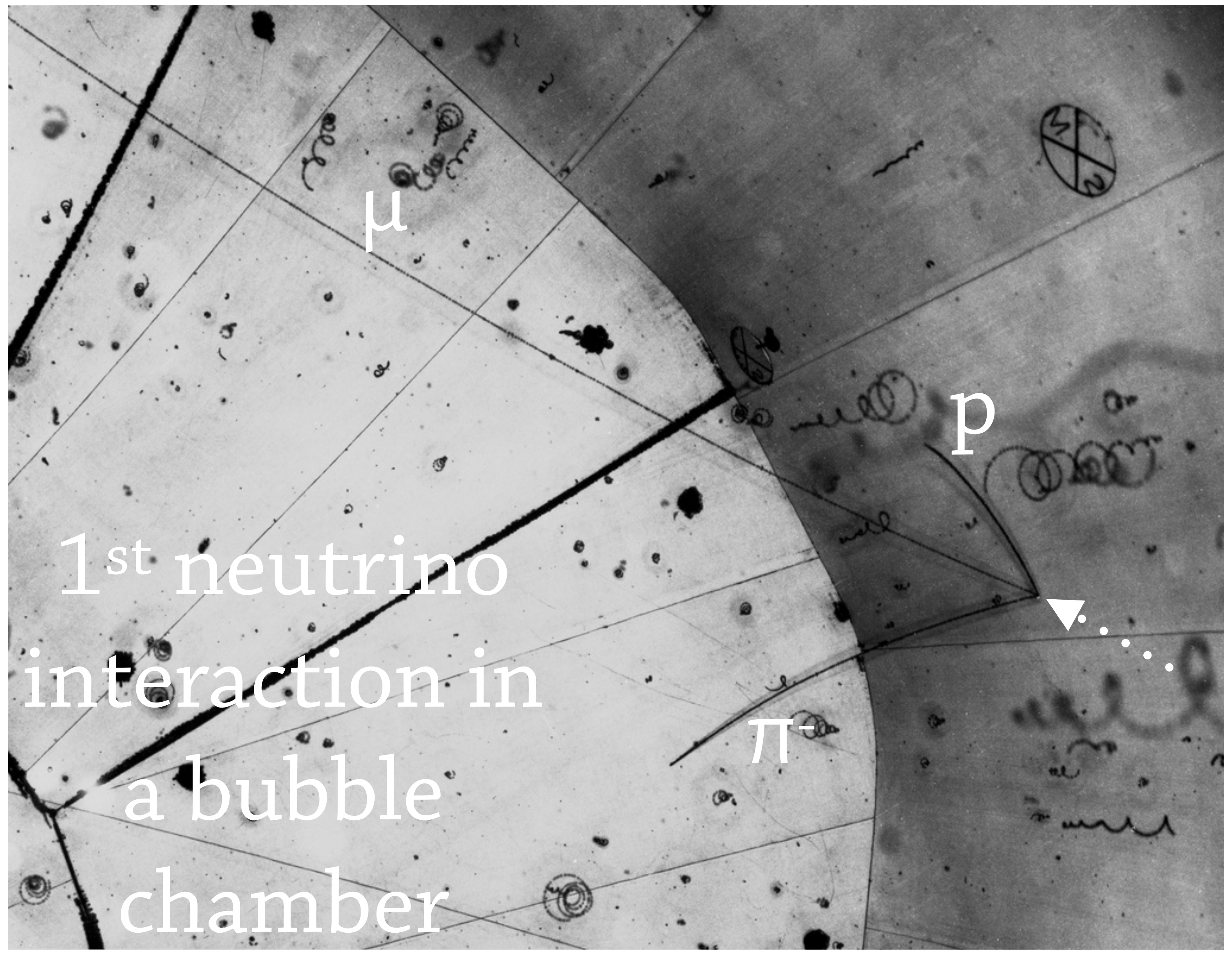
Bubble Chambers

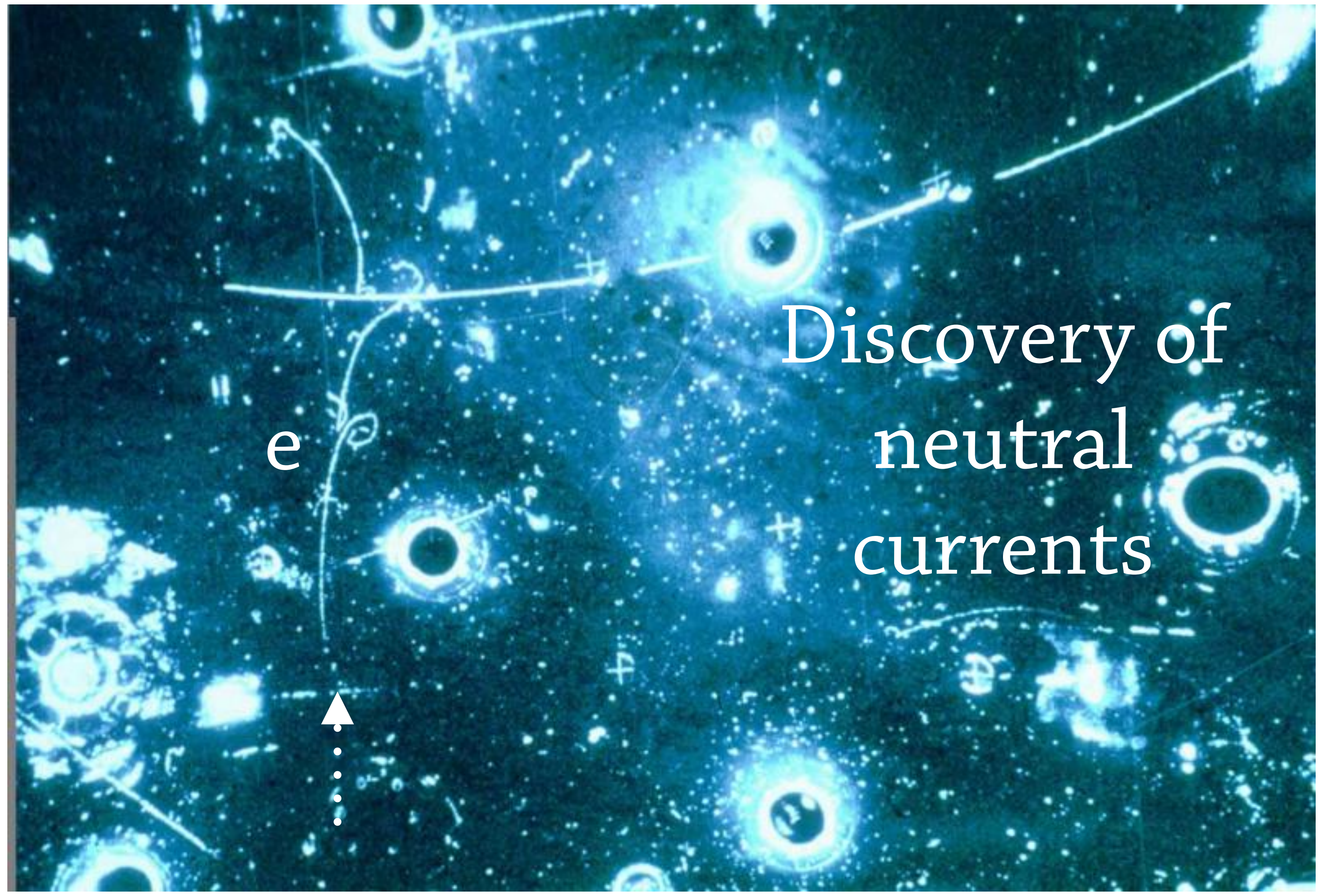
Superheated fluid turns locally to gas (bubbles) when energy is deposited by a charged tracks :



First bubble chambers where equipped with cameras, the pictures were scanned manually by the scanning ladies.


Scanning ladies



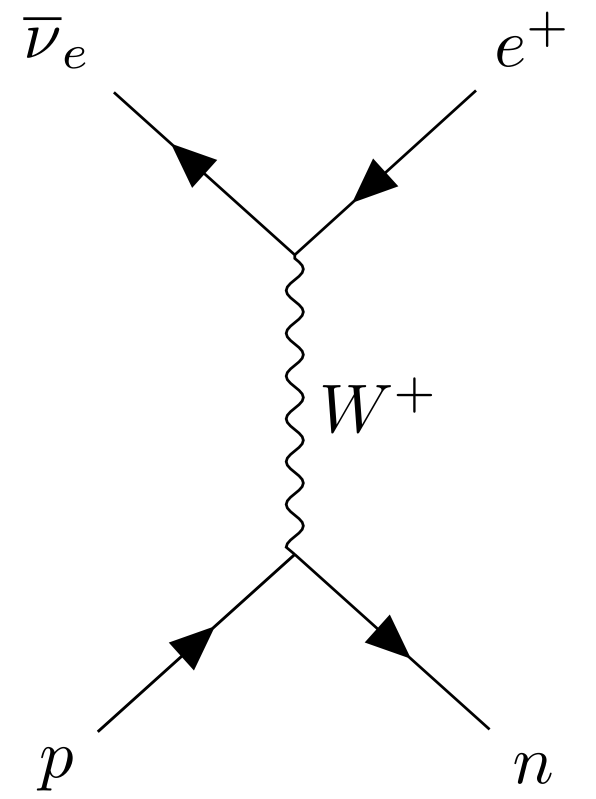


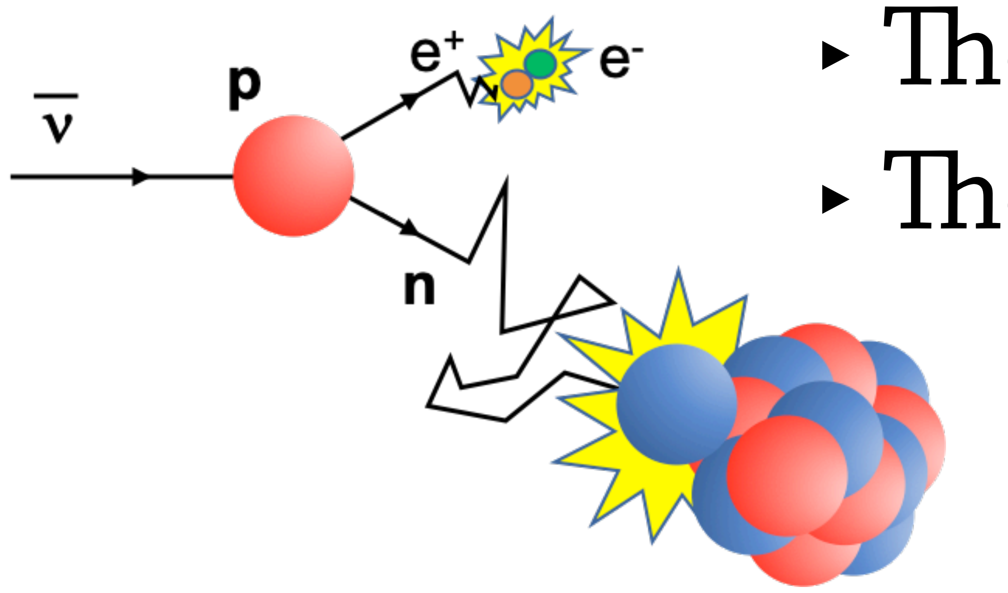
Using the Ionization potential

Liquid Scintillators

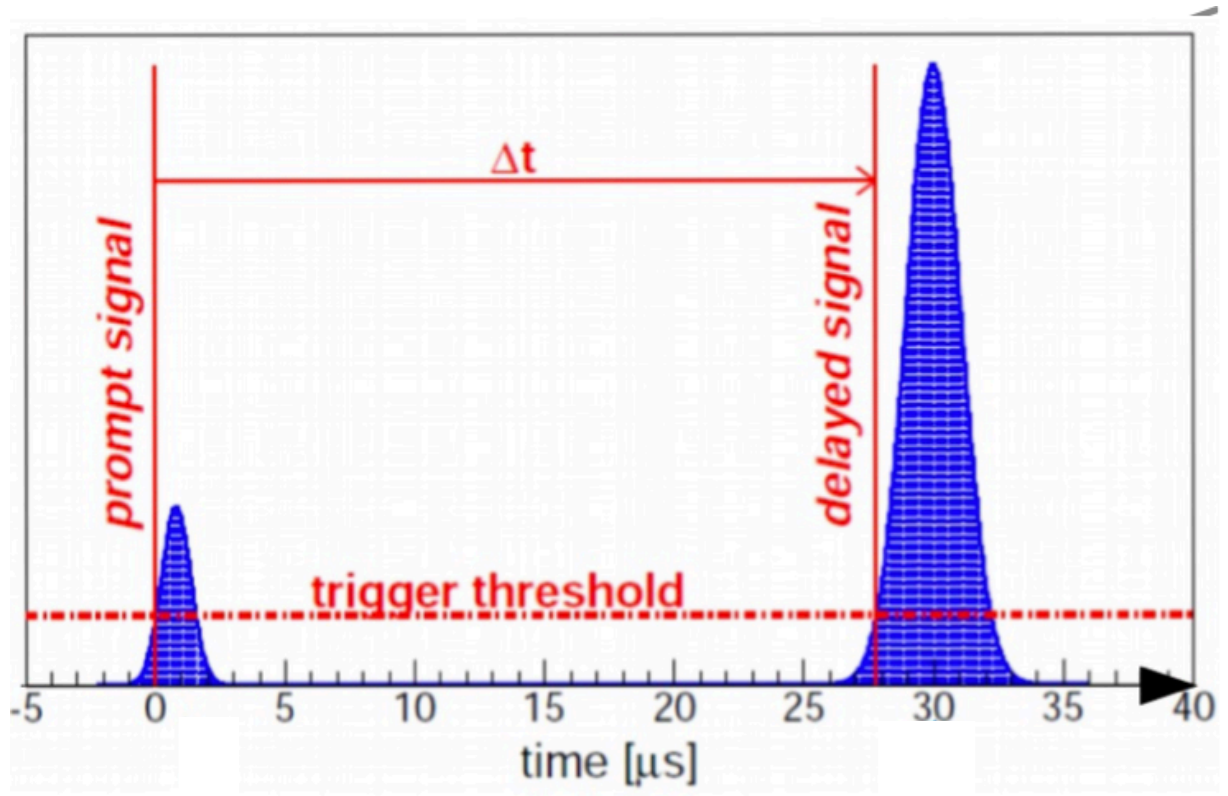
Organic liquid that scintillates when energy is deposited.

In neutrino physics, often used to tag inverse β-decay interactions



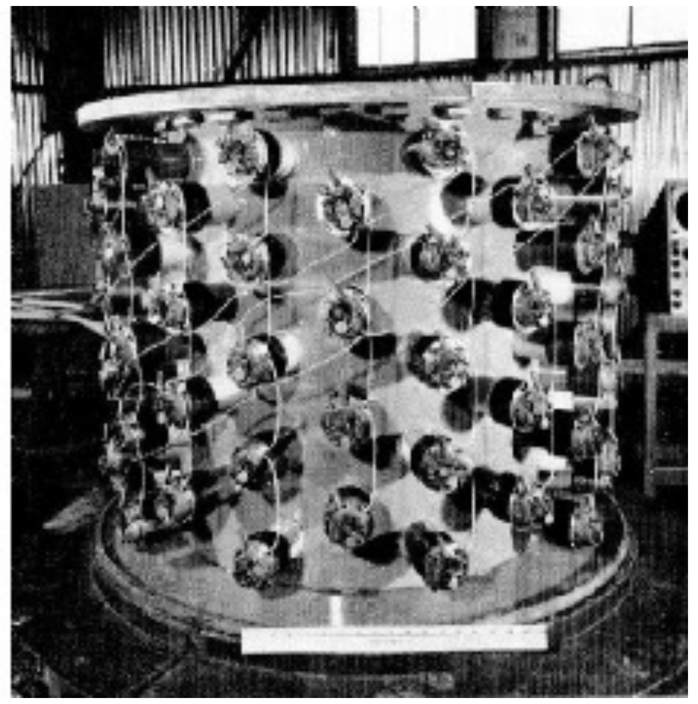


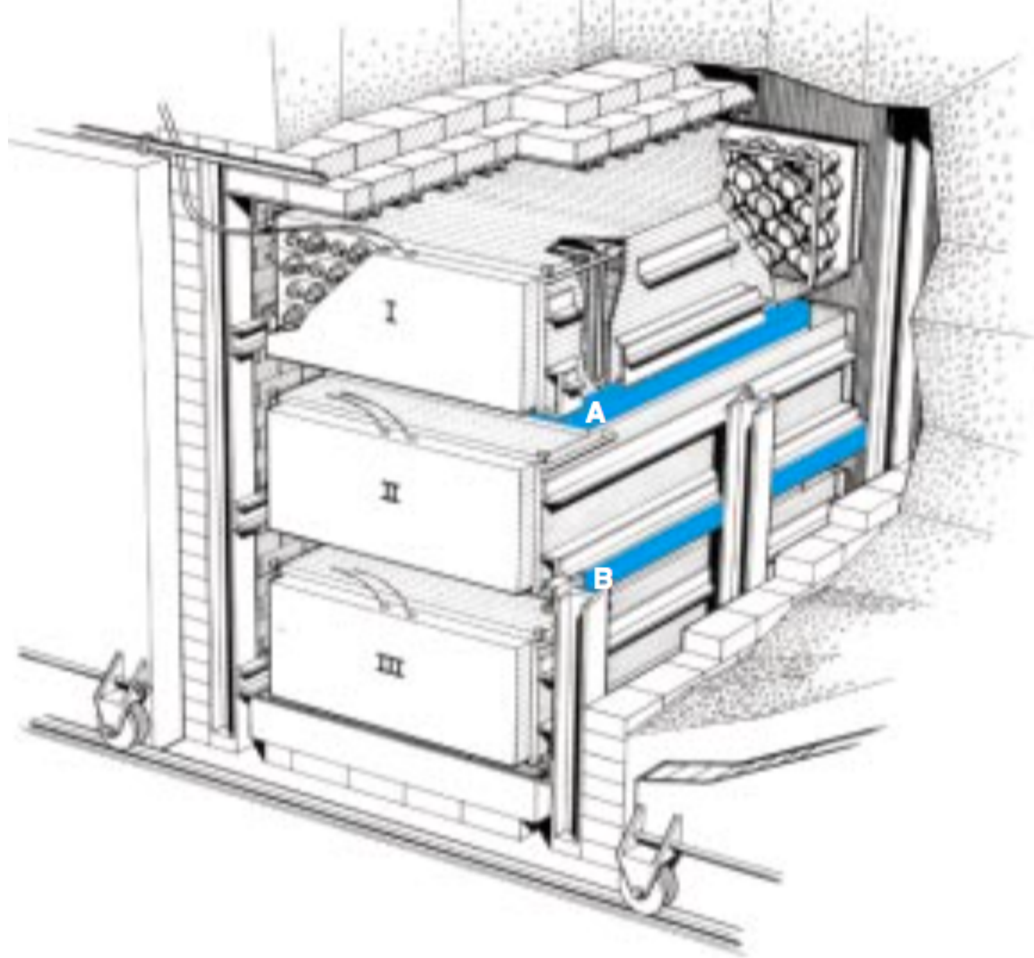
- The positron is quickly captured by an electron
- The neutron is captured later by a catcher-atom

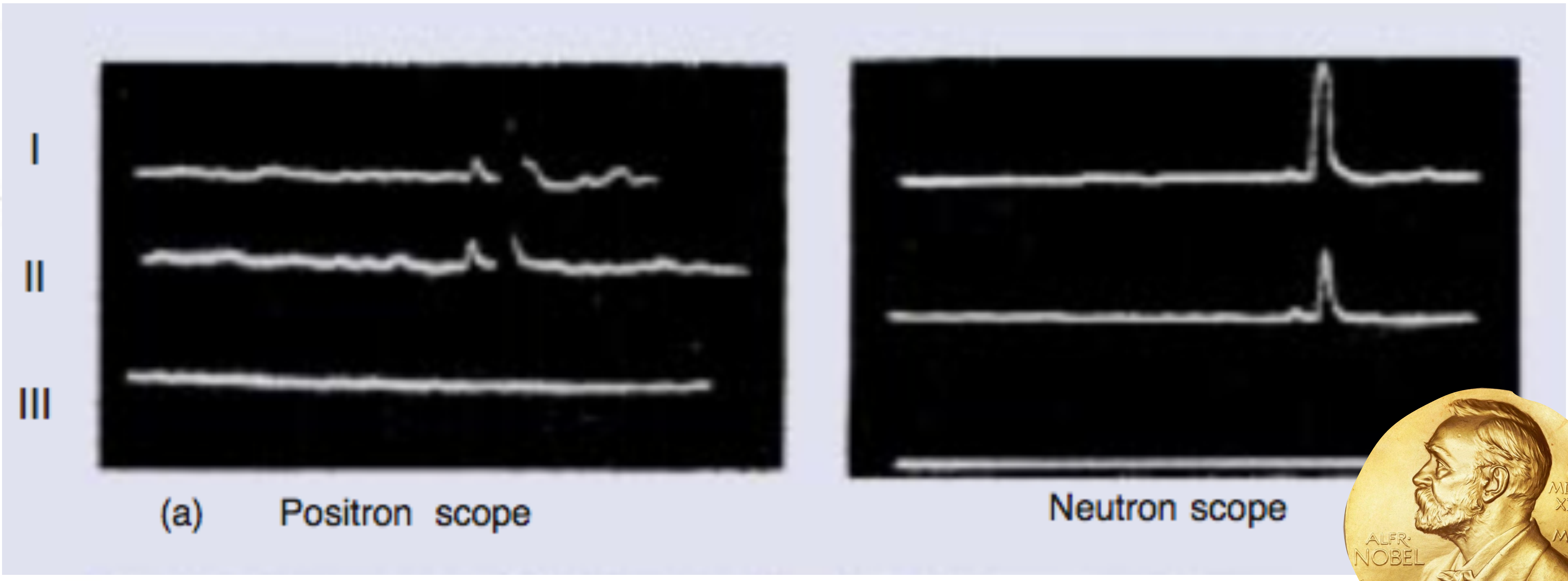


Savannah river experiment by Reines & Cowan in 1956

ν̄e discovery !





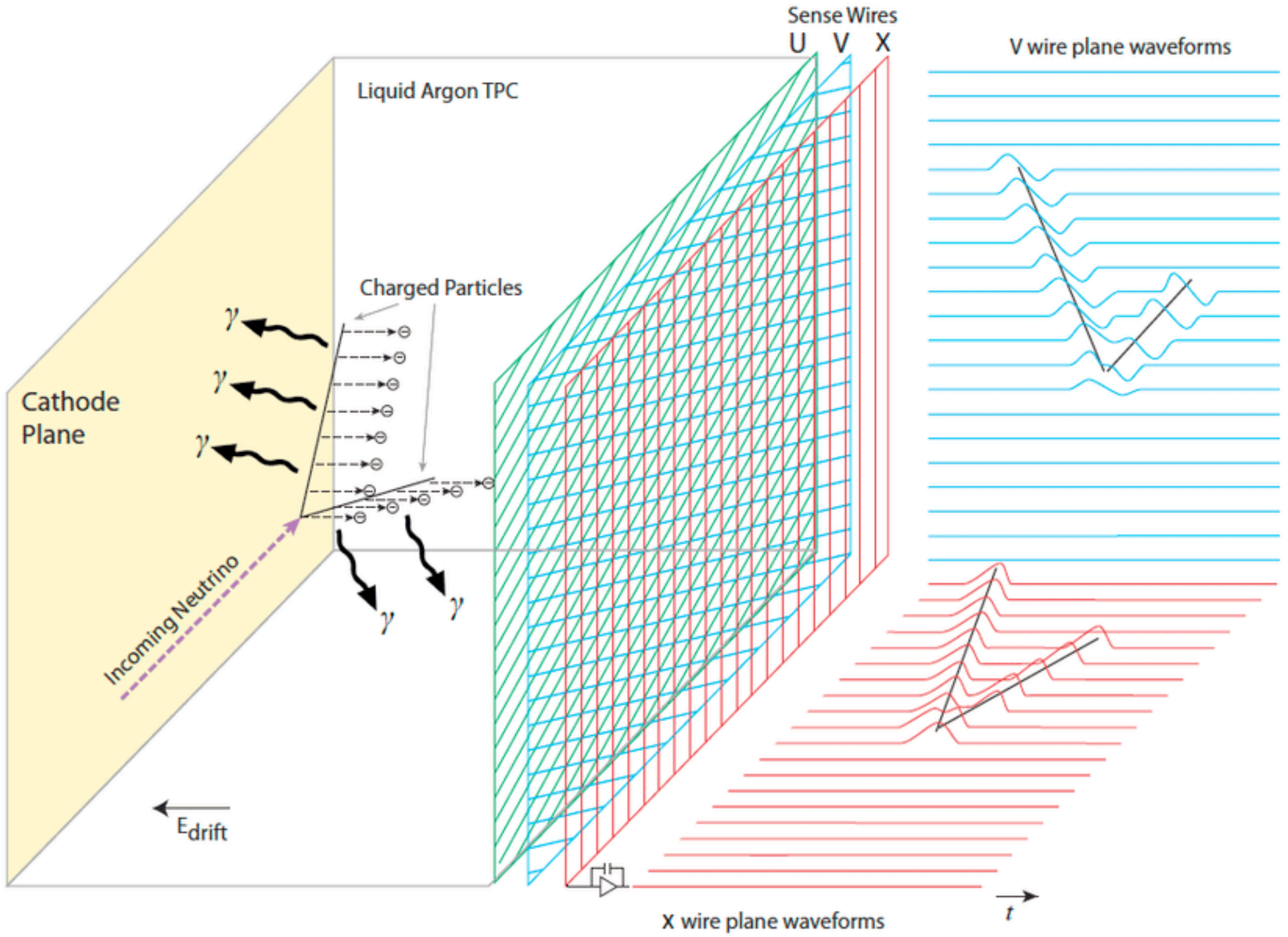


Using the Ionization potential

Time Projection Chamber [TPC]

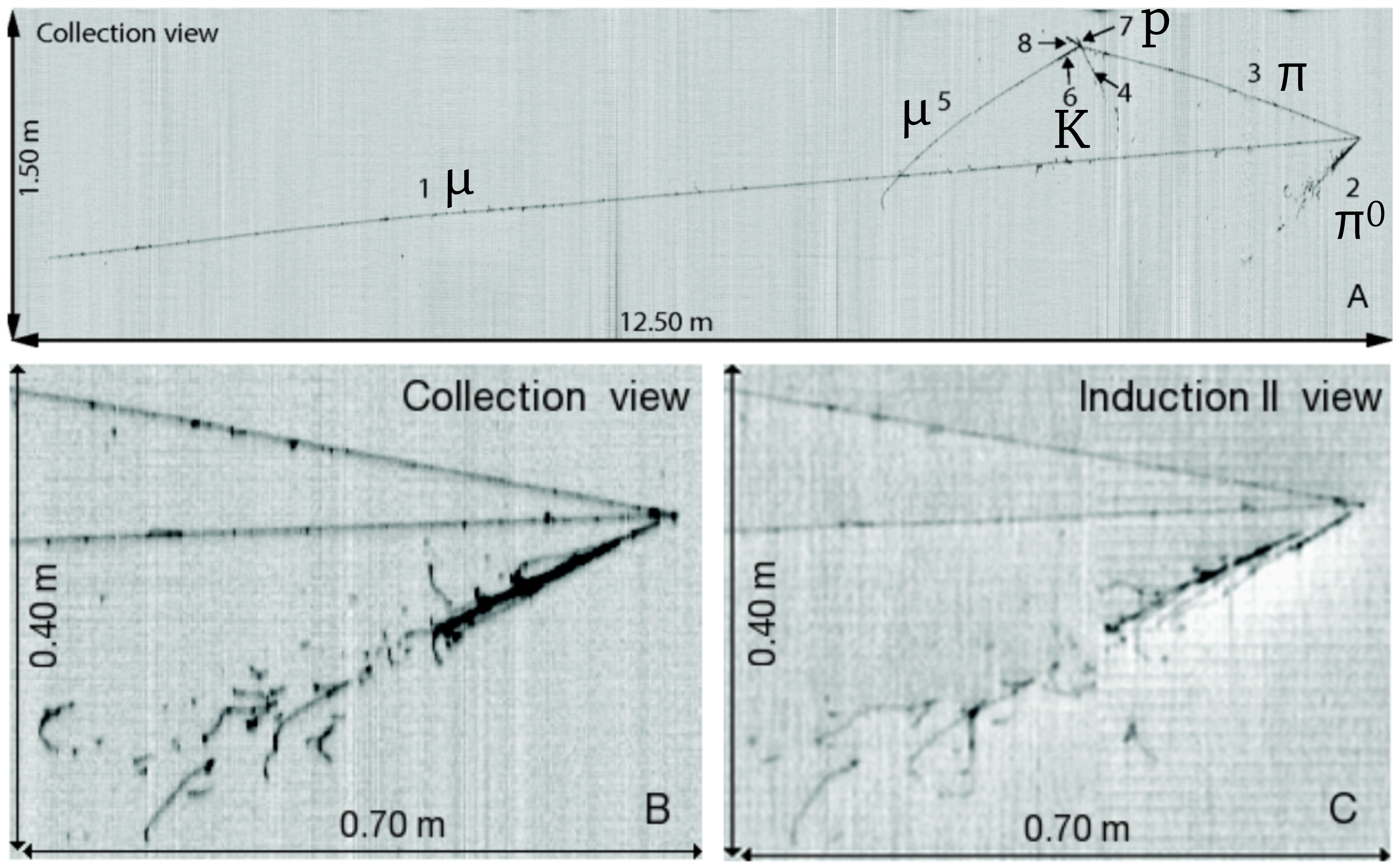
Uses a chamber filled with gas or liquid with an electric field applied across.

Free electrons from ionization are drifting towards the anode plane where they are collected : that gives a 2D image. The e^- arrival time provides the 3rd coordinates. The amount of e^- collected/cm is a handle to retrieve the particle identity/energy.



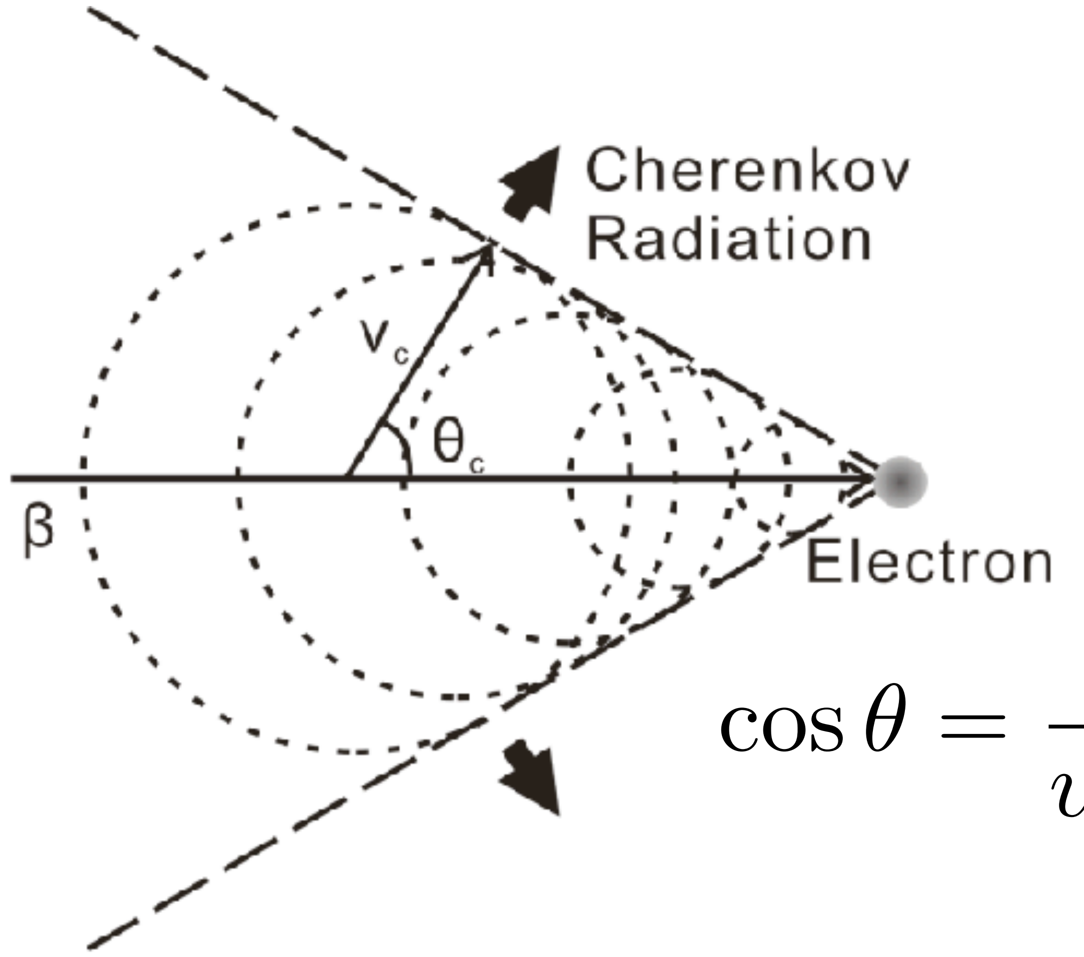
ICARUS experiment in Italy (now in USA)

- ν_μ interaction -



Using the Cherenkov effect

<ins>Principle</ins> : When a charged particle travels at a speed *v* higher than the speed of light in a medium *c/n* it radiates a cone of light :



$$\cos\theta = \frac{c}{vn}$$

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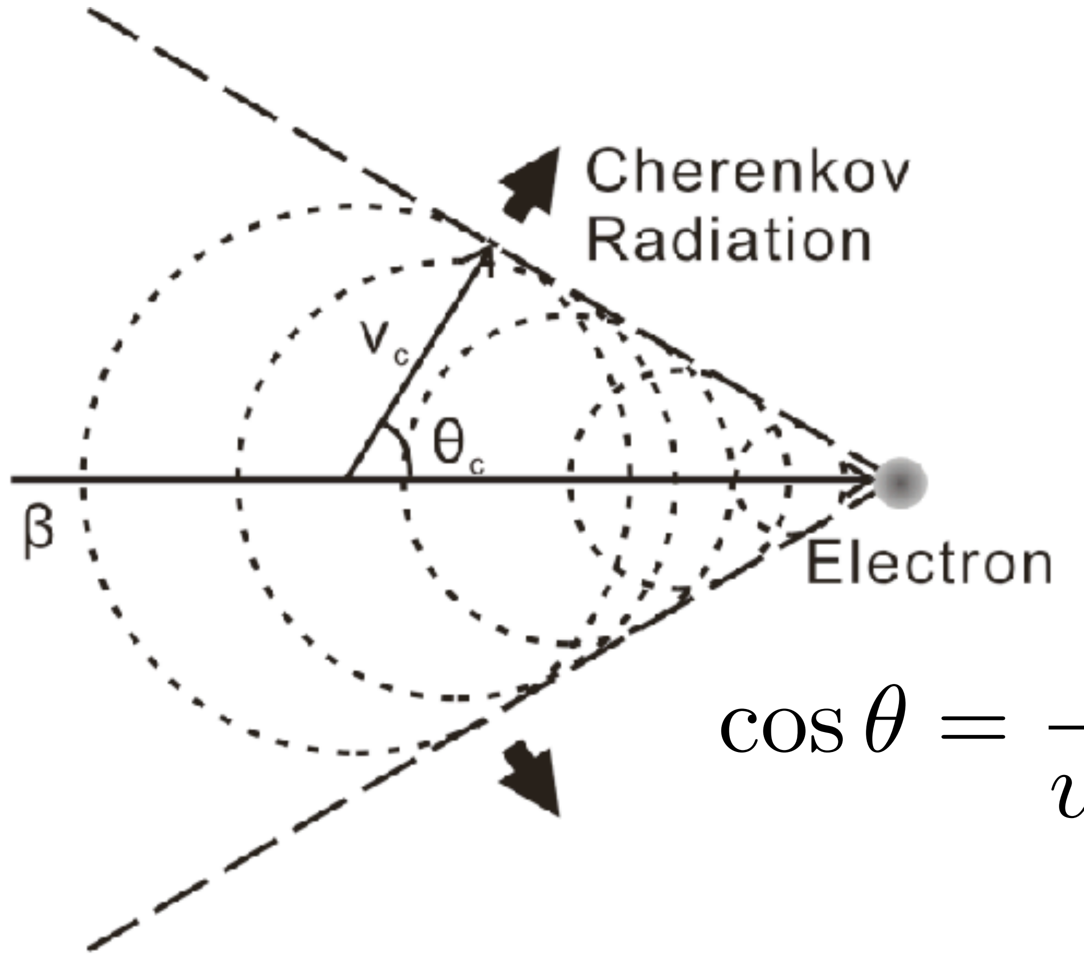

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Cherenkov detectors are widely used in neutrino physics :

-> Can use cheap/free medium (ultra pure water, ice, sea)

-> Use photomultipliers to detect the light, very well known device

-> Can have large volume : bigger volume = more chances to catch a neutrino

-> Ring shape allows particle identification ; ring characteristics (diameter, nb of photons) is linked to the particle energy => Excellent e/μ separation

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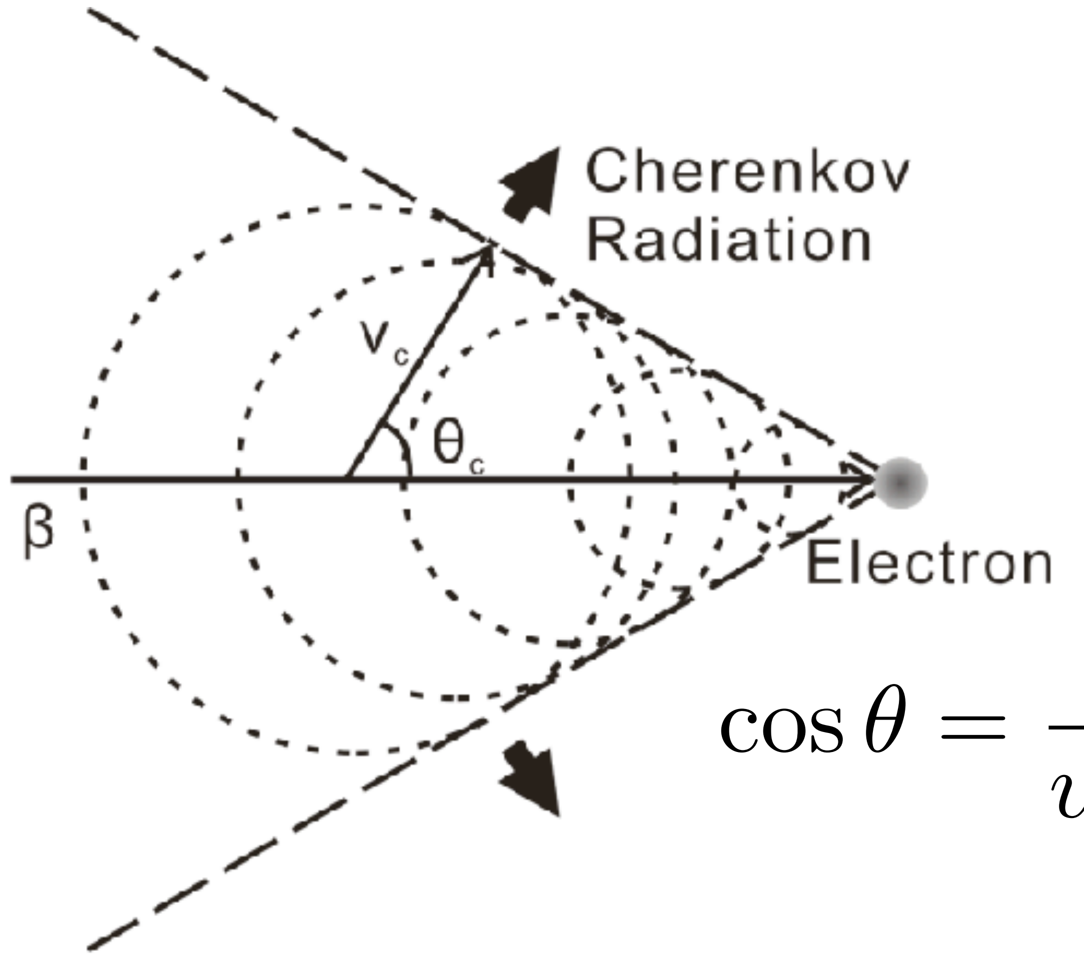

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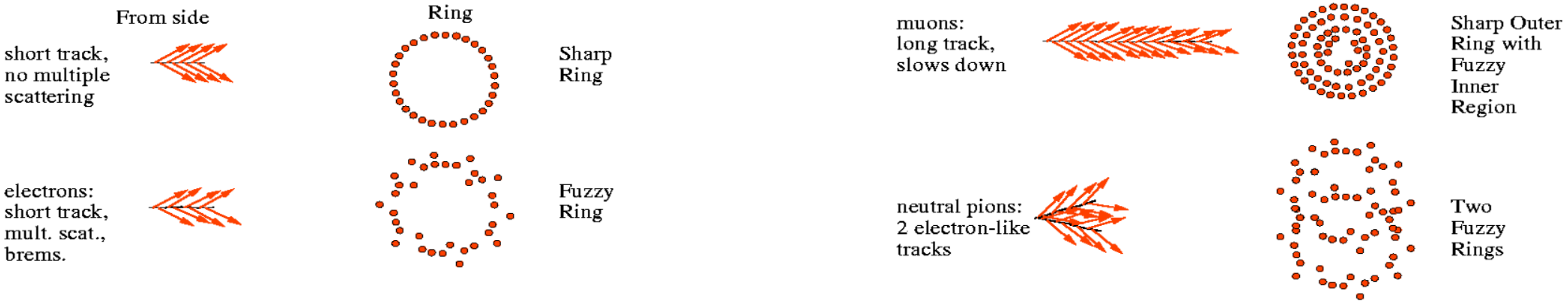
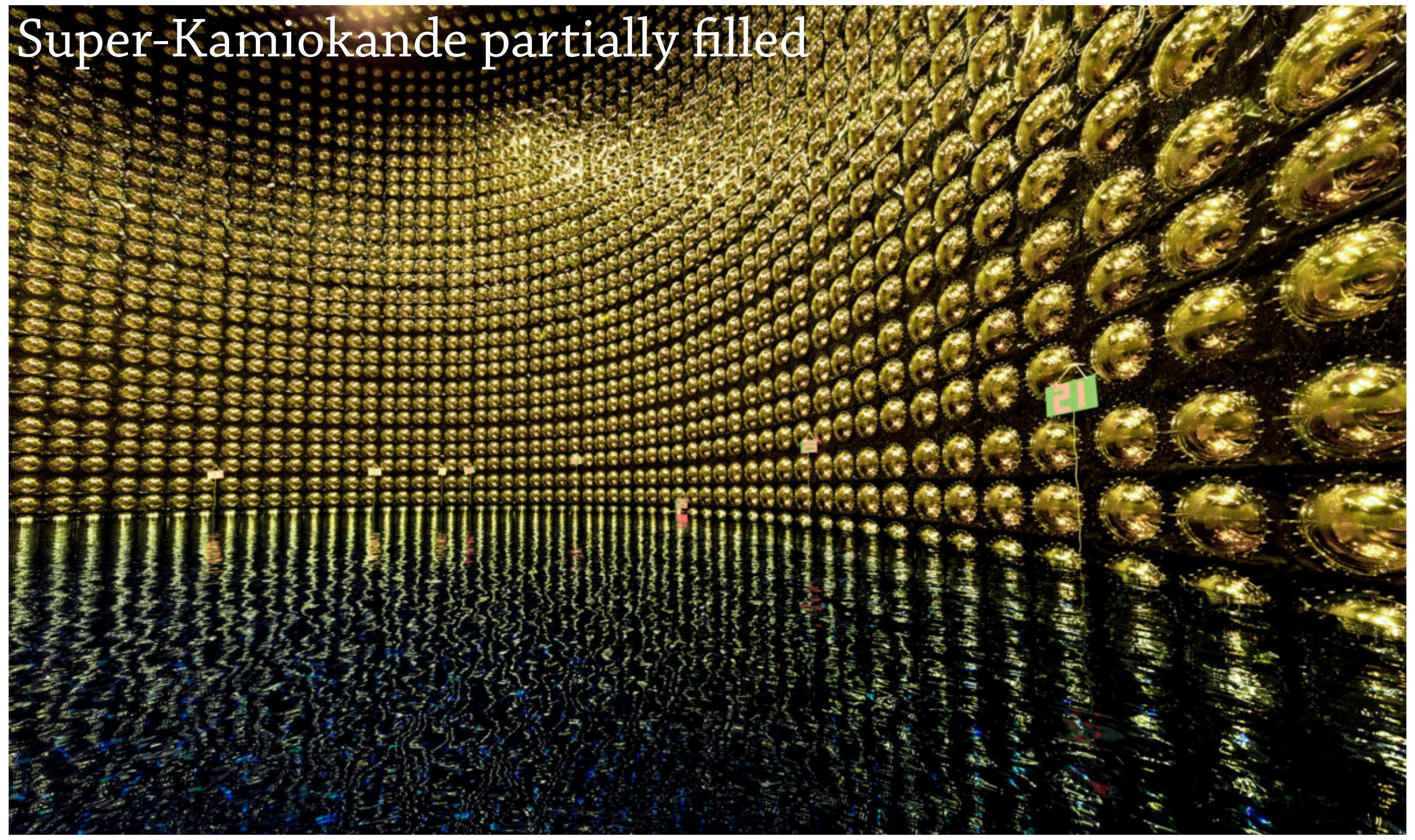
Particle identification using ring shape :



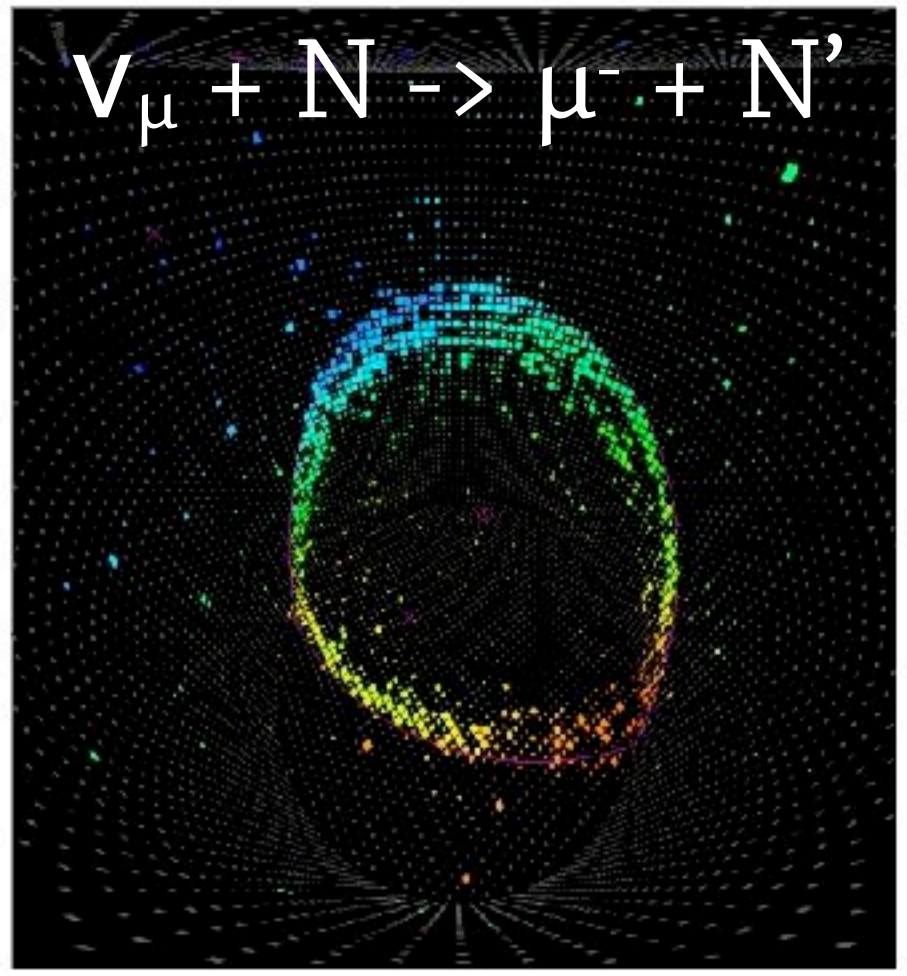
Using the Cherenkov effect

Super-Kamiokande in Japan

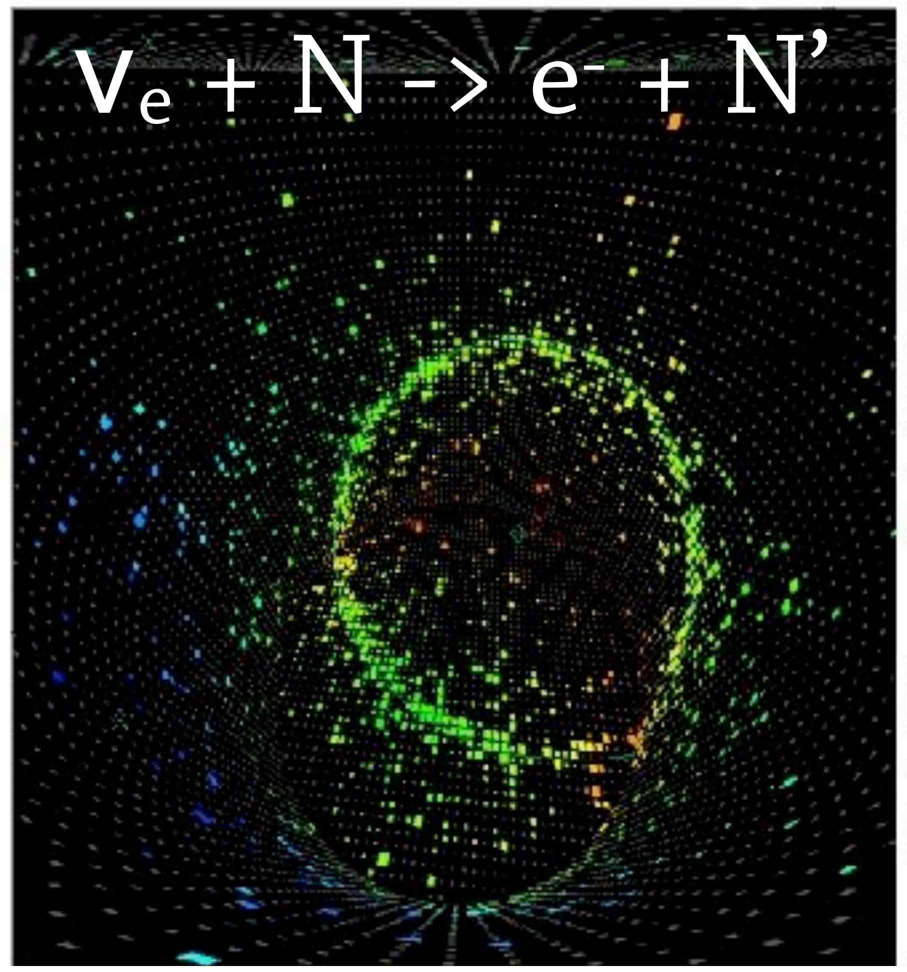
Tank of 50 kt of ultra pure water underneath a mountain, equipped with ~11k PMTs



Super-Kamiokande partially filled



$\nu_\mu + N \to \mu^- + N'$



$\nu_e + N \to e^- + N'$

Color = nb of photons collected

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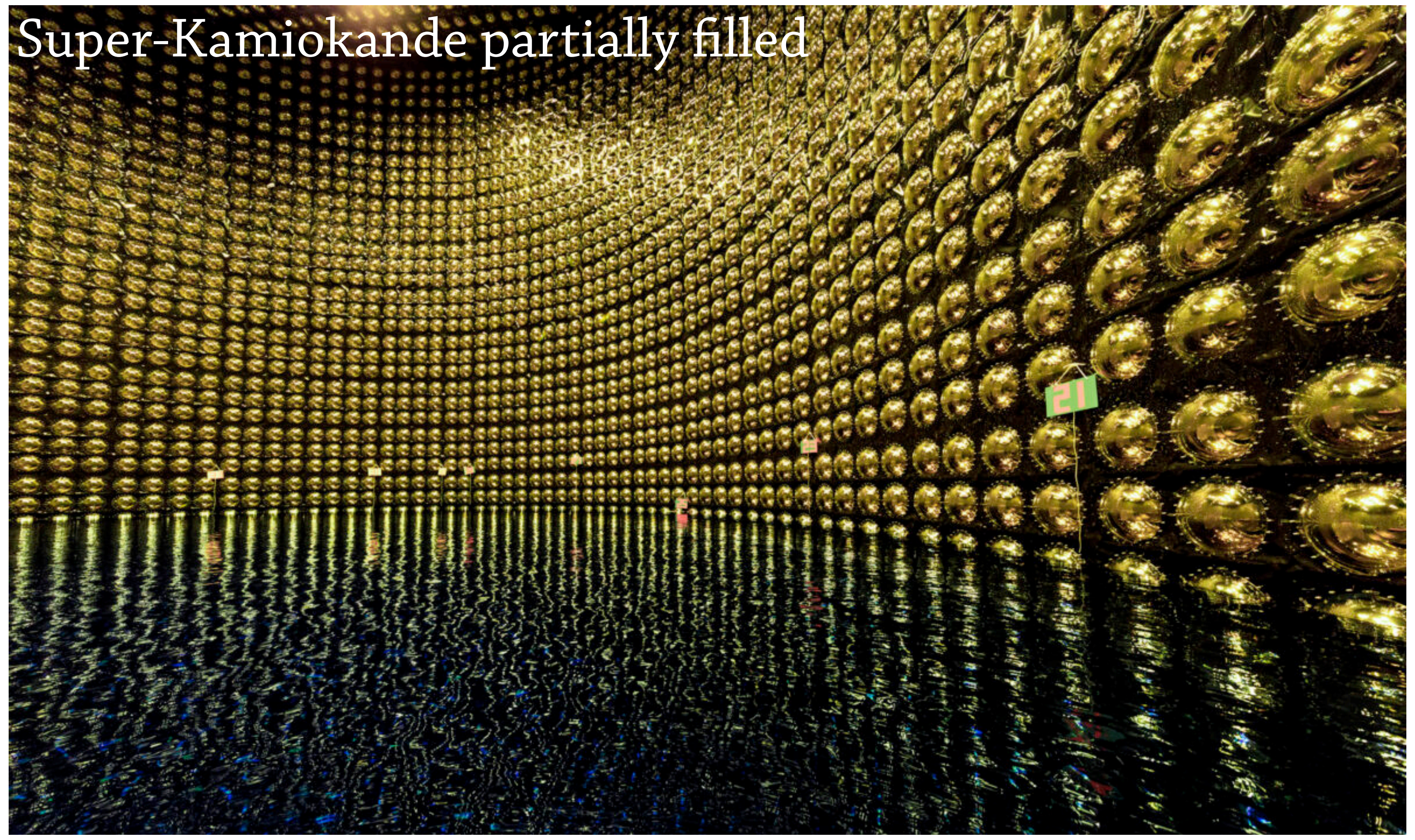
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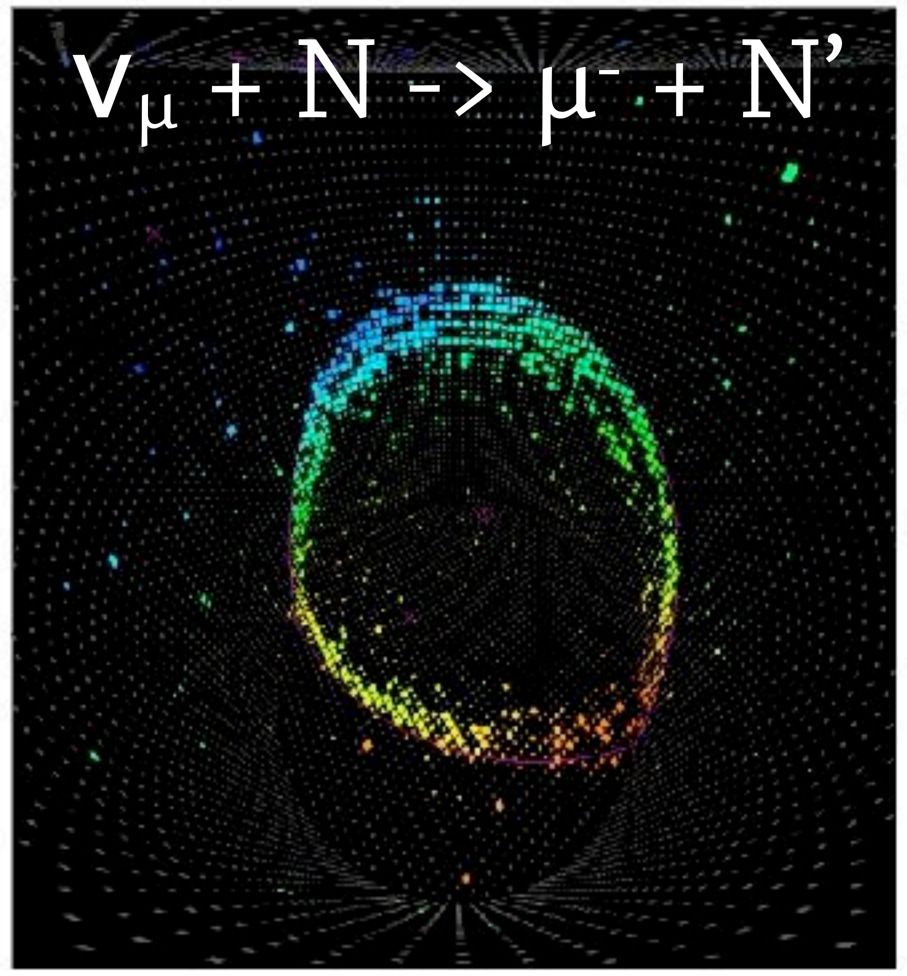
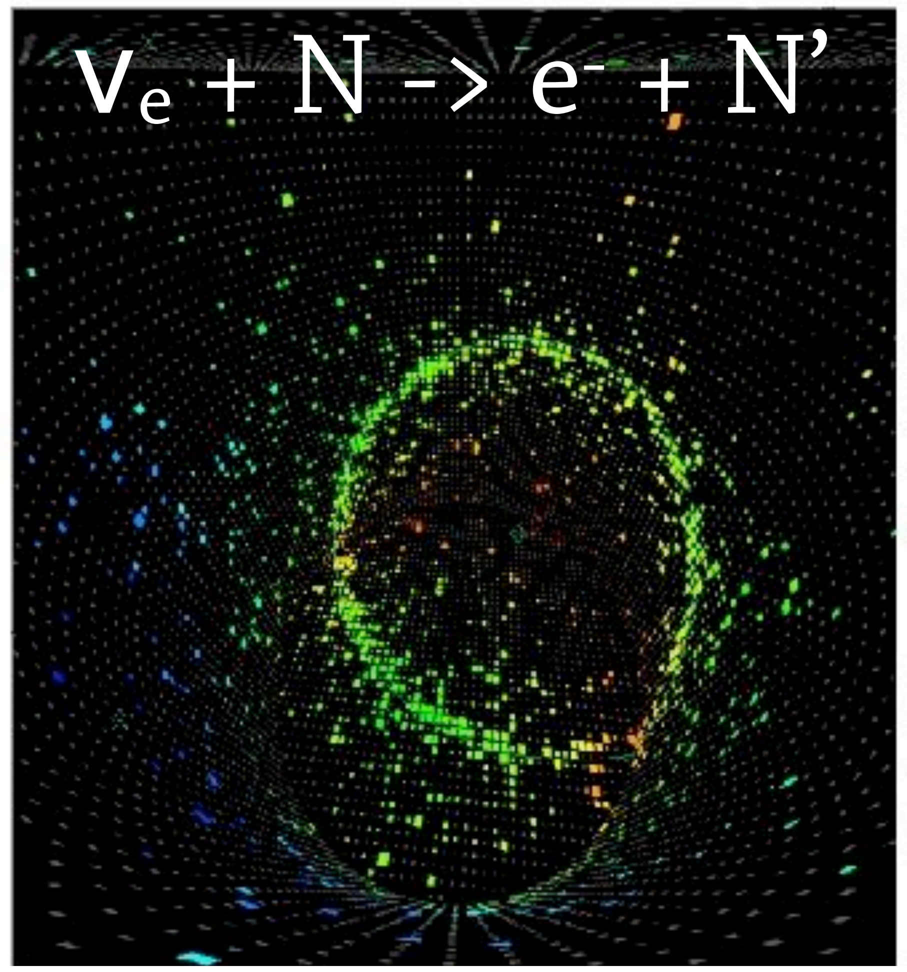

ν_μ + N -> μ^- + N’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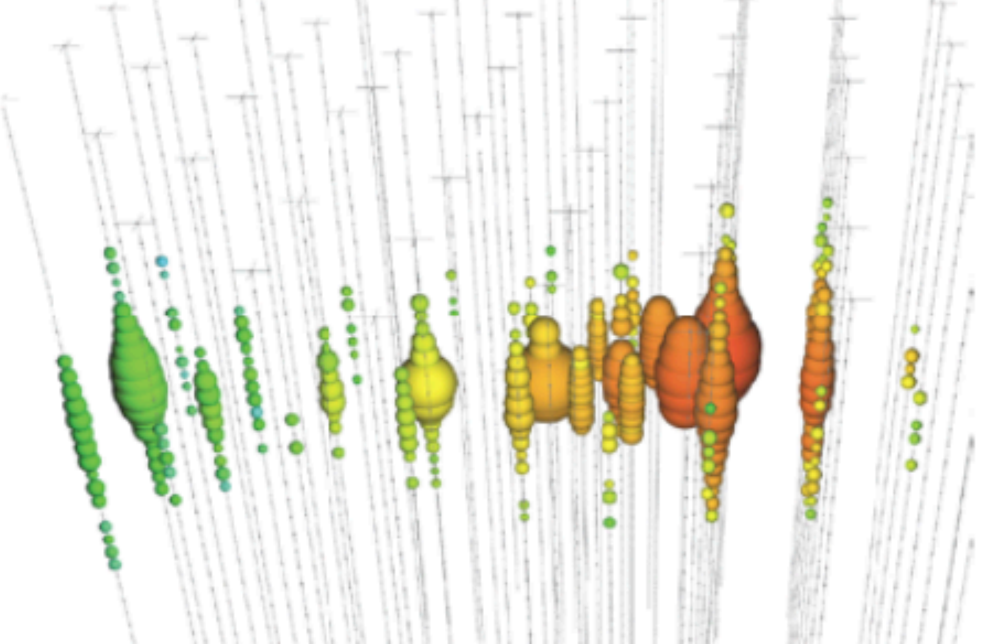
ICECUBE in Antartica

Giant detector in ice, equipped with 5k PTMs along 86 strings up to 2.4 km below the surface

ν_μ + N -> μ^- + N’



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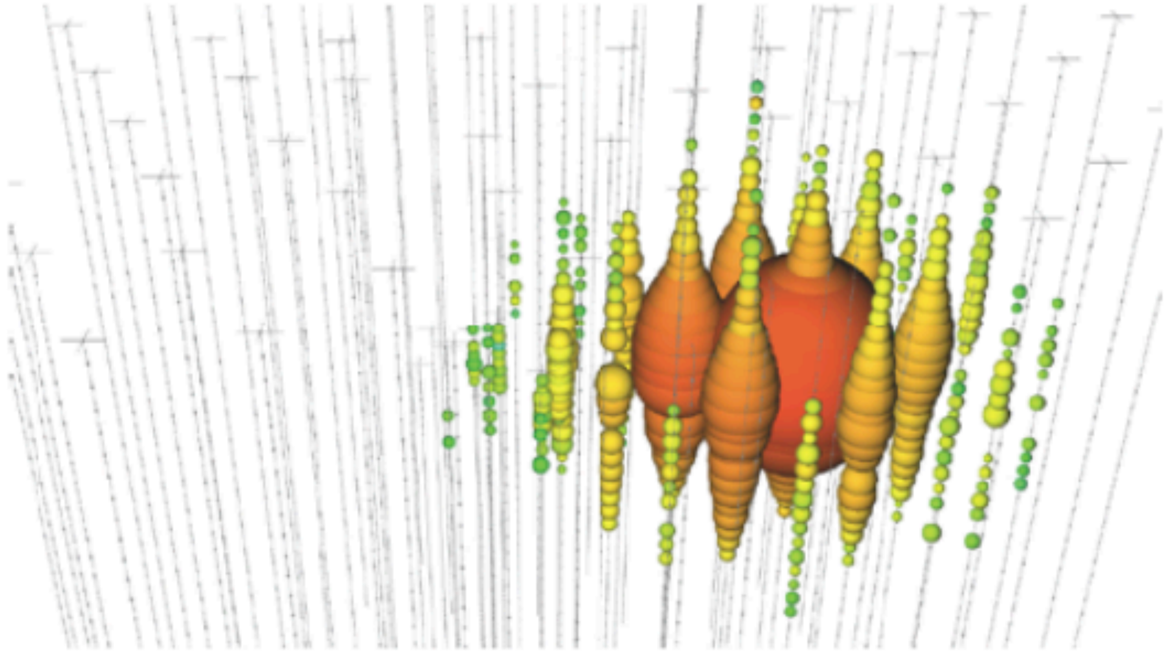
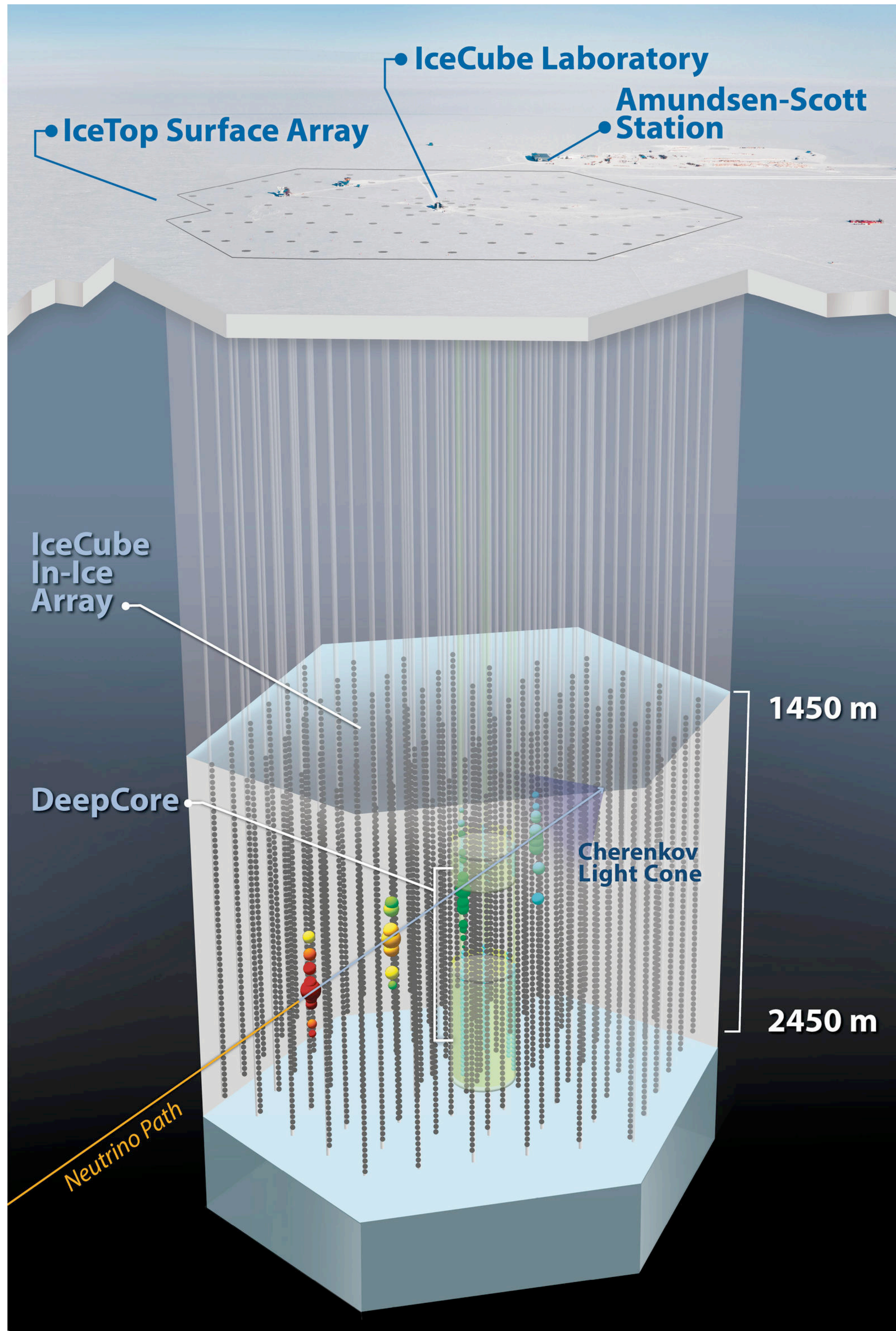


Color = time

size = nb of photons collected



DISCOVERY OF NEUTRINO OSCILLATION

Solar neutrino flux

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Among all the consequences, it helped to understand stellar nucleosynthesis that powers stars.

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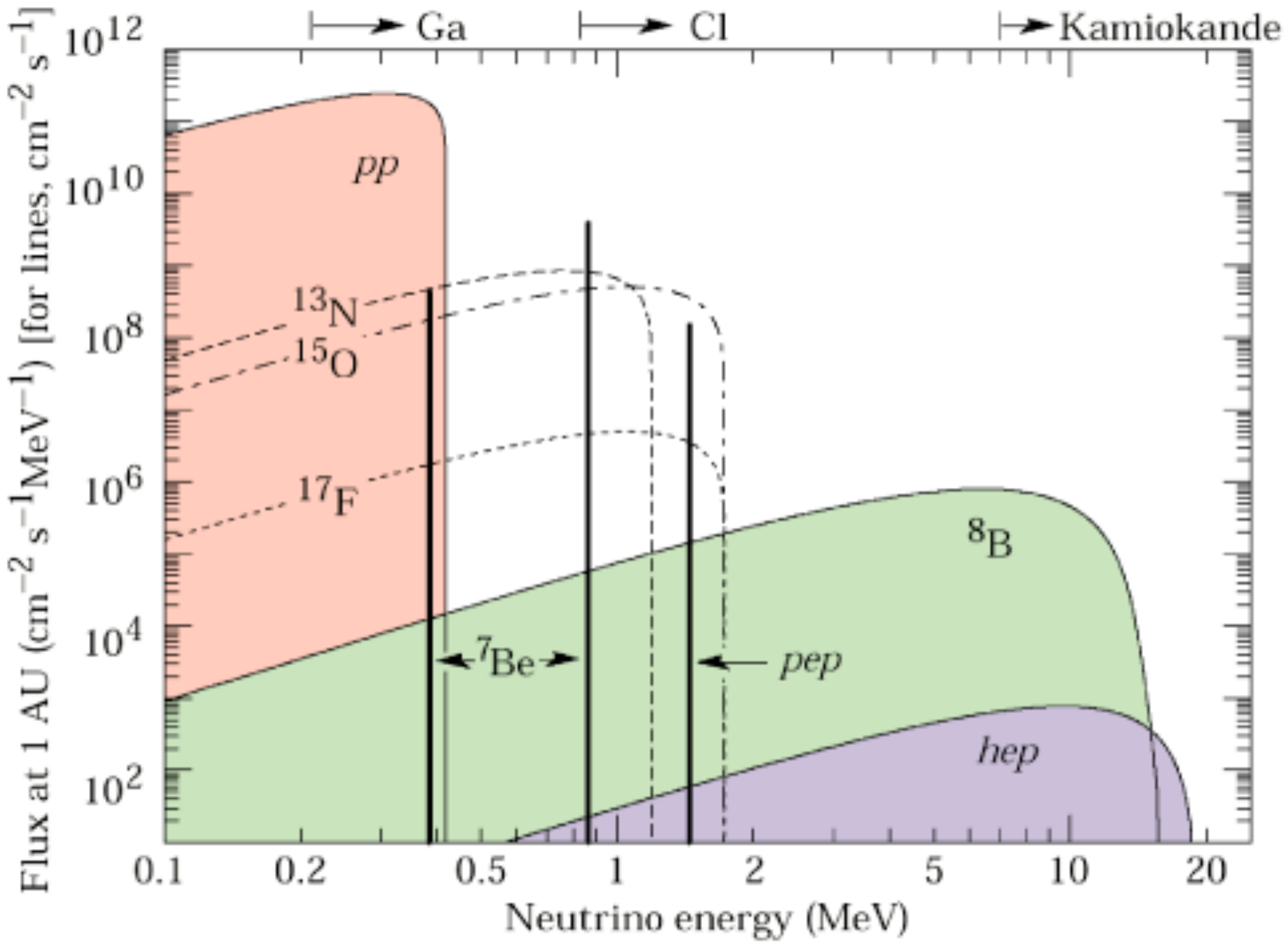
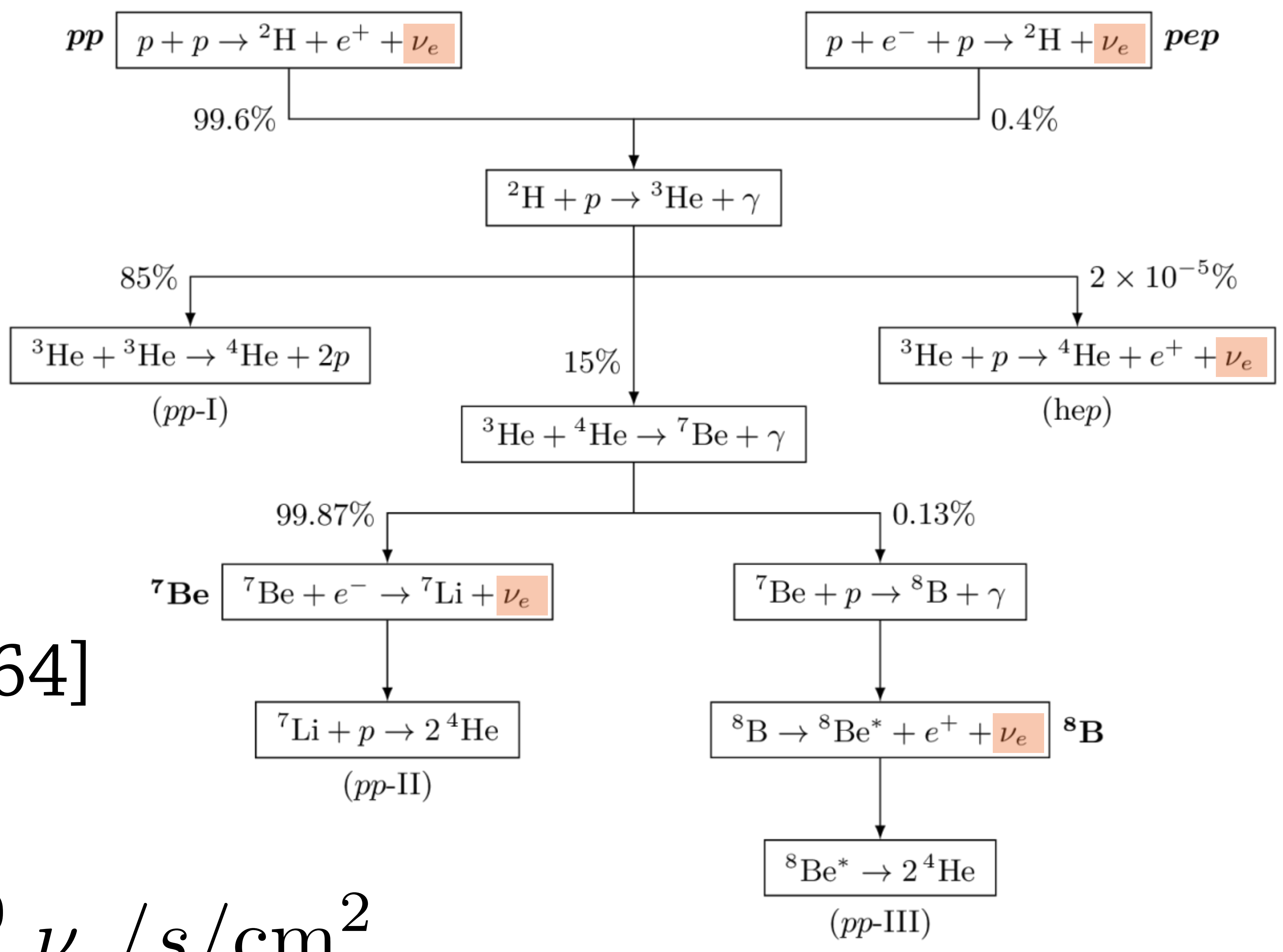
Bahcall made a prediction on the ν_e flux from the sun [1964]



Proton-proton fusion chain in sun-like stars



pp $p + p \to {}^2\text{H} + e^+ + \nu_e$

$p + e^- + p \to {}^2\text{H} + \nu_e$ **pep**

99.6%

0.4%

${}^2\text{H} + p \to {}^3\text{He} + \gamma$

85%

15%

$2 \times 10^{-5}\%$

${}^3\text{He} + {}^3\text{He} \to {}^4\text{He} + 2p$

(pp-I)

${}^3\text{He} + {}^4\text{He} \to {}^7\text{Be} + \gamma$

${}^3\text{He} + p \to {}^4\text{He} + e^+ + \nu_e$

(hep)

99.87%

0.13%

^{7}Be ${}^7\text{Be} + e^- \to {}^7\text{Li} + \nu_e$

${}^7\text{Be} + p \to {}^8\text{B} + \gamma$

${}^7\text{Li} + p \to 2\,{}^4\text{He}$

(pp-II)

${}^8\text{B} \to {}^8\text{Be}^* + e^+ + \nu_e$ **^{8}B**

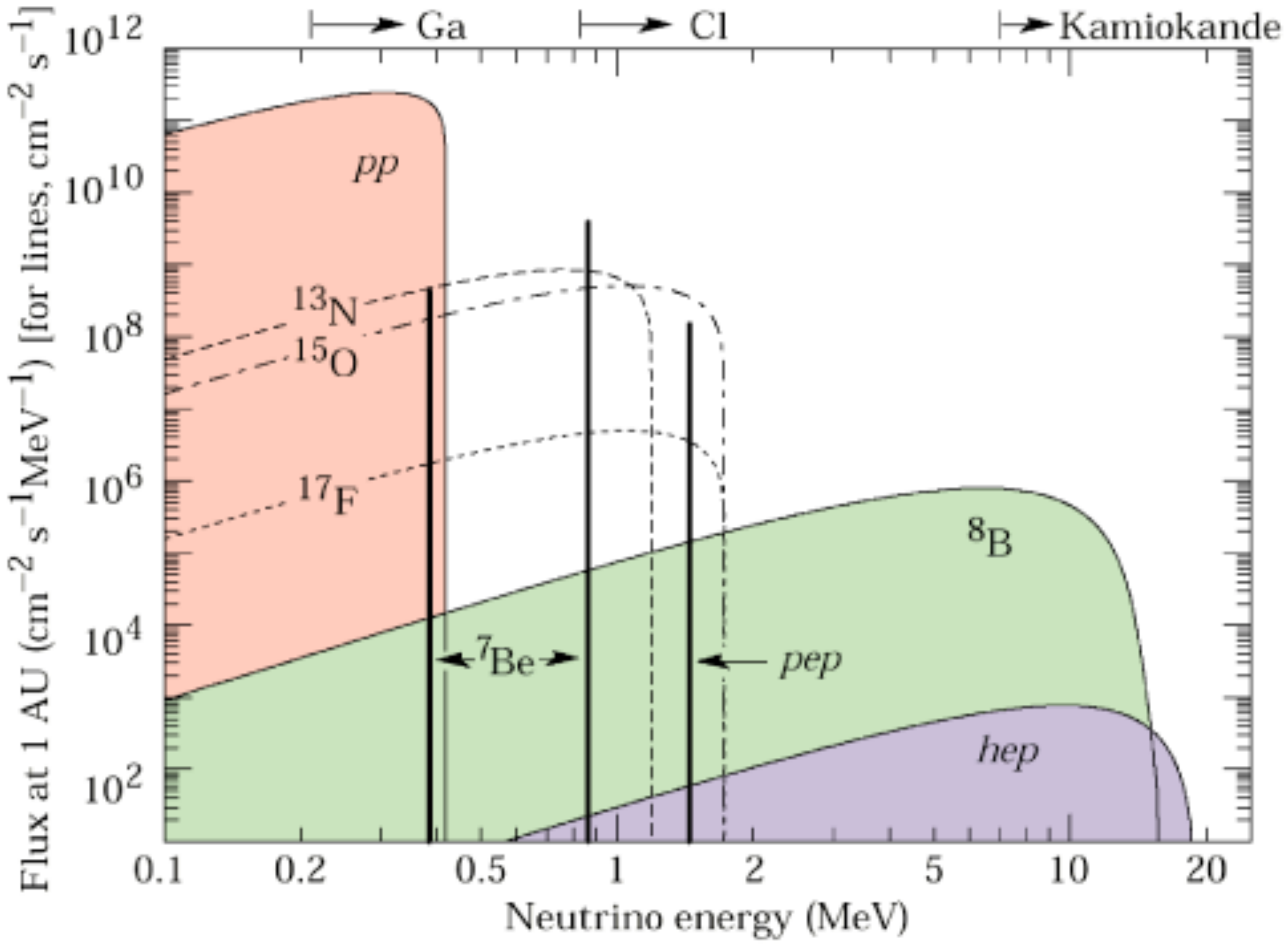
${}^8\text{Be}^* \to 2\,{}^4\text{He}$

(pp-III)

$$\phi_{\nu_e}^{\text{sun}} = 6.4 \times 10^{10}\ \nu_e/s/\text{cm}^2$$

Neutrinos can escape the sun plasma unaffected. Detecting those neutrino would prove the fusion chain happening inside sun's core.

Solar neutrino deficit



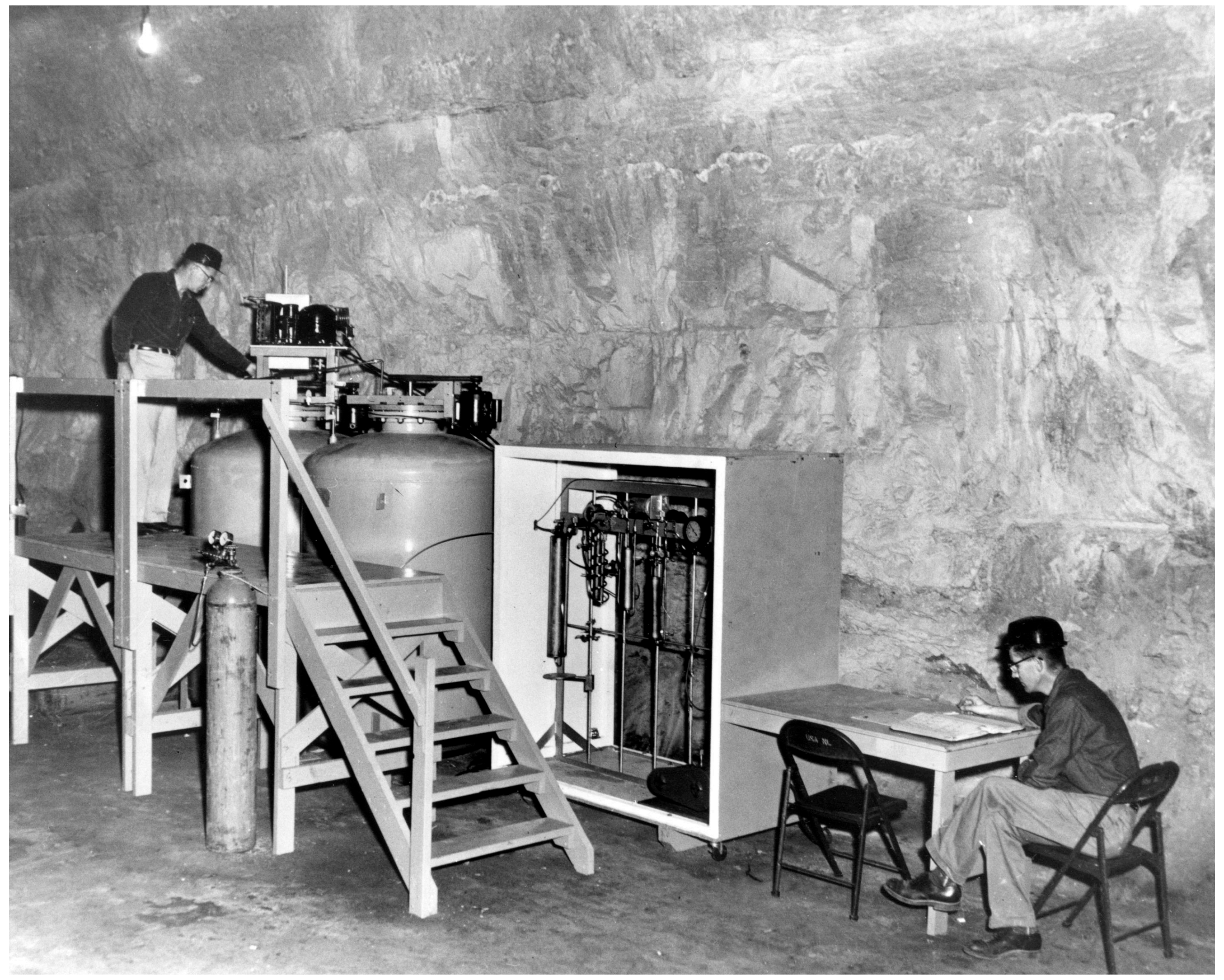
Solar Neutrino Unit = 10^{-36} interaction/s/atom

Solar neutrino deficit

Homestake Experiment designed to detect solar neutrinos

Underground detector observing

Cl to Ar conversion by: $\nu_e + {}^{37}\mathrm{Cl} \rightarrow {}^{37}\mathrm{Ar}^{+} + e^{-}$

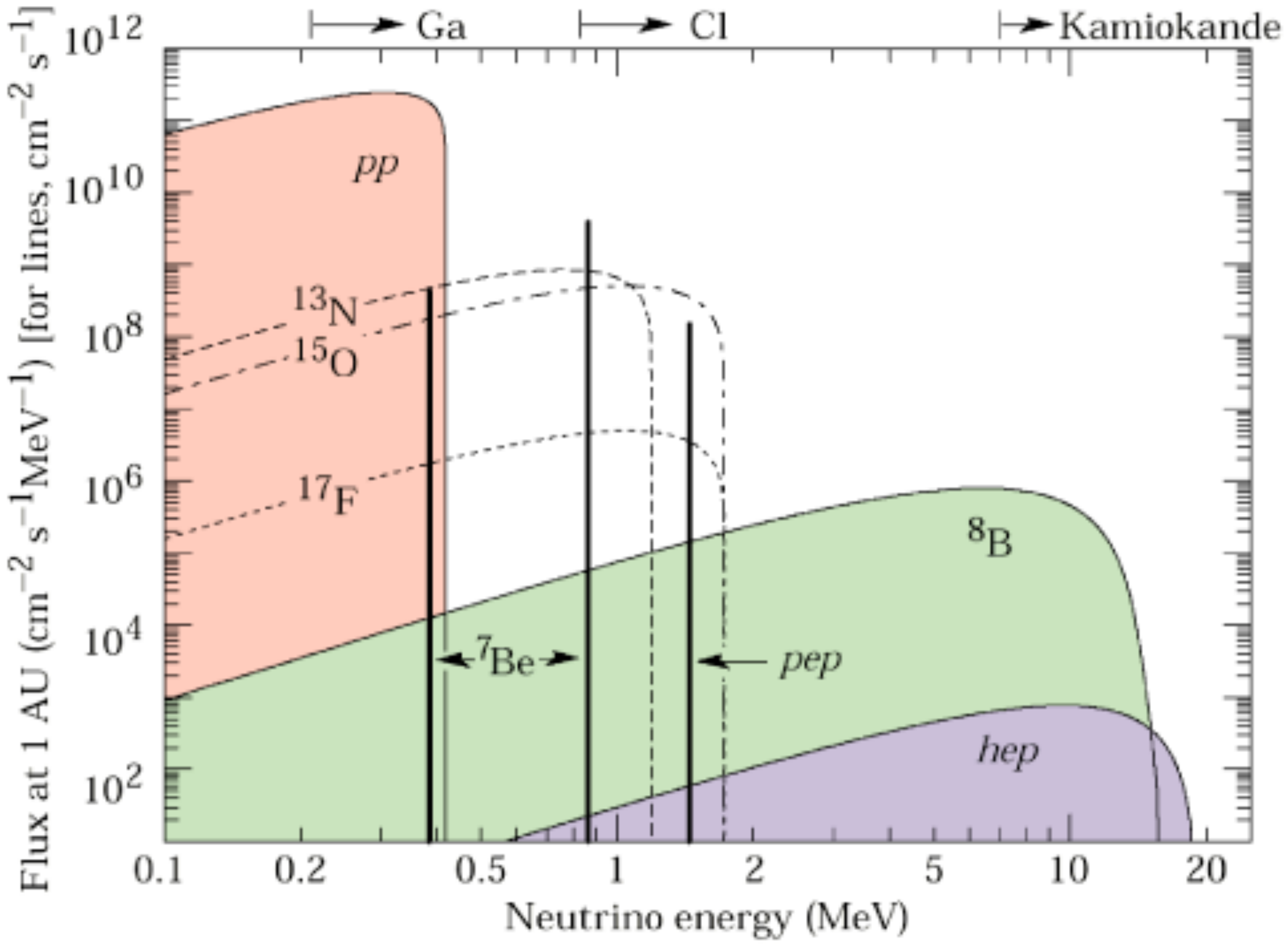


Number of Ar atom in the chamber was counted every few weeks with filters

Expected : 8.2 ± 1.8 SNU

Observed : 2.56 ± 0.23 SNU

60% ν_e missing



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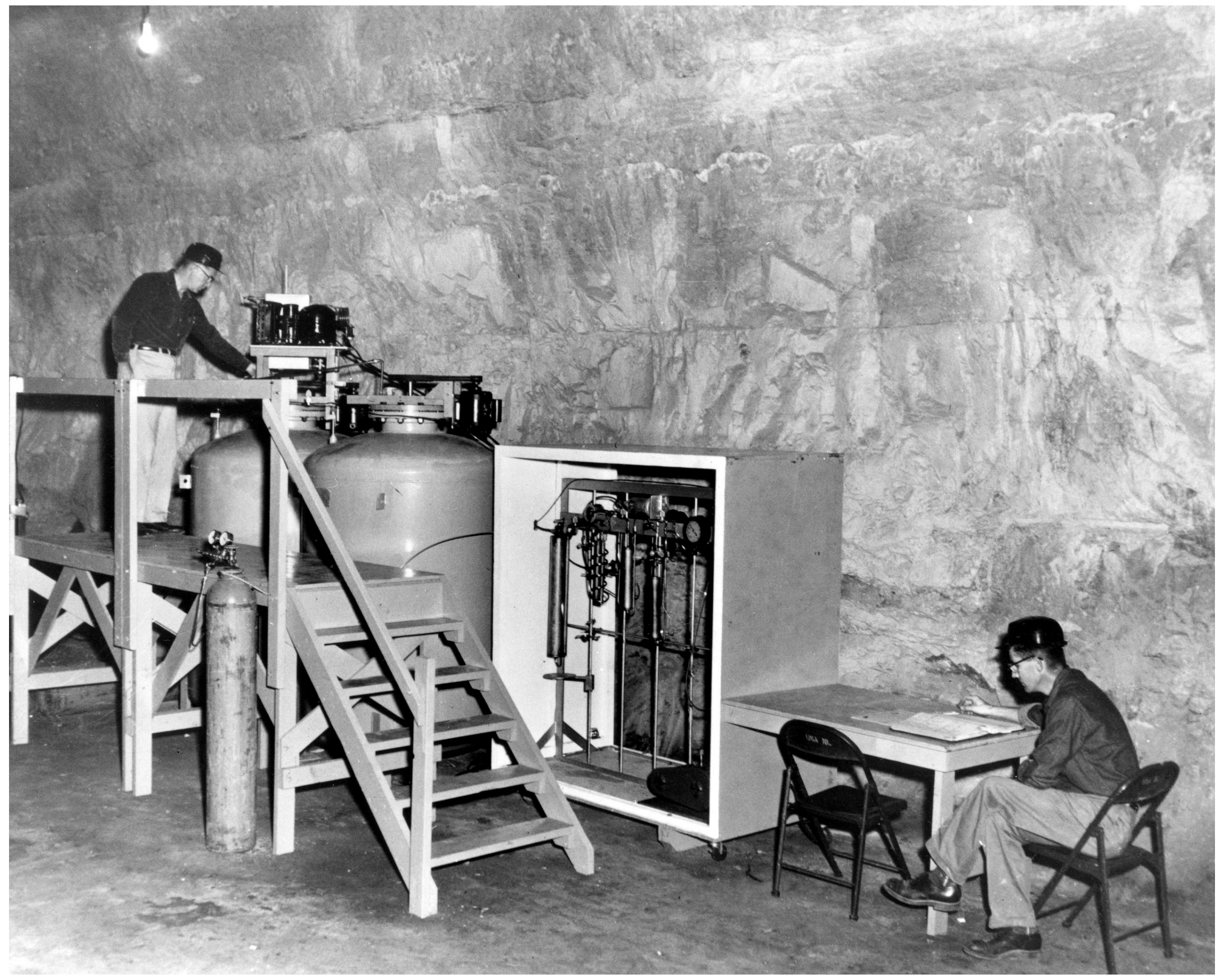

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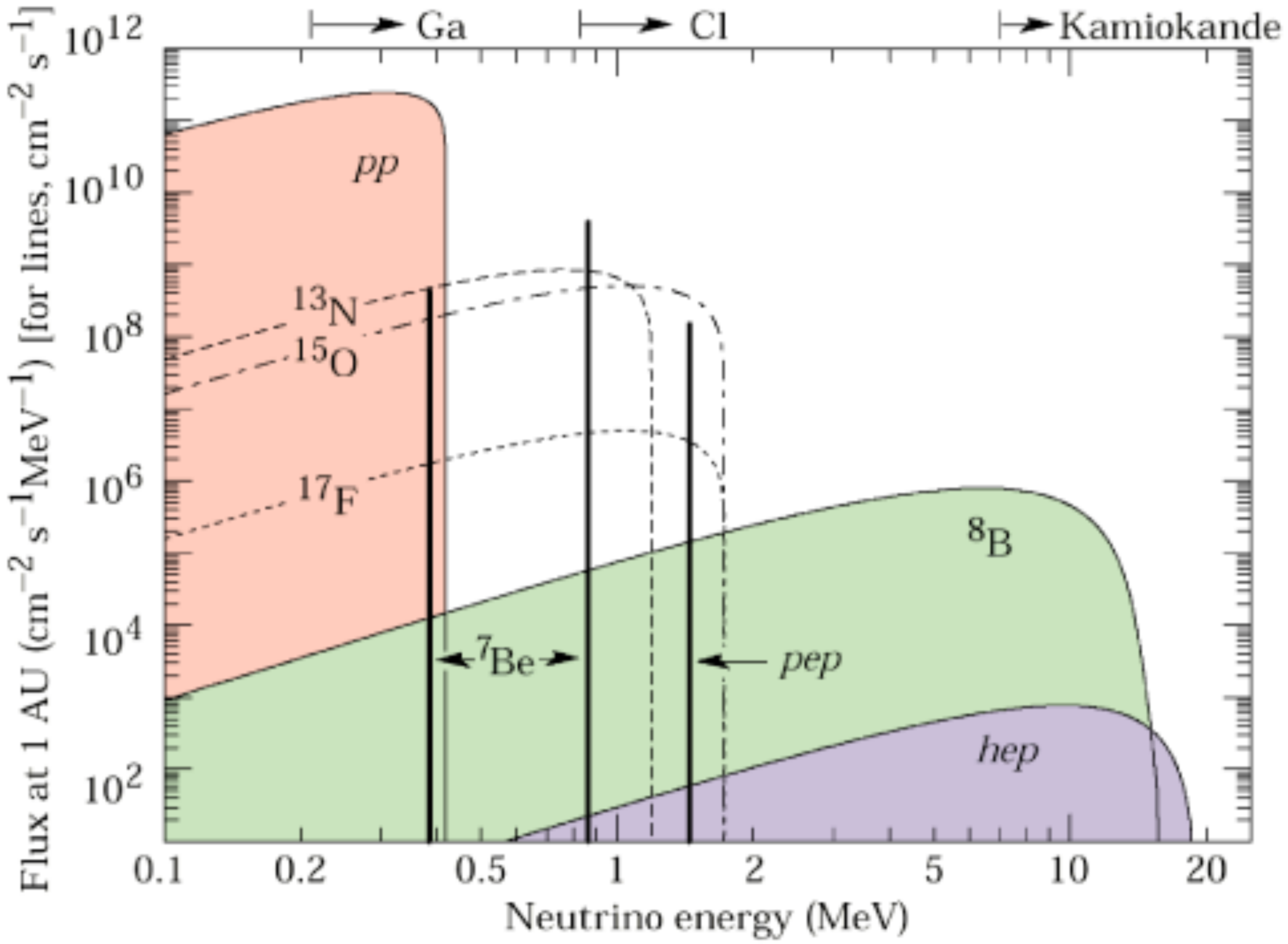

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The ***GALLEX*** and ***SAGE*** experiments used

Ga to Ge conversion : $\nu_e + {}^{71}\mathrm{Ga} \rightarrow {}^{71}\mathrm{Ge} + e^{-}$

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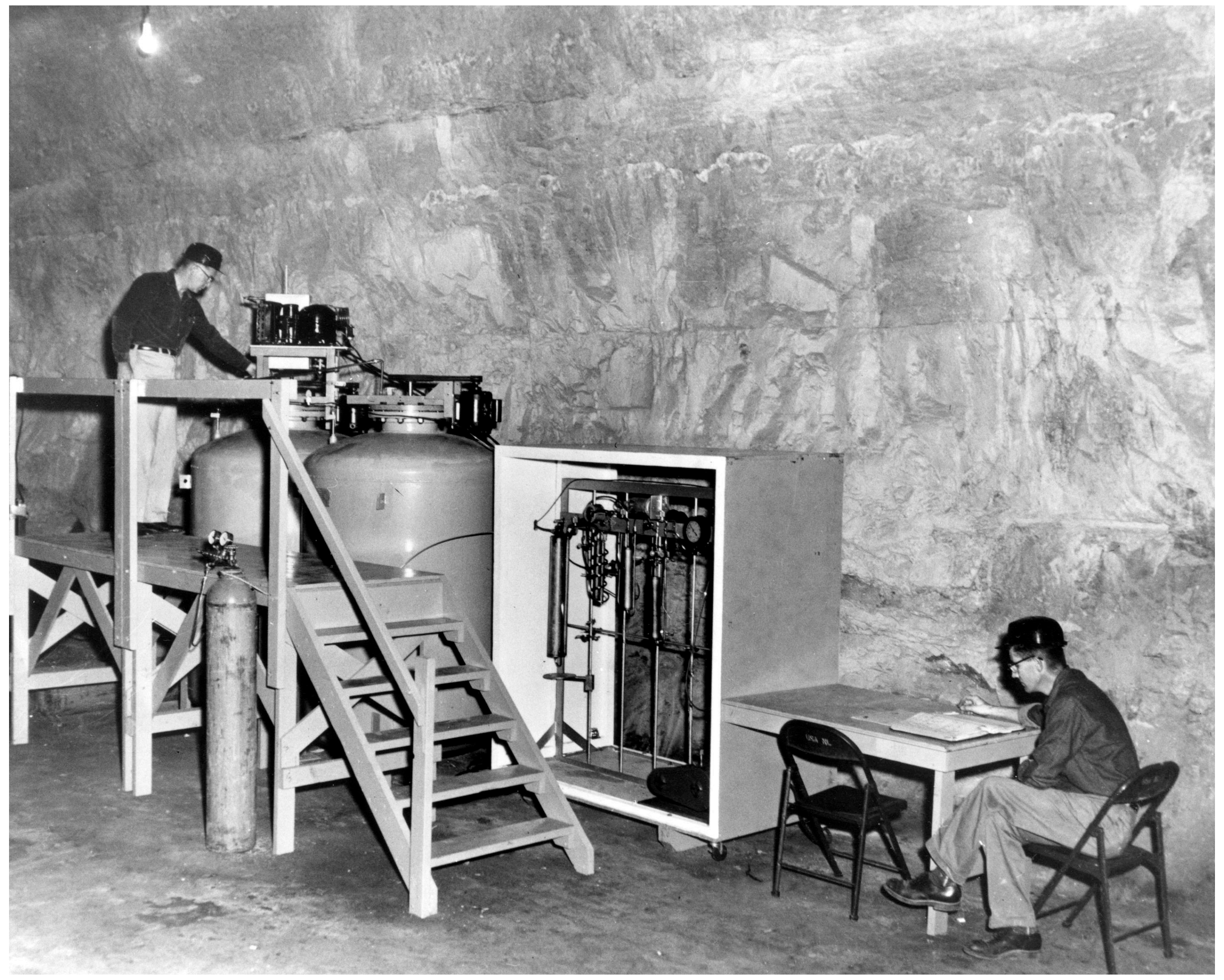

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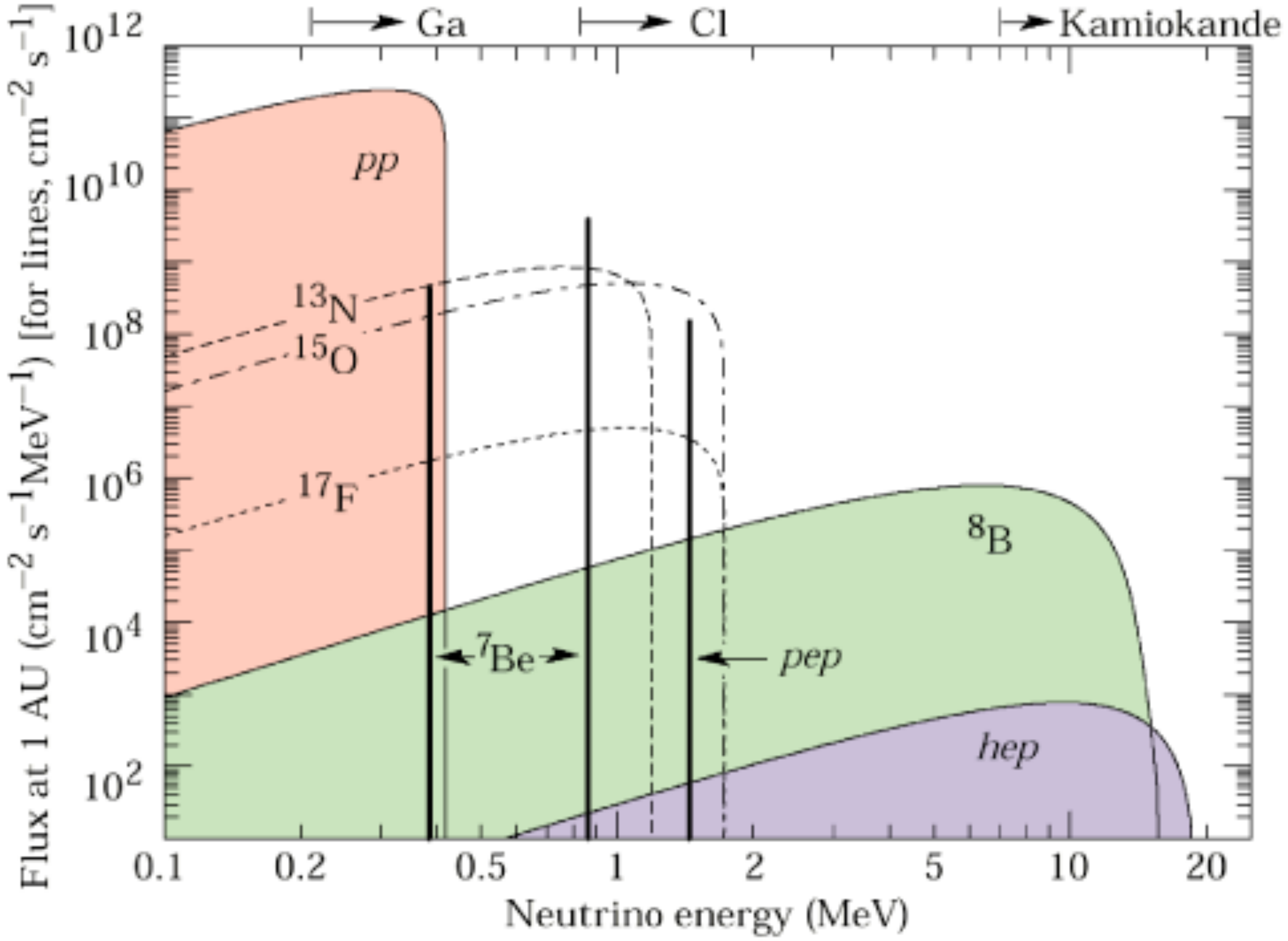

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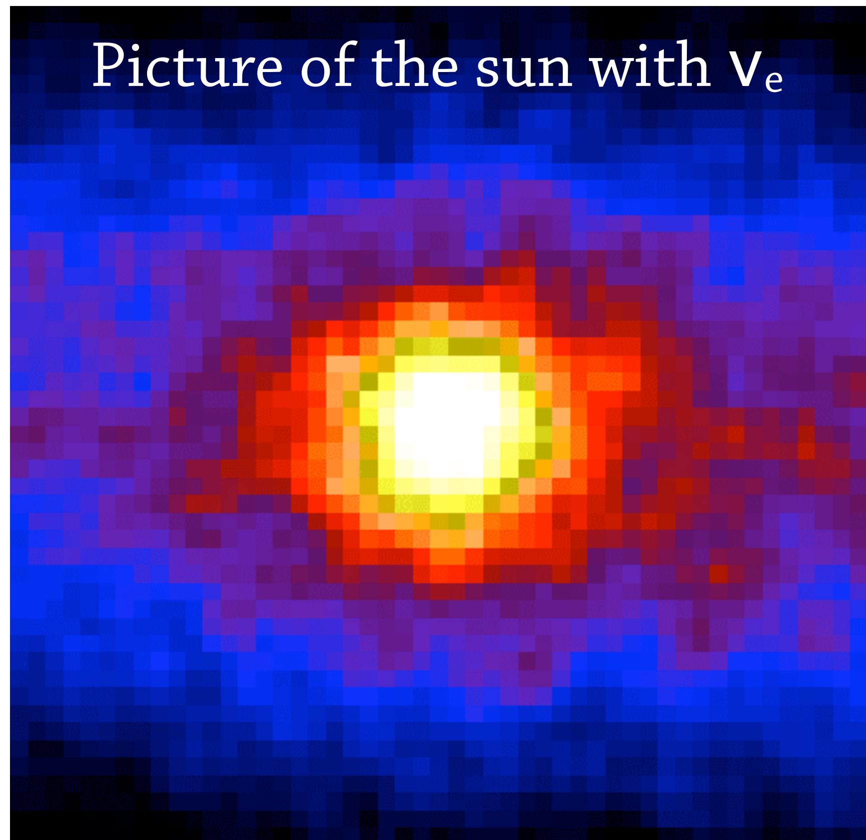
The ***Kamiokande*** experiment

observed solar neutrino through elastic scattering :

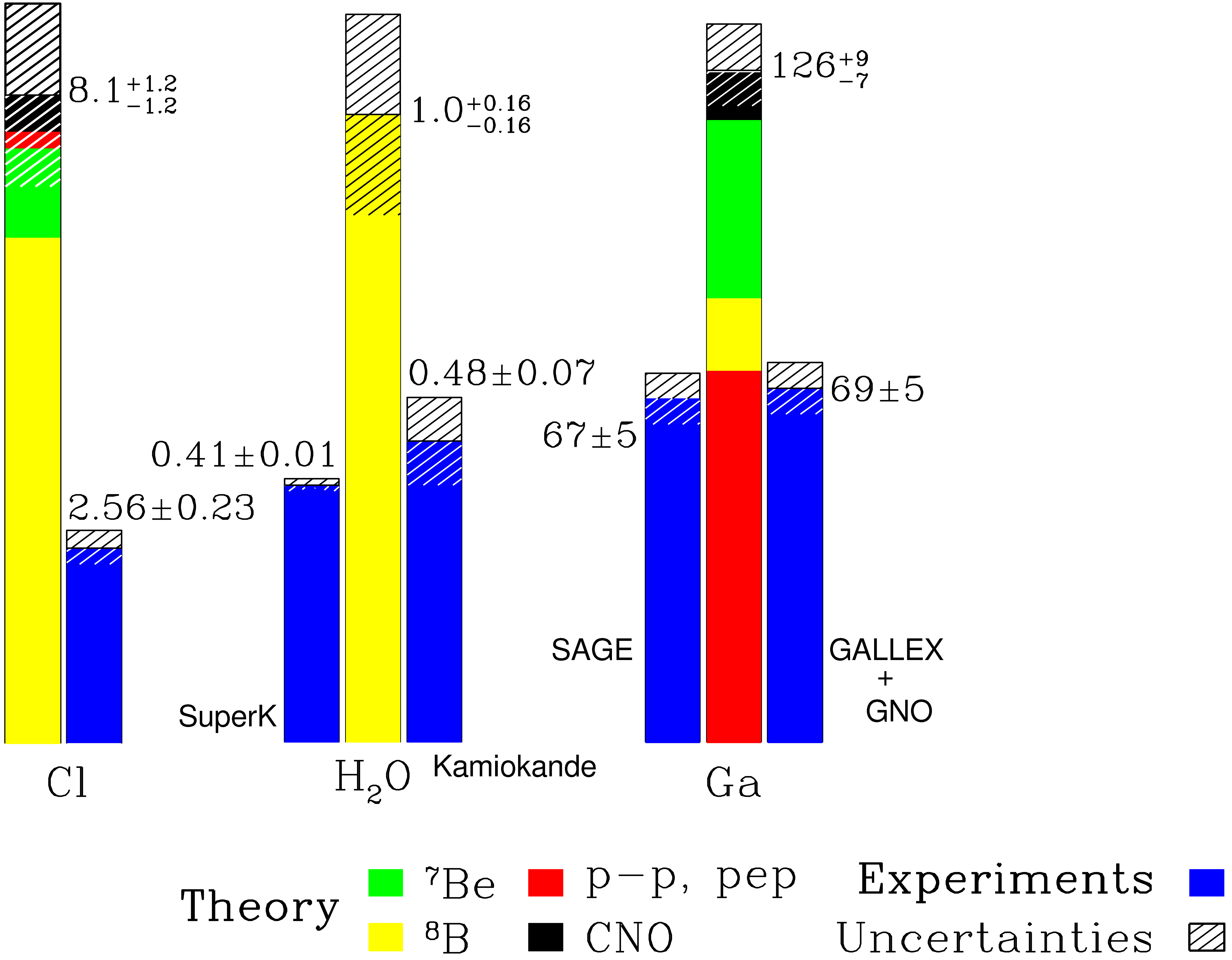
$$\nu_e + e^{-} \rightarrow \nu_e + e^{-}$$

45% ν_e missing

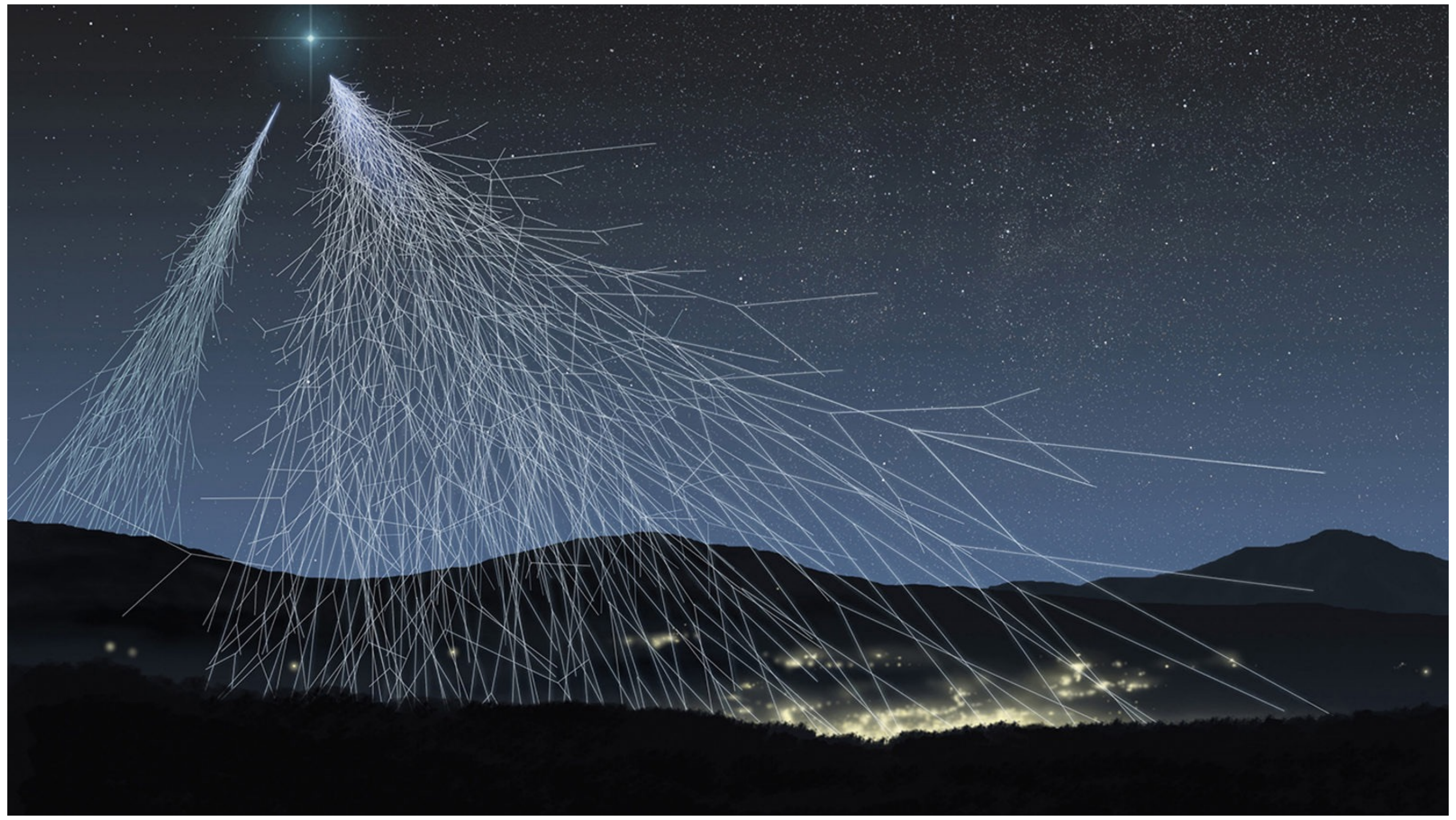


Picture of the sun with ν_e

Solar neutrino deficit



Atmospheric neutrino deficit



In parallel, interest in neutrinos from cosmic rays

→ When cosmic rays hit earth, they interact with the atmosphere and produce pions and muons

$$p + atm \rightarrow \pi^+ + \ldots$$
$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$
$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$$

At ground, we expect:

$\nu_\mu : \nu_e = 2 : 1$

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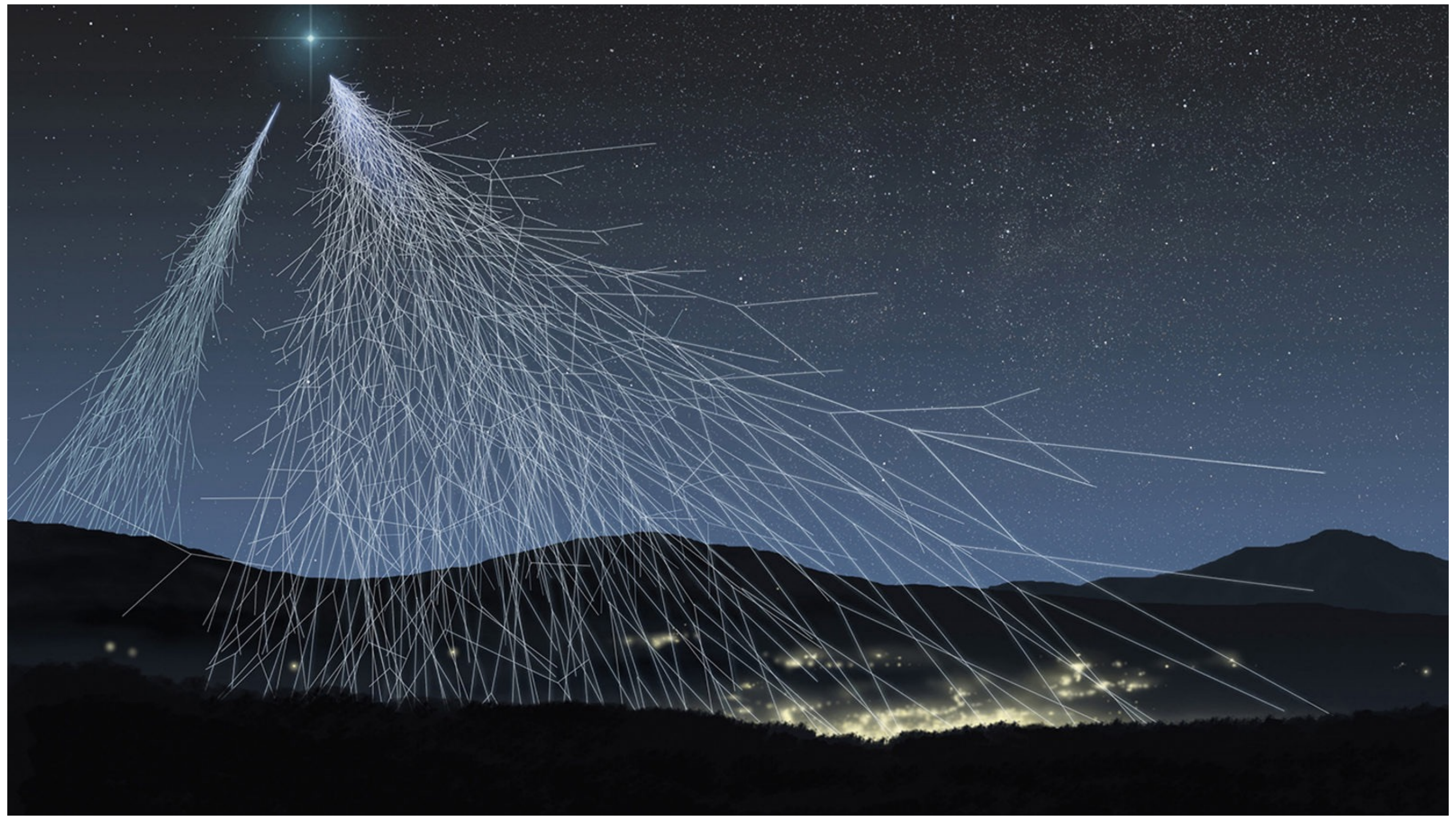

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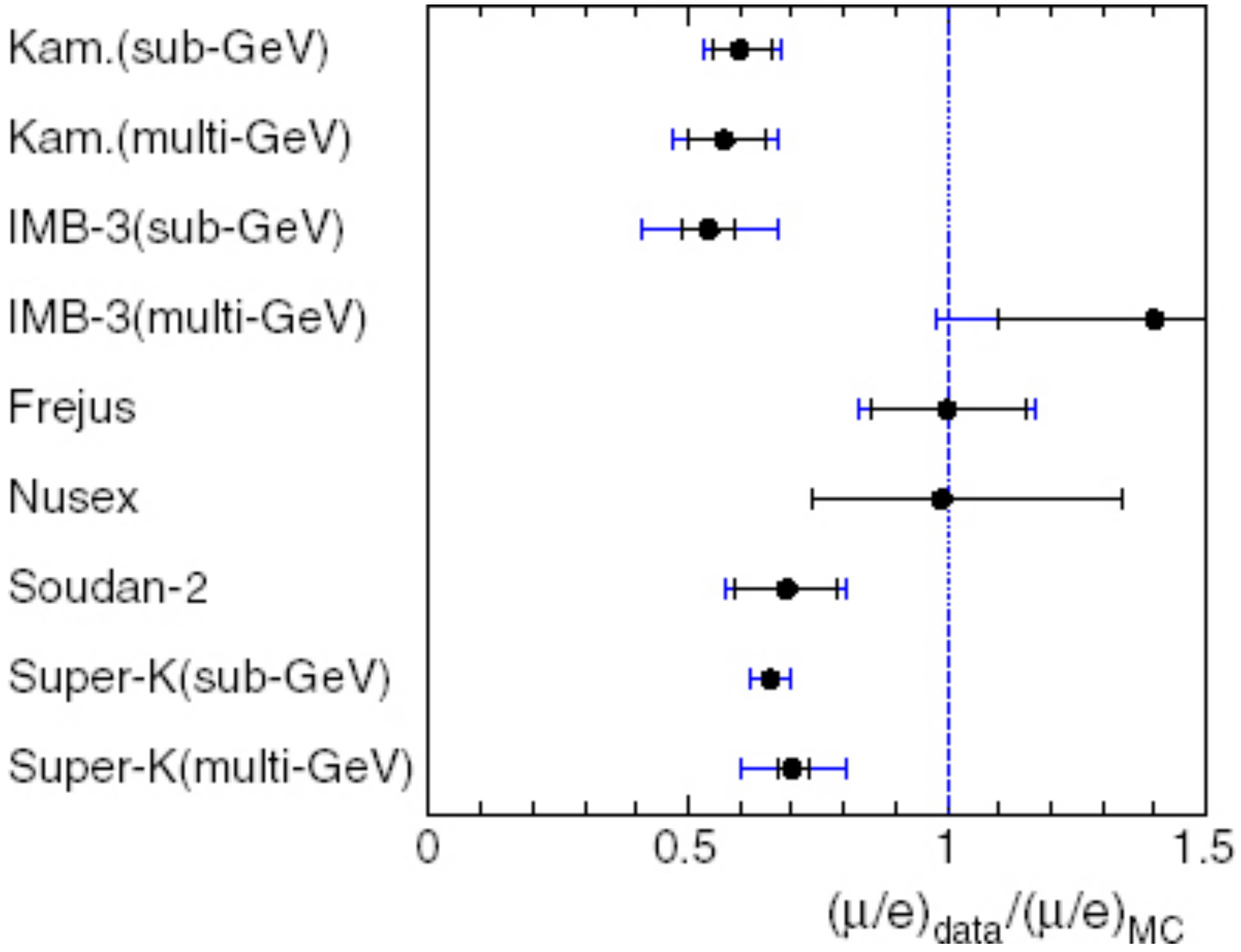


About 50% ν missing

Understanding the anomalies

○ Several hypothesis to explain the anomalies:

- Problems with fluxes computations, experiments
- Neutrino behavior: ν-decay, ν-decoherence, flavor changing neutral currents, oscillations, …

○ In 1957, Pontecorvo suggested the $\nu \rightarrow \bar{\nu}$ oscillations, in analogy with $K^0 \rightarrow \bar{K}^0$ mixing

○ Neutrino oscillation principle : Neutrino flavor and mass eigenstates are **not superimposed** but **linked** by a unitary mixing matrix

$$|\nu_\alpha\rangle = \sum_{i=1}^{3} U_{\alpha i} |\nu_i\rangle$$

α = (e, μ, τ) := Flavor states
i = (1, 2, 3) := Mass states
U = PMNS unitary mixing matrix

Where PMNS stands for

Pontecorvo-Maki-Nakagawa-Sakata

Understanding the anomalies

« Neutrino flavor and mass eigenstates are **not superimposed** but **linked** by a unitary mixing matrix »

This implies:

- The eigenstates of the Hamiltonian are $|\nu_j >$ with eigenvalues m_j for neutrinos at rest
- A produced neutrino of type *j* with momentum *p* is an energy (or mass) eigenstate with eigenvalue $E_j = \sqrt{p^2 + m_j^2}$

- Neutrinos are produced by weak interactions in weak eigenstates of a definite lepton number ($|\nu_e >$, $|\nu_\mu >$ or $|\nu_\tau >$) that are ***not*** energy eigenstates, the PMNS matrix links the weak/flavor eigenstates to the energy/mass eigenstates

$$|\nu_\alpha\rangle = \sum_{i=1}^{3} U_{\alpha i}|\nu_i\rangle$$

→ ***Neutrinos can be massive !***

Understanding the anomalies

« Neutrino flavor and mass eigenstates are **not superimposed** but **linked** by a unitary mixing matrix »

This implies:

- The matrix that connects the flavor eigenstates to the mass eigenstates is a rotation matrix
- The basis change should not create nor annihilate neutrinos !

$$|\nu_\alpha\rangle = \sum_{i=1}^{3} U_{\alpha i} |\nu_i\rangle$$

$$|\nu_i\rangle = \sum_{\alpha=1}^{3} U^*_{\alpha i} |\nu_\alpha\rangle$$

In practice:

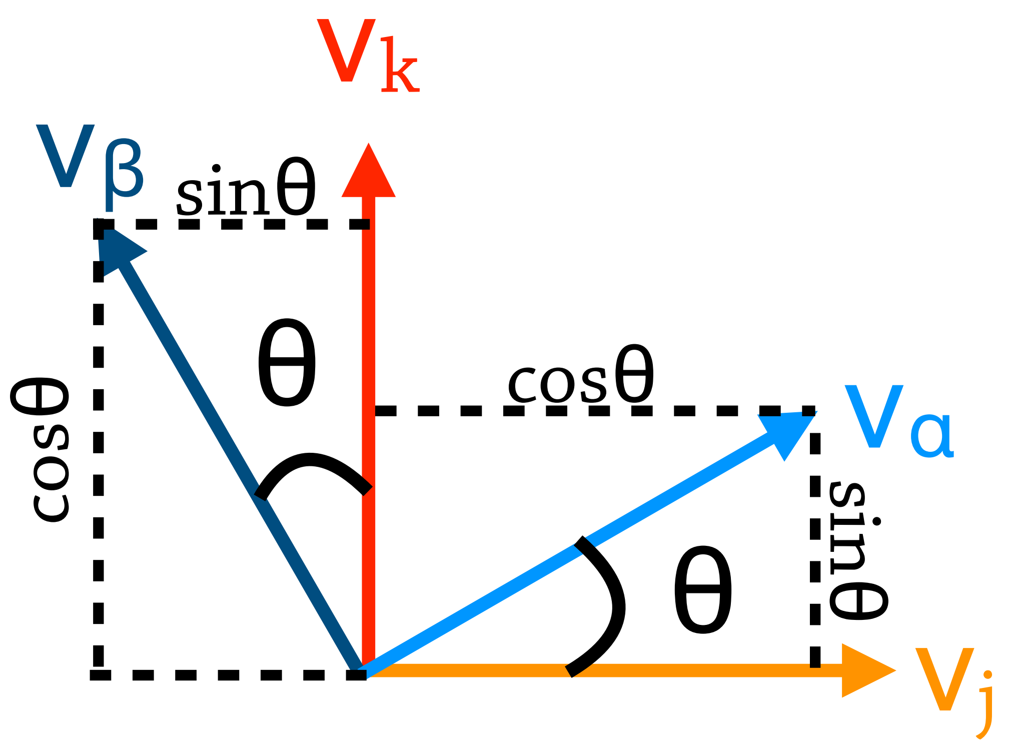
- $UU^\dagger = U^\dagger U = I$
- The sum of the squared values of each row and each column is 1

Simplified case with only two flavor

With only two ν flavors, the 2×2 unitary mixing matrix is equivalent to a rotation matrix:

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_j \\ \nu_k \end{pmatrix}$$

- ν_α ν_β are flavor eigenstates
- ν_j ν_k are mass eigenstates
- θ is the mixing angle



The flavor states can be written as :

$$|\nu_\alpha\rangle = \cos\theta|\nu_j\rangle + \sin\theta|\nu_k\rangle$$

$$|\nu_\beta\rangle = -\sin\theta|\nu_j\rangle + \cos\theta|\nu_k\rangle$$

Simplified case with only two flavor

To understand how state evolves with time, we apply the Schrödinger equation to each mass eigenstates in their reference system:

$$|\nu_j(\tau_j)\rangle = e^{-im_j\tau_j}|\nu_j\rangle$$

- m_j is the mass of ν_j
- τ_j is the time in the ν_j rest frame

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Simplified case with only two flavor

We are going to make a few approximations:

- Neutrinos are highly relativistic, such that t ~ L (with c = $\hbar$ = 1)

  The phase becomes:

$$-i(E_j t - p_j L) = -i(E_j - p_j)L$$

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- The ν_α is produced with a momentum p => all the mass eigenstates components of ν_α have the same momentum $p_j = p_k = p$

$$E_j = \sqrt{p_j^2 + m_j^2} \approx \sqrt{p^2 + m_j^2}$$

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- Finally, let's assume that E ~ p be the average energy of the neutrino mass eigenstates.

  The phase simplifies as:

$$-i(E_j - p_j)L \approx -i(E + \frac{m_j^2}{2E} - E) = -i\frac{m_j^2}{2E}$$

Simplified case with only two flavor

With those approximations, the flavor states evolution with distance now writes as :

$$|\nu_\alpha(L)\rangle = \cos\theta e^{-i(m_j^2 \frac{L}{2E})}|\nu_j\rangle + \sin\theta e^{-i(m_k^2 \frac{L}{2E})}|\nu_k\rangle$$

$$|\nu_\beta(L)\rangle = -\sin\theta e^{-i(m_j^2 \frac{L}{2E})}|\nu_j\rangle + \cos\theta e^{-i(m_k^2 \frac{L}{2E})}|\nu_k\rangle$$

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Let's assume a ν_α is produced at t = 0.

Given how the neutrino states evolve with time, there is a possibility that the neutrino you will detect has a different flavor from its flavor at production

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Let's assume a ν_α is produced at t = 0.

The probability of observing a neutrino of flavor α at t = L is defined as:

$$P(\nu_\alpha \to \nu_\alpha) = |\langle\nu_\alpha\,|\,\nu_\alpha(L)\rangle|^2$$

The probability of observing a neutrino of flavor β at t = L is defined as:

$$P(\nu_\alpha \to \nu_\beta) = |\langle\nu_\alpha\,|\,\nu_\beta(L)\rangle|^2$$

Simplified case with only two flavor

Let's derive the ν_α -> ν_α probability.

$$
\begin{aligned}
P(\nu_\alpha \to \nu_\alpha) &= |\langle \nu_\alpha \,|\, \nu_\alpha(L) \rangle|^2 \\
&= |(\cos\theta \langle \nu_j| + \sin\theta \langle \nu_k|)(\cos\theta e^{-i(m_j^2 \frac{L}{2E})}|\nu_j\rangle + \sin\theta e^{-i(m_k^2 \frac{L}{2E})}|\nu_k\rangle)|^2
\end{aligned}
$$

Simplified case with only two flavor

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Recalling that: $\langle a \,|\, b\rangle = \delta_{ab}$

The probability is:

$$P(\nu_\alpha \to \nu_\alpha) = \left|\cos^2\theta e^{-i(m_j^2 \frac{L}{2E})} + \sin^2\theta e^{-i(m_k^2 \frac{L}{2E})}\right|^2$$

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Recalling that for z and w being two complex numbers: $|z+w|^2 = |z|^2 + |w|^2 + 2zw^*$

$$P(\nu_\alpha \to \nu_\alpha) = \cos^4\theta + \sin^4\theta + 2\cos^2\theta\sin^2\theta\left|e^{-i\frac{L}{2E}(m_k^2 - m_j^2)}\right|$$

Simplified case with only two flavor

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Let's call Δm_{jk}^2 the mass squared difference: $\Delta m_{jk}^2 = m_k^2 - m_j^2$

$$\left| e^{-i\frac{L}{2E}(m_k^2 - m_j^2)} \right| = \left| e^{-i\frac{L}{2E}\Delta m_{jk}^2} \right| = \cos\left(\Delta m_{jk}^2 \frac{L}{2E} \right)$$

Simplified case with only two flavor

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Let's recall the trigonometry equations:

$$\cos^2 a + \sin^2 a = 1$$

$$(\cos^2 a + \sin^2 a)^2 = 1$$

$$\cos^4 a + \sin^4 a + 2\cos^2 a \sin^2 a = 1$$

$$\cos^4 a + \sin^4 a = 1 - 2\cos^2 a \sin^2 a$$

Simplified case with only two flavor

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We have:

$$P(\nu_\alpha \to \nu_\alpha) = 1 - 2\cos^2\theta\sin^2\theta + 2\cos^2\theta\sin^2\theta\cos(\Delta m_{jk}^2 \frac{L}{2E})$$

$$= 1 - 2\cos^2\theta\sin^2\theta \left(\cos(\Delta m_{jk}^2 \frac{L}{2E}) - 1\right)$$

Simplified case with only two flavor

The probability can be further simplified with trigonometry:

$$P(\nu_\alpha \to \nu_\alpha) = 1 - 2\cos^2\theta\sin^2\theta\left(\cos(\Delta m^2_{jk}\frac{L}{2E}) - 1\right)$$

Recalling that: $2\cos a\sin a = \sin 2a \longrightarrow 2\cos^2 a\sin^2 a = \frac{\sin 2a}{2}$

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Recalling that: $\frac{1-\cos 2a}{2} = \sin^2 a \longrightarrow \frac{1-\cos a}{2} = \sin^2\frac{a}{2}$

We finally have:

$$\boxed{P(\nu_\alpha \to \nu_\alpha) = 1 - \sin^2 2\theta\sin^2(\Delta m^2_{jk}\frac{L}{4E})}$$

Simplified case with only two flavor

This is the probability to detect a neutrino in the same flavor state as it was created:

$$P(\nu_\alpha \to \nu_\alpha) = 1 - \sin^2 2\theta \sin^2(\Delta m^2_{jk} \frac{L}{4E})$$

The probability to detect a neutrino with a different flavor state is:

$$\begin{aligned} P(\nu_\alpha \to \nu_\beta) &= |\langle \nu_\alpha \,|\, \nu_\beta(L) \rangle|^2 \\ &= \left| -\cos\theta \sin\theta e^{-i(m_j^2 \frac{L}{2E})} + \cos\theta \sin\theta e^{-i(m_k^2 \frac{L}{2E})} \right|^2 \end{aligned}$$

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$$P(\nu_\alpha \to \nu_\alpha) = 1 - \sin^2 2\theta \sin^2(\Delta m^2_{jk} \frac{L}{4E})$$

The probability to detect a neutrino with a different flavor state is:

$$\begin{aligned} P(\nu_\alpha \to \nu_\beta) &= |\langle \nu_\alpha \,|\, \nu_\beta(L) \rangle|^2 \\ &= \left| -\cos\theta \sin\theta e^{-i(m_j^2 \frac{L}{2E})} + \cos\theta \sin\theta e^{-i(m_k^2 \frac{L}{2E})} \right|^2 \\ &= \cos^2\theta \sin^2\theta + \cos^2\theta \sin^2\theta - 2\cos^2\theta \sin^2\theta \cos\left(\Delta m^2_{jk} \frac{L}{2E}\right) \\ &= \frac{\sin^2 2\theta}{2} \left(\cos(\Delta m^2_{jk} \frac{L}{2E}) - 1 \right) \\ &= \sin^2 2\theta \sin^2(\Delta m^2_{jk} \frac{L}{4E}) \end{aligned}$$

Simplified case with only two flavor

This is the probability to detect a neutrino in the same flavor state as it was created:

$$P(\nu_\alpha \to \nu_\alpha) = 1 - \sin^2 2\theta \sin^2(\Delta m^2_{jk} \frac{L}{4E})$$

The probability to detect a neutrino with a different flavor state is:

(...)

Or, using the fact that the mixing matrix is unitary and that we have only two neutrino flavor :

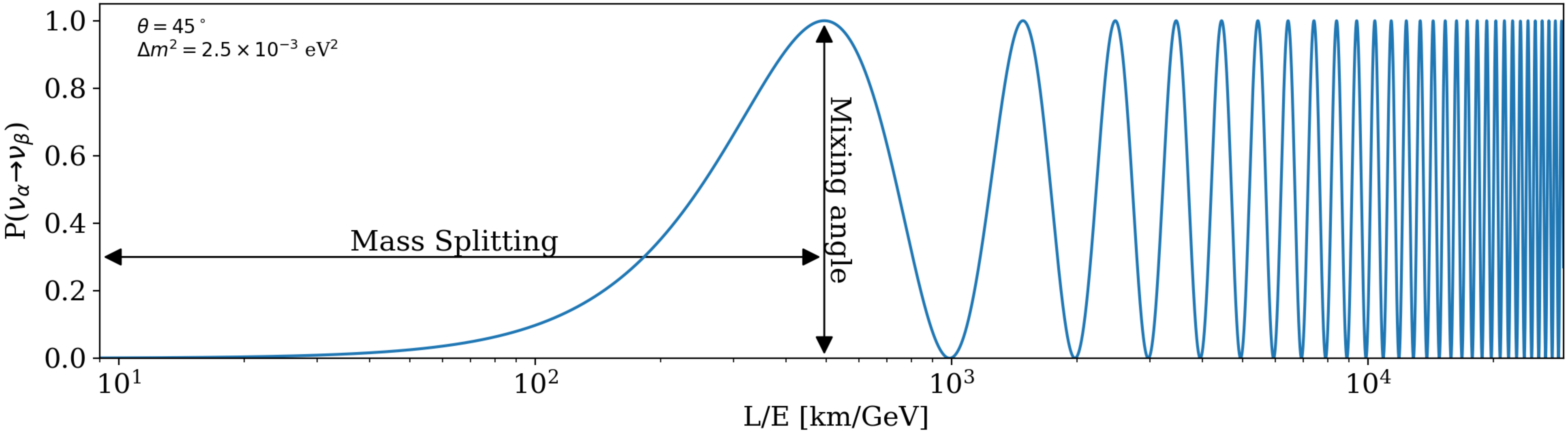
$$\begin{aligned} P(\nu_\alpha \to \nu_\beta) &= 1 - P(\nu_\alpha \to \nu_\alpha) \\ &= \sin^2 2\theta \sin^2(\Delta m^2_{jk} \frac{L}{4E}) \end{aligned}$$

Understanding the anomalies

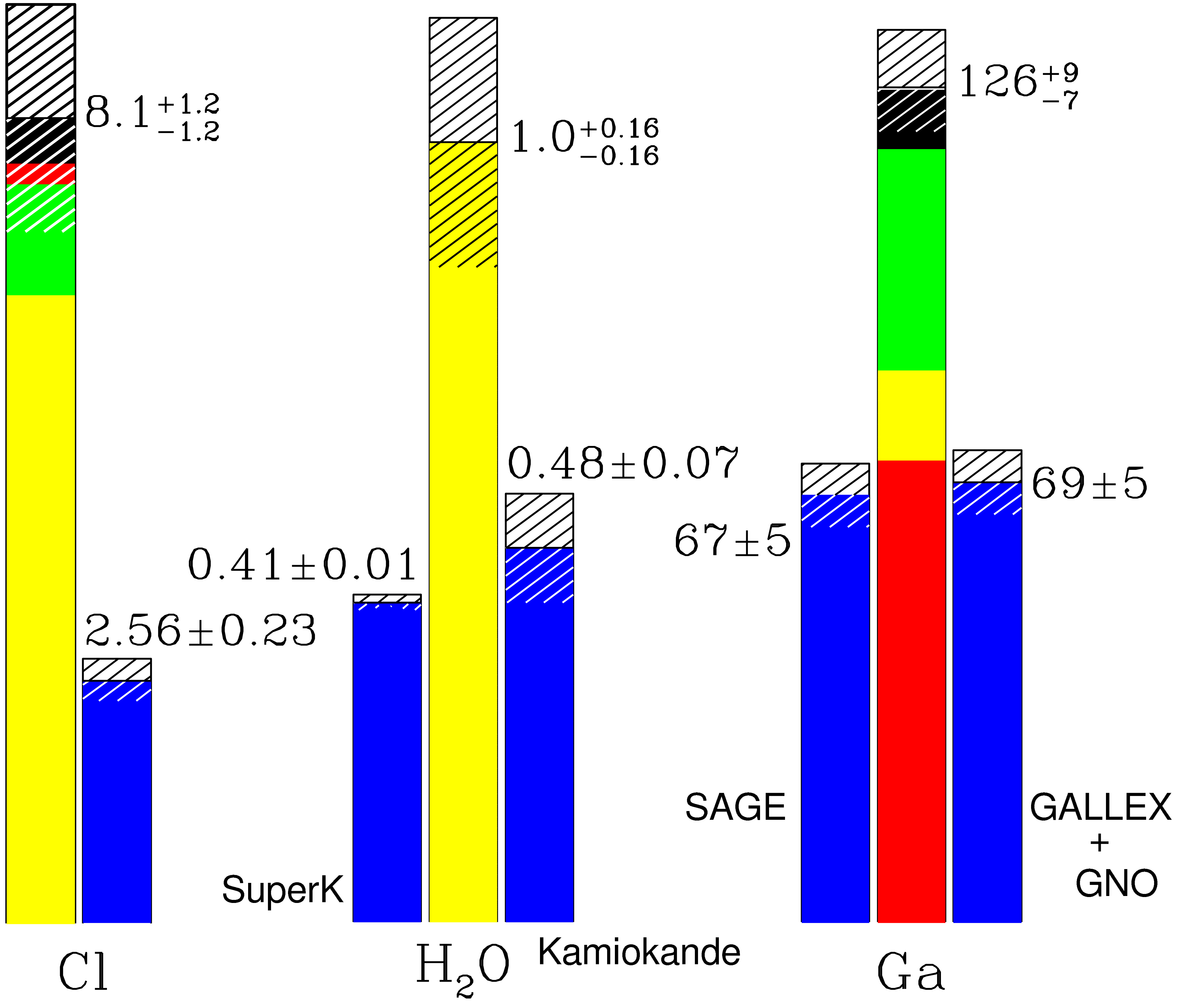
$$P(\nu_\alpha \to \nu_\beta) = \sin^2 2\theta \sin^2(\Delta m^2_{jk} \frac{L}{4E})$$

- L : source → detector distance
- E : neutrino energy

$\frac{L}{E} \ll \frac{1}{\Delta m^2}$ $\frac{L}{E} \simeq \frac{1}{\Delta m^2}$ $\frac{L}{E} \gg \frac{1}{\Delta m^2}$



Reminder : solar neutrino deficit



Theory 🟩 ^{7}Be 🟥 p−p, pep Experiments 🟦

🟨 ^{8}B ⬛ CNO Uncertainties ▨

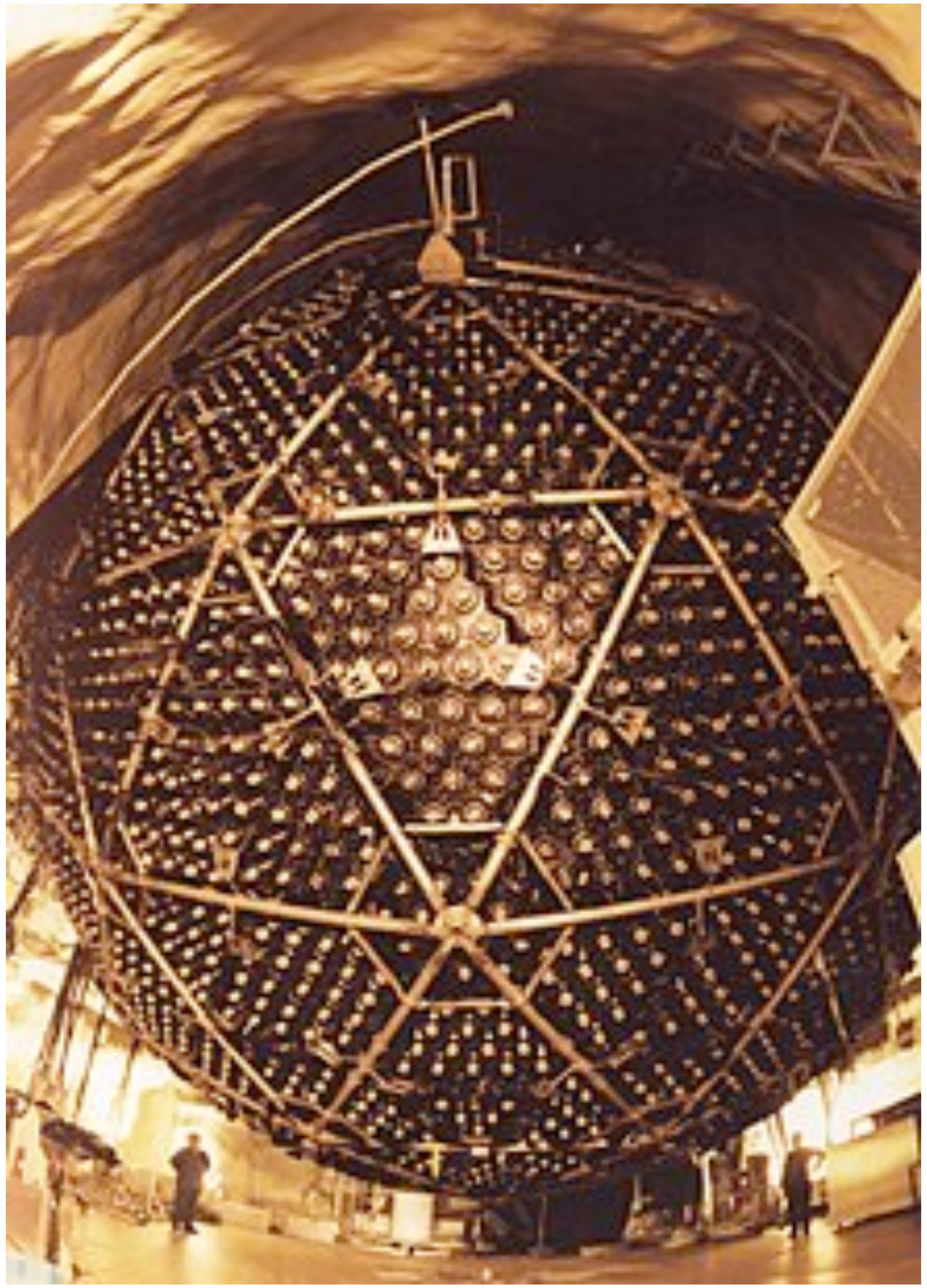
<u>Explanation proposal:</u>

Missing ν_e have oscillated into ν_μ and ν_τ and cannot be detected through CC interactions

$E_{thr}(\nu_\mu) = 110$ MeV

$E_{thr}(\nu_\tau) = 3.45$ GeV

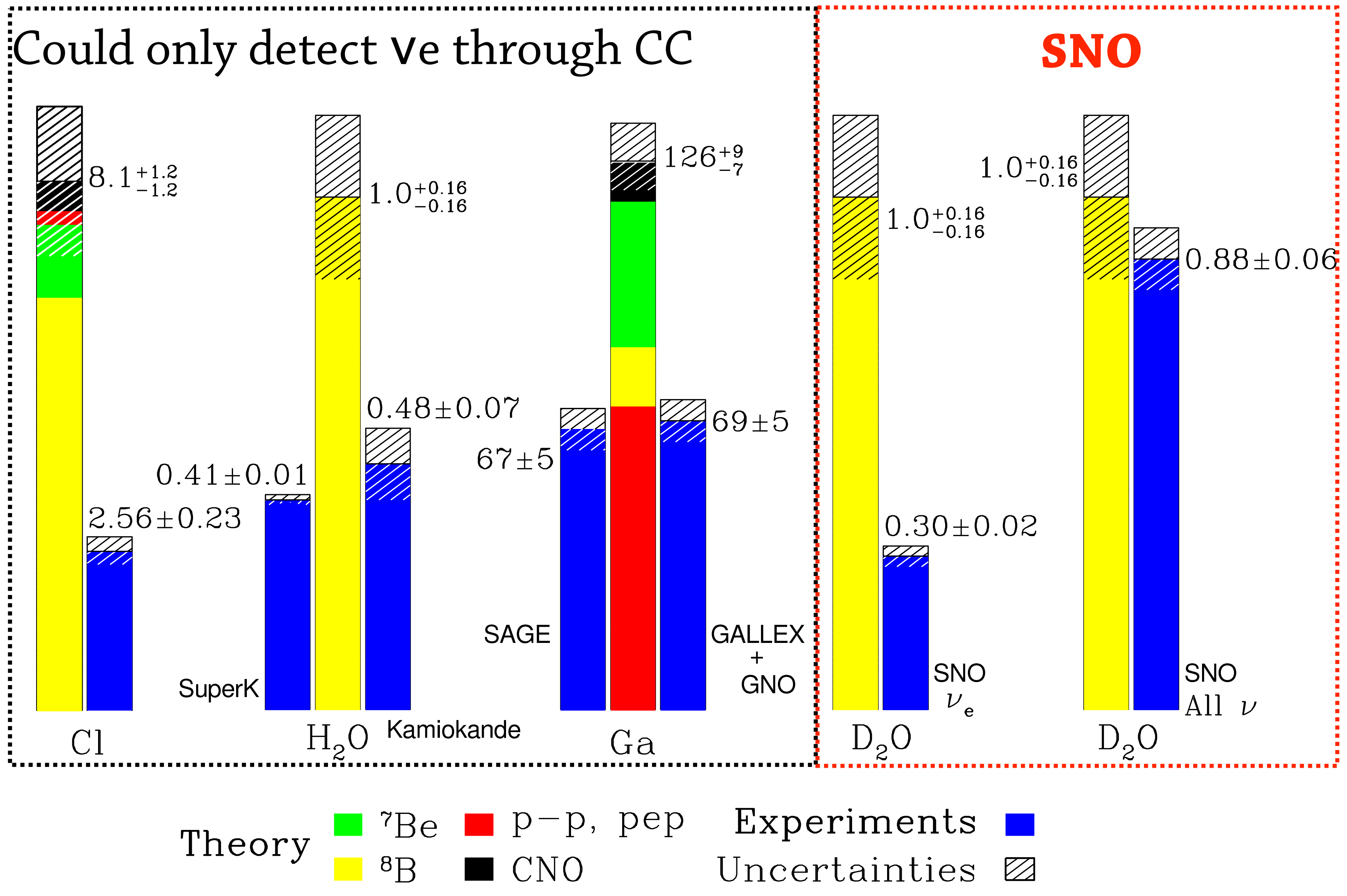
Proofs of neutrino oscillations - Solar

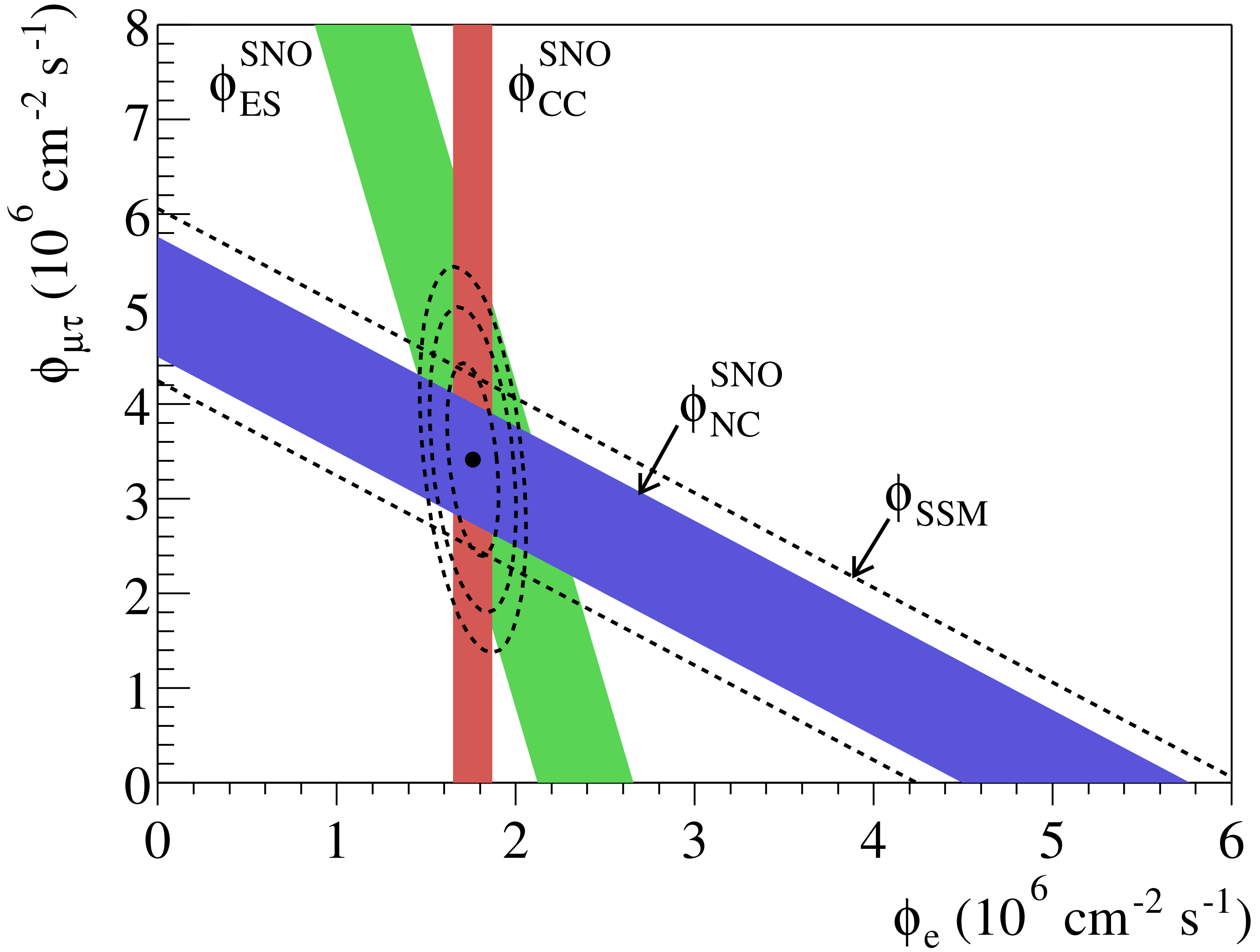


SNO (1kton of heavy water) was designed to detect solar neutrinos through:

- ○ **CC** interactions $\nu_e + d \rightarrow p + p + e^-$
  ν_e only (ν_μ & ν_τ don't have enough energy)
- ○ **ES** interactions $\nu_x + e^- \rightarrow \nu_x + e^-$
  all flavors
- ○ **NC** interactions $\nu_x + d \rightarrow p + n + \nu_x$
  all flavors

Proofs of neutrino oscillations - Solar





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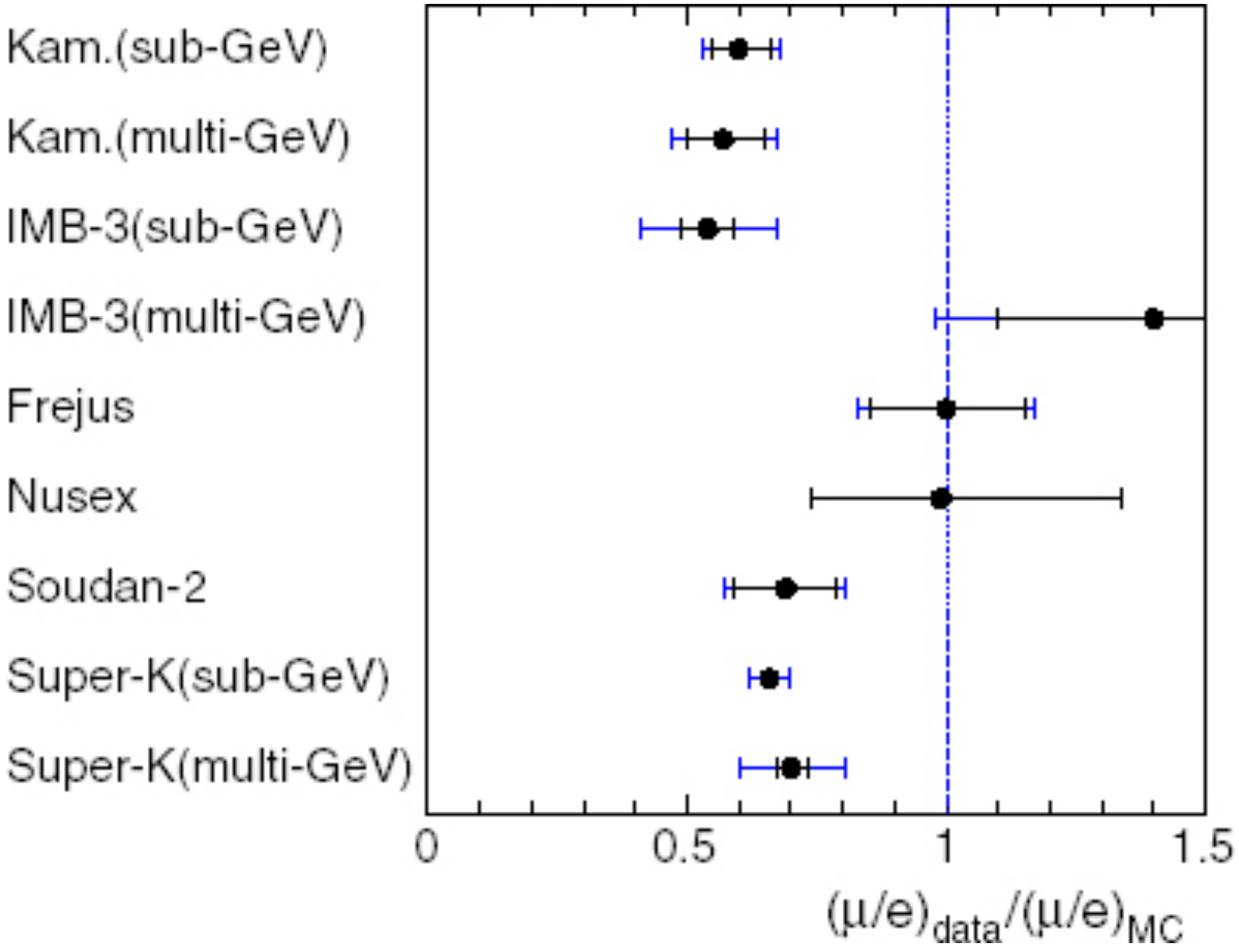
  all flavors
- ○ **NC** interactions $\nu_x + d \rightarrow p + n + \nu_x$
  all flavors

SNO measured the ratio : $\dfrac{\Phi_{CC}}{\Phi_{NC}} = 0.34 \pm 0.023(\text{stat.})^{+0.029}_{-0.031}$

And showed that the **total** flux of solar neutrino is **compatible** with the solar standard model

SNO proved that neutrino change flavors

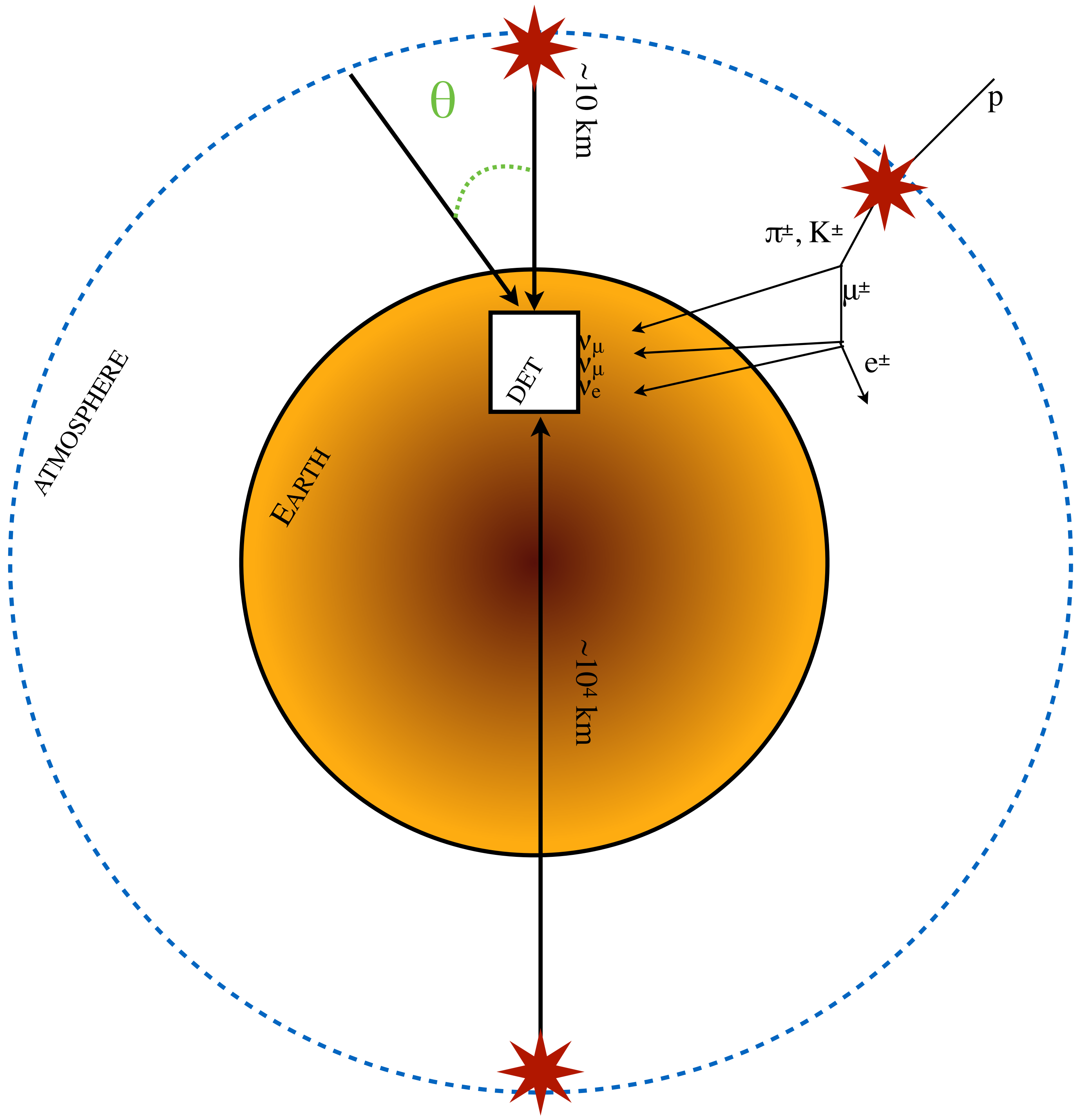
Reminder : Atmospheric neutrino deficit



Explanation proposal:

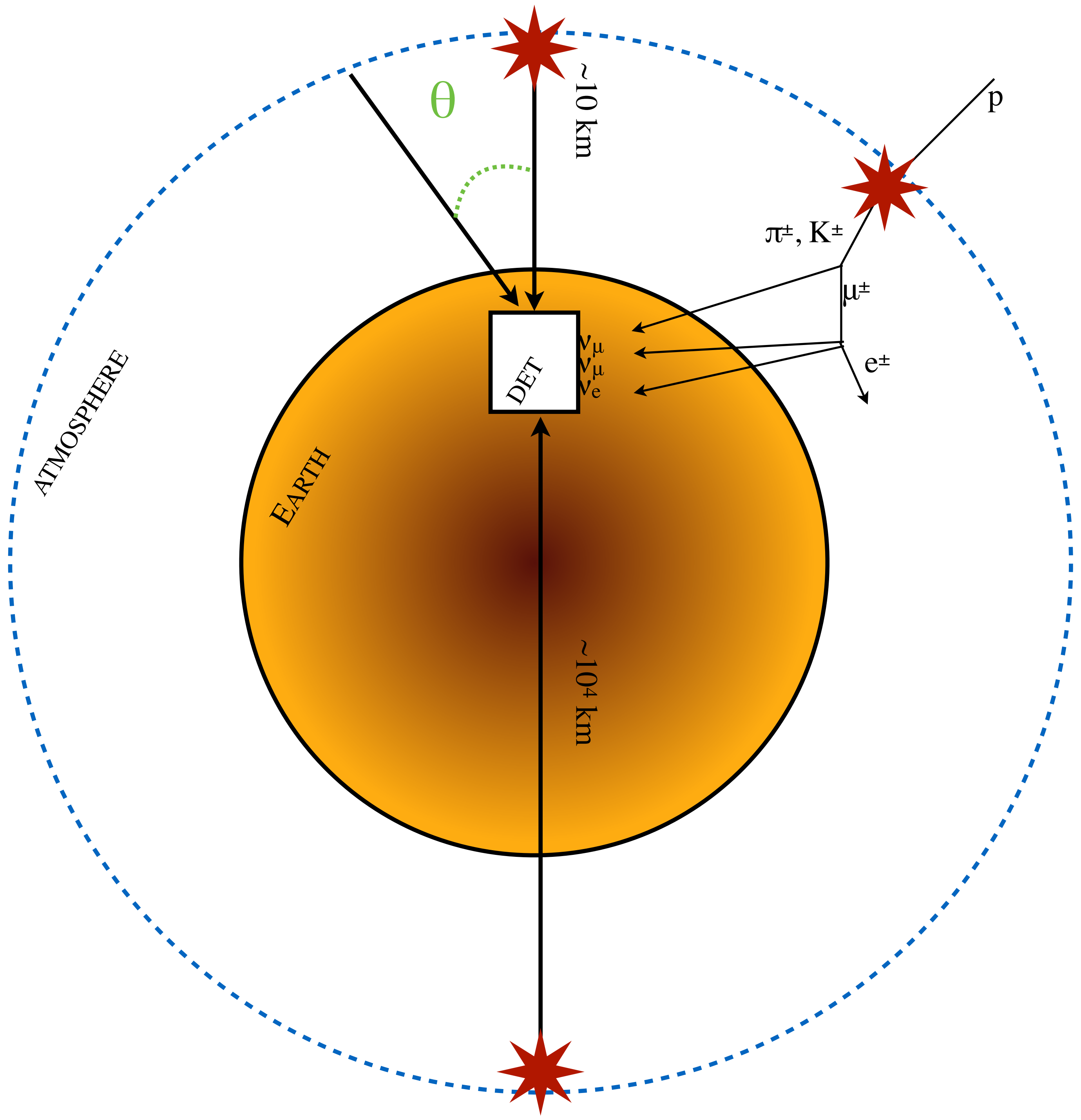
Produced ν_μ and ν_e oscillates, we should see the L/E dependance

Proofs of neutrino oscillations - Atmospheric



Super-Kamiokande measured the atmospheric ν_e and ν_μ energy as a function of cosθ ↔ L

Proofs of neutrino oscillations - Atmospheric



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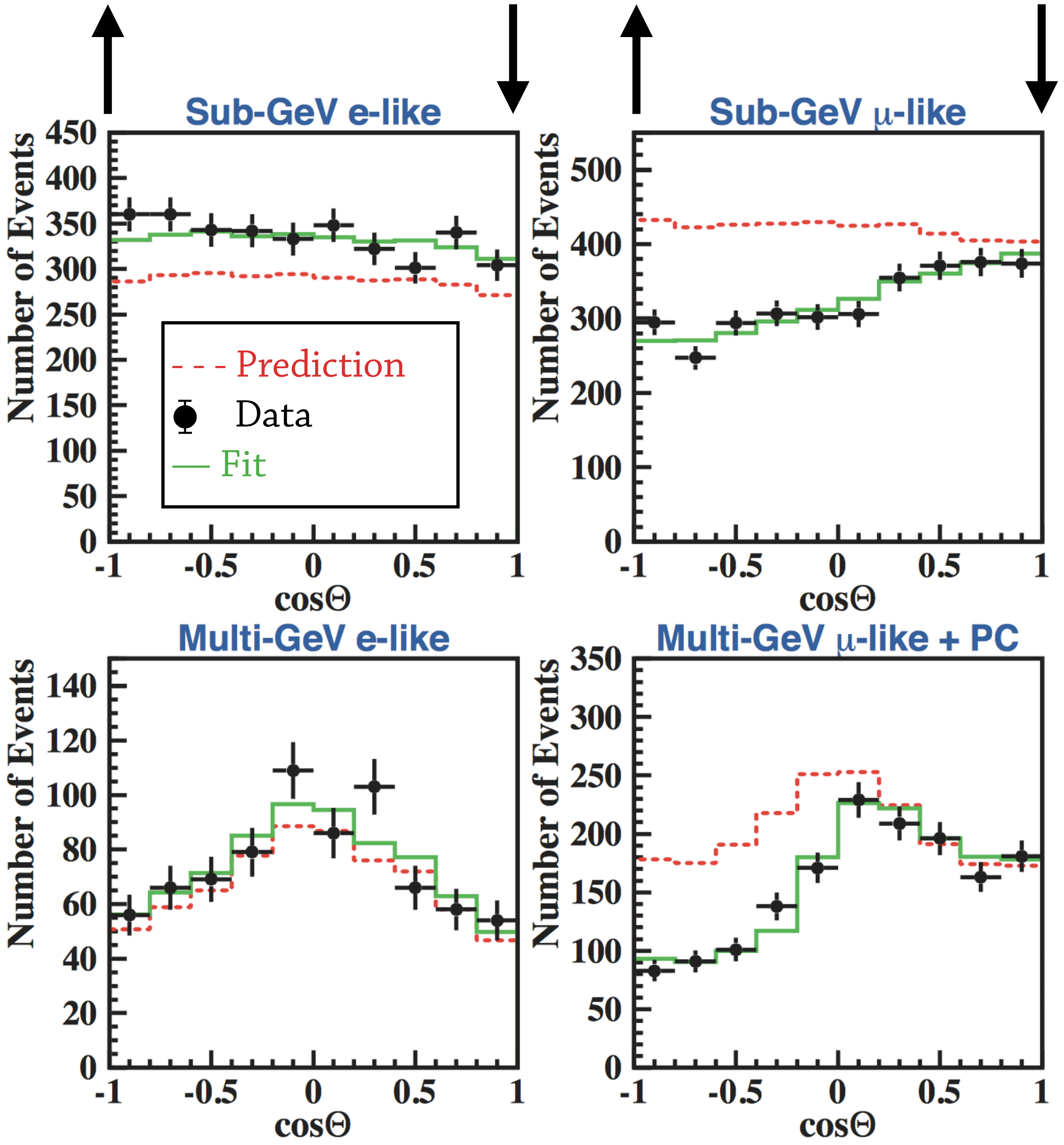


For ν_e :

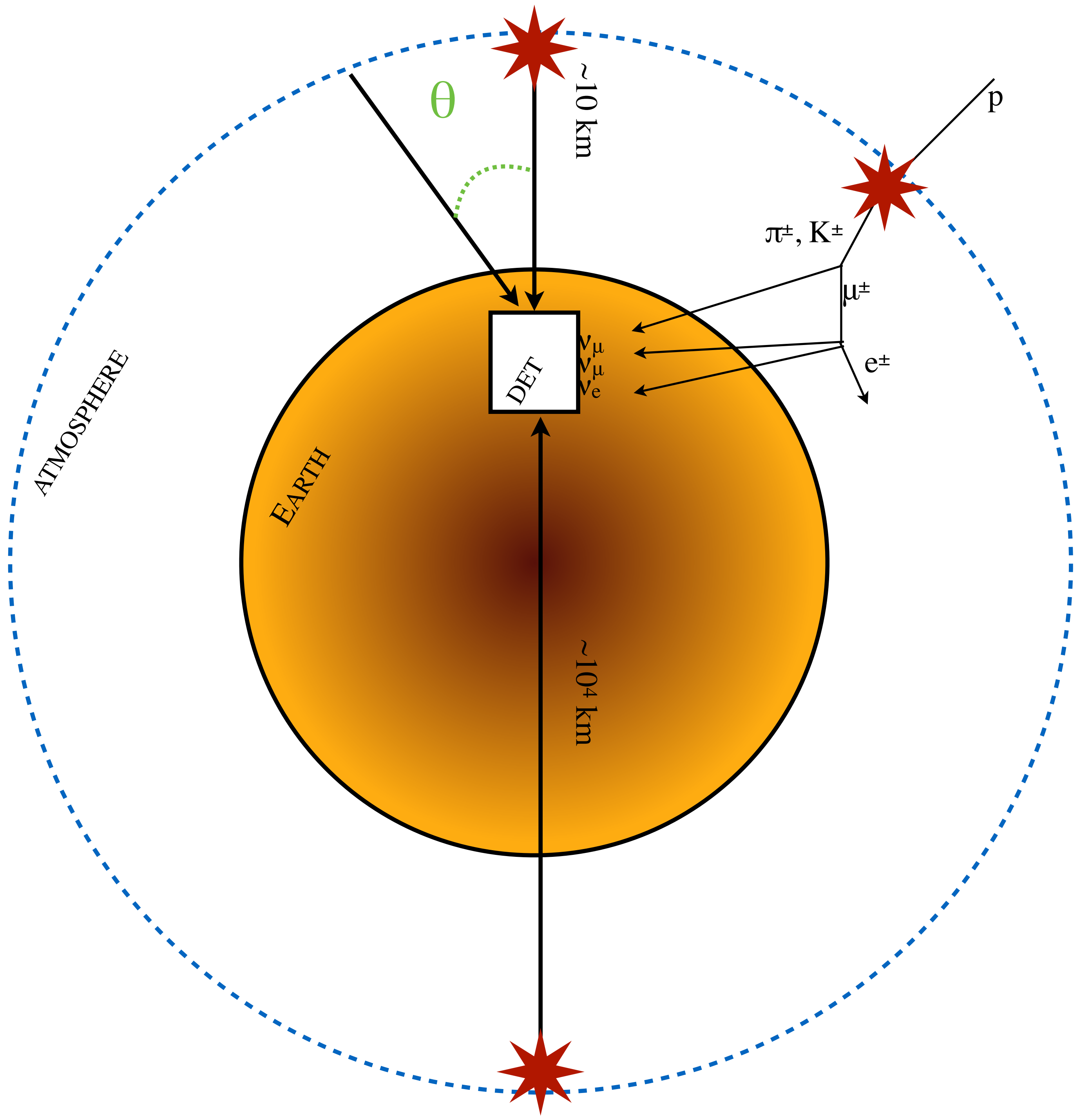
As predicted for all directions and energy

For ν_μ :

Loss of upwards ν_μ (L ~ 10^4 km)

As expected for downwards ν_μ (L ~ 10 km)

Proofs of neutrino oscillations - Atmospheric



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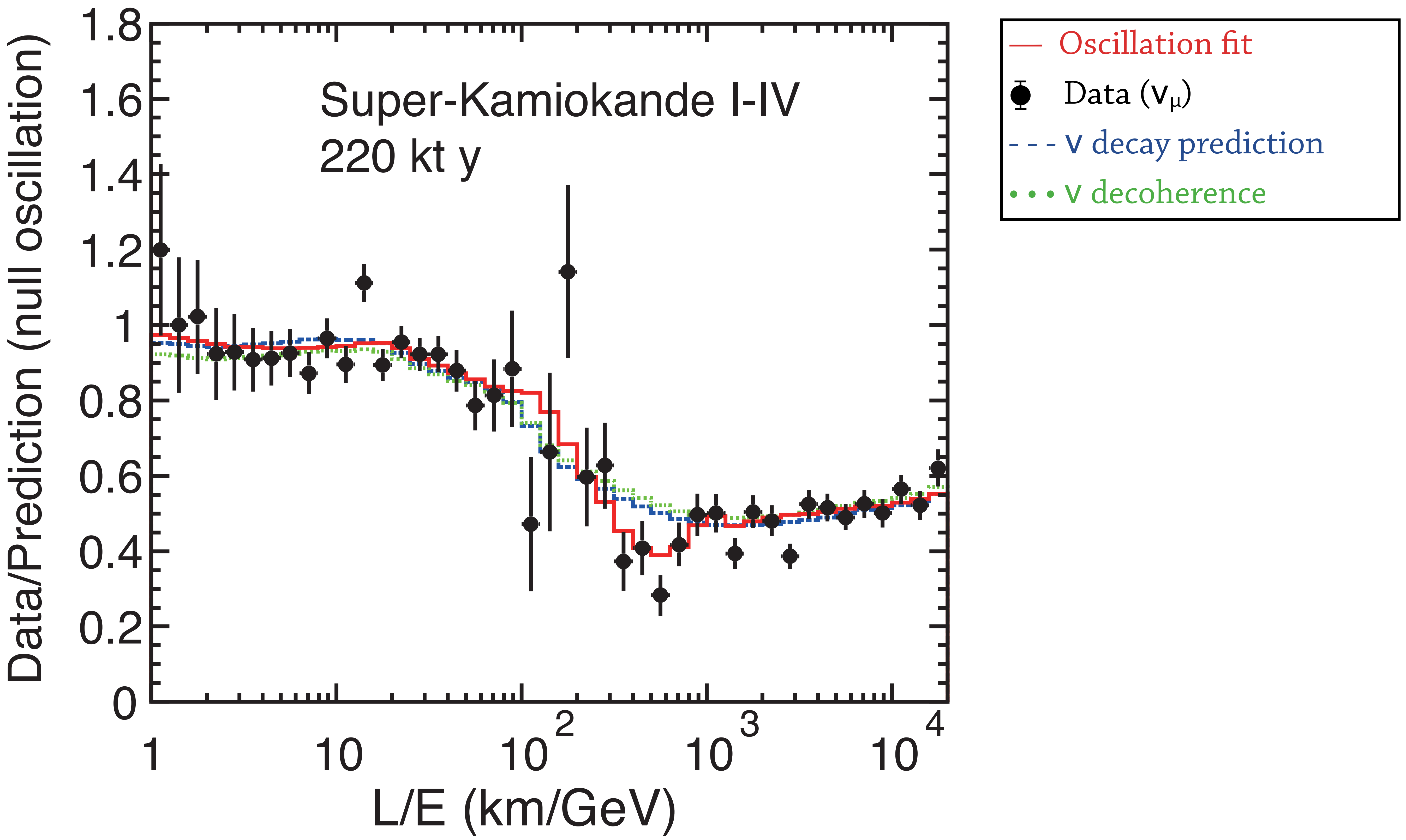
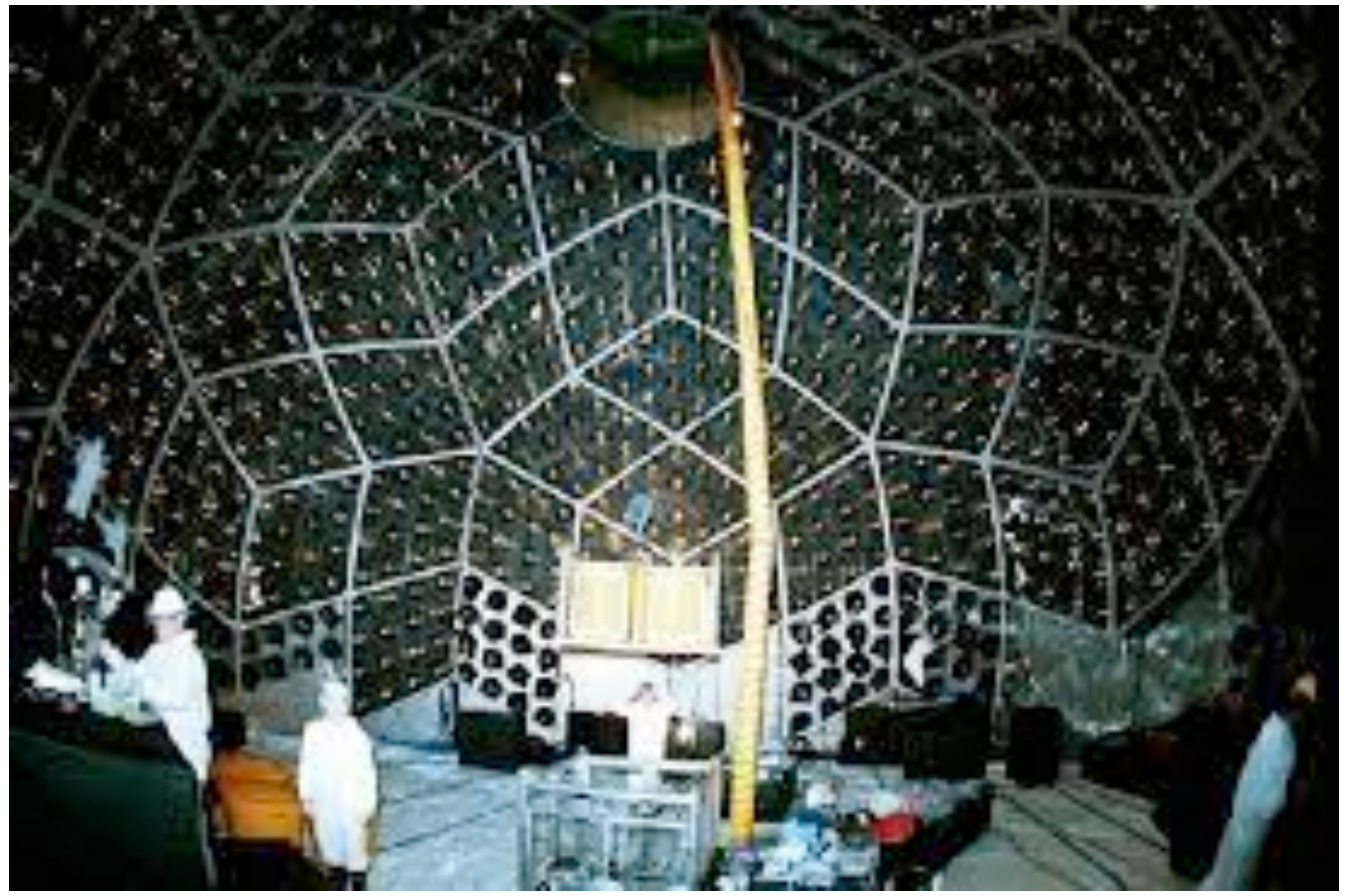
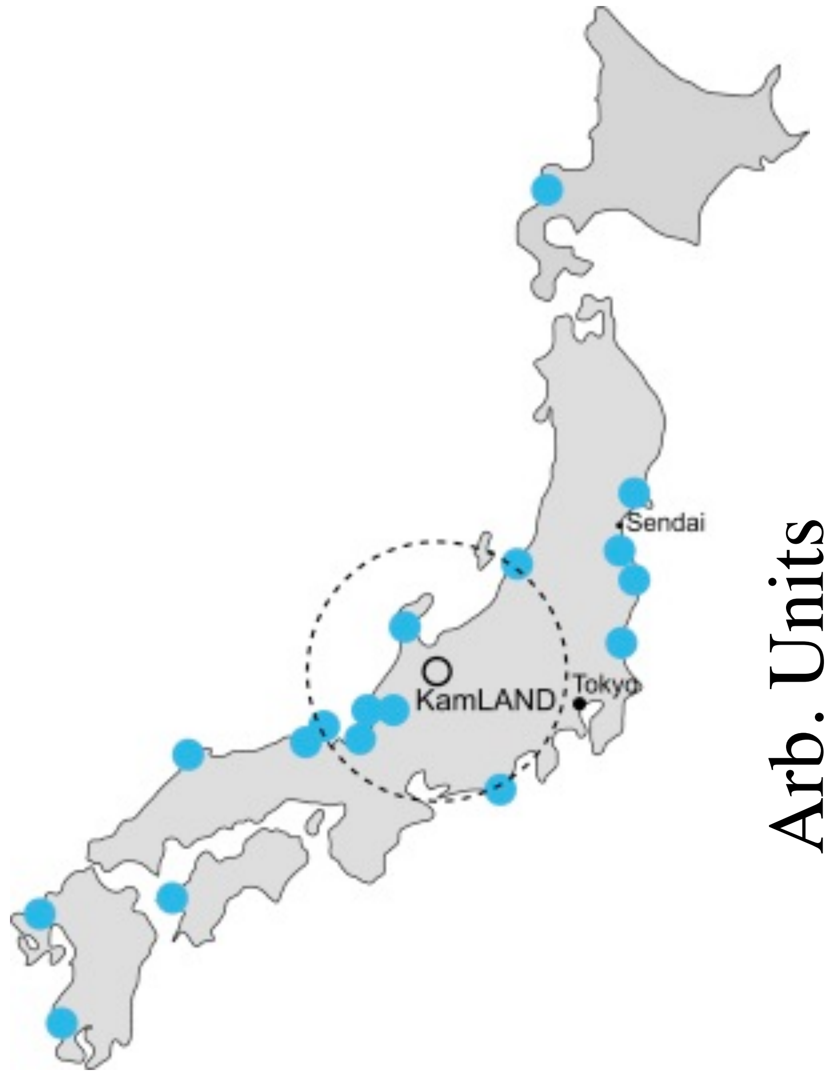


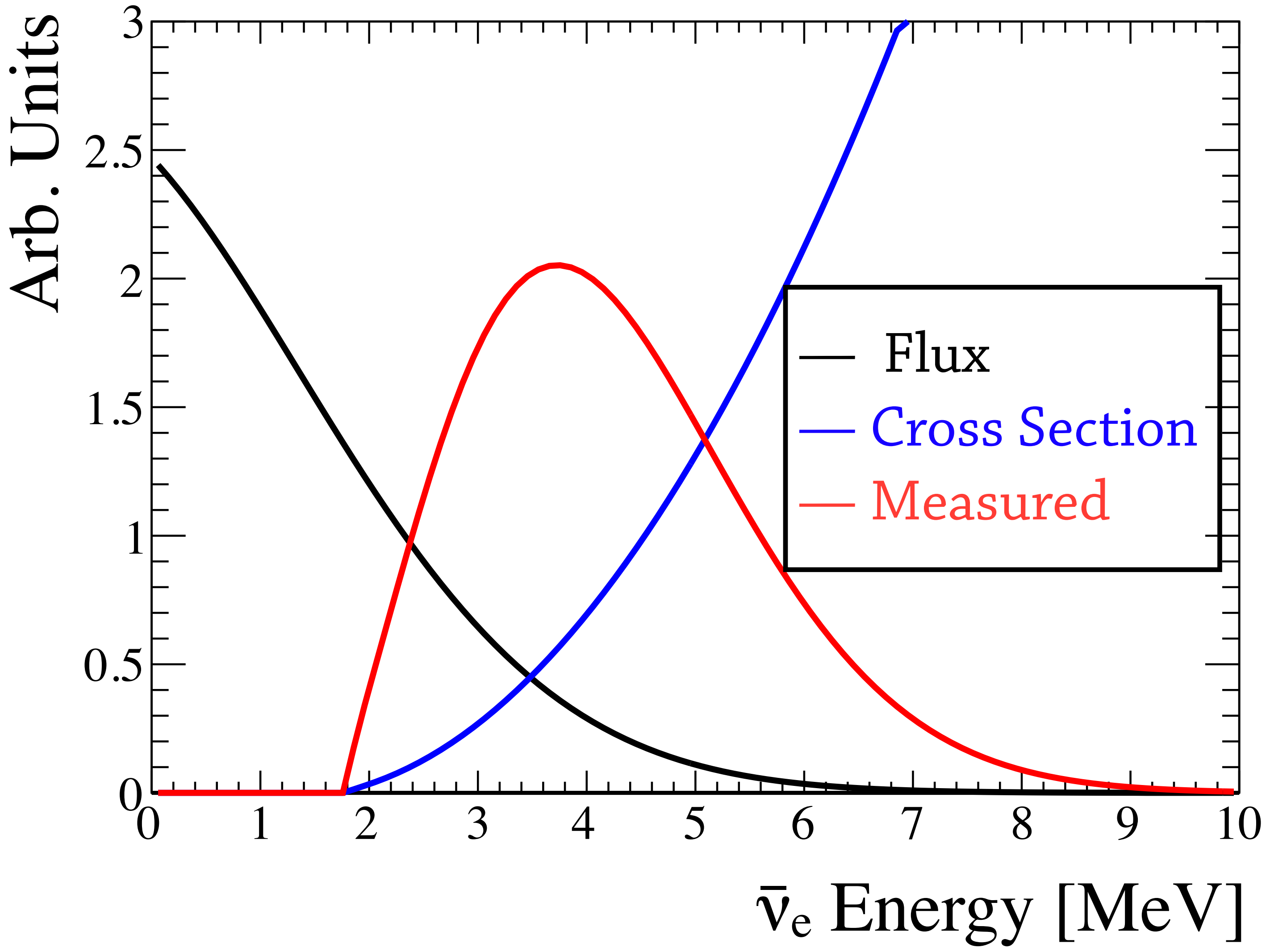
Super-Kamiokande proved that ν_μ disappear as a function of L/E (possibly into ν_τ)

Proofs of neutrino oscillations - Reactors



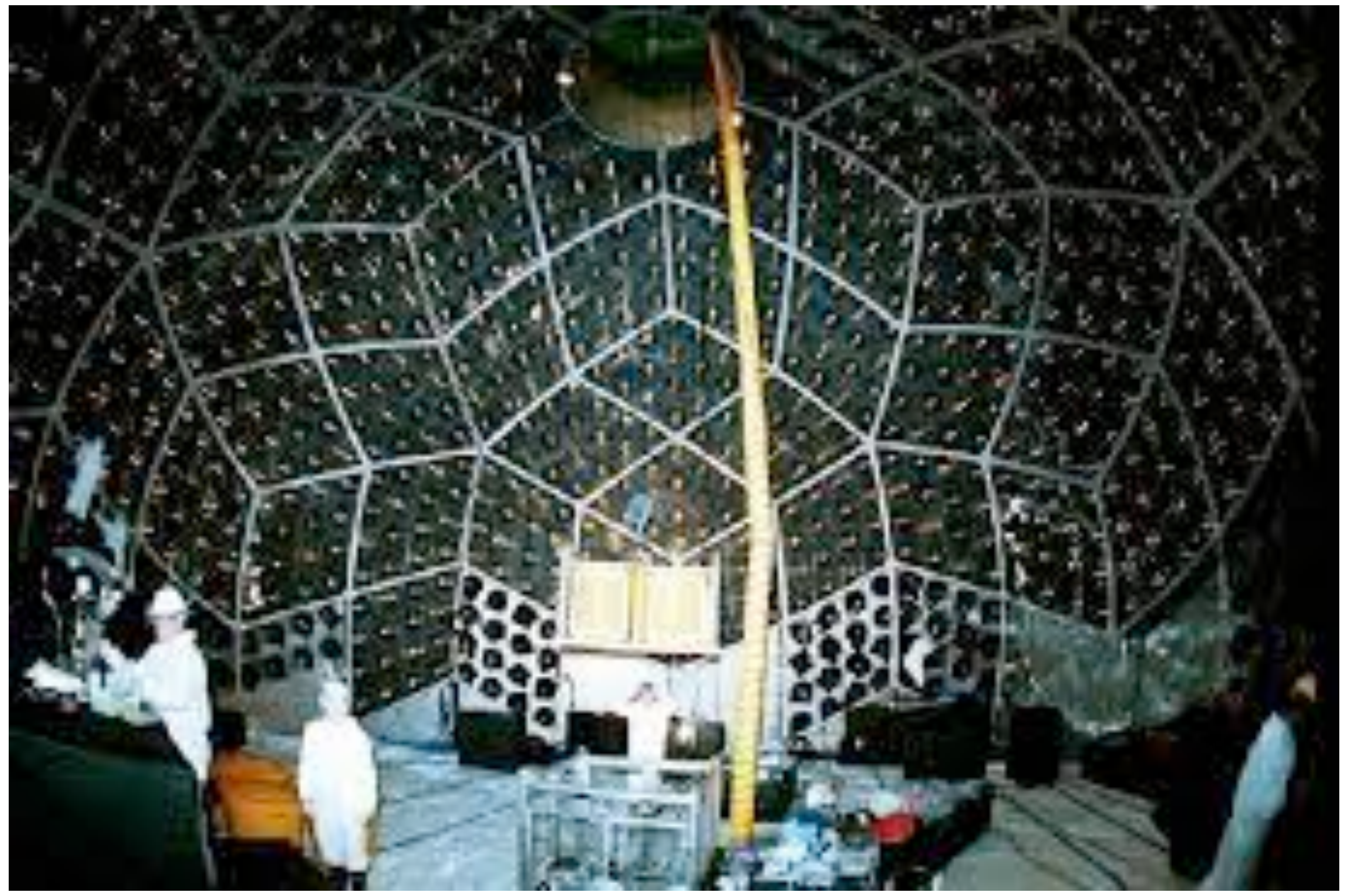


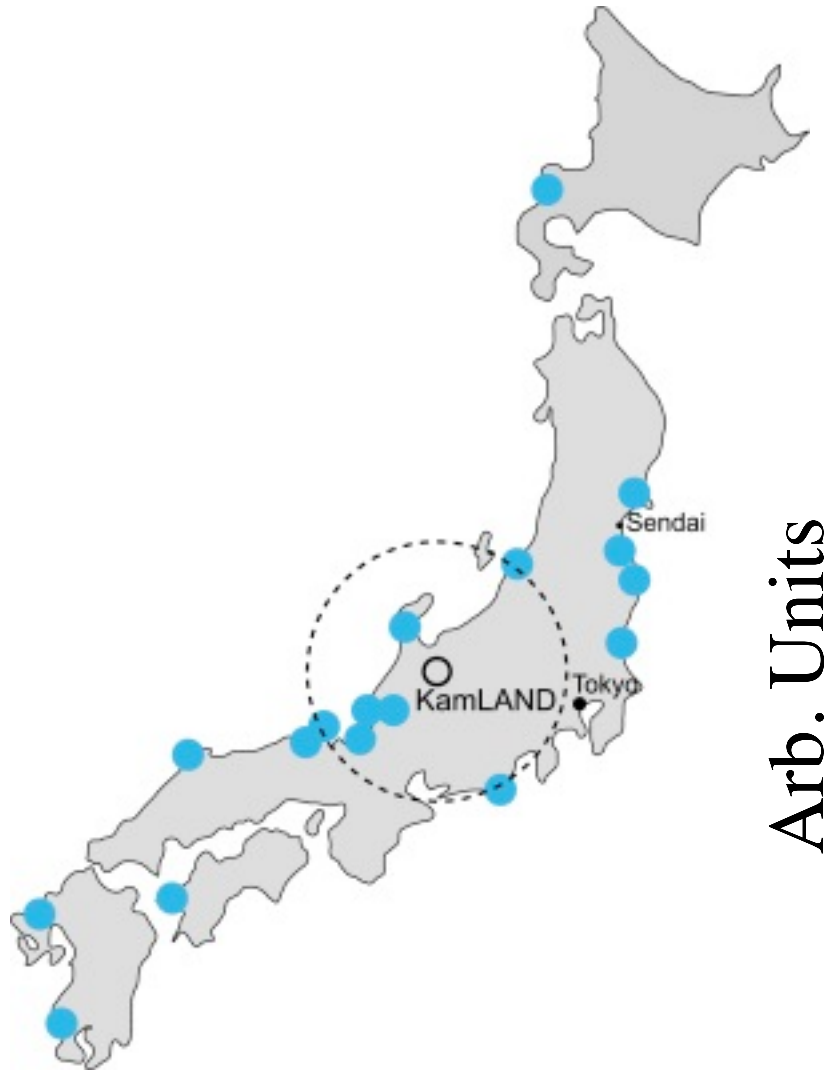
Kamland experiment in Japan measured the $\bar{\nu}_e$ flux from 53 nuclear reactors (L_{mean} ~ 180 km)



Reactor $\bar{\nu}_e$ spectrum up to ~10 MeV
-> Cannot measure appearance of new flavors

Proofs of neutrino oscillations - Reactors





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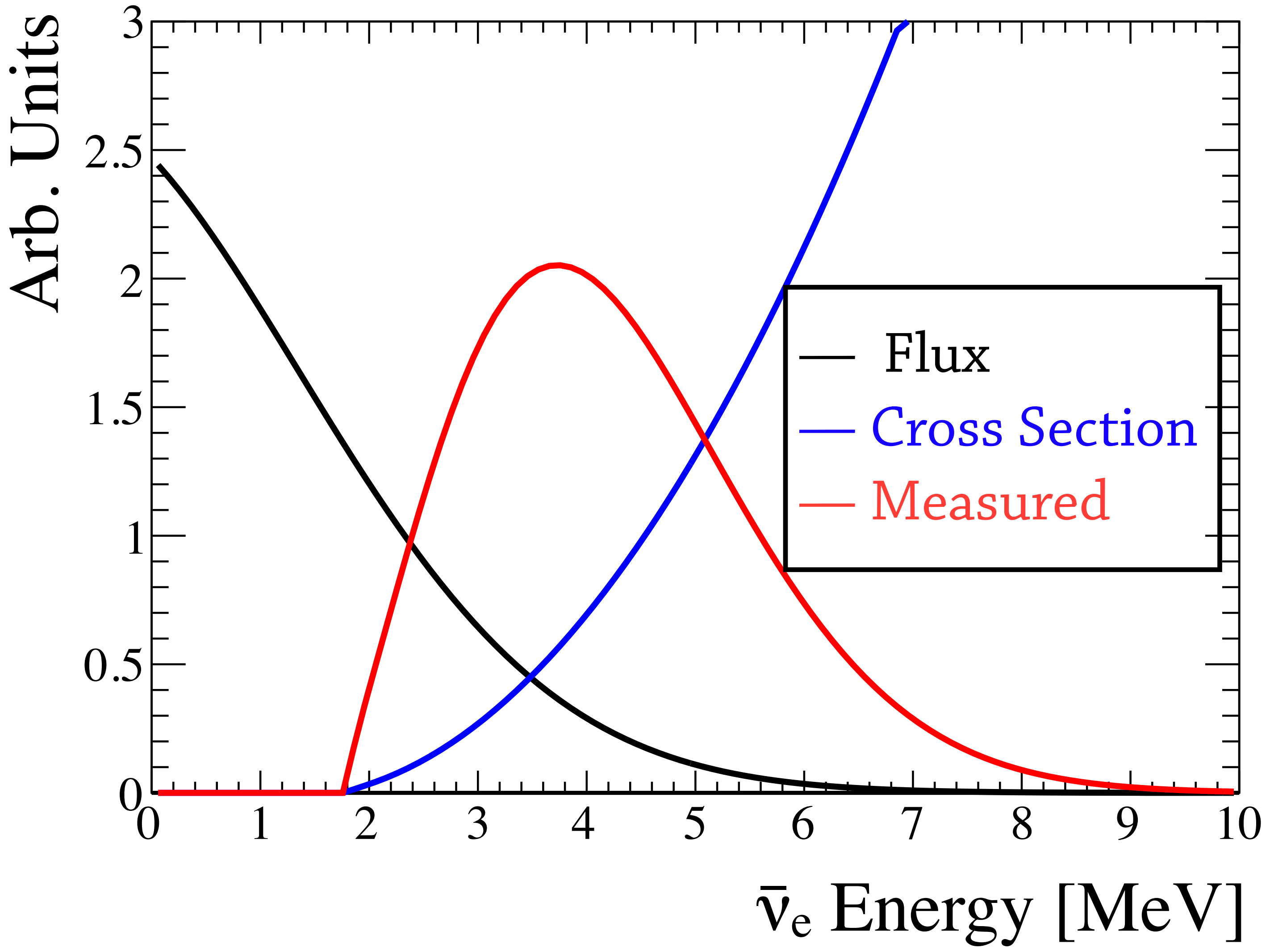

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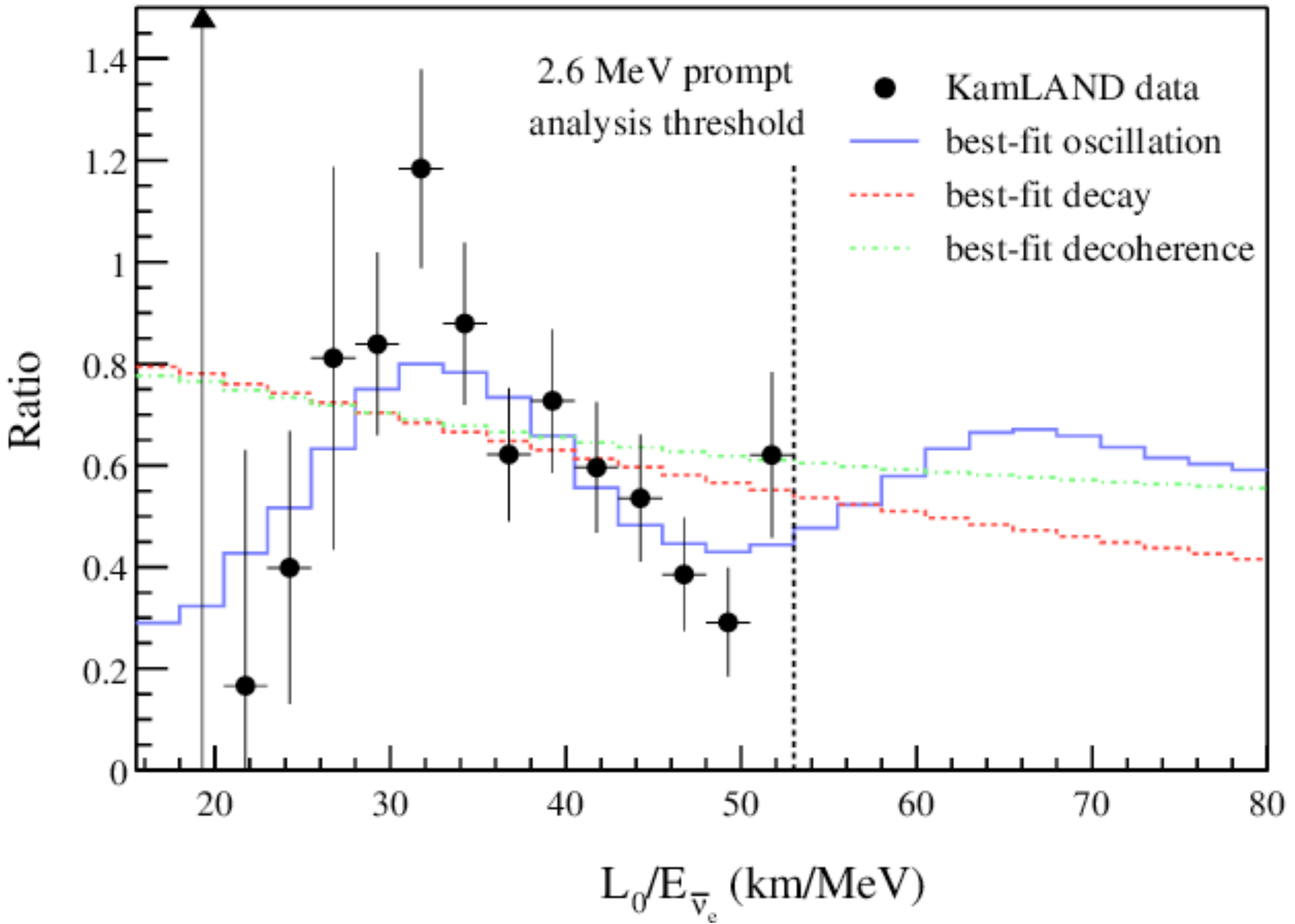
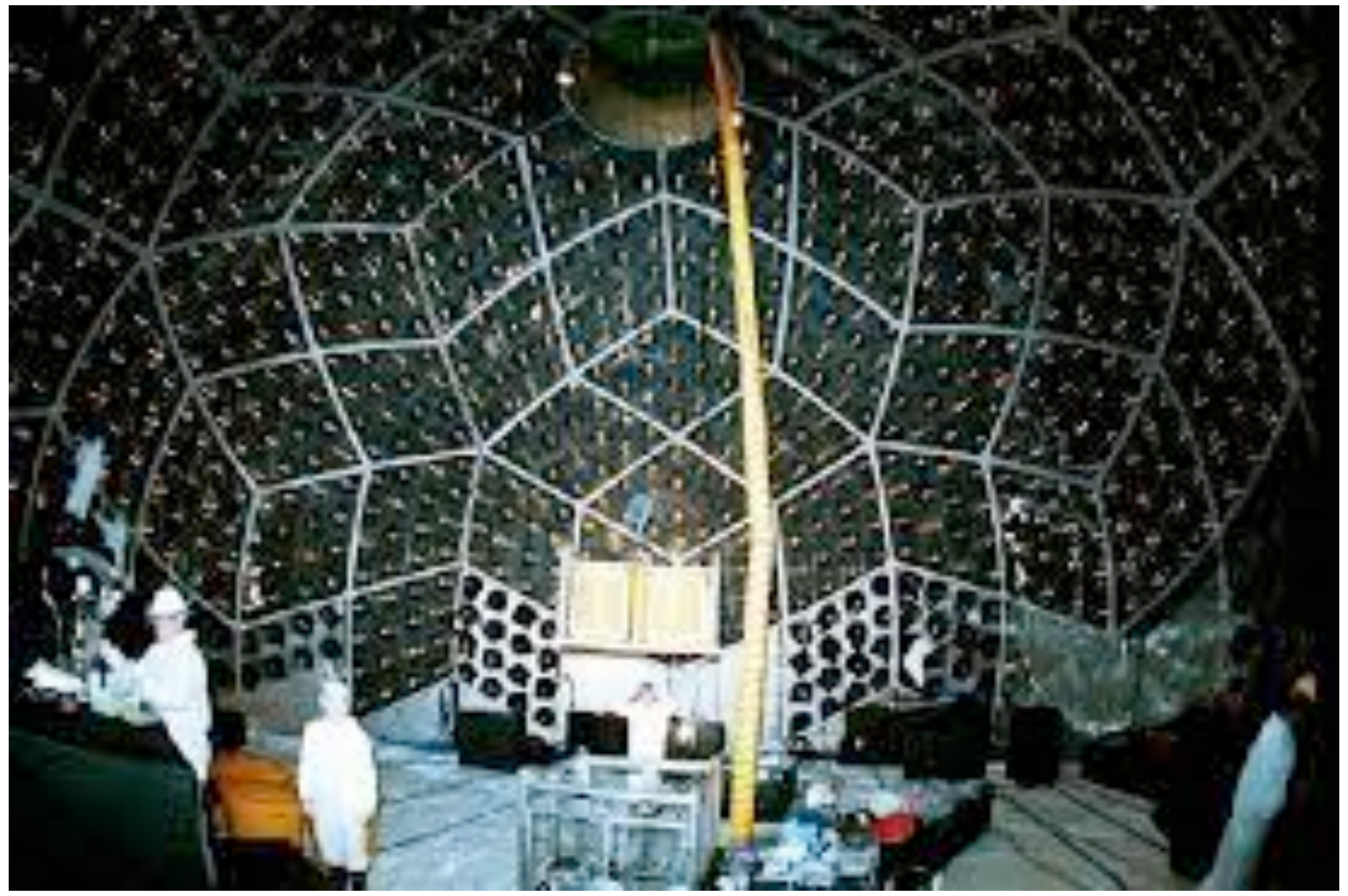


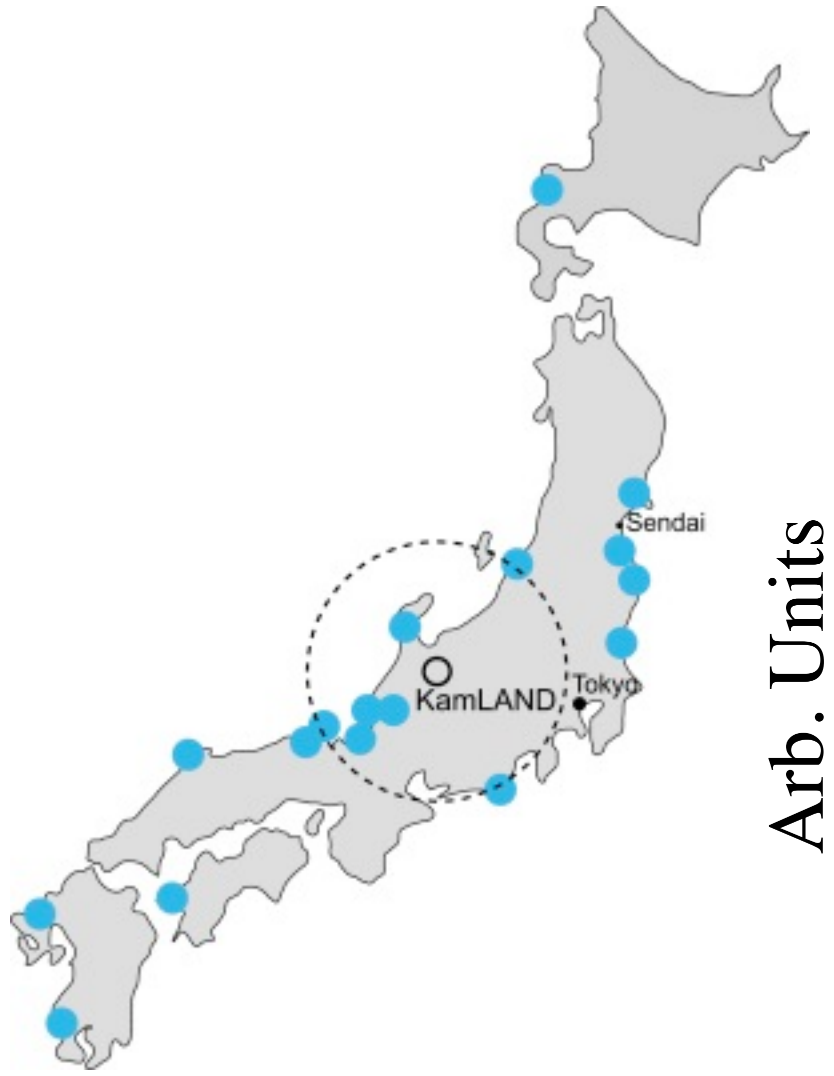
- First results -

○ Rejection of the ν-decay and ν-decoherence hypotheses

○ ν-oscillation preferred

Proofs of neutrino oscillations - Reactors





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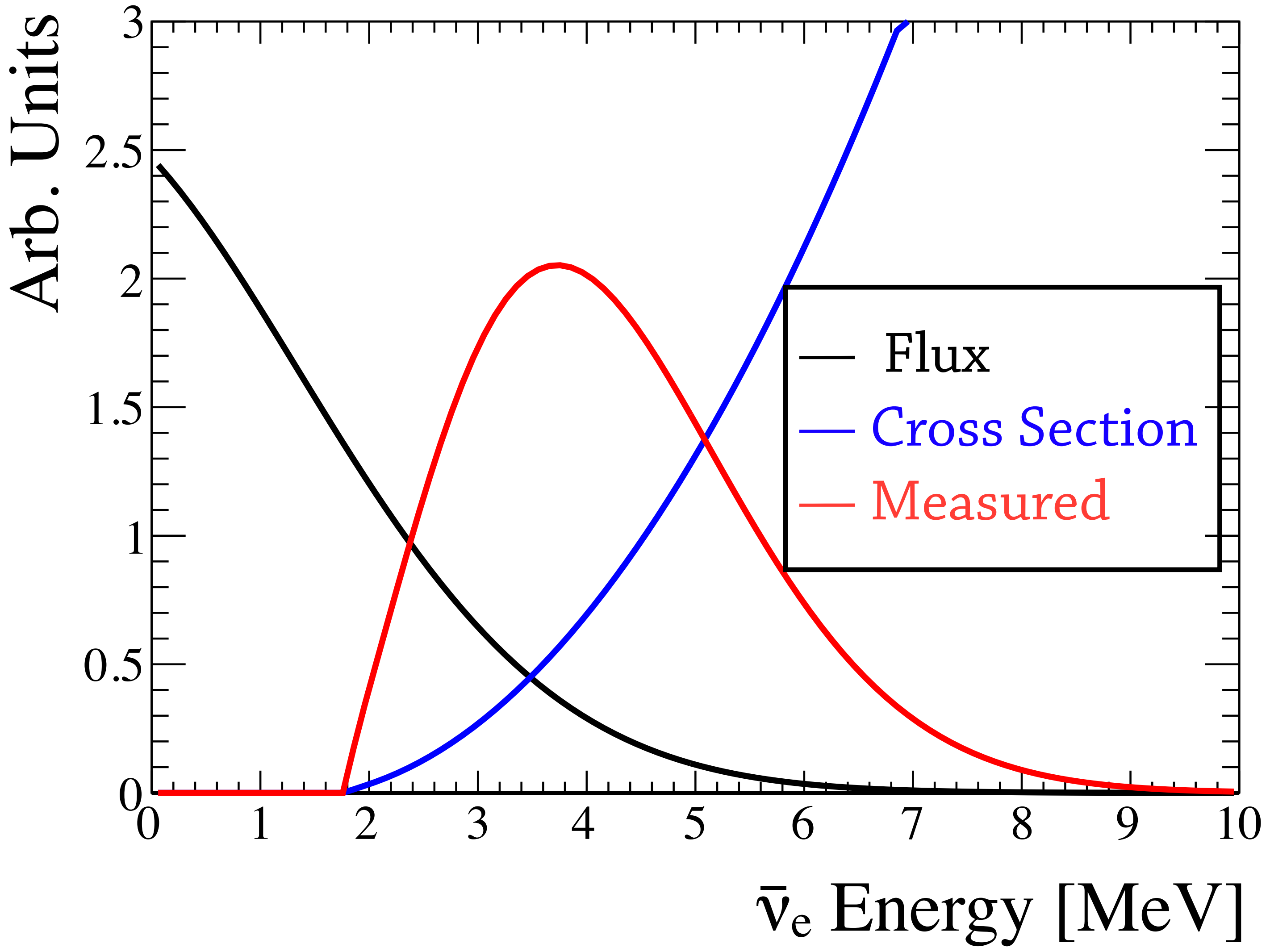

Reactor $\bar{\nu}_e$ spectrum up to ~10 MeV
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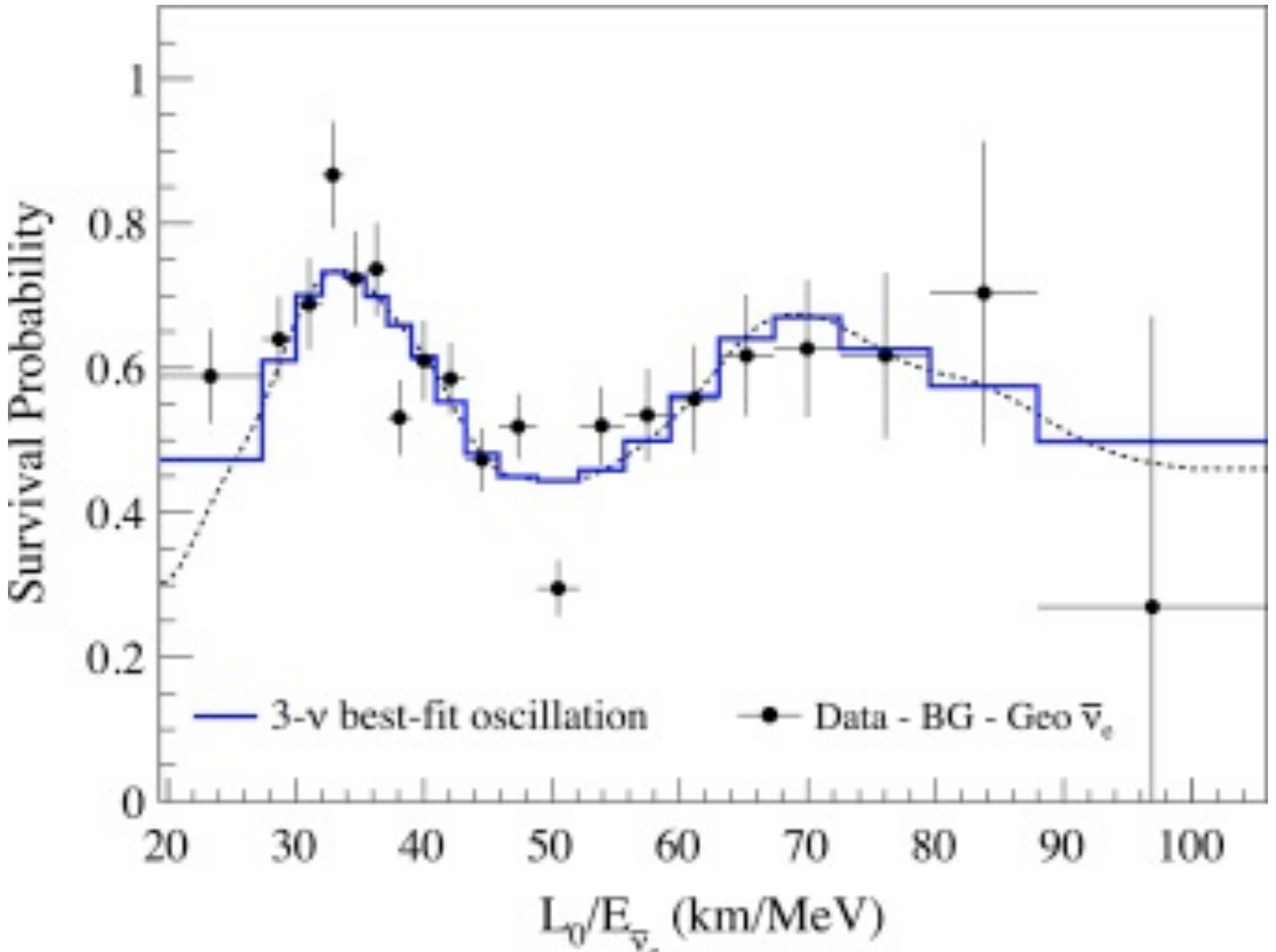

- Final results -

- ○ Very clear L/E pattern
- ○ Can see the disappearance dip, and re-appearance of $\bar{\nu}_e$!

KAMLAND proved that $\bar{\nu}_e$ oscillates !

NEUTRINO OSCILLATIONS

Oscillations with 3 flavors

With only two flavors, the mixing matrix was a simple rotation matrix:

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_j \\ \nu_k \end{pmatrix}$$

Let's now consider 3 neutrino flavors. In that case, the unitary mixing matrix U is written as :

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix}$$

$$s_{ij} = \sin\theta_{ij}$$

$$c_{ij} = \cos\theta_{ij}$$

We now have 3 rotation angles and one complex phase
This is the neutrino 3×3 unitary mixing matrix, the PMNS matrix

for Pontecorvo, Maki, Nakagawa, Sakata

Oscillations with 3 flavors

There is a more convenient way to write the PMNS matrix as a product of 3 rotation matrices:

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix}c_{13} & 0 & s_{13}e^{-i\delta}\\ 0 & 1 & 0\\ -s_{13}e^{i\delta} & 0 & c_{13}\end{pmatrix}\begin{pmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

$$s_{ij} = \sin\theta_{ij}$$
$$c_{ij} = \cos\theta_{ij}$$
$$\Delta m^2_{ij} = m^2_i - m^2_j$$

Three neutrinos -> 3 mass splittings : $\Delta m^2_{21},\ \Delta m^2_{31},\ \Delta m^2_{32}$

But only two are relevant, since : $\Delta m^2_{12} + \Delta m^2_{23} + \Delta m^2_{31} = 0$

Oscillations with 3 flavors

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Atmospheric Reactor/Accelerator Solar

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$$s_{ij} = \sin\theta_{ij}$$
$$c_{ij} = \cos\theta_{ij}$$
$$\Delta m_{ij}^2 = m_i^2 - m_j^2$$

Three neutrinos -> 3 mass splittings : $\Delta m_{21}^2,\ \Delta m_{31}^2,\ \Delta m_{32}^2$

But only two are relevant, since : $\Delta m_{12}^2 + \Delta m_{23}^2 + \Delta m_{31}^2 = 0$

The three mixing angles can be probed with different sources of neutrinos

Oscillations with 3 flavors

There is a more convenient way to write the PMNS matrix as a product of 3 rotation matrices:

Atmospheric Reactor/Accelerator Solar

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But only two are relevant, since : $\Delta m^2_{12} + \Delta m^2_{23} + \Delta m^2_{31} = 0$

In the 3 ν-flavor case, the oscillation phenomena is described by:

- ○ **3** mixing angles: θ_{12}, θ_{23} and θ_{13}
- ○ **2** mass splittings: $\Delta m^2_{21} = \Delta m^2_{sol}$ and $\Delta m^2_{31} = \Delta m^2_{atm}$
- ○ **1** CP violation phase δ

Oscillations with 3 flavors

A neutrino of flavor α, is written as the sum of the three mass eigenstates

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \qquad |\nu_\alpha\rangle = \sum_{i=1}^{3} U_{\alpha i} |\nu_i\rangle$$

With the same approximations as in the 2 neutrino case, we can write the time evolution of the mass eigenstates as:

$$|\nu_i(L)\rangle = e^{\left(-i m_i^2 \frac{L}{2E}\right)} |\nu_i\rangle$$

For the flavor eigenstates:

$$|\nu_\alpha(L)\rangle = \sum_{i=1}^{3} U_{\alpha i} |\nu_i(L)\rangle = \sum_{i=1}^{3} U_{\alpha i} e^{\left(-i m_i^2 \frac{L}{2E}\right)} |\nu_i\rangle$$

Oscillations with 3 flavors

The probability to detect a neutrino of flavor β at t=L while a neutrino of flavor α was created at t=0 is then:

$$\begin{aligned} P(\nu_\alpha \to \nu_\beta) &= |\langle \nu_\beta \,|\, \nu_\alpha(L) \rangle|^2 \\ &= \left| \sum_{j=1}^{3} U^*_{\alpha j} U_{\beta j} e^{(-i m_j^2 \frac{L}{2E})} \right|^2 \\ &= \left| U_{\alpha 1} U^*_{\beta 1} e^{-i\phi_1} + U_{\alpha 2} U^*_{\beta 2} e^{-i\phi_2} + U_{\alpha 3} U^*_{\beta 3} e^{-i\phi_3} \right|^2 \end{aligned}$$

Where: $\phi_j = m_j^2 \dfrac{L}{2E}$

To develop the expression, we will use the relationship (z, w and v are complex numbers):

$$|z + w + v|^2 = |z|^2 + |w|^2 + |v|^2 + 2\mathrm{Re}(zw^* + zv^* + wv^*)$$

Oscillations with 3 flavors

Without entering too much into details, we obtain:

$$P(\nu_\alpha \to \nu_\beta) = |\langle \nu_\beta \,|\, \nu_\alpha(L) \rangle|^2$$

$$= \left| \sum_{j=1}^{3} U_{\alpha j}^{*} U_{\beta j} e^{\left(-i m_j^2 \frac{L}{2E}\right)} \right|^2$$

No questions allowed

$$= \delta_{\alpha\beta} - 4 \sum_{j<k}^{3} \mathrm{Re}(U_{\alpha j} U_{\beta j}^{*} U_{\alpha k}^{*} U_{\beta k}) \sin^2 \left(\Delta m_{jk}^2 \frac{L}{4E} \right)$$

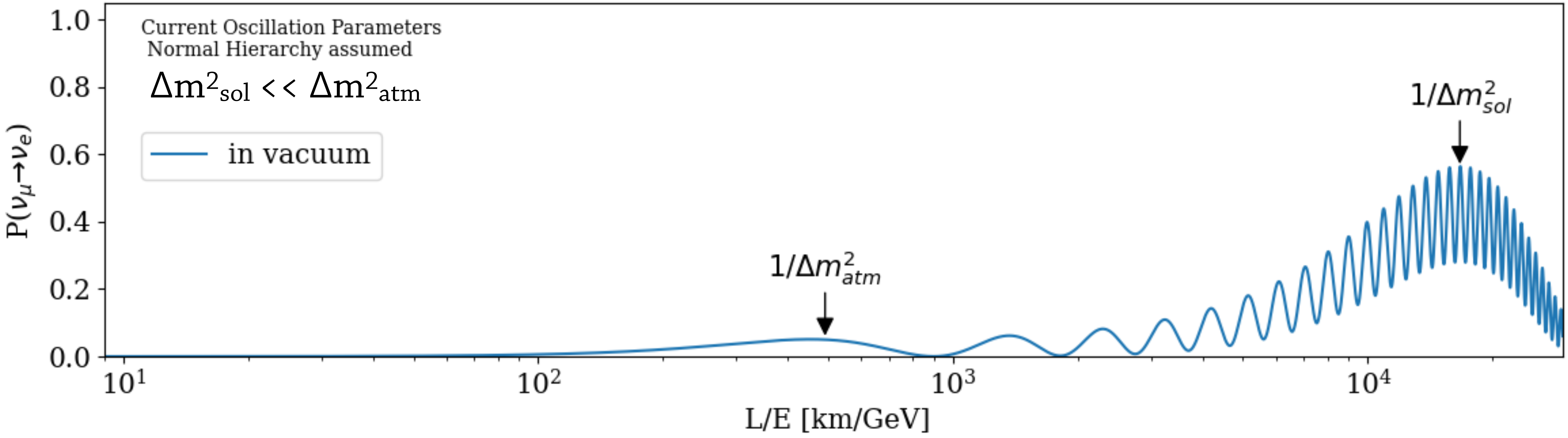
$$\pm 2 \sum_{j<k}^{3} \mathrm{Im}(U_{\alpha j} U_{\beta j}^{*} U_{\alpha k}^{*} U_{\beta k}) \sin \left(\Delta m_{jk}^2 \frac{L}{4E} \right)$$

‘+’ for neutrinos, ‘-‘ for anti-neutrinos

Oscillations with 3 flavors in vacuum

The oscillation probability is written as : $P(\nu_\alpha \to \nu_\beta) = |\langle \nu_\beta(L)|\nu_\alpha\rangle|^2 = \left|\sum_j U^*_{\alpha j} U_{\beta j} e^{-i\frac{m_j^2 L}{2E}}\right|^2$

As we have 2 mass splittings, we have 2 oscillation frequencies interfering :



$\Delta m^2_{21} = \Delta m^2_{sol}$

$\Delta m^2_{31} = \Delta m^2_{atm}$

Oscillations in matter

Up to here, we considered that the neutrino propagates <u>in vacuum, </u>let’s now consider the case of propagation in matter.

Oscillations in matter

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If we recall the 2 flavor case, the time evolution of the mass eigenstates can be written in the matrix format as:

$$i\frac{d}{dt}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = H\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = \begin{pmatrix}E_1 & 0\\ 0 & E_2\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} \simeq \frac{1}{2E}\begin{pmatrix}m_1^2 & 0\\ 0 & m_2^2\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$$

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Applying the 2×2 mixing matrix, we can write the flavor eigenstate evolution as:

Recall that:

$$\begin{pmatrix}\nu_\alpha\\\nu_\beta\end{pmatrix} = U\begin{pmatrix}\nu_1\\\nu_2\end{pmatrix}$$

$$UU^\dagger = U^\dagger U = I$$

$$i\frac{d}{dt}\begin{pmatrix}\nu_1\\\nu_2\end{pmatrix} = U^\dagger i\frac{d}{dt}\begin{pmatrix}\nu_\alpha\\\nu_\beta\end{pmatrix} = H\begin{pmatrix}\nu_1\\\nu_2\end{pmatrix} = HU^\dagger\begin{pmatrix}\nu_\alpha\\\nu_\beta\end{pmatrix}$$

$$i\frac{d}{dt}\begin{pmatrix}\nu_\alpha\\\nu_\beta\end{pmatrix} = UHU^\dagger\begin{pmatrix}\nu_1\\\nu_2\end{pmatrix}$$

Oscillations in matter

$$i\frac{d}{dt}\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix} = UHU^\dagger\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$$

In vacuum, the transformed hamiltonian can be written as:

No questions allowed

$$H_f = UHU^\dagger = H_0 + \frac{\Delta m^2}{4E}\begin{pmatrix}-\cos 2\theta & \sin 2\theta\\ \sin 2\theta & \cos 2\theta\end{pmatrix}$$

$$H_0 = \frac{m_1^2 + m_2^2}{4E}$$

Oscillations in matter

$$i\frac{d}{dt}\begin{pmatrix}\nu_\alpha \\ \nu_\beta\end{pmatrix} = UHU^\dagger \begin{pmatrix}\nu_1 \\ \nu_2\end{pmatrix}$$

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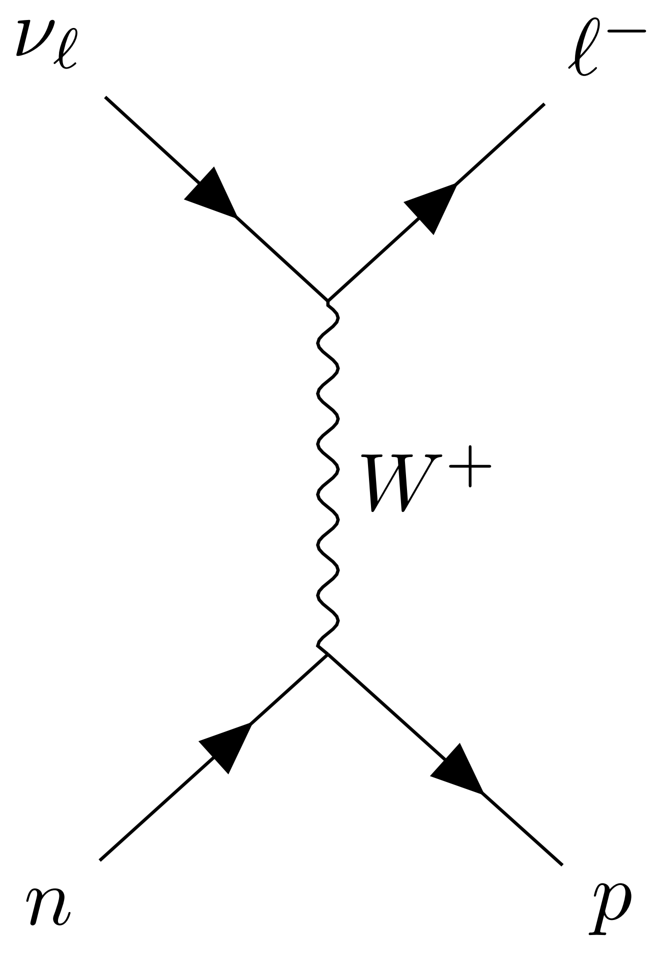
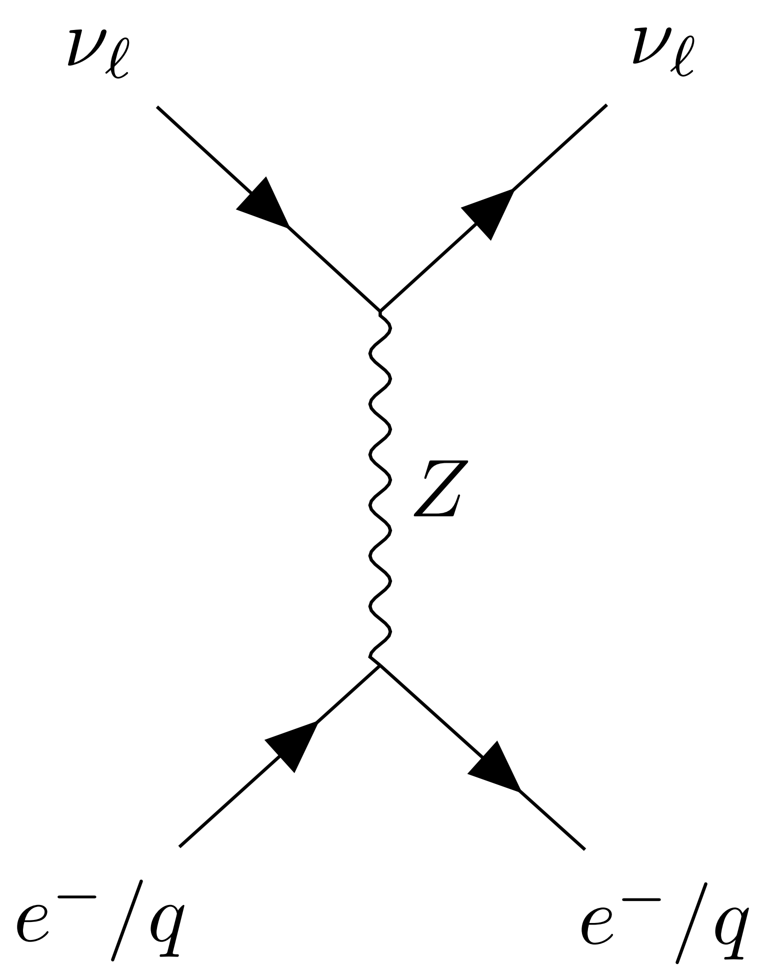
$$H_0 = \frac{m_1^2 + m_2^2}{4E}$$

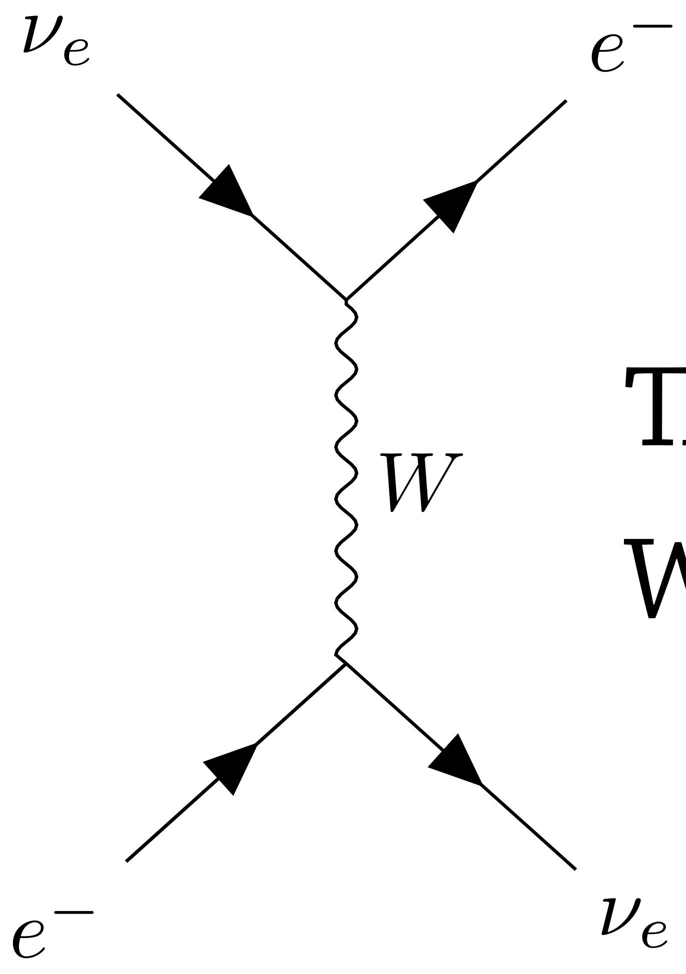
When in matter, the neutrinos can interact through CC or NC :



All ν_α



All ν_α



Only ν_e

But

The ν_e can interact with e^- through CC and NC

While ν_μ and ν_τ can only interact with e^- through NC

Oscillations in matter

This implies that in matter, the Hamiltonian have an extra potential V for the ν_e-ν_e interactions

$$V = \pm\sqrt{2} G_F N_e$$

- G_F is the Fermi constant
- N_e is the number of electrons per unit of volume
- ‘+’ for neutrinos, ‘-‘ for anti-neutrinos

$$i\frac{d}{dt}\begin{pmatrix}\nu_\alpha \\ \nu_\beta\end{pmatrix} = (H_f + V)\begin{pmatrix}\nu_\alpha \\ \nu_\beta\end{pmatrix}$$
$$= H_0\begin{pmatrix}\nu_\alpha \\ \nu_\beta\end{pmatrix} + \frac{\Delta m^2}{4E}\begin{pmatrix}-\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta\end{pmatrix} + V\begin{pmatrix}1 & 0 \\ 0 & 0\end{pmatrix}\begin{pmatrix}\nu_\alpha \\ \nu_\beta\end{pmatrix}$$

Oscillations in matter

This implies that in matter, the Hamiltonian have an extra potential V for the ν$_e$-ν$_e$ interactions

$$V = \pm\sqrt{2}G_F N_e$$

- G_F is the Fermi constant
- N_e is the number of electrons per unit of volume
- '+' for neutrinos, '-' for anti-neutrinos

$$i\frac{d}{dt}\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix} = (H_f + V)\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix}$$

$$= H_0\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix} + \frac{\Delta m^2}{4E}\begin{pmatrix}-\cos 2\theta & \sin 2\theta\\ \sin 2\theta & \cos 2\theta\end{pmatrix} + V\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix}$$

No questions allowed

Some simplifications to be done:
- H_0 is a constant
- Subtract V/2×I

$$H_M = \frac{\Delta m_M^2}{4E}\begin{pmatrix}-\cos 2\theta_M & \sin 2\theta_M\\ \sin 2\theta_M & \cos 2\theta_M\end{pmatrix}$$

Oscillations in matter

The Hamiltonian we obtain has the same form as in vacuum, except that we now have *effective* masses and mixing angles

$$H_M = \frac{\Delta m_M^2}{4E} \begin{pmatrix} -\cos 2\theta_M & \sin 2\theta_M \\ \sin 2\theta_M & \cos 2\theta_M \end{pmatrix}$$

$$A = \pm \frac{2\sqrt{2} G_F N_e E}{\Delta m^2}$$

$$\Delta m_M^2 = C \Delta m^2$$

$$\sin 2\theta_M = \frac{\sin 2\theta}{C}$$

$$C = \sqrt{(\cos 2\theta - A)^2 + \sin^2 2\theta}$$

Oscillations in matter

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$$\Delta m_M^2 = C \Delta m^2$$

$$\sin 2\theta_M = \frac{\sin 2\theta}{C}$$

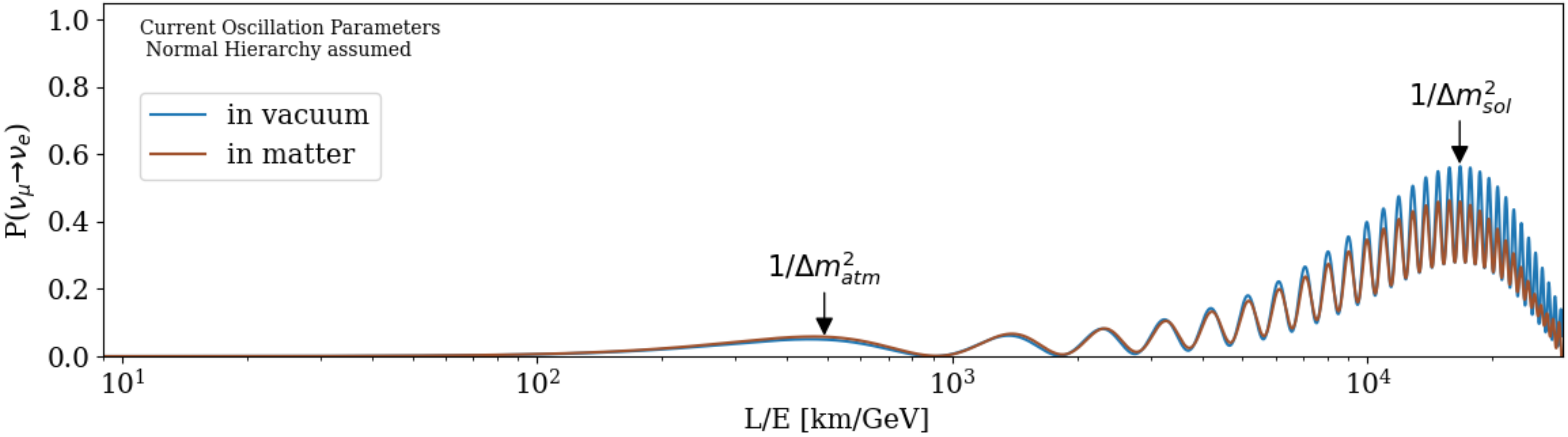
$$C = \sqrt{(\cos 2\theta - A)^2 + \sin^2 2\theta}$$

This implies that in matter, the oscillation probabilities will be modified

With 3 neutrino families, the derivation is more complex but the implications are the same: propagation in matter will modify the oscillation probabilities

Oscillations with 3 flavors in matter

Even earth crust has an impact on neutrino oscillation (denser matter -> stronger effect)



The earth density is not constant -> stronger modification of the neutrino oscillation probability when crossing the earth core

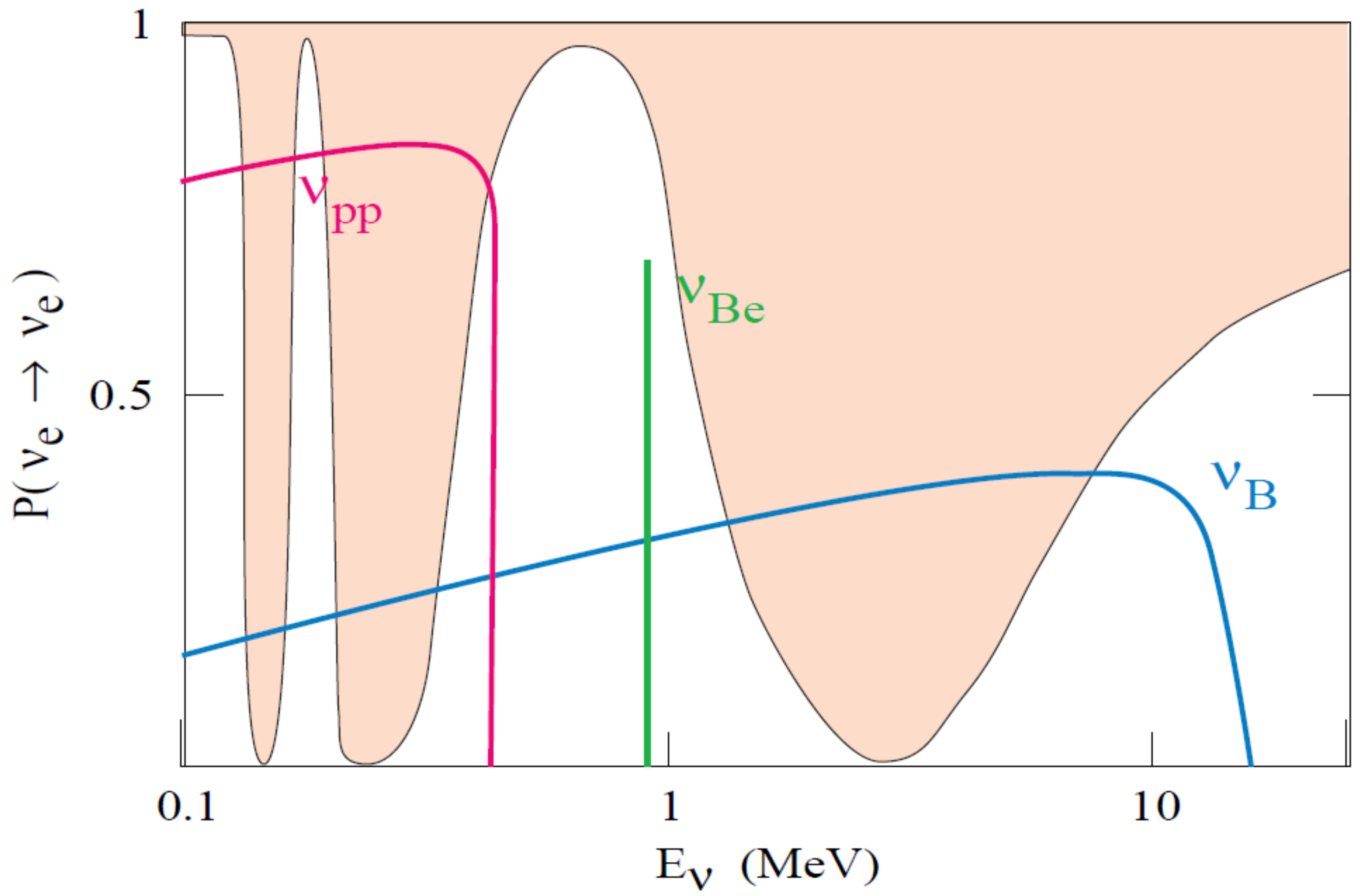
Oscillations parameters

Current values of the oscillation parameters:

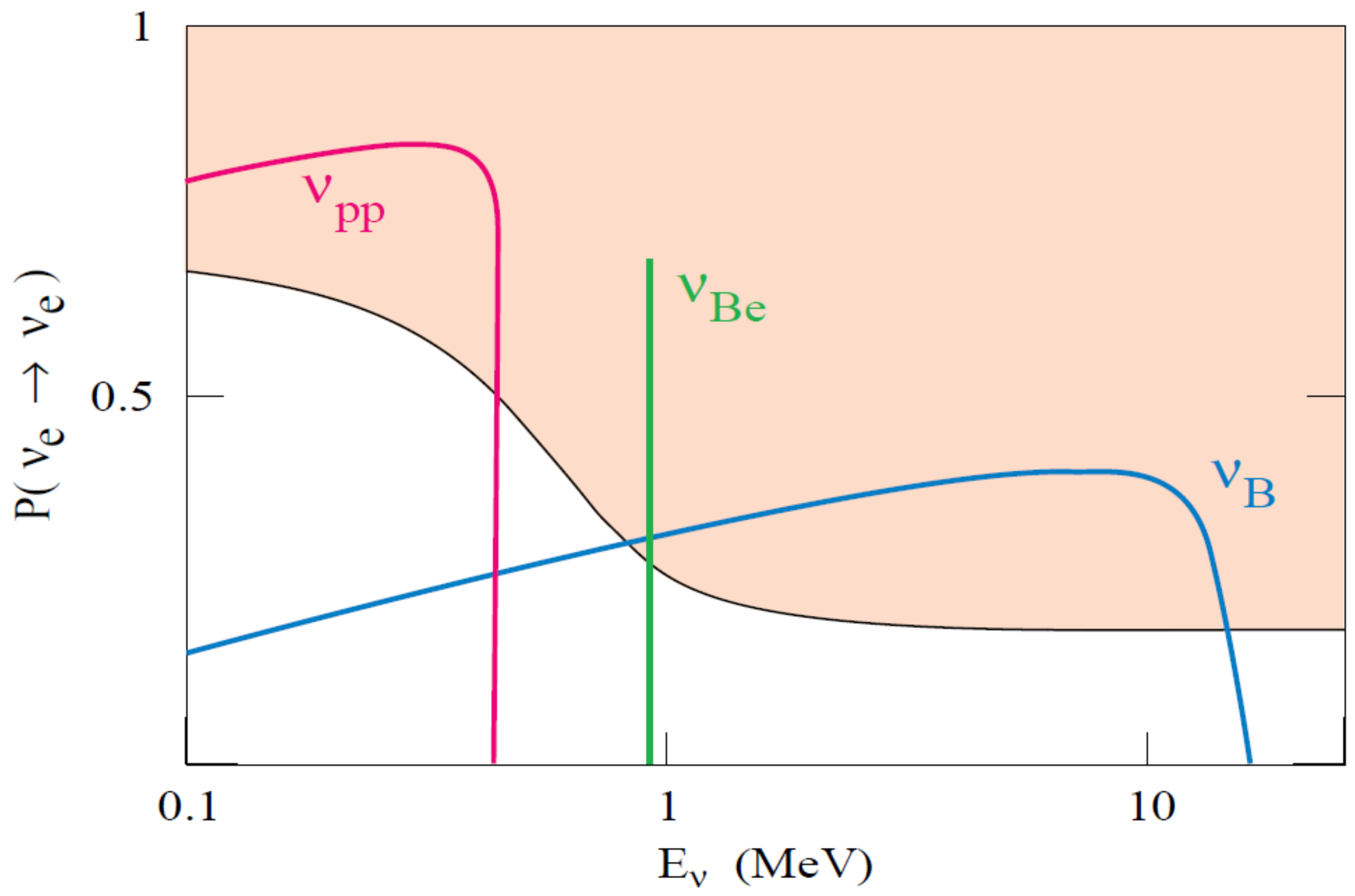
$$\begin{aligned}\theta_{12} &= 33.45^{+0.77}_{-0.75}\\ \theta_{23} &= 42.1^{+1.1}_{-0.9}\\ \theta_{13} &= 8.62^{+0.12}_{-0.12}\\ \Delta m^2_{\mathrm{sol}} = \Delta m^2_{12} &= 7.42^{+0.21}_{-0.20}\times 10^{-5}\ \mathrm{eV}^2\\ |\Delta m^2_{\mathrm{atm}}| = |\Delta m^2_{3\ell}| &= 2.510^{+0.027}_{-0.027}\times 10^{-3}\ \mathrm{eV}^2\end{aligned}$$

- Oscillations in vacuum are not sensitive to the sign of Δm^2
- Matter effects helps to determine Δm^2 sign:
  - $m_2 > m_1$ from solar ν_e
  - Not yet resolved for m_3

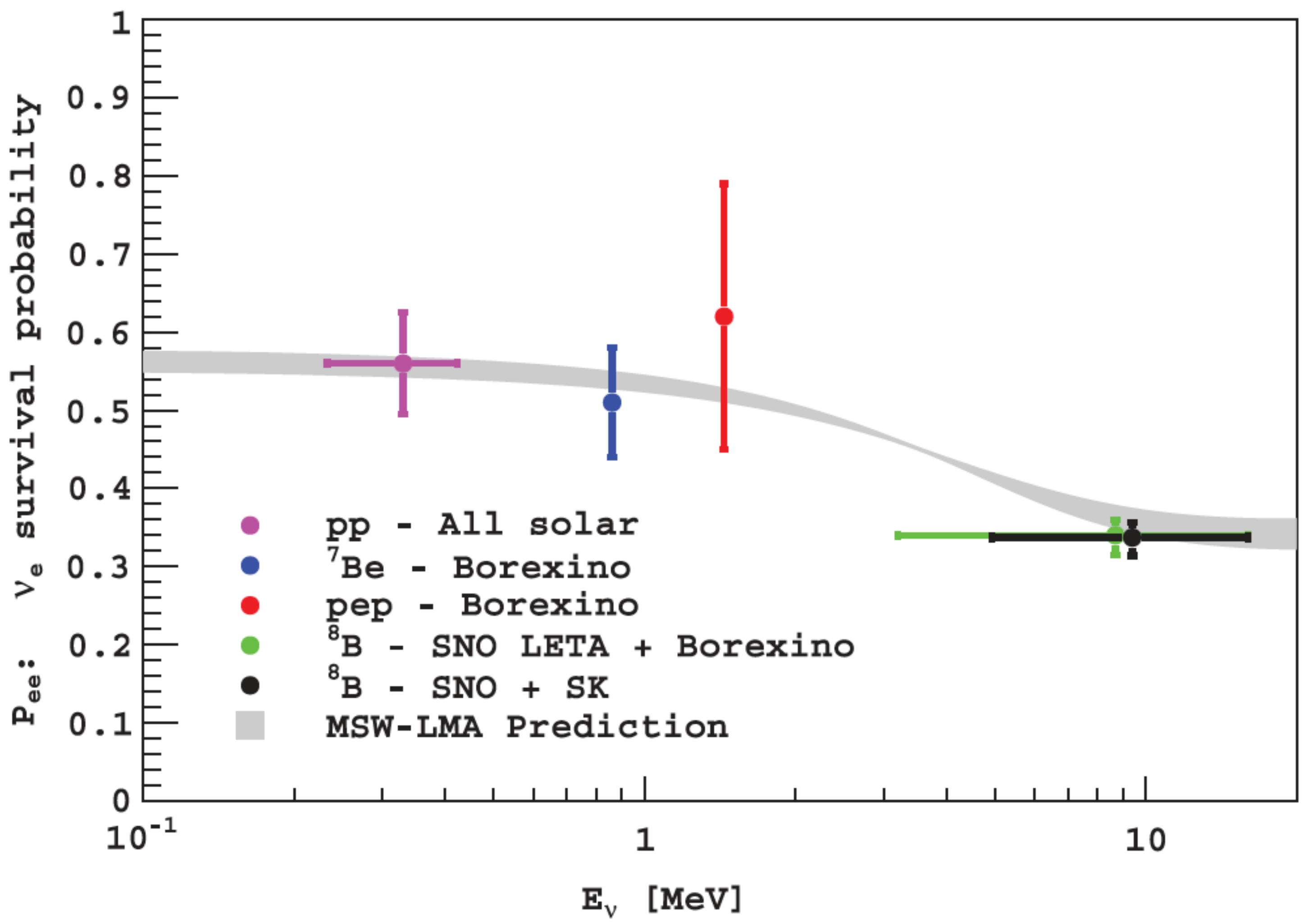
ν_e survival in vacuum



ν_e survival in matter

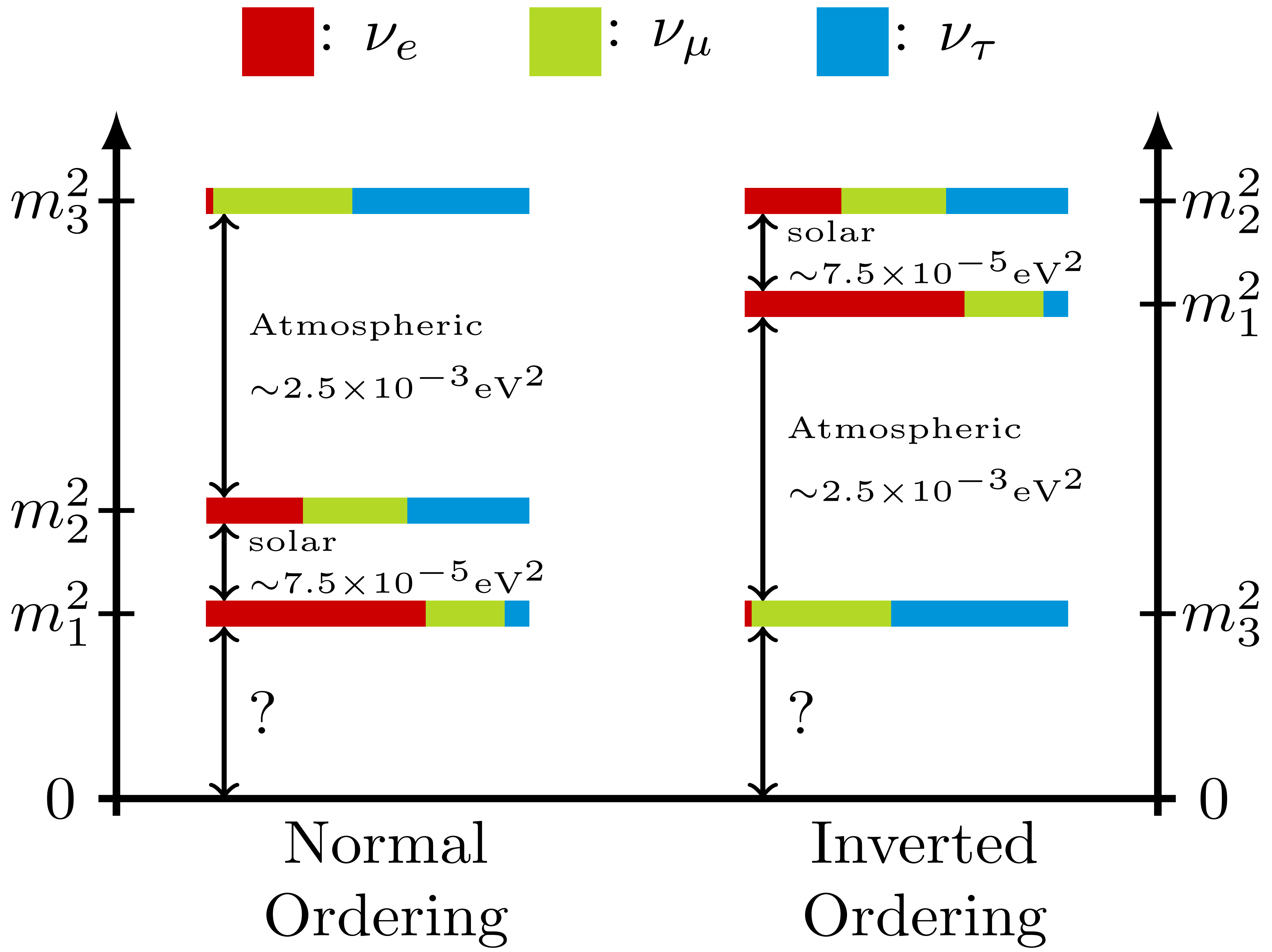


Solar ν_e flux measurement



Oscillations parameters

	Normal Ordering	Inverted Ordering
$\theta_{12} =$	$33.45^{+0.77}_{-0.75}$	$33.45^{+0.78}_{-0.75}$
$\theta_{23} =$	$42.1^{+1.1}_{-0.9}$	$49.0^{+0.9}_{-1.3}$
$\theta_{13} =$	$8.62^{+0.12}_{-0.12}$	$8.61^{+0.14}_{-0.12}$
$\Delta m^2_{\mathrm{sol}} = \Delta m^2_{12} =$	$7.42^{+0.21}_{-0.20} \times 10^{-5}\ \mathrm{eV}^2$	$7.42^{+0.21}_{-0.20} \times 10^{-5}\ \mathrm{eV}^2$
$\Delta m^2_{\mathrm{atm}} = \Delta m^2_{3\ell} =$	$+2.510^{+0.027}_{-0.027} \times 10^{-3}\ \mathrm{eV}^2$	$-2.490^{+0.026}_{-0.028} \times 10^{-3}\ \mathrm{eV}^2$

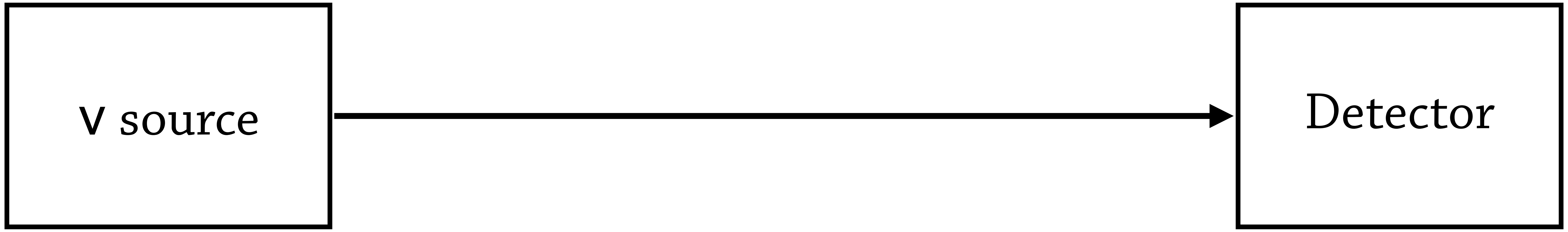


Three unknowns of neutrino oscillations :

- **Mass Hierarchy** : Normal or inverted ?
- **θ_{23} octant** : $\theta_{23} < 45°$ or $\theta_{23} > 45°$?
- **δ_{CP}** : Do ν behaves as $\bar{\nu}$?

CURRENT AND
FUTURE
EXPERIMENTS

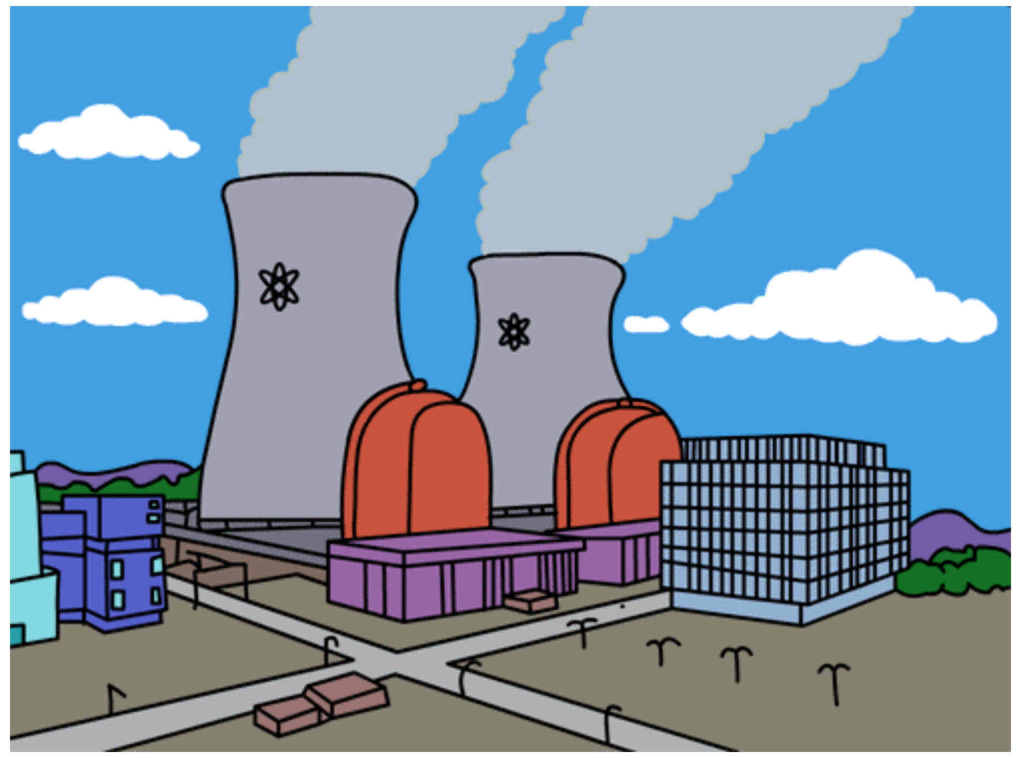
Principle for precision measurement



<u>Requirements :</u>
- Powerful source
- Initial location known
- Initial flavor content known
- Initial energy spectrum known

<u>Requirements :</u>
- At L/E for oscillation
- Able to distinguish e/μ/τ
- Energy reconstruction
- Big and/or dense

Experiments Using Reactors



Continuous powerful emission of $\bar{\nu}_e$ through the fission of ^{235}U, ^{239}Pu and ^{239}Pu.

Energy spectrum from 2 to 8 MeV

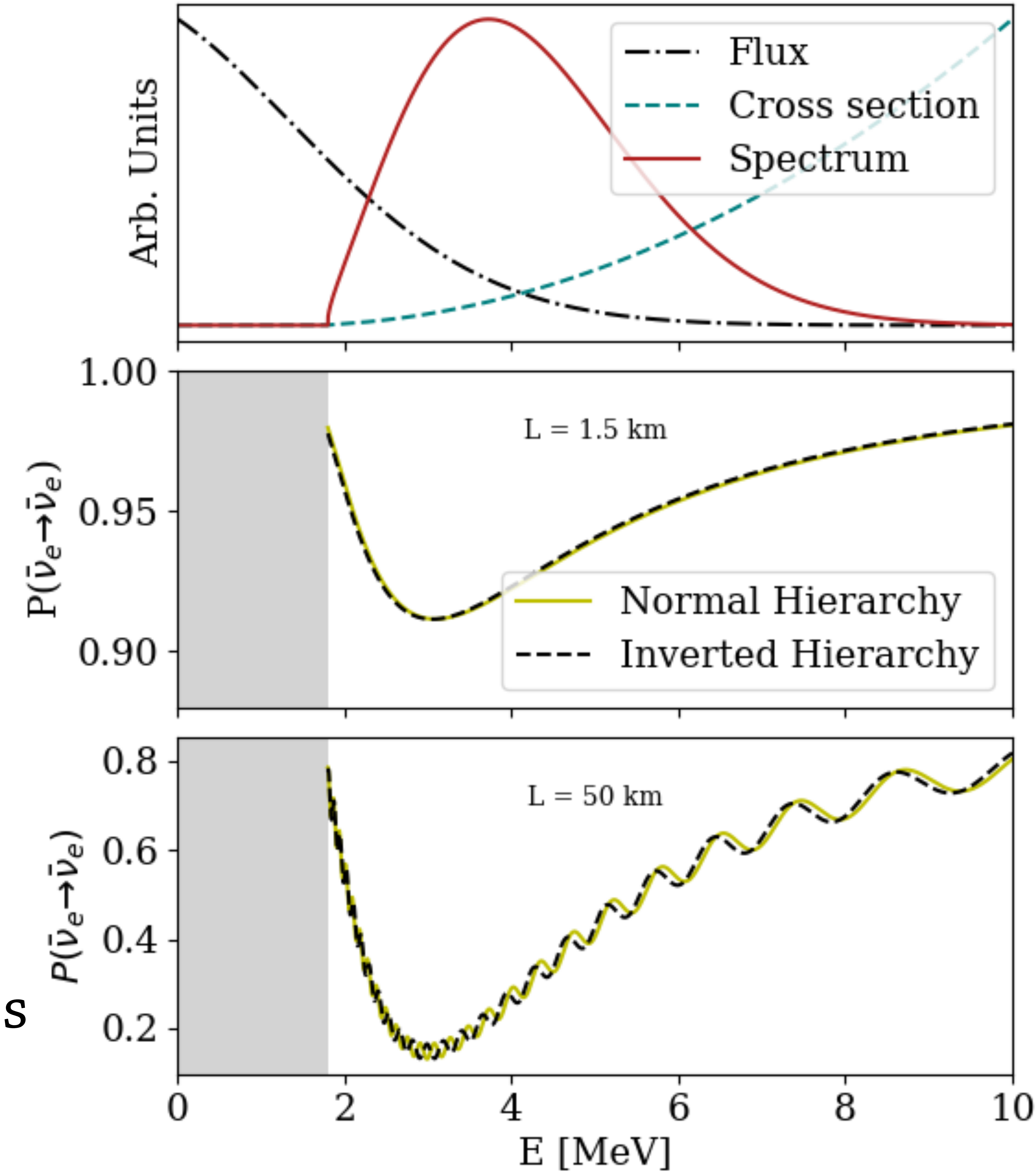
- ○ Cannot tag new flavor appearance ($\bar{\nu}_\mu$ or $\bar{\nu}_\tau$)
- ○ Only the disappearance measurement is possible

L ~ 1 km to be at atmospheric oscillation

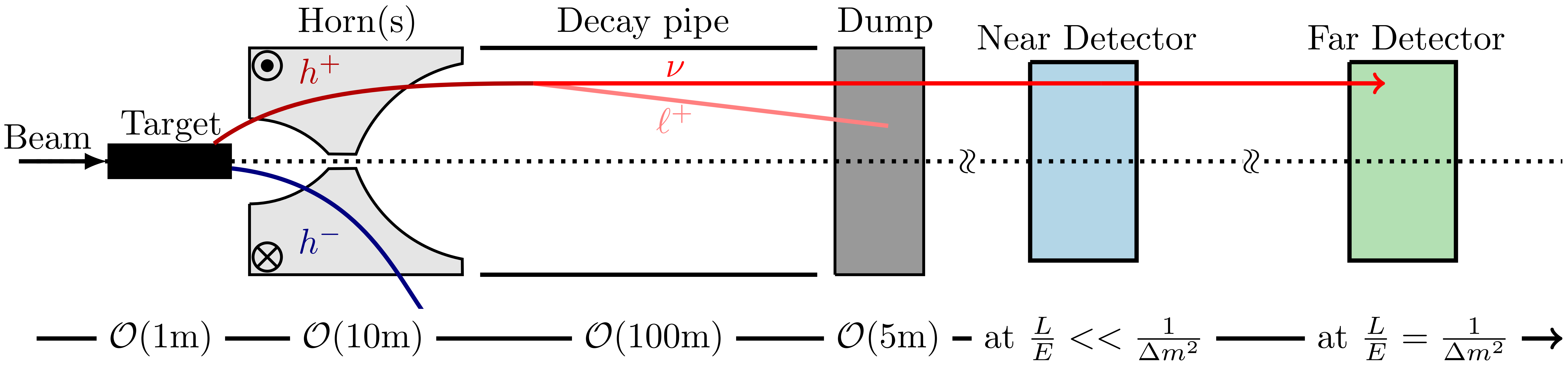
- ○ Considered to be in vacuum, the 2 flavor approximation is valid. No sensitivity to δ_{CP} or MH

L ~ 50 km to be at solar oscillation

- ○ Study of the interference between the 2 oscillations gives sensitivity to MH [***JUNO*** experiment]



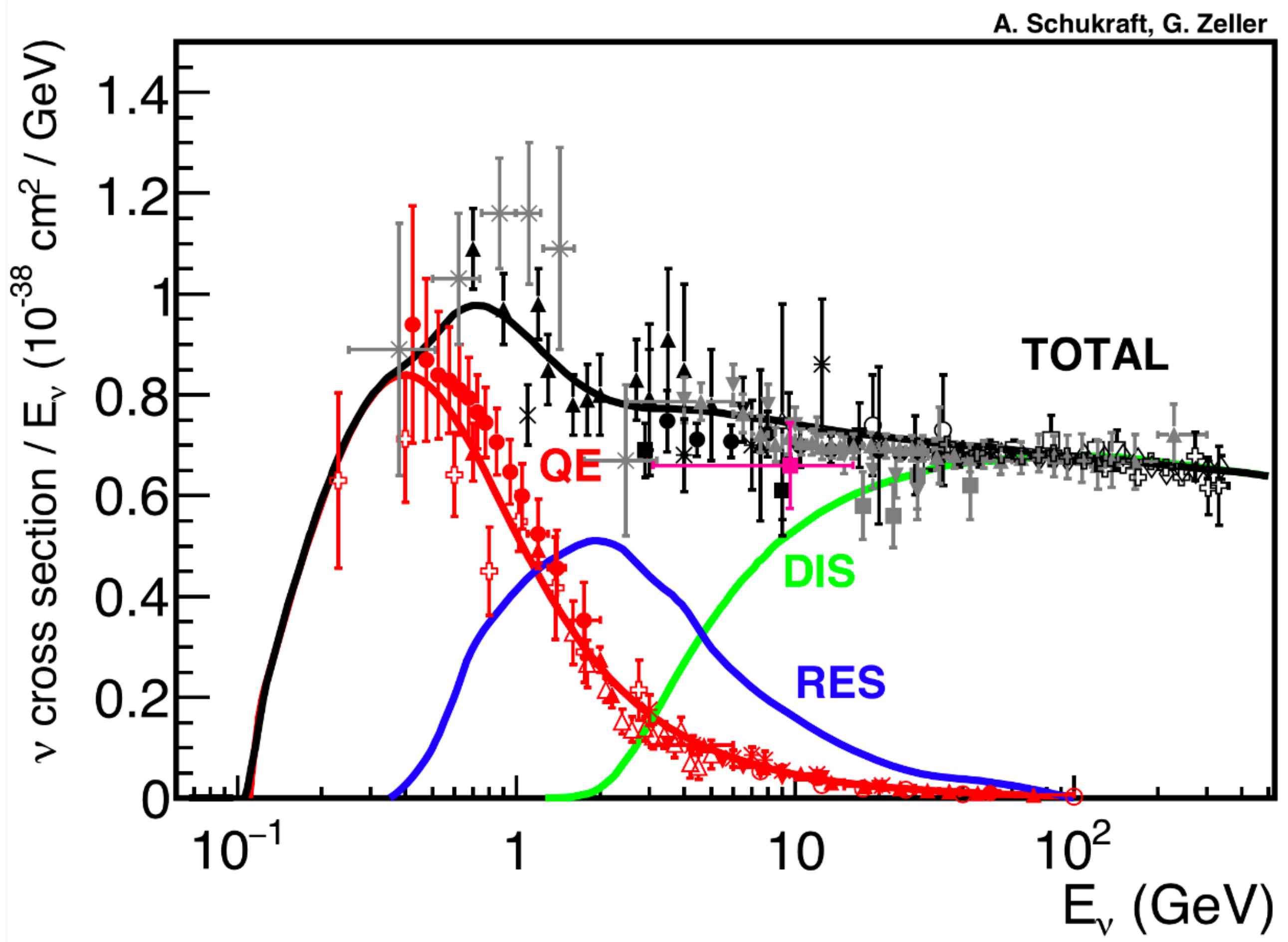
Experiments Using Accelerators



Principle :

- Accelerated proton collides into a target, produces mostly $\pi^{\pm}$.
- Pions main decay channel (99%) : $\pi \rightarrow \mu + \nu_\mu$
- Focussing horns to select π^+ (ν_μ flux) or π^- ($\bar{\nu}_\mu$ flux)
- A near detector to measure the flux *before* oscillations
- A far detector at the L/E to observe oscillations

 - ν beamline parameters tuned for optimal E



Experiments Using Accelerators

Three Channels possible (same for $\bar{\nu}_\mu$):

○ **$\nu_\mu \to \nu_\mu$:**

- No CP violation : $P(\nu_\mu \to \nu_\mu) \equiv P(\bar{\nu}_\mu \to \bar{\nu}_\mu)$
- Negligible matter effects

Experiments Using Accelerators

Three Channels possible (same for $\bar{\nu}_\mu$):

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- Negligible matter effects

○ **$\nu_\mu \rightarrow \nu_e$:** The Golden Channel

- Very sensitive to CP
- Very sensitive to MH with matter
- Very sensitive to θ_{23} octant

Experiments Using Accelerators

Three Channels possible (same for $\bar{\nu}_\mu$):

○ **$\nu_\mu \rightarrow \nu_\mu$:**

- No CP violation : $P(\nu_\mu \to \nu_\mu) \equiv P(\bar{\nu}_\mu \to \bar{\nu}_\mu)$
- Negligible matter effects

○ **$\nu_\mu \rightarrow \nu_e$:** The Golden Channel

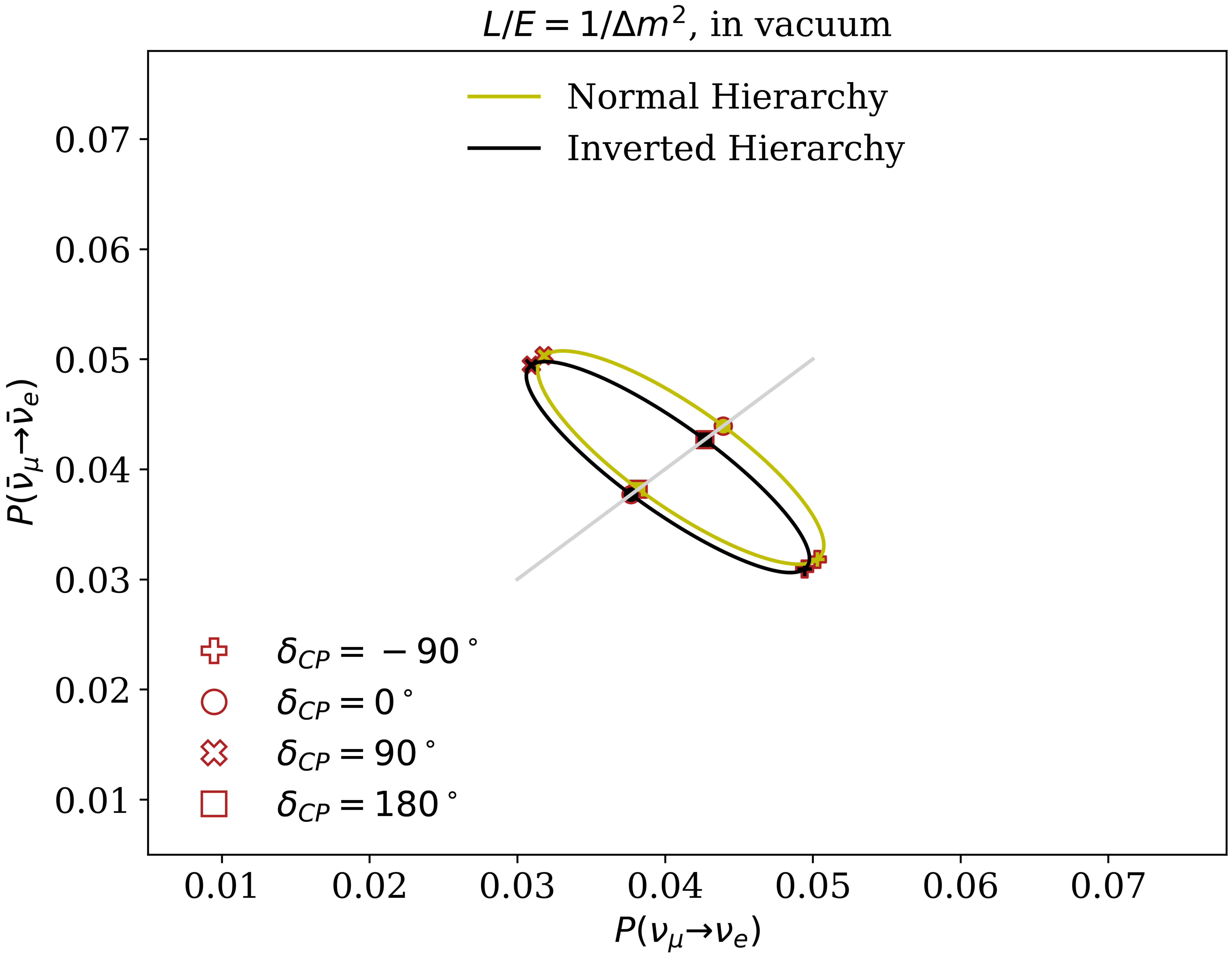
- Very sensitive to CP
- Very sensitive to MH with matter
- Very sensitive to θ_{23} octant

○ **$\nu_\mu \rightarrow \nu_\tau$:**

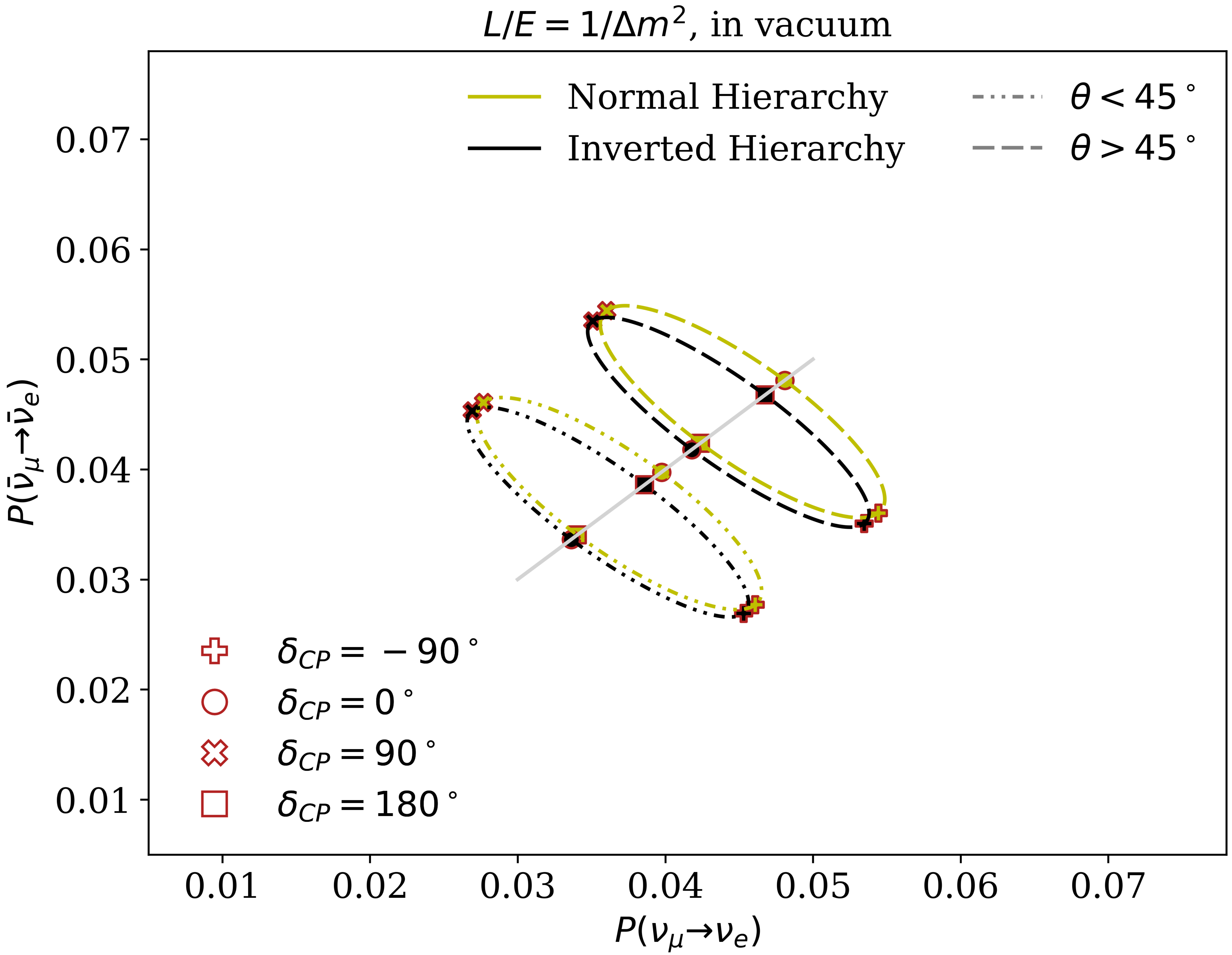
- Similar discovery potential as ν_e appearance but:

$m_\tau = 1.7$ GeV, $c\tau_\tau = 87$ μm and $\tau^\pm$ have hundreds of complicated decay channels

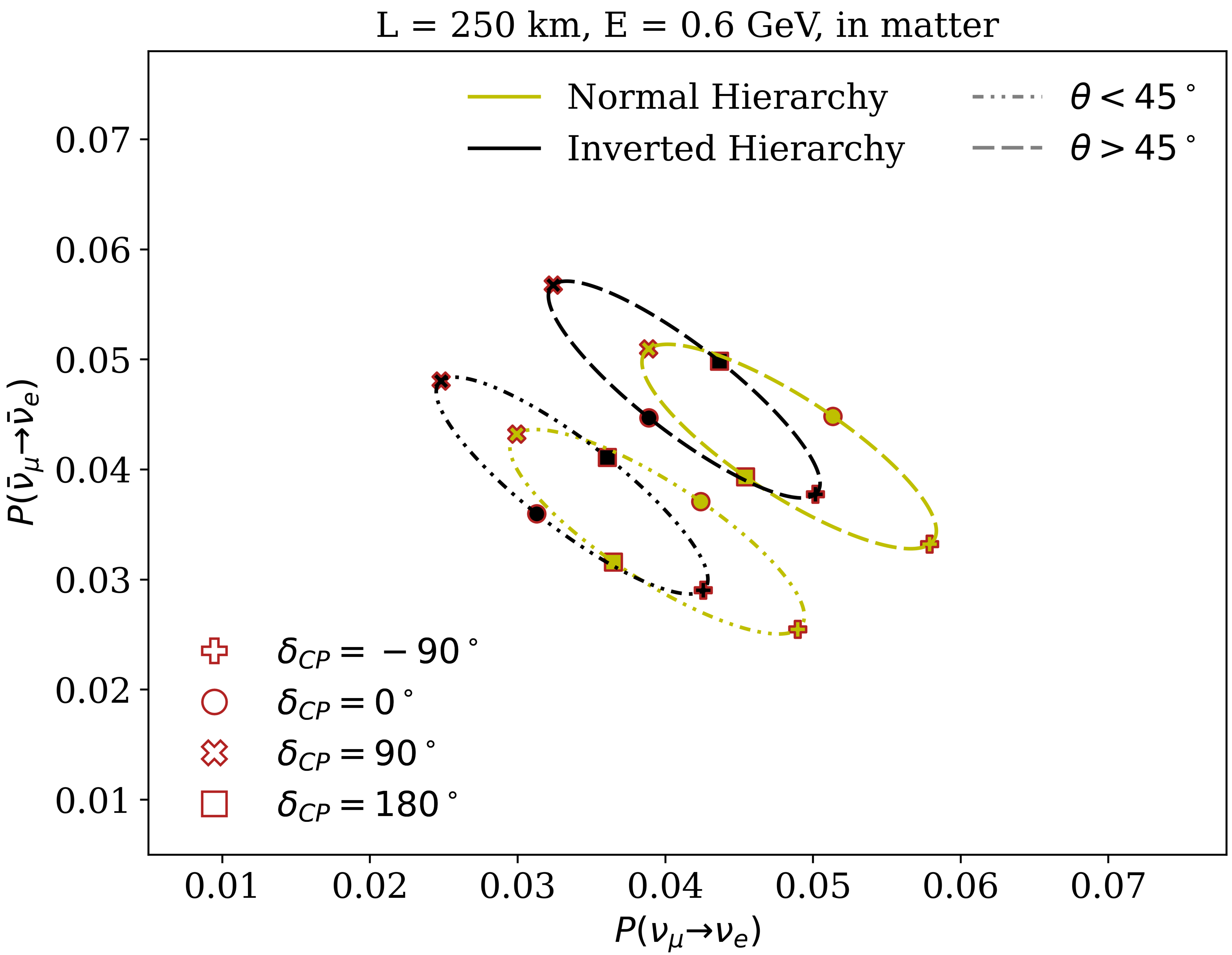
Experiments Using Accelerators



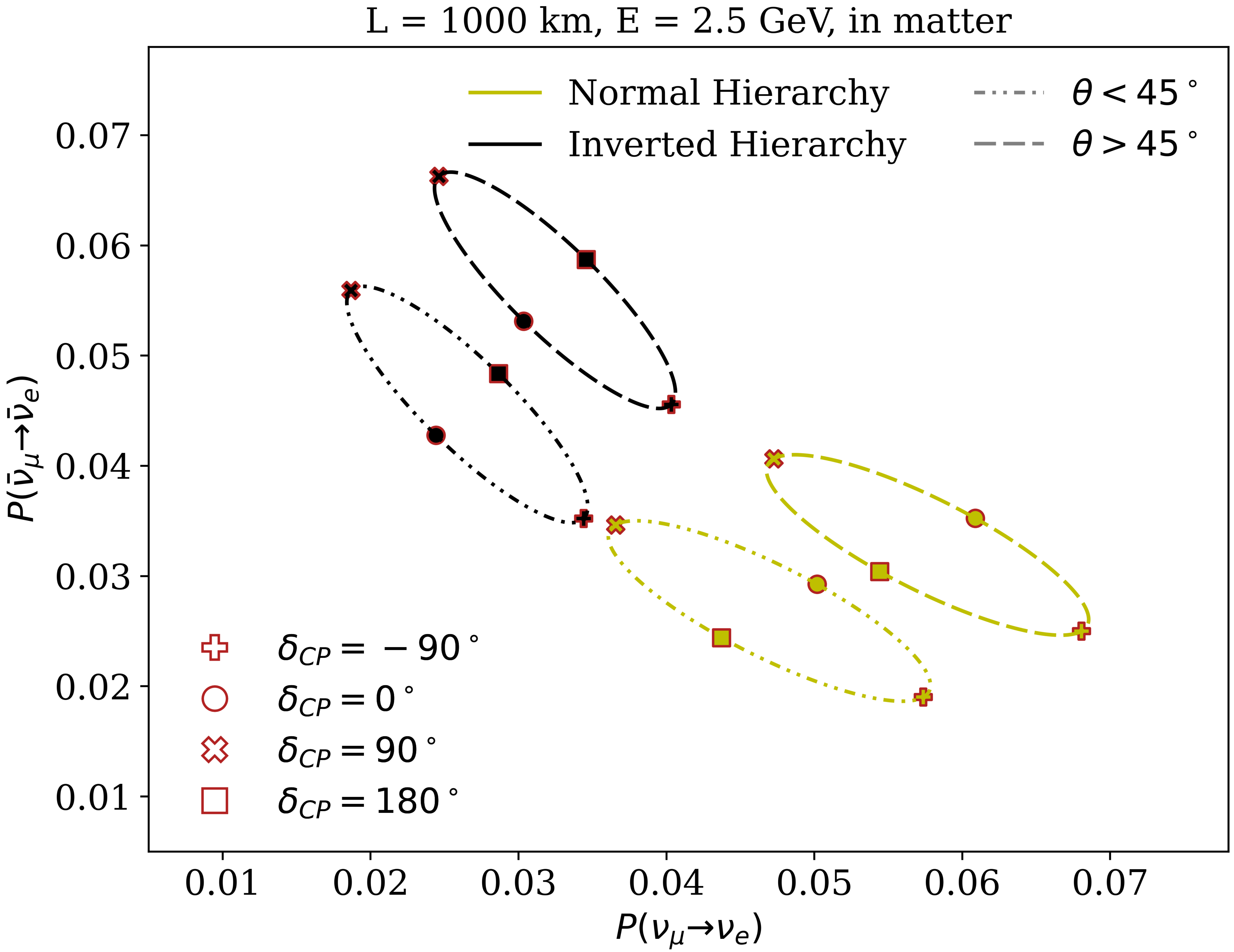
Experiments Using Accelerators



Experiments Using Accelerators

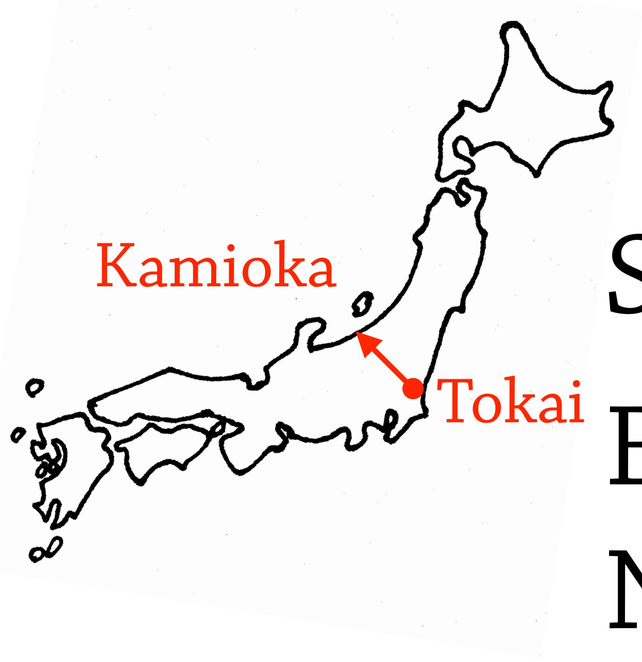


Experiments Using Accelerators



Current ν accelerator experiments

T2K in Japan

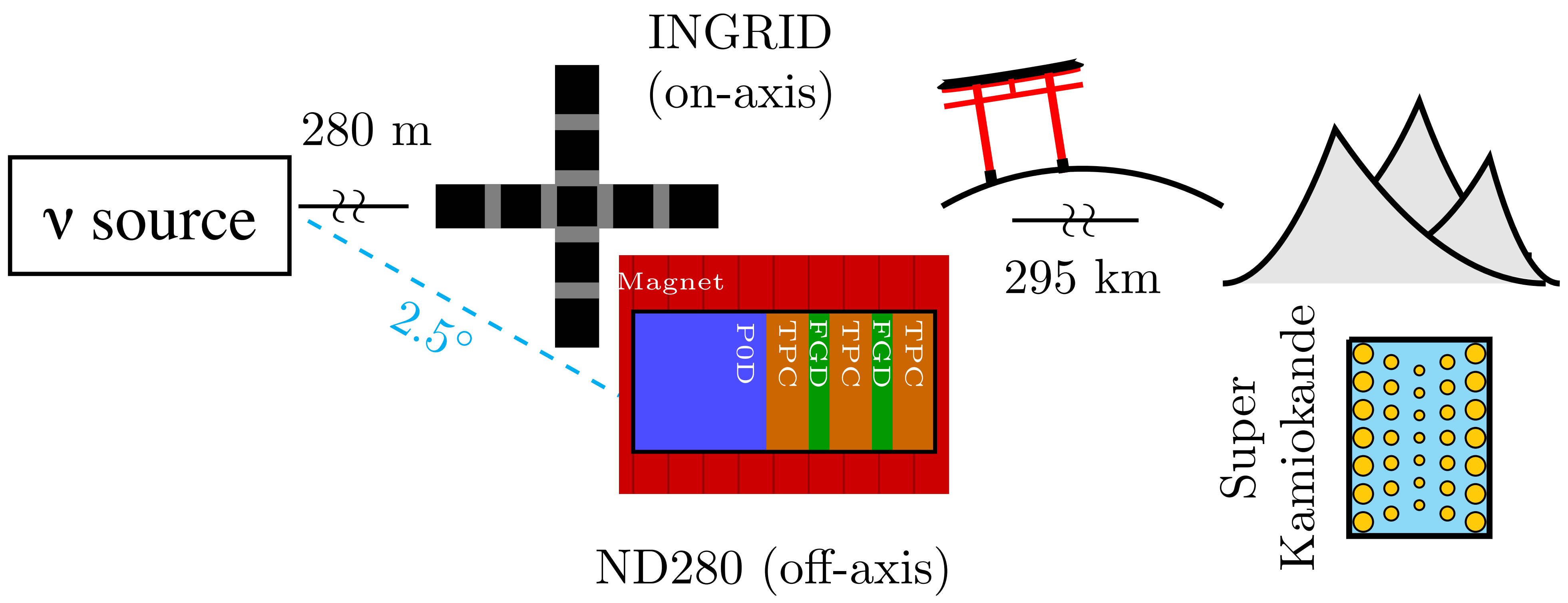


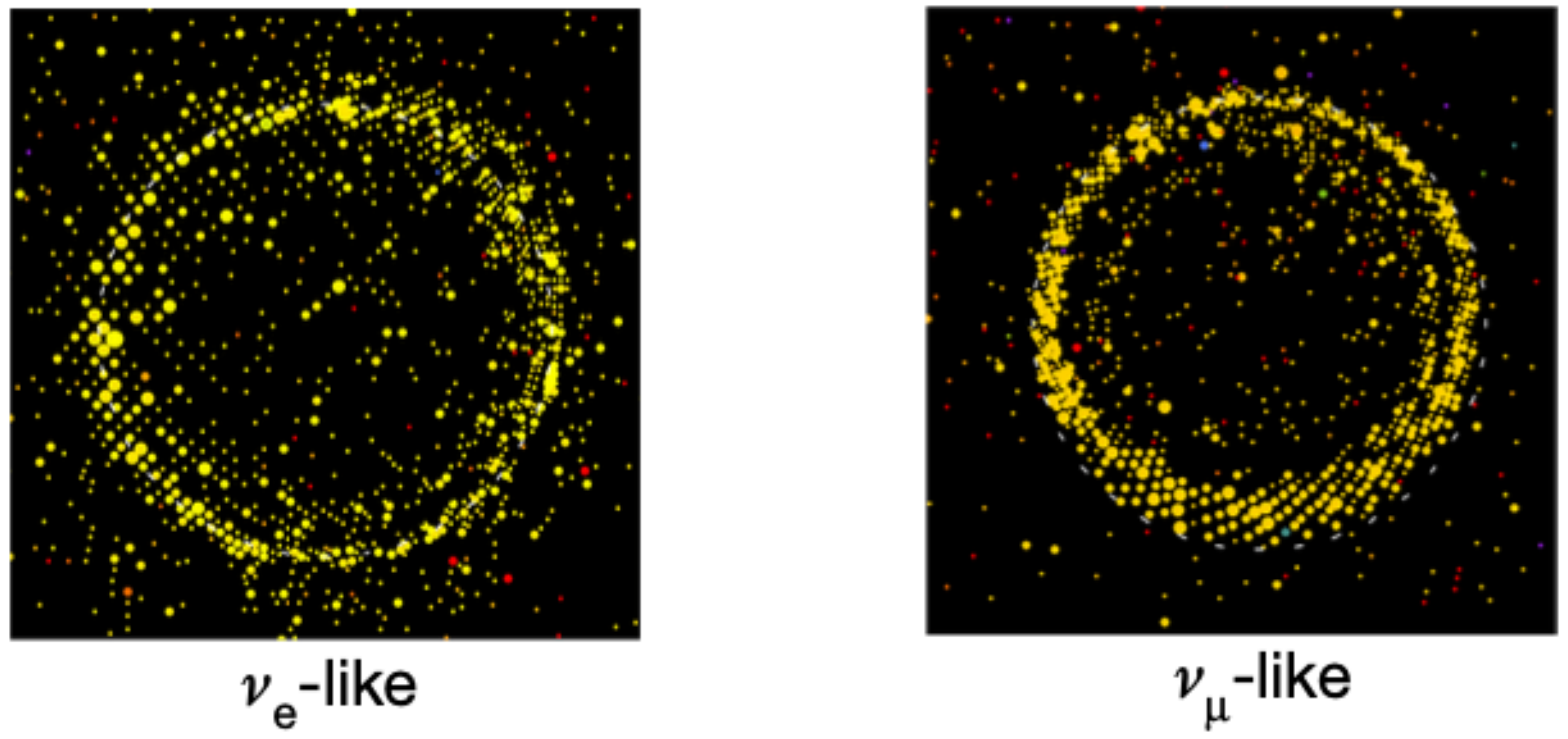
Since 2010, L = 295 km, E = 0.6 GeV

Equal ν and $\bar{\nu}$ runs

Near detector is a gaseous TPC

Far detector is Super-Kamiokande





ν_e-like

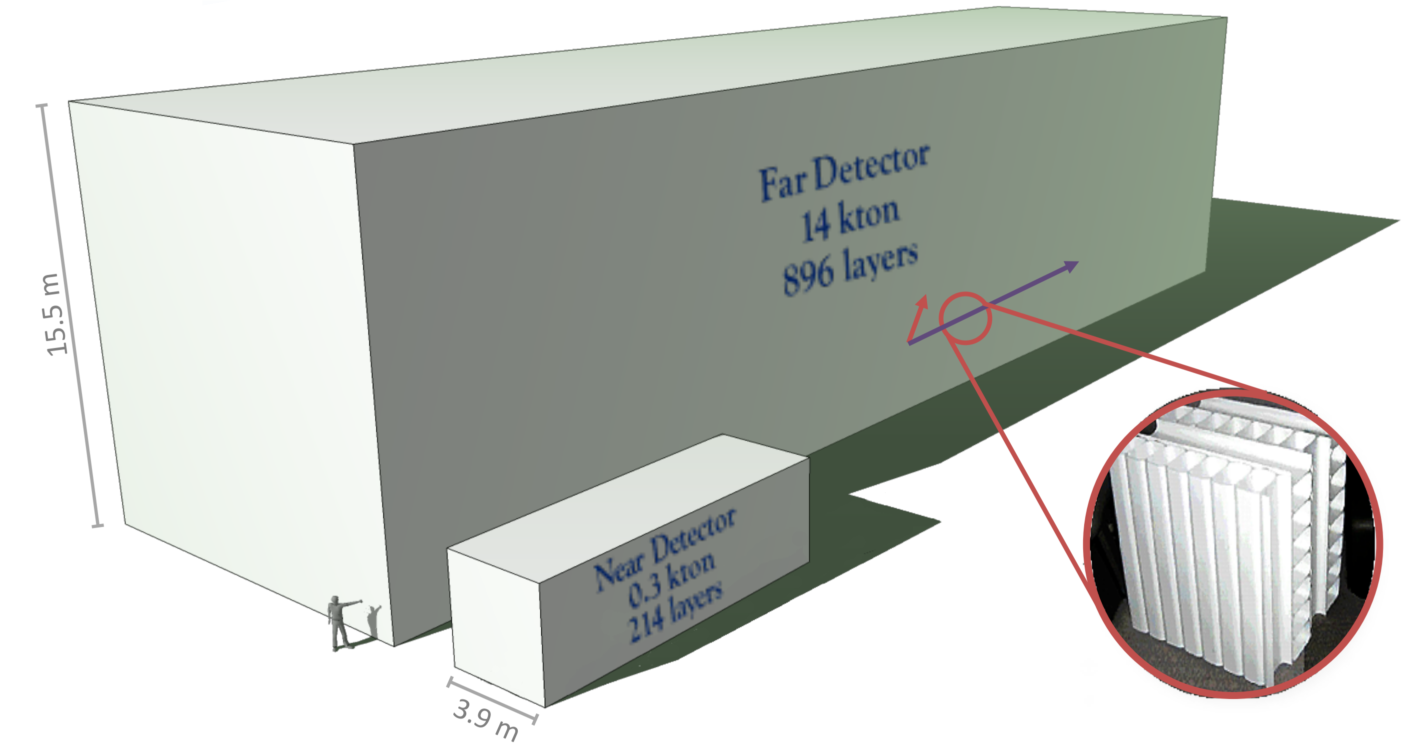
ν_μ-like

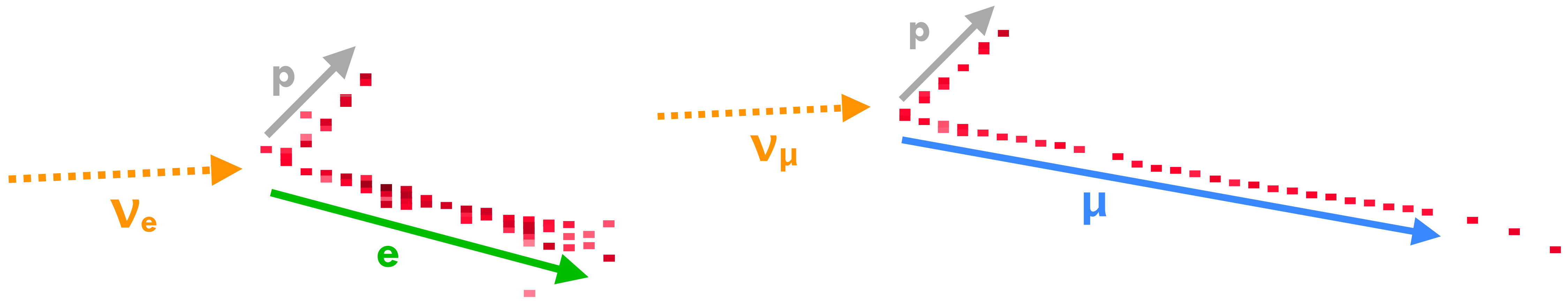
NOνA in the US

Since 2013, L = 810 km, E = 2 GeV

Equal ν and $\bar{\nu}$ runs

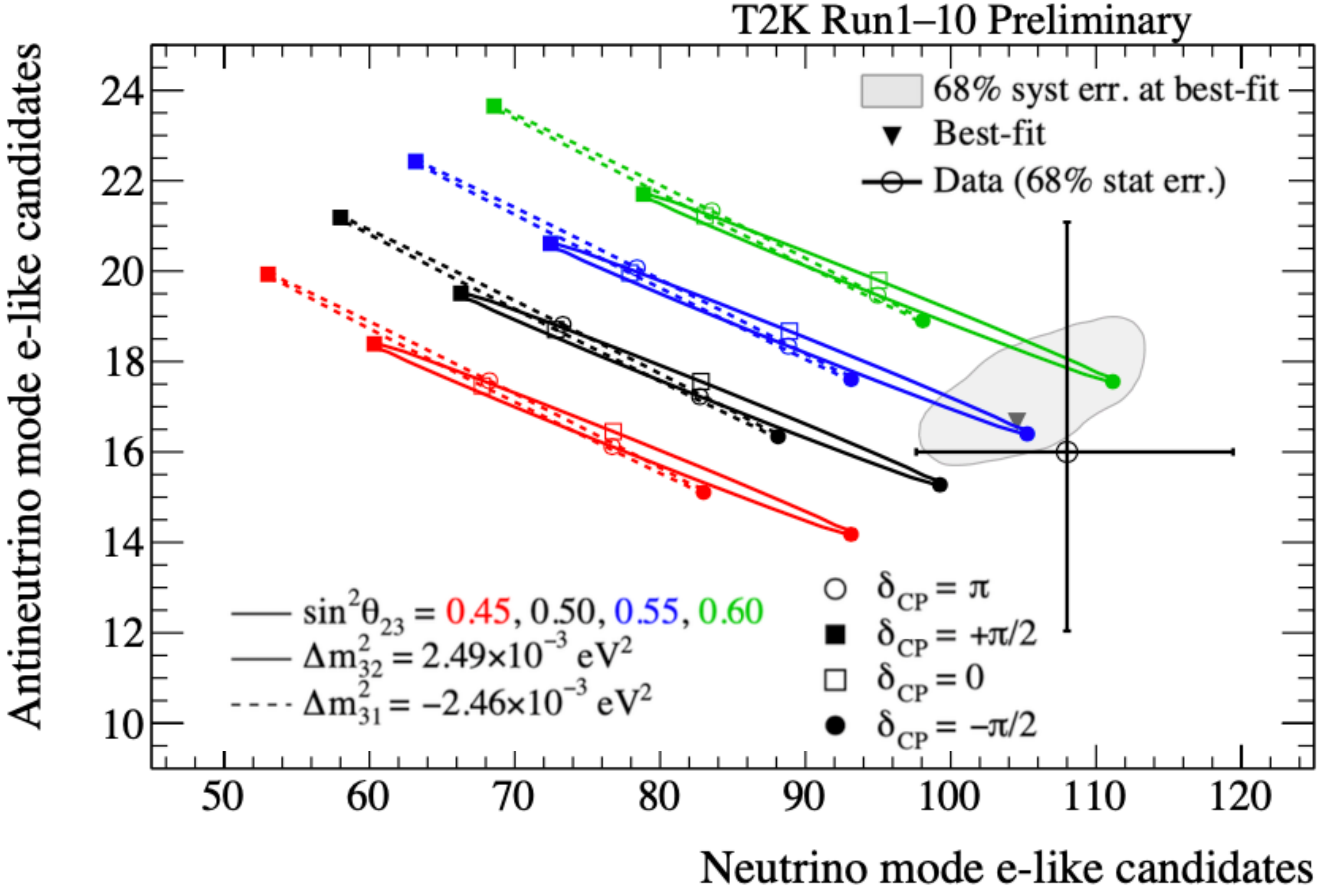
Near and Far detectors are plastic scintillators



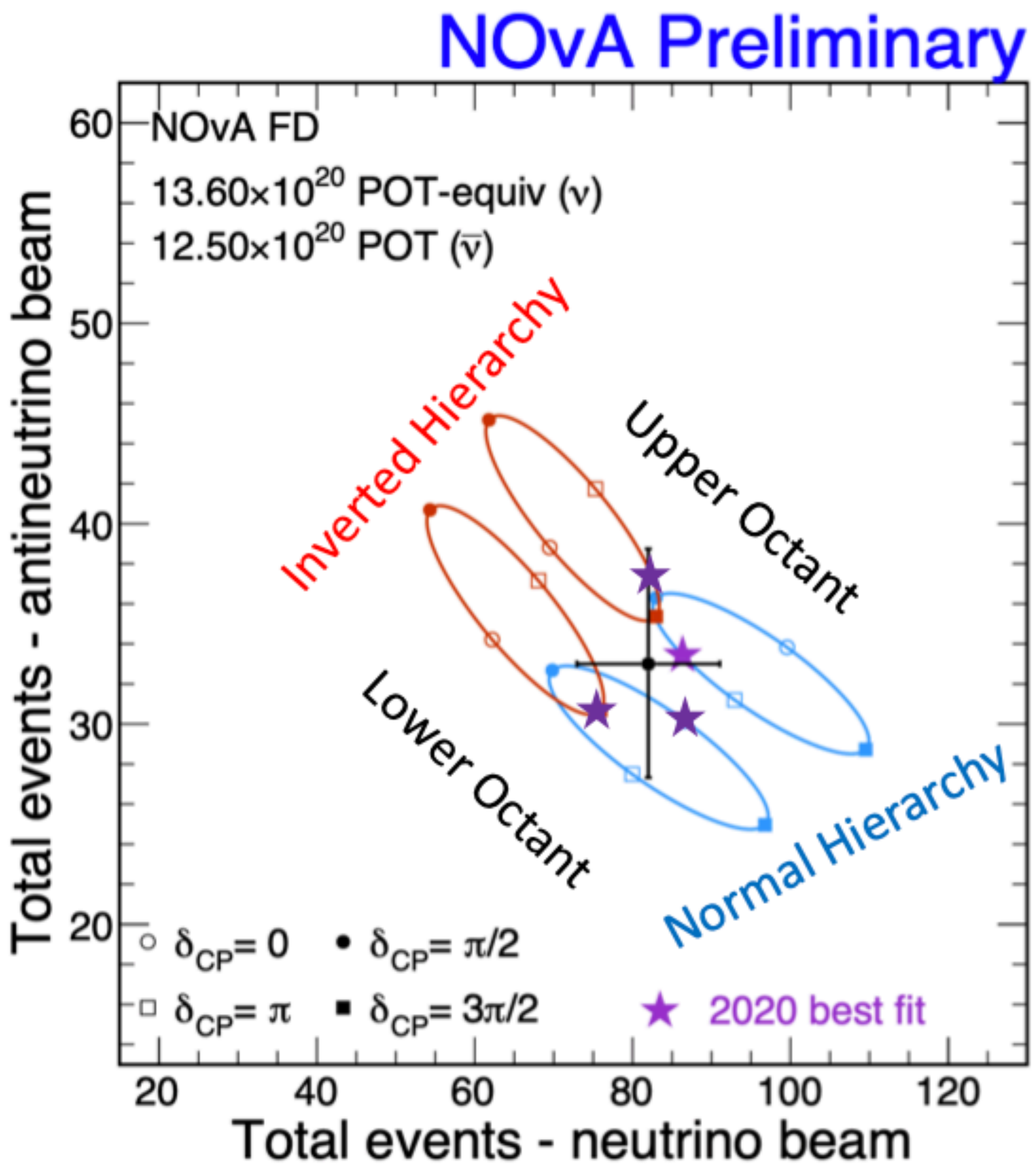


Current ν accelerator experiments

T2K in Japan

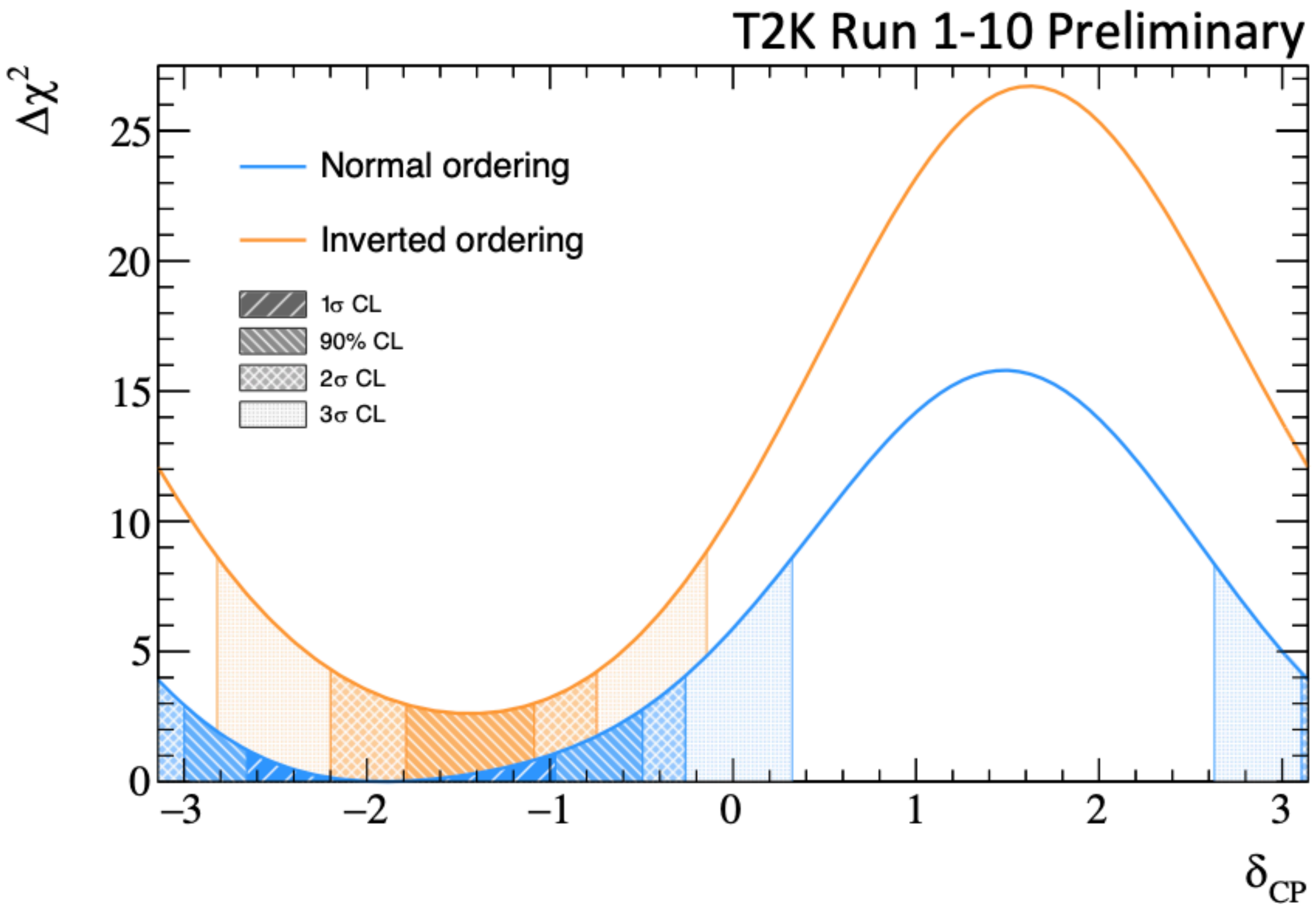


NOνA in the US



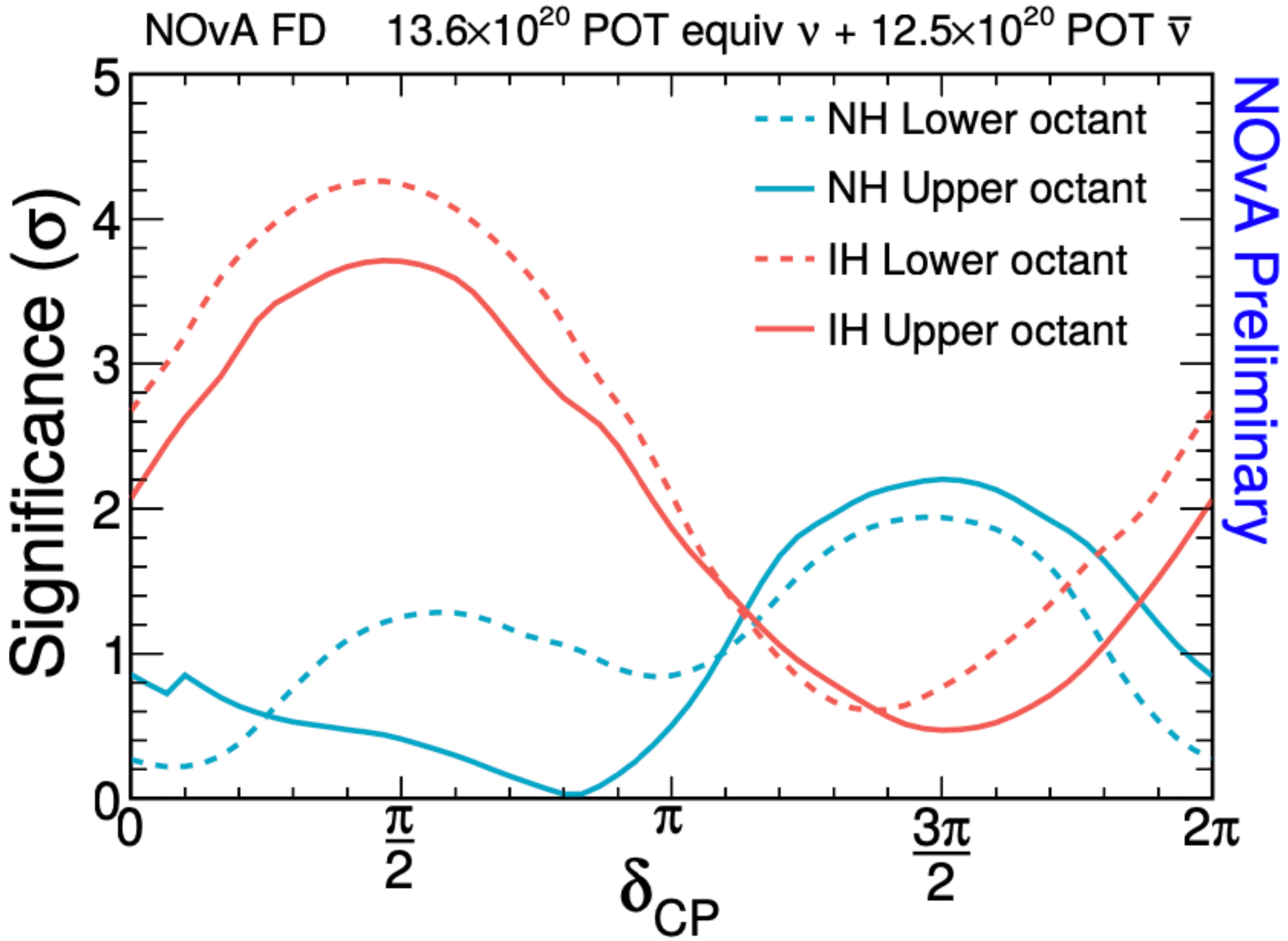
Current ν accelerator experiments

T2K in Japan



- ○Slight preference for Normal Hierarchy
- ○δ_{CP} = (0, π) excluded at 95% C.L. for both MH
- ○Large range around δ_{CP} = +π/2 excluded at 3σ

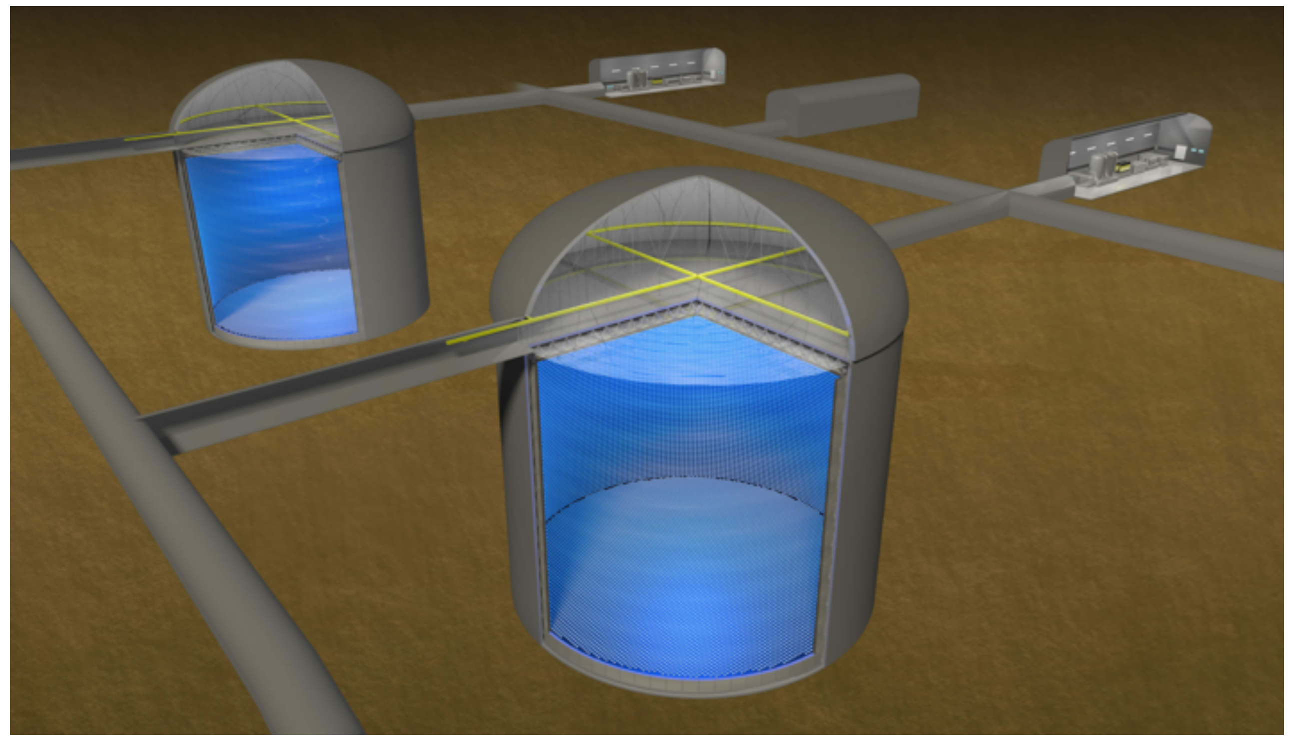
NOvA in the US



- ○Prefers Normal Hierarchy at 1.0σ
- ○Exclude δ_{CP} = π/2 + IH at >3σ
- ○Exclude δ_{CP} = 3π/2 + NH at 2σ

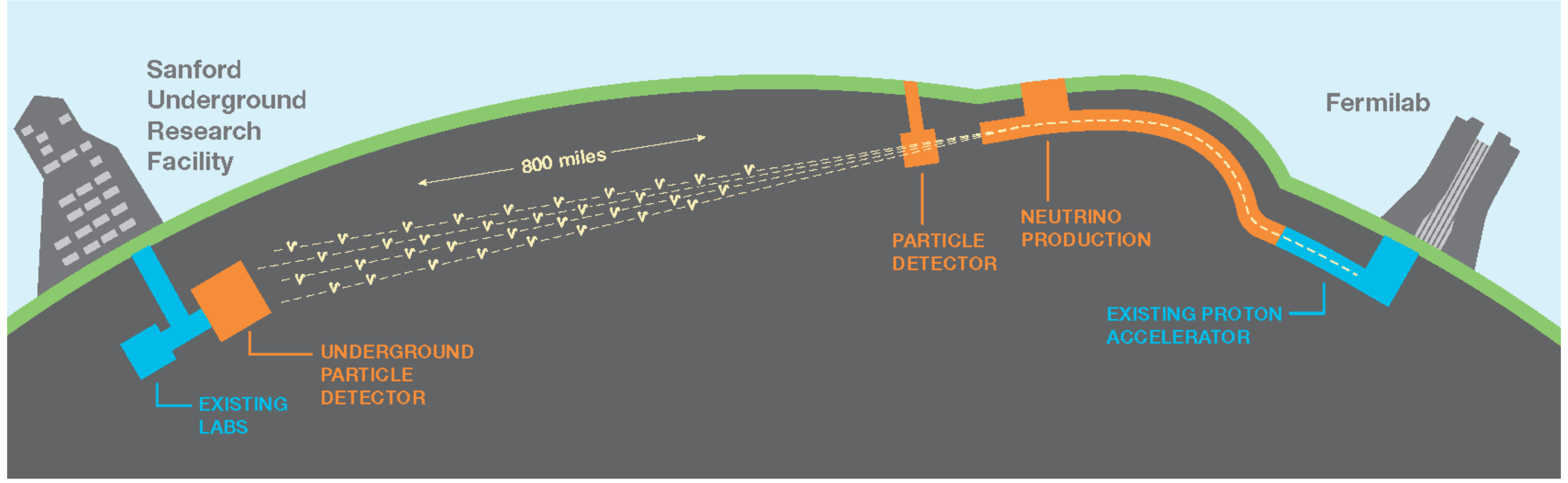
Future ν accelerator experiments

T2HK in Japan



- L=300 km, E ~ 0.6 GeV
- 260 kt water Cherenkov detector
- Proven and scalable technology
- Excellent e-μ ring separation
- Little R&D foreseen
- Only low energy beam possible (< 1 GeV)

DUNE in the US

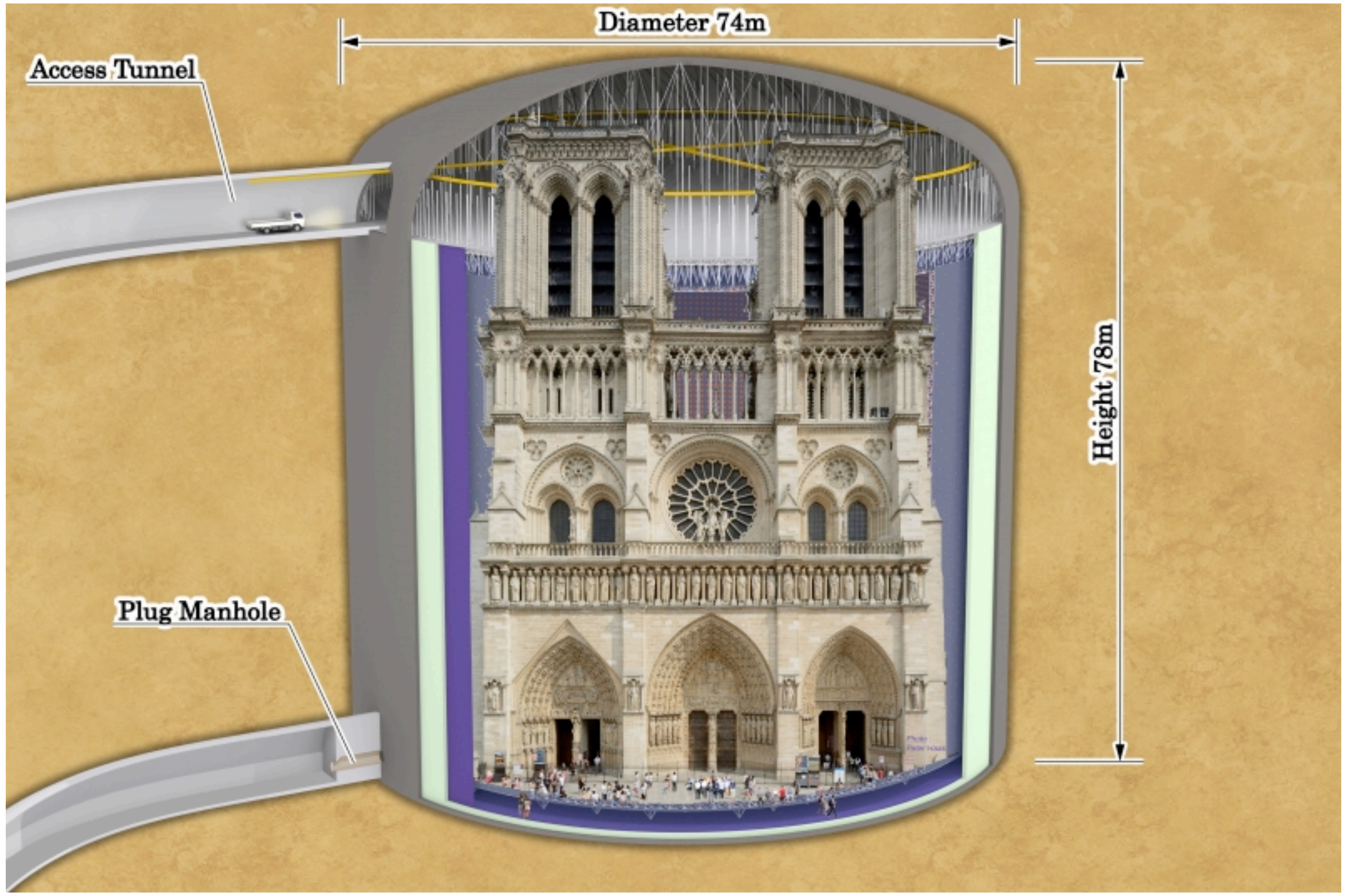


- L=1300 km, E ~ 1-3 GeV
- 40 kt liquid argon TPC detector
- 3D imaging with high granularity for precise tracking
- Low energy threshold (~10s MeV)
- Important R&D efforts ongoing : Scalability, Engineering

Both planning of starting data taking in ~2027

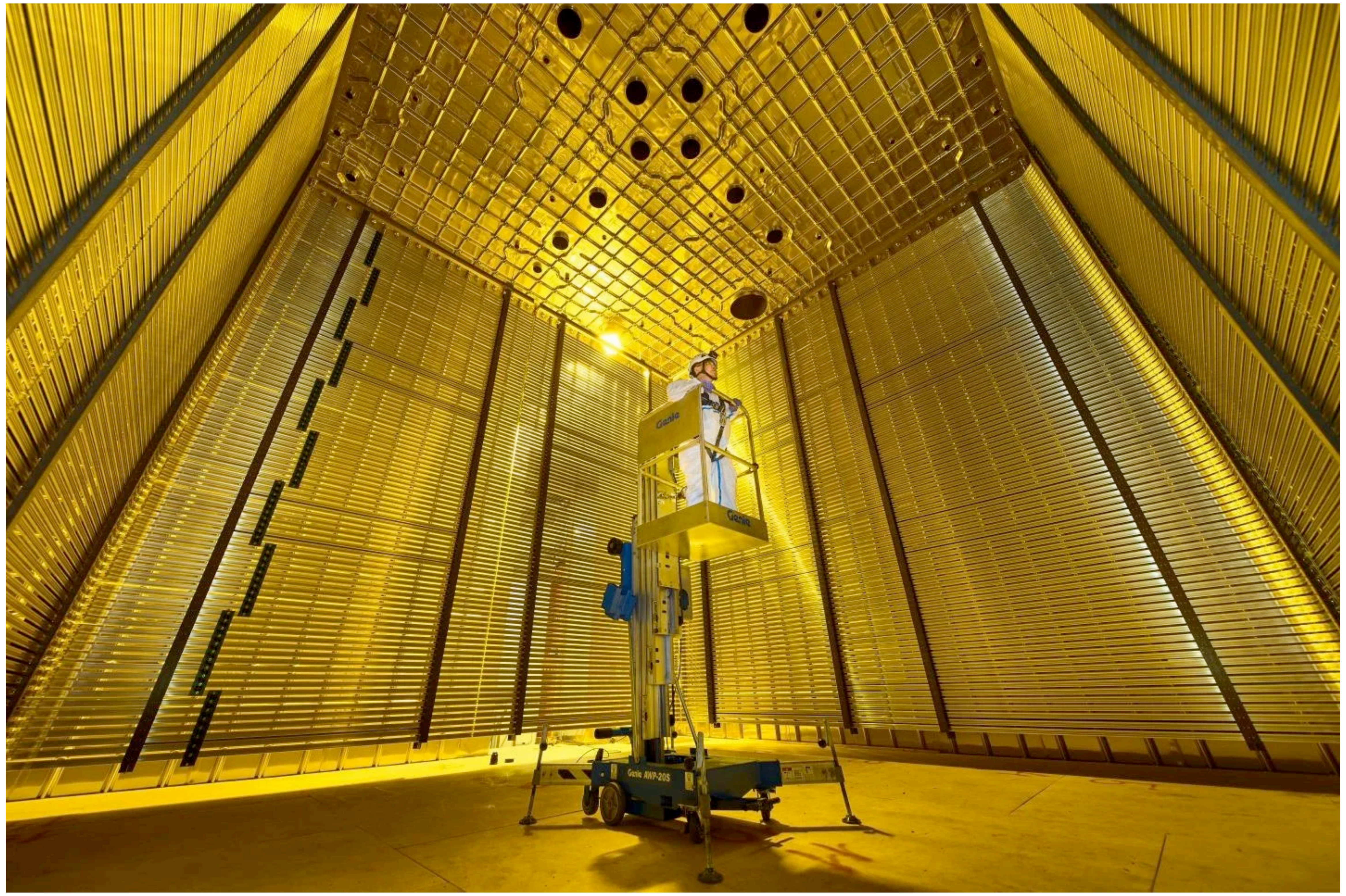
Future ν accelerator experiments

T2HK in Japan



Notre-Dame will fit inside Hyper-Kamiokande !

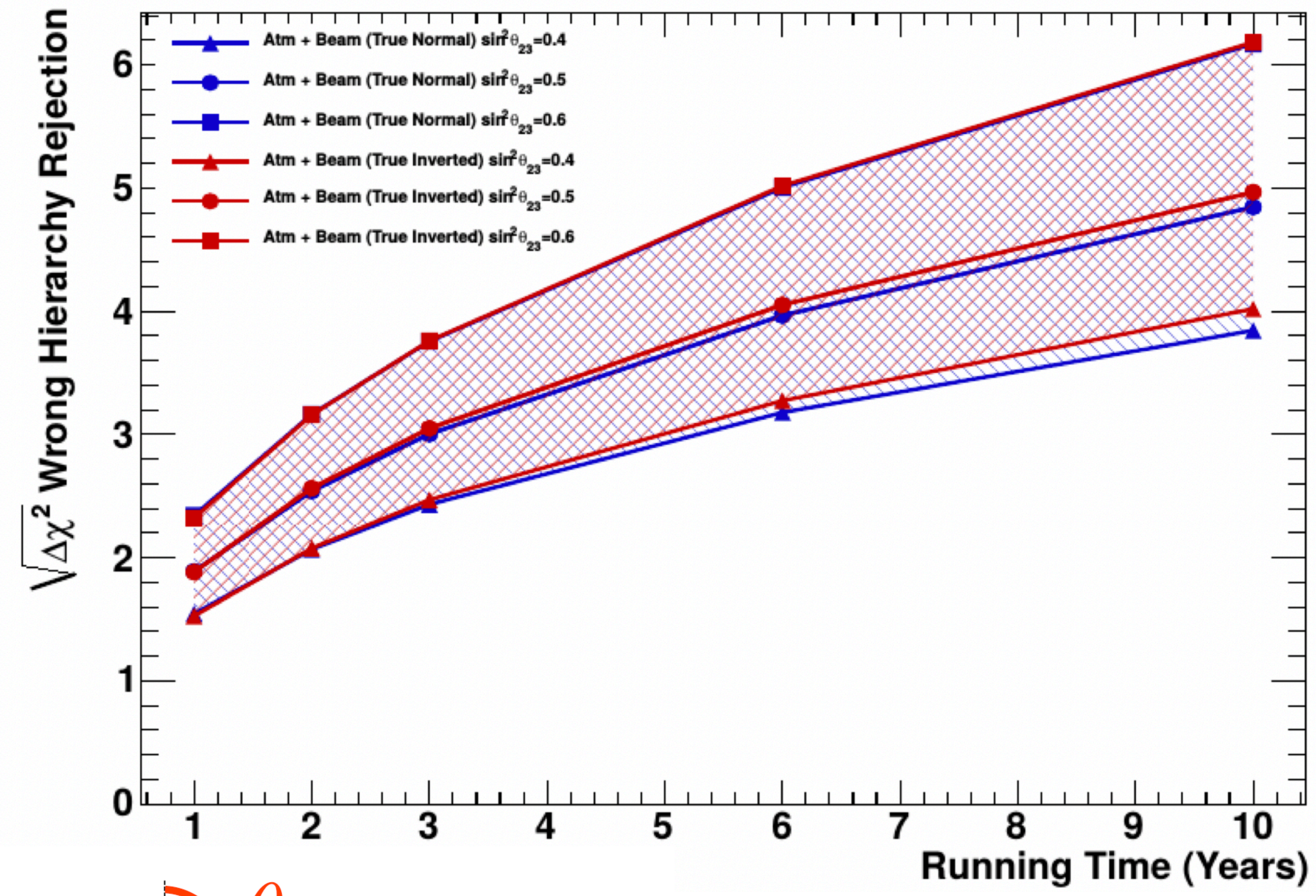
DUNE in the US

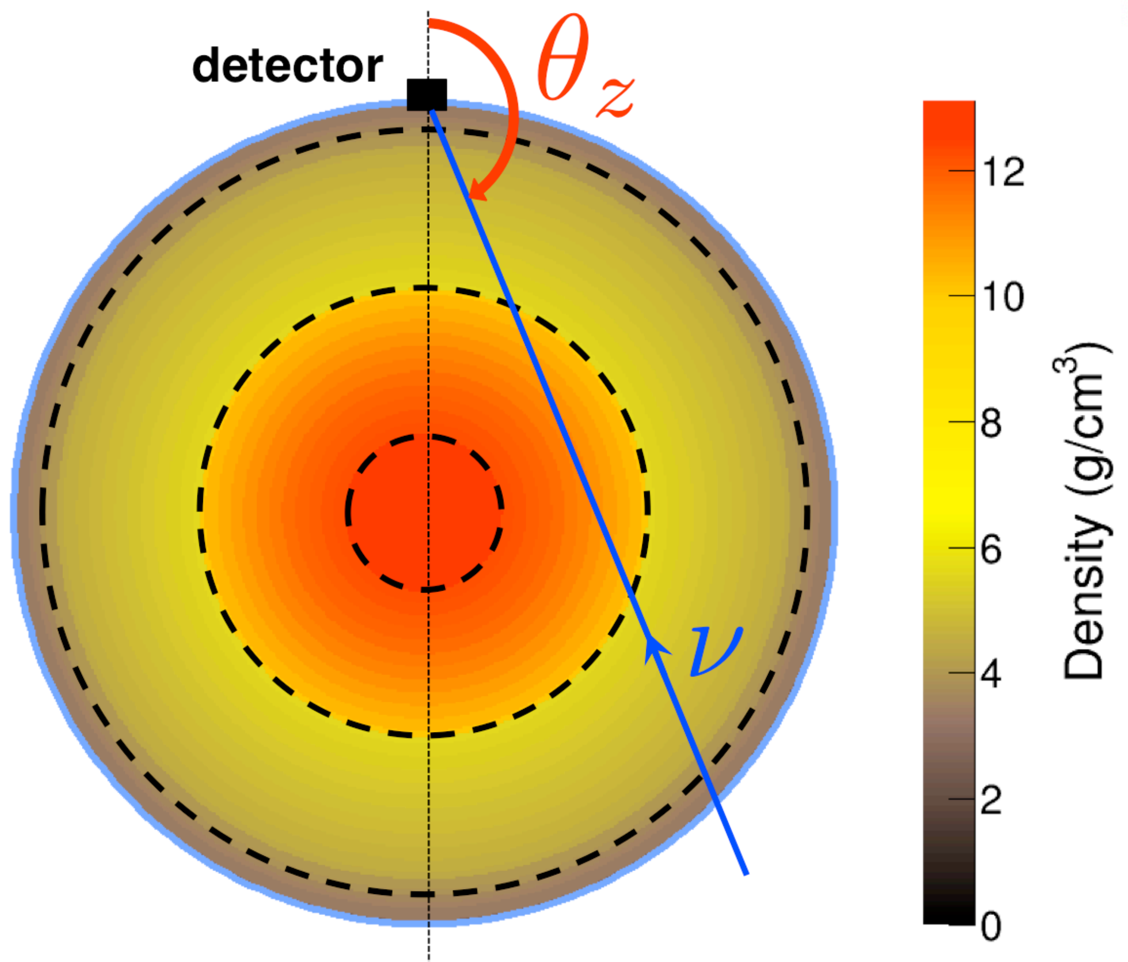


Inside DUNE prototype (6×6×6 m^3) at CERN
-> Future : 4 modules of 60×12×12 m^3 each

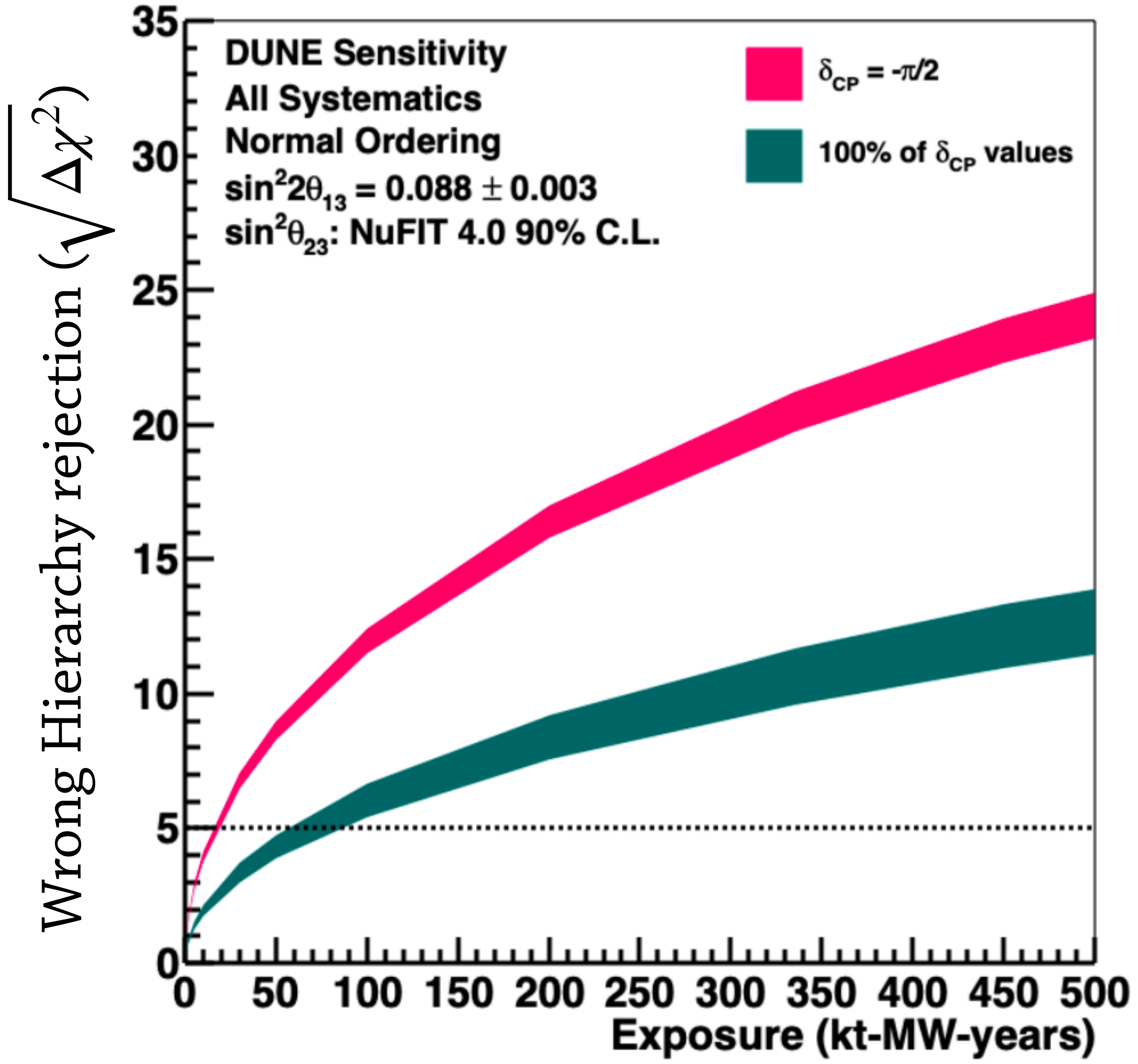
Future ν accelerator experiments

T2HK in Japan





DUNE in the US



DUNE default operation :
40 kton of LAr staged
Beam power at 1.2 ~ 2.4 MW

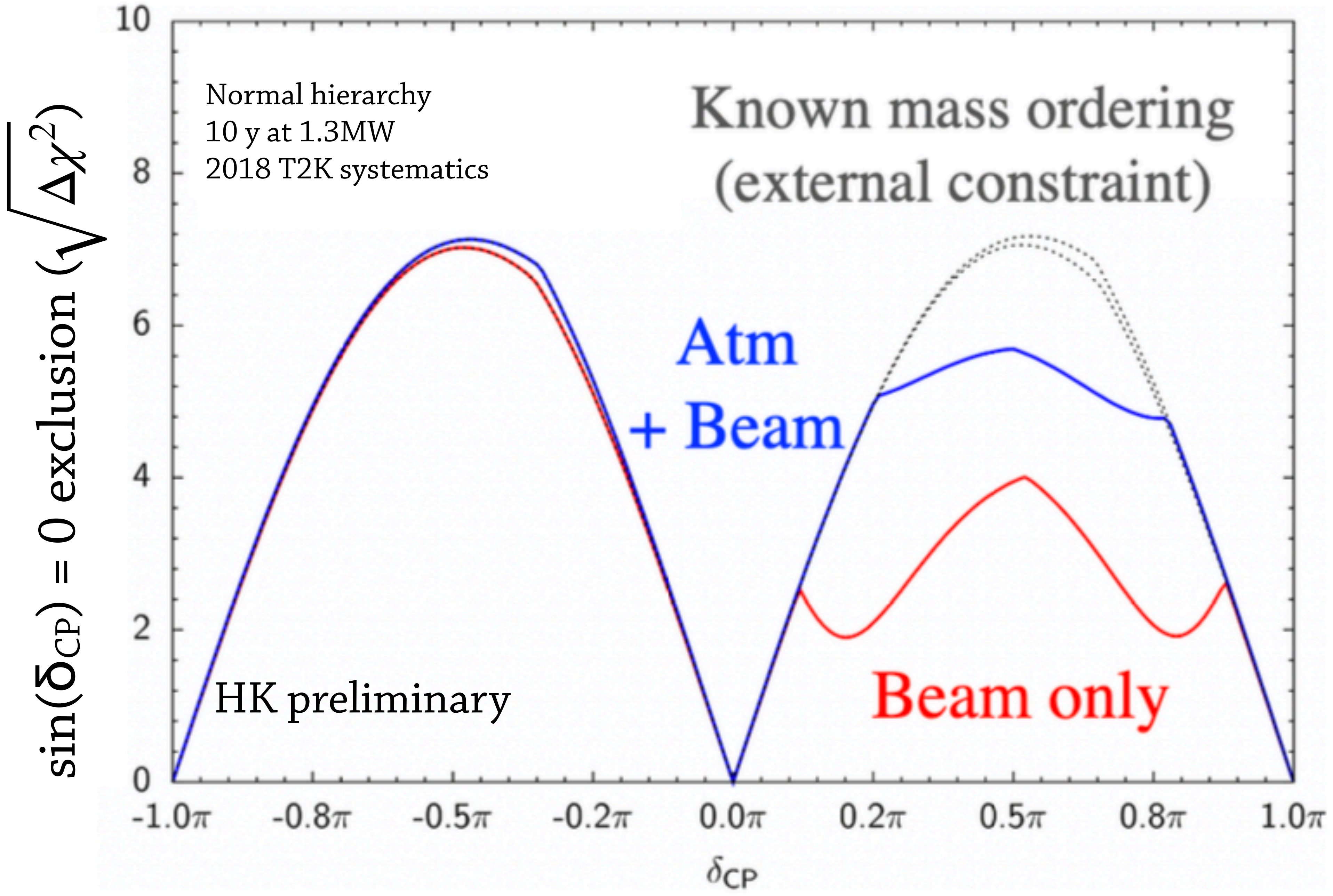
kt•MW•yr	Staged years
30	1.2
100	3.1
200	5.2
336	7
624	10
1104	15

In the ideal case of $\delta_{CP} = -\pi/2$

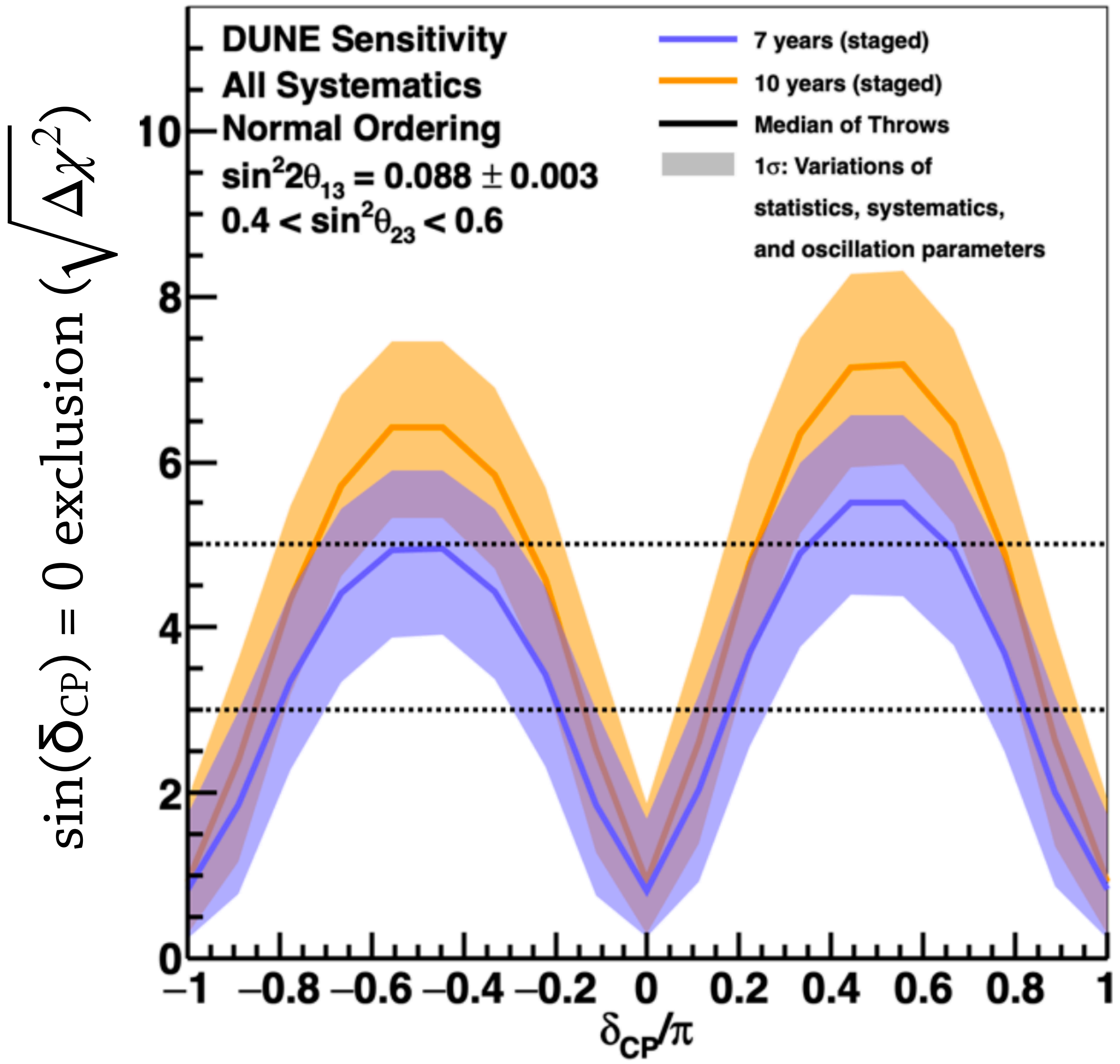
- ○ ***DUNE*** will resolve the MH at 5σ in ~1.5y
  [3y to exclude the wrong MH for any δ_{CP} value]
- ○ ***T2HK*** itself do not have a lot of sensitivity
  [can reach 5σ in 10y with beam + atmospheric ν]

Future ν accelerator experiments

T2HK in Japan



DUNE in the US



In 10 years of operation, if the MH is known:

○ ***DUNE*** can exclude $\delta_{CP} = (0, \pi)$ for 50% of δ_{CP} values

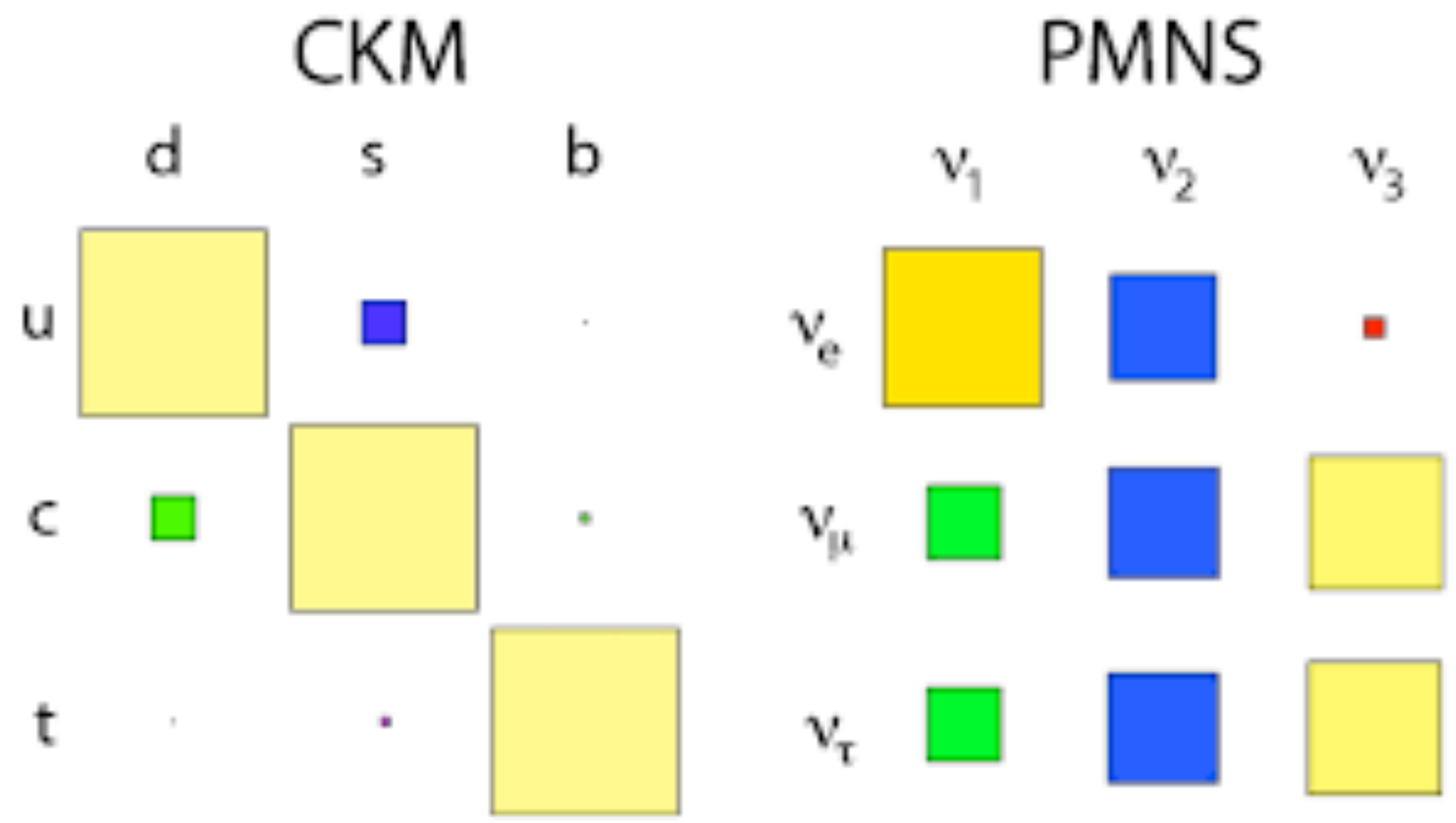
○ ***T2HK*** can reach 5σ for 60% of δ_{CP} values

CONCLUSIONS

Conclusions

Neutrinos **oscillates** :

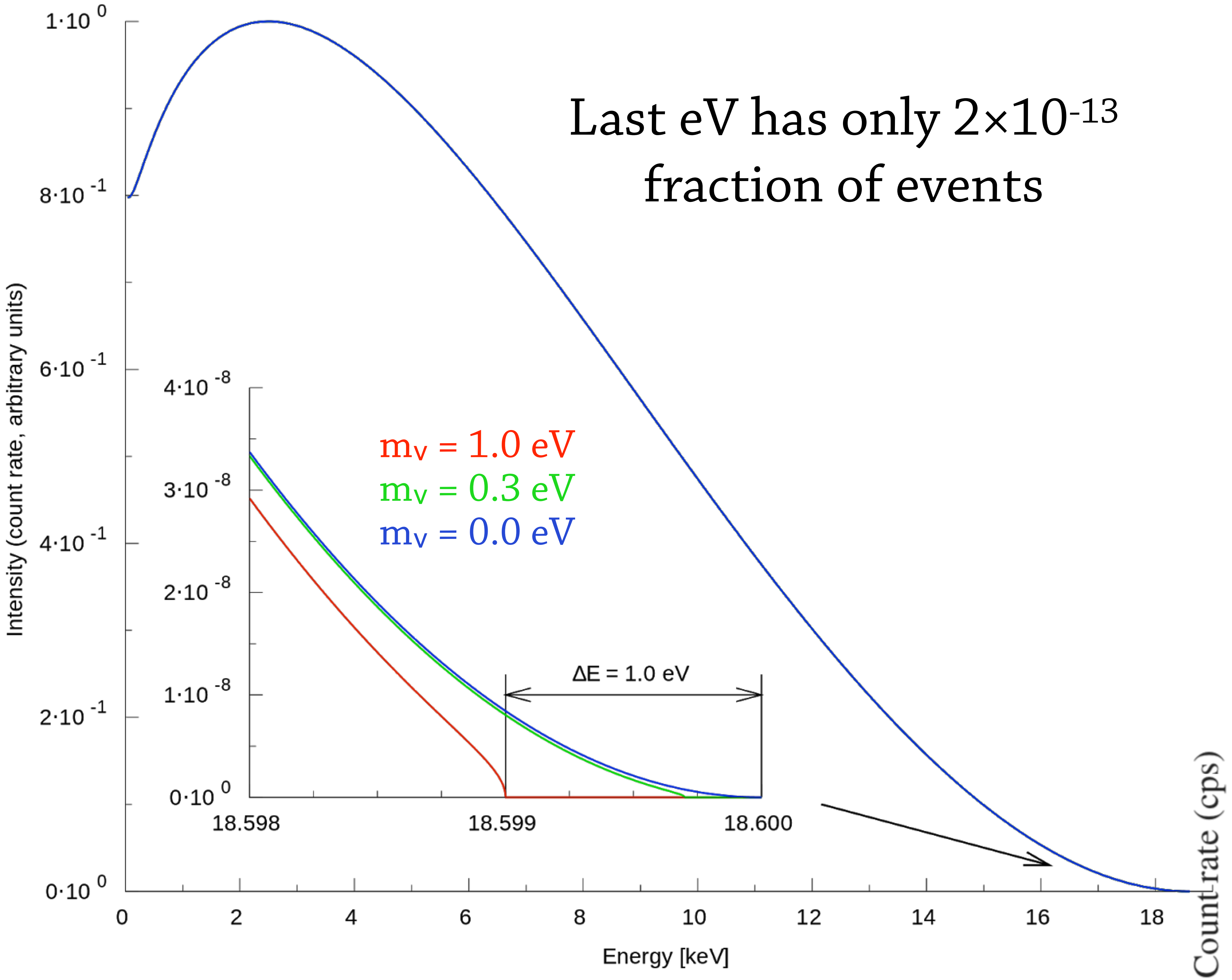
- ○ $\nu_e, \nu_\mu, \nu_\tau \neq \nu_1, \nu_2, \nu_3$
- ○ Two oscillation frequencies:
  fast (solar) and slow (atmospheric)
- ○ Neutrinos **mix** a lot more than quarks
- ○ In the next decade(s), all parameters measured:
  - matter/anti-matter asymmetry in the leptonic sector
  - neutrino mass ordering
- ○ Neutrinos are **massive** - and it raises many other questions !
  - ‣ What mass ?
  - ‣ Mass mechanism ?
  - ‣ Could there be other neutrinos ?



Neutrino absolute mass ? ***KATRIN experiment in Germany***

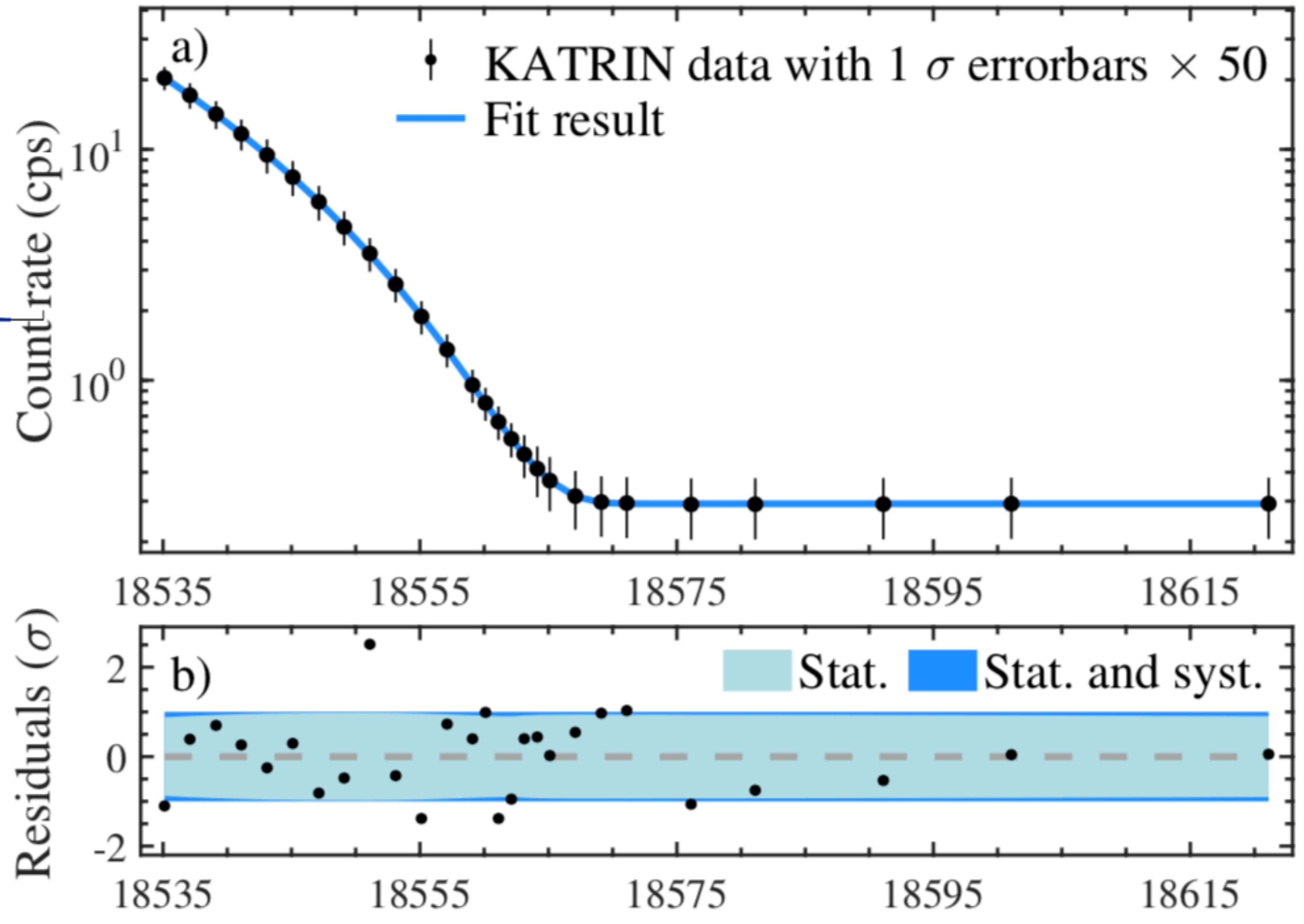
Look at the **end-point** of the β spectrum

↳ rare cases were the e^- takes most of the available energy



Last eV has only 2×10^{-13}
fraction of events


Installation of KATRIN experiment



Current limit :

$m_{\bar{\nu}_e} \lesssim 0.45$ eV at 90% CL

How neutrinos get massive ? ***ββ0ν experiments (SuperNEMO, CUORE, SNO+)***

- The **Dirac** way

Through Higgs coupling

Need a sterile right handed ν

$$\mathcal{L}^{D}_{mass} = -m_D(\bar{\nu}_R\nu_L + \bar{\nu}_L\nu_R)$$

$$m_D = \frac{v}{\sqrt{2}}Y_v \leftarrow \sim 10^{-12} \text{ (why?)}$$

- The **Majorana** way

No distinction between ν and ν̄

Mass given by seesaw mechanism

Need massive neutrinos

$$\nu_R = C\bar{\nu}_L^T = \nu_L^C$$

$$m = \frac{m_D^2 \leftarrow \text{Dirac term}}{m_R \quad \leftarrow \text{Very big}}$$

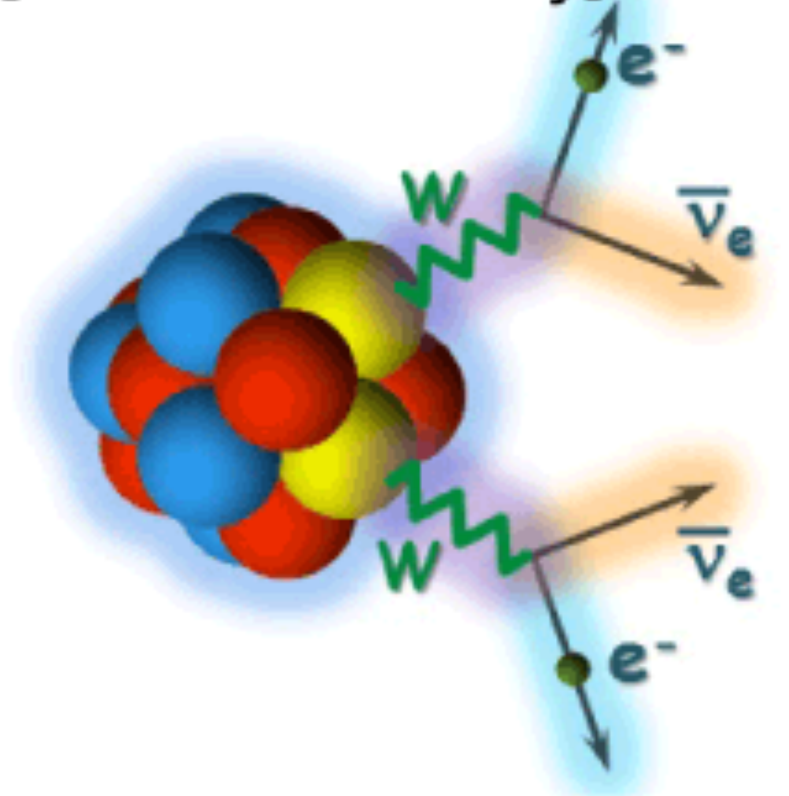
→ ***Only one way to prove that neutrino are Majorana particles :***

Double β decay with **no** neutrino emission

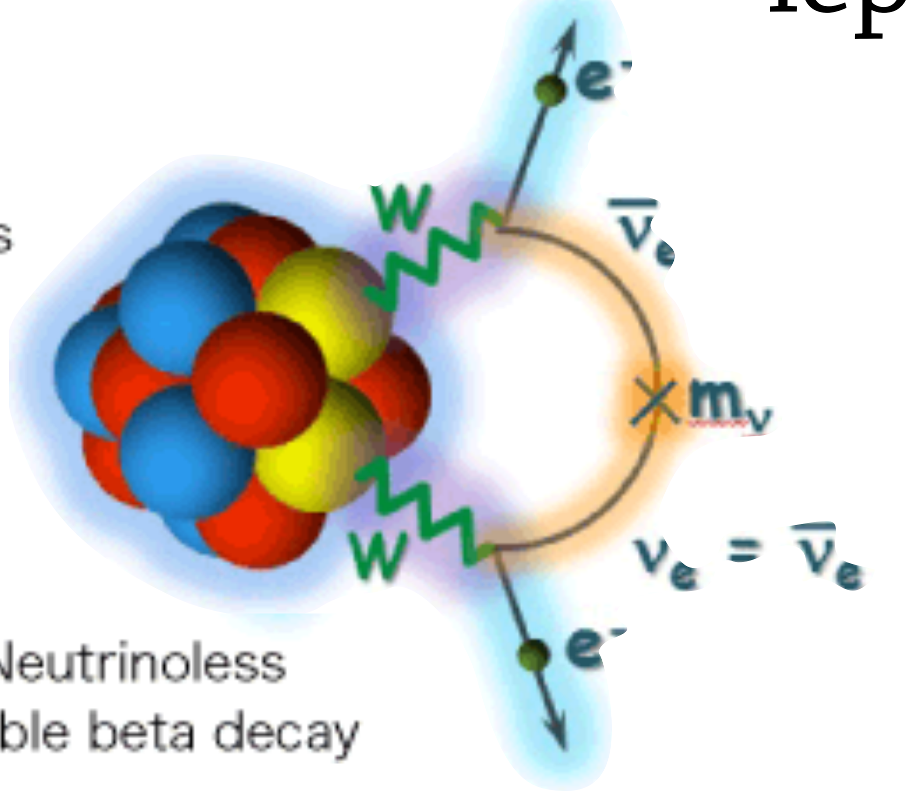
- ββ2ν is very rare (half life ~10^{18} - 10^{24} y)
- ββ0ν is **forbidden** in SM

↳lepton number violated by 2 units

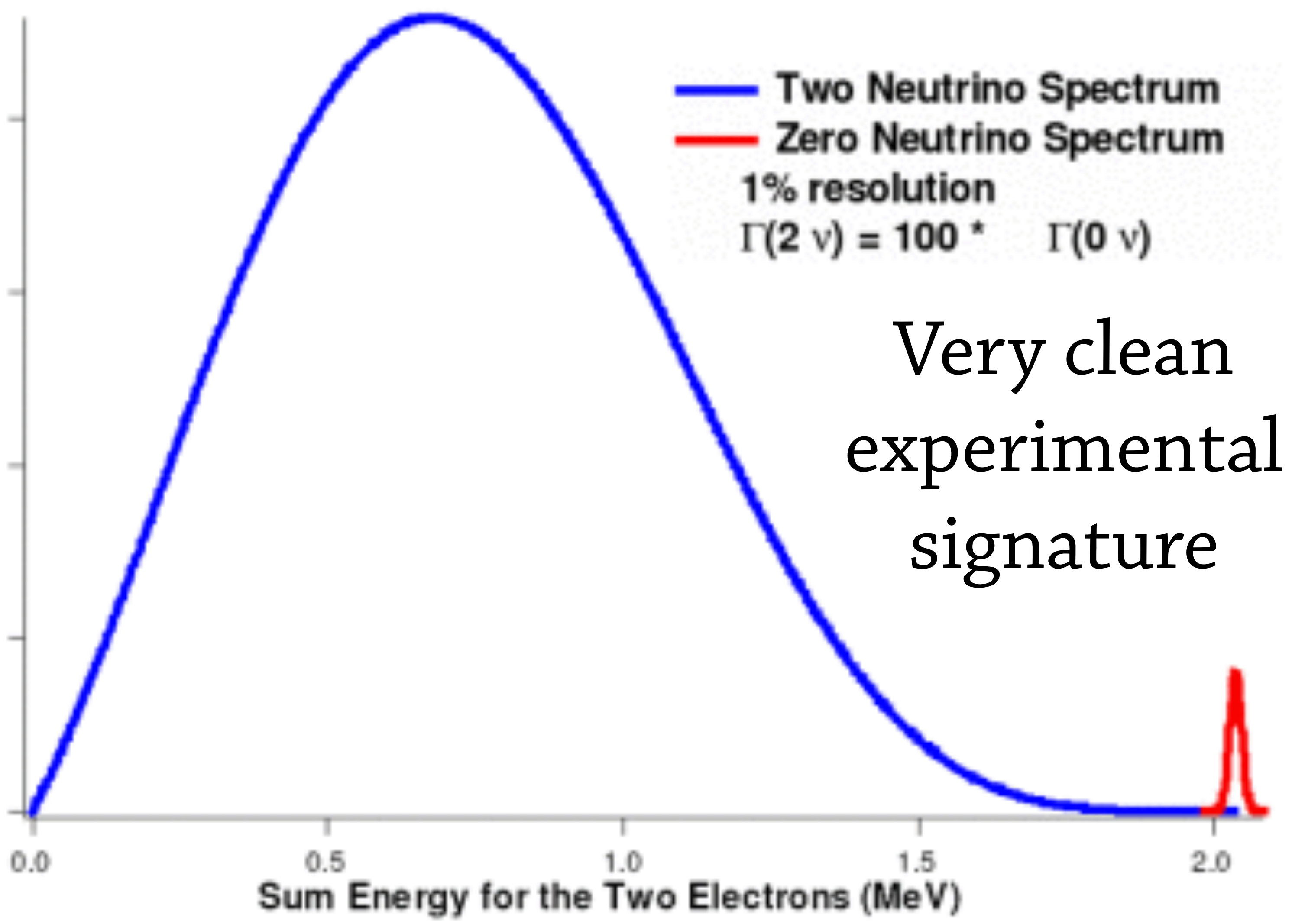
[Double beta decay]



Double beta decay
which emits anti-neutrinos

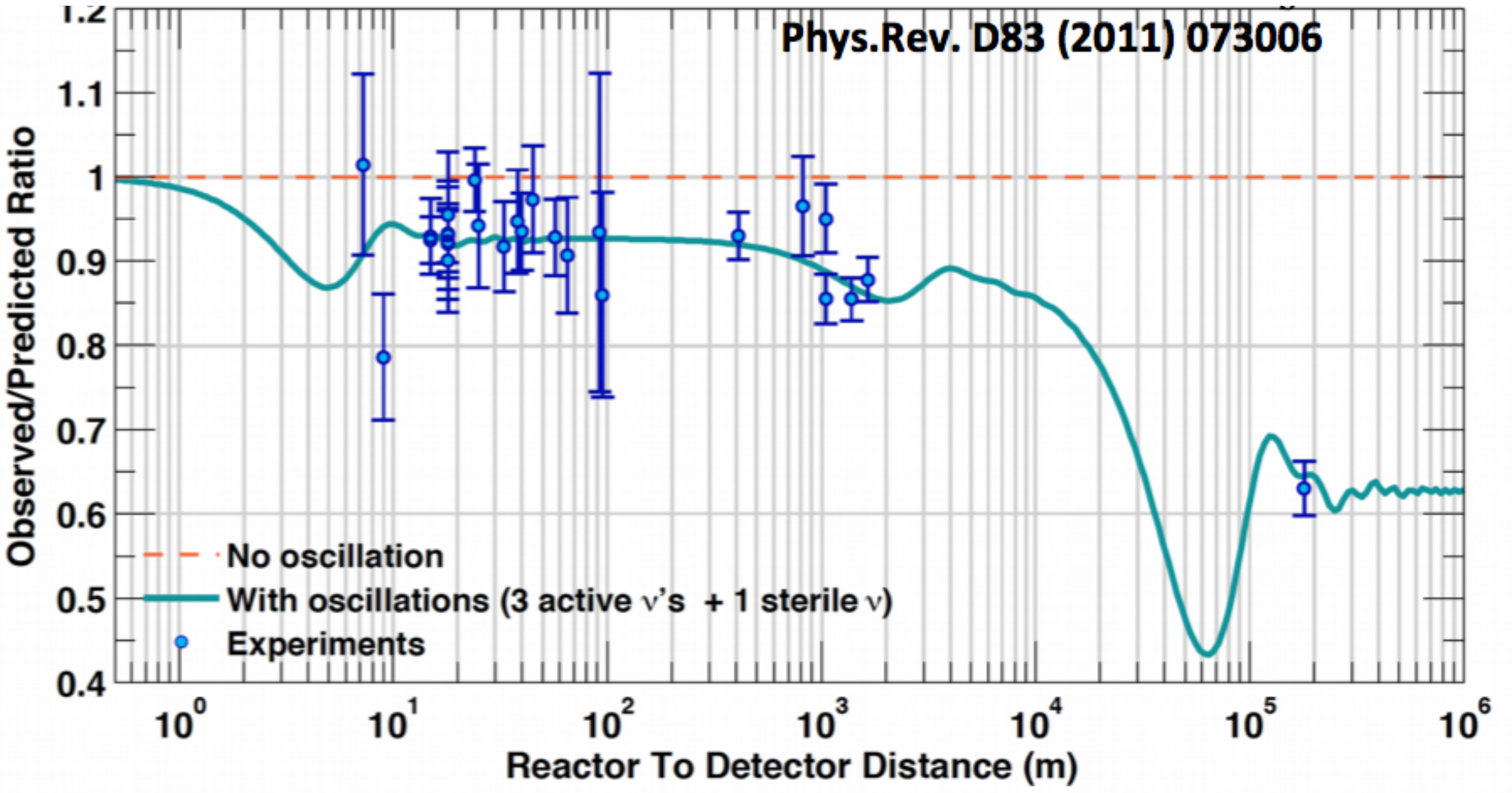


Neutrinoless
double beta decay



Very clean
experimental
signature

Only 3 Neutrinos ? ***STEREO, SOLID, PROSPECT,…***

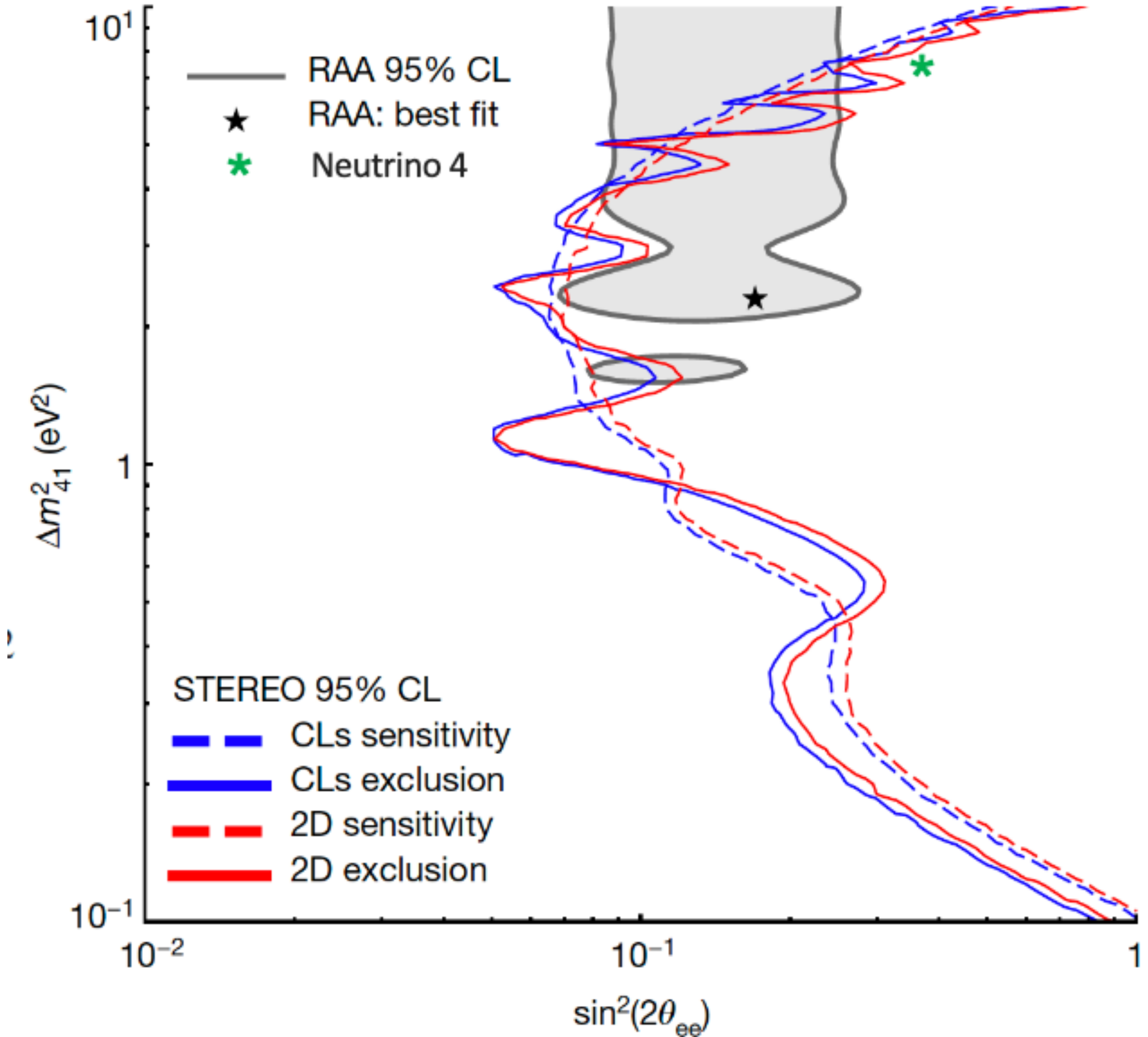


Phys.Rev. D83 (2011) 073006

A revised reactor $\bar{\nu}$e flux analysis showed that all past ν experiments had a **~6% deficit** at small distances (3σ)

-> Problem with reactor flux ?
-> Existence of a sterile neutrinos ?

Latest results from STEREO



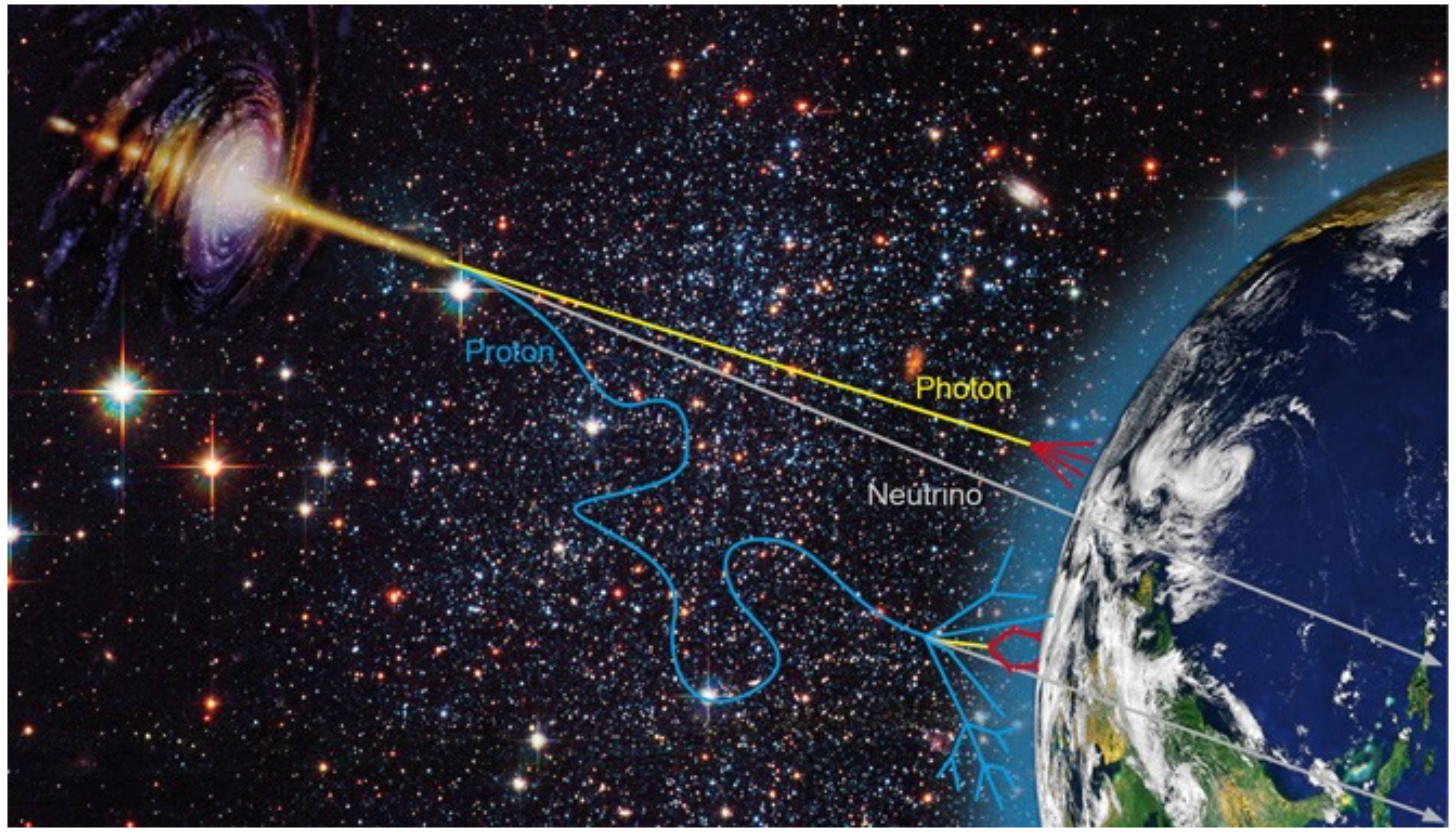
- Sterile because this neutrino cannot interact with weak force: it would be invisible
- But all 4 neutrinos could oscillate within each others

→ ***New mass splitting and new mixing angle***

Best fit parameters of reactor anomaly:

$\Delta m^2 \sim 2$ eV^2
$\sin^2(2\theta) \sim 0.15$
$L_{osc} \sim$ few m

Neutrino Astronomy: ***ICECUBE, KM3NET***

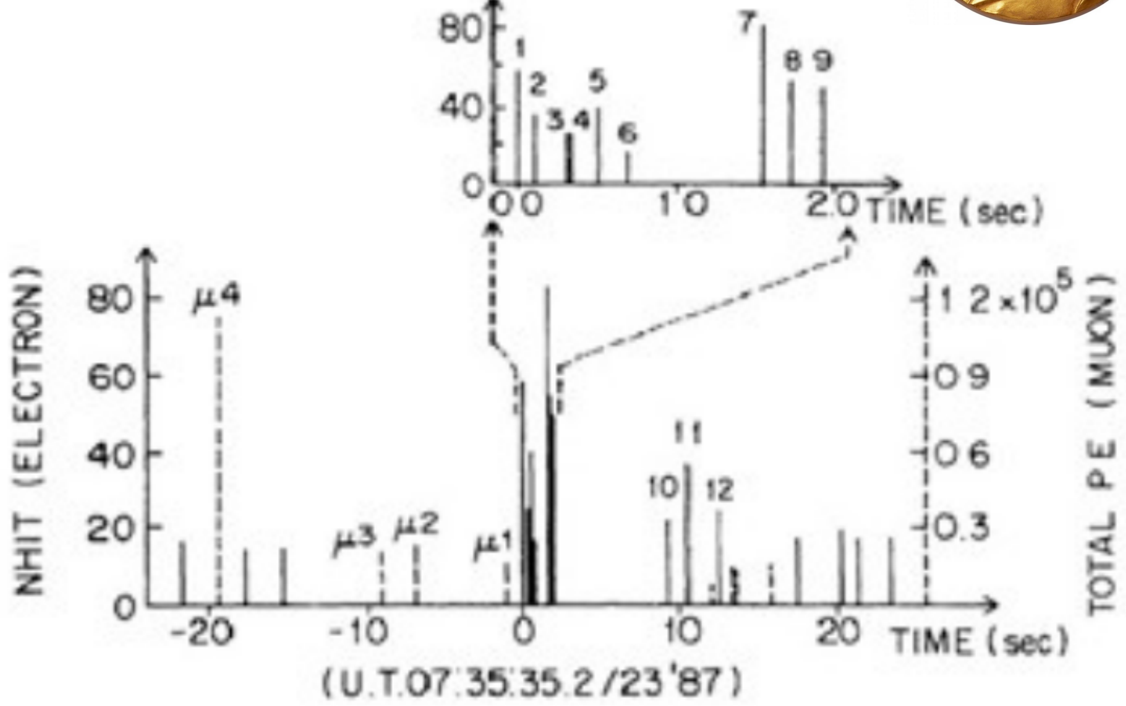


- Unlike protons & gammas, neutrinos **points to their sources**
- Can probe the inside of the structure
- **No GZK threshold** : can probe far away objects

On February 23rd 1987, a supernova exploded in the large magellan cloud (170 000 l.y.)

3h before the light signal, three neutrino detectors observed a large number of events in a very short time (**24 events in 13s**)

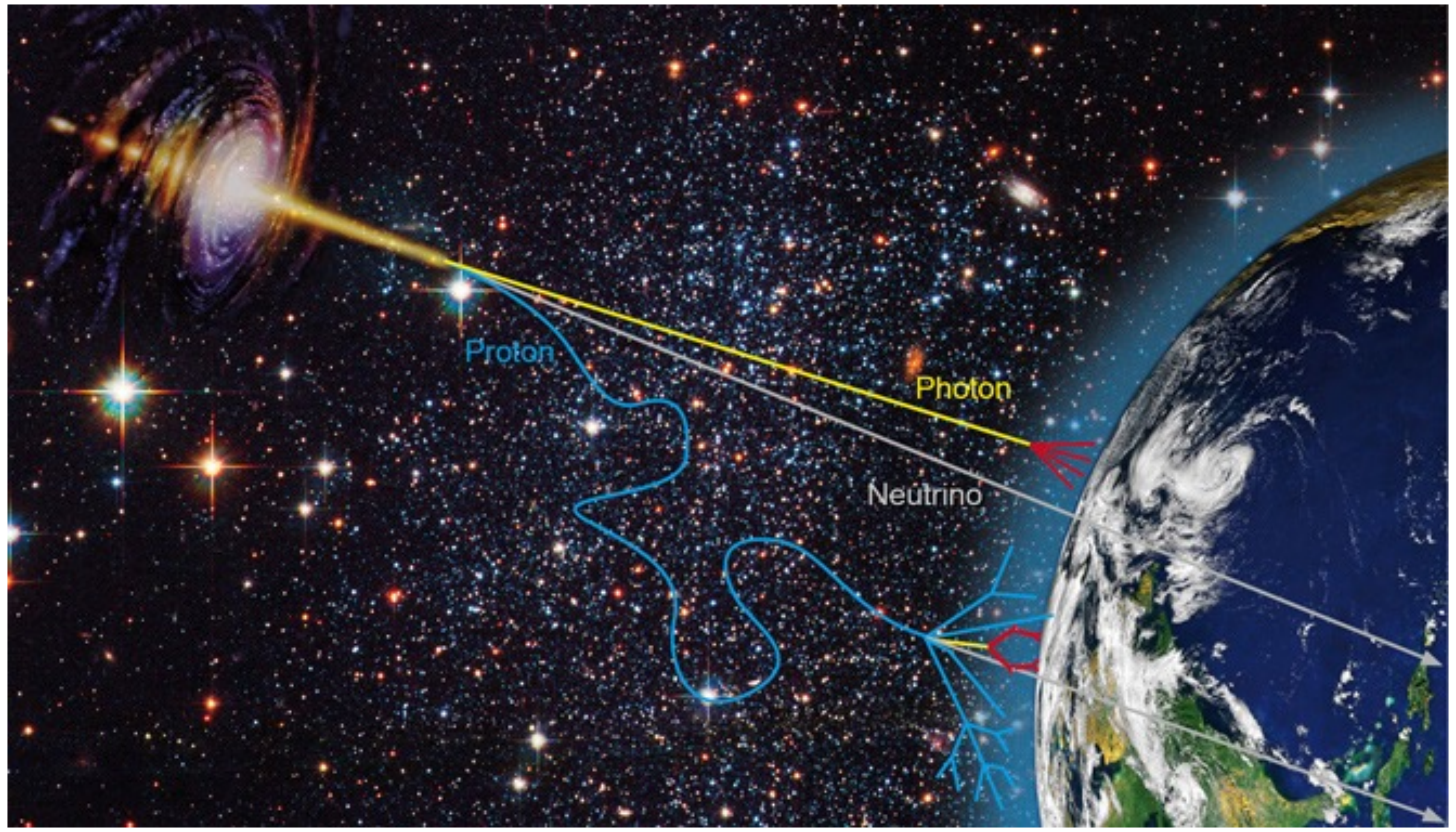
9 @ Kamiokande



- 99% of the SN energy is released as neutrinos
- 1st case of neutrino astronomy and multi-messenger
- all ν experiments waiting for next nearby SN explosion

SN1987A

Neutrino Astronomy: ***ICECUBE, KM3NET***



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On September 22nd 2017 : **Simultaneous** light & neutrino detection from the TXS 0506+056 blazar (3σ, $E_\nu = 290$ TeV)

(blazar = Active Galactic Nucleus with one jet pointing to earth)

- 1st case of planned **multi-messenger** astronomy
- Confirmed that blazar emits neutrinos