Supersymmetry Breaking Cascade Flow

from $\mathcal{N} = 2$ to adjoint QCD

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Costas $\cap$ Eric

1978 start graduate school in Princeton

– graduate college, shared an office, working on QCD

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1984 Costas and Pascale's wedding in Lille

1995 Eric's engagement party in Santa Barbara

– both working on supersymmetry breaking

1999 celebrate Costas and Pascale fifteenth wedding anniversary

– we collaborate on non-commutative QFT with Boris Pioline

2000's both working on holographic interfaces and defects

2013 we *finally* co-author one paper with John Estes and Darya Krym

"M-theory solutions invariant under $D(2,1;\gamma) \times D(2,1;\gamma)$*"*

2024 both *retire* !

Motivation

Seiberg-Witten solution for $\mathcal{N} = 2$ super Yang-Mills

- ⋆ provides exact low energy effective action and BPS spectrum

Softly breaking supersymmetry

- ⋆ **exploit the enhanced control provided by the SW solution**

Earlier investigations into softly breaking $\mathcal{N} = 2$

- ⋆ confinement via magnetic monopole condensation [Seiberg, Witten 1994]

  [Alvarez-Gaumé, Distler, Kounnas, Marino, 1996; Luty, Ratazzi 1999; Edelstein, Fuertes, Mas, Guilarte, 2000]
- ⋆ embedding $SU(2)$ adjoint QCD into $\mathcal{N} = 2$ [Cordova, Dumitrescu 2018]

This talk

Pure $\mathcal{N} = 2$ super-Yang-Mills with gauge group $SU(N)$

$\star$ gauge multiplet $(\phi, \lambda^1, \lambda^2, v_\mu)$ in adjoint representation

Add mass term $M^2\, tr(\phi^\dagger \phi)$ for gauge scalars ϕ

$\star$ softly breaks all supersymmetries

$\star$ preserves all other symmetries and 't Hooft anomalies

$\star$ ϕ decouples as $M \to \infty$ to adjoint QCD $(\lambda^1, \lambda^2, v_\mu)$ with two flavors

Phase structure along the flow $0 < M < \infty$?

Proposal: a magnetic dual Abelian Higgs model

$\Rightarrow$ **Cascade of phase transitions through partial Coulomb/Higgs phases**

with Thomas Dumitrescu, Efrat Gerchkovitz and Emily Nardoni

2012.11843; 2208.11502; 2408.xxxxx

$\mathcal{N}=2$ super Yang-Mills with gauge group $SU(N)$

Coulomb branch vacua $[\phi^\dagger, \phi] = 0$ **have unbroken** $\mathcal{N}=2$

$$SU(N) \longrightarrow U(1)^{N-1}$$

Low energy $\mathcal{N}=2$ **gauge** $U(1)_k$ **multiplets for** $k = 1, \cdots, N-1$

- $\star$ in terms of $\mathcal{N}=1$ chiral superfields A_k and gauge field strength W_k

$$\mathcal{L}_{\mathsf{SW}} = \frac{\mathrm{Im}}{4\pi} \sum_{k=1}^{N-1} \int d^4\theta \, A_{Dk} \bar{A}_k + \frac{\mathrm{Im}}{8\pi} \sum_{k,\ell=1}^{n-1} \int d^2\theta \, \tau_{k\ell} \, W_k W_\ell$$

- $\star$ specified by a locally holomorphic pre-potential $\mathcal{F}(A_1, \cdots, A_{N-1})$

$$A_{Dk} = \frac{\partial \mathcal{F}}{\partial A_k} \qquad\qquad \tau_{k\ell} = \frac{\partial^2 \mathcal{F}}{\partial A_k \, \partial A_\ell}$$

- $\star$ subject to $\mathrm{Im}\, \tau > 0$ for positive kinetic term of $U(1)_k$ gauge fields

The Seiberg-Witten solution

The Seiberg-Witten solution

- ⋆ determines the pre-potential $\mathcal{F}$ on the Coulomb branch
- ⋆ as a function of the vevs $\langle A_k \rangle = a_k$ and $\langle A_{Dk} \rangle = a_{Dk}$

SW curve $\mathcal{C}$ and differential λ for $SU(N)$

[Klemm, Lerche, Yankielowicz, Theisen; Argyres, Faraggi 1994]

- ⋆ in terms of a degree N polynomial $C(x)$

$$\mathcal{C} = \Big\{ (x,y),\ y^2 = C(x)^2 - \Lambda^{2N} \Big\} \qquad \lambda = \frac{xC' dx}{2\pi i \, y}$$

- ⋆ vevs a_k, a_{Dk} in terms of periods over canonical homology basis

$$a_k = \oint_{\mathfrak{A}_k} \lambda \qquad a_{Dk} = \oint_{\mathfrak{B}_k} \lambda$$

- ⋆ masses of BPS dyons with electric/magnetic charges $\mathbf{q}, \mathbf{m} \in \mathbb{Z}^{N-1}$

$$M_{\text{BPS}} = |Z| \qquad Z = \sum_{k=1}^{N-1} \big(q_k a_k + m_k a_{Dk} \big)$$

Multi-monopole points

- **Near a multi-monopole point** [Douglas, Shenker 1995]
  - ⋆ $N-1$ *mutually local* magnetic monopoles become massless
  - ⋆ magnetic periods $a_{Dk} \to 0$ so that $M_{\text{BPS}}(\mathbf{0}, \mathbf{m}) \to 0$
  - ⋆ electric periods $a_k \not\to 0$ so that $M_{\text{BPS}}(\mathbf{q}, \mathbf{0})$ remains finite

- **Massless states produce singularities in τ as $a_D \to 0$**

$$-2\pi i\, \tau_{k\ell} \approx \delta_{k\ell} \ln \frac{\Lambda}{a_{Dk}}$$

  - ⋆ the SW low energy effective Lagrangian breaks down because it integrated out light magnetic monopoles

- **Viable low energy effective theory**
  - ⋆ requires keeping the fields of massless states

Effective Abelian Higgs model

"Integrate in" magnetic monopole fields below RG scale μ

- ⋆ hyper-multiplets $\mathcal{H}_k$ for gauge group $U(1)_k$ for $k = 1, \cdots, N-1$
- ⋆ in terms of $\mathcal{N} = 1$ chiral superfields $\mathcal{H}_k^\pm$

Effective Lagrangian including magnetic monopole fields

- ⋆ dictated by $\mathcal{N} = 2$ supersymmetry

$$\mathcal{L}^{\rm eff}_{\sf SW} \quad \supset \quad \bar{\mathcal{H}}_k^\pm e^{\mp 2V_k} \mathcal{H}_k^\pm, \qquad \bar{A}_k A_{Dk}, \qquad A_{Dk} \mathcal{H}_k^+ \mathcal{H}_k^-, \qquad \tau^{\rm eff}_{Dk\ell} W_k W_\ell$$

- ⋆ Gauge couplings $\tau_{k\ell}$ are now free of singularities as $a_{Dk} \to 0$

$$-2\pi i\, \tau^{\rm eff}_{k\ell} = \delta_{k\ell} \ln \frac{\Lambda}{\mu} + \ln L_{k\ell} + \mathcal{O}(a_D)$$

- ⋆ where $L_{k\ell}$ is given by [ED, Dumitrescu, Gerchkovitz, Nardoni 2020]

$$L_{kk} = 16 N s_k^3 \qquad\qquad L_{k\neq\ell} = \frac{1 - c_{k+\ell}}{1 - c_{k-\ell}} \qquad\qquad \begin{cases} c_k = \cos(k\pi/N) \\ s_k = \sin(k\pi/N) \end{cases}$$

Soft supersymmetry breaking operator

The operator $\mathcal{T} = M^2\, tr(\phi^\dagger \phi)$ in the $\mathcal{N}=2$ theory

- ⋆ dimension protected since component of $\mathcal{N}=2$ stress tensor multiplet
- ⋆ IR behavior of $\mathcal{T}$ governed by SW theory
- ⋆ flows towards the Kähler potential

$$\mathcal{T} \to \frac{i}{4\pi} \sum_{k=1}^{N-1} \Big(\bar a_{Dk} a_k - \bar a_k a_{Dk} \Big)$$

- ⋆ with magnetic monopole hyper-multiplet fields $\mathcal{H}_k^\pm$ integrated in

$$\mathcal{T} \to \frac{i}{4\pi} \sum_{k=1}^{N-1} \Big(\bar a_{Dk}^{\rm eff}\, a_k^{\rm eff} - \bar a_k^{\rm eff}\, a_{Dk}^{\rm eff} + 2\pi i \bar h_k h_k \Big)$$

Flow from $\mathcal{N}=2$ to adjoint QCD by the operator $\mathcal{T}$

- ⋆ adding $\mathcal{N}=2$ to the Lagrangian breaks susy completely
- ⋆ in practice: ignore back-reaction of $\mathcal{T}$ on RG flow

The Abelian Higgs model with susy breaking

The operator $M^2\mathcal{T}$ drives vacuum towards minimum of $\mathcal{T}$

- ⋆ at the $\mathbb{Z}_{2N}$ symmetric point [ED, Dumitrescu, Nardoni 2022]
- ⋆ use self-consistent approximation for a_k linear in $a_{D\ell}$
- ⋆ treat higher order terms perturbatively [ED, Phong 1997]

Assembling effective potential for the Abelian Higgs model $+\mathcal{T}$

$$M^4 V_{\text{red}} = \sum_{k=1}^{N-1} \left(\frac{M^2 N \Lambda}{\pi^2} s_k \text{Im}\,(a_{Dk}) + \left\{ 2|a_{Dk}|^2 - \frac{1}{2} M^2 \right\} \bar{h}_k h_k \right)$$
$$+ \sum_{k,\ell=1}^{N-1} \left(M^2 t_{k\ell} a_{Dk} \bar{a}_{D\ell} + (t^{-1})_{k\ell} \left\{ (\bar{h}_k h_\ell)(\bar{h}_\ell h_k) - \frac{1}{2} (\bar{h}_k h_k)(\bar{h}_\ell h_\ell) \right\} \right)$$

Proposal: Abelian Higgs model is dual to flow from $\mathcal{N}=2$ to adjoint QCD

- ⋆ for small M back-reaction of $\mathcal{T}$ on flow can be ignored
- ⋆ for larger M we present evidence for a coherent picture

Vacuum alignment

Minima of $\mathcal{V}$ occur at $\mathrm{Re}\,(a_{Dk}) = 0$

The Higgs fields h_k align perfectly as $SU(2)_R$ doublets

- ⋆ minimize $\mathcal{V}$ for given values of $h_k^\dagger h_k$;

$$\mathcal{V}\Big|_{\bar h_k h_k} = \sum_{k,\ell} (t^{-1})_{k\ell}\, \mathbf{v}_k \cdot \mathbf{v}_\ell \qquad \mathbf{v}_k = h_k^\dagger \boldsymbol{\sigma} h_k \qquad h_k = \begin{pmatrix} h_k^1 \\ h_k^2 \end{pmatrix}$$

- ⋆ ground state is ferromagnetic under the mild assumption

$$(t^{-1})_{k\ell} < 0 \qquad\qquad k \neq \ell$$

- ⋆ holds automatically for sufficiently small μ/Λ

$SU(2)_R$ is spontaneously broken as soon as any $h_k \neq 0$

- ⋆ producing two Goldstone bosons $SU(2)_R \to U(1)_R$ i.e. CP^1-phase
- ⋆ matches expected chiral symmetry breaking in adjoint QCD

Organization of semi-classical analysis

Reduced field equations for Higgs VEVs

$$h_k\Big(M^2 - 4|a_{Dk}|^2 - 2\sum_{\ell=1}^{N-1}(t^{-1})_{k\ell}|h_\ell|^2\Big) = 0$$

Solutions to Higgs eqs organized by partitions of $k \in \{1, \cdots, N-1\}$

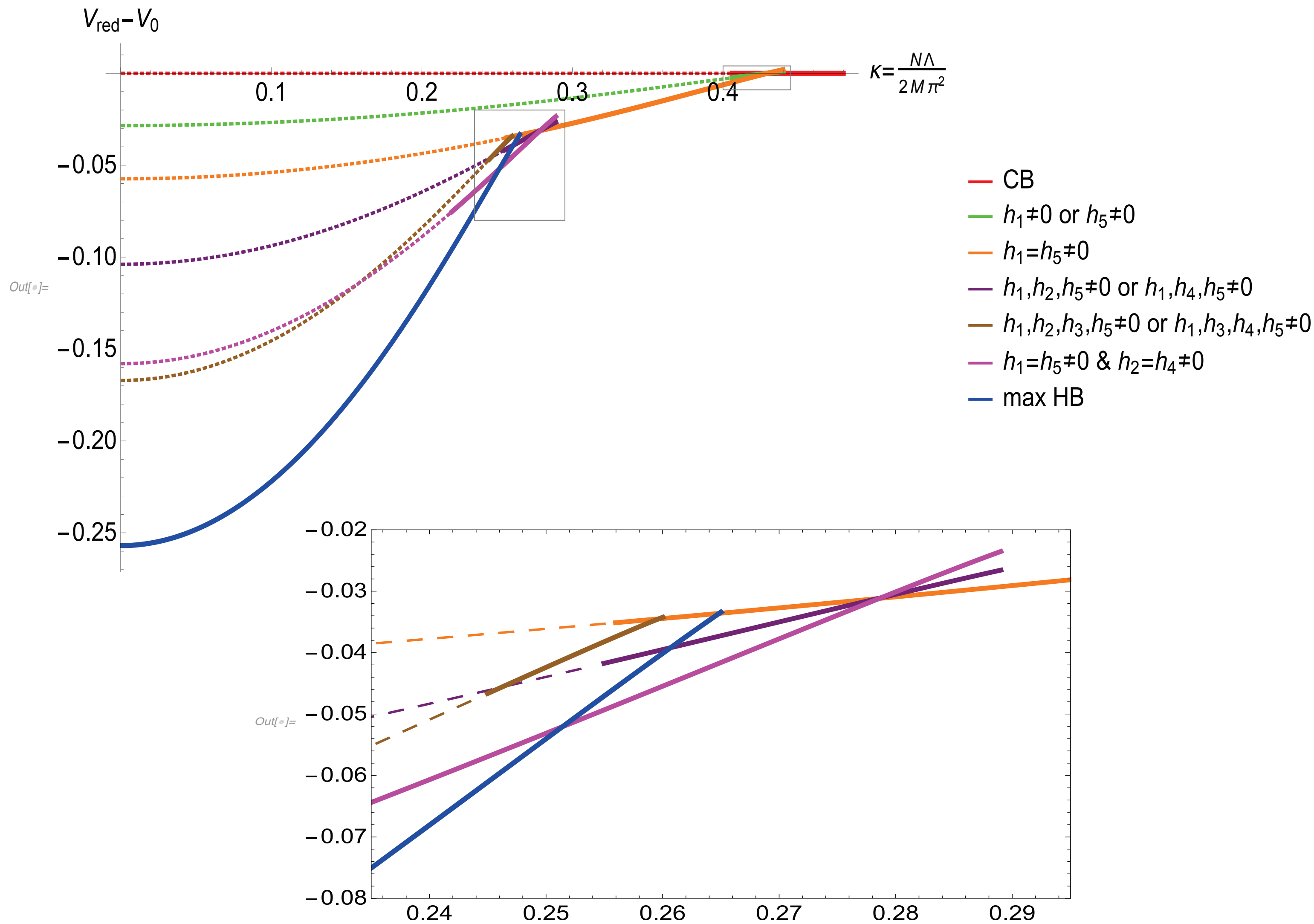
$$h_k = 0 \qquad \text{versus} \qquad M^2 - 4|a_{Dk}|^2 - 2\sum_{\ell=1}^{N-1}(t^{-1})_{k\ell}|h_\ell|^2 = 0$$

- ⋆ Solve for h_k within a given partition in terms of a_D
- ⋆ Results in coupled cubics in a_{Dk} for $h_k \neq 0$

Steps in semi-classical analysis

- ⋆ Existence of solutions for given $N, M/\Lambda$
- ⋆ Local stability of solutions: positive Hessian
- ⋆ Global stability of solutions: global minimum of $\mathcal{V}$ for given $N, M/\Lambda$

The example of SU(6) gauge group



Summary

Magnetic Abelian Higgs model dual for flow from $\mathcal{N}=2$ to adjoint QCD

- $\star$ Magnetic monopole fields have been “integrated in”
- $\star$ Soft susy breaking operator flows to Kähler potential of SW theory

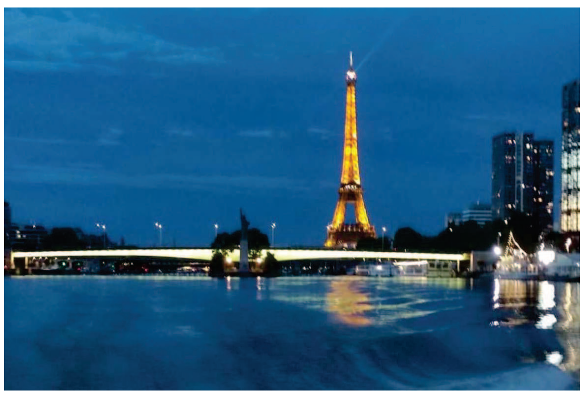
Semi-classical Abelian Higgs model matches $SU(N)$ theory

- $\star$ Small M : Coulomb branch
- $\star$ Large M : monopole condensation
  - – matches confinement in adjoint QCD

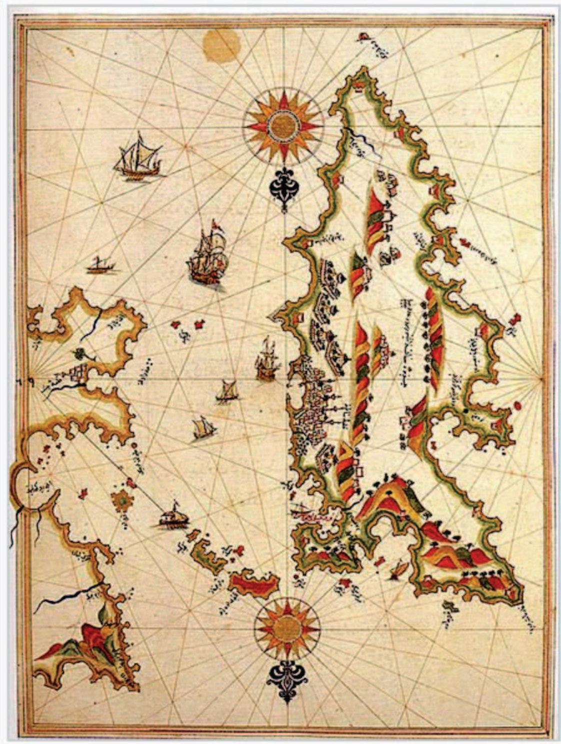
Semi-classical analysis of Abelian Higgs model predicts

- $\star$ Cascade of phases transitions
  - – through mixed Coulomb - Higgs phases

Happy retirement Costas !



... boating on the Seine ...



16th-century detailed map of Chios by Piri Reis

... on Chios with Pascale ...

... diving for octopus ...